Computer algebra independent integration tests

4-Trig-functions/4.3-Tangent/4.3.7-d-trig-^m-a+b-c-tan-^n-^p

Nasser M. Abbasi

July 17, 2021 Compiled on July 17, 2021 at 2:38pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [499]. This is test number [106].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (499)	% 0.00 (0)
Mathematica	% 99.60 (497)	% 0.40 (2)
Maple	% 81.36 (406)	% 18.64 (93)
Maxima	% 53.91 (269)	% 46.09 (230)
Fricas	% 81.36 (406)	% 18.64 (93)
Sympy	% 18.24 (91)	% 81.76 (408)
Giac	% 41.88 (209)	% 58.12 (290)
Mupad	% 56.71 (283)	% 43.29 (216)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

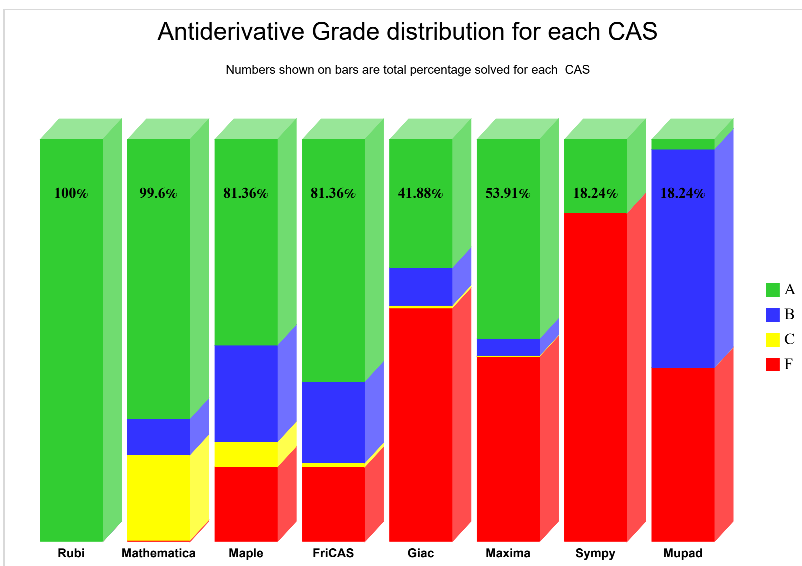
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

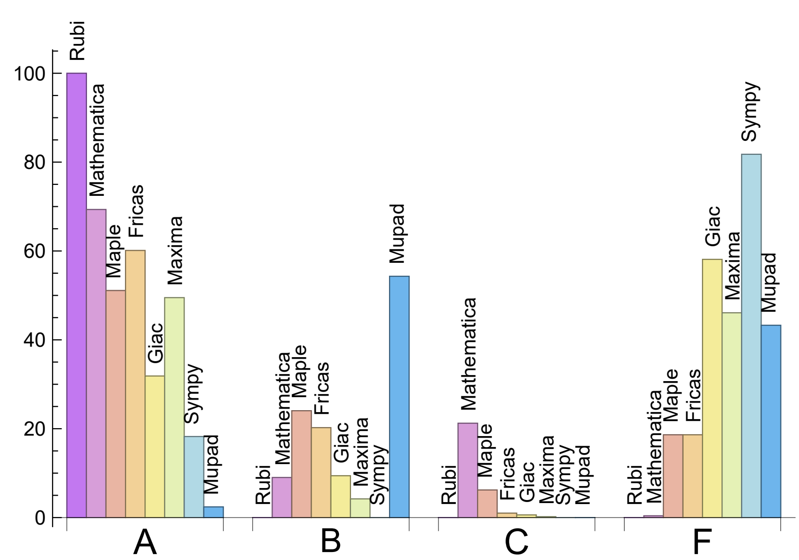
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	69.34	9.02	21.24	0.40
Maple	51.10	24.05	6.21	18.64
Maxima	49.50	4.21	0.20	46.09
Fricas	60.12	20.24	1.00	18.64
Sympy	18.24	0.00	0.00	81.76
Giac	31.86	9.42	0.60	58.12
Mupad	2.40	54.31	0.00	43.29

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	93	90.32 %	9.68 %	0.00 %
Maxima	230	66.52 %	12.61 %	20.87 %
Fricas	93	81.72 %	11.83 %	6.45 %
Sympy	408	72.55 %	27.45 %	0.00 %
Giac	290	52.41 %	8.62 %	38.97 %
Mupad	216	95.83 %	4.17 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

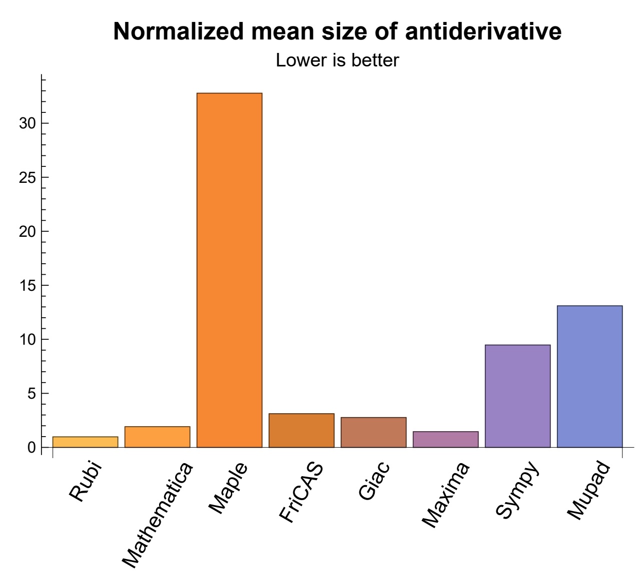
1.3 Performance

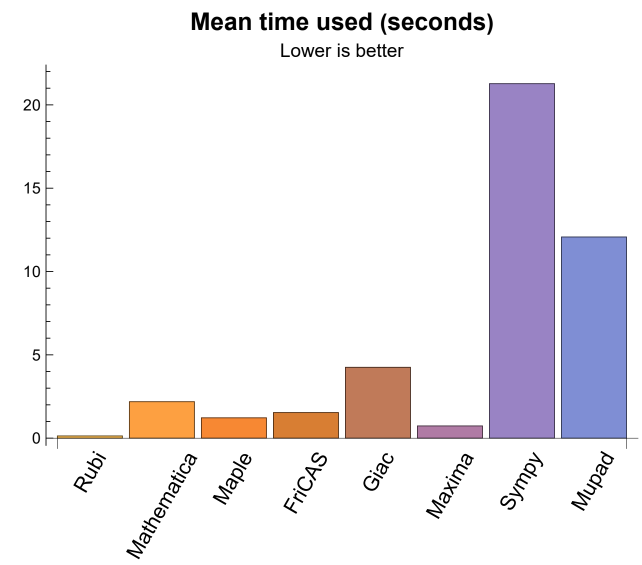
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	109.13	0.98	90.00	1.00
Mathematica	2.19	203.44	1.92	93.00	1.00
Maple	1.22	5998.49	32.78	152.00	1.56
Maxima	0.73	127.06	1.45	83.00	1.09
Fricas	1.53	410.97	3.11	222.50	2.67
Sympy	21.27	1077.24	9.47	129.00	1.92
Giac	4.25	222.52	2.76	113.00	1.30
Mupad	12.07	1597.68	13.10	108.00	1.31

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{174, 178, 420, 424, 486, 487, 488, 489, 490, 494, 495, 499}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {84, 85, 97, 136, 145, 148, 152, 153, 154, 155, 157, 158, 159, 160, 164, 165, 170, 171, 172, 173, 176, 304, 305, 318, 330, 331, 339, 342, 343, 345, 354, 355, 356, 357, 358, 360, 368, 369, 370, 371, 372, 422, 437, 438, 475, 481, 484, 485, 496, 497, 498}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

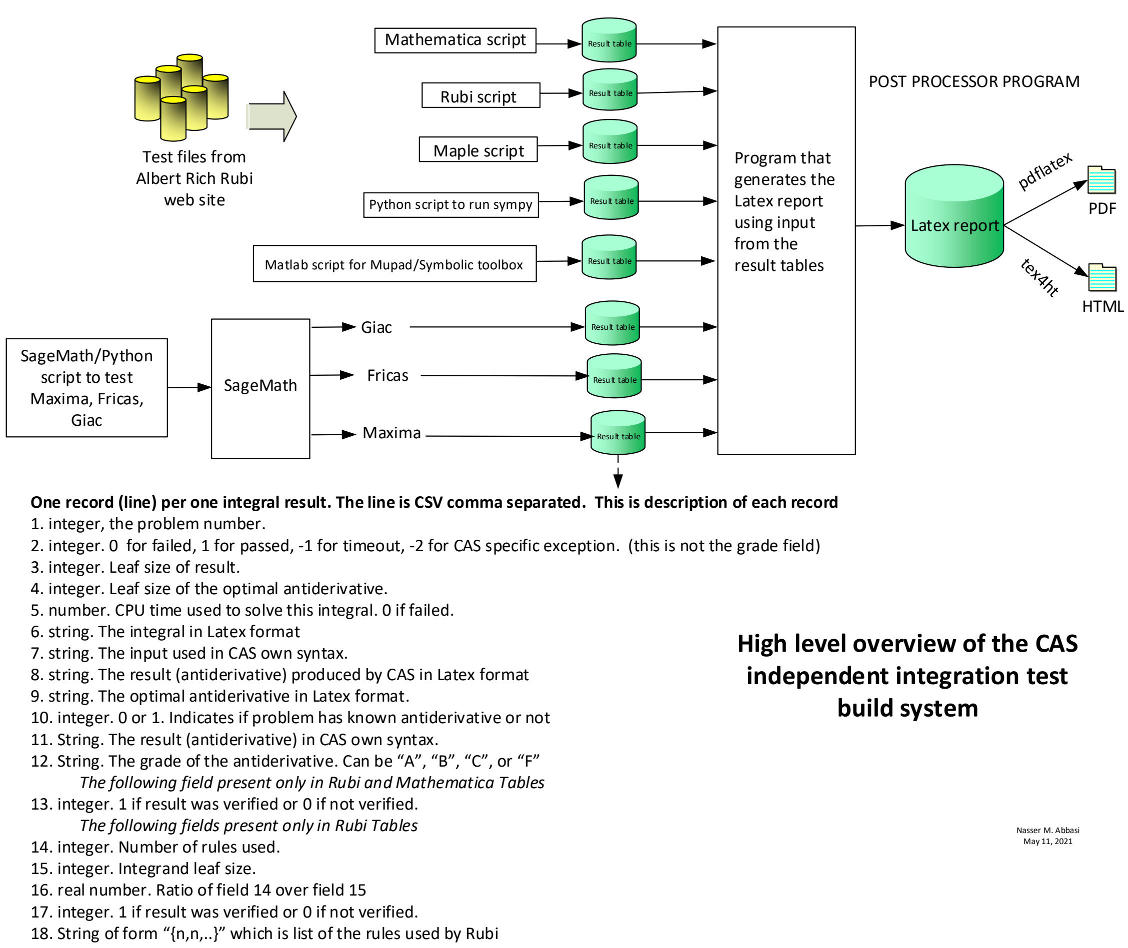
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 104, 105, 106, 109, 116, 117, 118, 121, 124, 125, 126, 127, 128, 129, 130, 133, 137, 138, 139, 140, 141, 142, 149, 150, 151, 154, 155, 156, 161, 162, 163, 166, 167, 168, 169, 172, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 206, 207, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 289, 292, 293, 294, 295, 296, 297, 298, 306, 307, 308, 309, 310, 311, 320, 321, 322, 323, 324, 325, 329, 334, 341, 353, 359, 360, 361, 362, 363, 364, 365, 366, 377, 381, 382, 383, 384, 385, 389, 390, 391, 394, 395, 396, 397, 398, 400, 401, 403, 404, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 477, 478, 479, 480, 482, 483, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499 }

B grade: { 33, 34, 35, 47, 48, 57, 58, 59, 60, 72, 84, 95, 96, 97, 107, 108, 119, 120, 131, 132, 143, 144, 145, 153, 157, 158, 160, 176, 204, 205, 249, 287, 288, 290, 291, 368, 369, 370, 371, 372, 393, 422, 450, 475, 497 }

C grade: { 8, 10, 11, 12, 16, 17, 18, 98, 99, 100, 101, 102, 103, 110, 111, 112, 113, 114, 115, 122, 123, 134, 135, 136, 146, 147, 148, 152, 159, 164, 165, 170, 171, 173, 195, 196, 197, 208, 210, 270, 299, 300, 301, 302, 303, 304, 305, 312, 313, 314, 315, 316, 317, 318, 319, 326, 327, 328, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 354, 355, 356, 357, 358, 374, 375, 376, 378, 379, 380, 386, 387, 388, 392, 399, 402, 405, 407, 437, 438, 449, 481, 484, 485, 496, 498 }

F grade: { 367, 373 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 63, 64, 65, 66, 70, 71, 72, 75, 76, 77, 78, 79, 82, 89, 90, 91, 116, 117, 124, 125, 126, 127, 130, 136, 137, 138, 139, 140, 141, 142, 148, 149, 150, 151, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 248, 249, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 320, 321, 322, 326, 327, 328, 329, 333, 334, 335, 339, 340, 341, 342, 346, 347, 348, 355, 374, 375, 376, 377, 378, 380, 382, 383, 384, 385, 386, 390, 396, 397, 420, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 438, 440, 441, 442, 443, 444, 445, 446, 448, 449, 452, 453, 454, 455, 457, 458, 459, 460, 464, 465, 466, 467, 469, 470, 471, 472, 486, 487, 488, 489, 490, 494, 495, 499 }

B grade: { 44, 45, 46, 50, 59, 60, 61, 62, 67, 68, 69, 73, 74, 80, 81, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 100, 104, 105, 106, 107, 108, 109, 112, 118, 119, 120, 121, 128, 129, 131, 132, 133, 143, 144, 145, 146, 147, 198, 204, 205, 206, 216, 229, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 256, 279, 296, 297, 298, 299, 300, 301, 302, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 319, 323, 324, 325, 336, 337, 338, 349, 350, 351, 352, 353, 354, 379, 381, 389, 393, 394, 400, 401, 403, 404, 437, 439, 447, 450, 451, 456, 461, 462, 463, 468, 473 }

C grade: { 98, 99, 101, 102, 103, 110, 111, 113, 114, 115, 122, 123, 134, 135, 169, 303, 304, 305, 316, 317, 318, 330, 331, 332, 343, 344, 345, 387, 388, 392, 399 }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173, 175, 176, 177, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 391, 395, 398, 402, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 491, 492, 493, 496, 497, 498 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 74, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 93, 94, 101, 102, 103, 104, 105, 106, 113, 114, 115, 116, 117, 118, 125, 126, 127, 129, 130, 137, 138, 139, 141, 142, 149, 150, 151, 167, 168, 169, 174, 178, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 262, 264, 273, 274, 276, 278, 279, 281, 283, 284, 285, 286, 287, 288, 289, 291, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 420, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 456, 457, 458, 459, 460, 461, 468, 469, 470, 471, 472, 473, 477, 478, 479, 486, 487, 488, 489, 490, 494, 495, 499 }

B grade: { 128, 140, 179, 180, 238, 239, 242, 257, 258, 259, 261, 263, 265, 266, 267, 269, 270, 271, 272, 277, 282 }

C grade: { 290 }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 80, 81, 82, 83, 84, 85, 95, 96, 97, 98, 99, 100, 107, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 124, 131, 132, 133, 134, 135, 136, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 170, 171, 172, 173, 175, 176, 177, 260, 268, 275, 280, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 450, 451, 452, 453, 454, 455, 462, 463, 464, 465, 466, 467, 474, 475, 476, 480, 481, 482, 483, 484, 485, 491, 492, 493, 496, 497, 498 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 17, 18, 29, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 68, 69, 70, 74, 75, 76, 92, 93, 94, 95, 96, 97, 100, 104, 105, 106, 107, 108, 109, 112, 114, 115, 116, 117, 118, 119, 120, 121, 124, 125, 126, 127, 128, 129, 130, 132, 136, 137, 138, 139, 140, 141, 142, 149, 167, 168, 169, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 230, 231, 233, 234, 235, 236, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 275, 277, 278, 282, 283, 284, 285, 286, 289, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 338, 339, 341, 342, 343, 344, 345, 353, 357, 358, 374, 375, 376, 377, 380, 381, 382, 383, 384, 389, 390, 391, 393, 394, 395, 396, 397, 398, 420, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 460, 461, 462, 463, 464, 465, 472, 473, 477, 478, 479, 486, 487, 488, 489, 490, 494, 495, 499 }

B grade: { 33, 34, 35, 47, 48, 60, 65, 66, 67, 71, 72, 73, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 101, 102, 103, 110, 111, 113, 122, 123, 131, 133, 134, 135, 143, 144, 145, 148, 224, 228, 229, 232, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 256, 274, 276, 279, 280, 281, 287, 288, 333, 334, 335, 336, 337, 340, 346, 347, 348, 349, 350, 351, 352, 354, 355, 356, 385, 386, 400, 401, 402, 403, 404, 405, 458, 459, 466, 467, 468, 469, 470, 471 }

C grade: { 290, 291, 292, 378, 379 }

F grade: { 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 170, 171, 172, 173, 175, 176, 177, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 387, 388, 392, 399, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 474, 475, 476, 480, 481, 482, 483, 484, 485, 491, 492, 493, 496, 497, 498 }

2.1.6 Sympy

A grade: { 39, 51, 64, 76, 88, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 226, 230, 231, 232, 233, 237, 238, 239, 243, 244, 245, 246, 250, 251, 252, 253, 254, 255, 256, 260, 268, 275, 278, 280, 289, 335, 348, 374, 375, 376, 377, 380, 381, 382, 383, 384, 420, 433, 488, 494 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 223, 227, 228, 229, 234, 235, 236, 240, 241, 242, 247, 248, 249, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 276, 277, 279, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 378, 379, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 489, 490, 491, 492, 493, 495, 496, 497, 498, 499 }

2.1.7 Giac

A grade: { 16, 17, 18, 31, 32, 40, 41, 42, 44, 45, 52, 53, 54, 57, 61, 62, 63, 64, 65, 66, 67, 70, 74, 75, 76, 77, 78, 79, 82, 86, 87, 88, 89, 90, 91, 174, 182, 183, 184, 208, 217, 218, 219, 220, 221, 222, 223, 230, 231, 232, 233, 234, 235, 236, 243, 244, 245, 246, 247, 248, 249, 254, 255, 256, 257, 258, 259, 260, 261, 263, 267, 268, 269, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 289, 293, 294, 295, 306, 307, 308, 320, 321, 322, 333, 334, 335, 346, 347, 348, 378, 379, 380, 385, 386, 389, 390, 393, 394, 396, 397, 400, 401, 420, 424, 425, 426, 427, 428, 431, 432, 433, 437, 438, 439, 440, 444, 445, 446, 450, 451, 452, 453, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 486, 487, 488, 494, 495, 499 }

B grade: { 38, 50, 51, 55, 56, 68, 69, 80, 81, 179, 180, 181, 185, 186, 187, 191, 192, 193, 195, 196, 197, 199, 200, 205, 206, 207, 209, 210, 250, 251, 252, 262, 264, 265, 266, 270, 288, 376, 377, 383, 403, 404, 434, 447, 454, 455, 467 }

C grade: { 290, 291, 292 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 39, 43, 46, 47, 48, 49, 58, 59, 60, 71, 72, 73, 83, 84, 85, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 188, 189, 190, 194, 198, 201, 202, 203, 204, 211, 212, 213, 214, 215, 216, 224, 225, 226, 227, 228, 229, 237, 238, 239, 240, 241, 242, 253, 271, 272, 283, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 381, 382, 384, 387, 388, 391, 392, 395, 398, 399, 402, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 429, 430, 435, 436, 441, 442, 443, 448, 449, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 489, 490, 491, 492, 493, 496, 497, 498 }

2.1.8 Mupad

A grade: { 174, 178, 420, 424, 486, 487, 488, 489, 490, 494, 495, 499 }

B grade: { 4, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 124, 125, 126, 127, 137, 138, 149, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 260, 261, 262, 263, 264, 265, 266, 268, 269, 273, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 306, 307, 308, 309, 310, 311, 320, 321, 322, 323, 324, 325, 329, 333, 334, 335, 336, 337, 338, 346, 347, 348, 349, 350, 351, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 479, 493 }

C grade: { }

F grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 257, 259, 267, 270, 271, 272, 274, 282, 299, 300, 301, 302, 303, 304, 305, 312, 313, 314, 315, 316, 317, 318, 319, 326, 327, 328, 330, 331, 332, 339, 340, 341, 342, 343, 344, 345, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 474, 475, 476, 477, 478, 480, 481, 482, 483, 484, 485, 491, 492, 496, 497, 498 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	56	58	47	74	0	0	-1
normalized size	1	1.00	0.57	0.59	0.48	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.359	0.287	0.611	0.742	0.000	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	48	34	52	0	0	-1
normalized size	1	1.00	0.77	0.79	0.56	0.85	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.109	0.263	0.825	0.402	0.000	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	19	38	0	0	-1
normalized size	1	1.00	1.00	1.16	0.59	1.19	0.00	0.00	-0.03
time (sec)	N/A	0.017	0.039	0.325	0.688	0.414	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	47	33	50	0	0	34
normalized size	1	1.00	1.26	1.52	1.06	1.61	0.00	0.00	1.10
time (sec)	N/A	0.023	0.081	0.408	0.647	0.413	0.000	0.000	11.444

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	56	64	46	69	0	0	-1
normalized size	1	1.00	0.85	0.97	0.70	1.05	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.360	0.307	0.991	0.411	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	68	74	66	82	0	0	-1
normalized size	1	1.00	0.70	0.76	0.68	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.259	0.348	1.215	0.860	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	199	263	178	0	0	0	-1
normalized size	1	1.00	0.55	0.72	0.49	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.146	0.831	0.253	1.266	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	54	236	140	0	0	0	-1
normalized size	1	1.00	0.19	0.83	0.49	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.127	0.078	0.165	0.807	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	161	206	133	0	0	0	-1
normalized size	1	1.00	0.63	0.81	0.52	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.115	0.259	0.229	2.158	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	43	211	126	0	0	0	-1
normalized size	1	1.00	0.17	0.83	0.49	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.115	0.033	0.310	1.333	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	45	233	163	0	0	0	-1
normalized size	1	1.00	0.15	0.78	0.55	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.128	0.074	0.211	3.344	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	45	272	172	0	0	0	-1
normalized size	1	1.00	0.12	0.75	0.47	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.148	0.057	0.231	1.916	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	86	84	79	96	0	0	-1
normalized size	1	1.00	0.47	0.46	0.43	0.53	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.736	0.218	0.948	0.982	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	66	64	53	62	0	0	-1
normalized size	1	1.00	0.60	0.58	0.48	0.56	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.745	0.208	0.850	0.761	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	42	26	37	0	0	-1
normalized size	1	1.00	0.82	0.84	0.52	0.74	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.096	0.230	1.012	0.850	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	43	40	27	39	0	48	-1
normalized size	1	1.00	0.84	0.78	0.53	0.76	0.00	0.94	-0.02
time (sec)	N/A	0.021	0.053	0.270	1.192	0.845	0.000	0.568	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	45	63	50	62	0	131	-1
normalized size	1	1.00	0.38	0.53	0.42	0.52	0.00	1.10	-0.01
time (sec)	N/A	0.045	0.050	0.227	0.878	0.744	0.000	2.253	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	45	83	70	82	0	196	-1
normalized size	1	1.00	0.25	0.45	0.38	0.45	0.00	1.07	-0.01
time (sec)	N/A	0.065	0.035	0.246	0.684	0.966	0.000	4.875	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	62	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.112	12.309	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.071	1.216	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.039	1.274	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	60	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.051	1.308	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	60	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.073	1.234	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	62	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.072	1.075	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.042	180.000	0.000	0.450	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.048	1.228	0.000	0.694	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.042	1.579	0.000	0.440	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.033	1.186	0.000	0.431	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	23	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.72	0.00	0.00	-0.03
time (sec)	N/A	0.019	0.023	180.000	0.000	0.473	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	104	92	62	64	0	0	92
normalized size	1	1.00	1.49	1.31	0.89	0.91	0.00	0.00	1.31
time (sec)	N/A	0.062	0.068	0.669	0.724	0.413	0.000	0.000	12.228

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	72	72	44	46	0	76	68
normalized size	1	1.00	1.50	1.50	0.92	0.96	0.00	1.58	1.42
time (sec)	N/A	0.047	0.050	0.602	0.307	0.485	0.000	1.886	12.010

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	46	52	31	31	0	41	39
normalized size	1	1.00	1.64	1.86	1.11	1.11	0.00	1.46	1.39
time (sec)	N/A	0.026	0.052	0.448	0.751	0.443	0.000	2.309	11.785

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	51	36	40	56	0	0	37
normalized size	1	1.00	2.04	1.44	1.60	2.24	0.00	0.00	1.48
time (sec)	N/A	0.028	0.028	0.433	0.672	0.443	0.000	0.000	11.563

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	123	76	76	124	0	0	95
normalized size	1	1.00	2.41	1.49	1.49	2.43	0.00	0.00	1.86
time (sec)	N/A	0.053	0.046	0.565	0.461	0.645	0.000	0.000	12.096

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	276	120	101	178	0	0	138
normalized size	1	1.00	3.49	1.52	1.28	2.25	0.00	0.00	1.75
time (sec)	N/A	0.070	6.057	0.526	0.580	0.700	0.000	0.000	11.975

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	89	122	111	90	0	0	105
normalized size	1	1.00	0.87	1.20	1.09	0.88	0.00	0.00	1.03
time (sec)	N/A	0.118	0.360	0.679	0.644	0.595	0.000	0.000	11.890

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	58	102	82	72	0	0	79
normalized size	1	1.00	0.78	1.38	1.11	0.97	0.00	0.00	1.07
time (sec)	N/A	0.073	0.372	0.581	0.809	0.578	0.000	0.000	11.429

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	81	51	54	0	395	41
normalized size	1	1.00	0.93	1.76	1.11	1.17	0.00	8.59	0.89
time (sec)	N/A	0.048	0.229	0.447	1.004	0.421	0.000	1.966	11.316

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	29	23	21	20	0	21
normalized size	1	1.00	1.47	1.53	1.21	1.11	1.05	0.00	1.11
time (sec)	N/A	0.013	0.013	0.025	0.749	0.424	0.139	0.000	11.276

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	24	37	0	26	26
normalized size	1	1.00	1.00	0.96	1.00	1.54	0.00	1.08	1.08
time (sec)	N/A	0.032	0.022	0.568	0.572	0.437	0.000	1.360	11.270

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	60	54	40	66	0	53	41
normalized size	1	1.00	1.43	1.29	0.95	1.57	0.00	1.26	0.98
time (sec)	N/A	0.043	0.077	0.712	0.480	0.408	0.000	1.774	11.486

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	106	83	59	91	0	79	59
normalized size	1	1.00	1.66	1.30	0.92	1.42	0.00	1.23	0.92
time (sec)	N/A	0.053	0.051	0.707	0.630	0.436	0.000	1.368	11.411

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	97	185	104	105	0	0	183
normalized size	1	1.00	0.91	1.73	0.97	0.98	0.00	0.00	1.71
time (sec)	N/A	0.107	0.723	0.744	0.438	0.442	0.000	0.000	12.648

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	72	155	80	80	0	144	128
normalized size	1	1.00	0.90	1.94	1.00	1.00	0.00	1.80	1.60
time (sec)	N/A	0.083	0.526	0.867	0.526	0.483	0.000	2.126	15.396

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	125	71	59	0	94	126
normalized size	1	1.00	0.89	2.31	1.31	1.09	0.00	1.74	2.33
time (sec)	N/A	0.046	0.307	0.716	0.724	0.452	0.000	2.132	14.122

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	66	124	68	87	0	0	86
normalized size	1	1.00	1.27	2.38	1.31	1.67	0.00	0.00	1.65
time (sec)	N/A	0.055	0.172	0.551	0.310	0.564	0.000	0.000	12.644

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	376	100	111	168	0	0	188
normalized size	1	1.00	4.59	1.22	1.35	2.05	0.00	0.00	2.29
time (sec)	N/A	0.109	6.130	0.739	0.617	0.438	0.000	0.000	12.609

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	447	183	163	284	0	0	243
normalized size	1	1.00	3.63	1.49	1.33	2.31	0.00	0.00	1.98
time (sec)	N/A	0.130	6.190	0.642	0.686	0.545	0.000	0.000	12.062

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	96	199	130	120	0	0	128
normalized size	1	1.00	0.79	1.63	1.07	0.98	0.00	0.00	1.05
time (sec)	N/A	0.131	1.459	0.776	0.591	0.441	0.000	0.000	12.330

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	71	168	87	94	0	1411	114
normalized size	1	1.00	0.84	1.98	1.02	1.11	0.00	16.60	1.34
time (sec)	N/A	0.107	0.733	0.705	0.838	0.559	0.000	22.942	11.796

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	73	87	58	51	68	382	76
normalized size	1	1.00	1.59	1.89	1.26	1.11	1.48	8.30	1.65
time (sec)	N/A	0.033	0.611	0.025	0.755	0.459	0.320	6.874	11.897

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	48	41	71	0	44	67
normalized size	1	1.00	0.96	1.04	0.89	1.54	0.00	0.96	1.46
time (sec)	N/A	0.054	0.516	0.633	0.549	0.444	0.000	3.896	11.856

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	59	81	66	92	0	84	69
normalized size	1	1.00	0.84	1.16	0.94	1.31	0.00	1.20	0.99
time (sec)	N/A	0.072	0.470	0.903	0.668	0.514	0.000	3.754	11.820

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	88	136	88	137	0	128	90
normalized size	1	1.00	0.95	1.46	0.95	1.47	0.00	1.38	0.97
time (sec)	N/A	0.090	0.806	0.740	0.333	0.526	0.000	5.949	12.192

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	177	205	0	294	0	377	643
normalized size	1	1.00	1.51	1.75	0.00	2.51	0.00	3.22	5.50
time (sec)	N/A	0.184	3.077	0.519	0.000	0.476	0.000	2.705	14.414

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	149	107	0	206	0	180	382
normalized size	1	1.00	1.77	1.27	0.00	2.45	0.00	2.14	4.55
time (sec)	N/A	0.123	0.659	0.570	0.000	0.547	0.000	1.385	13.369

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	121	63	0	158	0	81	112
normalized size	1	1.00	2.02	1.05	0.00	2.63	0.00	1.35	1.87
time (sec)	N/A	0.055	0.264	0.382	0.000	0.504	0.000	1.703	11.790

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	144	75	0	184	0	0	91
normalized size	1	1.00	2.40	1.25	0.00	3.07	0.00	0.00	1.52
time (sec)	N/A	0.070	0.243	0.565	0.000	0.536	0.000	0.000	11.861

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	195	189	0	327	0	0	591
normalized size	1	1.00	2.19	2.12	0.00	3.67	0.00	0.00	6.64
time (sec)	N/A	0.103	0.653	0.694	0.000	0.577	0.000	0.000	13.390

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	326	344	0	630	0	0	740
normalized size	1	1.00	2.51	2.65	0.00	4.85	0.00	0.00	5.69
time (sec)	N/A	0.176	6.261	0.597	0.000	0.573	0.000	0.000	14.718

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	140	545	305	521	0	292	4910
normalized size	1	1.00	0.79	3.06	1.71	2.93	0.00	1.64	27.58
time (sec)	N/A	0.293	0.571	0.524	0.807	0.560	0.000	2.085	17.016

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	99	304	183	383	0	190	3588
normalized size	1	1.00	0.77	2.36	1.42	2.97	0.00	1.47	27.81
time (sec)	N/A	0.152	0.273	0.623	0.847	0.505	0.000	2.440	15.739

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	137	93	274	0	113	190
normalized size	1	1.00	0.84	1.67	1.13	3.34	0.00	1.38	2.32
time (sec)	N/A	0.097	0.158	0.515	0.960	0.475	0.000	2.879	12.749

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	52	48	182	280	68	948
normalized size	1	1.00	0.98	1.04	0.96	3.64	5.60	1.36	18.96
time (sec)	N/A	0.075	0.063	0.219	0.856	0.539	2.354	1.955	11.687

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	46	42	257	0	62	40
normalized size	1	1.00	1.00	0.96	0.88	5.35	0.00	1.29	0.83
time (sec)	N/A	0.062	0.105	0.579	0.956	0.471	0.000	2.898	10.985

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	107	68	373	0	97	67
normalized size	1	1.00	0.96	1.41	0.89	4.91	0.00	1.28	0.88
time (sec)	N/A	0.090	0.315	0.581	0.878	0.473	0.000	2.993	11.053

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	103	191	104	543	0	151	115
normalized size	1	1.00	0.98	1.82	0.99	5.17	0.00	1.44	1.10
time (sec)	N/A	0.115	0.803	0.552	0.802	0.464	0.000	1.902	11.393

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	215	388	0	593	0	561	1049
normalized size	1	1.00	1.05	1.90	0.00	2.91	0.00	2.75	5.14
time (sec)	N/A	0.309	3.827	0.576	0.000	0.560	0.000	4.706	15.486

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	182	269	0	456	0	368	737
normalized size	1	1.00	1.37	2.02	0.00	3.43	0.00	2.77	5.54
time (sec)	N/A	0.180	3.316	0.560	0.000	0.567	0.000	2.526	14.720

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	146	114	0	307	0	153	436
normalized size	1	1.00	1.45	1.13	0.00	3.04	0.00	1.51	4.32
time (sec)	N/A	0.073	0.829	0.419	0.000	0.478	0.000	2.886	13.856

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	184	179	0	470	0	0	1140
normalized size	1	1.00	1.67	1.63	0.00	4.27	0.00	0.00	10.36
time (sec)	N/A	0.128	0.902	0.644	0.000	0.619	0.000	0.000	13.721

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	325	229	0	672	0	0	917
normalized size	1	1.00	2.21	1.56	0.00	4.57	0.00	0.00	6.24
time (sec)	N/A	0.181	6.307	0.652	0.000	0.561	0.000	0.000	12.373

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	392	428	0	1052	0	0	1113
normalized size	1	1.00	1.87	2.04	0.00	5.01	0.00	0.00	5.30
time (sec)	N/A	0.275	6.361	0.568	0.000	0.593	0.000	0.000	12.314

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	136	411	312	705	0	266	4616
normalized size	1	1.00	0.69	2.10	1.59	3.60	0.00	1.36	23.55
time (sec)	N/A	0.250	1.653	0.586	1.023	0.639	0.000	2.697	16.143

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	111	240	185	568	0	195	3301
normalized size	1	1.00	0.80	1.74	1.34	4.12	0.00	1.41	23.92
time (sec)	N/A	0.156	1.792	0.513	0.727	0.596	0.000	3.095	14.937

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	160	114	390	2322	127	2489
normalized size	1	1.00	0.91	1.65	1.18	4.02	23.94	1.31	25.66
time (sec)	N/A	0.083	1.113	0.181	0.582	0.498	28.540	1.803	13.532

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	75	73	373	0	93	70
normalized size	1	1.00	1.01	0.91	0.89	4.55	0.00	1.13	0.85
time (sec)	N/A	0.074	0.674	0.585	0.932	0.518	0.000	1.477	11.499

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	112	169	115	587	0	142	108
normalized size	1	1.00	0.97	1.46	0.99	5.06	0.00	1.22	0.93
time (sec)	N/A	0.146	0.991	0.623	0.620	0.654	0.000	3.340	11.370

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	151	281	161	855	0	212	178
normalized size	1	1.00	0.83	1.54	0.88	4.70	0.00	1.16	0.98
time (sec)	N/A	0.223	1.895	0.578	0.790	0.578	0.000	3.071	12.366

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	278	844	0	1018	0	885	1536
normalized size	1	1.00	1.05	3.20	0.00	3.86	0.00	3.35	5.82
time (sec)	N/A	0.411	5.811	0.642	0.000	0.776	0.000	5.395	16.328

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	230	504	0	775	0	563	1154
normalized size	1	1.00	1.28	2.80	0.00	4.31	0.00	3.13	6.41
time (sec)	N/A	0.246	5.954	0.592	0.000	0.669	0.000	7.402	15.454

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	170	221	0	556	0	223	780
normalized size	1	1.00	1.23	1.60	0.00	4.03	0.00	1.62	5.65
time (sec)	N/A	0.091	1.767	0.474	0.000	0.608	0.000	4.684	14.875

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	247	408	0	1050	0	0	1844
normalized size	1	1.00	1.49	2.46	0.00	6.33	0.00	0.00	11.11
time (sec)	N/A	0.217	3.439	0.789	0.000	0.788	0.000	0.000	16.268

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	414	435	0	1419	0	0	1652
normalized size	1	1.00	2.02	2.12	0.00	6.92	0.00	0.00	8.06
time (sec)	N/A	0.293	6.549	0.815	0.000	0.960	0.000	0.000	13.219

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	468	560	0	1693	0	0	1357
normalized size	1	1.00	1.81	2.16	0.00	6.54	0.00	0.00	5.24
time (sec)	N/A	0.376	6.576	0.674	0.000	0.887	0.000	0.000	12.824

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	194	598	460	1191	0	399	5965
normalized size	1	1.00	0.78	2.39	1.84	4.76	0.00	1.60	23.86
time (sec)	N/A	0.330	0.931	0.627	0.705	0.974	0.000	3.116	16.390

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	164	430	346	1076	0	282	4997
normalized size	1	1.00	0.85	2.23	1.79	5.58	0.00	1.46	25.89
time (sec)	N/A	0.246	2.563	0.560	0.642	0.882	0.000	2.926	16.487

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	138	350	227	742	9629	213	3901
normalized size	1	1.00	0.92	2.33	1.51	4.95	64.19	1.42	26.01
time (sec)	N/A	0.159	1.968	0.266	0.544	0.697	138.502	1.412	14.931

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	144	108	105	555	0	109	102
normalized size	1	1.00	1.29	0.96	0.94	4.96	0.00	0.97	0.91
time (sec)	N/A	0.088	0.993	0.751	0.583	0.855	0.000	2.782	11.221

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	146	235	158	857	0	175	147
normalized size	1	1.00	0.95	1.53	1.03	5.56	0.00	1.14	0.95
time (sec)	N/A	0.206	1.828	0.793	0.698	0.763	0.000	2.169	12.283

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	346	380	212	1199	0	263	199
normalized size	1	1.00	1.50	1.65	0.92	5.19	0.00	1.14	0.86
time (sec)	N/A	0.303	1.676	0.727	0.727	0.576	0.000	4.128	13.310

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	208	7044	203	378	0	0	-1
normalized size	1	1.00	1.29	43.75	1.26	2.35	0.00	0.00	-0.01
time (sec)	N/A	0.165	3.289	5.177	0.805	0.658	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	170	4296	132	277	0	0	-1
normalized size	1	1.00	1.50	38.02	1.17	2.45	0.00	0.00	-0.01
time (sec)	N/A	0.105	1.065	1.015	0.859	0.611	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	140	144	97	203	0	0	-1
normalized size	1	1.00	1.94	2.00	1.35	2.82	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.561	0.394	0.445	0.704	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	295	721	0	514	0	0	-1
normalized size	1	1.00	3.51	8.58	0.00	6.12	0.00	0.00	-0.01
time (sec)	N/A	0.087	6.655	1.191	0.000	0.680	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	460	2075	0	849	0	0	-1
normalized size	1	1.00	3.62	16.34	0.00	6.69	0.00	0.00	-0.01
time (sec)	N/A	0.139	3.739	1.235	0.000	1.385	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	1059	5378	0	1273	0	0	-1
normalized size	1	1.00	5.66	28.76	0.00	6.81	0.00	0.00	-0.01
time (sec)	N/A	0.232	6.704	1.343	0.000	2.015	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	330	2498	0	2068	0	0	-1
normalized size	1	1.00	1.75	13.22	0.00	10.94	0.00	0.00	-0.01
time (sec)	N/A	0.237	3.802	1.987	0.000	13.417	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	273	1342	0	1847	0	0	-1
normalized size	1	1.00	2.13	10.48	0.00	14.43	0.00	0.00	-0.01
time (sec)	N/A	0.135	3.836	0.779	0.000	1.367	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	203	169	0	410	0	0	-1
normalized size	1	1.00	2.39	1.99	0.00	4.82	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.893	0.249	0.000	0.548	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	156	1215	47	331	0	0	-1
normalized size	1	1.00	2.36	18.41	0.71	5.02	0.00	0.00	-0.02
time (sec)	N/A	0.079	2.180	4.516	0.627	0.559	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	204	2444	76	435	0	0	-1
normalized size	1	1.00	2.04	24.44	0.76	4.35	0.00	0.00	-0.01
time (sec)	N/A	0.095	4.410	1.710	0.499	0.685	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	287	3769	131	587	0	0	-1
normalized size	1	1.00	2.04	26.73	0.93	4.16	0.00	0.00	-0.01
time (sec)	N/A	0.128	3.501	1.327	0.492	1.368	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	233	2399	330	426	0	0	-1
normalized size	1	1.00	1.03	10.57	1.45	1.88	0.00	0.00	-0.00
time (sec)	N/A	0.216	5.570	1.179	0.810	0.907	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	188	1104	296	307	0	0	-1
normalized size	1	1.00	1.01	5.94	1.59	1.65	0.00	0.00	-0.01
time (sec)	N/A	0.165	1.894	0.800	0.980	0.822	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	170	357	176	268	0	0	-1
normalized size	1	1.00	1.50	3.16	1.56	2.37	0.00	0.00	-0.01
time (sec)	N/A	0.082	1.334	0.322	0.846	1.003	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	418	1249	0	747	0	0	-1
normalized size	1	1.00	3.29	9.83	0.00	5.88	0.00	0.00	-0.01
time (sec)	N/A	0.141	5.186	0.973	0.000	1.520	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	1022	2904	0	994	0	0	-1
normalized size	1	1.00	6.12	17.39	0.00	5.95	0.00	0.00	-0.01
time (sec)	N/A	0.206	6.765	0.991	0.000	1.972	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	409	6194	0	1365	0	0	-1
normalized size	1	1.00	1.83	27.78	0.00	6.12	0.00	0.00	-0.00
time (sec)	N/A	0.362	5.238	1.337	0.000	2.056	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	278	2630	0	2163	0	0	-1
normalized size	1	1.00	1.25	11.85	0.00	9.74	0.00	0.00	-0.00
time (sec)	N/A	0.322	5.011	1.166	0.000	120.241	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	324	2261	0	1931	0	0	-1
normalized size	1	1.00	1.96	13.70	0.00	11.70	0.00	0.00	-0.01
time (sec)	N/A	0.202	5.594	0.774	0.000	8.001	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	233	297	0	537	0	0	-1
normalized size	1	1.00	1.86	2.38	0.00	4.30	0.00	0.00	-0.01
time (sec)	N/A	0.100	1.475	0.257	0.000	0.879	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	220	1355	73	387	0	0	-1
normalized size	1	1.00	2.20	13.55	0.73	3.87	0.00	0.00	-0.01
time (sec)	N/A	0.099	2.683	0.986	0.666	0.765	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	177	4594	175	497	0	0	-1
normalized size	1	1.00	1.09	28.36	1.08	3.07	0.00	0.00	-0.01
time (sec)	N/A	0.137	1.930	1.310	0.515	1.586	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	213	6988	202	655	0	0	-1
normalized size	1	1.00	1.09	35.65	1.03	3.34	0.00	0.00	-0.01
time (sec)	N/A	0.171	2.169	1.834	0.686	5.220	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	112	169	214	124	0	0	-1
normalized size	1	1.00	0.78	1.17	1.49	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.144	2.137	1.086	0.492	0.851	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	104	106	75	0	0	-1
normalized size	1	1.00	0.84	1.18	1.20	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.101	1.493	0.874	0.353	0.585	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	52	78	35	45	0	0	-1
normalized size	1	1.00	1.41	2.11	0.95	1.22	0.00	0.00	-0.03
time (sec)	N/A	0.047	0.599	0.373	0.622	0.619	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	226	351	0	134	0	0	-1
normalized size	1	1.00	5.38	8.36	0.00	3.19	0.00	0.00	-0.02
time (sec)	N/A	0.068	2.662	1.216	0.000	0.759	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	303	2801	0	284	0	0	-1
normalized size	1	1.00	3.33	30.78	0.00	3.12	0.00	0.00	-0.01
time (sec)	N/A	0.115	3.196	1.200	0.000	0.631	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	278	6334	0	437	0	0	-1
normalized size	1	1.00	1.94	44.29	0.00	3.06	0.00	0.00	-0.01
time (sec)	N/A	0.160	4.756	1.424	0.000	0.653	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	314	1169	0	788	0	0	-1
normalized size	1	1.00	2.15	8.01	0.00	5.40	0.00	0.00	-0.01
time (sec)	N/A	0.167	4.244	1.080	0.000	4.691	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	270	795	0	696	0	0	-1
normalized size	1	1.00	2.90	8.55	0.00	7.48	0.00	0.00	-0.01
time (sec)	N/A	0.107	3.256	0.844	0.000	0.703	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	67	0	125	0	0	40
normalized size	1	1.00	1.00	1.46	0.00	2.72	0.00	0.00	0.87
time (sec)	N/A	0.034	0.081	0.353	0.000	0.463	0.000	0.000	12.364

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	49	57	30	49	0	0	36
normalized size	1	1.00	1.63	1.90	1.00	1.63	0.00	0.00	1.20
time (sec)	N/A	0.070	0.254	1.044	0.373	0.447	0.000	0.000	12.858

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	86	87	90	0	0	145
normalized size	1	1.00	0.92	1.16	1.18	1.22	0.00	0.00	1.96
time (sec)	N/A	0.090	0.424	1.082	0.779	0.637	0.000	0.000	18.816

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	90	148	173	141	0	0	761
normalized size	1	1.00	0.73	1.20	1.41	1.15	0.00	0.00	6.19
time (sec)	N/A	0.137	1.905	1.329	0.781	1.108	0.000	0.000	22.108

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	186	67748	389	233	0	0	-1
normalized size	1	1.00	0.93	340.44	1.95	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.187	1.937	7.052	0.511	0.653	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	106	14991	216	158	0	0	-1
normalized size	1	1.00	0.81	114.44	1.65	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.134	1.187	1.990	0.763	0.654	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	72	103	83	104	0	0	-1
normalized size	1	1.00	0.95	1.36	1.09	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.064	1.615	0.349	0.629	0.569	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	333	3491	0	355	0	0	-1
normalized size	1	1.00	3.96	41.56	0.00	4.23	0.00	0.00	-0.01
time (sec)	N/A	0.097	5.766	1.191	0.000	0.748	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	308	5633	0	455	0	0	-1
normalized size	1	1.00	2.43	44.35	0.00	3.58	0.00	0.00	-0.01
time (sec)	N/A	0.158	4.610	1.191	0.000	0.774	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	350	10582	0	705	0	0	-1
normalized size	1	1.00	1.87	56.59	0.00	3.77	0.00	0.00	-0.01
time (sec)	N/A	0.236	5.091	1.246	0.000	1.121	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	325	6027	0	1046	0	0	-1
normalized size	1	1.00	1.74	32.23	0.00	5.59	0.00	0.00	-0.01
time (sec)	N/A	0.223	3.591	16.158	0.000	128.623	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	282	1614	0	908	0	0	-1
normalized size	1	1.00	2.10	12.04	0.00	6.78	0.00	0.00	-0.01
time (sec)	N/A	0.157	2.918	5.316	0.000	6.274	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	214	104	0	310	0	0	-1
normalized size	1	1.00	2.52	1.22	0.00	3.65	0.00	0.00	-0.01
time (sec)	N/A	0.057	7.459	0.293	0.000	0.482	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	74	109	58	90	0	0	2978
normalized size	1	1.00	1.19	1.76	0.94	1.45	0.00	0.00	48.03
time (sec)	N/A	0.098	0.750	0.941	0.535	0.754	0.000	0.000	18.300

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	119	170	141	155	0	0	269040
normalized size	1	1.00	1.04	1.49	1.24	1.36	0.00	0.00	2360.00
time (sec)	N/A	0.122	0.929	1.077	0.698	3.410	0.000	0.000	35.944

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	135	264	255	232	0	0	-1
normalized size	1	1.00	0.79	1.54	1.49	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.179	1.465	1.095	0.748	31.862	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	294	391	534	370	0	0	-1
normalized size	1	1.00	1.19	1.58	2.15	1.49	0.00	0.00	-0.00
time (sec)	N/A	0.231	2.434	4.522	0.717	0.878	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	205	262	308	270	0	0	-1
normalized size	1	1.00	1.22	1.56	1.83	1.61	0.00	0.00	-0.01
time (sec)	N/A	0.159	1.553	1.505	1.048	0.726	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	124	147	135	202	0	0	-1
normalized size	1	1.00	1.05	1.25	1.14	1.71	0.00	0.00	-0.01
time (sec)	N/A	0.078	1.267	0.335	0.474	0.535	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	305	27448	0	696	0	0	-1
normalized size	1	1.00	2.24	201.82	0.00	5.12	0.00	0.00	-0.01
time (sec)	N/A	0.146	5.459	2.457	0.000	0.625	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	385	38486	0	889	0	0	-1
normalized size	1	1.00	2.18	217.44	0.00	5.02	0.00	0.00	-0.01
time (sec)	N/A	0.209	4.663	3.841	0.000	0.702	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	B	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	1142	49917	0	1037	0	0	-1
normalized size	1	1.00	4.82	210.62	0.00	4.38	0.00	0.00	-0.00
time (sec)	N/A	0.324	6.978	7.125	0.000	0.799	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	378	7943	0	0	0	0	-1
normalized size	1	1.00	1.54	32.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.333	5.669	15.659	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	309	2511	0	0	0	0	-1
normalized size	1	1.00	1.71	13.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	4.620	6.091	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	1331	176	0	561	0	0	-1
normalized size	1	1.00	9.93	1.31	0.00	4.19	0.00	0.00	-0.01
time (sec)	N/A	0.113	9.457	0.327	0.000	1.941	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	133	153	85	156	0	0	324
normalized size	1	1.00	1.37	1.58	0.88	1.61	0.00	0.00	3.34
time (sec)	N/A	0.106	1.049	1.123	0.793	10.474	0.000	0.000	27.443

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	140	245	195	0	0	0	-1
normalized size	1	1.00	0.96	1.68	1.34	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	1.172	1.314	0.781	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	174	371	337	0	0	0	-1
normalized size	1	1.00	0.79	1.69	1.54	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	2.335	1.098	0.730	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	292	0	0	0	0	0	-1
normalized size	1	1.00	3.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	2.127	4.956	0.000	0.718	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	275	0	0	0	0	0	-1
normalized size	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	2.427	2.144	0.000	1.376	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	283	0	0	0	0	0	-1
normalized size	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.223	7.841	5.199	0.000	0.565	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	184	0	0	0	0	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	4.112	4.457	0.000	0.796	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	80	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.911	0.911	0.000	0.768	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	1215	0	0	0	0	0	-1
normalized size	1	1.00	13.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	15.125	1.108	0.000	0.487	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	252	0	0	0	0	0	-1
normalized size	1	1.00	2.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	20.745	1.117	0.000	0.921	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	3698	0	0	0	0	0	-1
normalized size	1	1.00	44.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	18.779	4.824	0.000	0.424	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	192	0	0	0	0	0	-1
normalized size	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.586	0.734	0.000	0.409	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.721	1.053	0.000	0.452	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	111	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	1.576	1.079	0.000	0.498	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	176	141	0	0	0	0	0	-1
normalized size	1	0.98	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	1.340	1.051	0.000	0.551	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	295	0	0	0	0	0	-1
normalized size	1	1.00	3.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	2.027	7.974	0.000	0.490	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-1)	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	517	0	0	0	0	0	-1
normalized size	1	1.00	8.21	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.113	2.471	180.000	0.000	0.418	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.048	180.000	0.000	0.433	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	0	37	51	0	0	-1
normalized size	1	1.00	0.94	0.00	1.12	1.55	0.00	0.00	-0.03
time (sec)	N/A	0.097	0.052	180.000	0.771	0.450	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	59	0	73	104	0	0	-1
normalized size	1	1.00	0.86	0.00	1.06	1.51	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.157	180.000	0.372	0.426	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	89	171293	108	180	0	0	-1
normalized size	1	1.00	0.86	1647.05	1.04	1.73	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.295	26.464	0.515	0.438	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	506	0	0	0	0	0	-1
normalized size	1	1.00	5.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	2.842	33.920	0.000	0.441	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	284	0	0	0	0	0	-1
normalized size	1	1.00	3.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	1.308	21.642	0.000	0.438	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.203	25.659	0.000	0.462	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	1399	0	0	0	0	0	-1
normalized size	1	1.00	15.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	16.280	21.941	0.000	0.444	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	2.801	7.662	0.000	0.492	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	81	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.516	4.901	0.000	0.464	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	2033	0	0	0	0	0	-1
normalized size	1	1.00	18.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	16.881	2.810	0.000	0.492	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.529	1.404	0.000	0.445	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.127	2.777	7.982	0.000	0.455	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	43	157	56	68	519	53
normalized size	1	1.00	0.71	0.66	2.42	0.86	1.05	7.98	0.82
time (sec)	N/A	0.035	0.196	0.024	0.466	0.398	1.062	20.740	11.525

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	38	35	102	43	54	297	36
normalized size	1	1.00	0.76	0.70	2.04	0.86	1.08	5.94	0.72
time (sec)	N/A	0.030	0.103	0.028	0.462	0.395	0.545	7.950	11.468

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	25	59	29	37	133	24
normalized size	1	1.00	0.81	0.78	1.84	0.91	1.16	4.16	0.75
time (sec)	N/A	0.025	0.042	0.024	0.530	0.400	0.285	1.383	11.566

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	43	36	40	87	37	26
normalized size	1	1.00	0.84	1.39	1.16	1.29	2.81	1.19	0.84
time (sec)	N/A	0.022	0.026	0.173	0.476	0.424	0.543	1.084	11.830

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	69	67	84	248	51	35
normalized size	1	1.00	0.65	1.25	1.22	1.53	4.51	0.93	0.64
time (sec)	N/A	0.034	0.043	0.354	0.783	0.405	0.897	1.374	11.896

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	46	95	90	120	454	61	51
normalized size	1	1.00	0.58	1.20	1.14	1.52	5.75	0.77	0.65
time (sec)	N/A	0.048	0.039	0.339	0.943	0.394	1.400	1.630	11.932

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	106	99	67	116	1719	68
normalized size	1	1.00	0.85	1.43	1.34	0.91	1.57	23.23	0.92
time (sec)	N/A	0.049	0.258	0.035	0.443	0.449	0.730	49.425	11.712

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	65	78	70	51	88	1071	57
normalized size	1	1.00	1.23	1.47	1.32	0.96	1.66	20.21	1.08
time (sec)	N/A	0.039	0.168	0.029	0.544	0.416	0.394	9.920	11.704

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	40	50	37	36	60	500	37
normalized size	1	1.00	1.18	1.47	1.09	1.06	1.76	14.71	1.09
time (sec)	N/A	0.022	0.072	0.031	0.594	0.420	0.199	2.586	11.747

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	34	27	31	46	58	0	36
normalized size	1	1.00	1.31	1.04	1.19	1.77	2.23	0.00	1.38
time (sec)	N/A	0.029	0.041	0.569	0.632	0.459	0.412	0.000	11.658

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	56	41	31	61	97	0	54
normalized size	1	1.00	1.65	1.21	0.91	1.79	2.85	0.00	1.59
time (sec)	N/A	0.032	0.159	0.662	0.315	0.420	1.293	0.000	11.618

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	56	69	52	85	124	0	74
normalized size	1	1.00	1.06	1.30	0.98	1.60	2.34	0.00	1.40
time (sec)	N/A	0.042	0.229	0.534	0.504	0.414	2.932	0.000	11.649

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	129	120	72	69	109	1087	70
normalized size	1	1.00	1.61	1.50	0.90	0.86	1.36	13.59	0.88
time (sec)	N/A	0.052	0.050	0.028	0.920	0.411	0.963	86.830	11.566

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	97	92	57	54	82	633	53
normalized size	1	1.00	1.62	1.53	0.95	0.90	1.37	10.55	0.88
time (sec)	N/A	0.043	0.044	0.035	1.756	0.419	0.534	20.502	11.635

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	65	64	41	38	54	289	37
normalized size	1	1.00	1.62	1.60	1.02	0.95	1.35	7.22	0.92
time (sec)	N/A	0.034	0.029	0.035	0.775	0.432	0.287	2.533	11.540

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	29	23	21	20	0	21
normalized size	1	1.00	1.47	1.53	1.21	1.11	1.05	0.00	1.11
time (sec)	N/A	0.012	0.007	0.031	0.793	0.400	0.140	0.000	11.435

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	34	31	27	29	46	46	21
normalized size	1	1.00	1.62	1.48	1.29	1.38	2.19	2.19	1.00
time (sec)	N/A	0.025	0.019	0.512	0.665	0.408	0.782	2.318	11.470

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	65	47	46	49	70	106	40
normalized size	1	1.00	1.67	1.21	1.18	1.26	1.79	2.72	1.03
time (sec)	N/A	0.037	0.041	0.586	0.767	0.422	1.714	2.721	11.735

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	69	67	61	64	97	168	57
normalized size	1	1.00	1.13	1.10	1.00	1.05	1.59	2.75	0.93
time (sec)	N/A	0.046	0.051	0.445	0.794	0.397	4.422	3.260	11.937

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	89	198	162	107	206	0	113
normalized size	1	1.00	0.85	1.89	1.54	1.02	1.96	0.00	1.08
time (sec)	N/A	0.109	0.350	0.030	0.560	0.459	1.343	0.000	11.587

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	72	151	127	86	160	2600	97
normalized size	1	1.00	0.88	1.84	1.55	1.05	1.95	31.71	1.18
time (sec)	N/A	0.095	0.284	0.042	0.781	0.471	0.789	33.842	11.520

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	104	82	64	112	1510	68
normalized size	1	1.00	0.87	1.68	1.32	1.03	1.81	24.35	1.10
time (sec)	N/A	0.060	0.217	0.029	0.556	0.424	0.421	9.908	11.893

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	65	60	59	69	97	0	62
normalized size	1	1.00	1.27	1.18	1.16	1.35	1.90	0.00	1.22
time (sec)	N/A	0.064	0.127	0.640	0.605	0.424	1.138	0.000	11.952

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	51	62	51	93	129	0	68
normalized size	1	1.00	0.91	1.11	0.91	1.66	2.30	0.00	1.21
time (sec)	N/A	0.082	0.253	0.736	0.688	0.445	2.798	0.000	12.094

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	91	61	99	172	0	91
normalized size	1	1.00	0.80	1.20	0.80	1.30	2.26	0.00	1.20
time (sec)	N/A	0.088	0.302	0.543	0.316	0.439	8.543	0.000	12.176

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	243	226	118	115	212	0	155
normalized size	1	1.00	2.15	2.00	1.04	1.02	1.88	0.00	1.37
time (sec)	N/A	0.088	0.084	0.032	0.980	0.441	1.872	0.000	12.093

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	190	179	97	94	165	1684	127
normalized size	1	1.00	2.09	1.97	1.07	1.03	1.81	18.51	1.40
time (sec)	N/A	0.079	0.078	0.034	0.648	0.414	1.075	35.411	12.130

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	137	132	76	73	117	937	100
normalized size	1	1.00	1.99	1.91	1.10	1.06	1.70	13.58	1.45
time (sec)	N/A	0.074	0.052	0.029	1.040	0.414	0.626	8.869	11.674

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	73	87	58	51	68	382	76
normalized size	1	1.00	1.59	1.89	1.26	1.11	1.48	8.30	1.65
time (sec)	N/A	0.031	0.587	0.030	0.853	0.422	0.317	1.839	12.019

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	66	53	46	50	73	49	70
normalized size	1	1.00	1.74	1.39	1.21	1.32	1.92	1.29	1.84
time (sec)	N/A	0.065	0.115	0.480	0.559	0.410	1.704	3.070	12.020

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	71	60	57	60	90	122	58
normalized size	1	1.00	1.61	1.36	1.30	1.36	2.05	2.77	1.32
time (sec)	N/A	0.071	1.308	0.534	0.784	0.423	4.120	4.733	11.607

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	104	91	78	81	134	222	76
normalized size	1	1.00	1.53	1.34	1.15	1.19	1.97	3.26	1.12
time (sec)	N/A	0.077	0.101	0.694	0.795	0.408	8.465	7.941	11.518

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	64	72	76	92	348	0	74
normalized size	1	1.00	0.90	1.01	1.07	1.30	4.90	0.00	1.04
time (sec)	N/A	0.099	0.167	0.182	0.761	0.476	16.361	0.000	11.856

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	54	53	65	240	0	54
normalized size	1	1.00	0.82	1.08	1.06	1.30	4.80	0.00	1.08
time (sec)	N/A	0.086	0.034	0.143	0.307	0.450	3.675	0.000	11.779

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	50	30	38	143	0	66
normalized size	1	1.00	1.03	1.39	0.83	1.06	3.97	0.00	1.83
time (sec)	N/A	0.053	0.042	0.188	0.307	0.428	2.112	0.000	11.852

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	76	49	72	398	0	68
normalized size	1	1.00	0.89	1.19	0.77	1.12	6.22	0.00	1.06
time (sec)	N/A	0.082	0.060	0.726	0.830	0.452	8.379	0.000	11.712

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	63	150	68	128	743	0	89
normalized size	1	1.00	0.71	1.69	0.76	1.44	8.35	0.00	1.00
time (sec)	N/A	0.113	0.255	0.779	0.541	0.461	28.089	0.000	11.601

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	83	264	96	163	908	0	118
normalized size	1	1.00	0.72	2.30	0.83	1.42	7.90	0.00	1.03
time (sec)	N/A	0.137	0.372	0.923	0.459	0.455	94.468	0.000	11.782

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	92	102	83	278	685	118	1310
normalized size	1	1.00	1.08	1.20	0.98	3.27	8.06	1.39	15.41
time (sec)	N/A	0.188	0.839	0.176	0.898	0.489	35.000	21.517	11.997

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	70	70	65	220	493	86	1212
normalized size	1	1.00	1.11	1.11	1.03	3.49	7.83	1.37	19.24
time (sec)	N/A	0.106	0.289	0.130	1.008	0.452	6.762	2.980	12.183

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	52	47	181	292	67	135
normalized size	1	1.00	0.98	1.04	0.94	3.62	5.84	1.34	2.70
time (sec)	N/A	0.079	0.029	0.158	0.737	0.448	2.289	2.078	11.568

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	52	48	182	280	68	948
normalized size	1	1.00	0.98	1.04	0.96	3.64	5.60	1.36	18.96
time (sec)	N/A	0.074	0.050	0.255	0.764	0.462	2.307	1.547	12.297

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	68	73	65	243	570	86	438
normalized size	1	1.00	1.06	1.14	1.02	3.80	8.91	1.34	6.84
time (sec)	N/A	0.111	0.259	0.610	1.075	0.473	14.922	2.476	11.774

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	92	104	86	308	823	118	484
normalized size	1	1.00	1.10	1.24	1.02	3.67	9.80	1.40	5.76
time (sec)	N/A	0.173	0.696	0.845	0.965	0.507	57.334	3.246	12.200

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	121	158	112	352	0	164	524
normalized size	1	1.00	1.07	1.40	0.99	3.12	0.00	1.45	4.64
time (sec)	N/A	0.241	1.924	0.889	0.708	0.459	0.000	3.845	13.742

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	73	149	128	186	1583	0	90
normalized size	1	1.00	0.81	1.66	1.42	2.07	17.59	0.00	1.00
time (sec)	N/A	0.121	0.720	0.179	0.335	0.446	55.479	0.000	11.592

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	109	88	98	930	0	270
normalized size	1	1.00	0.88	1.58	1.28	1.42	13.48	0.00	3.91
time (sec)	N/A	0.100	0.579	0.268	0.345	0.427	26.506	0.000	11.601

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	57	104	88	98	816	0	195
normalized size	1	1.00	0.88	1.60	1.35	1.51	12.55	0.00	3.00
time (sec)	N/A	0.072	0.657	0.210	0.644	0.446	26.390	0.000	11.641

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	90	160	124	197	0	0	104
normalized size	1	1.00	0.87	1.55	1.20	1.91	0.00	0.00	1.01
time (sec)	N/A	0.121	2.105	0.984	0.542	0.475	0.000	0.000	11.733

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	98	234	187	292	0	0	144
normalized size	1	1.00	0.74	1.77	1.42	2.21	0.00	0.00	1.09
time (sec)	N/A	0.156	0.883	0.999	1.387	0.493	0.000	0.000	11.883

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	121	347	236	347	0	0	191
normalized size	1	1.00	0.75	2.16	1.47	2.16	0.00	0.00	1.19
time (sec)	N/A	0.179	1.092	0.988	1.190	0.521	0.000	0.000	12.369

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	118	184	135	474	3198	149	2581
normalized size	1	1.00	0.91	1.42	1.04	3.65	24.60	1.15	19.85
time (sec)	N/A	0.197	1.319	0.201	1.642	0.474	78.023	34.533	13.010

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	94	160	114	381	2416	127	2358
normalized size	1	1.00	0.99	1.68	1.20	4.01	25.43	1.34	24.82
time (sec)	N/A	0.118	0.816	0.204	0.752	0.487	28.759	3.695	13.519

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	87	151	97	393	2375	112	2136
normalized size	1	1.00	0.97	1.68	1.08	4.37	26.39	1.24	23.73
time (sec)	N/A	0.104	0.588	0.216	0.766	0.472	28.288	2.038	13.035

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	160	114	390	2322	127	2489
normalized size	1	1.00	0.91	1.65	1.18	4.02	23.94	1.31	25.66
time (sec)	N/A	0.080	1.044	0.291	0.758	0.464	27.875	2.285	13.523

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	117	187	151	503	0	171	2674
normalized size	1	1.00	0.91	1.46	1.18	3.93	0.00	1.34	20.89
time (sec)	N/A	0.192	3.216	0.834	0.715	0.477	0.000	2.597	14.234

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	137	218	193	596	0	179	2000
normalized size	1	1.00	0.81	1.29	1.14	3.53	0.00	1.06	11.83
time (sec)	N/A	0.287	3.601	0.944	0.769	0.517	0.000	6.539	15.751

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	231	272	239	672	0	225	3030
normalized size	1	1.00	1.06	1.25	1.10	3.08	0.00	1.03	13.90
time (sec)	N/A	0.342	6.282	0.936	0.754	0.537	0.000	7.349	16.004

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	97	234	189	206	3346	0	577
normalized size	1	1.00	0.90	2.17	1.75	1.91	30.98	0.00	5.34
time (sec)	N/A	0.152	1.156	0.197	0.543	0.451	133.671	0.000	12.518

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	87	193	194	212	2849	0	532
normalized size	1	1.00	0.90	1.99	2.00	2.19	29.37	0.00	5.48
time (sec)	N/A	0.117	0.814	0.204	0.435	0.455	132.460	0.000	12.515

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	82	190	192	206	2876	0	375
normalized size	1	1.00	0.88	2.04	2.06	2.22	30.92	0.00	4.03
time (sec)	N/A	0.088	0.748	0.243	0.527	0.455	133.352	0.000	12.462

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	126	289	250	422	0	0	181
normalized size	1	1.00	0.85	1.95	1.69	2.85	0.00	0.00	1.22
time (sec)	N/A	0.165	1.825	0.865	0.411	0.504	0.000	0.000	12.593

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	144	362	345	545	0	0	229
normalized size	1	1.00	0.80	2.00	1.91	3.01	0.00	0.00	1.27
time (sec)	N/A	0.214	2.063	0.997	0.968	0.579	0.000	0.000	13.465

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	178	477	416	611	0	0	269
normalized size	1	1.00	0.85	2.27	1.98	2.91	0.00	0.00	1.28
time (sec)	N/A	0.238	2.654	0.921	0.383	0.619	0.000	0.000	13.514

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	142	351	229	743	9823	215	3838
normalized size	1	1.00	0.93	2.29	1.50	4.86	64.20	1.41	25.08
time (sec)	N/A	0.229	2.374	0.240	0.838	0.485	141.171	33.171	15.523

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	136	338	212	749	9811	199	3667
normalized size	1	1.00	0.94	2.33	1.46	5.17	67.66	1.37	25.29
time (sec)	N/A	0.181	2.086	0.214	0.878	0.498	140.857	4.024	15.486

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	139	339	213	759	9763	200	3817
normalized size	1	1.00	0.97	2.35	1.48	5.27	67.80	1.39	26.51
time (sec)	N/A	0.154	2.113	0.237	0.881	0.757	140.009	3.582	15.488

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	138	350	227	742	9629	213	3901
normalized size	1	1.00	0.92	2.33	1.51	4.95	64.19	1.42	26.01
time (sec)	N/A	0.152	1.937	0.350	1.050	0.527	138.068	1.747	15.345

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	174	379	275	881	0	231	915
normalized size	1	1.00	0.92	2.01	1.46	4.66	0.00	1.22	4.84
time (sec)	N/A	0.290	2.395	0.980	0.821	0.513	0.000	5.475	15.047

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	184	413	332	1006	0	261	986
normalized size	1	1.00	0.77	1.72	1.38	4.19	0.00	1.09	4.11
time (sec)	N/A	0.365	4.978	0.808	0.580	0.554	0.000	7.727	15.359

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	949	466	396	1114	0	307	2507
normalized size	1	1.00	3.20	1.57	1.33	3.75	0.00	1.03	8.44
time (sec)	N/A	0.468	6.373	1.085	1.386	0.609	0.000	10.512	16.126

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	137	242	162	134	209	2209	164
normalized size	1	1.00	1.19	2.10	1.41	1.17	1.82	19.21	1.43
time (sec)	N/A	0.073	1.914	0.037	0.417	0.406	1.268	35.012	11.384

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	102	154	104	90	126	1027	115
normalized size	1	1.00	1.32	2.00	1.35	1.17	1.64	13.34	1.49
time (sec)	N/A	0.049	0.928	0.031	0.910	0.417	0.674	5.618	11.475

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	73	87	58	51	68	359	76
normalized size	1	1.00	1.59	1.89	1.26	1.11	1.48	7.80	1.65
time (sec)	N/A	0.031	0.597	0.035	1.128	0.441	0.314	1.483	11.420

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	29	23	21	20	0	21
normalized size	1	1.00	1.47	1.53	1.21	1.11	1.05	0.00	1.11
time (sec)	N/A	0.013	0.007	0.030	0.822	0.426	0.139	0.000	11.438

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	52	48	182	280	65	948
normalized size	1	1.00	0.98	1.04	0.96	3.64	5.60	1.30	18.96
time (sec)	N/A	0.074	0.076	0.205	0.781	0.476	2.397	0.819	11.904

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	160	114	390	2322	122	2489
normalized size	1	1.00	0.91	1.65	1.18	4.02	23.94	1.26	25.66
time (sec)	N/A	0.092	1.047	0.306	0.859	0.449	20.594	0.885	13.150

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	138	350	227	742	9629	205	3901
normalized size	1	1.00	0.92	2.33	1.51	4.95	64.19	1.37	26.01
time (sec)	N/A	0.144	1.929	0.333	1.008	0.488	99.076	1.159	13.979

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	32	56	860	56	0	48	-1
normalized size	1	1.00	0.59	1.04	15.93	1.04	0.00	0.89	-0.02
time (sec)	N/A	0.105	0.077	0.302	1.550	0.422	0.000	0.423	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	29	276	18	0	29	19
normalized size	1	1.00	0.67	0.97	9.20	0.60	0.00	0.97	0.63
time (sec)	N/A	0.087	0.026	0.196	0.788	0.379	0.000	0.300	11.643

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	39	295	47	0	40	-1
normalized size	1	1.00	0.67	1.08	8.19	1.31	0.00	1.11	-0.03
time (sec)	N/A	0.093	0.049	0.164	0.884	0.421	0.000	0.310	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	0	10	10	10	10
normalized size	1	1.00	1.00	1.10	0.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.043	0.009	0.123	0.000	0.416	0.599	0.300	11.626

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	23	38	63	0	24	12
normalized size	1	1.00	1.25	0.96	1.58	2.62	0.00	1.00	0.50
time (sec)	N/A	0.073	0.019	0.589	1.044	0.443	0.000	0.265	0.177

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	17	16	0	32	25
normalized size	1	1.00	1.00	1.21	1.21	1.14	0.00	2.29	1.79
time (sec)	N/A	0.080	0.014	0.526	0.848	0.386	0.000	0.455	11.979

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	38	51	303	58	0	42	37
normalized size	1	1.00	0.84	1.13	6.73	1.29	0.00	0.93	0.82
time (sec)	N/A	0.090	0.084	0.557	0.862	0.426	0.000	0.272	12.037

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	22	25	29	24	0	59	40
normalized size	1	1.00	0.65	0.74	0.85	0.71	0.00	1.74	1.18
time (sec)	N/A	0.100	0.023	0.635	0.975	0.417	0.000	0.270	11.846

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	31	34	65	90	0	66	41
normalized size	1	1.00	0.86	0.94	1.81	2.50	0.00	1.83	1.14
time (sec)	N/A	0.035	0.030	0.542	1.143	0.435	0.000	1.333	11.859

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	22	29	559	29	0	72	19
normalized size	1	1.00	0.69	0.91	17.47	0.91	0.00	2.25	0.59
time (sec)	N/A	0.100	0.049	0.131	1.634	0.402	0.000	0.299	12.014

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	34	54	934	57	0	49	-1
normalized size	1	1.00	0.58	0.92	15.83	0.97	0.00	0.83	-0.02
time (sec)	N/A	0.120	0.067	0.142	1.590	0.426	0.000	0.295	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	12	12	12	11
normalized size	1	1.00	1.00	0.93	0.00	0.86	0.86	0.86	0.79
time (sec)	N/A	0.051	0.015	0.076	0.000	0.454	1.940	0.340	0.176

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	32	134	49	0	42	33
normalized size	1	1.00	0.92	0.86	3.62	1.32	0.00	1.14	0.89
time (sec)	N/A	0.091	0.037	0.421	0.957	0.442	0.000	0.416	11.666

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	55	134	53	0	62	-1
normalized size	1	1.00	0.82	1.67	4.06	1.61	0.00	1.88	-0.03
time (sec)	N/A	0.109	0.027	0.548	1.008	0.410	0.000	0.750	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	43	62	556	72	0	0	-1
normalized size	1	1.00	0.63	0.91	8.18	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.070	0.282	1.010	0.412	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	65	90	1769	91	0	0	-1
normalized size	1	1.00	0.66	0.92	18.05	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.221	0.260	1.643	0.403	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	17	26	37	17	0	27	22
normalized size	1	1.00	0.68	1.04	1.48	0.68	0.00	1.08	0.88
time (sec)	N/A	0.092	0.025	0.225	0.694	0.387	0.000	0.386	0.288

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	49	38	42	64	0	40	-1
normalized size	1	1.00	1.58	1.23	1.35	2.06	0.00	1.29	-0.03
time (sec)	N/A	0.093	0.038	0.226	1.882	0.422	0.000	0.394	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	12	14	12	11
normalized size	1	1.00	1.00	1.08	0.00	1.00	1.17	1.00	0.92
time (sec)	N/A	0.046	0.013	0.148	0.000	0.409	0.570	0.292	11.800

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	29	42	66	0	34	31
normalized size	1	1.00	0.91	0.83	1.20	1.89	0.00	0.97	0.89
time (sec)	N/A	0.080	0.034	0.625	0.896	0.438	0.000	0.353	0.140

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	24	128	33	0	47	40
normalized size	1	1.00	0.71	0.77	4.13	1.06	0.00	1.52	1.29
time (sec)	N/A	0.092	0.028	0.575	1.482	0.395	0.000	0.316	11.818

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	23	29	38	43	36	26	29
normalized size	1	1.00	0.77	0.97	1.27	1.43	1.20	0.87	0.97
time (sec)	N/A	0.097	0.025	0.171	0.536	0.410	3.575	0.331	11.755

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	56	14	39	0	16	28
normalized size	1	1.00	0.78	2.43	0.61	1.70	0.00	0.70	1.22
time (sec)	N/A	0.101	0.017	0.175	0.949	0.389	0.000	0.598	11.650

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	35	15	12	23
normalized size	1	1.00	1.00	0.93	0.00	2.50	1.07	0.86	1.64
time (sec)	N/A	0.049	0.012	0.118	0.000	0.402	3.316	0.260	0.147

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	47	38	48	94	0	53	46
normalized size	1	1.00	0.89	0.72	0.91	1.77	0.00	1.00	0.87
time (sec)	N/A	0.091	0.061	0.458	0.883	0.417	0.000	0.543	11.686

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	31	31	225	52	0	55	-1
normalized size	1	1.00	0.52	0.52	3.75	0.87	0.00	0.92	-0.02
time (sec)	N/A	0.115	0.058	0.534	1.344	0.422	0.000	0.516	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	25	13	38	0	0	55
normalized size	1	1.00	1.00	1.04	0.54	1.58	0.00	0.00	2.29
time (sec)	N/A	0.029	0.051	0.372	1.441	0.396	0.000	0.000	12.074

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	57	26	70	0	81	35
normalized size	1	1.00	0.69	0.98	0.45	1.21	0.00	1.40	0.60
time (sec)	N/A	0.035	0.062	0.340	0.706	0.430	0.000	4.021	11.707

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	52	88	39	94	0	109	47
normalized size	1	1.00	0.59	1.00	0.44	1.07	0.00	1.24	0.53
time (sec)	N/A	0.044	0.103	0.382	0.612	0.420	0.000	6.246	0.198

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	62	119	50	118	0	137	161
normalized size	1	1.00	0.53	1.01	0.42	1.00	0.00	1.16	1.36
time (sec)	N/A	0.053	0.175	0.365	0.595	0.393	0.000	8.881	11.785

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	52	19	18	72	0	29	18
normalized size	1	1.00	2.36	0.86	0.82	3.27	0.00	1.32	0.82
time (sec)	N/A	0.015	0.067	0.144	0.547	0.393	0.000	0.310	0.109

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	44	4	3	60	0	16	3
normalized size	1	1.00	14.67	1.33	1.00	20.00	0.00	5.33	1.00
time (sec)	N/A	0.011	0.010	0.135	0.754	0.405	0.000	0.394	0.059

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	12	11	9
normalized size	1	1.00	1.00	1.09	1.00	1.00	1.09	1.00	0.82
time (sec)	N/A	0.013	0.007	0.087	0.381	0.449	0.361	0.444	0.028

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	72	32	20	73	0	29	31
normalized size	1	1.00	2.06	0.91	0.57	2.09	0.00	0.83	0.89
time (sec)	N/A	0.022	0.067	0.249	0.576	0.446	0.000	0.300	11.590

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	46	17	17	19	0	16	16
normalized size	1	1.00	2.88	1.06	1.06	1.19	0.00	1.00	1.00
time (sec)	N/A	0.017	0.007	0.257	0.855	0.414	0.000	0.510	0.111

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	0	12	0	12	13
normalized size	1	1.00	1.00	1.08	0.00	0.92	0.00	0.92	1.00
time (sec)	N/A	0.020	0.007	0.240	0.000	0.388	0.000	0.353	11.978

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	109	166	0	320	0	143	157
normalized size	1	1.00	0.93	1.42	0.00	2.74	0.00	1.22	1.34
time (sec)	N/A	0.146	1.383	0.395	0.000	0.585	0.000	0.858	20.970

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	82	114	0	254	0	96	76
normalized size	1	1.00	0.93	1.30	0.00	2.89	0.00	1.09	0.86
time (sec)	N/A	0.114	0.349	0.409	0.000	0.511	0.000	0.632	14.379

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	59	91	0	214	0	60	54
normalized size	1	1.00	0.95	1.47	0.00	3.45	0.00	0.97	0.87
time (sec)	N/A	0.071	0.050	0.235	0.000	0.509	0.000	0.517	12.512

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	72	615	0	382	0	0	83
normalized size	1	1.00	0.97	8.31	0.00	5.16	0.00	0.00	1.12
time (sec)	N/A	0.101	0.062	1.695	0.000	0.432	0.000	0.000	0.285

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	2135	0	592	0	0	238
normalized size	1	1.00	1.00	18.57	0.00	5.15	0.00	0.00	2.07
time (sec)	N/A	0.148	0.369	1.660	0.000	0.448	0.000	0.000	11.870

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	5676	0	729	0	0	542
normalized size	1	1.00	0.85	34.82	0.00	4.47	0.00	0.00	3.33
time (sec)	N/A	0.212	1.319	1.284	0.000	0.446	0.000	0.000	12.018

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	823	451	0	826	0	0	-1
normalized size	1	1.00	3.71	2.03	0.00	3.72	0.00	0.00	-0.00
time (sec)	N/A	0.337	6.335	0.369	0.000	2.099	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	767	323	0	671	0	0	-1
normalized size	1	1.00	4.54	1.91	0.00	3.97	0.00	0.00	-0.01
time (sec)	N/A	0.210	6.252	0.383	0.000	1.201	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	708	230	0	539	0	0	-1
normalized size	1	1.00	5.76	1.87	0.00	4.38	0.00	0.00	-0.01
time (sec)	N/A	0.134	6.178	0.295	0.000	0.723	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	203	169	0	410	0	0	-1
normalized size	1	1.00	2.39	1.99	0.00	4.82	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.884	0.419	0.000	0.509	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	64	2233	0	257	0	0	-1
normalized size	1	1.00	0.85	29.77	0.00	3.43	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.284	1.625	0.000	0.529	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	241	4518	0	311	0	0	-1
normalized size	1	1.00	2.06	38.62	0.00	2.66	0.00	0.00	-0.01
time (sec)	N/A	0.154	6.983	1.420	0.000	0.610	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	339	6894	0	375	0	0	-1
normalized size	1	1.00	2.03	41.28	0.00	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.231	14.387	1.362	0.000	0.543	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	139	256	0	414	0	196	233
normalized size	1	1.00	0.96	1.77	0.00	2.86	0.00	1.35	1.61
time (sec)	N/A	0.175	1.394	0.343	0.000	0.551	0.000	0.891	40.837

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	112	204	0	334	0	150	156
normalized size	1	1.00	0.97	1.76	0.00	2.88	0.00	1.29	1.34
time (sec)	N/A	0.145	0.890	0.231	0.000	0.533	0.000	0.863	22.575

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	80	181	0	255	0	114	91
normalized size	1	1.00	0.89	2.01	0.00	2.83	0.00	1.27	1.01
time (sec)	N/A	0.097	0.401	0.196	0.000	0.594	0.000	0.997	14.989

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	90	1765	0	591	0	0	546
normalized size	1	1.00	0.95	18.58	0.00	6.22	0.00	0.00	5.75
time (sec)	N/A	0.131	0.246	1.553	0.000	1.223	0.000	0.000	12.030

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	109	2011	0	584	0	0	447
normalized size	1	1.00	0.94	17.34	0.00	5.03	0.00	0.00	3.85
time (sec)	N/A	0.174	0.335	1.506	0.000	0.480	0.000	0.000	12.026

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	140	5224	0	748	0	0	578
normalized size	1	1.00	0.87	32.45	0.00	4.65	0.00	0.00	3.59
time (sec)	N/A	0.222	1.404	1.399	0.000	0.492	0.000	0.000	12.253

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	908	669	0	1059	0	0	-1
normalized size	1	1.00	3.09	2.28	0.00	3.60	0.00	0.00	-0.00
time (sec)	N/A	0.448	6.550	0.284	0.000	4.944	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	833	510	0	861	0	0	-1
normalized size	1	1.00	3.72	2.28	0.00	3.84	0.00	0.00	-0.00
time (sec)	N/A	0.355	6.440	0.377	0.000	2.987	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	771	386	0	708	0	0	-1
normalized size	1	1.00	4.48	2.24	0.00	4.12	0.00	0.00	-0.01
time (sec)	N/A	0.249	6.309	0.312	0.000	1.245	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	233	297	0	537	0	0	-1
normalized size	1	1.00	1.86	2.38	0.00	4.30	0.00	0.00	-0.01
time (sec)	N/A	0.095	1.427	0.311	0.000	0.935	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	724	3333	0	710	0	0	-1
normalized size	1	1.00	6.35	29.24	0.00	6.23	0.00	0.00	-0.01
time (sec)	N/A	0.138	6.241	1.547	0.000	1.276	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	78	6591	0	308	0	0	-1
normalized size	1	1.00	0.68	57.31	0.00	2.68	0.00	0.00	-0.01
time (sec)	N/A	0.173	0.341	1.352	0.000	0.554	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	140	10026	0	385	0	0	-1
normalized size	1	1.00	0.85	60.76	0.00	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.245	9.496	1.413	0.000	0.570	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	259	461	0	703	0	0	-1
normalized size	1	1.00	1.52	2.71	0.00	4.14	0.00	0.00	-0.01
time (sec)	N/A	0.179	1.343	0.567	0.000	2.224	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	87	111	0	314	0	114	97
normalized size	1	1.00	0.92	1.17	0.00	3.31	0.00	1.20	1.02
time (sec)	N/A	0.140	2.452	0.347	0.000	0.554	0.000	1.996	12.877

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	58	0	248	0	62	56
normalized size	1	1.00	0.97	0.91	0.00	3.88	0.00	0.97	0.88
time (sec)	N/A	0.111	0.284	0.300	0.000	0.543	0.000	1.911	12.328

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	35	0	185	0	35	35
normalized size	1	1.00	1.00	0.85	0.00	4.51	0.00	0.85	0.85
time (sec)	N/A	0.064	0.030	0.203	0.000	0.570	0.000	1.975	12.344

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	72	496	0	446	0	0	232
normalized size	1	1.00	0.97	6.70	0.00	6.03	0.00	0.00	3.14
time (sec)	N/A	0.108	0.084	1.488	0.000	0.463	0.000	0.000	12.027

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	135	3601	0	697	0	0	830
normalized size	1	1.00	1.16	31.04	0.00	6.01	0.00	0.00	7.16
time (sec)	N/A	0.163	0.757	1.349	0.000	0.461	0.000	0.000	0.437

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	162	7641	0	857	0	0	1215
normalized size	1	1.00	0.98	46.03	0.00	5.16	0.00	0.00	7.32
time (sec)	N/A	0.214	1.961	1.958	0.000	0.493	0.000	0.000	12.139

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	768	261	0	817	0	0	-1
normalized size	1	1.00	4.34	1.47	0.00	4.62	0.00	0.00	-0.01
time (sec)	N/A	0.222	6.335	0.326	0.000	2.079	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	713	165	0	647	0	0	-1
normalized size	1	1.00	5.70	1.32	0.00	5.18	0.00	0.00	-0.01
time (sec)	N/A	0.139	6.257	0.268	0.000	1.186	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	149	102	0	479	0	0	-1
normalized size	1	1.00	1.73	1.19	0.00	5.57	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.788	0.328	0.000	0.547	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	67	0	125	0	0	40
normalized size	1	1.00	1.00	1.46	0.00	2.72	0.00	0.00	0.87
time (sec)	N/A	0.031	0.070	0.359	0.000	0.444	0.000	0.000	12.687

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	212	1195	0	289	0	0	-1
normalized size	1	1.00	2.72	15.32	0.00	3.71	0.00	0.00	-0.01
time (sec)	N/A	0.119	9.498	1.349	0.000	0.566	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	263	2433	0	359	0	0	-1
normalized size	1	1.00	2.19	20.28	0.00	2.99	0.00	0.00	-0.01
time (sec)	N/A	0.165	11.071	1.769	0.000	0.564	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	794	3741	0	437	0	0	-1
normalized size	1	1.00	4.67	22.01	0.00	2.57	0.00	0.00	-0.01
time (sec)	N/A	0.246	16.311	1.699	0.000	0.579	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	84	141	0	432	0	99	112
normalized size	1	1.00	0.86	1.44	0.00	4.41	0.00	1.01	1.14
time (sec)	N/A	0.169	0.378	0.266	0.000	0.555	0.000	3.146	13.732

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	92	0	358	0	76	90
normalized size	1	1.00	1.03	1.26	0.00	4.90	0.00	1.04	1.23
time (sec)	N/A	0.133	0.359	0.296	0.000	0.550	0.000	2.124	13.100

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	56	68	0	332	56	69	85
normalized size	1	1.00	0.81	0.99	0.00	4.81	0.81	1.00	1.23
time (sec)	N/A	0.088	0.073	0.154	0.000	0.576	24.733	2.543	13.074

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	91	32888	0	920	0	0	1922
normalized size	1	1.00	0.86	310.26	0.00	8.68	0.00	0.00	18.13
time (sec)	N/A	0.149	0.136	2.190	0.000	0.491	0.000	0.000	12.640

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	115	54353	0	1252	0	0	2483
normalized size	1	1.00	0.73	346.20	0.00	7.97	0.00	0.00	15.82
time (sec)	N/A	0.246	0.428	3.119	0.000	0.544	0.000	0.000	12.538

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	142	79934	0	1522	0	0	2118
normalized size	1	1.00	0.66	371.79	0.00	7.08	0.00	0.00	9.85
time (sec)	N/A	0.346	1.227	5.348	0.000	0.564	0.000	0.000	13.133

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	787	286	0	1207	0	0	-1
normalized size	1	1.00	4.32	1.57	0.00	6.63	0.00	0.00	-0.01
time (sec)	N/A	0.251	6.445	0.394	0.000	2.337	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	250	193	0	974	0	0	-1
normalized size	1	1.00	2.03	1.57	0.00	7.92	0.00	0.00	-0.01
time (sec)	N/A	0.159	3.195	0.273	0.000	1.360	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	154	131	0	285	0	0	-1
normalized size	1	1.00	1.90	1.62	0.00	3.52	0.00	0.00	-0.01
time (sec)	N/A	0.111	3.227	0.243	0.000	0.472	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	214	104	0	310	0	0	-1
normalized size	1	1.00	2.52	1.22	0.00	3.65	0.00	0.00	-0.01
time (sec)	N/A	0.065	6.284	0.441	0.000	0.453	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	882	1305	0	471	0	0	-1
normalized size	1	1.00	6.89	10.20	0.00	3.68	0.00	0.00	-0.01
time (sec)	N/A	0.184	13.599	1.431	0.000	1.025	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	802	2577	0	579	0	0	-1
normalized size	1	1.00	4.36	14.01	0.00	3.15	0.00	0.00	-0.01
time (sec)	N/A	0.259	16.446	1.211	0.000	0.579	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	850	3925	0	687	0	0	-1
normalized size	1	1.00	3.37	15.58	0.00	2.73	0.00	0.00	-0.00
time (sec)	N/A	0.368	16.557	1.778	0.000	0.563	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	91	169	0	608	0	137	148
normalized size	1	1.00	0.79	1.47	0.00	5.29	0.00	1.19	1.29
time (sec)	N/A	0.204	0.456	0.344	0.000	0.552	0.000	4.220	16.029

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	84	118	0	572	0	116	138
normalized size	1	1.00	0.82	1.15	0.00	5.55	0.00	1.13	1.34
time (sec)	N/A	0.156	0.303	0.277	0.000	0.580	0.000	4.353	15.720

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	58	94	0	544	83	105	131
normalized size	1	1.00	0.59	0.95	0.00	5.49	0.84	1.06	1.32
time (sec)	N/A	0.108	0.137	0.186	0.000	0.565	31.490	4.162	15.696

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	94	331597	0	1649	0	0	2788
normalized size	1	1.00	0.64	2255.76	0.00	11.22	0.00	0.00	18.97
time (sec)	N/A	0.214	0.371	21.095	0.000	0.483	0.000	0.000	12.456

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	138	531560	0	2083	0	0	3429
normalized size	1	1.00	0.67	2580.39	0.00	10.11	0.00	0.00	16.65
time (sec)	N/A	0.350	0.620	38.085	0.000	0.630	0.000	0.000	13.100

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	165	790286	0	2433	0	0	4652
normalized size	1	1.00	0.61	2905.46	0.00	8.94	0.00	0.00	17.10
time (sec)	N/A	0.443	1.991	64.737	0.000	0.566	0.000	0.000	14.006

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	295	382	0	1714	0	0	-1
normalized size	1	1.00	1.73	2.23	0.00	10.02	0.00	0.00	-0.01
time (sec)	N/A	0.267	4.576	0.352	0.000	2.582	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	260	291	0	498	0	0	-1
normalized size	1	1.00	1.98	2.22	0.00	3.80	0.00	0.00	-0.01
time (sec)	N/A	0.159	6.054	0.319	0.000	0.499	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	365	232	0	529	0	0	-1
normalized size	1	1.00	2.85	1.81	0.00	4.13	0.00	0.00	-0.01
time (sec)	N/A	0.153	7.774	0.259	0.000	0.550	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	1331	176	0	561	0	0	-1
normalized size	1	1.00	9.93	1.31	0.00	4.19	0.00	0.00	-0.01
time (sec)	N/A	0.101	7.601	0.434	0.000	0.636	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	1890	0	0	753	0	0	-1
normalized size	1	1.00	10.16	0.00	0.00	4.05	0.00	0.00	-0.01
time (sec)	N/A	0.279	15.918	1.134	0.000	0.684	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	871	0	0	879	0	0	-1
normalized size	1	1.00	3.50	0.00	0.00	3.53	0.00	0.00	-0.00
time (sec)	N/A	0.378	16.591	1.041	0.000	0.730	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	921	0	0	1023	0	0	-1
normalized size	1	1.00	2.82	0.00	0.00	3.13	0.00	0.00	-0.00
time (sec)	N/A	0.496	16.772	1.385	0.000	0.984	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	74	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.089	3.059	0.000	0.545	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	101	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.309	2.197	0.000	0.609	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	106	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.908	1.155	0.000	0.603	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	73	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.136	0.970	0.000	0.699	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.067	0.100	0.921	0.000	0.787	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	98	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.174	1.323	0.000	0.536	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	142	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.741	1.078	0.000	0.680	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	172	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	2.758	1.376	0.000	0.756	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	3.539	1.436	0.000	0.626	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1896	0	0	0	0	0	-1
normalized size	1	1.00	22.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	16.592	1.008	0.000	0.577	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1992	0	0	0	0	0	-1
normalized size	1	1.00	24.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	15.103	0.896	0.000	0.698	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	192	0	0	0	0	0	-1
normalized size	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.487	0.022	0.000	0.562	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	1989	0	0	0	0	0	-1
normalized size	1	1.00	25.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	14.934	1.065	0.000	0.580	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1887	0	0	0	0	0	-1
normalized size	1	1.00	22.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	6.867	1.062	0.000	0.670	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	4.621	1.120	0.000	0.762	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	224	321	260	225	301	0	310
normalized size	1	1.00	0.88	1.26	1.02	0.88	1.18	0.00	1.22
time (sec)	N/A	0.147	1.037	0.027	0.991	0.600	3.584	0.000	11.849

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	160	201	183	148	194	0	174
normalized size	1	1.00	0.95	1.20	1.09	0.88	1.15	0.00	1.04
time (sec)	N/A	0.098	0.508	0.026	0.655	0.475	1.651	0.000	11.600

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	107	108	83	85	94	1065	117
normalized size	1	1.00	1.20	1.21	0.93	0.96	1.06	11.97	1.31
time (sec)	N/A	0.060	0.519	0.025	0.924	0.465	0.597	14.425	11.598

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	36	36	36	37	251	34
normalized size	1	1.00	0.94	1.12	1.12	1.12	1.16	7.84	1.06
time (sec)	N/A	0.019	0.089	0.025	0.303	0.532	0.182	1.617	11.530

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	278	355	291	4817	0	334	342
normalized size	1	1.00	1.09	1.39	1.14	18.82	0.00	1.30	1.34
time (sec)	N/A	0.380	0.663	0.280	1.131	2.095	0.000	1.792	12.653

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	575	1086	502	11554	0	605	988
normalized size	1	1.00	1.03	1.95	0.90	20.71	0.00	1.08	1.77
time (sec)	N/A	0.728	6.312	0.352	1.233	3.402	0.000	2.754	12.519

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	57	34	33	48	34	34	41
normalized size	1	1.00	1.54	0.92	0.89	1.30	0.92	0.92	1.11
time (sec)	N/A	0.061	0.026	0.102	0.752	0.536	0.180	0.592	11.618

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	196	412	265	225	386	0	271
normalized size	1	1.00	0.91	1.91	1.23	1.04	1.79	0.00	1.25
time (sec)	N/A	0.129	5.385	0.033	0.790	0.678	7.070	0.000	11.586

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	128	252	167	145	224	0	180
normalized size	1	1.00	0.89	1.75	1.16	1.01	1.56	0.00	1.25
time (sec)	N/A	0.082	1.066	0.027	0.768	0.868	3.176	0.000	11.662

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	75	134	91	81	116	1181	109
normalized size	1	1.00	0.91	1.63	1.11	0.99	1.41	14.40	1.33
time (sec)	N/A	0.055	0.577	0.027	0.587	0.618	1.084	31.858	11.471

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	44	43	34	32	32	0	31
normalized size	1	1.00	1.26	1.23	0.97	0.91	0.91	0.00	0.89
time (sec)	N/A	0.025	0.027	0.024	0.460	0.640	0.221	0.000	11.590

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	228	374	261	1541	0	354	4038
normalized size	1	1.00	0.75	1.24	0.86	5.10	0.00	1.17	13.37
time (sec)	N/A	0.321	0.572	0.207	0.861	0.736	0.000	3.187	15.043

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	648	648	598	886	394	4291	0	517	11516
normalized size	1	1.00	0.92	1.37	0.61	6.62	0.00	0.80	17.77
time (sec)	N/A	0.661	6.293	0.207	0.804	1.154	0.000	3.406	15.685

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	650	219	531	0	0	0	0	-1
normalized size	1	1.00	0.34	0.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.560	0.812	0.574	0.000	1.149	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	106	123	0	0	0	0	-1
normalized size	1	1.00	0.30	0.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.224	0.408	0.311	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	145	181	0	555	0	107	-1
normalized size	1	1.00	1.41	1.76	0.00	5.39	0.00	1.04	-0.01
time (sec)	N/A	0.207	4.094	0.307	0.000	0.699	0.000	0.456	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	86	139	0	475	0	89	-1
normalized size	1	1.00	0.96	1.54	0.00	5.28	0.00	0.99	-0.01
time (sec)	N/A	0.118	0.045	0.245	0.000	0.660	0.000	0.423	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	98	0	0	1021	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	10.01	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.087	0.520	0.000	0.988	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	643	643	550	537	0	0	0	0	-1
normalized size	1	1.00	0.86	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.496	17.876	0.231	0.000	38.640	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	324	374	0	758	0	176	-1
normalized size	1	1.00	2.19	2.53	0.00	5.12	0.00	1.19	-0.01
time (sec)	N/A	0.310	6.078	0.209	0.000	0.866	0.000	0.600	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	166	313	0	593	0	138	-1
normalized size	1	1.00	1.32	2.48	0.00	4.71	0.00	1.10	-0.01
time (sec)	N/A	0.208	4.843	0.220	0.000	0.851	0.000	0.698	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	190	0	0	1269	0	0	-1
normalized size	1	1.00	1.23	0.00	0.00	8.19	0.00	0.00	-0.01
time (sec)	N/A	0.267	3.232	0.437	0.000	37.866	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	91	0	483	0	75	-1
normalized size	1	1.00	1.00	1.23	0.00	6.53	0.00	1.01	-0.01
time (sec)	N/A	0.129	0.060	0.275	0.000	0.712	0.000	0.652	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	65	0	150	0	46	-1
normalized size	1	1.00	1.00	1.59	0.00	3.66	0.00	1.12	-0.02
time (sec)	N/A	0.068	0.017	0.231	0.000	0.637	0.000	0.461	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	0	475	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	6.79	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.064	0.515	0.000	0.752	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	122	179	0	0	0	0	-1
normalized size	1	1.00	0.42	0.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	2.360	0.352	0.000	0.855	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	67	267	0	292	0	103	-1
normalized size	1	1.00	0.94	3.76	0.00	4.11	0.00	1.45	-0.01
time (sec)	N/A	0.162	0.334	0.385	0.000	0.777	0.000	0.550	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	248	0	319	0	119	-1
normalized size	1	1.00	0.99	3.35	0.00	4.31	0.00	1.61	-0.01
time (sec)	N/A	0.115	0.308	0.230	0.000	0.746	0.000	0.471	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	108	0	0	954	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	7.88	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.620	0.443	0.000	0.992	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	104	654	0	556	0	597	-1
normalized size	1	1.00	0.95	6.00	0.00	5.10	0.00	5.48	-0.01
time (sec)	N/A	0.228	0.868	0.286	0.000	0.796	0.000	0.686	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	113	602	0	599	0	618	-1
normalized size	1	1.00	0.97	5.15	0.00	5.12	0.00	5.28	-0.01
time (sec)	N/A	0.187	0.833	0.247	0.000	0.960	0.000	0.593	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	149	0	0	1749	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	9.56	0.00	0.00	-0.01
time (sec)	N/A	0.302	1.543	0.461	0.000	1.192	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	151	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.713	1.430	1.928	0.000	0.433	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	304	0	0	0	0	0	-1
normalized size	1	1.00	2.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.331	0.653	1.332	0.000	0.521	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	385	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.283	6.312	1.324	0.000	0.438	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	617	617	381	0	0	0	0	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.579	6.412	2.118	0.000	0.717	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	76	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.085	12.612	0.000	0.525	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.094	0.085	12.824	0.000	0.571	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.042	12.279	0.000	0.415	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.110	0.051	12.760	0.000	0.410	0.000	0.000	0.000

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.114	0.083	12.633	0.000	0.416	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.112	0.102	11.127	0.000	0.450	0.000	0.000	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.092	0.069	2.006	0.000	0.450	0.000	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.069	0.063	12.880	0.000	0.422	0.000	0.000	0.000

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.080	0.017	13.428	0.000	0.509	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	59	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.108	0.059	10.357	0.000	0.529	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.059	3.763	1.763	0.000	0.546	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.158	2.002	0.000	0.530	0.000	0.000	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	265	0	0	0	0	0	-1
normalized size	1	1.00	2.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	2.579	1.916	0.000	0.646	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.124	180.000	0.000	0.452	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.139	9.864	1.998	0.000	0.500	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	93	116	95	95	0	98	147
normalized size	1	1.00	1.33	1.66	1.36	1.36	0.00	1.40	2.10
time (sec)	N/A	0.051	0.075	0.567	0.518	0.514	0.000	1.588	14.406

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	48	75	62	76	0	64	79
normalized size	1	1.00	1.14	1.79	1.48	1.81	0.00	1.52	1.88
time (sec)	N/A	0.034	0.028	0.294	1.104	0.518	0.000	1.390	12.294

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	47	44	46	44	0	48	32
normalized size	1	1.00	1.68	1.57	1.64	1.57	0.00	1.71	1.14
time (sec)	N/A	0.033	0.029	0.374	0.319	0.557	0.000	1.492	11.872

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	44	36	29	30	0	36	47
normalized size	1	1.00	1.38	1.12	0.91	0.94	0.00	1.12	1.47
time (sec)	N/A	0.035	0.017	0.631	0.311	0.456	0.000	1.480	12.095

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	72	47	47	0	0	71
normalized size	1	1.00	0.96	1.33	0.87	0.87	0.00	0.00	1.31
time (sec)	N/A	0.050	0.190	0.801	0.712	0.549	0.000	0.000	12.069

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	92	64	63	0	0	95
normalized size	1	1.00	0.99	1.21	0.84	0.83	0.00	0.00	1.25
time (sec)	N/A	0.059	0.310	0.719	0.438	0.543	0.000	0.000	11.998

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	75	94	56	74	0	70	56
normalized size	1	1.00	1.10	1.38	0.82	1.09	0.00	1.03	0.82
time (sec)	N/A	0.051	0.264	0.615	0.340	0.466	0.000	1.889	12.013

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	53	66	39	56	0	48	40
normalized size	1	1.00	1.15	1.43	0.85	1.22	0.00	1.04	0.87
time (sec)	N/A	0.041	0.146	0.586	0.308	0.487	0.000	1.519	11.937

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	33	25	37	36	25	25
normalized size	1	1.00	1.00	1.18	0.89	1.32	1.29	0.89	0.89
time (sec)	N/A	0.029	0.012	0.496	0.502	0.503	1.745	1.739	11.862

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	54	39	30	0	169	32
normalized size	1	1.00	0.97	1.64	1.18	0.91	0.00	5.12	0.97
time (sec)	N/A	0.039	0.058	0.339	0.555	0.512	0.000	1.497	11.942

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	46	81	69	49	0	0	67
normalized size	1	1.00	0.75	1.33	1.13	0.80	0.00	0.00	1.10
time (sec)	N/A	0.046	0.131	0.586	1.688	0.445	0.000	0.000	12.064

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	74	102	97	66	0	0	93
normalized size	1	1.00	0.85	1.17	1.11	0.76	0.00	0.00	1.07
time (sec)	N/A	0.055	0.212	0.737	1.038	0.530	0.000	0.000	12.540

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	875	248	156	137	0	167	269
normalized size	1	1.00	6.84	1.94	1.22	1.07	0.00	1.30	2.10
time (sec)	N/A	0.165	8.333	0.612	0.569	0.533	0.000	2.946	15.643

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	347	178	119	116	0	120	177
normalized size	1	1.00	3.61	1.85	1.24	1.21	0.00	1.25	1.84
time (sec)	N/A	0.086	7.334	0.378	0.437	0.459	0.000	2.697	14.568

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	66	125	105	106	0	104	148
normalized size	1	1.00	1.06	2.02	1.69	1.71	0.00	1.68	2.39
time (sec)	N/A	0.090	0.460	0.400	0.470	0.467	0.000	2.597	14.313

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	71	104	72	79	0	96	136
normalized size	1	1.00	1.27	1.86	1.29	1.41	0.00	1.71	2.43
time (sec)	N/A	0.064	0.430	0.615	0.650	0.512	0.000	3.017	14.043

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	52	89	56	71	0	0	119
normalized size	1	1.00	0.91	1.56	0.98	1.25	0.00	0.00	2.09
time (sec)	N/A	0.060	0.164	0.849	0.381	0.490	0.000	0.000	12.195

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	153	81	95	0	0	160
normalized size	1	1.00	0.90	1.78	0.94	1.10	0.00	0.00	1.86
time (sec)	N/A	0.080	0.377	0.984	0.314	0.509	0.000	0.000	12.269

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	116	183	104	117	0	0	188
normalized size	1	1.00	1.02	1.61	0.91	1.03	0.00	0.00	1.65
time (sec)	N/A	0.099	0.618	1.062	0.492	0.514	0.000	0.000	12.396

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	106	157	85	114	0	118	80
normalized size	1	1.00	1.10	1.64	0.89	1.19	0.00	1.23	0.83
time (sec)	N/A	0.084	0.389	0.765	0.490	0.507	0.000	3.139	12.223

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	83	111	66	94	0	80	60
normalized size	1	1.00	1.12	1.50	0.89	1.27	0.00	1.08	0.81
time (sec)	N/A	0.069	0.556	0.647	0.621	0.555	0.000	3.991	12.281

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	57	42	69	0	42	40
normalized size	1	1.00	1.00	1.16	0.86	1.41	0.00	0.86	0.82
time (sec)	N/A	0.054	0.145	0.638	0.502	0.510	0.000	2.753	12.134

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	111	66	69	0	594	91
normalized size	1	1.00	1.00	2.02	1.20	1.25	0.00	10.80	1.65
time (sec)	N/A	0.077	0.421	0.561	0.760	0.625	0.000	3.360	12.250

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	65	122	97	75	0	0	93
normalized size	1	1.00	0.75	1.40	1.11	0.86	0.00	0.00	1.07
time (sec)	N/A	0.085	0.331	0.789	0.757	0.529	0.000	0.000	12.239

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	87	166	131	98	0	0	126
normalized size	1	1.00	0.71	1.36	1.07	0.80	0.00	0.00	1.03
time (sec)	N/A	0.131	0.374	0.832	0.498	0.486	0.000	0.000	13.237

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	207	224	0	292	0	131	268
normalized size	1	1.00	2.30	2.49	0.00	3.24	0.00	1.46	2.98
time (sec)	N/A	0.143	1.382	0.708	0.000	0.496	0.000	2.962	13.761

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	111	0	169	0	88	67
normalized size	1	1.00	0.90	1.88	0.00	2.86	0.00	1.49	1.14
time (sec)	N/A	0.081	0.107	0.607	0.000	0.489	0.000	1.838	12.668

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	36	0	122	0	47	32
normalized size	1	1.00	1.00	0.90	0.00	3.05	0.00	1.18	0.80
time (sec)	N/A	0.046	0.048	0.550	0.000	0.437	0.000	1.726	12.571

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	61	0	182	0	73	61
normalized size	1	1.00	1.00	1.02	0.00	3.03	0.00	1.22	1.02
time (sec)	N/A	0.080	0.102	0.552	0.000	0.460	0.000	2.119	12.321

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	115	98	0	276	0	161	251
normalized size	1	1.00	1.31	1.11	0.00	3.14	0.00	1.83	2.85
time (sec)	N/A	0.122	0.510	0.795	0.000	0.498	0.000	1.541	15.436

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	148	165	0	395	0	319	1493
normalized size	1	1.00	1.17	1.31	0.00	3.13	0.00	2.53	11.85
time (sec)	N/A	0.148	1.821	0.832	0.000	0.550	0.000	1.504	15.077

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	103	206	110	425	0	151	136
normalized size	1	1.00	0.95	1.91	1.02	3.94	0.00	1.40	1.26
time (sec)	N/A	0.110	0.903	0.685	0.508	0.562	0.000	1.429	12.293

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	74	127	69	339	0	96	90
normalized size	1	1.00	0.96	1.65	0.90	4.40	0.00	1.25	1.17
time (sec)	N/A	0.091	0.337	0.703	0.665	0.492	0.000	1.460	12.229

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	66	45	267	0	62	44
normalized size	1	1.00	1.00	1.27	0.87	5.13	0.00	1.19	0.85
time (sec)	N/A	0.067	0.145	0.587	0.742	0.555	0.000	1.382	12.396

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	23	205	0	40	24
normalized size	1	1.00	1.00	0.75	0.72	6.41	0.00	1.25	0.75
time (sec)	N/A	0.053	0.056	0.591	0.469	0.537	0.000	1.604	12.503

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	137	95	290	0	110	254
normalized size	1	1.00	0.94	1.65	1.14	3.49	0.00	1.33	3.06
time (sec)	N/A	0.104	0.188	0.648	0.438	0.560	0.000	1.525	13.894

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	113	303	185	401	0	183	3681
normalized size	1	1.00	0.88	2.35	1.43	3.11	0.00	1.42	28.53
time (sec)	N/A	0.167	0.423	0.671	0.645	0.565	0.000	1.456	15.867

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	254	389	0	635	0	245	4304
normalized size	1	1.00	1.52	2.33	0.00	3.80	0.00	1.47	25.77
time (sec)	N/A	0.267	4.082	0.737	0.000	0.611	0.000	2.258	15.237

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	191	236	0	407	0	153	946
normalized size	1	1.00	1.75	2.17	0.00	3.73	0.00	1.40	8.68
time (sec)	N/A	0.141	0.856	0.767	0.000	0.659	0.000	2.105	14.371

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	75	80	0	266	0	91	187
normalized size	1	1.00	0.95	1.01	0.00	3.37	0.00	1.15	2.37
time (sec)	N/A	0.080	0.246	0.683	0.000	0.662	0.000	2.333	12.937

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	92	102	0	337	0	112	239
normalized size	1	1.00	0.98	1.09	0.00	3.59	0.00	1.19	2.54
time (sec)	N/A	0.083	0.253	0.593	0.000	0.594	0.000	1.852	12.777

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	119	118	0	451	0	152	269
normalized size	1	1.00	1.04	1.04	0.00	3.96	0.00	1.33	2.36
time (sec)	N/A	0.181	0.555	0.616	0.000	0.569	0.000	1.894	15.552

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	147	164	0	600	0	329	1690
normalized size	1	1.00	1.03	1.15	0.00	4.20	0.00	2.30	11.82
time (sec)	N/A	0.213	1.616	0.812	0.000	0.700	0.000	2.547	16.012

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	135	275	137	597	0	180	167
normalized size	1	1.00	1.06	2.17	1.08	4.70	0.00	1.42	1.31
time (sec)	N/A	0.142	0.793	0.919	0.809	0.624	0.000	2.288	12.175

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	181	100	479	0	128	119
normalized size	1	1.00	1.00	1.74	0.96	4.61	0.00	1.23	1.14
time (sec)	N/A	0.136	0.660	0.652	0.937	0.537	0.000	2.445	12.414

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	83	112	69	367	0	92	65
normalized size	1	1.00	1.08	1.45	0.90	4.77	0.00	1.19	0.84
time (sec)	N/A	0.076	0.310	0.675	0.571	0.546	0.000	2.093	12.192

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	63	57	53	327	0	70	54
normalized size	1	1.00	0.95	0.86	0.80	4.95	0.00	1.06	0.82
time (sec)	N/A	0.062	0.281	0.641	0.477	0.527	0.000	1.997	12.140

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	116	248	209	614	0	211	3843
normalized size	1	1.00	0.78	1.68	1.41	4.15	0.00	1.43	25.97
time (sec)	N/A	0.186	1.254	0.767	1.047	0.573	0.000	2.181	16.295

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	148	413	355	801	0	269	5272
normalized size	1	1.00	0.70	1.95	1.67	3.78	0.00	1.27	24.87
time (sec)	N/A	0.301	2.121	0.946	0.766	0.595	0.000	2.414	17.275

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	81	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.186	2.845	0.000	0.422	0.000	0.000	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	2033	0	0	0	0	0	-1
normalized size	1	1.00	18.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	16.230	2.873	0.000	0.533	0.000	0.000	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	89	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.183	1.252	0.000	0.474	0.000	0.000	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	122	0	106	107	0	0	-1
normalized size	1	1.00	1.23	0.00	1.07	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.135	2.180	2.179	0.740	0.447	0.000	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	87	0	71	75	0	0	-1
normalized size	1	1.00	1.34	0.00	1.09	1.15	0.00	0.00	-0.02
time (sec)	N/A	0.111	2.129	2.143	0.758	0.510	0.000	0.000	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	35	49	0	0	31
normalized size	1	1.00	1.00	0.00	1.13	1.58	0.00	0.00	1.00
time (sec)	N/A	0.093	0.024	180.000	0.890	0.546	0.000	0.000	13.291

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.038	0.022	0.000	0.567	0.000	0.000	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-1)	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	1060	0	0	0	0	0	-1
normalized size	1	1.00	17.38	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.104	5.505	180.000	0.000	0.558	0.000	0.000	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	81	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.113	2.065	0.000	0.651	0.000	0.000	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	70	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.082	11.997	0.000	0.496	0.000	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	482	0	0	0	0	0	-1
normalized size	1	1.00	6.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	3.608	15.824	0.000	0.471	0.000	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	1552	0	0	0	0	0	-1
normalized size	1	1.00	18.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	6.372	24.817	0.000	0.459	0.000	0.000	0.000

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.056	2.845	5.478	0.000	0.502	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	6.081	1.730	0.000	0.585	0.000	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	1.872	1.287	0.000	0.526	0.000	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	3.482	1.440	0.000	0.545	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	11.873	4.514	0.000	0.523	0.000	0.000	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	165	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.191	3.132	1.803	0.000	0.502	0.000	0.000	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	122	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	4.115	1.819	0.000	0.488	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0	76
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.01
time (sec)	N/A	0.077	0.127	1.608	0.000	0.485	0.000	0.000	14.046

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.014	0.807	1.462	0.000	0.442	0.000	0.000	0.000

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	8.622	2.767	0.000	0.523	0.000	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	299	0	0	0	0	0	-1
normalized size	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	2.020	3.359	0.000	0.500	0.000	0.000	0.000

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	292	0	0	0	0	0	-1
normalized size	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	3.674	2.711	0.000	0.701	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	319	0	0	0	0	0	-1
normalized size	1	1.00	3.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	2.187	1.373	0.000	0.505	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.132	2.908	5.107	0.000	0.630	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [379] had the largest ratio of [.9286]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	14	0.214
2	A	3	3	1.00	14	0.214
3	A	2	2	1.00	14	0.143
4	A	2	2	1.00	14	0.143
5	A	3	3	1.00	14	0.214
6	A	4	3	1.00	14	0.214
7	A	16	10	1.00	14	0.714
8	A	14	10	1.00	14	0.714
9	A	13	10	1.00	14	0.714
10	A	13	10	1.00	14	0.714
11	A	14	10	1.00	14	0.714
12	A	16	10	1.00	14	0.714
13	A	7	3	1.00	14	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
14	A	5	3	1.00	14	0.214
15	A	3	3	1.00	14	0.214
16	A	3	3	1.00	14	0.214
17	A	5	3	1.00	14	0.214
18	A	7	3	1.00	14	0.214
19	A	3	3	1.00	14	0.214
20	A	3	3	1.00	14	0.214
21	A	3	3	1.00	14	0.214
22	A	3	3	1.00	14	0.214
23	A	3	3	1.00	14	0.214
24	A	3	3	1.00	14	0.214
25	A	3	3	1.00	12	0.250
26	A	3	3	1.00	12	0.250
27	A	3	3	1.00	12	0.250
28	A	3	3	1.00	12	0.250
29	A	2	2	1.00	14	0.143
30	A	3	2	1.00	21	0.095
31	A	3	2	1.00	21	0.095
32	A	3	2	1.00	19	0.105
33	A	3	3	1.00	19	0.158
34	A	4	4	1.00	21	0.190
35	A	5	5	1.00	21	0.238
36	A	6	6	1.00	21	0.286
37	A	5	5	1.00	21	0.238
38	A	4	4	1.00	21	0.190
39	A	3	2	1.00	12	0.167
40	A	3	2	1.00	21	0.095
41	A	3	2	1.00	21	0.095
42	A	3	2	1.00	21	0.095
43	A	3	2	1.00	23	0.087
44	A	3	2	1.00	23	0.087
45	A	3	2	1.00	21	0.095
46	A	4	3	1.00	21	0.143
47	A	5	5	1.00	23	0.217
48	A	6	5	1.00	23	0.217
49	A	6	5	1.00	23	0.217

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
50	A	5	5	1.00	23	0.217
51	A	4	3	1.00	14	0.214
52	A	3	2	1.00	23	0.087
53	A	3	2	1.00	23	0.087
54	A	3	2	1.00	23	0.087
55	A	4	3	1.00	23	0.130
56	A	4	4	1.00	23	0.174
57	A	3	3	1.00	21	0.143
58	A	4	4	1.00	21	0.190
59	A	5	5	1.00	23	0.217
60	A	6	6	1.00	23	0.261
61	A	7	7	1.00	23	0.304
62	A	6	6	1.00	23	0.261
63	A	5	5	1.00	23	0.217
64	A	3	3	1.00	14	0.214
65	A	3	3	1.00	23	0.130
66	A	4	4	1.00	23	0.174
67	A	4	3	1.00	23	0.130
68	A	6	5	1.00	23	0.217
69	A	5	4	1.00	23	0.174
70	A	4	4	1.00	21	0.190
71	A	5	5	1.00	21	0.238
72	A	6	6	1.00	23	0.261
73	A	7	6	1.00	23	0.261
74	A	7	6	1.00	23	0.261
75	A	6	6	1.00	23	0.261
76	A	5	5	1.00	14	0.357
77	A	4	4	1.00	23	0.174
78	A	5	4	1.00	23	0.174
79	A	6	5	1.00	23	0.217
80	A	7	6	1.00	23	0.261
81	A	6	5	1.00	23	0.217
82	A	5	4	1.00	21	0.190
83	A	6	6	1.00	21	0.286
84	A	7	6	1.00	23	0.261
85	A	8	6	1.00	23	0.261

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	8	6	1.00	23	0.261
87	A	7	6	1.00	23	0.261
88	A	6	6	1.00	14	0.429
89	A	5	4	1.00	23	0.174
90	A	6	5	1.00	23	0.217
91	A	7	6	1.00	23	0.261
92	A	6	6	1.00	25	0.240
93	A	5	5	1.00	25	0.200
94	A	4	4	1.00	23	0.174
95	A	6	6	1.00	23	0.261
96	A	7	7	1.00	25	0.280
97	A	8	8	1.00	25	0.320
98	A	8	8	1.00	25	0.320
99	A	7	7	1.00	25	0.280
100	A	6	6	1.00	16	0.375
101	A	4	4	1.00	25	0.160
102	A	5	5	1.00	25	0.200
103	A	6	6	1.00	25	0.240
104	A	7	7	1.00	25	0.280
105	A	6	6	1.00	25	0.240
106	A	5	5	1.00	23	0.217
107	A	7	7	1.00	23	0.304
108	A	8	8	1.00	25	0.320
109	A	9	9	1.00	25	0.360
110	A	9	9	1.00	25	0.360
111	A	8	8	1.00	25	0.320
112	A	7	7	1.00	16	0.438
113	A	5	5	1.00	25	0.200
114	A	6	6	1.00	25	0.240
115	A	7	7	1.00	25	0.280
116	A	4	4	1.00	25	0.160
117	A	3	3	1.00	25	0.120
118	A	2	2	1.00	23	0.087
119	A	3	3	1.00	23	0.130
120	A	5	5	1.00	25	0.200
121	A	6	6	1.00	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
122	A	6	6	1.00	25	0.240
123	A	5	5	1.00	25	0.200
124	A	3	3	1.00	16	0.188
125	A	2	2	1.00	25	0.080
126	A	3	3	1.00	25	0.120
127	A	4	4	1.00	25	0.160
128	A	5	5	1.00	25	0.200
129	A	4	4	1.00	25	0.160
130	A	3	3	1.00	23	0.130
131	A	4	4	1.00	23	0.174
132	A	6	6	1.00	25	0.240
133	A	7	6	1.00	25	0.240
134	A	7	6	1.00	25	0.240
135	A	6	6	1.00	25	0.240
136	A	4	4	1.00	16	0.250
137	A	3	3	1.00	25	0.120
138	A	4	4	1.00	25	0.160
139	A	5	5	1.00	25	0.200
140	A	6	6	1.00	25	0.240
141	A	5	5	1.00	25	0.200
142	A	4	4	1.00	23	0.174
143	A	6	6	1.00	23	0.261
144	A	7	6	1.00	25	0.240
145	A	8	6	1.00	25	0.240
146	A	8	6	1.00	25	0.240
147	A	7	6	1.00	25	0.240
148	A	6	6	1.00	16	0.375
149	A	4	4	1.00	25	0.160
150	A	5	5	1.00	25	0.200
151	A	6	6	1.00	25	0.240
152	A	3	3	1.00	23	0.130
153	A	3	3	1.00	25	0.120
154	A	5	5	1.00	23	0.217
155	A	4	4	1.00	23	0.174
156	A	3	3	1.00	21	0.143
157	A	3	3	1.00	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
158	A	3	3	1.00	23	0.130
159	A	3	3	1.00	23	0.130
160	A	3	3	1.00	14	0.214
161	A	3	3	1.00	23	0.130
162	A	4	4	1.00	23	0.174
163	A	5	5	0.98	23	0.217
164	A	3	3	1.00	25	0.120
165	A	3	3	1.00	23	0.130
166	A	3	3	1.00	14	0.214
167	A	3	3	1.00	23	0.130
168	A	4	3	1.00	23	0.130
169	A	4	3	1.00	23	0.130
170	A	3	3	1.00	23	0.130
171	A	3	3	1.00	21	0.143
172	A	3	3	1.00	21	0.143
173	A	3	3	1.00	23	0.130
174	A	0	0	0.00	0	0.000
175	A	3	3	1.00	23	0.130
176	A	4	4	1.00	25	0.160
177	A	3	3	1.00	25	0.120
178	A	0	0	0.00	0	0.000
179	A	4	3	1.00	14	0.214
180	A	4	3	1.00	14	0.214
181	A	4	3	1.00	14	0.214
182	A	4	4	1.00	14	0.286
183	A	5	4	1.00	14	0.286
184	A	6	4	1.00	14	0.286
185	A	4	3	1.00	21	0.143
186	A	3	3	1.00	21	0.143
187	A	2	2	1.00	19	0.105
188	A	3	2	1.00	19	0.105
189	A	3	3	1.00	21	0.143
190	A	4	4	1.00	21	0.190
191	A	5	3	1.00	21	0.143
192	A	4	3	1.00	21	0.143
193	A	3	3	1.00	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
194	A	3	2	1.00	12	0.167
195	A	2	2	1.00	21	0.095
196	A	4	4	1.00	21	0.190
197	A	5	4	1.00	21	0.190
198	A	4	3	1.00	23	0.130
199	A	4	3	1.00	23	0.130
200	A	4	3	1.00	21	0.143
201	A	4	3	1.00	21	0.143
202	A	4	3	1.00	23	0.130
203	A	4	3	1.00	23	0.130
204	A	4	3	1.00	23	0.130
205	A	4	3	1.00	23	0.130
206	A	4	3	1.00	23	0.130
207	A	4	3	1.00	14	0.214
208	A	4	3	1.00	23	0.130
209	A	4	3	1.00	23	0.130
210	A	4	3	1.00	23	0.130
211	A	4	3	1.00	23	0.130
212	A	4	3	1.00	23	0.130
213	A	5	4	1.00	21	0.190
214	A	4	3	1.00	21	0.143
215	A	4	3	1.00	23	0.130
216	A	4	3	1.00	23	0.130
217	A	6	6	1.00	23	0.261
218	A	5	5	1.00	23	0.217
219	A	4	4	1.00	23	0.174
220	A	3	3	1.00	14	0.214
221	A	5	5	1.00	23	0.217
222	A	6	6	1.00	23	0.261
223	A	7	6	1.00	23	0.261
224	A	4	3	1.00	23	0.130
225	A	4	3	1.00	23	0.130
226	A	4	3	1.00	21	0.143
227	A	4	3	1.00	21	0.143
228	A	4	3	1.00	23	0.130
229	A	4	3	1.00	23	0.130

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
230	A	6	6	1.00	23	0.261
231	A	5	5	1.00	23	0.217
232	A	5	5	1.00	23	0.217
233	A	5	5	1.00	14	0.357
234	A	6	6	1.00	23	0.261
235	A	7	6	1.00	23	0.261
236	A	8	6	1.00	23	0.261
237	A	4	3	1.00	23	0.130
238	A	4	3	1.00	23	0.130
239	A	4	3	1.00	21	0.143
240	A	4	3	1.00	21	0.143
241	A	4	3	1.00	23	0.130
242	A	4	3	1.00	23	0.130
243	A	6	6	1.00	23	0.261
244	A	6	6	1.00	23	0.261
245	A	6	6	1.00	23	0.261
246	A	6	6	1.00	14	0.429
247	A	7	7	1.00	23	0.304
248	A	8	7	1.00	23	0.304
249	A	9	7	1.00	23	0.304
250	A	4	3	1.00	14	0.214
251	A	4	3	1.00	14	0.214
252	A	4	3	1.00	14	0.214
253	A	3	2	1.00	12	0.167
254	A	3	3	1.00	14	0.214
255	A	5	5	1.00	14	0.357
256	A	6	6	1.00	14	0.429
257	A	5	4	1.00	17	0.235
258	A	4	3	1.00	17	0.176
259	A	4	4	1.00	17	0.235
260	A	3	3	1.00	15	0.200
261	A	4	4	1.00	15	0.267
262	A	4	4	1.00	17	0.235
263	A	5	5	1.00	17	0.294
264	A	4	3	1.00	17	0.176
265	A	4	4	1.00	16	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
266	A	4	3	1.00	17	0.176
267	A	5	5	1.00	17	0.294
268	A	3	3	1.00	15	0.200
269	A	5	5	1.00	15	0.333
270	A	5	5	1.00	17	0.294
271	A	5	5	1.00	16	0.312
272	A	6	5	1.00	16	0.312
273	A	4	3	1.00	17	0.176
274	A	5	5	1.00	17	0.294
275	A	3	3	1.00	15	0.200
276	A	5	5	1.00	15	0.333
277	A	5	4	1.00	17	0.235
278	A	4	3	1.00	17	0.176
279	A	4	4	1.00	17	0.235
280	A	3	3	1.00	15	0.200
281	A	6	5	1.00	15	0.333
282	A	5	4	1.00	17	0.235
283	A	3	3	1.00	16	0.188
284	A	4	4	1.00	16	0.250
285	A	5	4	1.00	16	0.250
286	A	6	4	1.00	16	0.250
287	A	4	4	1.00	10	0.400
288	A	3	3	1.00	10	0.300
289	A	3	3	1.00	10	0.300
290	A	5	5	1.00	12	0.417
291	A	4	4	1.00	12	0.333
292	A	3	3	1.00	12	0.250
293	A	7	6	1.00	25	0.240
294	A	6	6	1.00	25	0.240
295	A	5	5	1.00	23	0.217
296	A	7	5	1.00	23	0.217
297	A	8	6	1.00	25	0.240
298	A	9	7	1.00	25	0.280
299	A	9	8	1.00	25	0.320
300	A	8	8	1.00	25	0.320
301	A	7	7	1.00	25	0.280

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
302	A	6	6	1.00	16	0.375
303	A	5	5	1.00	25	0.200
304	A	6	6	1.00	25	0.240
305	A	7	6	1.00	25	0.240
306	A	8	6	1.00	25	0.240
307	A	7	6	1.00	25	0.240
308	A	6	5	1.00	23	0.217
309	A	8	6	1.00	23	0.261
310	A	8	6	1.00	25	0.240
311	A	9	7	1.00	25	0.280
312	A	10	8	1.00	25	0.320
313	A	9	8	1.00	25	0.320
314	A	8	8	1.00	25	0.320
315	A	7	7	1.00	16	0.438
316	A	7	7	1.00	25	0.280
317	A	6	6	1.00	25	0.240
318	A	7	6	1.00	25	0.240
319	A	8	8	1.00	16	0.500
320	A	6	5	1.00	25	0.200
321	A	5	5	1.00	25	0.200
322	A	4	4	1.00	23	0.174
323	A	7	5	1.00	23	0.217
324	A	8	6	1.00	25	0.240
325	A	9	7	1.00	25	0.280
326	A	8	8	1.00	25	0.320
327	A	7	7	1.00	25	0.280
328	A	6	6	1.00	25	0.240
329	A	3	3	1.00	16	0.188
330	A	5	5	1.00	25	0.200
331	A	6	6	1.00	25	0.240
332	A	7	6	1.00	25	0.240
333	A	6	5	1.00	25	0.200
334	A	5	5	1.00	25	0.200
335	A	5	5	1.00	23	0.217
336	A	8	6	1.00	23	0.261
337	A	9	7	1.00	25	0.280

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
338	A	10	8	1.00	25	0.320
339	A	8	8	1.00	25	0.320
340	A	7	7	1.00	25	0.280
341	A	4	4	1.00	25	0.160
342	A	4	4	1.00	16	0.250
343	A	6	6	1.00	25	0.240
344	A	7	6	1.00	25	0.240
345	A	8	6	1.00	25	0.240
346	A	6	5	1.00	25	0.200
347	A	6	6	1.00	25	0.240
348	A	6	5	1.00	23	0.217
349	A	9	7	1.00	23	0.304
350	A	10	7	1.00	25	0.280
351	A	11	8	1.00	25	0.320
352	A	8	8	1.00	25	0.320
353	A	6	6	1.00	25	0.240
354	A	6	6	1.00	25	0.240
355	A	6	6	1.00	16	0.375
356	A	7	7	1.00	25	0.280
357	A	8	7	1.00	25	0.280
358	A	9	7	1.00	25	0.280
359	A	4	4	1.00	23	0.174
360	A	3	3	1.00	25	0.120
361	A	5	4	1.00	23	0.174
362	A	4	4	1.00	23	0.174
363	A	3	3	1.00	21	0.143
364	A	5	5	1.00	21	0.238
365	A	6	6	1.00	23	0.261
366	A	7	7	1.00	23	0.304
367	A	3	3	1.00	23	0.130
368	A	3	3	1.00	23	0.130
369	A	3	3	1.00	23	0.130
370	A	3	3	1.00	14	0.214
371	A	3	3	1.00	23	0.130
372	A	3	3	1.00	23	0.130
373	A	3	3	1.00	23	0.130

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	6	5	1.00	14	0.357
375	A	6	5	1.00	14	0.357
376	A	6	5	1.00	14	0.357
377	A	3	2	1.00	12	0.167
378	A	14	12	1.00	14	0.857
379	A	21	13	1.00	14	0.929
380	A	7	6	1.00	8	0.750
381	A	4	3	1.00	14	0.214
382	A	4	3	1.00	14	0.214
383	A	4	3	1.00	14	0.214
384	A	4	2	1.00	12	0.167
385	A	13	9	1.00	14	0.643
386	A	23	10	1.00	14	0.714
387	A	8	7	1.00	16	0.438
388	A	4	4	1.00	16	0.250
389	A	8	7	1.00	17	0.412
390	A	8	7	1.00	15	0.467
391	A	11	10	1.00	15	0.667
392	A	12	9	1.00	17	0.529
393	A	9	7	1.00	17	0.412
394	A	9	8	1.00	15	0.533
395	A	13	12	1.00	15	0.800
396	A	7	6	1.00	17	0.353
397	A	4	4	1.00	15	0.267
398	A	9	8	1.00	15	0.533
399	A	4	4	1.00	17	0.235
400	A	6	6	1.00	17	0.353
401	A	6	6	1.00	15	0.400
402	A	12	11	1.00	15	0.733
403	A	7	6	1.00	17	0.353
404	A	7	7	1.00	15	0.467
405	A	14	12	1.00	15	0.800
406	A	9	5	1.00	29	0.172
407	A	7	4	1.00	27	0.148
408	A	14	7	1.00	29	0.241
409	A	15	7	1.00	29	0.241

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
410	A	4	4	1.00	25	0.160
411	A	4	4	1.00	23	0.174
412	A	3	3	1.00	14	0.214
413	A	4	4	1.00	23	0.174
414	A	4	4	1.00	23	0.174
415	A	4	4	1.00	23	0.174
416	A	4	4	1.00	23	0.174
417	A	4	4	1.00	21	0.190
418	A	4	4	1.00	21	0.190
419	A	4	4	1.00	23	0.174
420	A	0	0	0.00	0	0.000
421	A	4	4	1.00	23	0.174
422	A	4	4	1.00	25	0.160
423	A	4	4	1.00	25	0.160
424	A	0	0	0.00	0	0.000
425	A	4	4	1.00	21	0.190
426	A	3	3	1.00	19	0.158
427	A	3	3	1.00	19	0.158
428	A	2	1	1.00	21	0.048
429	A	3	2	1.00	21	0.095
430	A	3	2	1.00	21	0.095
431	A	3	2	1.00	21	0.095
432	A	3	2	1.00	21	0.095
433	A	2	1	1.00	21	0.048
434	A	3	3	1.00	21	0.143
435	A	4	4	1.00	21	0.190
436	A	5	4	1.00	21	0.190
437	A	5	5	1.00	23	0.217
438	A	4	4	1.00	21	0.190
439	A	5	4	1.00	21	0.190
440	A	4	3	1.00	23	0.130
441	A	3	2	1.00	23	0.087
442	A	3	2	1.00	23	0.087
443	A	3	2	1.00	23	0.087
444	A	3	2	1.00	23	0.087
445	A	3	2	1.00	23	0.087

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	3	2	1.00	23	0.087
447	A	5	4	1.00	23	0.174
448	A	4	4	1.00	23	0.174
449	A	5	5	1.00	23	0.217
450	A	5	5	1.00	23	0.217
451	A	4	4	1.00	23	0.174
452	A	2	2	1.00	21	0.095
453	A	3	3	1.00	21	0.143
454	A	4	3	1.00	23	0.130
455	A	4	3	1.00	23	0.130
456	A	4	3	1.00	23	0.130
457	A	4	3	1.00	23	0.130
458	A	3	3	1.00	23	0.130
459	A	2	2	1.00	23	0.087
460	A	5	5	1.00	23	0.217
461	A	6	6	1.00	23	0.261
462	A	6	6	1.00	23	0.261
463	A	5	5	1.00	23	0.217
464	A	3	3	1.00	23	0.130
465	A	3	3	1.00	21	0.143
466	A	5	4	1.00	21	0.190
467	A	5	4	1.00	23	0.174
468	A	5	4	1.00	23	0.174
469	A	5	4	1.00	23	0.174
470	A	3	3	1.00	23	0.130
471	A	3	3	1.00	23	0.130
472	A	6	6	1.00	23	0.261
473	A	7	6	1.00	23	0.261
474	A	2	2	1.00	23	0.087
475	A	3	3	1.00	25	0.120
476	A	2	2	1.00	25	0.080
477	A	4	3	1.00	23	0.130
478	A	4	3	1.00	23	0.130
479	A	3	3	1.00	23	0.130
480	A	3	3	1.00	14	0.214
481	A	3	3	1.00	23	0.130

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
482	A	2	2	1.00	23	0.087
483	A	2	2	1.00	21	0.095
484	A	2	2	1.00	21	0.095
485	A	2	2	1.00	23	0.087
486	A	0	0	0.00	0	0.000
487	A	0	0	0.00	0	0.000
488	A	0	0	0.00	0	0.000
489	A	0	0	0.00	0	0.000
490	A	0	0	0.00	0	0.000
491	A	9	6	1.00	25	0.240
492	A	7	6	1.00	25	0.240
493	A	3	3	1.00	25	0.120
494	A	0	0	0.00	0	0.000
495	A	0	0	0.00	0	0.000
496	A	4	4	1.00	23	0.174
497	A	4	4	1.00	25	0.160
498	A	4	4	1.00	25	0.160
499	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int \left(b\tan^2(e+fx)\right)^{5/2}\,dx$

Optimal. Leaf size=98

$$-\frac{b^2\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}+\frac{b^2\tan^3(e+fx)\sqrt{b\tan^2(e+fx)}}{4f}-\frac{b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\log(\cos(e}{f}$$

```
[Out] -b^2*cot(f*x+e)*ln(cos(f*x+e))*(b*tan(f*x+e)^2)^(1/2)/f-1/2*b^2*(b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e)/f+1/4*b^2*(b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$\frac{b^2\tan^3(e+fx)\sqrt{b\tan^2(e+fx)}}{4f}-\frac{b^2\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}-\frac{b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\log(\cos(e+}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((b^2*Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f) - (b^2*Tan
[e + f*x]*Sqrt[b*Tan[e + f*x]^2])/(2*f) + (b^2*Tan[e + f*x]^3*Sqrt[b*Tan[e
+ f*x]^2])/(4*f)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^2(e+fx)\right)^{5/2} dx = \left(b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan^5(e+fx)\,dx$$
$$= \frac{b^2\tan^3(e+fx)\sqrt{b\tan^2(e+fx)}}{4f} - \left(b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan^3(e+fx)\,d$$
$$= -\frac{b^2\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f} + \frac{b^2\tan^3(e+fx)\sqrt{b\tan^2(e+fx)}}{4f} + \left(b^2\cot(e+fx\right.$$
$$= -\frac{b^2\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f} - \frac{b^2\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}$$

Mathematica [A] time = 0.36, size = 56, normalized size = 0.57

$$-\frac{\cot(e+fx)\left(b\tan^2(e+fx)\right)^{5/2}\left(2\cot^2(e+fx)+4\cot^4(e+fx)\log(\cos(e+fx))-1\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/4*(Cot[e + f*x]*(-1 + 2*Cot[e + f*x]^2 + 4*Cot[e + f*x]^4*Log[Cos[e + f*
x]])*(b*Tan[e + f*x]^2)^(5/2))/f
```

fricas [A] time = 0.74, size = 74, normalized size = 0.76

$$\frac{\left(b^2\tan\left(fx+e\right)^4 - 2\,b^2\tan\left(fx+e\right)^2 - 2\,b^2\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right) - 3\,b^2\right)\sqrt{b\tan\left(fx+e\right)^2}}{4\,f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*tan(f*x + e)^4 - 2*b^2*tan(f*x + e)^2 - 2*b^2*log(1/(tan(f*x + e)^
2 + 1)) - 3*b^2)*sqrt(b*tan(f*x + e)^2)/(f*tan(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
```

```
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)sqrt(b)*(-3*b^2*sign(tan(f*x+exp(1)))-
2*b^2*sign(tan(f*x+exp(1)))*tan(f*x)^2+b^2*sign(tan(f*x+exp(1)))*tan(f*x)^4
-2*b^2*sign(tan(f*x+exp(1)))*tan(exp(1))^2+b^2*sign(tan(f*x+exp(1)))*tan(ex
p(1))^4-2*b^2*sign(tan(f*x+exp(1)))*ln((4*tan(f*x)^2*tan(exp(1))^2-8*tan(f*
x)^3*tan(exp(1))+4*tan(f*x)^4*tan(exp(1))^2+4*tan(f*x)^2-8*tan(f*x)*tan(exp
(1))+4)/(tan(exp(1))^2+1))-4*b^2*sign(tan(f*x+exp(1)))*tan(f*x)^2*tan(exp(1
))^2-2*b^2*sign(tan(f*x+exp(1)))*tan(f*x)^2*tan(exp(1))^4+8*b^2*sign(tan(f*
x+exp(1)))*tan(f*x)^3*tan(exp(1))^3+8*b^2*sign(tan(f*x+exp(1)))*tan(f*x)^3*
tan(exp(1))-2*b^2*sign(tan(f*x+exp(1)))*tan(f*x)^4*tan(exp(1))^2-3*b^2*sign
(tan(f*x+exp(1)))*tan(f*x)^4*tan(exp(1))^4+8*b^2*sign(tan(f*x+exp(1)))*tan(
f*x)*tan(exp(1))^3+8*b^2*sign(tan(f*x+exp(1)))*tan(f*x)*tan(exp(1))-12*b^2*
sign(tan(f*x+exp(1)))*tan(f*x)^2*tan(exp(1))^2*ln((4*tan(f*x)^2*tan(exp(1))
^2-8*tan(f*x)^3*tan(exp(1))+4*tan(f*x)^4*tan(exp(1))^2+4*tan(f*x)^2-8*tan(f
*x)*tan(exp(1))+4)/(tan(exp(1))^2+1))+8*b^2*sign(tan(f*x+exp(1)))*tan(f*x)^
3*tan(exp(1))^3*ln((4*tan(f*x)^2*tan(exp(1))^2-8*tan(f*x)^3*tan(exp(1))+4*t
an(f*x)^4*tan(exp(1))^2+4*tan(f*x)^2-8*tan(f*x)*tan(exp(1))+4)/(tan(exp(1))
^2+1))-2*b^2*sign(tan(f*x+exp(1)))*tan(f*x)^4*tan(exp(1))^4*ln((4*tan(f*x)^
2*tan(exp(1))^2-8*tan(f*x)^3*tan(exp(1))+4*tan(f*x)^4*tan(exp(1))^2+4*tan(f
*x)^2-8*tan(f*x)*tan(exp(1))+4)/(tan(exp(1))^2+1))+8*b^2*sign(tan(f*x+exp(1
)))*tan(f*x)*tan(exp(1))*ln((4*tan(f*x)^2*tan(exp(1))^2-8*tan(f*x)^3*tan(ex
p(1))+4*tan(f*x)^4*tan(exp(1))^2+4*tan(f*x)^2-8*tan(f*x)*tan(exp(1))+4)/(ta
n(exp(1))^2+1)))/(4*f*tan(f*x)^4*tan(exp(1))^4-16*f*tan(f*x)^3*tan(exp(1))^
3+24*f*tan(f*x)^2*tan(exp(1))^2-16*f*tan(f*x)*tan(exp(1))+4*f)
```

maple [A] time = 0.29, size = 58, normalized size = 0.59

$$\frac{\left(b\left(\tan^{2}\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(\tan^{4}\left(fx+e\right)-2\left(\tan^{2}\left(fx+e\right)\right)+2\ln\left(1+\tan^{2}\left(fx+e\right)\right)\right)}{4f\tan\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/4/f*(b*tan(f*x+e)^2)^(5/2)*(tan(f*x+e)^4-2*tan(f*x+e)^2+2*ln(1+tan(f*x+e)
^2))/tan(f*x+e)^5
```

maxima [A] time = 0.61, size = 47, normalized size = 0.48

$$\frac{b^{\frac{5}{2}}\tan\left(fx+e\right)^{4}-2\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{2}+2\,b^{\frac{5}{2}}\log\left(\tan\left(fx+e\right)^{2}+1\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b^(5/2)*tan(f*x + e)^4 - 2*b^(5/2)*tan(f*x + e)^2 + 2*b^(5/2)*log(tan(
f*x + e)^2 + 1))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int((b*tan(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**(5/2), x)
```

3.2 $\int \left(b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=61

$$\frac{b\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}+\frac{b\cot(e+fx)\sqrt{b\tan^2(e+fx)}\,\log(\cos(e+fx))}{f}$$

```
[Out] b*cot(f*x+e)*ln(cos(f*x+e))*(b*tan(f*x+e)^2)^(1/2)/f+1/2*b*(b*tan(f*x+e)^2)
^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$\frac{b\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}+\frac{b\cot(e+fx)\sqrt{b\tan^2(e+fx)}\,\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f + (b*Tan[e + f*
x]*Sqrt[b*Tan[e + f*x]^2])/(2*f)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(b\tan^2(e+fx)\right)^{3/2}\,dx &= \left(b\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan^3(e+fx)\,dx\\ &= \frac{b\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}-\left(b\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan(e+fx)\,dx\\ &= \frac{b\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f}+\frac{b\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}\end{aligned}$$

Mathematica [A] time = 0.11, size = 47, normalized size = 0.77

$$\frac{\cot^3(e+fx)\left(b\tan^2(e+fx)\right)^{3/2}\left(\tan^2(e+fx)+2\log(\cos(e+fx))\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cot[e + f*x]^3*(b*Tan[e + f*x]^2)^(3/2)*(2*Log[Cos[e + f*x]] + Tan[e + f*x
]^2))/(2*f)
```

fricas [A] time = 0.40, size = 52, normalized size = 0.85

$$\frac{\left(b\tan\left(fx+e\right)^2+b\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)+b\right)\sqrt{b\tan\left(fx+e\right)^2}}{2\,f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*tan(f*x + e)^2 + b*log(1/(tan(f*x + e)^2 + 1)) + b)*sqrt(b*tan(f*x +
 e)^2)/(f*tan(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)sqrt(b)*b*(tan(f*x)^2*tan(exp(1))^2+
tan(f*x)^2+tan(exp(1))^2+tan(f*x)^2*tan(exp(1))^2*ln((4*tan(f*x)^2*tan(exp(
1))^2-8*tan(f*x)^3*tan(exp(1))+4*tan(f*x)^4*tan(exp(1))^2+4*tan(f*x)^2-8*ta
n(f*x)*tan(exp(1))+4)/(tan(exp(1))^2+1))-2*tan(f*x)*tan(exp(1))*ln((4*tan(f
*x)^2*tan(exp(1))^2-8*tan(f*x)^3*tan(exp(1))+4*tan(f*x)^4*tan(exp(1))^2+4*t
an(f*x)^2-8*tan(f*x)*tan(exp(1))+4)/(tan(exp(1))^2+1))+ln((-8*tan(f*x)^3*ta
n(exp(1))+4*tan(f*x)^4*tan(exp(1))^2+4*tan(f*x)^2*tan(exp(1))^2+4*tan(f*x)^
2-8*tan(f*x)*tan(exp(1))+4)/(tan(exp(1))^2+1))+1)*sign(tan(f*x+exp(1))))/(2*
f*tan(f*x)^2*tan(exp(1))^2-4*f*tan(f*x)*tan(exp(1))+2*f)
```

maple [A] time = 0.26, size = 48, normalized size = 0.79

$$-\frac{\left(b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(-\left(\tan^2\left(fx+e\right)\right)+\ln\left(1+\tan^2\left(fx+e\right)\right)\right)}{2f\tan\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f*(b*tan(f*x+e)^2)^(3/2)*(-tan(f*x+e)^2+ln(1+tan(f*x+e)^2))/tan(f*x+e)
^3
```

maxima [A] time = 0.83, size = 34, normalized size = 0.56

$$\frac{b^{\frac{3}{2}}\tan\left(fx+e\right)^2-b^{\frac{3}{2}}\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^(3/2)*tan(f*x + e)^2 - b^(3/2)*log(tan(f*x + e)^2 + 1))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\tan\left(e+f\,x\right)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int((b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**(3/2), x)
```

3.3 $\int \sqrt{b \tan^2(e+fx)}\, dx$

Optimal. Leaf size=32

$$-\frac{\cot(e+fx)\sqrt{b\tan^2(e+fx)}\,\log(\cos(e+fx))}{f}$$

```
[Out] -cot(f*x+e)*ln(cos(f*x+e))*(b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3658, 3475}

$$-\frac{\cot(e+fx)\sqrt{b\tan^2(e+fx)}\,\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\tan^2(e+fx)}\, dx &= \left(\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan(e+fx)\, dx\\ &= -\frac{\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f}\end{aligned}$$

Mathematica [A] time = 0.04, size = 32, normalized size = 1.00

$$-\frac{\cot(e+fx)\sqrt{b\tan^2(e+fx)}\,\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f)
```

fricas [A] time = 0.41, size = 38, normalized size = 1.19

$$-\frac{\sqrt{b\tan\left(fx+e\right)^{2}}\log\left(\frac{1}{\tan\left(fx+e\right)^{2}+1}\right)}{2\,f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*tan(f*x + e)^2)*log(1/(tan(f*x + e)^2 + 1))/(f*tan(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)-sqrt(b)*sign(tan(f*x+exp(1)))*ln(abs(cos(f*x+exp(1))))/f
```

maple [A] time = 0.32, size = 37, normalized size = 1.16

$$\frac{\sqrt{b\left(\tan^{2}\left(fx+e\right)\right)}\,\ln\left(1+\tan^{2}\left(fx+e\right)\right)}{2f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f*(b*tan(f*x+e)^2)^(1/2)/tan(f*x+e)*ln(1+tan(f*x+e)^2)
```

maxima [A] time = 0.69, size = 19, normalized size = 0.59

$$\frac{\sqrt{b}\log\left(\tan\left(fx+e\right)^{2}+1\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b)*log(tan(f*x + e)^2 + 1)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{b\tan\left(e+f\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int((b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x)**2), x)
```

3.4 $\int \frac{1}{\sqrt{b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=31

$$\frac{\tan(e+fx)\log(\sin(e+fx))}{f\sqrt{b\tan^2(e+fx)}}$$

```
[Out] ln(sin(f*x+e))*tan(f*x+e)/f/(b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3658, 3475}

$$\frac{\tan(e+fx)\log(\sin(e+fx))}{f\sqrt{b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[e + f*x]^2],x]
```

```
[Out] (Log[Sin[e + f*x]]*Tan[e + f*x])/(f*Sqrt[b*Tan[e + f*x]^2])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\tan^2(e+fx)}}\,dx &= \frac{\tan(e+fx)\int \cot(e+fx)\,dx}{\sqrt{b\tan^2(e+fx)}}\\ &= \frac{\log(\sin(e+fx))\tan(e+fx)}{f\sqrt{b\tan^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 39, normalized size = 1.26

$$\frac{\tan(e+fx)(\log(\tan(e+fx))+\log(\cos(e+fx)))}{f\sqrt{b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[e + f*x]^2],x]
```

```
[Out] ((Log[Cos[e + f*x]] + Log[Tan[e + f*x]])*Tan[e + f*x])/(f*Sqrt[b*Tan[e + f*
x]^2])
```

fricas [A] time = 0.41, size = 50, normalized size = 1.61

$$\frac{\sqrt{b\tan\left(fx+e\right)^{2}}\log\left(\frac{\tan\left(fx+e\right)^{2}}{\tan\left(fx+e\right)^{2}+1}\right)}{2\,bf\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*tan(f*x + e)^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))/(b*f*ta
n(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*(1/4*sqr
t(b)*ln(abs(-cos(f*x+exp(1))+1)/abs(cos(f*x+exp(1))+1))/b/sign(tan(f*x+exp(
1)))-1/2*sqrt(b)*ln(abs((-cos(f*x+exp(1))+1)/(cos(f*x+exp(1))+1)+1))/b/sign
(tan(f*x+exp(1))))/f
```

maple [A] time = 0.41, size = 47, normalized size = 1.52

$$\frac{\tan\left(fx+e\right)\left(2\ln\left(\tan\left(fx+e\right)\right)-\ln\left(1+\tan^{2}\left(fx+e\right)\right)\right)}{2f\sqrt{b\left(\tan^{2}\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f*tan(f*x+e)*(2*ln(tan(f*x+e))-ln(1+tan(f*x+e)^2))/(b*tan(f*x+e)^2)^(1/
2)
```

maxima [A] time = 0.65, size = 33, normalized size = 1.06

$$-\frac{\frac{\log\left(\tan\left(fx+e\right)^{2}+1\right)}{\sqrt{b}}-\frac{2\log\left(\tan\left(fx+e\right)\right)}{\sqrt{b}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(tan(f*x + e)^2 + 1)/sqrt(b) - 2*log(tan(f*x + e))/sqrt(b))/f
```

mupad [B] time = 11.44, size = 34, normalized size = 1.10

$$\frac{\mathrm{atan}\left(\frac{\sqrt{-b}\,\tan\left(e+f\,x\right)}{\sqrt{b\,{\tan\left(e+f\,x\right)}^{2}}}\right)}{\sqrt{-b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] atan(((-b)^(1/2)*tan(e + f*x))/(b*tan(e + f*x)^2)^(1/2))/((-b)^(1/2)*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \tan^2\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(e + f*x)**2), x)
```

3.5 $\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\cot(e+fx)}{2bf\sqrt{b\tan^2(e+fx)}}-\frac{\tan(e+fx)\log(\sin(e+fx))}{bf\sqrt{b\tan^2(e+fx)}}$$

```
[Out] -1/2*cot(f*x+e)/b/f/(b*tan(f*x+e)^2)^(1/2)-ln(sin(f*x+e))*tan(f*x+e)/b/f/(b
*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$-\frac{\cot(e+fx)}{2bf\sqrt{b\tan^2(e+fx)}}-\frac{\tan(e+fx)\log(\sin(e+fx))}{bf\sqrt{b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] -Cot[e + f*x]/(2*b*f*Sqrt[b*Tan[e + f*x]^2]) - (Log[Sin[e + f*x]]*Tan[e + f
*x])/(b*f*Sqrt[b*Tan[e + f*x]^2])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx &= \frac{\tan(e+fx)\int \cot^3(e+fx)\,dx}{b\sqrt{b\tan^2(e+fx)}}\\
&= -\frac{\cot(e+fx)}{2bf\sqrt{b\tan^2(e+fx)}}-\frac{\tan(e+fx)\int \cot(e+fx)\,dx}{b\sqrt{b\tan^2(e+fx)}}\\
&= -\frac{\cot(e+fx)}{2bf\sqrt{b\tan^2(e+fx)}}-\frac{\log(\sin(e+fx))\tan(e+fx)}{bf\sqrt{b\tan^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 56, normalized size = 0.85

$$-\frac{\tan^3(e+fx)\left(\cot^2(e+fx)+2\log(\tan(e+fx))+2\log(\cos(e+fx))\right)}{2f\left(b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] -1/2*((Cot[e + f*x]^2 + 2*Log[Cos[e + f*x]] + 2*Log[Tan[e + f*x]])*Tan[e +
f*x]^3)/(f*(b*Tan[e + f*x]^2)^(3/2))
```

fricas [A] time = 0.41, size = 69, normalized size = 1.05

$$-\frac{\sqrt{b\tan\left(fx+e\right)^2}\left(\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2+\tan\left(fx+e\right)^2+1\right)}{2\,b^2f\tan\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*tan(f*x + e)^2)*(log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f
*x + e)^2 + tan(f*x + e)^2 + 1)/(b^2*f*tan(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/b/f/sqrt(b)/2
*(1/8*(4*tan((f*x+exp(1))/2)^2*sign(-tan((f*x+exp(1))/2)^2+1)-sign(-tan((f*
x+exp(1))/2)^2+1))/tan((f*x+exp(1))/2)^2/sign(tan((f*x+exp(1))/2))-1/8*tan(
(f*x+exp(1))/2)^2*sign(-tan((f*x+exp(1))/2)^2+1)/sign(tan((f*x+exp(1))/2))-
1/2*sign(-tan((f*x+exp(1))/2)^2+1)*ln(tan((f*x+exp(1))/2)^2)/sign(tan((f*x+
exp(1))/2))+sign(-tan((f*x+exp(1))/2)^2+1)*ln(tan((f*x+exp(1))/2)^2+1)/sign
(tan((f*x+exp(1))/2)))
```

maple [A] time = 0.31, size = 64, normalized size = 0.97

$$-\frac{\tan\left(fx+e\right)\left(2\ln\left(\tan\left(fx+e\right)\right)\left(\tan^2\left(fx+e\right)\right)-\ln\left(1+\tan^2\left(fx+e\right)\right)\left(\tan^2\left(fx+e\right)\right)+1\right)}{2f\left(b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f*tan(f*x+e)*(2*ln(tan(f*x+e))*tan(f*x+e)^2-ln(1+tan(f*x+e)^2)*tan(f*x
+e)^2+1)/(b*tan(f*x+e)^2)^(3/2)
```

maxima [A] time = 0.99, size = 46, normalized size = 0.70

$$\frac{\frac{\log\left(\tan\left(fx+e\right)^{2}+1\right)}{b^{\frac{3}{2}}}-\frac{2\,\log\left(\tan\left(fx+e\right)\right)}{b^{\frac{3}{2}}}-\frac{1}{b^{\frac{3}{2}}\tan\left(fx+e\right)^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(tan(f*x + e)^2 + 1)/b^(3/2) - 2*log(tan(f*x + e))/b^(3/2) - 1/(b^(
3/2)*tan(f*x + e)^2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \tan^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**(-3/2), x)
```

3.6 $\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}}+\frac{\cot(e+fx)}{2b^2f\sqrt{b\tan^2(e+fx)}}+\frac{\tan(e+fx)\log(\sin(e+fx))}{b^2f\sqrt{b\tan^2(e+fx)}}$$

```
[Out] 1/2*cot(f*x+e)/b^2/f/(b*tan(f*x+e)^2)^(1/2)-1/4*cot(f*x+e)^3/b^2/f/(b*tan(f
*x+e)^2)^(1/2)+ln(sin(f*x+e))*tan(f*x+e)/b^2/f/(b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$-\frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}}+\frac{\cot(e+fx)}{2b^2f\sqrt{b\tan^2(e+fx)}}+\frac{\tan(e+fx)\log(\sin(e+fx))}{b^2f\sqrt{b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] Cot[e + f*x]/(2*b^2*f*Sqrt[b*Tan[e + f*x]^2]) - Cot[e + f*x]^3/(4*b^2*f*Sqr
t[b*Tan[e + f*x]^2]) + (Log[Sin[e + f*x]]*Tan[e + f*x])/(b^2*f*Sqrt[b*Tan[e
 + f*x]^2])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\tan(e+fx)\int \cot^5(e+fx)\,dx}{b^2\sqrt{b\tan^2(e+fx)}}$$
$$= -\frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}} - \frac{\tan(e+fx)\int \cot^3(e+fx)\,dx}{b^2\sqrt{b\tan^2(e+fx)}}$$
$$= \frac{\cot(e+fx)}{2b^2f\sqrt{b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}} + \frac{\tan(e+fx)\int \cot(e+fx)\,dx}{b^2\sqrt{b\tan^2(e+fx)}}$$
$$= \frac{\cot(e+fx)}{2b^2f\sqrt{b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}} + \frac{\log(\sin(e+fx))\tan(e+fx)}{b^2f\sqrt{b\tan^2(e+fx)}}$$

Mathematica [A] time = 0.26, size = 68, normalized size = 0.70

$$\frac{\tan^5(e+fx)\left(-\cot^4(e+fx)+2\cot^2(e+fx)+4\log(\tan(e+fx))+4\log(\cos(e+fx))\right)}{4f\left(b\tan^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] ((2*Cot[e + f*x]^2 - Cot[e + f*x]^4 + 4*Log[Cos[e + f*x]] + 4*Log[Tan[e + f
*x]])*Tan[e + f*x]^5)/(4*f*(b*Tan[e + f*x]^2)^(5/2))
```

fricas [A] time = 0.86, size = 82, normalized size = 0.85

$$\frac{\left(2\,\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^4+3\,\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2-1\right)\sqrt{b\tan\left(fx+e\right)^2}}{4\,b^3f\tan\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + 3*tan(f*x
+ e)^4 + 2*tan(f*x + e)^2 - 1)*sqrt(b*tan(f*x + e)^2)/(b^3*f*tan(f*x + e)^5
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f/sqrt(b)/b^2/2*(1/1024*(192*ta
```

```
n((f*x+exp(1))/2)^2*sign(tan((f*x+exp(1))/2))*sign(-tan((f*x+exp(1))/2)^2+1
)-16*tan((f*x+exp(1))/2)^4*sign(tan((f*x+exp(1))/2))*sign(-tan((f*x+exp(1))
/2)^2+1))+1/64*(12*tan((f*x+exp(1))/2)^2*sign(-tan((f*x+exp(1))/2)^2+1)-48*
tan((f*x+exp(1))/2)^4*sign(-tan((f*x+exp(1))/2)^2+1)-sign(-tan((f*x+exp(1))
/2)^2+1))/tan((f*x+exp(1))/2)^4/sign(tan((f*x+exp(1))/2))+1/2*sign(-tan((f*
x+exp(1))/2)^2+1)*ln(tan((f*x+exp(1))/2)^2)/sign(tan((f*x+exp(1))/2))-sign(
-tan((f*x+exp(1))/2)^2+1)*ln(tan((f*x+exp(1))/2)^2+1)/sign(tan((f*x+exp(1))
/2)))
```

maple [A] time = 0.35, size = 74, normalized size = 0.76

$$\frac{\tan\left(fx+e\right)\left(4\ln\left(\tan\left(fx+e\right)\right)\left(\tan^{4}\left(fx+e\right)\right)-2\ln\left(1+\tan^{2}\left(fx+e\right)\right)\left(\tan^{4}\left(fx+e\right)\right)+2\left(\tan^{2}\left(fx+e\right)\right)}{4f\left(b\left(\tan^{2}\left(fx+e\right)\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/4/f*tan(f*x+e)*(4*ln(tan(f*x+e))*tan(f*x+e)^4-2*ln(1+tan(f*x+e)^2)*tan(f*
x+e)^4+2*tan(f*x+e)^2-1)/(b*tan(f*x+e)^2)^(5/2)
```

maxima [A] time = 1.21, size = 66, normalized size = 0.68

$$-\frac{\frac{2\log\left(\tan\left(fx+e\right)^{2}+1\right)}{b^{\frac{5}{2}}}-\frac{4\log\left(\tan\left(fx+e\right)\right)}{b^{\frac{5}{2}}}-\frac{2\sqrt{b}\tan\left(fx+e\right)^{2}-\sqrt{b}}{b^{3}\tan\left(fx+e\right)^{4}}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*log(tan(f*x + e)^2 + 1)/b^(5/2) - 4*log(tan(f*x + e))/b^(5/2) - (2*
sqrt(b)*tan(f*x + e)^2 - sqrt(b))/(b^3*tan(f*x + e)^4))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**(-5/2), x)
```

3.7 $\int \left(b \tan^3(e+fx)\right)^{5/2} dx$

Optimal. Leaf size=364

$$-\frac{2b^2\tan^3(e+fx)\sqrt{b\tan^3(e+fx)}}{9f}+\frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f}+\frac{2b^2\tan^5(e+fx)\sqrt{b\tan^3(e+fx)}}{13f}-\frac{b^2\tan}{}$$

```
[Out] -2*b^2*cot(f*x+e)*(b*tan(f*x+e)^3)^(1/2)/f+1/2*b^2*arctan(-1+2^(1/2)*tan(f*
x+e)^(1/2))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)+1/2*b^2*arcta
n(1+2^(1/2)*tan(f*x+e)^(1/2))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(
3/2)-1/4*b^2*ln(1-2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*(b*tan(f*x+e)^3)^(1/
2)/f*2^(1/2)/tan(f*x+e)^(3/2)+1/4*b^2*ln(1+2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x
+e))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)+2/5*b^2*(b*tan(f*x+e
)^3)^(1/2)*tan(f*x+e)/f-2/9*b^2*(b*tan(f*x+e)^3)^(1/2)*tan(f*x+e)^3/f+2/13*
b^2*(b*tan(f*x+e)^3)^(1/2)*tan(f*x+e)^5/f
```

Rubi [A] time = 0.15, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2b^2\tan^5(e+fx)\sqrt{b\tan^3(e+fx)}}{13f}-\frac{2b^2\tan^3(e+fx)\sqrt{b\tan^3(e+fx)}}{9f}+\frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f}-\frac{b^2\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^(5/2),x]
```

```
[Out] (-2*b^2*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^3])/f - (b^2*ArcTan[1 - Sqrt[2]*Sq
rt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x]^(3/2)) +
(b^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2
]*f*Tan[e + f*x]^(3/2)) - (b^2*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e +
 f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]^(3/2)) + (b^2*Log[
1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*S
qrt[2]*f*Tan[e + f*x]^(3/2)) + (2*b^2*Tan[e + f*x]*Sqrt[b*Tan[e + f*x]^3])/
(5*f) - (2*b^2*Tan[e + f*x]^3*Sqrt[b*Tan[e + f*x]^3])/(9*f) + (2*b^2*Tan[e
+ f*x]^5*Sqrt[b*Tan[e + f*x]^3])/(13*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^3(e+fx)\right)^{5/2} dx = \frac{\left(b^2\sqrt{b\tan^3(e+fx)}\right)\int \tan^{\frac{15}{2}}(e+fx)\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2b^2\tan^5(e+fx)\sqrt{b\tan^3(e+fx)}}{13f} - \frac{\left(b^2\sqrt{b\tan^3(e+fx)}\right)\int \tan^{\frac{11}{2}}(e+fx)\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= -\frac{2b^2\tan^3(e+fx)\sqrt{b\tan^3(e+fx)}}{9f} + \frac{2b^2\tan^5(e+fx)\sqrt{b\tan^3(e+fx)}}{13f} + \frac{\left(b^2\sqrt{b\tan\right.}{}$$

$$= \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - \frac{2b^2\tan^3(e+fx)\sqrt{b\tan^3(e+fx)}}{9f} + \frac{2b^2\tan^5(e+}{}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - \frac{2b^2\tan^3(e+}{}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - \frac{2b^2\tan^3(e+}{}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - \frac{2b^2\tan^3(e+}{}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - \frac{2b^2\tan^3(e+}{}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - \frac{2b^2\tan^3(e+}{}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{b^2\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\sqrt{b}}{2\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{b^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}$$

Mathematica [A] time = 0.83, size = 199, normalized size = 0.55

$$\frac{b\left(b\tan^3(e+fx)\right)^{3/2}\left(-1170\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)+1170\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)+360\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^3)^(5/2),x]
```

```
[Out] (b*(b*Tan[e + f*x]^3)^(3/2)*(-1170*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e +
f*x]]] + 1170*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - 585*Sqrt[2]*
Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] + 585*Sqrt[2]*Log[1 + Sq
rt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 4680*Sqrt[Tan[e + f*x]] + 936*Ta
n[e + f*x]^(5/2) - 520*Tan[e + f*x]^(9/2) + 360*Tan[e + f*x]^(13/2)))/(2340
*f*Tan[e + f*x]^(9/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2*b*((1/13*b^60*f^12*sqrt(b*tan(f*x+
exp(1)))*(b*tan(f*x+exp(1)))^6-1/9*b^62*f^12*sqrt(b*tan(f*x+exp(1)))*(b*tan
(f*x+exp(1)))^4+1/5*b^64*f^12*sqrt(b*tan(f*x+exp(1)))*(b*tan(f*x+exp(1)))^2
-b^66*f^12*sqrt(b*tan(f*x+exp(1))))/b^65/f^13+1/2*b*sqrt(abs(b))*atan(sqrt(
2)*(1/2*sqrt(2)*sqrt(abs(b))+sqrt(b*tan(f*x+exp(1))))/sqrt(abs(b)))/sqrt(2)
/f+1/2*b*sqrt(abs(b))*atan(sqrt(2)*(-1/2*sqrt(2)*sqrt(abs(b))+sqrt(b*tan(f*
x+exp(1))))/sqrt(abs(b)))/sqrt(2)/f+1/4*b*sqrt(abs(b))*ln(b*tan(f*x+exp(1))
+sqrt(2)*sqrt(b*tan(f*x+exp(1)))*sqrt(abs(b))+abs(b))/sqrt(2)/f-1/4*b*sqrt(
abs(b))*ln(b*tan(f*x+exp(1))-sqrt(2)*sqrt(b*tan(f*x+exp(1)))*sqrt(abs(b))+a
bs(b))/sqrt(2)/f)*sign(tan(f*x+exp(1)))
```

maple [A] time = 0.25, size = 263, normalized size = 0.72

$$\left(b\left(\tan^3\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(360\left(b\tan\left(fx+e\right)\right)^{\frac{13}{2}}-520b^2\left(b\tan\left(fx+e\right)\right)^{\frac{9}{2}}+585b^6\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{b\tan\left(fx+e\right)+\left(b^2\right)^{\frac{1}{4}}\sqrt{}}{b\tan\left(fx+e\right)-\left(b^2\right)^{\frac{1}{4}}\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^3)^(5/2),x)
```

```
[Out] 1/2340/f*(b*tan(f*x+e)^3)^(5/2)*(360*(b*tan(f*x+e))^(13/2)-520*b^2*(b*tan(f
*x+e))^(9/2)+585*b^6*(b^2)^(1/4)*2^(1/2)*ln((b*tan(f*x+e)+(b^2)^(1/4)*(b*ta
n(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(f*x+e)-(b^2)^(1/4)*(b*tan(f*x+e
))^(1/2)*2^(1/2)+(b^2)^(1/2)))+1170*b^6*(b^2)^(1/4)*2^(1/2)*arctan((2^(1/2)
*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+1170*b^6*(b^2)^(1/4)*2^(1/2
)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))+936*(b*tan
(f*x+e))^(5/2)*b^4-4680*b^6*(b*tan(f*x+e))^(1/2))/tan(f*x+e)^5/(b*tan(f*x+e
))^(5/2)/b^4
```

maxima [A] time = 1.27, size = 178, normalized size = 0.49

$$360\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{\frac{13}{2}}-520\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{\frac{9}{2}}+936\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{\frac{5}{2}}+585\left(2\sqrt{2}\sqrt{b}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2340*(360*b^(5/2)*tan(f*x + e)^(13/2) - 520*b^(5/2)*tan(f*x + e)^(9/2) +
936*b^(5/2)*tan(f*x + e)^(5/2) + 585*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*
```

```
(sqrt(2) + 2*sqrt(tan(f*x + e)))) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(
sqrt(2) - 2*sqrt(tan(f*x + e)))) + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(f*x
 + e)) + tan(f*x + e) + 1) - sqrt(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(f*x + e)
) + tan(f*x + e) + 1))*b^2 - 4680*b^(5/2)*sqrt(tan(f*x + e)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\tan\left(e+f\,x\right)^3\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^3)^(5/2),x)
[Out] int((b*tan(e + f*x)^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^3\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**3)**(5/2),x)
[Out] Integral((b*tan(e + f*x)**3)**(5/2), x)
```

3.8 $\int \left(b\tan^3(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=286

$$-\frac{2b\sqrt{b\tan^3(e+fx)}}{3f}+\frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f}-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}+\frac{b\,t}{}$$

```
[Out] -2/3*b*(b*tan(f*x+e)^3)^(1/2)/f+1/2*b*arctan(-1+2^(1/2)*tan(f*x+e)^(1/2))*(
b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)+1/2*b*arctan(1+2^(1/2)*tan
(f*x+e)^(1/2))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)+1/4*b*ln(1
-2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(
f*x+e)^(3/2)-1/4*b*ln(1+2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*(b*tan(f*x+e)^
3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)+2/7*b*(b*tan(f*x+e)^3)^(1/2)*tan(f*x+e)
^2/f
```

Rubi [A] time = 0.13, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f}-\frac{2b\sqrt{b\tan^3(e+fx)}}{3f}-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}+\frac{b\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b*Tan[e + f*x]^3])/(3*f) - (b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*
x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x]^(3/2)) + (b*ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x
]^(3/2)) + (b*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan
[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]^(3/2)) - (b*Log[1 + Sqrt[2]*Sqrt[Ta
n[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*
x]^(3/2)) + (2*b*Tan[e + f*x]^2*Sqrt[b*Tan[e + f*x]^3])/(7*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^3(e+fx)\right)^{3/2}\,dx = \frac{\left(b\sqrt{b\tan^3(e+fx)}\right)\int \tan^{\frac{9}{2}}(e+fx)\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} - \frac{\left(b\sqrt{b\tan^3(e+fx)}\right)\int \tan^{\frac{5}{2}}(e+fx)\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} + \frac{\left(b\sqrt{b\tan^3(e+fx)}\right)\int}{\tan^{\frac{3}{2}}(e+}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} + \frac{\left(b\sqrt{b\tan^3(e+fx)}\right)\text{Su}}{f\,\text{t}}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} + \frac{\left(2b\sqrt{b\tan^3(e+fx)}\right)\text{S}}{f}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} - \frac{\left(b\sqrt{b\tan^3(e+fx)}\right)\text{Su}}{f}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} + \frac{\left(b\sqrt{b\tan^3(e+fx)}\right)\text{Su}}{}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{b\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\sqrt{b\tan^3(e+f}}{2\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} - \frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} + \frac{b\tan^{-}}{}$$

Mathematica [C] time = 0.08, size = 54, normalized size = 0.19

$$\frac{2b\sqrt{b\tan^3(e+fx)}\left(7\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)+3\tan^2(e+fx)-7\right)}{21f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^3)^(3/2),x]
```

```
[Out] (2*b*Sqrt[b*Tan[e + f*x]^3]*(-7 + 7*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e +
 f*x]^2] + 3*Tan[e + f*x]^2))/(21*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)b*2*((1/7*b^18*f^6*sqrt(b*tan(f*x+ex
p(1)))*(b*tan(f*x+exp(1)))^3-1/3*b^21*f^6*sqrt(b*tan(f*x+exp(1)))*tan(f*x+e
xp(1)))/b^21/f^7+1/2*sqrt(abs(b))*abs(b)*atan(sqrt(2)*(1/2*sqrt(2)*sqrt(abs
(b))+sqrt(b*tan(f*x+exp(1))))/sqrt(abs(b)))/sqrt(2)/b/f+1/2*sqrt(abs(b))*ab
s(b)*atan(sqrt(2)*(-1/2*sqrt(2)*sqrt(abs(b))+sqrt(b*tan(f*x+exp(1))))/sqrt(
abs(b)))/sqrt(2)/b/f-1/4*sqrt(abs(b))*abs(b)*ln(b*tan(f*x+exp(1))+sqrt(2)*s
qrt(b*tan(f*x+exp(1)))*sqrt(abs(b))+abs(b))/sqrt(2)/b/f+1/4*sqrt(abs(b))*ab
s(b)*ln(b*tan(f*x+exp(1))-sqrt(2)*sqrt(b*tan(f*x+exp(1)))*sqrt(abs(b))+abs(
b))/sqrt(2)/b/f)*sign(tan(f*x+exp(1)))
```

maple [A] time = 0.16, size = 236, normalized size = 0.83

$$\frac{\left(b\left(\tan^{3}\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(24\left(b\tan\left(fx+e\right)\right)^{\frac{7}{2}}\left(b^{2}\right)^{\frac{1}{4}}+21b^{4}\sqrt{2}\,\ln\left(-\frac{\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(fx+e\right)}\,\sqrt{2}-b\tan\left(fx+e\right)-\sqrt{b^{2}}}{b\tan\left(fx+e\right)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(fx+e\right)}\,\sqrt{2}+\sqrt{b^{2}}}\right)+42b^{4}\sqrt{2}\ \mathrm{a}\right.}{84f\tan\left(fx+e\right)^{3}\left(b\tan\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^3)^(3/2),x)
```

```
[Out] 1/84/f*(b*tan(f*x+e)^3)^(3/2)*(24*(b*tan(f*x+e))^(7/2)*(b^2)^(1/4)+21*b^4*2
^(1/2)*ln(-((b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/
2))/(b*tan(f*x+e)+(b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+42
*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))
+42*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/
4))-56*(b*tan(f*x+e))^(3/2)*b^2*(b^2)^(1/4))/tan(f*x+e)^3/(b*tan(f*x+e))^(3
/2)/b^2/(b^2)^(1/4)
```

maxima [A] time = 0.81, size = 140, normalized size = 0.49

$$\frac{24\,b^{\frac{3}{2}}\tan\left(fx+e\right)^{\frac{7}{2}}-56\,b^{\frac{3}{2}}\tan\left(fx+e\right)^{\frac{3}{2}}+21\left(2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{\tan\left(fx+e\right)}\right)\right)+2\,\sqrt{2}\,\arctan\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/84*(24*b^(3/2)*tan(f*x + e)^(7/2) - 56*b^(3/2)*tan(f*x + e)^(3/2) + 21*(2
*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))) + 2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) - sqrt(2)*log(sqrt(2)*
sqrt(tan(f*x + e)) + tan(f*x + e) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(tan(f*x
+ e)) + tan(f*x + e) + 1))*b^(3/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\tan\left(e+fx\right)^{3}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^3)^(3/2),x)
```

```
[Out] int((b*tan(e + f*x)^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^3\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**3)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**3)**(3/2), x)
```

3.9 $\int \sqrt{b \tan^3(e+fx)}\, dx$

Optimal. Leaf size=255

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}+\frac{\sqrt{b\tan^3(e+fx}}{}$$

```
[Out] 2*cot(f*x+e)*(b*tan(f*x+e)^3)^(1/2)/f-1/2*arctan(-1+2^(1/2)*tan(f*x+e)^(1/2
))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)-1/2*arctan(1+2^(1/2)*t
an(f*x+e)^(1/2))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)+1/4*ln(1
-2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(
f*x+e)^(3/2)-1/4*ln(1+2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*(b*tan(f*x+e)^3)
^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)
```

Rubi [A] time = 0.12, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}+\frac{\sqrt{b\tan^3(e+fx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]^3],x]
```

```
[Out] (2*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^3])/f + (ArcTan[1 - Sqrt[2]*Sqrt[Tan[e
+ f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x]^(3/2)) - (ArcTan[1
 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f
*x]^(3/2)) + (Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan
[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]^(3/2)) - (Log[1 + Sqrt[2]*Sqrt[Tan[
e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]
^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{b\tan^3(e+fx)}\,dx = \frac{\sqrt{b\tan^3(e+fx)}\int \tan^{\frac{3}{2}}(e+fx)\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$
$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\sqrt{b\tan^3(e+fx)}\int \frac{1}{\sqrt{\tan(e+fx)}}\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$
$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\sqrt{b\tan^3(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f\tan^{\frac{3}{2}}(e+fx)}$$
$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\left(2\sqrt{b\tan^3(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(e+fx)}\right)}{f\tan^{\frac{3}{2}}(e+fx)}$$
$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\sqrt{b\tan^3(e+fx)}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(e+fx)}\right)}{f\tan^{\frac{3}{2}}(e+fx)} -$$
$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\sqrt{b\tan^3(e+fx)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(e+f}\right.}{2f\tan^{\frac{3}{2}}(e+fx)}$$
$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\sqrt{b\tan^3(e+}}{2\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}$$
$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} - \frac{\tan^{-}}{}$$

Mathematica [A] time = 0.26, size = 161, normalized size = 0.63

$$\frac{\sqrt{b\tan^3(e+fx)}\left(2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)-2\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)+8\sqrt{\tan(e+fx)}+\sqrt{}\right.}{4f\tan^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]^3],x]
```

```
[Out] ((2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 2*Sqrt[2]*ArcTan[1 + S
qrt[2]*Sqrt[Tan[e + f*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + T
an[e + f*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] +
 8*Sqrt[Tan[e + f*x]])*Sqrt[b*Tan[e + f*x]^3])/(4*f*Tan[e + f*x]^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2*b*(sqrt(b*tan(f*x+exp(1)))/f-1/2*s
qrt(abs(b))*atan(sqrt(2)*(1/2*sqrt(2)*sqrt(abs(b))+sqrt(b*tan(f*x+exp(1))))
/sqrt(abs(b)))/sqrt(2)/f-1/2*sqrt(abs(b))*atan(sqrt(2)*(-1/2*sqrt(2)*sqrt(a
bs(b))+sqrt(b*tan(f*x+exp(1))))/sqrt(abs(b)))/sqrt(2)/f-1/4*sqrt(abs(b))*ln
(b*tan(f*x+exp(1))+sqrt(2)*sqrt(b*tan(f*x+exp(1)))*sqrt(abs(b))+abs(b))/sqr
t(2)/f+1/4*sqrt(abs(b))*ln(b*tan(f*x+exp(1))-sqrt(2)*sqrt(b*tan(f*x+exp(1))
)*sqrt(abs(b))+abs(b))/sqrt(2)/f)*sign(tan(f*x+exp(1)))/b
```

maple [A] time = 0.23, size = 206, normalized size = 0.81

$$\frac{\sqrt{b\left(\tan^{3}\left(fx+e\right)\right)}\left(-2\left(b^{2}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan\left(fx+e\right)}+\left(b^{2}\right)^{\frac{1}{4}}}{\left(b^{2}\right)^{\frac{1}{4}}}\right)-2\left(b^{2}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan\left(fx+e\right)}-\left(b^{2}\right)^{\frac{1}{4}}}{\left(b^{2}\right)^{\frac{1}{4}}}\right)\cdots}{4f\tan\left(fx+e\right)\sqrt{b\tan\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^3)^(1/2),x)
```

```
[Out] 1/4/f*(b*tan(f*x+e)^3)^(1/2)*(-2*(b^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(b*tan
(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))-2*(b^2)^(1/4)*2^(1/2)*arctan((2^(1
/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))-(b^2)^(1/4)*2^(1/2)*ln((
b*tan(f*x+e)+(b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(f
*x+e)-(b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+8*(b*tan(f*x+e
))^(1/2))/tan(f*x+e)/(b*tan(f*x+e))^(1/2)
```

maxima [A] time = 2.16, size = 133, normalized size = 0.52

$$-\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(fx+e\right)}\right)\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(fx+e\right)}\right)\right)+\sqrt{2}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))
) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e))))
 + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) - sqr
t(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) - 8*sqrt(b
)*sqrt(tan(f*x + e)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{b\tan\left(e+f\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^3)^(1/2),x)
```

```
[Out] int((b*tan(e + f*x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan^{3}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x)**3), x)
```

3.10 $\int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx$

Optimal. Leaf size=255

$$-\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}}$$

```
[Out] -2*tan(f*x+e)/f/(b*tan(f*x+e)^3)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(f*x+e)^(1/
2))*tan(f*x+e)^(3/2)/f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)-1/2*arctan(1+2^(1/2)*
tan(f*x+e)^(1/2))*tan(f*x+e)^(3/2)/f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)-1/4*ln(
1-2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*tan(f*x+e)^(3/2)/f*2^(1/2)/(b*tan(f*
x+e)^3)^(1/2)+1/4*ln(1+2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*tan(f*x+e)^(3/2
)/f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}}-\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[e + f*x]^3],x]
```

```
[Out] (-2*Tan[e + f*x])/(f*Sqrt[b*Tan[e + f*x]^3]) + (ArcTan[1 - Sqrt[2]*Sqrt[Tan
[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sqrt[2]*f*Sqrt[b*Tan[e + f*x]^3]) - (ArcTa
n[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sqrt[2]*f*Sqrt[b*Tan
[e + f*x]^3]) - (Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Tan[e +
 f*x]^(3/2))/(2*Sqrt[2]*f*Sqrt[b*Tan[e + f*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[T
an[e + f*x]] + Tan[e + f*x]]*Tan[e + f*x]^(3/2))/(2*Sqrt[2]*f*Sqrt[b*Tan[e
+ f*x]^3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx = \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{3}{2}}(e+fx)}\,dx}{\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\int \sqrt{\tan(e+fx)}\,dx}{\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\,\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\left(2\tan^{\frac{3}{2}}(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(e+fx)}\right)}{f\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(e+fx)}\right)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+\ldots}{\ldots}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(e+fx)}\right)}{2f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}\ldots}{\ldots}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\tan^{\frac{3}{2}}(e+fx)}{2\sqrt{2}f\sqrt{b\tan^3(e+fx)}} + \frac{\log\ldots}{\ldots}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{-1}\left(1+\sqrt{2}\ldots\right.}{\sqrt{2}\ldots}$$

Mathematica [C] time = 0.03, size = 43, normalized size = 0.17

$$-\frac{2\tan(e+fx)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(e+fx)\right)}{f\sqrt{b\tan^3(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[e + f*x]^3],x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*Sqrt[
b*Tan[e + f*x]^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*b^2*(-1/
2*sqrt(abs(b))*abs(b)*atan(sqrt(2)*(1/2*sqrt(2)*sqrt(abs(b))+sqrt(b*tan(f*x
+exp(1))))/sqrt(abs(b)))/sqrt(2)/b^4/f/sign(tan(f*x+exp(1)))-1/2*sqrt(abs(b
))*abs(b)*atan(sqrt(2)*(-1/2*sqrt(2)*sqrt(abs(b))+sqrt(b*tan(f*x+exp(1))))/
sqrt(abs(b)))/sqrt(2)/b^4/f/sign(tan(f*x+exp(1)))+1/4*sqrt(abs(b))*abs(b)*l
n(b*tan(f*x+exp(1))+sqrt(2)*sqrt(b*tan(f*x+exp(1)))*sqrt(abs(b))+abs(b))/sq
rt(2)/b^4/f/sign(tan(f*x+exp(1)))-1/4*sqrt(abs(b))*abs(b)*ln(b*tan(f*x+exp(
1))-sqrt(2)*sqrt(b*tan(f*x+exp(1)))*sqrt(abs(b))+abs(b))/sqrt(2)/b^4/f/sign
(tan(f*x+exp(1)))-1/b^2/sqrt(b*tan(f*x+exp(1)))/f/sign(tan(f*x+exp(1))))
```

maple [A] time = 0.31, size = 211, normalized size = 0.83

$$-\frac{\tan\left(fx+e\right)\left(\sqrt{2}\,\sqrt{b\tan\left(fx+e\right)}\,\ln\left(-\frac{\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(fx+e\right)}\,\sqrt{2}-b\tan\left(fx+e\right)-\sqrt{b^{2}}}{b\tan\left(fx+e\right)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(fx+e\right)}\,\sqrt{2}+\sqrt{b^{2}}}\right)+2\sqrt{2}\,\sqrt{b\tan\left(fx+e\right)}\,\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right)\cdots\right)}{4f\sqrt{b\left(\tan^{3}\left(fx+e\right)\right)}\,\left(b^{2}\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^3)^(1/2),x)
```

```
[Out] -1/4/f*tan(f*x+e)*(2^(1/2)*(b*tan(f*x+e))^(1/2)*ln(-((b^2)^(1/4)*(b*tan(f*x
+e))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/2))/(b*tan(f*x+e)+(b^2)^(1/4)*(b*t
an(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*2^(1/2)*(b*tan(f*x+e))^(1/2)*arcta
n((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+2*2^(1/2)*(b*tan(
f*x+e))^(1/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4)
)+8*(b^2)^(1/4))/(b*tan(f*x+e)^3)^(1/2)/(b^2)^(1/4)
```

maxima [A] time = 1.33, size = 126, normalized size = 0.49

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(fx+e\right)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(fx+e\right)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(fx+e\right)}+\tan\left(fx+e\right)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\cdots}\right.\cdots}{\sqrt{b}}\cdots}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) - sqrt(2)*log(
sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(
tan(f*x + e)) + tan(f*x + e) + 1))/sqrt(b) + 8/(sqrt(b)*sqrt(tan(f*x + e)))
)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{b\tan\left(e+f\,x\right)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^3)^(1/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \tan^{3}\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**3)**(1/2),x)
[Out] Integral(1/sqrt(b*tan(e + f*x)**3), x)
```

3.11 $\int \frac{1}{\left(b\tan^3(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=298

$$\frac{2}{3bf\sqrt{b\tan^3(e+fx)}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}bf\sqrt{b\tan^3(e+fx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}bf\sqrt{b\tan^3(e+fx)}}-$$

```
[Out] 2/3/b/f/(b*tan(f*x+e)^3)^(1/2)-2/7*cot(f*x+e)^2/b/f/(b*tan(f*x+e)^3)^(1/2)+
1/2*arctan(-1+2^(1/2)*tan(f*x+e)^(1/2))*tan(f*x+e)^(3/2)/b/f*2^(1/2)/(b*tan
(f*x+e)^3)^(1/2)+1/2*arctan(1+2^(1/2)*tan(f*x+e)^(1/2))*tan(f*x+e)^(3/2)/b/
f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)-1/4*ln(1-2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+
e))*tan(f*x+e)^(3/2)/b/f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)+1/4*ln(1+2^(1/2)*ta
n(f*x+e)^(1/2)+tan(f*x+e))*tan(f*x+e)^(3/2)/b/f*2^(1/2)/(b*tan(f*x+e)^3)^(1
/2)
```

Rubi [A] time = 0.13, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}bf\sqrt{b\tan^3(e+fx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}bf\sqrt{b\tan^3(e+fx)}}+\frac{2}{3bf\sqrt{b\tan^3(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^(-3/2),x]
```

```
[Out] 2/(3*b*f*Sqrt[b*Tan[e + f*x]^3]) - (2*Cot[e + f*x]^2)/(7*b*f*Sqrt[b*Tan[e +
 f*x]^3]) - (ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sq
rt[2]*b*f*Sqrt[b*Tan[e + f*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]
*Tan[e + f*x]^(3/2))/(Sqrt[2]*b*f*Sqrt[b*Tan[e + f*x]^3]) - (Log[1 - Sqrt[2
]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Tan[e + f*x]^(3/2))/(2*Sqrt[2]*b*f*Sqr
t[b*Tan[e + f*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*
Tan[e + f*x]^(3/2))/(2*Sqrt[2]*b*f*Sqrt[b*Tan[e + f*x]^3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{3/2}}\,dx = \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{9}{2}}(e+fx)}\,dx}{b\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{5}{2}}(e+fx)}\,dx}{b\sqrt{b\tan^3(e+fx)}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\sqrt{\tan(e+fx)}}\,dx}{b\sqrt{b\tan^3(e+fx)}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx, x,$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\left(2\tan^{\frac{3}{2}}(e+fx)\right)\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x,$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\mathrm{ta}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e$$

Mathematica [C] time = 0.07, size = 45, normalized size = 0.15

$$-\frac{2\tan(e+fx)\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\tan^2(e+fx)\right)}{7f\left(b\tan^3(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^3)^(-3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-7/4, 1, -3/4, -Tan[e + f*x]^2]*Tan[e + f*x])/(7*f*(b
*Tan[e + f*x]^3)^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^3)^(-3/2), x)
```

maple [A] time = 0.21, size = 233, normalized size = 0.78

$$\frac{\tan\left(fx+e\right)\left(21\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\left(b\tan\left(fx+e\right)\right)^{\frac{7}{2}}\ln\left(\frac{b\tan\left(fx+e\right)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan\left(fx+e\right)}\,\sqrt{2}+\sqrt{b^2}}{b\tan\left(fx+e\right)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan\left(fx+e\right)}\,\sqrt{2}+\sqrt{b^2}}\right)+42\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\left(b\tan\left(fx\right.\right.}{84}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^3)^(3/2),x)
```

```
[Out] 1/84/f*tan(f*x+e)/b^4*(21*(b^2)^(1/4)*2^(1/2)*(b*tan(f*x+e))^(7/2)*ln((b*ta
n(f*x+e)+(b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(f*x+e
)-(b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+42*(b^2)^(1/4)*2^(
1/2)*(b*tan(f*x+e))^(7/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))
/(b^2)^(1/4))+42*(b^2)^(1/4)*2^(1/2)*(b*tan(f*x+e))^(7/2)*arctan((2^(1/2)*(
b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))+56*tan(f*x+e)^2*b^4-24*b^4)/(
b*tan(f*x+e)^3)^(3/2)
```

maxima [A] time = 3.34, size = 163, normalized size = 0.55

$$\frac{\frac{21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(fx+e\right)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(fx+e\right)}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(fx+e\right)}+\tan\left(fx+e\right)+1\right)-\sqrt{2}\log\left(-\right.}{b^{\frac{3}{2}}}}{84\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/84*(21*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) + sqrt(2)*l
og(sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) - sqrt(2)*log(-sqrt(2)*sq
rt(tan(f*x + e)) + tan(f*x + e) + 1))/b^(3/2) + 8*(21*sqrt(tan(f*x + e)) +
7/tan(f*x + e)^(3/2) - 3/tan(f*x + e)^(7/2))/b^(3/2) - 168*sqrt(tan(f*x + e
))/b^(3/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,{\tan\left(e+f\,x\right)}^3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^3)^(3/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan^3\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**3)**(3/2),x)
[Out] Integral((b*tan(e + f*x)**3)**(-3/2), x)
```

3.12 $\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=364

$$\frac{2\tan(e+fx)}{b^2 f\sqrt{b\tan^3(e+fx)}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}\,b^2 f\sqrt{b\tan^3(e+fx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}\,b^2 f\sqrt{b\tan^3(e+fx)}}$$

```
[Out] -2/5*cot(f*x+e)/b^2/f/(b*tan(f*x+e)^3)^(1/2)+2/9*cot(f*x+e)^3/b^2/f/(b*tan(
f*x+e)^3)^(1/2)-2/13*cot(f*x+e)^5/b^2/f/(b*tan(f*x+e)^3)^(1/2)+2*tan(f*x+e)
/b^2/f/(b*tan(f*x+e)^3)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(f*x+e)^(1/2))*tan(f
*x+e)^(3/2)/b^2/f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)+1/2*arctan(1+2^(1/2)*tan(f
*x+e)^(1/2))*tan(f*x+e)^(3/2)/b^2/f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)+1/4*ln(1
-2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*tan(f*x+e)^(3/2)/b^2/f*2^(1/2)/(b*tan
(f*x+e)^3)^(1/2)-1/4*ln(1+2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*tan(f*x+e)^(
3/2)/b^2/f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}\,b^2 f\sqrt{b\tan^3(e+fx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}\,b^2 f\sqrt{b\tan^3(e+fx)}}+\frac{2\tan(e+fx)}{b^2 f\sqrt{b\tan^3(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^(-5/2),x]
```

```
[Out] (-2*Cot[e + f*x])/(5*b^2*f*Sqrt[b*Tan[e + f*x]^3]) + (2*Cot[e + f*x]^3)/(9*
b^2*f*Sqrt[b*Tan[e + f*x]^3]) - (2*Cot[e + f*x]^5)/(13*b^2*f*Sqrt[b*Tan[e +
 f*x]^3]) + (2*Tan[e + f*x])/(b^2*f*Sqrt[b*Tan[e + f*x]^3]) - (ArcTan[1 - S
qrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sqrt[2]*b^2*f*Sqrt[b*Tan[e
+ f*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(S
qrt[2]*b^2*f*Sqrt[b*Tan[e + f*x]^3]) + (Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]
+ Tan[e + f*x]]*Tan[e + f*x]^(3/2))/(2*Sqrt[2]*b^2*f*Sqrt[b*Tan[e + f*x]^3]
) - (Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Tan[e + f*x]^(3/2))
/(2*Sqrt[2]*b^2*f*Sqrt[b*Tan[e + f*x]^3])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx = \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{15}{2}}(e+fx)}\,dx}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{11}{2}}(e+fx)}\,dx}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$= \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{7}{2}}(e+fx)}\,dx}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} - \tan^{\frac{3}{2}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2}{b^2f\sqrt{}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2}{b^2f\sqrt{}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2}{b^2f\sqrt{}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2}{b^2f\sqrt{}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2}{b^2f\sqrt{}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2}{b^2f\sqrt{}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2}{b^2f\sqrt{}}$$

Mathematica [C] time = 0.06, size = 45, normalized size = 0.12

$$-\frac{2\tan(e+fx)\,{}_2F_1\left(-\frac{13}{4},1;-\frac{9}{4};-\tan^2(e+fx)\right)}{13f\left(b\tan^3(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^3)^(-5/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-13/4, 1, -9/4, -Tan[e + f*x]^2]*Tan[e + f*x])/(13*f*
(b*Tan[e + f*x]^3)^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \tan\left(fx+e\right)^{3}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^3)^(-5/2), x)
```

maple [A] time = 0.23, size = 272, normalized size = 0.75

$$\tan\left(fx+e\right)\left(585\sqrt{2}\,\left(b\tan\left(fx+e\right)\right)^{\frac{13}{2}}\ln\left(-\frac{\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(fx+e\right)}\,\sqrt{2}-b\tan\left(fx+e\right)-\sqrt{b^{2}}}{b\tan\left(fx+e\right)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(fx+e\right)}\,\sqrt{2}+\sqrt{b^{2}}}\right)+1170\sqrt{2}\,\left(b\tan\left(fx+e\right)\right)^{\frac{13}{2}}\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^3)^(5/2),x)
```

```
[Out] 1/2340/f*tan(f*x+e)/b^6*(585*2^(1/2)*(b*tan(f*x+e))^(13/2)*ln(-((b^2)^(1/4)
*(b*tan(f*x+e))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/2))/(b*tan(f*x+e)+(b^2)
^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+1170*2^(1/2)*(b*tan(f*x+e
))^(13/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+11
70*2^(1/2)*(b*tan(f*x+e))^(13/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)
^(1/4))/(b^2)^(1/4))+4680*(b^2)^(1/4)*tan(f*x+e)^6*b^6-936*b^6*(b^2)^(1/4)*
tan(f*x+e)^4+520*b^6*(b^2)^(1/4)*tan(f*x+e)^2-360*b^6*(b^2)^(1/4))/(b*tan(f
*x+e)^3)^(5/2)/(b^2)^(1/4)
```

maxima [A] time = 1.92, size = 172, normalized size = 0.47

$$\frac{\frac{585\left(2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{\tan\left(fx+e\right)}\right)\right)+2\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{\tan\left(fx+e\right)}\right)\right)-\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt{\tan\left(fx+e\right)}+\tan\left(fx+e\right)+1\right)+\sqrt{2}\,\log\left(-\sqrt{2}\right.\right.}{b^{\frac{5}{2}}}}{2340\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2340*(585*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e))))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) - sqrt(2
)*log(sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) + sqrt(2)*log(-sqrt(2)
*sqrt(tan(f*x + e)) + tan(f*x + e) + 1))/b^(5/2) + 8*(585*sqrt(b)/sqrt(tan(
f*x + e)) - 117*sqrt(b)/tan(f*x + e)^(5/2) + 65*sqrt(b)/tan(f*x + e)^(9/2)
- 45*sqrt(b)/tan(f*x + e)^(13/2))/b^3)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\tan\left(e+f\,x\right)^3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^3)^(5/2),x)
[Out] int(1/(b*tan(e + f*x)^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan^3\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**3)**(5/2),x)
[Out] Integral((b*tan(e + f*x)**3)**(-5/2), x)
```

3.13 $\int \left(b\tan^4(e+fx)\right)^{5/2}\,dx$

Optimal. Leaf size=182

$$-\frac{b^2\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f}+\frac{b^2\tan^7(e+fx)\sqrt{b\tan^4(e+fx)}}{9f}-\frac{b^2\tan^5(e+fx)\sqrt{b\tan^4(e+fx)}}{7f}+\frac{b^2\tan^3(e\cdots}{}$$

```
[Out] b^2*cot(f*x+e)*(b*tan(f*x+e)^4)^(1/2)/f-b^2*x*cot(f*x+e)^2*(b*tan(f*x+e)^4)
^(1/2)-1/3*b^2*(b*tan(f*x+e)^4)^(1/2)*tan(f*x+e)/f+1/5*b^2*(b*tan(f*x+e)^4)
^(1/2)*tan(f*x+e)^3/f-1/7*b^2*(b*tan(f*x+e)^4)^(1/2)*tan(f*x+e)^5/f+1/9*b^2
*(b*tan(f*x+e)^4)^(1/2)*tan(f*x+e)^7/f
```

Rubi [A] time = 0.06, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{b^2\tan^7(e+fx)\sqrt{b\tan^4(e+fx)}}{9f}-\frac{b^2\tan^5(e+fx)\sqrt{b\tan^4(e+fx)}}{7f}+\frac{b^2\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f}-\frac{b^2\tan(e+\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^(5/2),x]
```

```
[Out] (b^2*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/f - b^2*x*Cot[e + f*x]^2*Sqrt[b*T
an[e + f*x]^4] - (b^2*Tan[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/(3*f) + (b^2*Tan
[e + f*x]^3*Sqrt[b*Tan[e + f*x]^4])/(5*f) - (b^2*Tan[e + f*x]^5*Sqrt[b*Tan[
e + f*x]^4])/(7*f) + (b^2*Tan[e + f*x]^7*Sqrt[b*Tan[e + f*x]^4])/(9*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^4(e+fx)\right)^{5/2}\,dx = \left(b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^{10}(e+fx)\,dx$$
$$= \frac{b^2\tan^7(e+fx)\sqrt{b\tan^4(e+fx)}}{9f} - \left(b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^8(e+$$
$$= -\frac{b^2\tan^5(e+fx)\sqrt{b\tan^4(e+fx)}}{7f} + \frac{b^2\tan^7(e+fx)\sqrt{b\tan^4(e+fx)}}{9f} + \left(b^2\cot^2(e\right.$$
$$= \frac{b^2\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f} - \frac{b^2\tan^5(e+fx)\sqrt{b\tan^4(e+fx)}}{7f} + \frac{b^2\tan^7(e+}{}$$
$$= -\frac{b^2\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b^2\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f} - \frac{b^2\tan^5(e+}{}$$
$$= \frac{b^2\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - \frac{b^2\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b^2\tan^3(e+fx}{}$$
$$= \frac{b^2\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - b^2x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)} - \frac{b^2\tan(e+f}{}$$

Mathematica [A] time = 0.74, size = 86, normalized size = 0.47

$$\frac{\cot(e+fx)\left(b\tan^4(e+fx)\right)^{5/2}\left(315\cot^8(e+fx) - 105\cot^6(e+fx) + 63\cot^4(e+fx) - 45\cot^2(e+fx) - 315\right.}{315f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^(5/2),x]
```

```
[Out] (Cot[e + f*x]*(35 - 45*Cot[e + f*x]^2 + 63*Cot[e + f*x]^4 - 105*Cot[e + f*x
]^6 + 315*Cot[e + f*x]^8 - 315*ArcTan[Tan[e + f*x]]*Cot[e + f*x]^9)*(b*Tan[
e + f*x]^4)^(5/2))/(315*f)
```

fricas [A] time = 0.98, size = 96, normalized size = 0.53

$$\frac{\left(35\,b^2\tan\left(fx+e\right)^9 - 45\,b^2\tan\left(fx+e\right)^7 + 63\,b^2\tan\left(fx+e\right)^5 - 105\,b^2\tan\left(fx+e\right)^3 - 315\,b^2fx + 315\,b^2\tan\right.}{315\,f\tan\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 - 45*b^2*tan(f*x + e)^7 + 63*b^2*tan(f*x + e)^
5 - 105*b^2*tan(f*x + e)^3 - 315*b^2*f*x + 315*b^2*tan(f*x + e))*sqrt(b*tan
(f*x + e)^4)/(f*tan(f*x + e)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.22, size = 84, normalized size = 0.46

$$-\frac{\left(b\left(\tan^4\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(-35\left(\tan^9\left(fx+e\right)\right)+45\left(\tan^7\left(fx+e\right)\right)-63\left(\tan^5\left(fx+e\right)\right)+105\left(\tan^3\left(fx+e\right)\right)+315\right.}{315f\tan\left(fx+e\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^4)^(5/2),x)
```

```
[Out] -1/315/f*(b*tan(f*x+e)^4)^(5/2)*(-35*tan(f*x+e)^9+45*tan(f*x+e)^7-63*tan(f*
x+e)^5+105*tan(f*x+e)^3+315*arctan(tan(f*x+e))-315*tan(f*x+e))/tan(f*x+e)^1
0
```

maxima [A] time = 0.95, size = 79, normalized size = 0.43

$$\frac{35\,b^{\frac{5}{2}}\tan\left(fx+e\right)^9-45\,b^{\frac{5}{2}}\tan\left(fx+e\right)^7+63\,b^{\frac{5}{2}}\tan\left(fx+e\right)^5-105\,b^{\frac{5}{2}}\tan\left(fx+e\right)^3-315\left(fx+e\right)b^{\frac{5}{2}}+315\,b^{\frac{5}{2}}}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^(5/2)*tan(f*x + e)^9 - 45*b^(5/2)*tan(f*x + e)^7 + 63*b^(5/2)*t
an(f*x + e)^5 - 105*b^(5/2)*tan(f*x + e)^3 - 315*(f*x + e)*b^(5/2) + 315*b^
(5/2)*tan(f*x + e))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^4\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^4)^(5/2),x)
```

```
[Out] int((b*tan(e + f*x)^4)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan^{4}{\left(e + f x \right)}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**4)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**4)**(5/2), x)
```

3.14 $\int \left(b\tan^4(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=110

$$-\frac{b\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f}+\frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f}-bx\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}+\frac{b\cot(e+f}{}$$

```
[Out] b*cot(f*x+e)*(b*tan(f*x+e)^4)^(1/2)/f-b*x*cot(f*x+e)^2*(b*tan(f*x+e)^4)^(1/
2)-1/3*b*(b*tan(f*x+e)^4)^(1/2)*tan(f*x+e)/f+1/5*b*(b*tan(f*x+e)^4)^(1/2)*t
an(f*x+e)^3/f
```

Rubi [A] time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f}-\frac{b\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f}-bx\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}+\frac{b\cot(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^(3/2),x]
```

```
[Out] (b*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/f - b*x*Cot[e + f*x]^2*Sqrt[b*Tan[e
 + f*x]^4] - (b*Tan[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/(3*f) + (b*Tan[e + f*x
]^3*Sqrt[b*Tan[e + f*x]^4])/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^4(e+fx)\right)^{3/2} dx = \left(b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^6(e+fx)\,dx$$
$$= \frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f} - \left(b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^4(e+fx)\,dx$$
$$= -\frac{b\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f} + \left(b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^2(e+fx)\,dx$$
$$= \frac{b\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - \frac{b\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f} - \left(b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int 1\,dx$$
$$= \frac{b\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - bx\cot^2(e+fx)\sqrt{b\tan^4(e+fx)} - \frac{b\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f}$$

Mathematica [A] time = 0.75, size = 66, normalized size = 0.60

$$\frac{\cot(e+fx)\left(b\tan^4(e+fx)\right)^{3/2}\left(15\cot^4(e+fx)-5\cot^2(e+fx)-15\tan^{-1}(\tan(e+fx))\cot^5(e+fx)+3\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^(3/2),x]
```

```
[Out] (Cot[e + f*x]*(3 - 5*Cot[e + f*x]^2 + 15*Cot[e + f*x]^4 - 15*ArcTan[Tan[e +
 f*x]]*Cot[e + f*x]^5)*(b*Tan[e + f*x]^4)^(3/2))/(15*f)
```

fricas [A] time = 0.76, size = 62, normalized size = 0.56

$$\frac{\left(3\,b\tan\left(fx+e\right)^5-5\,b\tan\left(fx+e\right)^3-15\,bfx+15\,b\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^4}}{15\,f\tan\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 - 5*b*tan(f*x + e)^3 - 15*b*f*x + 15*b*tan(f*x + e
))*sqrt(b*tan(f*x + e)^4)/(f*tan(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)sq
rt(b)*b*(-60*f*x*tan(exp(1))^5*tan(f*x)^5+300*f*x*tan(exp(1))^4*tan(f*x)^4-
```

```
600*f*x*tan(exp(1))^3*tan(f*x)^3+600*f*x*tan(exp(1))^2*tan(f*x)^2-300*f*x*t
an(exp(1))*tan(f*x)+60*f*x-15*pi*sign(2*tan(exp(1))^2*tan(f*x)+2*tan(exp(1)
)*tan(f*x)^2-2*tan(exp(1))-2*tan(f*x))*tan(exp(1))^5*tan(f*x)^5+75*pi*sign(
2*tan(exp(1))^2*tan(f*x)+2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))-2*tan(f*x))
*tan(exp(1))^4*tan(f*x)^4-150*pi*sign(2*tan(exp(1))^2*tan(f*x)+2*tan(exp(1)
)*tan(f*x)^2-2*tan(exp(1))-2*tan(f*x))*tan(exp(1))^3*tan(f*x)^3+150*pi*sign
(2*tan(exp(1))^2*tan(f*x)+2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))-2*tan(f*x)
)*tan(exp(1))^2*tan(f*x)^2-75*pi*sign(2*tan(exp(1))^2*tan(f*x)+2*tan(exp(1)
)*tan(f*x)^2-2*tan(exp(1))-2*tan(f*x))*tan(exp(1))*tan(f*x)+15*pi*sign(2*ta
n(exp(1))^2*tan(f*x)+2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))-2*tan(f*x))-15*
pi*tan(exp(1))^5*tan(f*x)^5+75*pi*tan(exp(1))^4*tan(f*x)^4-150*pi*tan(exp(1
))^3*tan(f*x)^3+150*pi*tan(exp(1))^2*tan(f*x)^2-75*pi*tan(exp(1))*tan(f*x)+
15*pi+30*atan((tan(exp(1))*tan(f*x)-1)/(tan(exp(1))+tan(f*x)))*tan(exp(1))^
5*tan(f*x)^5-150*atan((tan(exp(1))*tan(f*x)-1)/(tan(exp(1))+tan(f*x)))*tan(
exp(1))^4*tan(f*x)^4+300*atan((tan(exp(1))*tan(f*x)-1)/(tan(exp(1))+tan(f*x
)))*tan(exp(1))^3*tan(f*x)^3-300*atan((tan(exp(1))*tan(f*x)-1)/(tan(exp(1))
+tan(f*x)))*tan(exp(1))^2*tan(f*x)^2+150*atan((tan(exp(1))*tan(f*x)-1)/(tan
(exp(1))+tan(f*x)))*tan(exp(1))*tan(f*x)-30*atan((tan(exp(1))*tan(f*x)-1)/(
tan(exp(1))+tan(f*x)))+30*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)
-1))*tan(exp(1))^5*tan(f*x)^5-150*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*
tan(f*x)-1))*tan(exp(1))^4*tan(f*x)^4+300*atan((tan(exp(1))+tan(f*x))/(tan(
exp(1))*tan(f*x)-1))*tan(exp(1))^3*tan(f*x)^3-300*atan((tan(exp(1))+tan(f*x
))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^2*tan(f*x)^2+150*atan((tan(exp(1))
+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))*tan(f*x)-30*atan((tan(exp(
1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))-60*tan(exp(1))^5*tan(f*x)^4+20*tan(
exp(1))^5*tan(f*x)^2-12*tan(exp(1))^5-60*tan(exp(1))^4*tan(f*x)^5+300*tan(e
xp(1))^4*tan(f*x)^3-100*tan(exp(1))^4*tan(f*x)+300*tan(exp(1))^3*tan(f*x)^4
-600*tan(exp(1))^3*tan(f*x)^2+20*tan(exp(1))^3+20*tan(exp(1))^2*tan(f*x)^5-
600*tan(exp(1))^2*tan(f*x)^3+300*tan(exp(1))^2*tan(f*x)-100*tan(exp(1))*tan
(f*x)^4+300*tan(exp(1))*tan(f*x)^2-60*tan(exp(1))-12*tan(f*x)^5+20*tan(f*x)
^3-60*tan(f*x))/(60*f*tan(exp(1))^5*tan(f*x)^5-300*f*tan(exp(1))^4*tan(f*x)
^4+600*f*tan(exp(1))^3*tan(f*x)^3-600*f*tan(exp(1))^2*tan(f*x)^2+300*f*tan(
exp(1))*tan(f*x)-60*f)
```

maple [A] time = 0.21, size = 64, normalized size = 0.58

$$-\frac{\left(b\left(\tan^4\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(-3\left(\tan^5\left(fx+e\right)\right)+5\left(\tan^3\left(fx+e\right)\right)+15\arctan\left(\tan\left(fx+e\right)\right)-15\tan\left(fx+e\right)\right)}{15f\tan\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^4)^(3/2),x)
```

```
[Out] -1/15/f*(b*tan(f*x+e)^4)^(3/2)*(-3*tan(f*x+e)^5+5*tan(f*x+e)^3+15*arctan(ta
n(f*x+e))-15*tan(f*x+e))/tan(f*x+e)^6
```

maxima [A] time = 0.85, size = 53, normalized size = 0.48

$$\frac{3\,b^{\frac{3}{2}}\tan\left(fx+e\right)^5-5\,b^{\frac{3}{2}}\tan\left(fx+e\right)^3-15\left(fx+e\right)b^{\frac{3}{2}}+15\,b^{\frac{3}{2}}\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^(3/2)*tan(f*x + e)^5 - 5*b^(3/2)*tan(f*x + e)^3 - 15*(f*x + e)*b^
(3/2) + 15*b^(3/2)*tan(f*x + e))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan\left(e+f\,x\right)^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^4)^(3/2),x)
[Out] int((b*tan(e + f*x)^4)^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^4\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**4)**(3/2),x)
[Out] Integral((b*tan(e + f*x)**4)**(3/2), x)
```

3.15 $\int \sqrt{b\tan^4(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}-x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}$$

```
[Out] cot(f*x+e)*(b*tan(f*x+e)^4)^(1/2)/f-x*cot(f*x+e)^2*(b*tan(f*x+e)^4)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}-x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]^4],x]
```

```
[Out] (Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/f - x*Cot[e + f*x]^2*Sqrt[b*Tan[e + f
*x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\tan^4(e+fx)}\,dx &= \left(\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^2(e+fx)\,dx\\ &= \frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}-\left(\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int 1\,dx\\ &= \frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}-x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\end{aligned}$$

Mathematica [A] time = 0.10, size = 41, normalized size = 0.82

$$-\frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}\left(\tan^{-1}(\tan(e+fx))\cot(e+fx)-1\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]^4],x]
```

```
[Out] -((Cot[e + f*x]*(-1 + ArcTan[Tan[e + f*x]]*Cot[e + f*x])*Sqrt[b*Tan[e + f*x
]^4])/f)
```

fricas [A] time = 0.85, size = 37, normalized size = 0.74

$$-\frac{\sqrt{b\tan\left(fx+e\right)^{4}}\left(fx-\tan\left(fx+e\right)\right)}{f\tan\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*tan(f*x + e)^4)*(f*x - tan(f*x + e))/(f*tan(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)sqrt(b)*(-4*f*x*tan(exp(1))*tan(f*x)+4*f*x-pi*sign(2*t
an(exp(1))^2*tan(f*x)+2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))-2*tan(f*x))*ta
n(exp(1))*tan(f*x)+pi*sign(2*tan(exp(1))^2*tan(f*x)+2*tan(exp(1))*tan(f*x)^
2-2*tan(exp(1))-2*tan(f*x))-pi*tan(exp(1))*tan(f*x)+pi+2*atan((tan(exp(1))*
tan(f*x)-1)/(tan(exp(1))+tan(f*x)))*tan(exp(1))*tan(f*x)-2*atan((tan(exp(1)
)*tan(f*x)-1)/(tan(exp(1))+tan(f*x)))+2*atan((tan(exp(1))+tan(f*x))/(tan(ex
p(1))*tan(f*x)-1))*tan(exp(1))*tan(f*x)-2*atan((tan(exp(1))+tan(f*x))/(tan(
exp(1))*tan(f*x)-1))-4*tan(exp(1))-4*tan(f*x))/(4*f*tan(exp(1))*tan(f*x)-4*
f)
```

maple [A] time = 0.23, size = 42, normalized size = 0.84

$$-\frac{\sqrt{b\left(\tan^{4}\left(fx+e\right)\right)}\,\left(-\tan\left(fx+e\right)+\arctan\left(\tan\left(fx+e\right)\right)\right)}{f\tan\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^4)^(1/2),x)
```

```
[Out] -1/f*(b*tan(f*x+e)^4)^(1/2)*(-tan(f*x+e)+arctan(tan(f*x+e)))/tan(f*x+e)^2
```

maxima [A] time = 1.01, size = 26, normalized size = 0.52

$$-\frac{\left(fx+e\right)\sqrt{b}\;-\;\sqrt{b}\,\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -((f*x + e)*sqrt(b) - sqrt(b)*tan(f*x + e))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\tan\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^4)^(1/2),x)
[Out] int((b*tan(e + f*x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan^4\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**4)**(1/2),x)
[Out] Integral(sqrt(b*tan(e + f*x)**4), x)
```

3.16 $\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}}-\frac{x\tan^2(e+fx)}{\sqrt{b\tan^4(e+fx)}}$$

```
[Out] -tan(f*x+e)/f/(b*tan(f*x+e)^4)^(1/2)-x*tan(f*x+e)^2/(b*tan(f*x+e)^4)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$-\frac{x\tan^2(e+fx)}{\sqrt{b\tan^4(e+fx)}}-\frac{\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[e + f*x]^4],x]
```

```
[Out] -(Tan[e + f*x]/(f*Sqrt[b*Tan[e + f*x]^4])) - (x*Tan[e + f*x]^2)/Sqrt[b*Tan[
e + f*x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx &= \frac{\tan^2(e+fx)\int \cot^2(e+fx)\,dx}{\sqrt{b\tan^4(e+fx)}}\\ &= -\frac{\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}}-\frac{\tan^2(e+fx)\int 1\,dx}{\sqrt{b\tan^4(e+fx)}}\\ &= -\frac{\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}}-\frac{x\tan^2(e+fx)}{\sqrt{b\tan^4(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.05, size = 43, normalized size = 0.84

$$-\frac{\tan(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(e+fx)\right)}{f\sqrt{b\tan^4(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[e + f*x]^4],x]
```

```
[Out] -((Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*Sqrt[b
*Tan[e + f*x]^4]))
```

fricas [A] time = 0.85, size = 39, normalized size = 0.76

$$-\frac{\sqrt{b\tan\left(fx+e\right)^4}\left(fx\tan\left(fx+e\right)+1\right)}{bf\tan\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*tan(f*x + e)^4)*(f*x*tan(f*x + e) + 1)/(b*f*tan(f*x + e)^3)
```

giac [A] time = 0.57, size = 48, normalized size = 0.94

$$-\frac{\frac{2\left(fx+e\right)}{\sqrt{b}}-\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{b}}+\frac{1}{\sqrt{b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(f*x + e)/sqrt(b) - tan(1/2*f*x + 1/2*e)/sqrt(b) + 1/(sqrt(b)*tan(1
/2*f*x + 1/2*e)))/f
```

maple [A] time = 0.27, size = 40, normalized size = 0.78

$$-\frac{\tan\left(fx+e\right)\left(\arctan\left(\tan\left(fx+e\right)\right)\tan\left(fx+e\right)+1\right)}{f\sqrt{b\left(\tan^4\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^4)^(1/2),x)
```

```
[Out] -1/f*tan(f*x+e)*(arctan(tan(f*x+e))*tan(f*x+e)+1)/(b*tan(f*x+e)^4)^(1/2)
```

maxima [A] time = 1.19, size = 27, normalized size = 0.53

$$-\frac{\frac{fx+e}{\sqrt{b}}+\frac{1}{\sqrt{b}\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -((f*x + e)/sqrt(b) + 1/(sqrt(b)*tan(f*x + e)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\tan\left(e+f\,x\right)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^4)^(1/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tan^4\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(e + f*x)**4), x)
```

3.17 $\int \frac{1}{\left(b\tan^4(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{\tan(e+fx)}{bf\sqrt{b\tan^4(e+fx)}}-\frac{x\tan^2(e+fx)}{b\sqrt{b\tan^4(e+fx)}}-\frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}}+\frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}}$$

```
[Out] 1/3*cot(f*x+e)/b/f/(b*tan(f*x+e)^4)^(1/2)-1/5*cot(f*x+e)^3/b/f/(b*tan(f*x+e
)^4)^(1/2)-tan(f*x+e)/b/f/(b*tan(f*x+e)^4)^(1/2)-x*tan(f*x+e)^2/b/(b*tan(f*
x+e)^4)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$-\frac{x\tan^2(e+fx)}{b\sqrt{b\tan^4(e+fx)}}-\frac{\tan(e+fx)}{bf\sqrt{b\tan^4(e+fx)}}-\frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}}+\frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^(-3/2),x]
```

```
[Out] Cot[e + f*x]/(3*b*f*Sqrt[b*Tan[e + f*x]^4]) - Cot[e + f*x]^3/(5*b*f*Sqrt[b*
Tan[e + f*x]^4]) - Tan[e + f*x]/(b*f*Sqrt[b*Tan[e + f*x]^4]) - (x*Tan[e + f
*x]^2)/(b*Sqrt[b*Tan[e + f*x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{3/2}}\,dx = \frac{\tan^2(e+fx)\int \cot^6(e+fx)\,dx}{b\sqrt{b\tan^4(e+fx)}}$$
$$= -\frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}} - \frac{\tan^2(e+fx)\int \cot^4(e+fx)\,dx}{b\sqrt{b\tan^4(e+fx)}}$$
$$= \frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}} + \frac{\tan^2(e+fx)\int \cot^2(e+fx)\,dx}{b\sqrt{b\tan^4(e+fx)}}$$
$$= \frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}} - \frac{\tan(e+fx)}{bf\sqrt{b\tan^4(e+fx)}} - \frac{\tan^2(e+fx)\int}{b\sqrt{b\tan^4(e+}$$
$$= \frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}} - \frac{\tan(e+fx)}{bf\sqrt{b\tan^4(e+fx)}} - \frac{x\tan^2(e+fx}{b\sqrt{b\tan^4(e+f}$$

Mathematica [C] time = 0.05, size = 45, normalized size = 0.38

$$-\frac{\tan(e+fx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(e+fx)\right)}{5f\left(b\tan^4(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^(-3/2),x]
```

```
[Out] -1/5*(Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*(b
*Tan[e + f*x]^4)^(3/2))
```

fricas [A] time = 0.74, size = 62, normalized size = 0.52

$$-\frac{\left(15\,fx\tan\left(fx+e\right)^5+15\,\tan\left(fx+e\right)^4-5\,\tan\left(fx+e\right)^2+3\right)\sqrt{b\tan\left(fx+e\right)^4}}{15\,b^2f\tan\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(15*f*x*tan(f*x + e)^5 + 15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)*sq
rt(b*tan(f*x + e)^4)/(b^2*f*tan(f*x + e)^7)
```

giac [A] time = 2.25, size = 131, normalized size = 1.10

$$-\frac{\frac{480\left(fx+e\right)}{\sqrt{b}} - \frac{3\,b^{\frac{9}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,b^{\frac{9}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+330\,b^{\frac{9}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{b^5} + \frac{330\,\sqrt{b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-35\,\sqrt{b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,\sqrt{b}}{b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}}{480\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -1/480*(480*(f*x + e)/sqrt(b) - (3*b^(9/2)*tan(1/2*f*x + 1/2*e)^5 - 35*b^(9
/2)*tan(1/2*f*x + 1/2*e)^3 + 330*b^(9/2)*tan(1/2*f*x + 1/2*e))/b^5 + (330*s
qrt(b)*tan(1/2*f*x + 1/2*e)^4 - 35*sqrt(b)*tan(1/2*f*x + 1/2*e)^2 + 3*sqrt(
b))/(b*tan(1/2*f*x + 1/2*e)^5))/(b*f)
```

maple [A]   time = 0.23, size = 63, normalized size = 0.53

$$-\frac{\tan\left(fx+e\right)\left(15\arctan\left(\tan\left(fx+e\right)\right)\left(\tan^{5}\left(fx+e\right)\right)+15\left(\tan^{4}\left(fx+e\right)\right)-5\left(\tan^{2}\left(fx+e\right)\right)+3\right)}{15f\left(b\left(\tan^{4}\left(fx+e\right)\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^4)^(3/2),x)
```

```
[Out] -1/15/f*tan(f*x+e)*(15*arctan(tan(f*x+e))*tan(f*x+e)^5+15*tan(f*x+e)^4-5*ta
n(f*x+e)^2+3)/(b*tan(f*x+e)^4)^(3/2)
```

maxima [A]   time = 0.88, size = 50, normalized size = 0.42

$$-\frac{\frac{15\left(fx+e\right)}{b^{\frac{3}{2}}}+\frac{15\tan\left(fx+e\right)^{4}-5\tan\left(fx+e\right)^{2}+3}{b^{\frac{3}{2}}\tan\left(fx+e\right)^{5}}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(f*x + e)/b^(3/2) + (15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)/(b
^(3/2)*tan(f*x + e)^5))/f
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\tan\left(e+f\,x\right)^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^4)^(3/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^4)^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan^{4}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**4)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**4)**(-3/2), x)
```

3.18 $\int \frac{1}{\left(b\tan^4(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{\tan(e+fx)}{b^2f\sqrt{b\tan^4(e+fx)}}-\frac{x\tan^2(e+fx)}{b^2\sqrt{b\tan^4(e+fx)}}-\frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}}+\frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}}-\frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}}$$

```
[Out] 1/3*cot(f*x+e)/b^2/f/(b*tan(f*x+e)^4)^(1/2)-1/5*cot(f*x+e)^3/b^2/f/(b*tan(f
*x+e)^4)^(1/2)+1/7*cot(f*x+e)^5/b^2/f/(b*tan(f*x+e)^4)^(1/2)-1/9*cot(f*x+e)
^7/b^2/f/(b*tan(f*x+e)^4)^(1/2)-tan(f*x+e)/b^2/f/(b*tan(f*x+e)^4)^(1/2)-x*t
an(f*x+e)^2/b^2/(b*tan(f*x+e)^4)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$-\frac{x\tan^2(e+fx)}{b^2\sqrt{b\tan^4(e+fx)}}-\frac{\tan(e+fx)}{b^2f\sqrt{b\tan^4(e+fx)}}-\frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}}+\frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}}-\frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^(-5/2),x]
```

```
[Out] Cot[e + f*x]/(3*b^2*f*Sqrt[b*Tan[e + f*x]^4]) - Cot[e + f*x]^3/(5*b^2*f*Sqr
t[b*Tan[e + f*x]^4]) + Cot[e + f*x]^5/(7*b^2*f*Sqrt[b*Tan[e + f*x]^4]) - Co
t[e + f*x]^7/(9*b^2*f*Sqrt[b*Tan[e + f*x]^4]) - Tan[e + f*x]/(b^2*f*Sqrt[b*
Tan[e + f*x]^4]) - (x*Tan[e + f*x]^2)/(b^2*Sqrt[b*Tan[e + f*x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{5/2}}\,dx = \frac{\tan^2(e+fx)\int \cot^{10}(e+fx)\,dx}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$= -\frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\tan^2(e+fx)\int \cot^8(e+fx)\,dx}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$= \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\tan^2(e+fx)\int \cot^6(e+fx)\,dx}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$= -\frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}} - \tan^2(e$$

$$= \frac{\cot(e+fx)}{3b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot}{9b^2f\sqrt{b}}$$

$$= \frac{\cot(e+fx)}{3b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot}{9b^2f\sqrt{b}}$$

$$= \frac{\cot(e+fx)}{3b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot}{9b^2f\sqrt{b}}$$

Mathematica [C] time = 0.03, size = 45, normalized size = 0.25

$$-\frac{\tan(e+fx)\,{}_2F_1\left(-\frac{9}{2},1;-\frac{7}{2};-\tan^2(e+fx)\right)}{9f\left(b\tan^4(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^(-5/2),x]
```

```
[Out] -1/9*(Hypergeometric2F1[-9/2, 1, -7/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*(b
*Tan[e + f*x]^4)^(5/2))
```

fricas [A] time = 0.97, size = 82, normalized size = 0.45

$$-\frac{\left(315\,fx\tan\left(fx+e\right)^9 + 315\,\tan\left(fx+e\right)^8 - 105\,\tan\left(fx+e\right)^6 + 63\,\tan\left(fx+e\right)^4 - 45\,\tan\left(fx+e\right)^2 + 35\right)}{315\,b^3 f\tan\left(fx+e\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/315*(315*f*x*tan(f*x + e)^9 + 315*tan(f*x + e)^8 - 105*tan(f*x + e)^6 +
63*tan(f*x + e)^4 - 45*tan(f*x + e)^2 + 35)*sqrt(b*tan(f*x + e)^4)/(b^3*f*t
an(f*x + e)^11)
```

giac [A] time = 4.88, size = 196, normalized size = 1.07

$$-\frac{\frac{161280\left(fx+e\right)}{b^{\frac{5}{2}}} + \frac{121590\sqrt{b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8 - 18480\sqrt{b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6 + 3528\sqrt{b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4 - 495\sqrt{b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 35\sqrt{b}}{b^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9} - \frac{35\,b}{}}{161280\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/161280*(161280*(f*x + e)/b^(5/2) + (121590*sqrt(b)*tan(1/2*f*x + 1/2*e)^
8 - 18480*sqrt(b)*tan(1/2*f*x + 1/2*e)^6 + 3528*sqrt(b)*tan(1/2*f*x + 1/2*e
)^4 - 495*sqrt(b)*tan(1/2*f*x + 1/2*e)^2 + 35*sqrt(b))/(b^3*tan(1/2*f*x + 1
/2*e)^9) - (35*b^(49/2)*tan(1/2*f*x + 1/2*e)^9 - 495*b^(49/2)*tan(1/2*f*x +
 1/2*e)^7 + 3528*b^(49/2)*tan(1/2*f*x + 1/2*e)^5 - 18480*b^(49/2)*tan(1/2*f
*x + 1/2*e)^3 + 121590*b^(49/2)*tan(1/2*f*x + 1/2*e))/b^27)/f
```

maple [A] time = 0.25, size = 83, normalized size = 0.45

$$-\frac{\tan\left(fx+e\right)\left(315\arctan\left(\tan\left(fx+e\right)\right)\left(\tan^{9}\left(fx+e\right)\right)+315\left(\tan^{8}\left(fx+e\right)\right)-105\left(\tan^{6}\left(fx+e\right)\right)+63\left(\tan^{4}\right.\right.}{315f\left(b\left(\tan^{4}\left(fx+e\right)\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^4)^(5/2),x)
```

```
[Out] -1/315/f*tan(f*x+e)*(315*arctan(tan(f*x+e))*tan(f*x+e)^9+315*tan(f*x+e)^8-1
05*tan(f*x+e)^6+63*tan(f*x+e)^4-45*tan(f*x+e)^2+35)/(b*tan(f*x+e)^4)^(5/2)
```

maxima [A] time = 0.68, size = 70, normalized size = 0.38

$$-\frac{\frac{315\left(fx+e\right)}{b^{\frac{5}{2}}}+\frac{315\tan\left(fx+e\right)^{8}-105\tan\left(fx+e\right)^{6}+63\tan\left(fx+e\right)^{4}-45\tan\left(fx+e\right)^{2}+35}{b^{\frac{5}{2}}\tan\left(fx+e\right)^{9}}}{315f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/315*(315*(f*x + e)/b^(5/2) + (315*tan(f*x + e)^8 - 105*tan(f*x + e)^6 +
63*tan(f*x + e)^4 - 45*tan(f*x + e)^2 + 35)/(b^(5/2)*tan(f*x + e)^9))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^4\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^4)^(5/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^4)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan^{4}\left(e+fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**4)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**4)**(-5/2), x)
```

3.19 $\int \left(b\tan^n(e+fx)\right)^{5/2}\,dx$

Optimal. Leaf size=71

$$\frac{2b^2\tan^{2n+1}(e+fx)\sqrt{b\tan^n(e+fx)}\,{}_2F_1\left(1,\frac{1}{4}(5n+2);\frac{1}{4}(5n+6);-\tan^2(e+fx)\right)}{f(5n+2)}$$

```
[Out] 2*b^2*hypergeom([1, 1/2+5/4*n],[3/2+5/4*n],-tan(f*x+e)^2)*(b*tan(f*x+e)^n)^
(1/2)*tan(f*x+e)^(1+2*n)/f/(2+5*n)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3659, 3476, 364}

$$\frac{2b^2\tan^{2n+1}(e+fx)\sqrt{b\tan^n(e+fx)}\,{}_2F_1\left(1,\frac{1}{4}(5n+2);\frac{1}{4}(5n+6);-\tan^2(e+fx)\right)}{f(5n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^(5/2),x]
```

```
[Out] (2*b^2*Hypergeometric2F1[1, (2 + 5*n)/4, (6 + 5*n)/4, -Tan[e + f*x]^2]*Tan[
e + f*x]^(1 + 2*n)*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + 5*n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(b\tan^n(e+fx)\right)^{5/2}\,dx &= \left(b^2\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\int \tan^{\frac{5n}{2}}(e+fx)\,dx\\
&= \frac{\left(b^2\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\text{Subst}\left(\int \frac{x^{5n/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{2b^2\,{}_2F_1\left(1,\frac{1}{4}(2+5n);\frac{1}{4}(6+5n);-\tan^2(e+fx)\right)\tan^{1+2n}(e+fx)\sqrt{b\tan^n(e+fx)}}{f(2+5n)}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 62, normalized size = 0.87

$$\frac{2\tan(e+fx)\left(b\tan^n(e+fx)\right)^{5/2}\,{}_2F_1\left(1,\frac{1}{4}(5n+2);\frac{1}{4}(5n+6);-\tan^2(e+fx)\right)}{f(5n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^(5/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + 5*n)/4, (6 + 5*n)/4, -Tan[e + f*x]^2]*Tan[e +
f*x]*(b*Tan[e + f*x]^n)^(5/2))/(f*(2 + 5*n))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^n\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(5/2), x)
```

maple [F] time = 12.31, size = 0, normalized size = 0.00

$$\int \left(b\left(\tan^n\left(fx+e\right)\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^(5/2),x)
```

```
[Out] int((b*tan(f*x+e)^n)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^n\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^n\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^n)^(5/2),x)
```

```
[Out] int((b*tan(e + f*x)^n)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^n\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**n)**(5/2), x)
```

3.20 $\int \left(b \tan^n(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=65

$$\frac{2b \tan^{n+1}(e+fx)\sqrt{b \tan^n(e+fx)}\,{}_2F_1\left(1,\frac{1}{4}(3n+2);\frac{3(n+2)}{4};-\tan^2(e+fx)\right)}{f(3n+2)}$$

```
[Out] 2*b*hypergeom([1, 1/2+3/4*n],[3/2+3/4*n],-tan(f*x+e)^2)*(b*tan(f*x+e)^n)^(1
/2)*tan(f*x+e)^(1+n)/f/(2+3*n)
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2b \tan^{n+1}(e+fx)\sqrt{b \tan^n(e+fx)}\,{}_2F_1\left(1,\frac{1}{4}(3n+2);\frac{3(n+2)}{4};-\tan^2(e+fx)\right)}{f(3n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^(3/2),x]
```

```
[Out] (2*b*Hypergeometric2F1[1, (2 + 3*n)/4, (3*(2 + n))/4, -Tan[e + f*x]^2]*Tan[
e + f*x]^(1 + n)*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + 3*n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(b \tan^n(e+fx)\right)^{3/2} dx &= \left(b \tan^{-\frac{n}{2}}(e+fx)\sqrt{b \tan^n(e+fx)}\right)\int \tan^{\frac{3n}{2}}(e+fx)\,dx\\
&= \frac{\left(b \tan^{-\frac{n}{2}}(e+fx)\sqrt{b \tan^n(e+fx)}\right)\operatorname{Subst}\left(\int \frac{x^{3n/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{2b\,{}_2F_1\left(1,\frac{1}{4}(2+3n);\frac{3(2+n)}{4};-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)\sqrt{b \tan^n(e+fx)}}{f(2+3n)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 0.92

$$\frac{2\tan(e+fx)\left(b\tan^n(e+fx)\right)^{3/2}\,{}_2F_1\left(1,\frac{1}{4}(3n+2);\frac{3(n+2)}{4};-\tan^2(e+fx)\right)}{f(3n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^(3/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + 3*n)/4, (3*(2 + n))/4, -Tan[e + f*x]^2]*Tan[e
+ f*x]*(b*Tan[e + f*x]^n)^(3/2))/(f*(2 + 3*n))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(3/2), x)
```

maple [F] time = 1.22, size = 0, normalized size = 0.00

$$\int \left(b\left(\tan^{n}\left(fx+e\right)\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^(3/2),x)
```

```
[Out] int((b*tan(f*x+e)^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^n\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^n)^(3/2),x)
```

```
[Out] int((b*tan(e + f*x)^n)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^{n}\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**n)**(3/2), x)
```

3.21 $\int \sqrt{b \tan^n(e+fx)}\,dx$

Optimal. Leaf size=56

$$\frac{2\tan(e+fx)\sqrt{b\tan^n(e+fx)}\;{}_2F_1\left(1,\frac{n+2}{4};\frac{n+6}{4};-\tan^2(e+fx)\right)}{f(n+2)}$$

```
[Out] 2*hypergeom([1, 1/2+1/4*n],[3/2+1/4*n],-tan(f*x+e)^2)*(b*tan(f*x+e)^n)^(1/2
)*tan(f*x+e)/f/(2+n)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3659, 3476, 364}

$$\frac{2\tan(e+fx)\sqrt{b\tan^n(e+fx)}\;{}_2F_1\left(1,\frac{n+2}{4};\frac{n+6}{4};-\tan^2(e+fx)\right)}{f(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]^n],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + n)/4, (6 + n)/4, -Tan[e + f*x]^2]*Tan[e + f*x]
*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{b\tan^n(e+fx)}\,dx &= \left(\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\int \tan^{\frac{n}{2}}(e+fx)\,dx\\
&= \frac{\left(\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\text{Subst}\left(\int \frac{x^{n/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{2\,{}_2F_1\left(1,\frac{2+n}{4};\frac{6+n}{4};-\tan^2(e+fx)\right)\tan(e+fx)\sqrt{b\tan^n(e+fx)}}{f(2+n)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 1.00

$$\frac{2\tan(e+fx)\sqrt{b\tan^n(e+fx)}\,{}_2F_1\left(1,\frac{n+2}{4};\frac{n+6}{4};-\tan^2(e+fx)\right)}{f(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]^n],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + n)/4, (6 + n)/4, -Tan[e + f*x]^2]*Tan[e + f*x]
*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + n))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan\left(fx+e\right)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^n), x)
```

maple [F] time = 1.27, size = 0, normalized size = 0.00

$$\int \sqrt{b\left(\tan^n\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^(1/2),x)
```

```
[Out] int((b*tan(f*x+e)^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan\left(fx+e\right)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\,{\tan\left(e+f\,x\right)}^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^n)^(1/2),x)
```

```
[Out] int((b*tan(e + f*x)^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan^{n}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**(1/2),x)
[Out] Integral(sqrt(b*tan(e + f*x)**n), x)
```

3.22 $\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\tan(e+fx)\,{}_2F_1\left(1,\frac{2-n}{4};\frac{6-n}{4};-\tan^2(e+fx)\right)}{f(2-n)\sqrt{b\tan^n(e+fx)}}$$

```
[Out] 2*hypergeom([1, 1/2-1/4*n],[3/2-1/4*n],-tan(f*x+e)^2)*tan(f*x+e)/f/(2-n)/(b
*tan(f*x+e)^n)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\tan(e+fx)\,{}_2F_1\left(1,\frac{2-n}{4};\frac{6-n}{4};-\tan^2(e+fx)\right)}{f(2-n)\sqrt{b\tan^n(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[e + f*x]^n],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - n)/4, (6 - n)/4, -Tan[e + f*x]^2]*Tan[e + f*x]
)/(f*(2 - n)*Sqrt[b*Tan[e + f*x]^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx = \frac{\tan^{\frac{n}{2}}(e+fx)\int \tan^{-\frac{n}{2}}(e+fx)\,dx}{\sqrt{b\tan^n(e+fx)}}$$
$$= \frac{\tan^{\frac{n}{2}}(e+fx)\,\text{Subst}\left(\int \frac{x^{-n/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f\sqrt{b\tan^n(e+fx)}}$$
$$= \frac{2\,{}_2F_1\left(1, \frac{2-n}{4}; \frac{6-n}{4}; -\tan^2(e+fx)\right)\tan(e+fx)}{f(2-n)\sqrt{b\tan^n(e+fx)}}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 0.97

$$-\frac{2\tan(e+fx)\,{}_2F_1\left(1, \frac{2-n}{4}; \frac{6-n}{4}; -\tan^2(e+fx)\right)}{f(n-2)\sqrt{b\tan^n(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[e + f*x]^n],x]
[Out] (-2*Hypergeometric2F1[1, (2 - n)/4, (6 - n)/4, -Tan[e + f*x]^2]*Tan[e + f*x
])/(f*(-2 + n)*Sqrt[b*Tan[e + f*x]^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tan\left(fx+e\right)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*tan(f*x + e)^n), x)
```

maple [F] time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\left(\tan^n\left(fx+e\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^n)^(1/2),x)
[Out] int(1/(b*tan(f*x+e)^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tan\left(fx+e\right)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*tan(f*x + e)^n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\tan\left(e+f\,x\right)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^n)^(1/2),x)
[Out] int(1/(b*tan(e + f*x)^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tan^{n}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**n)**(1/2),x)
[Out] Integral(1/sqrt(b*tan(e + f*x)**n), x)
```

3.23 $\int \frac{1}{\left(b\tan^n(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan^{1-n}(e+fx)\,{}_2F_1\left(1,\frac{1}{4}(2-3n);\frac{3(2-n)}{4};-\tan^2(e+fx)\right)}{bf(2-3n)\sqrt{b\tan^n(e+fx)}}$$

```
[Out] 2*hypergeom([1, 1/2-3/4*n],[3/2-3/4*n],-tan(f*x+e)^2)*tan(f*x+e)^(1-n)/b/f/
(2-3*n)/(b*tan(f*x+e)^n)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\tan^{1-n}(e+fx)\,{}_2F_1\left(1,\frac{1}{4}(2-3n);\frac{3(2-n)}{4};-\tan^2(e+fx)\right)}{bf(2-3n)\sqrt{b\tan^n(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^(-3/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - 3*n)/4, (3*(2 - n))/4, -Tan[e + f*x]^2]*Tan[e
+ f*x]^(1 - n))/(b*f*(2 - 3*n)*Sqrt[b*Tan[e + f*x]^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^n(e+fx)\right)^{3/2}}\,dx = \frac{\tan^{\frac{n}{2}}(e+fx)\int \tan^{-\frac{3n}{2}}(e+fx)\,dx}{b\sqrt{b\tan^n(e+fx)}}$$
$$= \frac{\tan^{\frac{n}{2}}(e+fx)\,\text{Subst}\left(\int \frac{x^{-3n/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{bf\sqrt{b\tan^n(e+fx)}}$$
$$= \frac{2\,{}_2F_1\left(1,\frac{1}{4}(2-3n);\frac{3(2-n)}{4};-\tan^2(e+fx)\right)\tan^{1-n}(e+fx)}{bf(2-3n)\sqrt{b\tan^n(e+fx)}}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 0.85

$$-\frac{2\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{4}(2-3n);-\frac{3}{4}(n-2);-\tan^2(e+fx)\right)}{f(3n-2)\left(b\tan^n(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^(-3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[1, (2 - 3*n)/4, (-3*(-2 + n))/4, -Tan[e + f*x]^2]*Tan
[e + f*x])/(f*(-2 + 3*n)*(b*Tan[e + f*x]^n)^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(-3/2), x)
```

maple [F] time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\tan^n\left(fx+e\right)\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^n)^(3/2),x)
```

```
[Out] int(1/(b*tan(f*x+e)^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\tan\left(e+f\,x\right)^{n}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^n)^(3/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan^{n}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**n)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**n)**(-3/2), x)
```

3.24 $\int \frac{1}{\left(b\tan^n(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan^{1-2n}(e+fx)\,{}_2F_1\left(1,\frac{1}{4}(2-5n);\frac{1}{4}(6-5n);-\tan^2(e+fx)\right)}{b^2f(2-5n)\sqrt{b\tan^n(e+fx)}}$$

```
[Out] 2*hypergeom([1, 1/2-5/4*n],[3/2-5/4*n],-tan(f*x+e)^2)*tan(f*x+e)^(1-2*n)/b^
2/f/(2-5*n)/(b*tan(f*x+e)^n)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\tan^{1-2n}(e+fx)\,{}_2F_1\left(1,\frac{1}{4}(2-5n);\frac{1}{4}(6-5n);-\tan^2(e+fx)\right)}{b^2f(2-5n)\sqrt{b\tan^n(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^(-5/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - 5*n)/4, (6 - 5*n)/4, -Tan[e + f*x]^2]*Tan[e +
f*x]^(1 - 2*n))/(b^2*f*(2 - 5*n)*Sqrt[b*Tan[e + f*x]^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^n(e+fx)\right)^{5/2}}\,dx = \frac{\tan^{\frac{n}{2}}(e+fx)\int \tan^{-\frac{5n}{2}}(e+fx)\,dx}{b^2\sqrt{b\tan^n(e+fx)}}$$
$$= \frac{\tan^{\frac{n}{2}}(e+fx)\,\text{Subst}\left(\int \frac{x^{-5n/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{b^2 f\sqrt{b\tan^n(e+fx)}}$$
$$= \frac{2\,{}_2F_1\left(1, \frac{1}{4}(2-5n); \frac{1}{4}(6-5n); -\tan^2(e+fx)\right)\tan^{1-2n}(e+fx)}{b^2 f(2-5n)\sqrt{b\tan^n(e+fx)}}$$

Mathematica [A] time = 0.07, size = 62, normalized size = 0.87

$$-\frac{2\tan(e+fx)\,{}_2F_1\left(1, \frac{1}{4}(2-5n); \frac{1}{4}(6-5n); -\tan^2(e+fx)\right)}{f(5n-2)\left(b\tan^n(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^(-5/2),x]
[Out] (-2*Hypergeometric2F1[1, (2 - 5*n)/4, (6 - 5*n)/4, -Tan[e + f*x]^2]*Tan[e +
 f*x])/(f*(-2 + 5*n)*(b*Tan[e + f*x]^n)^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^n\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^n)^(-5/2), x)
```

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\tan^n\left(fx+e\right)\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^n)^(5/2),x)
[Out] int(1/(b*tan(f*x+e)^n)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \tan\left(fx+e\right)^{n}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(5/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^n)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b \tan\left(e+f\,x\right)^{n}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^n)^(5/2),x)
[Out] int(1/(b*tan(e + f*x)^n)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \tan^{n}\left(e+fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**n)**(5/2),x)
[Out] Integral((b*tan(e + f*x)**n)**(-5/2), x)
```

3.25 $\int \left(b\tan^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=59

$$\frac{\tan(e+fx)\left(b\tan^n(e+fx)\right)^p\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)}{f(np+1)}$$

```
[Out] hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*tan(f
*x+e)^n)^p/f/(n*p+1)
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3659, 3476, 364}

$$\frac{\tan(e+fx)\left(b\tan^n(e+fx)\right)^p\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^n)^p)/(f*(1 + n*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(b\tan^n(e+fx)\right)^p\,dx &= \left(\tan^{-np}(e+fx)\left(b\tan^n(e+fx)\right)^p\right)\int \tan^{np}(e+fx)\,dx\\ &= \frac{\left(\tan^{-np}(e+fx)\left(b\tan^n(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{np}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^n(e+fx)\right)^p}{f(1+np)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.97

$$\frac{\tan(e+fx)\left(b\tan^n(e+fx)\right)^p\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)}{fnp+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^p,x]
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^n)^p)/(f + f*n*p)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\tan\left(fx+e\right)^n\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^n)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^n)^p, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int\left(b\left(\tan^n\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^p,x)
[Out] int((b*tan(f*x+e)^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^n)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(b\,\tan\left(e+f\,x\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^n)^p,x)
```

```
[Out] int((b*tan(e + f*x)^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^{n}\left(e+fx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**p,x)
[Out] Integral((b*tan(e + f*x)**n)**p, x)
```

3.26 $\int \left(b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=59

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p\,{}_2F_1\left(1,\frac{1}{2}(2p+1);\frac{1}{2}(2p+3);-\tan^2(e+fx)\right)}{f(2p+1)}$$

```
[Out] hypergeom([1, 1/2+p],[3/2+p],-tan(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+e)^2)^p/f
/(1+2*p)
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3658, 3476, 364}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p\,{}_2F_1\left(1,\frac{1}{2}(2p+1);\frac{1}{2}(2p+3);-\tan^2(e+fx)\right)}{f(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 2*p)/2, (3 + 2*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(b\tan^2(e+fx)\right)^p\,dx &= \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int \tan^{2p}(e+fx)\,dx\\
&= \frac{\left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{2p}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{{}_2F_1\left(1,\frac{1}{2}(1+2p);\frac{1}{2}(3+2p);-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^2(e+fx)\right)^p}{f(1+2p)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.83

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p {}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};-\tan^2(e+fx)\right)}{2fp+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 1/2 + p, 3/2 + p, -Tan[e + f*x]^2]*Tan[e + f*x]*(b*Ta
n[e + f*x]^2)^p)/(f + 2*f*p)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\tan\left(fx+e\right)^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p, x)
```

maple [F] time = 1.23, size = 0, normalized size = 0.00

$$\int\left(b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(b\tan\left(e+fx\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^2)^p,x)
```

```
[Out] int((b*tan(e + f*x)^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^2\left(e+fx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**p, x)
```

3.27 $\int \left(b\tan^3(e+fx)\right)^p\,dx$

Optimal. Leaf size=57

$$\frac{\tan(e+fx)\left(b\tan^3(e+fx)\right)^p\,{}_2F_1\left(1,\frac{1}{2}(3p+1);\frac{3(p+1)}{2};-\tan^2(e+fx)\right)}{f(3p+1)}$$

```
[Out] hypergeom([1, 1/2+3/2*p],[3/2+3/2*p],-tan(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+e
)^3)^p/f/(1+3*p)
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3658, 3476, 364}

$$\frac{\tan(e+fx)\left(b\tan^3(e+fx)\right)^p\,{}_2F_1\left(1,\frac{1}{2}(3p+1);\frac{3(p+1)}{2};-\tan^2(e+fx)\right)}{f(3p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 3*p)/2, (3*(1 + p))/2, -Tan[e + f*x]^2]*Tan[e +
f*x]*(b*Tan[e + f*x]^3)^p)/(f*(1 + 3*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(b\tan^3(e+fx)\right)^p\,dx &= \left(\tan^{-3p}(e+fx)\left(b\tan^3(e+fx)\right)^p\right)\int \tan^{3p}(e+fx)\,dx\\
&= \frac{\left(\tan^{-3p}(e+fx)\left(b\tan^3(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{3p}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{{}_2F_1\left(1,\frac{1}{2}(1+3p);\frac{3(1+p)}{2};-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^3(e+fx)\right)^p}{f(1+3p)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.96

$$\frac{\tan(e+fx)\left(b\tan^3(e+fx)\right)^p {}_2F_1\left(1,\frac{1}{2}(3p+1);\frac{3(p+1)}{2};-\tan^2(e+fx)\right)}{3fp+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^3)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 3*p)/2, (3*(1 + p))/2, -Tan[e + f*x]^2]*Tan[e +
f*x]*(b*Tan[e + f*x]^3)^p)/(f + 3*f*p)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\tan\left(fx+e\right)^3\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^3)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^3)^p, x)
```

maple [F] time = 1.58, size = 0, normalized size = 0.00

$$\int \left(b\left(\tan^3\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^3)^p,x)
```

```
[Out] int((b*tan(f*x+e)^3)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^3)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\tan\left(e+fx\right)^3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^3)^p,x)
```

```
[Out] int((b*tan(e + f*x)^3)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan^3 \left(e + fx\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**3)**p,x)
[Out] Integral((b*tan(e + f*x)**3)**p, x)
```

3.28 $\int \left(b\tan^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=59

$$\frac{\tan(e+fx)\left(b\tan^4(e+fx)\right)^p\,{}_2F_1\left(1,\frac{1}{2}(4p+1);\frac{1}{2}(4p+3);-\tan^2(e+fx)\right)}{f(4p+1)}$$

```
[Out] hypergeom([1, 1/2+2*p],[3/2+2*p],-tan(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+e)^4)
^p/f/(1+4*p)
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3658, 3476, 364}

$$\frac{\tan(e+fx)\left(b\tan^4(e+fx)\right)^p\,{}_2F_1\left(1,\frac{1}{2}(4p+1);\frac{1}{2}(4p+3);-\tan^2(e+fx)\right)}{f(4p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 4*p)/2, (3 + 4*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^4)^p)/(f*(1 + 4*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(b\tan^4(e+fx)\right)^p\,dx &= \left(\tan^{-4p}(e+fx)\left(b\tan^4(e+fx)\right)^p\right)\int \tan^{4p}(e+fx)\,dx\\
&= \frac{\left(\tan^{-4p}(e+fx)\left(b\tan^4(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{4p}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{{}_2F_1\left(1,\frac{1}{2}(1+4p);\frac{1}{2}(3+4p);-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^4(e+fx)\right)^p}{f(1+4p)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.90

$$\frac{\tan(e+fx)\left(b\tan^4(e+fx)\right)^p\,{}_2F_1\left(1,2p+\frac{1}{2};2p+\frac{3}{2};-\tan^2(e+fx)\right)}{4fp+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 1/2 + 2*p, 3/2 + 2*p, -Tan[e + f*x]^2]*Tan[e + f*x]*(
b*Tan[e + f*x]^4)^p)/(f + 4*f*p)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^4\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^4)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^4)^p, x)
```

maple [F] time = 1.19, size = 0, normalized size = 0.00

$$\int\left(b\left(\tan^4\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^4)^p,x)
```

```
[Out] int((b*tan(f*x+e)^4)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^4)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(b\tan\left(e+fx\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^4)^p,x)
```

```
[Out] int((b*tan(e + f*x)^4)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^4\left(e+fx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**4)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**4)**p, x)
```

3.29 $\int \left(b \tan^n(e+fx)\right)^{\frac{1}{n}} dx$

Optimal. Leaf size=32

$$-\frac{\cot(e+fx)\log(\cos(e+fx))\left(b\tan^n(e+fx)\right)^{\frac{1}{n}}}{f}$$

```
[Out] -cot(f*x+e)*ln(cos(f*x+e))*(b*tan(f*x+e)^n)^(1/n)/f
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3659, 3475}

$$-\frac{\cot(e+fx)\log(\cos(e+fx))\left(b\tan^n(e+fx)\right)^{\frac{1}{n}}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^n^(-1),x]
```

```
[Out] -((Cot[e + f*x]*Log[Cos[e + f*x]]*(b*Tan[e + f*x]^n)^n^(-1))/f)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(b\tan^n(e+fx)\right)^{\frac{1}{n}} dx &= \left(\cot(e+fx)\left(b\tan^n(e+fx)\right)^{\frac{1}{n}}\right)\int \tan(e+fx)\,dx\\
&= -\frac{\cot(e+fx)\log(\cos(e+fx))\left(b\tan^n(e+fx)\right)^{\frac{1}{n}}}{f}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$-\frac{\cot(e+fx)\log(\cos(e+fx))\left(b\tan^n(e+fx)\right)^{\frac{1}{n}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^n^(-1),x]
```

```
[Out] -((Cot[e + f*x]*Log[Cos[e + f*x]]*(b*Tan[e + f*x]^n)^n^(-1))/f)
```

fricas [A] time = 0.47, size = 23, normalized size = 0.72

$$-\frac{b^{\left(\frac{1}{n}\right)}\log\left(\frac{1}{\tan\left(fx+e\right)^{2}+1}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/n),x, algorithm="fricas")
[Out] -1/2*b^(1/n)*log(1/(tan(f*x + e)^2 + 1))/f
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{n}\right)^{\left(\frac{1}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/n),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^n)^(1/n), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(b\left(\tan^{n}\left(fx+e\right)\right)\right)^{\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^(1/n),x)
[Out] int((b*tan(f*x+e)^n)^(1/n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{n}\right)^{\left(\frac{1}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/n),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^n)^(1/n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^n\right)}^{1/n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^n)^(1/n),x)
[Out] int((b*tan(e + f*x)^n)^(1/n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^{n}\left(e+fx\right)\right)^{\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**(1/n),x)
[Out] Integral((b*tan(e + f*x)**n)**(1/n), x)
```

3.30 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)dx$

Optimal. Leaf size=70

$$-\frac{(a-b)\cos^5(e+fx)}{5f}+\frac{(2a-3b)\cos^3(e+fx)}{3f}-\frac{(a-3b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a-3*b)*cos(f*x+e)/f+1/3*(2*a-3*b)*cos(f*x+e)^3/f-1/5*(a-b)*cos(f*x+e)^5/f
+b*sec(f*x+e)/f
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3664, 448}

$$-\frac{(a-b)\cos^5(e+fx)}{5f}+\frac{(2a-3b)\cos^3(e+fx)}{3f}-\frac{(a-3b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - 3*b)*Cos[e + f*x])/f) + ((2*a - 3*b)*Cos[e + f*x]^3)/(3*f) - ((a -
b)*Cos[e + f*x]^5)/(5*f) + (b*Sec[e + f*x])/f
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a-b+bx^2\right)}{x^6}\,dx,x,\sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b+\frac{a-b}{x^6}+\frac{-2a+3b}{x^4}+\frac{a-3b}{x^2}\right)dx,x,\sec(e+fx)\right)}{f}\\ &= -\frac{(a-3b)\cos(e+fx)}{f}+\frac{(2a-3b)\cos^3(e+fx)}{3f}-\frac{(a-b)\cos^5(e+f}{5f}\end{aligned}$$

Mathematica [A] time = 0.07, size = 104, normalized size = 1.49

$$-\frac{5a\cos(e+fx)}{8f}+\frac{5a\cos(3(e+fx))}{48f}-\frac{a\cos(5(e+fx))}{80f}+\frac{19b\cos(e+fx)}{8f}-\frac{3b\cos(3(e+fx))}{16f}+\frac{b\cos(5(e+fx))}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-5*a*Cos[e + f*x])/(8*f) + (19*b*Cos[e + f*x])/(8*f) + (5*a*Cos[3*(e + f*x
)])/(48*f) - (3*b*Cos[3*(e + f*x)])/(16*f) - (a*Cos[5*(e + f*x)])/(80*f) +
(b*Cos[5*(e + f*x)])/(80*f) + (b*Sec[e + f*x])/f
```

fricas [A] time = 0.41, size = 64, normalized size = 0.91

$$-\frac{3 (a-b) \cos \left(fx+e\right)^{6} - 5 (2 a - 3 b) \cos \left(fx+e\right)^{4} + 15 (a - 3 b) \cos \left(fx+e\right)^{2} - 15 b}{15 f \cos \left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a - b)*cos(f*x + e)^6 - 5*(2*a - 3*b)*cos(f*x + e)^4 + 15*(a - 3*
b)*cos(f*x + e)^2 - 15*b)/(f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.67, size = 92, normalized size = 1.31

$$\frac{-\frac{a\left(\frac{8}{3}+\sin^{4}\left(fx+e\right)+\frac{4\left(\sin^{2}\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5} + b\left(\frac{\sin^{8}\left(fx+e\right)}{\cos\left(fx+e\right)} + \left(\frac{16}{5} + \sin^{6}\left(fx+e\right) + \frac{6\left(\sin^{4}\left(fx+e\right)\right)}{5} + \frac{8\left(\sin^{2}\left(fx+e\right)\right)}{5}\right)\cos\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(-1/5*a*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+b*(sin(f*x+e)^8/
cos(f*x+e)+(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)
))
```

maxima [A] time = 0.72, size = 62, normalized size = 0.89

$$-\frac{3 (a-b) \cos \left(fx+e\right)^{5} - 5 (2 a - 3 b) \cos \left(fx+e\right)^{3} + 15 (a - 3 b) \cos \left(fx+e\right) - \frac{15 b}{\cos\left(fx+e\right)}}{15 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(3*(a - b)*cos(f*x + e)^5 - 5*(2*a - 3*b)*cos(f*x + e)^3 + 15*(a - 3*
b)*cos(f*x + e) - 15*b/cos(f*x + e))/f
```

mupad [B] time = 12.23, size = 92, normalized size = 1.31

$$-\frac{\frac{5 a}{16} - \frac{35 b}{16} + \frac{25 a \cos\left(2 e+2 f x\right)}{96} - \frac{11 a \cos\left(4 e+4 f x\right)}{240} + \frac{a \cos\left(6 e+6 f x\right)}{160} - \frac{35 b \cos\left(2 e+2 f x\right)}{32} + \frac{7 b \cos\left(4 e+4 f x\right)}{80} - \frac{b \cos\left(6 e+6 f x\right)}{160}}{f \cos\left(e+f x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2),x)
```

```
[Out] -((5*a)/16 - (35*b)/16 + (25*a*cos(2*e + 2*f*x))/96 - (11*a*cos(4*e + 4*f*x
))/240 + (a*cos(6*e + 6*f*x))/160 - (35*b*cos(2*e + 2*f*x))/32 + (7*b*cos(4
*e + 4*f*x))/80 - (b*cos(6*e + 6*f*x))/160)/(f*cos(e + f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)\sin^5\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**5, x)
```

3.31 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=48

$$\frac{(a-b)\cos^3(e+fx)}{3f}-\frac{(a-2b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a-2*b)*cos(f*x+e)/f+1/3*(a-b)*cos(f*x+e)^3/f+b*sec(f*x+e)/f
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3664, 448}

$$\frac{(a-b)\cos^3(e+fx)}{3f}-\frac{(a-2b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - 2*b)*Cos[e + f*x])/f) + ((a - b)*Cos[e + f*x]^3)/(3*f) + (b*Sec[e +
 f*x])/f
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a-b+bx^2)}{x^4}\,dx,x,\sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b+\frac{-a+b}{x^4}+\frac{a-2b}{x^2}\right)\,dx,x,\sec(e+fx)\right)}{f}\\ &= -\frac{(a-2b)\cos(e+fx)}{f}+\frac{(a-b)\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 1.50

$$-\frac{3a\cos(e+fx)}{4f}+\frac{a\cos(3(e+fx))}{12f}+\frac{7b\cos(e+fx)}{4f}-\frac{b\cos(3(e+fx))}{12f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-3*a*Cos[e + f*x])/(4*f) + (7*b*Cos[e + f*x])/(4*f) + (a*Cos[3*(e + f*x)])
/(12*f) - (b*Cos[3*(e + f*x)])/(12*f) + (b*Sec[e + f*x])/f
```

fricas [A] time = 0.49, size = 46, normalized size = 0.96

$$\frac{(a-b)\cos\left(fx+e\right)^4-3\,(a-2\,b)\cos\left(fx+e\right)^2+3\,b}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((a - b)*cos(f*x + e)^4 - 3*(a - 2*b)*cos(f*x + e)^2 + 3*b)/(f*cos(f*x
+ e))
```

giac [A] time = 1.89, size = 76, normalized size = 1.58

$$\frac{b}{f\cos\left(fx+e\right)}+\frac{af^5\cos\left(fx+e\right)^3-bf^5\cos\left(fx+e\right)^3-3\,af^5\cos\left(fx+e\right)+6\,bf^5\cos\left(fx+e\right)}{3\,f^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] b/(f*cos(f*x + e)) + 1/3*(a*f^5*cos(f*x + e)^3 - b*f^5*cos(f*x + e)^3 - 3*a
*f^5*cos(f*x + e) + 6*b*f^5*cos(f*x + e))/f^6
```

maple [A] time = 0.60, size = 72, normalized size = 1.50

$$\frac{-\frac{a\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+b\left(\frac{\sin^6\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(-1/3*a*(2+sin(f*x+e)^2)*cos(f*x+e)+b*(sin(f*x+e)^6/cos(f*x+e)+(8/3+sin
(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)))
```

maxima [A] time = 0.31, size = 44, normalized size = 0.92

$$\frac{(a-b)\cos\left(fx+e\right)^3-3\,(a-2\,b)\cos\left(fx+e\right)+\frac{3\,b}{\cos\left(fx+e\right)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*((a - b)*cos(f*x + e)^3 - 3*(a - 2*b)*cos(f*x + e) + 3*b/cos(f*x + e))/
f
```

mupad [B] time = 12.01, size = 68, normalized size = 1.42

$$-\frac{\frac{3\,a}{8}-\frac{15\,b}{8}+\frac{a\,\cos\left(2\,e+2\,f\,x\right)}{3}-\frac{a\,\cos\left(4\,e+4\,f\,x\right)}{24}-\frac{5\,b\,\cos\left(2\,e+2\,f\,x\right)}{6}+\frac{b\,\cos\left(4\,e+4\,f\,x\right)}{24}}{f\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2),x)
```

```
[Out] -((3*a)/8 - (15*b)/8 + (a*cos(2*e + 2*f*x))/3 - (a*cos(4*e + 4*f*x))/24 - (
5*b*cos(2*e + 2*f*x))/6 + (b*cos(4*e + 4*f*x))/24)/(f*cos(e + f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)\sin^3\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**3, x)
```

3.32 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{b\sec(e+fx)}{f}-\frac{(a-b)\cos(e+fx)}{f}$$

```
[Out] -(a-b)*cos(f*x+e)/f+b*sec(f*x+e)/f
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3664, 14}

$$\frac{b\sec(e+fx)}{f}-\frac{(a-b)\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)*Cos[e + f*x])/f) + (b*Sec[e + f*x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a-b+bx^2}{x^2}\,dx,x,\sec(e+fx)\right)}{f}\\ &= \frac{\operatorname{Subst}\left(\int \left(b+\frac{a-b}{x^2}\right)\,dx,x,\sec(e+fx)\right)}{f}\\ &= -\frac{(a-b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.05, size = 46, normalized size = 1.64

$$\frac{a\sin(e)\sin(fx)}{f}-\frac{a\cos(e)\cos(fx)}{f}+\frac{b\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Cos[e]*Cos[f*x])/f) + (b*Cos[e + f*x])/f + (b*Sec[e + f*x])/f + (a*Sin
[e]*Sin[f*x])/f
```

fricas [A] time = 0.44, size = 31, normalized size = 1.11

$$-\frac{(a-b)\cos\left(fx+e\right)^{2}-b}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
[Out] -((a - b)*cos(f*x + e)^2 - b)/(f*cos(f*x + e))
```

giac [A] time = 2.31, size = 41, normalized size = 1.46

$$b\left(\frac{\cos\left(fx+e\right)}{f}+\frac{1}{f\cos\left(fx+e\right)}\right)-\frac{a\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
[Out] b*(cos(f*x + e)/f + 1/(f*cos(f*x + e))) - a*cos(f*x + e)/f
```

maple [A] time = 0.45, size = 52, normalized size = 1.86

$$\frac{-a\cos\left(fx+e\right)+b\left(\frac{\sin^{4}\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(2+\sin^{2}\left(fx+e\right)\right)\cos\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2),x)
[Out] 1/f*(-a*cos(f*x+e)+b*(sin(f*x+e)^4/cos(f*x+e)+(2+sin(f*x+e)^2)*cos(f*x+e)))
```

maxima [A] time = 0.75, size = 31, normalized size = 1.11

$$\frac{b\left(\frac{1}{\cos\left(fx+e\right)}+\cos\left(fx+e\right)\right)-a\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
[Out] (b*(1/cos(f*x + e) + cos(f*x + e)) - a*cos(f*x + e))/f
```

mupad [B] time = 11.78, size = 39, normalized size = 1.39

$$\frac{\left(\cos\left(e+f\,x\right)+1\right)\,\left(b-a\,\cos\left(e+f\,x\right)+b\,\cos\left(e+f\,x\right)\right)}{f\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*tan(e + f*x)^2),x)
[Out] ((cos(e + f*x) + 1)*(b - a*cos(e + f*x) + b*cos(e + f*x)))/(f*cos(e + f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{2}\left(e+fx\right)\right)\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2),x)
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x), x)
```

3.33 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=25

$$\frac{b\sec(e+fx)}{f}-\frac{a\tanh^{-1}(\cos(e+fx))}{f}$$

```
[Out] -a*arctanh(cos(f*x+e))/f+b*sec(f*x+e)/f
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3664, 388, 207}

$$\frac{b\sec(e+fx)}{f}-\frac{a\tanh^{-1}(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*ArcTanh[Cos[e + f*x]])/f) + (b*Sec[e + f*x])/f
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a-b+bx^2}{-1+x^2}\,dx,x,\sec(e+fx)\right)}{f}\\ &= \frac{b\sec(e+fx)}{f}+\frac{a\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sec(e+fx)\right)}{f}\\ &= -\frac{a\tanh^{-1}(\cos(e+fx))}{f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [B] time = 0.03, size = 51, normalized size = 2.04

$$\frac{a\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{a\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Log[Cos[e/2 + (f*x)/2]])/f) + (a*Log[Sin[e/2 + (f*x)/2]])/f + (b*Sec[e
 + f*x])/f
```

fricas [B] time = 0.44, size = 56, normalized size = 2.24

$$-\frac{a\cos\left(fx+e\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-a\cos\left(fx+e\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-2\,b}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) - a*cos(f*x + e)*log(-1/2*
cos(f*x + e) + 1/2) - 2*b)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-b/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1)+a/4*ln(abs(1-co
s(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [A] time = 0.43, size = 36, normalized size = 1.44

$$\frac{a\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}+\frac{b}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*a*ln(csc(f*x+e)-cot(f*x+e))+1/f*b/cos(f*x+e)
```

maxima [A] time = 0.67, size = 40, normalized size = 1.60

$$-\frac{a\log\left(\cos\left(fx+e\right)+1\right)-a\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\,b}{\cos\left(fx+e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(cos(f*x + e) + 1) - a*log(cos(f*x + e) - 1) - 2*b/cos(f*x + e))
/f
```

mupad [B] time = 11.56, size = 37, normalized size = 1.48

$$\frac{a\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}-\frac{2\,b}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)/sin(e + f*x),x)
[Out] (a*log(tan(e/2 + (f*x)/2)))/f - (2*b)/(f*(tan(e/2 + (f*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)\csc\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2),x)
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x), x)
```

3.34 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=51

$$-\frac{(a+2b)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -1/2*(a+2*b)*arctanh(cos(f*x+e))/f-1/2*a*cot(f*x+e)*csc(f*x+e)/f+b*sec(f*x+
e)/f
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3664, 455, 388, 207}

$$-\frac{(a+2b)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a + 2*b)*ArcTanh[Cos[e + f*x]])/(2*f) - (a*Cot[e + f*x]*Csc[e + f*x])/(2
*f) + (b*Sec[e + f*x])/f
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^2(a-b+bx^2)}{(-1+x^2)^2}dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{a\cot(e+fx)\csc(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{-a-2bx^2}{-1+x^2}dx, x, \sec(e+fx)\right)}{2f}$$
$$= -\frac{a\cot(e+fx)\csc(e+fx)}{2f} + \frac{b\sec(e+fx)}{f} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{-1+\dots}\right.}{2\dots}$$
$$= -\frac{(a+2b)\tanh^{-1}(\cos(e+fx))}{2f} - \frac{a\cot(e+fx)\csc(e+fx)}{2f} + \frac{b\sec(\dots}{\dots}$$

Mathematica [B] time = 0.05, size = 123, normalized size = 2.41

$$-\frac{a\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} - \frac{a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f} + \frac{b\sec(e+fx)}{f} + \frac{b\,l\dots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/8*(a*Csc[(e + f*x)/2]^2)/f - (a*Log[Cos[(e + f*x)/2]])/(2*f) - (b*Log[Co
s[(e + f*x)/2]])/f + (a*Log[Sin[(e + f*x)/2]])/(2*f) + (b*Log[Sin[(e + f*x)
/2]])/f + (a*Sec[(e + f*x)/2]^2)/(8*f) + (b*Sec[e + f*x])/f
```

fricas [B] time = 0.65, size = 124, normalized size = 2.43

$$\frac{2(a+2b)\cos\left(fx+e\right)^2 - \left((a+2b)\cos\left(fx+e\right)^3 - (a+2b)\cos\left(fx+e\right)\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right) + \left((a+2\dots\right.}{4\left(f\cos\left(fx+e\right)^3 - f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a + 2*b)*cos(f*x + e)^2 - ((a + 2*b)*cos(f*x + e)^3 - (a + 2*b)*cos
(f*x + e))*log(1/2*cos(f*x + e) + 1/2) + ((a + 2*b)*cos(f*x + e)^3 - (a + 2
*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) - 4*b)/(f*cos(f*x + e)^3 - f
*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a/16+(-((1-cos(f*x+exp
(1)))/(1+cos(f*x+exp(1))))^2*a-2*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^
2*b-14*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a)*1/16/(((1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1))))^2-(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))-(-a-2*b
)/8*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [A] time = 0.56, size = 76, normalized size = 1.49

$$-\frac{a\cot\left(fx+e\right)\csc\left(fx+e\right)}{2f}+\frac{a\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2f}+\frac{b}{f\cos\left(fx+e\right)}+\frac{b\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/2*a*cot(f*x+e)*csc(f*x+e)/f+1/2/f*a*ln(csc(f*x+e)-cot(f*x+e))+1/f*b/cos(
f*x+e)+1/f*b*ln(csc(f*x+e)-cot(f*x+e))
```

maxima [A] time = 0.46, size = 76, normalized size = 1.49

$$-\frac{(a+2b)\log\left(\cos\left(fx+e\right)+1\right)-(a+2b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left((a+2b)\cos\left(fx+e\right)^{2}-2b\right)}{\cos\left(fx+e\right)^{3}-\cos\left(fx+e\right)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/4*((a + 2*b)*log(cos(f*x + e) + 1) - (a + 2*b)*log(cos(f*x + e) - 1) - 2
*((a + 2*b)*cos(f*x + e)^2 - 2*b)/(cos(f*x + e)^3 - cos(f*x + e)))/f
```

mupad [B] time = 12.10, size = 95, normalized size = 1.86

$$\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}}{8f}-\frac{\frac{a}{2}-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\left(\frac{a}{2}+8b\right)}{f\left(4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}-4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\right)}+\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(\frac{a}{2}+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)/sin(e + f*x)^3,x)
```

```
[Out] (a*tan(e/2 + (f*x)/2)^2)/(8*f) - (a/2 - tan(e/2 + (f*x)/2)^2*(a/2 + 8*b))/(
f*(4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^4)) + (log(tan(e/2 + (f*x)
/2))*(a/2 + b))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{2}\left(e+fx\right)\right)\csc^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**3, x)
```

3.35 $\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=79

$$-\frac{3(a+4b)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f}-\frac{a\cot^3(e+fx)\csc(e+fx)}{4f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -3/8*(a+4*b)*arctanh(cos(f*x+e))/f-1/8*(5*a+4*b)*cot(f*x+e)*csc(f*x+e)/f-1/
4*a*cot(f*x+e)^3*csc(f*x+e)/f+b*sec(f*x+e)/f
```

Rubi [A] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3664, 455, 1157, 388, 207}

$$-\frac{3(a+4b)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f}-\frac{a\cot^3(e+fx)\csc(e+fx)}{4f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-3*(a + 4*b)*ArcTanh[Cos[e + f*x]])/(8*f) - ((5*a + 4*b)*Cot[e + f*x]*Csc[
e + f*x])/(8*f) - (a*Cot[e + f*x]^3*Csc[e + f*x])/(4*f) + (b*Sec[e + f*x])/
f
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^4(a-b+bx^2)}{(-1+x^2)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{a\cot^3(e+fx)\csc(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{-a-4ax^2-4bx^4}{(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{4f}$$

$$= -\frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{a\cot^3(e+fx)\csc(e+fx)}{4f} - \text{Sub}$$

$$= -\frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{a\cot^3(e+fx)\csc(e+fx)}{4f} + \frac{b\,\text{se}}{}$$

$$= -\frac{3(a+4b)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f} -$$

Mathematica [B] time = 6.06, size = 276, normalized size = 3.49

$$-\frac{a\csc^4\left(\frac{1}{2}(e+fx)\right)}{64f}-\frac{3a\csc^2\left(\frac{1}{2}(e+fx)\right)}{32f}+\frac{a\sec^4\left(\frac{1}{2}(e+fx)\right)}{64f}+\frac{3a\sec^2\left(\frac{1}{2}(e+fx)\right)}{32f}+\frac{3a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8f}-\frac{3a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-3*a*Csc[(e + f*x)/2]^2)/(32*f) - (b*Csc[(e + f*x)/2]^2)/(8*f) - (a*Csc[(e
 + f*x)/2]^4)/(64*f) - (3*a*Log[Cos[(e + f*x)/2]])/(8*f) - (3*b*Log[Cos[(e
+ f*x)/2]])/(2*f) + (3*a*Log[Sin[(e + f*x)/2]])/(8*f) + (3*b*Log[Sin[(e + f
*x)/2]])/(2*f) + (3*a*Sec[(e + f*x)/2]^2)/(32*f) + (b*Sec[(e + f*x)/2]^2)/(
8*f) + (a*Sec[(e + f*x)/2]^4)/(64*f) + (b*Sin[(e + f*x)/2])/(f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])) - (b*Sin[(e + f*x)/2])/(f*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]))
```

fricas [B] time = 0.70, size = 178, normalized size = 2.25

$$\frac{6\,(a+4\,b)\cos\left(fx+e\right)^4-10\,(a+4\,b)\cos\left(fx+e\right)^2-3\left((a+4\,b)\cos\left(fx+e\right)^5-2\,(a+4\,b)\cos\left(fx+e\right)^3+(a+\right.}{16\left(f\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(6*(a + 4*b)*cos(f*x + e)^4 - 10*(a + 4*b)*cos(f*x + e)^2 - 3*((a + 4*
b)*cos(f*x + e)^5 - 2*(a + 4*b)*cos(f*x + e)^3 + (a + 4*b)*cos(f*x + e))*lo
g(1/2*cos(f*x + e) + 1/2) + 3*((a + 4*b)*cos(f*x + e)^5 - 2*(a + 4*b)*cos(f
```

```
*x + e)^3 + (a + 4*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) + 16*b)/(f
*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-b/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1)+(-18*((1-cos(f*
x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-72*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1))))^2*b-8*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a-8*(1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1)))*b-a)*1/128/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2
+(32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+256*(1-cos(f*x+exp(1)))/
(1+cos(f*x+exp(1)))*a+256*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b)/4096+(
3*a+12*b)/32*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [A] time = 0.53, size = 120, normalized size = 1.52

$$-\frac{a\cot\left(fx+e\right)\left(\csc^{3}\left(fx+e\right)\right)}{4f}-\frac{3a\cot\left(fx+e\right)\csc\left(fx+e\right)}{8f}+\frac{3a\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{8f}-\frac{}{2f\sin\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/4/f*a*cot(f*x+e)*csc(f*x+e)^3-3/8*a*cot(f*x+e)*csc(f*x+e)/f+3/8/f*a*ln(c
sc(f*x+e)-cot(f*x+e))-1/2/f*b/sin(f*x+e)^2/cos(f*x+e)+3/2/f*b/cos(f*x+e)+3/
2/f*b*ln(csc(f*x+e)-cot(f*x+e))
```

maxima [A] time = 0.58, size = 101, normalized size = 1.28

$$-\frac{3\,(a+4\,b)\log\left(\cos\left(fx+e\right)+1\right)-3\,(a+4\,b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(a+4\,b)\cos\left(fx+e\right)^{4}-5\,(a+4\,b)\cos\left(fx+e\right)^{2}+8\,b\right.}{\cos\left(fx+e\right)^{5}-2\,\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/16*(3*(a + 4*b)*log(cos(f*x + e) + 1) - 3*(a + 4*b)*log(cos(f*x + e) - 1
) - 2*(3*(a + 4*b)*cos(f*x + e)^4 - 5*(a + 4*b)*cos(f*x + e)^2 + 8*b)/(cos(
f*x + e)^5 - 2*cos(f*x + e)^3 + cos(f*x + e)))/f
```

mupad [B] time = 11.97, size = 138, normalized size = 1.75

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}\,\left(\frac{a}{8}+\frac{b}{8}\right)}{f}-\frac{(-2\,a-34\,b)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}+\left(\frac{7\,a}{4}+2\,b\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+\frac{a}{4}}{f\,\left(16\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-16\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}\right)}+\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(\frac{3\,a}{8}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)/sin(e + f*x)^5,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^2*(a/8 + b/8))/f - (a/4 + tan(e/2 + (f*x)/2)^2*((7*a)/4
 + 2*b) - tan(e/2 + (f*x)/2)^4*(2*a + 34*b))/(f*(16*tan(e/2 + (f*x)/2)^4 -
```

```
16*tan(e/2 + (f*x)/2)^6)) + (log(tan(e/2 + (f*x)/2))*((3*a)/8 + (3*b)/2))/f
 + (a*tan(e/2 + (f*x)/2)^4)/(64*f)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)\csc^5\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**5, x)
```

3.36 $\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=102

$$-\frac{(a-b)\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{(13a-19b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{(11a-29b)\sin(e+fx)\cos(e+fx)}{16f}+$$

```
[Out] 5/16*(a-7*b)*x-1/16*(11*a-29*b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(13*a-19*b)*co
s(f*x+e)^3*sin(f*x+e)/f-1/6*(a-b)*cos(f*x+e)^5*sin(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A] time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3663, 455, 1814, 1157, 388, 203}

$$-\frac{(a-b)\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{(13a-19b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{(11a-29b)\sin(e+fx)\cos(e+fx)}{16f}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (5*(a - 7*b)*x)/16 - ((11*a - 29*b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((1
3*a - 19*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - ((a - b)*Cos[e + f*x]^5*S
in[e + f*x])/(6*f) + (b*Tan[e + f*x])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^6(a+bx^2)}{(1+x^2)^4}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cos^5(e+fx)\sin(e+fx)}{6f} - \frac{\text{Subst}\left(\int \frac{-a+b+6(a-b)x^2-6(a-b)x^4-6bx^6}{(1+x^2)^3}\right.}{6f}$$

$$= \frac{(13a-19b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{(a-b)\cos^5(e+fx)\sin(e+fx)}{6f}$$

$$= -\frac{(11a-29b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-19b)\cos^3(e+fx)\sin(e}{24f}$$

$$= -\frac{(11a-29b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-19b)\cos^3(e+fx)\sin(e}{24f}$$

$$= \frac{5}{16}(a-7b)x - \frac{(11a-29b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-19b)\cos^3(}{2}$$

Mathematica [A] time = 0.36, size = 89, normalized size = 0.87

$$\frac{(141b-45a)\sin(2(e+fx))+3(3a-5b)\sin(4(e+fx))-a\sin(6(e+fx))+60ae+60afx+b\sin(6(e+fx))+1}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (60*a*e - 420*b*e + 60*a*f*x - 420*b*f*x + (-45*a + 141*b)*Sin[2*(e + f*x)]
 + 3*(3*a - 5*b)*Sin[4*(e + f*x)] - a*Sin[6*(e + f*x)] + b*Sin[6*(e + f*x)]
 + 192*b*Tan[e + f*x])/(192*f)
```

fricas [A] time = 0.60, size = 90, normalized size = 0.88

$$\frac{15(a-7b)fx\cos\left(fx+e\right)-\left(8(a-b)\cos\left(fx+e\right)^6-2(13a-19b)\cos\left(fx+e\right)^4+3(11a-29b)\cos\left(fx+e\right)^2\right.}{48f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(15*(a - 7*b)*f*x*cos(f*x + e) - (8*(a - b)*cos(f*x + e)^6 - 2*(13*a -
 19*b)*cos(f*x + e)^4 + 3*(11*a - 29*b)*cos(f*x + e)^2 - 48*b)*sin(f*x + e)
)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
```

```
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)(60*a*f*x*tan(exp(1))^7*tan(f*x)^7+180*a*f*x*tan(exp(1))^7*tan(f*x)^5+180
*a*f*x*tan(exp(1))^7*tan(f*x)^3+60*a*f*x*tan(exp(1))^7*tan(f*x)-60*a*f*x*ta
n(exp(1))^6*tan(f*x)^6-180*a*f*x*tan(exp(1))^6*tan(f*x)^4-180*a*f*x*tan(exp
(1))^6*tan(f*x)^2-60*a*f*x*tan(exp(1))^6+180*a*f*x*tan(exp(1))^5*tan(f*x)^7
+540*a*f*x*tan(exp(1))^5*tan(f*x)^5+540*a*f*x*tan(exp(1))^5*tan(f*x)^3+180*
a*f*x*tan(exp(1))^5*tan(f*x)-180*a*f*x*tan(exp(1))^4*tan(f*x)^6-540*a*f*x*t
an(exp(1))^4*tan(f*x)^4-540*a*f*x*tan(exp(1))^4*tan(f*x)^2-180*a*f*x*tan(ex
p(1))^4+180*a*f*x*tan(exp(1))^3*tan(f*x)^7+540*a*f*x*tan(exp(1))^3*tan(f*x)
^5+540*a*f*x*tan(exp(1))^3*tan(f*x)^3+180*a*f*x*tan(exp(1))^3*tan(f*x)-180*
a*f*x*tan(exp(1))^2*tan(f*x)^6-540*a*f*x*tan(exp(1))^2*tan(f*x)^4-540*a*f*x
*tan(exp(1))^2*tan(f*x)^2-180*a*f*x*tan(exp(1))^2+60*a*f*x*tan(exp(1))*tan(
f*x)^7+180*a*f*x*tan(exp(1))*tan(f*x)^5+180*a*f*x*tan(exp(1))*tan(f*x)^3+60
*a*f*x*tan(exp(1))*tan(f*x)-60*a*f*x*tan(f*x)^6-180*a*f*x*tan(f*x)^4-180*a*
f*x*tan(f*x)^2-60*a*f*x+60*a*tan(exp(1))^7*tan(f*x)^6+160*a*tan(exp(1))^7*t
an(f*x)^4+132*a*tan(exp(1))^7*tan(f*x)^2+60*a*tan(exp(1))^6*tan(f*x)^7+120*
a*tan(exp(1))^6*tan(f*x)^5+20*a*tan(exp(1))^6*tan(f*x)^3-264*a*tan(exp(1))^
6*tan(f*x)+120*a*tan(exp(1))^5*tan(f*x)^6+300*a*tan(exp(1))^5*tan(f*x)^4-36
0*a*tan(exp(1))^5*tan(f*x)^2+132*a*tan(exp(1))^5+160*a*tan(exp(1))^4*tan(f*
x)^7+300*a*tan(exp(1))^4*tan(f*x)^5-960*a*tan(exp(1))^4*tan(f*x)^3+20*a*tan
(exp(1))^4*tan(f*x)+20*a*tan(exp(1))^3*tan(f*x)^6-960*a*tan(exp(1))^3*tan(f
*x)^4+300*a*tan(exp(1))^3*tan(f*x)^2+160*a*tan(exp(1))^3+132*a*tan(exp(1))^
2*tan(f*x)^7-360*a*tan(exp(1))^2*tan(f*x)^5+300*a*tan(exp(1))^2*tan(f*x)^3+
120*a*tan(exp(1))^2*tan(f*x)-264*a*tan(exp(1))*tan(f*x)^6+20*a*tan(exp(1))*
tan(f*x)^4+120*a*tan(exp(1))*tan(f*x)^2+60*a*tan(exp(1))+132*a*tan(f*x)^5+1
60*a*tan(f*x)^3+60*a*tan(f*x)-420*b*f*x*tan(exp(1))^7*tan(f*x)^7-1260*b*f*x
*tan(exp(1))^7*tan(f*x)^5-1260*b*f*x*tan(exp(1))^7*tan(f*x)^3-420*b*f*x*tan
(exp(1))^7*tan(f*x)+420*b*f*x*tan(exp(1))^6*tan(f*x)^6+1260*b*f*x*tan(exp(1
))^6*tan(f*x)^4+1260*b*f*x*tan(exp(1))^6*tan(f*x)^2+420*b*f*x*tan(exp(1))^6
-1260*b*f*x*tan(exp(1))^5*tan(f*x)^7-3780*b*f*x*tan(exp(1))^5*tan(f*x)^5-37
80*b*f*x*tan(exp(1))^5*tan(f*x)^3-1260*b*f*x*tan(exp(1))^5*tan(f*x)+1260*b*
```

```
f*x*tan(exp(1))^4*tan(f*x)^6+3780*b*f*x*tan(exp(1))^4*tan(f*x)^4+3780*b*f*x
*tan(exp(1))^4*tan(f*x)^2+1260*b*f*x*tan(exp(1))^4-1260*b*f*x*tan(exp(1))^3
*tan(f*x)^7-3780*b*f*x*tan(exp(1))^3*tan(f*x)^5-3780*b*f*x*tan(exp(1))^3*ta
n(f*x)^3-1260*b*f*x*tan(exp(1))^3*tan(f*x)+1260*b*f*x*tan(exp(1))^2*tan(f*x
)^6+3780*b*f*x*tan(exp(1))^2*tan(f*x)^4+3780*b*f*x*tan(exp(1))^2*tan(f*x)^2
+1260*b*f*x*tan(exp(1))^2-420*b*f*x*tan(exp(1))*tan(f*x)^7-1260*b*f*x*tan(e
xp(1))*tan(f*x)^5-1260*b*f*x*tan(exp(1))*tan(f*x)^3-420*b*f*x*tan(exp(1))*t
an(f*x)+420*b*f*x*tan(f*x)^6+1260*b*f*x*tan(f*x)^4+1260*b*f*x*tan(f*x)^2+42
0*b*f*x+21*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*ta
n(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^7*tan(
f*x)^7+63*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan
(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^7*tan(f
*x)^5+63*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(
exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^7*tan(f*
x)^3+21*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(e
xp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^7*tan(f*x
)-21*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(
1))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^6*tan(f*x)^6
-63*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1
))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^6*tan(f*x)^4-
63*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1)
)+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^6*tan(f*x)^2-2
1*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))
+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^6+63*b*pi*sign(
2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))
*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^5*tan(f*x)^7+189*b*pi*sign(
2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))
*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^5*tan(f*x)^5+189*b*pi*sign(
2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))
*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^5*tan(f*x)^3+63*b*pi*sign(2
*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*
sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^5*tan(f*x)-63*b*pi*sign(2*ta
n(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sig
n(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^4*tan(f*x)^6-189*b*pi*sign(2*ta
n(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sig
n(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^4*tan(f*x)^4-189*b*pi*sign(2*ta
n(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sig
n(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^4*tan(f*x)^2-63*b*pi*sign(2*tan
(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign
(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^4+63*b*pi*sign(2*tan(exp(1))^2*t
an(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1
))^2*tan(f*x)^2-2)*tan(exp(1))^3*tan(f*x)^7+189*b*pi*sign(2*tan(exp(1))^2*t
an(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1
))^2*tan(f*x)^2-2)*tan(exp(1))^3*tan(f*x)^5+189*b*pi*sign(2*tan(exp(1))^2*t
an(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1
))^2*tan(f*x)^2-2)*tan(exp(1))^3*tan(f*x)^3+63*b*pi*sign(2*tan(exp(1))^2*ta
n(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1)
)^2*tan(f*x)^2-2)*tan(exp(1))^3*tan(f*x)-63*b*pi*sign(2*tan(exp(1))^2*tan(f
*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2
*tan(f*x)^2-2)*tan(exp(1))^2*tan(f*x)^6-189*b*pi*sign(2*tan(exp(1))^2*tan(f
*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2
*tan(f*x)^2-2)*tan(exp(1))^2*tan(f*x)^4-189*b*pi*sign(2*tan(exp(1))^2*tan(f
*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2
*tan(f*x)^2-2)*tan(exp(1))^2*tan(f*x)^2-63*b*pi*sign(2*tan(exp(1))^2*tan(f*
x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*
tan(f*x)^2-2)*tan(exp(1))^2+21*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp
(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2
)*tan(exp(1))*tan(f*x)^7+63*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1)
)*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*t
```

```
an(exp(1))*tan(f*x)^5+63*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*t
an(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(
exp(1))*tan(f*x)^3+21*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(
f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp
(1))*tan(f*x)-21*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^
2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(f*x)^6-6
3*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))
+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(f*x)^4-63*b*pi*sign(2*t
an(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*si
gn(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(f*x)^2-21*b*pi*sign(2*tan(exp(1))^2*ta
n(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1)
)^2*tan(f*x)^2-2)+21*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f
*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^7*tan(f*x)^7+63*b*pi*sign(2*tan
(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(
exp(1))^7*tan(f*x)^5+63*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*ta
n(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^7*tan(f*x)^3+21*b*pi*sign(2*
tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*t
an(exp(1))^7*tan(f*x)-21*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*t
an(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^6*tan(f*x)^6-63*b*pi*sign(2
*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*
tan(exp(1))^6*tan(f*x)^4-63*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1)
)*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^6*tan(f*x)^2-21*b*pi*sig
n(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x
))*tan(exp(1))^6+63*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*
x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^5*tan(f*x)^7+189*b*pi*sign(2*tan
(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(
exp(1))^5*tan(f*x)^5+189*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*t
an(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^5*tan(f*x)^3+63*b*pi*sign(2
*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*
tan(exp(1))^5*tan(f*x)-63*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*
tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^4*tan(f*x)^6-189*b*pi*sign
(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x)
)*tan(exp(1))^4*tan(f*x)^4-189*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp
(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^4*tan(f*x)^2-63*b*pi*
sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(
f*x))*tan(exp(1))^4+63*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan
(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^3*tan(f*x)^7+189*b*pi*sign(2*
tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*t
an(exp(1))^3*tan(f*x)^5+189*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1)
)*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^3*tan(f*x)^3+63*b*pi*sig
n(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x
))*tan(exp(1))^3*tan(f*x)-63*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1
))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^2*tan(f*x)^6-189*b*pi*s
ign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f
*x))*tan(exp(1))^2*tan(f*x)^4-189*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(
exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^2*tan(f*x)^2-63*b*
pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*t
an(f*x))*tan(exp(1))^2+21*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*
tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))*tan(f*x)^7+63*b*pi*sign(2*
tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*t
an(exp(1))*tan(f*x)^5+63*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*t
an(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))*tan(f*x)^3+21*b*pi*sign(2*t
an(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*ta
n(exp(1))*tan(f*x)-21*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(
f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(f*x)^6-63*b*pi*sign(2*tan(exp(1))^2*ta
n(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(f*x)^4-63*b*p
i*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*ta
n(f*x))*tan(f*x)^2-21*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(
f*x)^2-2*tan(exp(1))+2*tan(f*x))+42*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(
```

```
1))*tan(f*x)-1))*tan(exp(1))^7*tan(f*x)^7+126*b*atan((tan(exp(1))+tan(f*x))
/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^7*tan(f*x)^5+126*b*atan((tan(exp(1))
+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^7*tan(f*x)^3+42*b*atan((ta
n(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^7*tan(f*x)-42*b*a
tan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^6*tan(f*x)
^6-126*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^
6*tan(f*x)^4-126*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*ta
n(exp(1))^6*tan(f*x)^2-42*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*
x)-1))*tan(exp(1))^6+126*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x
)-1))*tan(exp(1))^5*tan(f*x)^7+378*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1
))*tan(f*x)-1))*tan(exp(1))^5*tan(f*x)^5+378*b*atan((tan(exp(1))+tan(f*x))/
(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^5*tan(f*x)^3+126*b*atan((tan(exp(1))+
tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^5*tan(f*x)-126*b*atan((tan(
exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^4*tan(f*x)^6-378*b*
atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^4*tan(f*x
)^4-378*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))
^4*tan(f*x)^2-126*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*t
an(exp(1))^4+126*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*ta
n(exp(1))^3*tan(f*x)^7+378*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f
*x)-1))*tan(exp(1))^3*tan(f*x)^5+378*b*atan((tan(exp(1))+tan(f*x))/(tan(exp
(1))*tan(f*x)-1))*tan(exp(1))^3*tan(f*x)^3+126*b*atan((tan(exp(1))+tan(f*x)
)/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^3*tan(f*x)-126*b*atan((tan(exp(1))+
tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^2*tan(f*x)^6-378*b*atan((ta
n(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^2*tan(f*x)^4-378*
b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^2*tan(f
*x)^2-126*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1
))^2+42*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))
*tan(f*x)^7+126*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan
(exp(1))*tan(f*x)^5+126*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)
-1))*tan(exp(1))*tan(f*x)^3+42*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*t
an(f*x)-1))*tan(exp(1))*tan(f*x)-42*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(
1))*tan(f*x)-1))*tan(f*x)^6-126*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*
tan(f*x)-1))*tan(f*x)^4-126*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(
f*x)-1))*tan(f*x)^2-42*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-
1))-42*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^
7*tan(f*x)^7-126*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*ta
n(exp(1))^7*tan(f*x)^5-126*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f
*x)+1))*tan(exp(1))^7*tan(f*x)^3-42*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(
1))*tan(f*x)+1))*tan(exp(1))^7*tan(f*x)+42*b*atan((tan(exp(1))-tan(f*x))/(t
an(exp(1))*tan(f*x)+1))*tan(exp(1))^6*tan(f*x)^6+126*b*atan((tan(exp(1))-ta
n(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^6*tan(f*x)^4+126*b*atan((tan(
exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^6*tan(f*x)^2+42*b*a
tan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^6-126*b*at
an((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^5*tan(f*x)^
7-378*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^5
*tan(f*x)^5-378*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan
(exp(1))^5*tan(f*x)^3-126*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*
x)+1))*tan(exp(1))^5*tan(f*x)+126*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1)
)*tan(f*x)+1))*tan(exp(1))^4*tan(f*x)^6+378*b*atan((tan(exp(1))-tan(f*x))/(
tan(exp(1))*tan(f*x)+1))*tan(exp(1))^4*tan(f*x)^4+378*b*atan((tan(exp(1))-t
an(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^4*tan(f*x)^2+126*b*atan((tan
(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^4-126*b*atan((tan(
exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^3*tan(f*x)^7-378*b*
atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^3*tan(f*x
)^5-378*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))
^3*tan(f*x)^3-126*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*t
an(exp(1))^3*tan(f*x)+126*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*
x)+1))*tan(exp(1))^2*tan(f*x)^6+378*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(
1))*tan(f*x)+1))*tan(exp(1))^2*tan(f*x)^4+378*b*atan((tan(exp(1))-tan(f*x))
```

```
/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^2*tan(f*x)^2+126*b*atan((tan(exp(1))
-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^2-42*b*atan((tan(exp(1))-t
an(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))*tan(f*x)^7-126*b*atan((tan(e
xp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))*tan(f*x)^5-126*b*ata
n((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))*tan(f*x)^3-4
2*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))*tan(f
*x)+42*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(f*x)^6+1
26*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(f*x)^4+126*b
*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(f*x)^2+42*b*atan
((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))-420*b*tan(exp(1))^7*tan(f
*x)^6-1120*b*tan(exp(1))^7*tan(f*x)^4-924*b*tan(exp(1))^7*tan(f*x)^2-192*b*
tan(exp(1))^7-420*b*tan(exp(1))^6*tan(f*x)^7-840*b*tan(exp(1))^6*tan(f*x)^5
-140*b*tan(exp(1))^6*tan(f*x)^3+504*b*tan(exp(1))^6*tan(f*x)-840*b*tan(exp(
1))^5*tan(f*x)^6-2100*b*tan(exp(1))^5*tan(f*x)^4-1512*b*tan(exp(1))^5*tan(f
*x)^2-924*b*tan(exp(1))^5-1120*b*tan(exp(1))^4*tan(f*x)^7-2100*b*tan(exp(1)
)^4*tan(f*x)^5-140*b*tan(exp(1))^4*tan(f*x)-140*b*tan(exp(1))^3*tan(f*x)^6-
2100*b*tan(exp(1))^3*tan(f*x)^2-1120*b*tan(exp(1))^3-924*b*tan(exp(1))^2*ta
n(f*x)^7-1512*b*tan(exp(1))^2*tan(f*x)^5-2100*b*tan(exp(1))^2*tan(f*x)^3-84
0*b*tan(exp(1))^2*tan(f*x)+504*b*tan(exp(1))*tan(f*x)^6-140*b*tan(exp(1))*t
an(f*x)^4-840*b*tan(exp(1))*tan(f*x)^2-420*b*tan(exp(1))-192*b*tan(f*x)^7-9
24*b*tan(f*x)^5-1120*b*tan(f*x)^3-420*b*tan(f*x))/(192*f*tan(exp(1))^7*tan(
f*x)^7+576*f*tan(exp(1))^7*tan(f*x)^5+576*f*tan(exp(1))^7*tan(f*x)^3+192*f*
tan(exp(1))^7*tan(f*x)-192*f*tan(exp(1))^6*tan(f*x)^6-576*f*tan(exp(1))^6*t
an(f*x)^4-576*f*tan(exp(1))^6*tan(f*x)^2-192*f*tan(exp(1))^6+576*f*tan(exp(
1))^5*tan(f*x)^7+1728*f*tan(exp(1))^5*tan(f*x)^5+1728*f*tan(exp(1))^5*tan(f
*x)^3+576*f*tan(exp(1))^5*tan(f*x)-576*f*tan(exp(1))^4*tan(f*x)^6-1728*f*ta
n(exp(1))^4*tan(f*x)^4-1728*f*tan(exp(1))^4*tan(f*x)^2-576*f*tan(exp(1))^4+
576*f*tan(exp(1))^3*tan(f*x)^7+1728*f*tan(exp(1))^3*tan(f*x)^5+1728*f*tan(e
xp(1))^3*tan(f*x)^3+576*f*tan(exp(1))^3*tan(f*x)-576*f*tan(exp(1))^2*tan(f*
x)^6-1728*f*tan(exp(1))^2*tan(f*x)^4-1728*f*tan(exp(1))^2*tan(f*x)^2-576*f*
tan(exp(1))^2+192*f*tan(exp(1))*tan(f*x)^7+576*f*tan(exp(1))*tan(f*x)^5+576
*f*tan(exp(1))*tan(f*x)^3+192*f*tan(exp(1))*tan(f*x)-192*f*tan(f*x)^6-576*f
*tan(f*x)^4-576*f*tan(f*x)^2-192*f)
```

maple [A] time = 0.68, size = 122, normalized size = 1.20

$$\frac{a\left(-\frac{\left(\sin^5\left(fx+e\right)+\frac{5\left(\sin^3\left(fx+e\right)\right)}{4}+\frac{15\sin\left(fx+e\right)}{8}\right)\cos\left(fx+e\right)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\sin^9\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\sin^7\left(fx+e\right)+\frac{7\left(\sin^5\left(fx+e\right)\right)}{6}+\frac{35\left(\sin^3\left(f\cdots\right.\right.}{24}\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/1
6*f*x+5/16*e)+b*(sin(f*x+e)^9/cos(f*x+e)+(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/
24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)-35/16*f*x-35/16*e))
```

maxima [A] time = 0.64, size = 111, normalized size = 1.09

$$\frac{15\left(fx+e\right)(a-7b)+48\,b\tan\left(fx+e\right)-\frac{3\,(11\,a-29\,b)\tan\left(fx+e\right)^5+8\,(5\,a-17\,b)\tan\left(fx+e\right)^3+3\,(5\,a-19\,b)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^6+3\,\tan\left(fx+e\right)^4+3\,\tan\left(fx+e\right)^2+1}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(15*(f*x + e)*(a - 7*b) + 48*b*tan(f*x + e) - (3*(11*a - 29*b)*tan(f*x
 + e)^5 + 8*(5*a - 17*b)*tan(f*x + e)^3 + 3*(5*a - 19*b)*tan(f*x + e))/(tan
(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

mupad [B] time = 11.89, size = 105, normalized size = 1.03

$$x\left(\frac{5a}{16}-\frac{35b}{16}\right)-\frac{\left(\frac{11a}{16}-\frac{29b}{16}\right)\tan\left(e+fx\right)^5+\left(\frac{5a}{6}-\frac{17b}{6}\right)\tan\left(e+fx\right)^3+\left(\frac{5a}{16}-\frac{19b}{16}\right)\tan\left(e+fx\right)}{f\left(\tan\left(e+fx\right)^6+3\tan\left(e+fx\right)^4+3\tan\left(e+fx\right)^2+1\right)}+\frac{b\tan\left(e+\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6*(a + b*tan(e + f*x)^2),x)
```

```
[Out] x*((5*a)/16 - (35*b)/16) - (tan(e + f*x)^3*((5*a)/6 - (17*b)/6) + tan(e + f
*x)^5*((11*a)/16 - (29*b)/16) + tan(e + f*x)*((5*a)/16 - (19*b)/16))/(f*(3*
tan(e + f*x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) + (b*tan(e + f*x))
/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(e+fx\right)\right)\sin^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**6, x)
```

3.37 $\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)dx$

Optimal. Leaf size=74

$$\frac{(a-b)\sin(e+fx)\cos^3(e+fx)}{4f}-\frac{(5a-9b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}x(a-5b)+\frac{b\tan(e+fx)}{f}$$

```
[Out] 3/8*(a-5*b)*x-1/8*(5*a-9*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*(a-b)*cos(f*x+e)^3*
sin(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3663, 455, 1157, 388, 203}

$$\frac{(a-b)\sin(e+fx)\cos^3(e+fx)}{4f}-\frac{(5a-9b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}x(a-5b)+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (3*(a - 5*b)*x)/8 - ((5*a - 9*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) + ((a - b
)*Cos[e + f*x]^3*Sin[e + f*x])/(4*f) + (b*Tan[e + f*x])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)}{(1+x^2)^3}dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{(a-b)\cos^3(e+fx)\sin(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{a-b-4(a-b)x^2-4bx^4}{(1+x^2)^2}dx, x, \text{ta}\right.}{4f}$$
$$= -\frac{(5a-9b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{(a-b)\cos^3(e+fx)\sin(e+f}{4f}$$
$$= -\frac{(5a-9b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{(a-b)\cos^3(e+fx)\sin(e+f}{4f}$$
$$= \frac{3}{8}(a-5b)x - \frac{(5a-9b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{(a-b)\cos^3(e+f}{4f}$$

Mathematica [A] time = 0.37, size = 58, normalized size = 0.78

$$\frac{12(a-5b)(e+fx)-8(a-2b)\sin(2(e+fx))+(a-b)\sin(4(e+fx))+32b\tan(e+fx)}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (12*(a - 5*b)*(e + f*x) - 8*(a - 2*b)*Sin[2*(e + f*x)] + (a - b)*Sin[4*(e +
 f*x)] + 32*b*Tan[e + f*x])/(32*f)
```

fricas [A] time = 0.58, size = 72, normalized size = 0.97

$$\frac{3\,(a-5\,b)fx\cos\left(fx+e\right)+\left(2\,(a-b)\cos\left(fx+e\right)^4-(5\,a-9\,b)\cos\left(fx+e\right)^2+8\,b\right)\sin\left(fx+e\right)}{8\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a - 5*b)*f*x*cos(f*x + e) + (2*(a - b)*cos(f*x + e)^4 - (5*a - 9*b)
*cos(f*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
```

2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)(24*a*f*x*tan(exp(1))^5*tan(f*x)^5+48*a*f*x*tan(exp(1))^5*tan(f*x)^3+24*a*f*x*tan(exp(1))^5*tan(f*x)-24*a*f*x*tan(exp(1))^4*tan(f*x)^4-48*a*f*x*tan(exp(1))^4*tan(f*x)^2-24*a*f*x*tan(exp(1))^4+48*a*f*x*tan(exp(1))^3*tan(f*x)^5+96*a*f*x*tan(exp(1))^3*tan(f*x)^3+48*a*f*x*tan(exp(1))^3*tan(f*x)-48*a*f*x*tan(exp(1))^2*tan(f*x)^4-96*a*f*x*tan(exp(1))^2*tan(f*x)^2-48*a*f*x*tan(exp(1))^2+24*a*f*x*tan(exp(1))*tan(f*x)^5+48*a*f*x*tan(exp(1))*tan(f*x)^3+24*a*f*x*tan(exp(1))*tan(f*x)-24*a*f*x*tan(f*x)^4-48*a*f*x*tan(f*x)^2-24*a*f*x+24*a*tan(exp(1))^5*tan(f*x)^4+40*a*tan(exp(1))^5*tan(f*x)^2+24*a*tan(exp(1))^4*tan(f*x)^5+24*a*tan(exp(1))^4*tan(f*x)^3-80*a*tan(exp(1))^4*tan(f*x)+24*a*tan(exp(1))^3*tan(f*x)^4-96*a*tan(exp(1))^3*tan(f*x)^2+40*a*tan(exp(1))^3+40*a*tan(exp(1))^2*tan(f*x)^5-96*a*tan(exp(1))^2*tan(f*x)^3+24*a*tan(exp(1))^2*tan(f*x)-80*a*tan(exp(1))*tan(f*x)^4+24*a*tan(exp(1))*tan(f*x)^2+24*a*tan(exp(1))+40*a*tan(f*x)^3+24*a*tan(f*x)-120*b*f*x*tan(exp(1))^5*tan(f*x)^5-240*b*f*x*tan(exp(1))^5*tan(f*x)^3-120*b*f*x*tan(exp(1))^5*tan(f*x)+120*b*f*x*tan(exp(1))^4*tan(f*x)^4+240*b*f*x*tan(exp(1))^4*tan(f*x)^2+120*b*f*x*tan(exp(1))^4-240*b*f*x*tan(exp(1))^3*tan(f*x)^5-480*b*f*x*tan(exp(1))^3*tan(f*x)^3-240*b*f*x*tan(exp(1))^3*tan(f*x)+240*b*f*x*tan(exp(1))^2*tan(f*x)^4+480*b*f*x*tan(exp(1))^2*tan(f*x)^2+240*b*f*x*tan(exp(1))^2-120*b*f*x*tan(exp(1))*tan(f*x)^5-240*b*f*x*tan(exp(1))*tan(f*x)^3-120*b*f*

```
x*tan(exp(1))*tan(f*x)+120*b*f*x*tan(f*x)^4+240*b*f*x*tan(f*x)^2+120*b*f*x+
3*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))
+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^5*tan(f*x)^5+6*
b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2
*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^5*tan(f*x)^3+3*b*
pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*t
an(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^5*tan(f*x)-3*b*pi*s
ign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f
*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^4*tan(f*x)^4-6*b*pi*sig
n(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x
))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^4*tan(f*x)^2-3*b*pi*sign(
2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))
*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(1))^4+6*b*pi*sign(2*tan(exp(1))
^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(e
xp(1))^2*tan(f*x)^2-2)*tan(exp(1))^3*tan(f*x)^5+12*b*pi*sign(2*tan(exp(1))^
2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(ex
p(1))^2*tan(f*x)^2-2)*tan(exp(1))^3*tan(f*x)^3+6*b*pi*sign(2*tan(exp(1))^2*
tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(
1))^2*tan(f*x)^2-2)*tan(exp(1))^3*tan(f*x)-6*b*pi*sign(2*tan(exp(1))^2*tan(
f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^
2*tan(f*x)^2-2)*tan(exp(1))^2*tan(f*x)^4-12*b*pi*sign(2*tan(exp(1))^2*tan(f
*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2
*tan(f*x)^2-2)*tan(exp(1))^2*tan(f*x)^2-6*b*pi*sign(2*tan(exp(1))^2*tan(f*x
)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*t
an(f*x)^2-2)*tan(exp(1))^2+3*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1
))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*
tan(exp(1))*tan(f*x)^5+6*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*t
an(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(
exp(1))*tan(f*x)^3+3*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f
*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(exp(
1))*tan(f*x)-3*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-
2*tan(exp(1))+2*tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(f*x)^4-6*b
*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*
tan(f*x))*sign(2*tan(exp(1))^2*tan(f*x)^2-2)*tan(f*x)^2-3*b*pi*sign(2*tan(e
xp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*sign(2
*tan(exp(1))^2*tan(f*x)^2-2)+3*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp
(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^5*tan(f*x)^5+6*b*pi*s
ign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f
*x))*tan(exp(1))^5*tan(f*x)^3+3*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(ex
p(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^5*tan(f*x)-3*b*pi*si
gn(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*
x))*tan(exp(1))^4*tan(f*x)^4-6*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp
(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^4*tan(f*x)^2-3*b*pi*s
ign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f
*x))*tan(exp(1))^4+6*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f
*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^3*tan(f*x)^5+12*b*pi*sign(2*tan
(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(
exp(1))^3*tan(f*x)^3+6*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan
(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^3*tan(f*x)-6*b*pi*sign(2*tan(
exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(e
xp(1))^2*tan(f*x)^4-12*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan
(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))^2*tan(f*x)^2-6*b*pi*sign(2*ta
n(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan
(exp(1))^2+3*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*
tan(exp(1))+2*tan(f*x))*tan(exp(1))*tan(f*x)^5+6*b*pi*sign(2*tan(exp(1))^2*
tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(exp(1))*tan
(f*x)^3+3*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan
(exp(1))+2*tan(f*x))*tan(exp(1))*tan(f*x)-3*b*pi*sign(2*tan(exp(1))^2*tan(f
*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*x))*tan(f*x)^4-6*b*pi*si
```

```
gn(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))+2*tan(f*
x))*tan(f*x)^2-3*b*pi*sign(2*tan(exp(1))^2*tan(f*x)-2*tan(exp(1))*tan(f*x)^
2-2*tan(exp(1))+2*tan(f*x))+6*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*ta
n(f*x)-1))*tan(exp(1))^5*tan(f*x)^5+12*b*atan((tan(exp(1))+tan(f*x))/(tan(e
xp(1))*tan(f*x)-1))*tan(exp(1))^5*tan(f*x)^3+6*b*atan((tan(exp(1))+tan(f*x)
)/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^5*tan(f*x)-6*b*atan((tan(exp(1))+ta
n(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^4*tan(f*x)^4-12*b*atan((tan(e
xp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^4*tan(f*x)^2-6*b*ata
n((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^4+12*b*atan(
(tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^3*tan(f*x)^5+2
4*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))^3*tan
(f*x)^3+12*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*tan(exp(
1))^3*tan(f*x)-12*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f*x)-1))*t
an(exp(1))^2*tan(f*x)^4-24*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*tan(f
*x)-1))*tan(exp(1))^2*tan(f*x)^2-12*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(
1))*tan(f*x)-1))*tan(exp(1))^2+6*b*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))
*tan(f*x)-1))*tan(exp(1))*tan(f*x)^5+12*b*atan((tan(exp(1))+tan(f*x))/(tan(
exp(1))*tan(f*x)-1))*tan(exp(1))*tan(f*x)^3+6*b*atan((tan(exp(1))+tan(f*x))
/(tan(exp(1))*tan(f*x)-1))*tan(exp(1))*tan(f*x)-6*b*atan((tan(exp(1))+tan(f
*x))/(tan(exp(1))*tan(f*x)-1))*tan(f*x)^4-12*b*atan((tan(exp(1))+tan(f*x))/
(tan(exp(1))*tan(f*x)-1))*tan(f*x)^2-6*b*atan((tan(exp(1))+tan(f*x))/(tan(e
xp(1))*tan(f*x)-1))-6*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1
))*tan(exp(1))^5*tan(f*x)^5-12*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*t
an(f*x)+1))*tan(exp(1))^5*tan(f*x)^3-6*b*atan((tan(exp(1))-tan(f*x))/(tan(e
xp(1))*tan(f*x)+1))*tan(exp(1))^5*tan(f*x)+6*b*atan((tan(exp(1))-tan(f*x))/
(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^4*tan(f*x)^4+12*b*atan((tan(exp(1))-t
an(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^4*tan(f*x)^2+6*b*atan((tan(e
xp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^4-12*b*atan((tan(exp
(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^3*tan(f*x)^5-24*b*atan
((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^3*tan(f*x)^3-
12*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1))^3*ta
n(f*x)+12*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*tan(exp(1
))^2*tan(f*x)^4+24*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x)+1))*
tan(exp(1))^2*tan(f*x)^2+12*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(
f*x)+1))*tan(exp(1))^2-6*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*tan(f*x
)+1))*tan(exp(1))*tan(f*x)^5-12*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*
tan(f*x)+1))*tan(exp(1))*tan(f*x)^3-6*b*atan((tan(exp(1))-tan(f*x))/(tan(ex
p(1))*tan(f*x)+1))*tan(exp(1))*tan(f*x)+6*b*atan((tan(exp(1))-tan(f*x))/(ta
n(exp(1))*tan(f*x)+1))*tan(f*x)^4+12*b*atan((tan(exp(1))-tan(f*x))/(tan(exp
(1))*tan(f*x)+1))*tan(f*x)^2+6*b*atan((tan(exp(1))-tan(f*x))/(tan(exp(1))*t
an(f*x)+1))-120*b*tan(exp(1))^5*tan(f*x)^4-200*b*tan(exp(1))^5*tan(f*x)^2-6
4*b*tan(exp(1))^5-120*b*tan(exp(1))^4*tan(f*x)^5-120*b*tan(exp(1))^4*tan(f*
x)^3+80*b*tan(exp(1))^4*tan(f*x)-120*b*tan(exp(1))^3*tan(f*x)^4-160*b*tan(e
xp(1))^3*tan(f*x)^2-200*b*tan(exp(1))^3-200*b*tan(exp(1))^2*tan(f*x)^5-160*
b*tan(exp(1))^2*tan(f*x)^3-120*b*tan(exp(1))^2*tan(f*x)+80*b*tan(exp(1))*ta
n(f*x)^4-120*b*tan(exp(1))*tan(f*x)^2-120*b*tan(exp(1))-64*b*tan(f*x)^5-200
*b*tan(f*x)^3-120*b*tan(f*x))/(64*f*tan(exp(1))^5*tan(f*x)^5+128*f*tan(exp(
1))^5*tan(f*x)^3+64*f*tan(exp(1))^5*tan(f*x)-64*f*tan(exp(1))^4*tan(f*x)^4-
128*f*tan(exp(1))^4*tan(f*x)^2-64*f*tan(exp(1))^4+128*f*tan(exp(1))^3*tan(f
*x)^5+256*f*tan(exp(1))^3*tan(f*x)^3+128*f*tan(exp(1))^3*tan(f*x)-128*f*tan
(exp(1))^2*tan(f*x)^4-256*f*tan(exp(1))^2*tan(f*x)^2-128*f*tan(exp(1))^2+64
*f*tan(exp(1))*tan(f*x)^5+128*f*tan(exp(1))*tan(f*x)^3+64*f*tan(exp(1))*tan
(f*x)-64*f*tan(f*x)^4-128*f*tan(f*x)^2-64*f)
```

maple [A] time = 0.58, size = 102, normalized size = 1.38

$$\frac{a\left(-\frac{\left(\sin^{3}\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin^{7}\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\sin^{5}\left(fx+e\right)+\frac{5\left(\sin^{3}\left(fx+e\right)\right)}{4}+\frac{15\sin\left(fx+e\right)}{8}\right)\cos\left(fx\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+b*(sin
(f*x+e)^7/cos(f*x+e)+(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*
x+e)-15/8*f*x-15/8*e))
```

maxima [A] time = 0.81, size = 82, normalized size = 1.11

$$\frac{3\left(fx+e\right)(a-5\,b)+8\,b\tan\left(fx+e\right)-\frac{(5\,a-9\,b)\tan\left(fx+e\right)^{3}+(3\,a-7\,b)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{4}+2\,\tan\left(fx+e\right)^{2}+1}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(f*x + e)*(a - 5*b) + 8*b*tan(f*x + e) - ((5*a - 9*b)*tan(f*x + e)^3
 + (3*a - 7*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

mupad [B] time = 11.43, size = 79, normalized size = 1.07

$$x\left(\frac{3\,a}{8}-\frac{15\,b}{8}\right)-\frac{\left(\frac{5\,a}{8}-\frac{9\,b}{8}\right)\,\tan\left(e+f\,x\right)^3+\left(\frac{3\,a}{8}-\frac{7\,b}{8}\right)\,\tan\left(e+f\,x\right)}{f\,\left({\tan\left(e+f\,x\right)}^4+2\,{\tan\left(e+f\,x\right)}^2+1\right)}+\frac{b\,\tan\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2),x)
```

```
[Out] x*((3*a)/8 - (15*b)/8) - (tan(e + f*x)^3*((5*a)/8 - (9*b)/8) + tan(e + f*x)
*((3*a)/8 - (7*b)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (b*tan(
e + f*x))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tan^{2}{\left(e + f x \right)}\right) \sin^{4}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**4, x)
```

3.38 $\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=46

$$-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}x(a-3b)+\frac{b\tan(e+fx)}{f}$$

```
[Out] 1/2*(a-3*b)*x-1/2*(a-b)*cos(f*x+e)*sin(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3663, 455, 388, 203}

$$-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}x(a-3b)+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a - 3*b)*x)/2 - ((a - b)*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b*Tan[e + f*
x])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)}{(1+x^2)^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cos(e+fx)\sin(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{-a+b-2bx^2}{1+x^2}dx, x, \tan(e+\right.}{2f}$$

$$= -\frac{(a-b)\cos(e+fx)\sin(e+fx)}{2f} + \frac{b\tan(e+fx)}{f} + \frac{(a-3b)\,\text{Subst}\left(\right.}{}$$

$$= \frac{1}{2}(a-3b)x - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{2f} + \frac{b\tan(e+fx)}{f}$$

Mathematica [A] time = 0.23, size = 43, normalized size = 0.93

$$\frac{2(a-3b)(e+fx)+(b-a)\sin(2(e+fx))+4b\tan(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (2*(a - 3*b)*(e + f*x) + (-a + b)*Sin[2*(e + f*x)] + 4*b*Tan[e + f*x])/(4*f
)
```

fricas [A] time = 0.42, size = 54, normalized size = 1.17

$$\frac{(a-3\,b)fx\cos\left(fx+e\right)-\left((a-b)\cos\left(fx+e\right)^2-2\,b\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a - 3*b)*f*x*cos(f*x + e) - ((a - b)*cos(f*x + e)^2 - 2*b)*sin(f*x +
e))/(f*cos(f*x + e))
```

giac [B] time = 1.97, size = 395, normalized size = 8.59

$$\frac{afx\tan\left(fx\right)^3\tan(e)^3-3\,bfx\tan\left(fx\right)^3\tan(e)^3+afx\tan\left(fx\right)^3\tan(e)-3\,bfx\tan\left(fx\right)^3\tan(e)-afx\tan\left(fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*f*x*tan(f*x)^3*tan(e)^3 - 3*b*f*x*tan(f*x)^3*tan(e)^3 + a*f*x*tan(f*
x)^3*tan(e) - 3*b*f*x*tan(f*x)^3*tan(e) - a*f*x*tan(f*x)^2*tan(e)^2 + 3*b*f
*x*tan(f*x)^2*tan(e)^2 + a*f*x*tan(f*x)*tan(e)^3 - 3*b*f*x*tan(f*x)*tan(e)^
3 + a*tan(f*x)^3*tan(e)^2 - 3*b*tan(f*x)^3*tan(e)^2 + a*tan(f*x)^2*tan(e)^3
 - 3*b*tan(f*x)^2*tan(e)^3 - a*f*x*tan(f*x)^2 + 3*b*f*x*tan(f*x)^2 + a*f*x*
tan(f*x)*tan(e) - 3*b*f*x*tan(f*x)*tan(e) - a*f*x*tan(e)^2 + 3*b*f*x*tan(e)
^2 - 2*b*tan(f*x)^3 - 2*a*tan(f*x)^2*tan(e) - 2*a*tan(f*x)*tan(e)^2 - 2*b*t
an(e)^3 - a*f*x + 3*b*f*x + a*tan(f*x) - 3*b*tan(f*x) + a*tan(e) - 3*b*tan(
e))/(f*tan(f*x)^3*tan(e)^3 + f*tan(f*x)^3*tan(e) - f*tan(f*x)^2*tan(e)^2 +
f*tan(f*x)*tan(e)^3 - f*tan(f*x)^2 + f*tan(f*x)*tan(e) - f*tan(e)^2 - f)
```

maple [A] time = 0.45, size = 81, normalized size = 1.76

$$\frac{a\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b\left(\frac{\sin^{5}\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\sin^{3}\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)-\frac{3fx}{2}-\frac{3e}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+b*(sin(f*x+e)^5/cos(f*x+e
)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x-3/2*e))
```

maxima [A] time = 1.00, size = 51, normalized size = 1.11

$$\frac{\left(fx+e\right)(a-3\,b)+2\,b\tan\left(fx+e\right)-\frac{(a-b)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{2}+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((f*x + e)*(a - 3*b) + 2*b*tan(f*x + e) - (a - b)*tan(f*x + e)/(tan(f*x
 + e)^2 + 1))/f
```

mupad [B] time = 11.32, size = 41, normalized size = 0.89

$$\frac{b\tan\left(e+f\,x\right)-\sin\left(2\,e+2\,f\,x\right)\left(\frac{a}{4}-\frac{b}{4}\right)+f\,x\left(\frac{a}{2}-\frac{3\,b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (b*tan(e + f*x) - sin(2*e + 2*f*x)*(a/4 - b/4) + f*x*(a/2 - (3*b)/2))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{2}\left(e+fx\right)\right)\sin^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**2, x)
```

3.39 $\int \left(a+b \tan^2(e+fx)\right) dx$

Optimal. Leaf size=19

$$ax+\frac{b\tan(e+fx)}{f}-bx$$

```
[Out] a*x-b*x+b*tan(f*x+e)/f
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3473, 8}

$$ax+\frac{b\tan(e+fx)}{f}-bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[e + f*x]^2,x]
```

```
[Out] a*x - b*x + (b*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tan^2(e+fx)\right) dx &= ax+b\int \tan^2(e+fx)\,dx\\
&= ax+\frac{b\tan(e+fx)}{f}-b\int 1\,dx\\
&= ax-bx+\frac{b\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.47

$$ax-\frac{b\tan^{-1}(\tan(e+fx))}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[e + f*x]^2,x]
```

```
[Out] a*x - (b*ArcTan[Tan[e + f*x]])/f + (b*Tan[e + f*x])/f
```

fricas [A] time = 0.42, size = 21, normalized size = 1.11

$$\frac{(a-b)fx+b\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] ((a - b)*f*x + b*tan(f*x + e))/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)b*(-4*f*x*tan(exp(1))*tan(f*x)+4*f*x-pi*sign(2*tan(exp
(1))^2*tan(f*x)+2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))-2*tan(f*x))*tan(exp(
1))*tan(f*x)+pi*sign(2*tan(exp(1))^2*tan(f*x)+2*tan(exp(1))*tan(f*x)^2-2*ta
n(exp(1))-2*tan(f*x))-pi*tan(exp(1))*tan(f*x)+pi+2*atan((tan(exp(1))*tan(f*
x)-1)/(tan(exp(1))+tan(f*x)))*tan(exp(1))*tan(f*x)-2*atan((tan(exp(1))*tan(
f*x)-1)/(tan(exp(1))+tan(f*x)))+2*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*
tan(f*x)-1))*tan(exp(1))*tan(f*x)-2*atan((tan(exp(1))+tan(f*x))/(tan(exp(1)
)*tan(f*x)-1))-4*tan(exp(1))-4*tan(f*x))/(4*f*tan(exp(1))*tan(f*x)-4*f)+a*x
```

maple [A] time = 0.02, size = 29, normalized size = 1.53

$$ax+\frac{b\tan\left(fx+e\right)}{f}-\frac{b\arctan\left(\tan\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(f*x+e)^2,x)
```

```
[Out] a*x+b*tan(f*x+e)/f-b/f*arctan(tan(f*x+e))
```

maxima [A] time = 0.75, size = 23, normalized size = 1.21

$$ax-\frac{\left(fx+e-\tan\left(fx+e\right)\right)b}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x - (f*x + e - tan(f*x + e))*b/f
```

mupad [B] time = 11.28, size = 21, normalized size = 1.11

$$\frac{b\tan\left(e+f\,x\right)+f\,x\,\left(a-b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*tan(e + f*x)^2,x)
```

```
[Out] (b*tan(e + f*x) + f*x*(a - b))/f
```

sympy [A] time = 0.14, size = 20, normalized size = 1.05

$$ax+b\left(\begin{cases}-x+\frac{\tan\left(e+fx\right)}{f} & \text{for}\ f\neq 0\\ x\tan^{2}\left(e\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)**2,x)
```

```
[Out] a*x + b*Piecewise((-x + tan(e + f*x)/f, Ne(f, 0)), (x*tan(e)**2, True))
```

3.40 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=24

$$\frac{b\tan(e+fx)}{f}-\frac{a\cot(e+fx)}{f}$$

```
[Out] -a*cot(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3663, 14}

$$\frac{b\tan(e+fx)}{f}-\frac{a\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Cot[e + f*x])/f) + (b*Tan[e + f*x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx&=\frac{\text{Subst}\left(\int \frac{a+bx^2}{x^2}\,dx,x,\tan(e+fx)\right)}{f}\\&=\frac{\text{Subst}\left(\int \left(b+\frac{a}{x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\&=-\frac{a\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$\frac{b\tan(e+fx)}{f}-\frac{a\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Cot[e + f*x])/f) + (b*Tan[e + f*x])/f
```

fricas [A] time = 0.44, size = 37, normalized size = 1.54

$$-\frac{(a+b)\cos\left(fx+e\right)^2-b}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
[Out] -((a + b)*cos(f*x + e)^2 - b)/(f*cos(f*x + e)*sin(f*x + e))
```

giac [A] time = 1.36, size = 26, normalized size = 1.08

$$\frac{b\tan\left(fx+e\right)-\frac{a}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
[Out] (b*tan(f*x + e) - a/tan(f*x + e))/f
```

maple [A] time = 0.57, size = 23, normalized size = 0.96

$$\frac{-a\cot\left(fx+e\right)+b\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x)
[Out] 1/f*(-a*cot(f*x+e)+b*tan(f*x+e))
```

maxima [A] time = 0.57, size = 24, normalized size = 1.00

$$\frac{b\tan\left(fx+e\right)-\frac{a}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
[Out] (b*tan(f*x + e) - a/tan(f*x + e))/f
```

mupad [B] time = 11.27, size = 26, normalized size = 1.08

$$\frac{b\tan\left(e+fx\right)}{f}-\frac{a}{f\tan\left(e+fx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)/sin(e + f*x)^2,x)
[Out] (b*tan(e + f*x))/f - a/(f*tan(e + f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(e+fx\right)\right)\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**2, x)
```

3.41 $\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=42

$$-\frac{(a+b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(a+b)*cot(f*x+e)/f-1/3*a*cot(f*x+e)^3/f+b*tan(f*x+e)/f
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3663, 448}

$$-\frac{(a+b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a + b)*Cot[e + f*x])/f) - (a*Cot[e + f*x]^3)/(3*f) + (b*Tan[e + f*x])/f
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)\left(a+bx^2\right)}{x^4}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(b+\frac{a}{x^4}+\frac{a+b}{x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{(a+b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 60, normalized size = 1.43

$$-\frac{2a\cot(e+fx)}{3f}-\frac{a\cot(e+fx)\csc^2(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}-\frac{b\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-2*a*Cot[e + f*x])/(3*f) - (b*Cot[e + f*x])/f - (a*Cot[e + f*x]*Csc[e + f*
x]^2)/(3*f) + (b*Tan[e + f*x])/f
```

fricas [A] time = 0.41, size = 66, normalized size = 1.57

$$-\frac{2\,(a+3\,b)\cos\left(fx+e\right)^{4}-3\,(a+3\,b)\cos\left(fx+e\right)^{2}+3\,b}{3\left(f\cos\left(fx+e\right)^{3}-f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a + 3*b)*cos(f*x + e)^4 - 3*(a + 3*b)*cos(f*x + e)^2 + 3*b)/((f*co
s(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))
```

giac [A] time = 1.77, size = 53, normalized size = 1.26

$$\frac{3\,b\tan\left(fx+e\right)-\frac{3\,a\tan\left(fx+e\right)^{2}+3\,b\tan\left(fx+e\right)^{2}+a}{\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b*tan(f*x + e) - (3*a*tan(f*x + e)^2 + 3*b*tan(f*x + e)^2 + a)/tan(f
*x + e)^3)/f
```

maple [A] time = 0.71, size = 54, normalized size = 1.29

$$\frac{a\left(-\frac{2}{3}-\frac{\left(\csc^{2}\left(fx+e\right)\right)}{3}\right)\cot\left(fx+e\right)+b\left(\frac{1}{\sin\left(fx+e\right)\cos\left(fx+e\right)}-2\cot\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(
f*x+e)))
```

maxima [A] time = 0.48, size = 40, normalized size = 0.95

$$\frac{3\,b\tan\left(fx+e\right)-\frac{3\,(a+b)\tan\left(fx+e\right)^{2}+a}{\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*tan(f*x + e) - (3*(a + b)*tan(f*x + e)^2 + a)/tan(f*x + e)^3)/f
```

mupad [B] time = 11.49, size = 41, normalized size = 0.98

$$\frac{b\tan\left(e+f\,x\right)}{f}-\frac{(a+b)\,\tan\left(e+f\,x\right)^{2}+\frac{a}{3}}{f\tan\left(e+f\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)/sin(e + f*x)^4,x)
```

```
[Out] (b*tan(e + f*x))/f - (a/3 + tan(e + f*x)^2*(a + b))/(f*tan(e + f*x)^3)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)\csc^4\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**4, x)
```

3.42 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=64

$$-\frac{(2a+b)\cot^3(e+fx)}{3f}-\frac{(a+2b)\cot(e+fx)}{f}-\frac{a\cot^5(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(a+2*b)*cot(f*x+e)/f-1/3*(2*a+b)*cot(f*x+e)^3/f-1/5*a*cot(f*x+e)^5/f+b*tan
(f*x+e)/f
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3663, 448}

$$-\frac{(2a+b)\cot^3(e+fx)}{3f}-\frac{(a+2b)\cot(e+fx)}{f}-\frac{a\cot^5(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a + 2*b)*Cot[e + f*x])/f) - ((2*a + b)*Cot[e + f*x]^3)/(3*f) - (a*Cot[e
 + f*x]^5)/(5*f) + (b*Tan[e + f*x])/f
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2\left(a+bx^2\right)}{x^6}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b+\frac{a}{x^6}+\frac{2a+b}{x^4}+\frac{a+2b}{x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\ &= -\frac{(a+2b)\cot(e+fx)}{f}-\frac{(2a+b)\cot^3(e+fx)}{3f}-\frac{a\cot^5(e+fx)}{5f}+\frac{b}{}\end{aligned}$$

Mathematica [A] time = 0.05, size = 106, normalized size = 1.66

$$-\frac{8a\cot(e+fx)}{15f}-\frac{a\cot(e+fx)\csc^4(e+fx)}{5f}-\frac{4a\cot(e+fx)\csc^2(e+fx)}{15f}+\frac{b\tan(e+fx)}{f}-\frac{5b\cot(e+fx)}{3f}-\frac{b\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-8*a*Cot[e + f*x])/(15*f) - (5*b*Cot[e + f*x])/(3*f) - (4*a*Cot[e + f*x]*C
sc[e + f*x]^2)/(15*f) - (b*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) - (a*Cot[e +
f*x]*Csc[e + f*x]^4)/(5*f) + (b*Tan[e + f*x])/f
```

fricas [A] time = 0.44, size = 91, normalized size = 1.42

$$-\frac{8(a+5b)\cos\left(fx+e\right)^6-20(a+5b)\cos\left(fx+e\right)^4+15(a+5b)\cos\left(fx+e\right)^2-15b}{15\left(f\cos\left(fx+e\right)^5-2f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a + 5*b)*cos(f*x + e)^6 - 20*(a + 5*b)*cos(f*x + e)^4 + 15*(a + 5
*b)*cos(f*x + e)^2 - 15*b)/((f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(
f*x + e))*sin(f*x + e))
```

giac [A] time = 1.37, size = 79, normalized size = 1.23

$$\frac{15b\tan\left(fx+e\right)-\frac{15a\tan\left(fx+e\right)^4+30b\tan\left(fx+e\right)^4+10a\tan\left(fx+e\right)^2+5b\tan\left(fx+e\right)^2+3a}{\tan\left(fx+e\right)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*b*tan(f*x + e) - (15*a*tan(f*x + e)^4 + 30*b*tan(f*x + e)^4 + 10*a
*tan(f*x + e)^2 + 5*b*tan(f*x + e)^2 + 3*a)/tan(f*x + e)^5)/f
```

maple [A] time = 0.71, size = 83, normalized size = 1.30

$$\frac{a\left(-\frac{8}{15}-\frac{\left(\csc^4\left(fx+e\right)\right)}{5}-\frac{4\left(\csc^2\left(fx+e\right)\right)}{15}\right)\cot\left(fx+e\right)+b\left(-\frac{1}{3\sin\left(fx+e\right)^3\cos\left(fx+e\right)}+\frac{4}{3\sin\left(fx+e\right)\cos\left(fx+e\right)}-\frac{8\cot\left(fx+e\right)}{3}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-8/15-1/5*csc(f*x+e)^4-4/15*csc(f*x+e)^2)*cot(f*x+e)+b*(-1/3/sin(f*
x+e)^3/cos(f*x+e)+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e)))
```

maxima [A] time = 0.63, size = 59, normalized size = 0.92

$$\frac{15b\tan\left(fx+e\right)-\frac{15(a+2b)\tan\left(fx+e\right)^4+5(2a+b)\tan\left(fx+e\right)^2+3a}{\tan\left(fx+e\right)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*b*tan(f*x + e) - (15*(a + 2*b)*tan(f*x + e)^4 + 5*(2*a + b)*tan(f*
x + e)^2 + 3*a)/tan(f*x + e)^5)/f
```

mupad [B] time = 11.41, size = 59, normalized size = 0.92

$$\frac{b\tan\left(e+fx\right)}{f}-\frac{(a+2b)\tan\left(e+fx\right)^4+\left(\frac{2a}{3}+\frac{b}{3}\right)\tan\left(e+fx\right)^2+\frac{a}{5}}{f\tan\left(e+fx\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)/sin(e + f*x)^6,x)
```

```
[Out] (b*tan(e + f*x))/f - (a/5 + tan(e + f*x)^2*((2*a)/3 + b/3) + tan(e + f*x)^4
*(a + 2*b))/(f*tan(e + f*x)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)\csc^6\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**6, x)
```

3.43 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=107

$$-\frac{\left(a^2-6ab+6b^2\right)\cos(e+fx)}{f}-\frac{(a-b)^2\cos^5(e+fx)}{5f}+\frac{2(a-2b)(a-b)\cos^3(e+fx)}{3f}+\frac{2b(a-2b)\sec(e+fx)}{f}+\frac{b^2}{}$$

```
[Out] -(a^2-6*a*b+6*b^2)*cos(f*x+e)/f+2/3*(a-2*b)*(a-b)*cos(f*x+e)^3/f-1/5*(a-b)^
2*cos(f*x+e)^5/f+2*(a-2*b)*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3664, 448}

$$-\frac{\left(a^2-6ab+6b^2\right)\cos(e+fx)}{f}-\frac{(a-b)^2\cos^5(e+fx)}{5f}+\frac{2(a-2b)(a-b)\cos^3(e+fx)}{3f}+\frac{2b(a-2b)\sec(e+fx)}{f}+\frac{b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a^2 - 6*a*b + 6*b^2)*Cos[e + f*x])/f) + (2*(a - 2*b)*(a - b)*Cos[e + f*
x]^3)/(3*f) - ((a - b)^2*Cos[e + f*x]^5)/(5*f) + (2*(a - 2*b)*b*Sec[e + f*x
])/f + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^2}{x^6}\,dx,x,\sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(2(a-2b)b+\frac{(a-b)^2}{x^6}+\frac{2(a-2b)(-a+b)}{x^4}+\frac{a^2-6ab+6b^2}{x^2}+b^2x^2\right)dx,x\right.}{f}\\ &= -\frac{\left(a^2-6ab+6b^2\right)\cos(e+fx)}{f}+\frac{2(a-2b)(a-b)\cos^3(e+fx)}{3f}-\frac{(a-}{}\end{aligned}$$

Mathematica [A] time = 0.72, size = 97, normalized size = 0.91

$$\frac{-30\left(5a^2-38ab+41b^2\right)\cos(e+fx)+5(5a-13b)(a-b)\cos(3(e+fx))-3(a-b)^2\cos(5(e+fx))+480b(a-2}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-30*(5*a^2 - 38*a*b + 41*b^2)*Cos[e + f*x] + 5*(5*a - 13*b)*(a - b)*Cos[3*
(e + f*x)] - 3*(a - b)^2*Cos[5*(e + f*x)] + 480*(a - 2*b)*b*Sec[e + f*x] +
80*b^2*Sec[e + f*x]^3)/(240*f)
```

fricas [A] time = 0.44, size = 105, normalized size = 0.98

$$-\frac{3\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^8-10\left(a^2-3\,ab+2\,b^2\right)\cos\left(fx+e\right)^6+15\left(a^2-6\,ab+6\,b^2\right)\cos\left(fx+e\right)^4-3}{15\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^8 - 10*(a^2 - 3*a*b + 2*b^2)*cos(
f*x + e)^6 + 15*(a^2 - 6*a*b + 6*b^2)*cos(f*x + e)^4 - 30*(a*b - 2*b^2)*cos
(f*x + e)^2 - 5*b^2)/(f*cos(f*x + e)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.74, size = 185, normalized size = 1.73

$$\frac{-\frac{a^2\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5}+2ab\left(\frac{\sin^8\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\frac{16}{5}+\sin^6\left(fx+e\right)+\frac{6\left(\sin^4\left(fx+e\right)\right)}{5}+\frac{8\left(\sin^2\left(fx+e\right)\right)}{5}\right)\cos\left(\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-1/5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(sin(f*x
+e)^8/cos(f*x+e)+(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(
f*x+e))+b^2*(1/3*sin(f*x+e)^10/cos(f*x+e)^3-7/3*sin(f*x+e)^10/cos(f*x+e)-7/
3*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(f*x+e)^4+64/35*sin(f*x+e)
^2)*cos(f*x+e)))
```

maxima [A] time = 0.44, size = 104, normalized size = 0.97

$$-\frac{3\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^5-10\left(a^2-3\,ab+2\,b^2\right)\cos\left(fx+e\right)^3+15\left(a^2-6\,ab+6\,b^2\right)\cos\left(fx+e\right)-\frac{5(}{}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/15*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 10*(a^2 - 3*a*b + 2*b^2)*cos(
f*x + e)^3 + 15*(a^2 - 6*a*b + 6*b^2)*cos(f*x + e) - 5*(6*(a*b - 2*b^2)*cos
(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

mupad [B] time = 12.65, size = 183, normalized size = 1.71

$$\frac{2a^2\cos\left(e+fx\right)^3}{3f}-\frac{6b^2\cos\left(e+fx\right)}{f}-\frac{a^2\cos\left(e+fx\right)}{f}-\frac{a^2\cos\left(e+fx\right)^5}{5f}-\frac{4b^2}{f\cos\left(e+fx\right)}+\frac{b^2}{3f\cos\left(e+fx\right)^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (2*a^2*cos(e + f*x)^3)/(3*f) - (6*b^2*cos(e + f*x))/f - (a^2*cos(e + f*x))/
f - (a^2*cos(e + f*x)^5)/(5*f) - (4*b^2)/(f*cos(e + f*x)) + b^2/(3*f*cos(e
+ f*x)^3) + (4*b^2*cos(e + f*x)^3)/(3*f) - (b^2*cos(e + f*x)^5)/(5*f) + (6*
a*b*cos(e + f*x))/f + (2*a*b)/(f*cos(e + f*x)) - (2*a*b*cos(e + f*x)^3)/f +
 (2*a*b*cos(e + f*x)^5)/(5*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(e+fx\right)\right)^2\sin^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x)**5, x)
```

3.44 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=80

$$\frac{(a-b)^2\cos^3(e+fx)}{3f}-\frac{(a-3b)(a-b)\cos(e+fx)}{f}+\frac{b(2a-3b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(a-3*b)*(a-b)*cos(f*x+e)/f+1/3*(a-b)^2*cos(f*x+e)^3/f+(2*a-3*b)*b*sec(f*x+
e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3664, 448}

$$\frac{(a-b)^2\cos^3(e+fx)}{3f}-\frac{(a-3b)(a-b)\cos(e+fx)}{f}+\frac{b(2a-3b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a - 3*b)*(a - b)*Cos[e + f*x])/f) + ((a - b)^2*Cos[e + f*x]^3)/(3*f) +
((2*a - 3*b)*b*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)\left(a-b+bx^2\right)^2}{x^4}\,dx,x,\sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left((2a-3b)b-\frac{(a-b)^2}{x^4}+\frac{(a-3b)(a-b)}{x^2}+b^2x^2\right)dx,x,\sec(e+fx)\right)}{f}\\ &= -\frac{(a-3b)(a-b)\cos(e+fx)}{f}+\frac{(a-b)^2\cos^3(e+fx)}{3f}+\frac{(2a-3b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.53, size = 72, normalized size = 0.90

$$\frac{\left(-9a^2+42ab-33b^2\right)\cos(e+fx)+(a-b)^2\cos(3(e+fx))+4b\sec(e+fx)\left(6a+b\sec^2(e+fx)-9b\right)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((-9*a^2 + 42*a*b - 33*b^2)*Cos[e + f*x] + (a - b)^2*Cos[3*(e + f*x)] + 4*b
*Sec[e + f*x]*(6*a - 9*b + b*Sec[e + f*x]^2))/(12*f)
```

fricas [A] time = 0.48, size = 80, normalized size = 1.00

$$\frac{\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^6-3\left(a^2-4\,ab+3\,b^2\right)\cos\left(fx+e\right)^4+3\left(2\,ab-3\,b^2\right)\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^6 - 3*(a^2 - 4*a*b + 3*b^2)*cos(f*x +
 e)^4 + 3*(2*a*b - 3*b^2)*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

giac [A] time = 2.13, size = 144, normalized size = 1.80

$$\frac{6\,ab\cos\left(fx+e\right)^2-9\,b^2\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}+\frac{a^2f^{11}\cos\left(fx+e\right)^3-2\,abf^{11}\cos\left(fx+e\right)^3+b^2f^{11}\cos\left(fx+e\right)^3-}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a*b*cos(f*x + e)^2 - 9*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
+ 1/3*(a^2*f^11*cos(f*x + e)^3 - 2*a*b*f^11*cos(f*x + e)^3 + b^2*f^11*cos(f
*x + e)^3 - 3*a^2*f^11*cos(f*x + e) + 12*a*b*f^11*cos(f*x + e) - 9*b^2*f^11
*cos(f*x + e))/f^12
```

maple [B] time = 0.87, size = 155, normalized size = 1.94

$$\frac{-\frac{a^2\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+2ab\left(\frac{\sin^6\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)\right)+b^2\left(\frac{\sin^8\left(fx+e\right)}{3\cos\left(fx+e\right)^3}-\frac{5(}{3}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-1/3*a^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(sin(f*x+e)^6/cos(f*x+e)+(8
/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))+b^2*(1/3*sin(f*x+e)^8/cos(f*x
+e)^3-5/3*sin(f*x+e)^8/cos(f*x+e)-5/3*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8
/5*sin(f*x+e)^2)*cos(f*x+e)))
```

maxima [A] time = 0.53, size = 80, normalized size = 1.00

$$\frac{\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^3-3\left(a^2-4\,ab+3\,b^2\right)\cos\left(fx+e\right)+\frac{3\left(2\,ab-3\,b^2\right)\cos\left(fx+e\right)^2+b^2}{\cos\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - 3*(a^2 - 4*a*b + 3*b^2)*cos(f*x +
 e) + (3*(2*a*b - 3*b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

mupad [B] time = 15.40, size = 128, normalized size = 1.60

$$-\frac{32\,a\,b+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(64\,a\,b-32\,a^2\right)+12\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(24\,a^2-96\,a\,b+96\,b^2\right)-4\,a^2}{f\,\left(3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-9\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+9\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] -(32*a*b + tan(e/2 + (f*x)/2)^6*(64*a*b - 32*a^2) + 12*a^2*tan(e/2 + (f*x)/
2)^8 + tan(e/2 + (f*x)/2)^4*(24*a^2 - 96*a*b + 96*b^2) - 4*a^2 - 32*b^2)/(f
*(9*tan(e/2 + (f*x)/2)^4 - 9*tan(e/2 + (f*x)/2)^8 + 3*tan(e/2 + (f*x)/2)^12
 - 3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^2\sin^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x)**3, x)
```

3.45 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=54

$$-\frac{(a-b)^2\cos(e+fx)}{f}+\frac{2b(a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(a-b)^2*cos(f*x+e)/f+2*(a-b)*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3664, 270}

$$-\frac{(a-b)^2\cos(e+fx)}{f}+\frac{2b(a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a - b)^2*Cos[e + f*x])/f) + (2*(a - b)*b*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^2}{x^2}\,dx,x,\sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(2(a-b)b+\frac{(a-b)^2}{x^2}+b^2x^2\right)dx,x,\sec(e+fx)\right)}{f}\\ &= -\frac{(a-b)^2\cos(e+fx)}{f}+\frac{2(a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.31, size = 48, normalized size = 0.89

$$\frac{b\sec(e+fx)\left(6a+b\sec^2(e+fx)-6b\right)-3(a-b)^2\cos(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*(a - b)^2*Cos[e + f*x] + b*Sec[e + f*x]*(6*a - 6*b + b*Sec[e + f*x]^2))
/(3*f)
```

fricas [A] time = 0.45, size = 59, normalized size = 1.09

$$-\frac{3 \left(a^2 - 2 \, ab + b^2\right) \cos\left(fx + e\right)^4 - 6 \left(ab - b^2\right) \cos\left(fx + e\right)^2 - b^2}{3 \, f \cos\left(fx + e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 6*(a*b - b^2)*cos(f*x + e)^2 -
 b^2)/(f*cos(f*x + e)^3)
```

giac [A] time = 2.13, size = 94, normalized size = 1.74

$$-\frac{a^2 f^3 \cos\left(fx + e\right) - 2 \, abf^3 \cos\left(fx + e\right) + b^2 f^3 \cos\left(fx + e\right)}{f^4} + \frac{6 \, ab \cos\left(fx + e\right)^2 - 6 \, b^2 \cos\left(fx + e\right)^2 + b^2}{3 \, f \cos\left(fx + e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -(a^2*f^3*cos(f*x + e) - 2*a*b*f^3*cos(f*x + e) + b^2*f^3*cos(f*x + e))/f^4
 + 1/3*(6*a*b*cos(f*x + e)^2 - 6*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^
3)
```

maple [B] time = 0.72, size = 125, normalized size = 2.31

$$\frac{-\cos\left(fx + e\right) a^2 + 2ab \left(\frac{\sin^4\left(fx+e\right)}{\cos\left(fx+e\right)} + \left(2 + \sin^2\left(fx + e\right)\right) \cos\left(fx + e\right)\right) + b^2 \left(\frac{\sin^6\left(fx+e\right)}{3 \cos\left(fx+e\right)^3} - \frac{\sin^6\left(fx+e\right)}{\cos\left(fx+e\right)} - \left(\frac{8}{3} + \sin^4\left(fx+e\right) + \frac{4}{3}\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-cos(f*x+e)*a^2+2*a*b*(sin(f*x+e)^4/cos(f*x+e)+(2+sin(f*x+e)^2)*cos(f*
x+e))+b^2*(1/3*sin(f*x+e)^6/cos(f*x+e)^3-sin(f*x+e)^6/cos(f*x+e)-(8/3+sin(f
*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)))
```

maxima [A] time = 0.72, size = 71, normalized size = 1.31

$$\frac{6 \, ab \left(\frac{1}{\cos\left(fx+e\right)} + \cos\left(fx + e\right)\right) - b^2 \left(\frac{6 \, \cos\left(fx+e\right)^2 - 1}{\cos\left(fx+e\right)^3} + 3 \, \cos\left(fx + e\right)\right) - 3 \, a^2 \cos\left(fx + e\right)}{3 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(6*a*b*(1/cos(f*x + e) + cos(f*x + e)) - b^2*((6*cos(f*x + e)^2 - 1)/co
s(f*x + e)^3 + 3*cos(f*x + e)) - 3*a^2*cos(f*x + e))/f
```

mupad [B] time = 14.12, size = 126, normalized size = 2.33

$$-\frac{8 \, a \, b + \tan\left(\frac{e}{2} + \frac{f \, x}{2}\right)^4 \left(8 \, a \, b - 6 \, a^2\right) + 2 \, a^2 \tan\left(\frac{e}{2} + \frac{f \, x}{2}\right)^6 + \tan\left(\frac{e}{2} + \frac{f \, x}{2}\right)^2 \left(6 \, a^2 - 16 \, a \, b + \frac{32 \, b^2}{3}\right) - 2 \, a^2 - \frac{16 \, b^2}{3} \cdots}{f \left(\tan\left(\frac{e}{2} + \frac{f \, x}{2}\right)^8 - 2 \tan\left(\frac{e}{2} + \frac{f \, x}{2}\right)^6 + 2 \tan\left(\frac{e}{2} + \frac{f \, x}{2}\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] -(8*a*b + tan(e/2 + (f*x)/2)^4*(8*a*b - 6*a^2) + 2*a^2*tan(e/2 + (f*x)/2)^6
 + tan(e/2 + (f*x)/2)^2*(6*a^2 - 16*a*b + (32*b^2)/3) - 2*a^2 - (16*b^2)/3)
/(f*(2*tan(e/2 + (f*x)/2)^2 - 2*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8
 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^2 \sin\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x), x)
```

3.46 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=52

$$-\frac{a^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{b(2a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -a^2*arctanh(cos(f*x+e))/f+(2*a-b)*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3664, 390, 207}

$$-\frac{a^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{b(2a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTanh[Cos[e + f*x]])/f) + ((2*a - b)*b*Sec[e + f*x])/f + (b^2*Sec[
e + f*x]^3)/(3*f)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^2}{-1+x^2}\,dx,x,\sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{a^2}{-1+x^2}\right)dx,x,\sec(e+fx)\right)}{f}\\ &= \frac{(2a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\text{s}\right.}{f}\\ &= -\frac{a^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{(2a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.17, size = 66, normalized size = 1.27

$$\frac{3a^2\left(\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)+3b(2a-b)\sec(e+fx)+b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (3*a^2*(-Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]]) + 3*(2*a - b)*b*Sec
[e + f*x] + b^2*Sec[e + f*x]^3)/(3*f)
```

fricas [A] time = 0.56, size = 87, normalized size = 1.67

$$-\frac{3\,a^2\cos\left(fx+e\right)^3\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-3\,a^2\cos\left(fx+e\right)^3\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-6\left(2\,ab-b^2\right)\cos\left(f\right.}{6\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^2*cos(f*x + e)^3*log(1/2*cos(f*x + e) + 1/2) - 3*a^2*cos(f*x + e)
^3*log(-1/2*cos(f*x + e) + 1/2) - 6*(2*a*b - b^2)*cos(f*x + e)^2 - 2*b^2)/(
f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((-6*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a*b+12*(1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1)))*a*b-6*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1
)))*b^2-6*a*b+2*b^2)*1/3/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1)^3+a^2/
4*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [B] time = 0.55, size = 124, normalized size = 2.38

$$\frac{a^2\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}+\frac{2ab}{f\cos\left(fx+e\right)}+\frac{b^2\left(\sin^4\left(fx+e\right)\right)}{3f\cos\left(fx+e\right)^3}-\frac{b^2\left(\sin^4\left(fx+e\right)\right)}{3f\cos\left(fx+e\right)}-\frac{b^2\left(\sin^2\left(fx+e\right)\right)\cos}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*a^2*ln(csc(f*x+e)-cot(f*x+e))+2/f*a*b/cos(f*x+e)+1/3/f*b^2*sin(f*x+e)^4
/cos(f*x+e)^3-1/3/f*b^2*sin(f*x+e)^4/cos(f*x+e)-1/3/f*b^2*sin(f*x+e)^2*cos(
f*x+e)-2/3/f*cos(f*x+e)*b^2
```

maxima [A] time = 0.31, size = 68, normalized size = 1.31

$$-\frac{3\,a^2\log\left(\cos\left(fx+e\right)+1\right)-3\,a^2\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(2\,ab-b^2\right)\cos\left(fx+e\right)^2+b^2\right)}{\cos\left(fx+e\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a^2*log(cos(f*x + e) + 1) - 3*a^2*log(cos(f*x + e) - 1) - 2*(3*(2*a
*b - b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

mupad [B] time = 12.64, size = 86, normalized size = 1.65

$$\frac{a^2\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}-\frac{4\,a\,b-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(8\,a\,b-4\,b^2\right)-\frac{4\,b^2}{3}+4\,a\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2/sin(e + f*x),x)
```

```
[Out] (a^2*log(tan(e/2 + (f*x)/2)))/f - (4*a*b - tan(e/2 + (f*x)/2)^2*(8*a*b - 4*
b^2) - (4*b^2)/3 + 4*a*b*tan(e/2 + (f*x)/2)^4)/(f*(tan(e/2 + (f*x)/2)^2 - 1
)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^2\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x), x)
```

3.47 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=82

$$-\frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f}+\frac{a(a+4b)\sec(e+fx)}{2f}-\frac{a(a+4b)\tanh^{-1}(\cos(e+fx))}{2f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -1/2*a*(a+4*b)*arctanh(cos(f*x+e))/f+1/2*a*(a+4*b)*sec(f*x+e)/f-1/2*a^2*csc
(f*x+e)^2*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A] time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 463, 459, 321, 207}

$$-\frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f}+\frac{a(a+4b)\sec(e+fx)}{2f}-\frac{a(a+4b)\tanh^{-1}(\cos(e+fx))}{2f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(a*(a + 4*b)*ArcTanh[Cos[e + f*x]])/(2*f) + (a*(a + 4*b)*Sec[e + f*x])/(2*
f) - (a^2*Csc[e + f*x]^2*Sec[e + f*x])/(2*f) + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
```

```
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a-b+bx^2\right)^2}{\left(-1+x^2\right)^2} dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f} + \frac{\text{Subst}\left(\int \frac{x^2\left(a^2+4ab-2b^2+2b^2x^2\right)}{-1+x^2} dx, x, s\right.}{2f}$$

$$= -\frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f} + \frac{b^2\sec^3(e+fx)}{3f} + \frac{(a(a+4b))\,\text{Subs}}{}$$

$$= \frac{a(a+4b)\sec(e+fx)}{2f} - \frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f} + \frac{b^2\sec^3(e+}{3f}$$

$$= -\frac{a(a+4b)\tanh^{-1}(\cos(e+fx))}{2f} + \frac{a(a+4b)\sec(e+fx)}{2f} - \frac{a^2\csc^2(}{}$$

Mathematica [B] time = 6.13, size = 376, normalized size = 4.59

$$\frac{\left(a^2+4ab\right)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f}+\frac{\left(-a^2-4ab\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f}-\frac{a^2\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f}+\frac{a^2\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/8*(a^2*Csc[(e + f*x)/2]^2)/f + ((-a^2 - 4*a*b)*Log[Cos[(e + f*x)/2]])/(2
*f) + ((a^2 + 4*a*b)*Log[Sin[(e + f*x)/2]])/(2*f) + (a^2*Sec[(e + f*x)/2]^2
)/(8*f) + b^2/(12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (b^2*Sin[(e
+ f*x)/2])/(6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) - (b^2*Sin[(e + f*
x)/2])/(6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) + b^2/(12*f*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^2) + (-12*a*b*Sin[(e + f*x)/2] - b^2*Sin[(e + f
*x)/2])/(6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) + (12*a*b*Sin[(e + f*x)
/2] + b^2*Sin[(e + f*x)/2])/(6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [B] time = 0.44, size = 168, normalized size = 2.05

$$\frac{6\left(a^2+4\,ab\right)\cos\left(fx+e\right)^4-4\left(6\,ab-b^2\right)\cos\left(fx+e\right)^2-4\,b^2-3\left(\left(a^2+4\,ab\right)\cos\left(fx+e\right)^5-\left(a^2+4\,ab\right)\cos}{12\left(f\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2 + 4*a*b)*cos(f*x + e)^4 - 4*(6*a*b - b^2)*cos(f*x + e)^2 - 4*b
^2 - 3*((a^2 + 4*a*b)*cos(f*x + e)^5 - (a^2 + 4*a*b)*cos(f*x + e)^3)*log(1/
2*cos(f*x + e) + 1/2) + 3*((a^2 + 4*a*b)*cos(f*x + e)^5 - (a^2 + 4*a*b)*cos
(f*x + e)^3)*log(-1/2*cos(f*x + e) + 1/2))/(f*cos(f*x + e)^5 - f*cos(f*x +
e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2-8*(1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1)))*a*b-a^2)*1/16/(1-cos(f*x+exp(1)))*(1+cos(f*x+ex
p(1)))+(-6*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a*b-3*((1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1))))^2*b^2+12*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
)*a*b-6*a*b-b^2)*1/3/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1)^3+(1-cos(f
*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2/16+(a^2+4*a*b)/8*ln(abs(1-cos(f*x+exp(1
)))/abs(1+cos(f*x+exp(1)))))
```

maple [A] time = 0.74, size = 100, normalized size = 1.22

$$-\frac{a^2\csc\left(fx+e\right)\cot\left(fx+e\right)}{2f}+\frac{a^2\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2f}+\frac{2ab}{f\cos\left(fx+e\right)}+\frac{2ab\ln\left(\csc\left(fx+e\right)-\cot\left(fx\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*a^2*csc(f*x+e)*cot(f*x+e)+1/2/f*a^2*ln(csc(f*x+e)-cot(f*x+e))+2/f*a*
b/cos(f*x+e)+2/f*a*b*ln(csc(f*x+e)-cot(f*x+e))+1/3/f*b^2/cos(f*x+e)^3
```

maxima [A] time = 0.62, size = 111, normalized size = 1.35

$$-\frac{3\left(a^2+4\,ab\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(a^2+4\,ab\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(a^2+4\,ab\right)\cos\left(fx+e\right)^4-2\left(6\,ab-b^2\right)\cos\left(fx+\right.\right.}{\cos\left(fx+e\right)^5-\cos\left(fx+e\right)^3}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*(a^2 + 4*a*b)*log(cos(f*x + e) + 1) - 3*(a^2 + 4*a*b)*log(cos(f*x
+ e) - 1) - 2*(3*(a^2 + 4*a*b)*cos(f*x + e)^4 - 2*(6*a*b - b^2)*cos(f*x + e
)^2 - 2*b^2)/(cos(f*x + e)^5 - cos(f*x + e)^3))/f
```

mupad [B] time = 12.61, size = 188, normalized size = 2.29

$$\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(\frac{a^2}{2}+2\,b\,a\right)}{f}+\frac{a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{8\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{3\,a^2}{2}+32\,b\,a\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(\frac{a^2}{2}+16\,a\,b+8\right.}{f\,\left(-4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+12\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-12\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^3,x)
```

```
[Out] (log(tan(e/2 + (f*x)/2))*(2*a*b + a^2/2))/f + (a^2*tan(e/2 + (f*x)/2)^2)/(8
*f) - (tan(e/2 + (f*x)/2)^4*(32*a*b + (3*a^2)/2) - tan(e/2 + (f*x)/2)^6*(16
*a*b + a^2/2 + 8*b^2) - tan(e/2 + (f*x)/2)^2*(16*a*b + (3*a^2)/2 + (8*b^2)/
3) + a^2/2)/(f*(4*tan(e/2 + (f*x)/2)^2 - 12*tan(e/2 + (f*x)/2)^4 + 12*tan(e
/2 + (f*x)/2)^6 - 4*tan(e/2 + (f*x)/2)^8))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^2\csc^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x)**3, x)
```

3.48 $\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=123

$$\frac{\left(a^2+8ab+4b^2\right)\sec(e+fx)}{4f}-\frac{\left(3a^2+24ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{a^2\csc^4(e+fx)\sec(e+fx)}{4f}-\frac{a(a+8b)\,\mathrm{c}}{}$$

```
[Out] -1/8*(3*a^2+24*a*b+8*b^2)*arctanh(cos(f*x+e))/f-1/8*a*(a+8*b)*cot(f*x+e)*cs
c(f*x+e)/f+1/4*(a^2+8*a*b+4*b^2)*sec(f*x+e)/f-1/4*a^2*csc(f*x+e)^4*sec(f*x+
e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A] time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 463, 455, 1153, 207}

$$\frac{\left(a^2+8ab+4b^2\right)\sec(e+fx)}{4f}-\frac{\left(3a^2+24ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{a^2\csc^4(e+fx)\sec(e+fx)}{4f}-\frac{a(a+8b)\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((3*a^2 + 24*a*b + 8*b^2)*ArcTanh[Cos[e + f*x]])/(8*f) - (a*(a + 8*b)*Cot[
e + f*x]*Csc[e + f*x])/(8*f) + ((a^2 + 8*a*b + 4*b^2)*Sec[e + f*x])/(4*f) -
 (a^2*Csc[e + f*x]^4*Sec[e + f*x])/(4*f) + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a-b+bx^2\right)^2}{\left(-1+x^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{a^2\csc^4(e+fx)\sec(e+fx)}{4f} + \frac{\text{Subst}\left(\int \frac{x^4\left(a^2+8ab-4b^2+4b^2x^2\right)}{\left(-1+x^2\right)^2}\,dx, x, s\right.}{4f}$$

$$= -\frac{a(a+8b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{a^2\csc^4(e+fx)\sec(e+fx)}{4f} -$$

$$= -\frac{a(a+8b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{a^2\csc^4(e+fx)\sec(e+fx)}{4f} -$$

$$= -\frac{a(a+8b)\cot(e+fx)\csc(e+fx)}{8f} + \frac{\left(a^2+8ab+4b^2\right)\sec(e+fx)}{4f}$$

$$= -\frac{\left(3a^2+24ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{a(a+8b)\cot(e+fx)}{8f}$$

Mathematica [B] time = 6.19, size = 447, normalized size = 3.63

$$\frac{\left(3a^2+24ab+8b^2\right)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8f} + \frac{\left(-3a^2-24ab-8b^2\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8f} + \frac{\left(-3a^2-8ab\right)\csc^2\left(\frac{1}{2}(\right.}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((-3*a^2 - 8*a*b)*Csc[(e + f*x)/2]^2)/(32*f) - (a^2*Csc[(e + f*x)/2]^4)/(64
*f) + ((-3*a^2 - 24*a*b - 8*b^2)*Log[Cos[(e + f*x)/2]])/(8*f) + ((3*a^2 + 2
4*a*b + 8*b^2)*Log[Sin[(e + f*x)/2]])/(8*f) + ((3*a^2 + 8*a*b)*Sec[(e + f*x
)/2]^2)/(32*f) + (a^2*Sec[(e + f*x)/2]^4)/(64*f) + b^2/(12*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^2) + (b^2*Sin[(e + f*x)/2])/(6*f*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^3) - (b^2*Sin[(e + f*x)/2])/(6*f*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^3) + b^2/(12*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (
-12*a*b*Sin[(e + f*x)/2] - 7*b^2*Sin[(e + f*x)/2])/(6*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])) + (12*a*b*Sin[(e + f*x)/2] + 7*b^2*Sin[(e + f*x)/2])/(6
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [B] time = 0.54, size = 284, normalized size = 2.31

$$6\left(3a^2+24ab+8b^2\right)\cos\left(fx+e\right)^6 - 10\left(3a^2+24ab+8b^2\right)\cos\left(fx+e\right)^4 + 16\left(6ab+b^2\right)\cos\left(fx+e\right)^2 + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(6*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^6 - 10*(3*a^2 + 24*a*b + 8*b^
2)*cos(f*x + e)^4 + 16*(6*a*b + b^2)*cos(f*x + e)^2 + 16*b^2 - 3*((3*a^2 +
24*a*b + 8*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^5
+ (3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e) + 1/2) + 3*
((3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 24*a*b + 8*b^2)*cos(f
*x + e)^5 + (3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^3)*log(-1/2*cos(f*x + e)
+ 1/2))/(f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((-18*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2-144*((1-c
os(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a*b-48*((1-cos(f*x+exp(1)))/(1+cos(f
*x+exp(1))))^2*b^2-8*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2-16*(1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1)))*a*b-a^2)*1/128/((1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1))))^2+(-6*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a*b-6*((1-
cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2+12*(1-cos(f*x+exp(1)))/(1+cos(f
*x+exp(1)))*a*b+6*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2-6*a*b-4*b^2)*
1/3/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))-1)^3+(32*((1-cos(f*x+exp(1)))/
(1+cos(f*x+exp(1))))^2*a^2+256*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2+
512*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a*b)/4096+(3*a^2+24*a*b+8*b^2)/
32*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [A] time = 0.64, size = 183, normalized size = 1.49

$$-\frac{a^{2} \cot\left(fx+e\right)\left(\csc^{3}\left(fx+e\right)\right)}{4f}-\frac{3a^{2} \csc\left(fx+e\right)\cot\left(fx+e\right)}{8f}+\frac{3a^{2} \ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{8f}-\frac{\quad}{f\sin\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/4/f*a^2*cot(f*x+e)*csc(f*x+e)^3-3/8/f*a^2*csc(f*x+e)*cot(f*x+e)+3/8/f*a^
2*ln(csc(f*x+e)-cot(f*x+e))-1/f*a*b/sin(f*x+e)^2/cos(f*x+e)+3/f*a*b/cos(f*x
+e)+3/f*a*b*ln(csc(f*x+e)-cot(f*x+e))+1/3/f*b^2/cos(f*x+e)^3+1/f*b^2/cos(f*
x+e)+1/f*b^2*ln(csc(f*x+e)-cot(f*x+e))
```

maxima [A] time = 0.69, size = 163, normalized size = 1.33

$$-\frac{3\left(3a^{2}+24ab+8b^{2}\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(3a^{2}+24ab+8b^{2}\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(3a^{2}+24ab+8b^{2}\right)\right.}{}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(3*(3*a^2 + 24*a*b + 8*b^2)*log(cos(f*x + e) + 1) - 3*(3*a^2 + 24*a*b
 + 8*b^2)*log(cos(f*x + e) - 1) - 2*(3*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e
)^6 - 5*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(6*a*b + b^2)*cos(f*x +
 e)^2 + 8*b^2)/(cos(f*x + e)^7 - 2*cos(f*x + e)^5 + cos(f*x + e)^3))/f
```

mupad [B] time = 12.06, size = 243, normalized size = 1.98

$$\frac{a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4}{64f}+\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(\frac{3a^2}{8}+3ab+b^2\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{5a^2}{4}+4ba\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8\left(2a^2+\right.}{f\left(-16\tan\left(\frac{e}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^5,x)
```

```
[Out] (a^2*tan(e/2 + (f*x)/2)^4)/(64*f) + (log(tan(e/2 + (f*x)/2))*(3*a*b + (3*a^
2)/8 + b^2))/f - (tan(e/2 + (f*x)/2)^2*(4*a*b + (5*a^2)/4) - tan(e/2 + (f*x
)/2)^8*(68*a*b + 2*a^2 + 64*b^2) - tan(e/2 + (f*x)/2)^4*(76*a*b + (21*a^2)/
4 + (128*b^2)/3) + tan(e/2 + (f*x)/2)^6*(140*a*b + (23*a^2)/4 + 64*b^2) + a
^2/4)/(f*(16*tan(e/2 + (f*x)/2)^4 - 48*tan(e/2 + (f*x)/2)^6 + 48*tan(e/2 +
(f*x)/2)^8 - 16*tan(e/2 + (f*x)/2)^10)) + (tan(e/2 + (f*x)/2)^2*((a*b)/4 +
a^2/8))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.49 $\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=122

$$-\frac{\left(a^2-10ab+13b^2\right)\tan(e+fx)}{4f}+\frac{1}{8}x\left(3a^2-30ab+35b^2\right)+\frac{(a-b)^2\sin^4(e+fx)\tan(e+fx)}{4f}-\frac{(a-9b)(a-b)\sin}{8}$$

```
[Out] 1/8*(3*a^2-30*a*b+35*b^2)*x-1/8*(a-9*b)*(a-b)*cos(f*x+e)*sin(f*x+e)/f-1/4*(
a^2-10*a*b+13*b^2)*tan(f*x+e)/f+1/4*(a-b)^2*sin(f*x+e)^4*tan(f*x+e)/f+1/3*b
^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 463, 455, 1153, 203}

$$-\frac{\left(a^2-10ab+13b^2\right)\tan(e+fx)}{4f}+\frac{1}{8}x\left(3a^2-30ab+35b^2\right)+\frac{(a-b)^2\sin^4(e+fx)\tan(e+fx)}{4f}-\frac{(a-9b)(a-b)\sin}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((3*a^2 - 30*a*b + 35*b^2)*x)/8 - ((a - 9*b)*(a - b)*Cos[e + f*x]*Sin[e + f
*x])/(8*f) - ((a^2 - 10*a*b + 13*b^2)*Tan[e + f*x])/(4*f) + ((a - b)^2*Sin[
e + f*x]^4*Tan[e + f*x])/(4*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{x^4\left(a+bx^2\right)^2}{\left(1+x^2\right)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{(a-b)^2\sin^4(e+fx)\tan(e+fx)}{4f}-\frac{\text{Subst}\left(\int\frac{x^4\left(a^2-10ab+5b^2-4b^2x^2\right)}{\left(1+x^2\right)^2}\,dx\ldots\right.}{4f}$$

$$=-\frac{(a-9b)(a-b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{(a-b)^2\sin^4(e+fx)\tan\ldots}{4f}$$

$$=-\frac{(a-9b)(a-b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{(a-b)^2\sin^4(e+fx)\tan\ldots}{4f}$$

$$=-\frac{(a-9b)(a-b)\cos(e+fx)\sin(e+fx)}{8f}-\frac{\left(a^2-10ab+13b^2\right)\tan\ldots}{4f}$$

$$=\frac{1}{8}\left(3a^2-30ab+35b^2\right)x-\frac{(a-9b)(a-b)\cos(e+fx)\sin(e+fx)}{8f}\ldots$$

Mathematica [A] time = 1.46, size = 96, normalized size = 0.79

$$\frac{12\left(3a^2-30ab+35b^2\right)(e+fx)-24\left(a^2-4ab+3b^2\right)\sin(2(e+fx))+3(a-b)^2\sin(4(e+fx))+32b\tan(e+\ldots}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (12*(3*a^2 - 30*a*b + 35*b^2)*(e + f*x) - 24*(a^2 - 4*a*b + 3*b^2)*Sin[2*(e
 + f*x)] + 3*(a - b)^2*Sin[4*(e + f*x)] + 32*b*(6*a - 10*b + b*Sec[e + f*x]
^2)*Tan[e + f*x])/(96*f)
```

fricas [A] time = 0.44, size = 120, normalized size = 0.98

$$\frac{3\left(3a^2-30ab+35b^2\right)fx\cos\left(fx+e\right)^3+\left(6\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^6-3\left(5a^2-18ab+13b^2\right)\cos\left(fx\ldots\right.\right.}{24f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(3*a^2 - 30*a*b + 35*b^2)*f*x*cos(f*x + e)^3 + (6*(a^2 - 2*a*b + b^
2)*cos(f*x + e)^6 - 3*(5*a^2 - 18*a*b + 13*b^2)*cos(f*x + e)^4 + 16*(3*a*b
- 5*b^2)*cos(f*x + e)^2 + 8*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.78, size = 199, normalized size = 1.63

$$a^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin^7(fx+e)}{\cos(fx+e)}+\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2*a*
b*(sin(f*x+e)^7/cos(f*x+e)+(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*
cos(f*x+e)-15/8*f*x-15/8*e)+b^2*(1/3*sin(f*x+e)^9/cos(f*x+e)^3-2*sin(f*x+e)
^9/cos(f*x+e)-2*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin
(f*x+e))*cos(f*x+e)+35/8*f*x+35/8*e))
```

maxima [A] time = 0.59, size = 130, normalized size = 1.07

$$\frac{8\,b^2\tan\left(fx+e\right)^3+3\left(3\,a^2-30\,ab+35\,b^2\right)\left(fx+e\right)+24\left(2\,ab-3\,b^2\right)\tan\left(fx+e\right)-\frac{3\left(\left(5\,a^2-18\,ab+13\,b^2\right)\tan\left(fx+e\right)^3+\cdots\right.}{\tan\left(fx+e\right)^4+2\,\mathrm{ta}\cdots}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(8*b^2*tan(f*x + e)^3 + 3*(3*a^2 - 30*a*b + 35*b^2)*(f*x + e) + 24*(2*
a*b - 3*b^2)*tan(f*x + e) - 3*((5*a^2 - 18*a*b + 13*b^2)*tan(f*x + e)^3 + (
3*a^2 - 14*a*b + 11*b^2)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 +
 1))/f
```

mupad [B] time = 12.33, size = 128, normalized size = 1.05

$$x\left(\frac{3\,a^2}{8}-\frac{15\,a\,b}{4}+\frac{35\,b^2}{8}\right)+\frac{\tan\left(e+f\,x\right)\,\left(2\,a\,b-3\,b^2\right)}{f}+\frac{b^2\tan\left(e+f\,x\right)^3}{3\,f}-\frac{\left(\frac{5\,a^2}{8}-\frac{9\,a\,b}{4}+\frac{13\,b^2}{8}\right)\tan\left(e+f\,x\right)^3+\left(\cdots\right.}{f\,\left(\tan\left(e+f\,x\right)^4+2\,\mathrm{tan}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] x*((3*a^2)/8 - (15*a*b)/4 + (35*b^2)/8) + (tan(e + f*x)*(2*a*b - 3*b^2))/f
+ (b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)*((3*a^2)/8 - (7*a*b)/4 + (11*b
^2)/8) + tan(e + f*x)^3*((5*a^2)/8 - (9*a*b)/4 + (13*b^2)/8))/(f*(2*tan(e +
 f*x)^2 + tan(e + f*x)^4 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(e+fx\right)\right)^2\sin^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x)**4, x)
```

3.50 $\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=85

$$-\frac{(a-5b)(a-b)\tan(e+fx)}{2f}+\frac{(a-b)^2\sin^2(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}x(a-5b)(a-b)+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] 1/2*(a-5*b)*(a-b)*x-1/2*(a-5*b)*(a-b)*tan(f*x+e)/f+1/2*(a-b)^2*sin(f*x+e)^2
*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 463, 459, 321, 203}

$$-\frac{(a-5b)(a-b)\tan(e+fx)}{2f}+\frac{(a-b)^2\sin^2(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}x(a-5b)(a-b)+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((a - 5*b)*(a - b)*x)/2 - ((a - 5*b)*(a - b)*Tan[e + f*x])/(2*f) + ((a - b)
^2*Sin[e + f*x]^2*Tan[e + f*x])/(2*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
```

```
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^2}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a-b)^2\sin^2(e+fx)\tan(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{x^2\left(a^2-6ab+3b^2-2b^2x^2\right)}{1+x^2}\,dx, x,\right.}{2f}$$

$$= \frac{(a-b)^2\sin^2(e+fx)\tan(e+fx)}{2f} + \frac{b^2\tan^3(e+fx)}{3f} - \frac{((a-5b)(a-b)}{}$$

$$= -\frac{(a-5b)(a-b)\tan(e+fx)}{2f} + \frac{(a-b)^2\sin^2(e+fx)\tan(e+fx)}{2f} + \frac{b^2}{}$$

$$= \frac{1}{2}(a-5b)(a-b)x - \frac{(a-5b)(a-b)\tan(e+fx)}{2f} + \frac{(a-b)^2\sin^2(e+fx}{2f}$$

Mathematica [A] time = 0.73, size = 71, normalized size = 0.84

$$\frac{6\left(a^2-6ab+5b^2\right)(e+fx)-3(a-b)^2\sin(2(e+fx))+4b\tan(e+fx)\left(6a+b\sec^2(e+fx)-7b\right)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (6*(a^2 - 6*a*b + 5*b^2)*(e + f*x) - 3*(a - b)^2*Sin[2*(e + f*x)] + 4*b*(6*
a - 7*b + b*Sec[e + f*x]^2)*Tan[e + f*x])/(12*f)
```

fricas [A] time = 0.56, size = 94, normalized size = 1.11

$$\frac{3\left(a^2-6\,ab+5\,b^2\right)fx\cos\left(fx+e\right)^3-\left(3\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^4-2\left(6\,ab-7\,b^2\right)\cos\left(fx+e\right)^2-2\,b^2\right)\sin}{6\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2 - 6*a*b + 5*b^2)*f*x*cos(f*x + e)^3 - (3*(a^2 - 2*a*b + b^2)*co
s(f*x + e)^4 - 2*(6*a*b - 7*b^2)*cos(f*x + e)^2 - 2*b^2)*sin(f*x + e))/(f*c
os(f*x + e)^3)
```

giac [B] time = 22.94, size = 1411, normalized size = 16.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*a^2*f*x*tan(f*x)^5*tan(e)^5 - 18*a*b*f*x*tan(f*x)^5*tan(e)^5 + 15*b^
2*f*x*tan(f*x)^5*tan(e)^5 + 3*a^2*f*x*tan(f*x)^5*tan(e)^3 - 18*a*b*f*x*tan(
```

```
f*x)^5*tan(e)^3 + 15*b^2*f*x*tan(f*x)^5*tan(e)^3 - 9*a^2*f*x*tan(f*x)^4*tan
(e)^4 + 54*a*b*f*x*tan(f*x)^4*tan(e)^4 - 45*b^2*f*x*tan(f*x)^4*tan(e)^4 + 3
*a^2*f*x*tan(f*x)^3*tan(e)^5 - 18*a*b*f*x*tan(f*x)^3*tan(e)^5 + 15*b^2*f*x*
tan(f*x)^3*tan(e)^5 + 3*a^2*tan(f*x)^5*tan(e)^4 - 18*a*b*tan(f*x)^5*tan(e)^
4 + 15*b^2*tan(f*x)^5*tan(e)^4 + 3*a^2*tan(f*x)^4*tan(e)^5 - 18*a*b*tan(f*x
)^4*tan(e)^5 + 15*b^2*tan(f*x)^4*tan(e)^5 - 9*a^2*f*x*tan(f*x)^4*tan(e)^2 +
 54*a*b*f*x*tan(f*x)^4*tan(e)^2 - 45*b^2*f*x*tan(f*x)^4*tan(e)^2 + 12*a^2*f
*x*tan(f*x)^3*tan(e)^3 - 72*a*b*f*x*tan(f*x)^3*tan(e)^3 + 60*b^2*f*x*tan(f*
x)^3*tan(e)^3 - 9*a^2*f*x*tan(f*x)^2*tan(e)^4 + 54*a*b*f*x*tan(f*x)^2*tan(e
)^4 - 45*b^2*f*x*tan(f*x)^2*tan(e)^4 - 12*a*b*tan(f*x)^5*tan(e)^2 + 10*b^2*
tan(f*x)^5*tan(e)^2 - 12*a^2*tan(f*x)^4*tan(e)^3 + 36*a*b*tan(f*x)^4*tan(e)
^3 - 30*b^2*tan(f*x)^4*tan(e)^3 - 12*a^2*tan(f*x)^3*tan(e)^4 + 36*a*b*tan(f
*x)^3*tan(e)^4 - 30*b^2*tan(f*x)^3*tan(e)^4 - 12*a*b*tan(f*x)^2*tan(e)^5 +
10*b^2*tan(f*x)^2*tan(e)^5 + 9*a^2*f*x*tan(f*x)^3*tan(e) - 54*a*b*f*x*tan(f
*x)^3*tan(e) + 45*b^2*f*x*tan(f*x)^3*tan(e) - 12*a^2*f*x*tan(f*x)^2*tan(e)^
2 + 72*a*b*f*x*tan(f*x)^2*tan(e)^2 - 60*b^2*f*x*tan(f*x)^2*tan(e)^2 + 9*a^2
*f*x*tan(f*x)*tan(e)^3 - 54*a*b*f*x*tan(f*x)*tan(e)^3 + 45*b^2*f*x*tan(f*x)
*tan(e)^3 - 2*b^2*tan(f*x)^5 + 24*a*b*tan(f*x)^4*tan(e) - 30*b^2*tan(f*x)^4
*tan(e) + 18*a^2*tan(f*x)^3*tan(e)^2 - 36*a*b*tan(f*x)^3*tan(e)^2 + 10*b^2*
tan(f*x)^3*tan(e)^2 + 18*a^2*tan(f*x)^2*tan(e)^3 - 36*a*b*tan(f*x)^2*tan(e)
^3 + 10*b^2*tan(f*x)^2*tan(e)^3 + 24*a*b*tan(f*x)*tan(e)^4 - 30*b^2*tan(f*x
)*tan(e)^4 - 2*b^2*tan(e)^5 - 3*a^2*f*x*tan(f*x)^2 + 18*a*b*f*x*tan(f*x)^2
- 15*b^2*f*x*tan(f*x)^2 + 9*a^2*f*x*tan(f*x)*tan(e) - 54*a*b*f*x*tan(f*x)*t
an(e) + 45*b^2*f*x*tan(f*x)*tan(e) - 3*a^2*f*x*tan(e)^2 + 18*a*b*f*x*tan(e)
^2 - 15*b^2*f*x*tan(e)^2 - 12*a*b*tan(f*x)^3 + 10*b^2*tan(f*x)^3 - 12*a^2*t
an(f*x)^2*tan(e) + 36*a*b*tan(f*x)^2*tan(e) - 30*b^2*tan(f*x)^2*tan(e) - 12
*a^2*tan(f*x)*tan(e)^2 + 36*a*b*tan(f*x)*tan(e)^2 - 30*b^2*tan(f*x)*tan(e)^
2 - 12*a*b*tan(e)^3 + 10*b^2*tan(e)^3 - 3*a^2*f*x + 18*a*b*f*x - 15*b^2*f*x
 + 3*a^2*tan(f*x) - 18*a*b*tan(f*x) + 15*b^2*tan(f*x) + 3*a^2*tan(e) - 18*a
*b*tan(e) + 15*b^2*tan(e))/(f*tan(f*x)^5*tan(e)^5 + f*tan(f*x)^5*tan(e)^3 -
 3*f*tan(f*x)^4*tan(e)^4 + f*tan(f*x)^3*tan(e)^5 - 3*f*tan(f*x)^4*tan(e)^2
+ 4*f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan(e)^4 + 3*f*tan(f*x)^3*tan(e)
 - 4*f*tan(f*x)^2*tan(e)^2 + 3*f*tan(f*x)*tan(e)^3 - f*tan(f*x)^2 + 3*f*tan
(f*x)*tan(e) - f*tan(e)^2 - f)
```

maple [B] time = 0.70, size = 168, normalized size = 1.98

$$\frac{a^{2}\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab\left(\frac{\sin^{5}\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\sin^{3}\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)-\frac{3fx}{2}-\frac{3e}{2}\right)+b^{2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a*b*(sin(f*x+e)^5/cos
(f*x+e)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x-3/2*e)+b^2*(1/3*si
n(f*x+e)^7/cos(f*x+e)^3-4/3*sin(f*x+e)^7/cos(f*x+e)-4/3*(sin(f*x+e)^5+5/4*s
in(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/2*f*x+5/2*e))
```

maxima [A] time = 0.84, size = 87, normalized size = 1.02

$$\frac{2\,b^{2}\tan\left(fx+e\right)^{3}+3\left(a^{2}-6\,ab+5\,b^{2}\right)\left(fx+e\right)+12\left(ab-b^{2}\right)\tan\left(fx+e\right)-\frac{3\left(a^{2}-2\,ab+b^{2}\right)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{2}+1}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*tan(f*x + e)^3 + 3*(a^2 - 6*a*b + 5*b^2)*(f*x + e) + 12*(a*b - b
^2)*tan(f*x + e) - 3*(a^2 - 2*a*b + b^2)*tan(f*x + e)/(tan(f*x + e)^2 + 1))
/f
```

mupad [B] time = 11.80, size = 114, normalized size = 1.34

$$\frac{\tan\left(e+fx\right)\left(2ab-2b^2\right)}{f}+\frac{b^2\tan\left(e+fx\right)^3}{3f}-\frac{\sin\left(2e+2fx\right)\left(\frac{a^2}{2}-ab+\frac{b^2}{2}\right)}{2f}+\frac{\mathrm{atan}\left(\frac{\tan\left(e+fx\right)(a-b)(a-5b)}{2\left(\frac{a^2}{2}-3ab+\frac{5b^2}{2}\right)}\right)(a-b
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)*(2*a*b - 2*b^2))/f + (b^2*tan(e + f*x)^3)/(3*f) - (sin(2*e +
2*f*x)*(a^2/2 - a*b + b^2/2))/(2*f) + (atan((tan(e + f*x)*(a - b)*(a - 5*b)
)/(2*(a^2/2 - 3*a*b + (5*b^2)/2)))*(a - b)*(a - 5*b))/(2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(e+fx\right)\right)^2\sin^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x)**2, x)
```

3.51 $\int \left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=46

$$\frac{b(2a-b)\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] (a-b)^2*x+(2*a-b)*b*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 390, 203}

$$\frac{b(2a-b)\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a - b)^2*x + ((2*a - b)*b*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{(a-b)^2}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+\right.}{f}\\ &= (a-b)^2x+\frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.61, size = 73, normalized size = 1.59

$$\frac{\tan(e+fx)\left(b\left(6a-b\left(3-\tan^2(e+fx)\right)\right)+\frac{3(a-b)^2\tanh^{-1}\left(\sqrt{-\tan^2(e+fx)}\right)}{\sqrt{-\tan^2(e+fx)}}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (Tan[e + f*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Tan[e + f*x]^2]])/Sqrt[-Tan[e + f
*x]^2] + b*(6*a - b*(3 - Tan[e + f*x]^2))))/(3*f)
```

fricas [A] time = 0.46, size = 51, normalized size = 1.11

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(a^2-2\,ab+b^2\right)fx+3\left(2\,ab-b^2\right)\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*x + 3*(2*a*b - b^2)*tan(f
*x + e))/f
```

giac [B] time = 6.87, size = 382, normalized size = 8.30

$$\frac{3\,a^2fx\tan\left(fx\right)^3\tan(e)^3-6\,abfx\tan\left(fx\right)^3\tan(e)^3+3\,b^2fx\tan\left(fx\right)^3\tan(e)^3-9\,a^2fx\tan\left(fx\right)^2\tan(e)^2+18\,ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*f*x*tan(f*x)^3*tan(e)^3 - 6*a*b*f*x*tan(f*x)^3*tan(e)^3 + 3*b^2*
f*x*tan(f*x)^3*tan(e)^3 - 9*a^2*f*x*tan(f*x)^2*tan(e)^2 + 18*a*b*f*x*tan(f*
x)^2*tan(e)^2 - 9*b^2*f*x*tan(f*x)^2*tan(e)^2 - 6*a*b*tan(f*x)^3*tan(e)^2 +
 3*b^2*tan(f*x)^3*tan(e)^2 - 6*a*b*tan(f*x)^2*tan(e)^3 + 3*b^2*tan(f*x)^2*t
an(e)^3 + 9*a^2*f*x*tan(f*x)*tan(e) - 18*a*b*f*x*tan(f*x)*tan(e) + 9*b^2*f*
x*tan(f*x)*tan(e) - b^2*tan(f*x)^3 + 12*a*b*tan(f*x)^2*tan(e) - 9*b^2*tan(f
*x)^2*tan(e) + 12*a*b*tan(f*x)*tan(e)^2 - 9*b^2*tan(f*x)*tan(e)^2 - b^2*tan
(e)^3 - 3*a^2*f*x + 6*a*b*f*x - 3*b^2*f*x - 6*a*b*tan(f*x) + 3*b^2*tan(f*x)
 - 6*a*b*tan(e) + 3*b^2*tan(e))/(f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan
(e)^2 + 3*f*tan(f*x)*tan(e) - f)
```

maple [A] time = 0.02, size = 87, normalized size = 1.89

$$\frac{b^2\left(\tan^3\left(fx+e\right)\right)}{3f}+\frac{2ab\tan\left(fx+e\right)}{f}-\frac{b^2\tan\left(fx+e\right)}{f}+\frac{\arctan\left(\tan\left(fx+e\right)\right)a^2}{f}-\frac{2\arctan\left(\tan\left(fx+e\right)\right)ab}{f}+\frac{\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/3*b^2*tan(f*x+e)^3/f+2*a*b*tan(f*x+e)/f-b^2*tan(f*x+e)/f+1/f*arctan(tan(f
*x+e))*a^2-2/f*arctan(tan(f*x+e))*a*b+1/f*arctan(tan(f*x+e))*b^2
```

maxima [A] time = 0.76, size = 58, normalized size = 1.26

$$a^2x-\frac{2\left(fx+e-\tan\left(fx+e\right)\right)ab}{f}+\frac{\left(\tan\left(fx+e\right)^3+3\,fx+3\,e-3\,\tan\left(fx+e\right)\right)b^2}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 2*(f*x + e - tan(f*x + e))*a*b/f + 1/3*(tan(f*x + e)^3 + 3*f*x + 3*
e - 3*tan(f*x + e))*b^2/f
```

mupad [B] time = 11.90, size = 76, normalized size = 1.65

$$\frac{\tan\left(e+f\,x\right)\,\left(2\,a\,b-b^2\right)}{f}+\frac{\mathrm{atan}\left(\frac{\tan\left(e+f\,x\right)\,{\left(a-b\right)}^2}{a^2-2\,a\,b+b^2}\right)\,{\left(a-b\right)}^2}{f}+\frac{b^2\,{\tan\left(e+f\,x\right)}^3}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)*(2*a*b - b^2))/f + (atan((tan(e + f*x)*(a - b)^2)/(a^2 - 2*a*
b + b^2))*(a - b)^2)/f + (b^2*tan(e + f*x)^3)/(3*f)
```

sympy [A] time = 0.32, size = 68, normalized size = 1.48

$$\begin{cases} a^2x - 2abx + \frac{2ab\tan\left(e+fx\right)}{f} + b^2x + \frac{b^2\tan^3\left(e+fx\right)}{3f} - \frac{b^2\tan\left(e+fx\right)}{f} & \text{for}\ f \neq 0 \\ x\left(a + b\tan^2\left(e\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*x + 2*a*b*tan(e + f*x)/f + b**2*x + b**2*tan(e +
f*x)**3/(3*f) - b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)**2, Tr
ue))
```

3.52 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=46

$$-\frac{a^2\cot(e+fx)}{f}+\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -a^2*cot(f*x+e)/f+2*a*b*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3663, 270}

$$-\frac{a^2\cot(e+fx)}{f}+\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a^2*Cot[e + f*x])/f) + (2*a*b*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*
f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right)dx,x,\tan(e+fx)\right)}{f}\\ &= -\frac{a^2\cot(e+fx)}{f}+\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.52, size = 44, normalized size = 0.96

$$\frac{b\tan(e+fx)\left(6a+b\sec^2(e+fx)-b\right)-3a^2\cot(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*a^2*Cot[e + f*x] + b*(6*a - b + b*Sec[e + f*x]^2)*Tan[e + f*x])/(3*f)
```

fricas [A] time = 0.44, size = 71, normalized size = 1.54

$$-\frac{\left(3a^2+6ab-b^2\right)\cos\left(fx+e\right)^4-2\left(3ab-b^2\right)\cos\left(fx+e\right)^2-b^2}{3f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
[Out] -1/3*((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a*b - b^2)*cos(f*x + e)^2
 - b^2)/(f*cos(f*x + e)^3*sin(f*x + e))
```

giac [A] time = 3.90, size = 44, normalized size = 0.96

$$\frac{b^2\tan\left(fx+e\right)^3+6ab\tan\left(fx+e\right)-\frac{3a^2}{\tan\left(fx+e\right)}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) - 3*a^2/tan(f*x + e))/f
```

maple [A] time = 0.63, size = 48, normalized size = 1.04

$$\frac{-a^2\cot\left(fx+e\right)+2ab\tan\left(fx+e\right)+\frac{b^2\left(\sin^3\left(fx+e\right)\right)}{3\cos\left(fx+e\right)^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x)
[Out] 1/f*(-a^2*cot(f*x+e)+2*a*b*tan(f*x+e)+1/3*b^2*sin(f*x+e)^3/cos(f*x+e)^3)
```

maxima [A] time = 0.55, size = 41, normalized size = 0.89

$$\frac{b^2\tan\left(fx+e\right)^3+6ab\tan\left(fx+e\right)-\frac{3a^2}{\tan\left(fx+e\right)}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) - 3*a^2/tan(f*x + e))/f
```

mupad [B] time = 11.86, size = 67, normalized size = 1.46

$$\frac{-3a^2\cos\left(e+fx\right)^4+6ab\cos\left(e+fx\right)^2\sin\left(e+fx\right)^2+b^2\sin\left(e+fx\right)^4}{3f\cos\left(e+fx\right)^3\sin\left(e+fx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^2,x)
[Out] (b^2*sin(e + f*x)^4 - 3*a^2*cos(e + f*x)^4 + 6*a*b*cos(e + f*x)^2*sin(e + f
*x)^2)/(3*f*cos(e + f*x)^3*sin(e + f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^2 \csc^2\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x)**2, x)
```

3.53 $\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=70

$$-\frac{a^2\cot^3(e+fx)}{3f}+\frac{b(2a+b)\tan(e+fx)}{f}-\frac{a(a+2b)\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -a*(a+2*b)*cot(f*x+e)/f-1/3*a^2*cot(f*x+e)^3/f+b*(2*a+b)*tan(f*x+e)/f+1/3*b
^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3663, 448}

$$-\frac{a^2\cot^3(e+fx)}{3f}+\frac{b(2a+b)\tan(e+fx)}{f}-\frac{a(a+2b)\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a*(a + 2*b)*Cot[e + f*x])/f) - (a^2*Cot[e + f*x]^3)/(3*f) + (b*(2*a + b)
*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)\left(a+bx^2\right)^2}{x^4}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(b(2a+b)+\frac{a^2}{x^4}+\frac{a(a+2b)}{x^2}+b^2x^2\right)dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{a(a+2b)\cot(e+fx)}{f}-\frac{a^2\cot^3(e+fx)}{3f}+\frac{b(2a+b)\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.47, size = 59, normalized size = 0.84

$$\frac{b\tan(e+fx)\left(6a+b\sec^2(e+fx)+2b\right)-a\cot(e+fx)\left(a\csc^2(e+fx)+2a+6b\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-(a*Cot[e + f*x]*(2*a + 6*b + a*Csc[e + f*x]^2)) + b*(6*a + 2*b + b*Sec[e
+ f*x]^2)*Tan[e + f*x])/(3*f)
```

fricas [A] time = 0.51, size = 92, normalized size = 1.31

$$-\frac{2\left(a^{2}+6\,ab+b^{2}\right)\cos\left(fx+e\right)^{6}-3\left(a^{2}+6\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}+6\,ab\cos\left(fx+e\right)^{2}+b^{2}}{3\left(f\cos\left(fx+e\right)^{5}-f\cos\left(fx+e\right)^{3}\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 + 6*a*b + b^2)*cos(f*x + e)^6 - 3*(a^2 + 6*a*b + b^2)*cos(f*x
+ e)^4 + 6*a*b*cos(f*x + e)^2 + b^2)/((f*cos(f*x + e)^5 - f*cos(f*x + e)^3)
*sin(f*x + e))
```

giac [A] time = 3.75, size = 84, normalized size = 1.20

$$\frac{b^{2}\tan\left(fx+e\right)^{3}+6\,ab\tan\left(fx+e\right)+3\,b^{2}\tan\left(fx+e\right)-\frac{3\,a^{2}\tan\left(fx+e\right)^{2}+6\,ab\tan\left(fx+e\right)^{2}+a^{2}}{\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 3*b^2*tan(f*x + e) - (3*a^2*
tan(f*x + e)^2 + 6*a*b*tan(f*x + e)^2 + a^2)/tan(f*x + e)^3)/f
```

maple [A] time = 0.90, size = 81, normalized size = 1.16

$$\frac{a^{2}\left(-\frac{2}{3}-\frac{\left(\csc^{2}\left(fx+e\right)\right)}{3}\right)\cot\left(fx+e\right)+2ab\left(\frac{1}{\sin\left(fx+e\right)\cos\left(fx+e\right)}-2\cot\left(fx+e\right)\right)-b^{2}\left(-\frac{2}{3}-\frac{\left(\sec^{2}\left(fx+e\right)\right)}{3}\right)\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+2*a*b*(1/sin(f*x+e)/cos(f*x+e)-
2*cot(f*x+e))-b^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

maxima [A] time = 0.67, size = 66, normalized size = 0.94

$$\frac{b^{2}\tan\left(fx+e\right)^{3}+3\left(2\,ab+b^{2}\right)\tan\left(fx+e\right)-\frac{3\left(a^{2}+2\,ab\right)\tan\left(fx+e\right)^{2}+a^{2}}{\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(2*a*b + b^2)*tan(f*x + e) - (3*(a^2 + 2*a*b)*t
an(f*x + e)^2 + a^2)/tan(f*x + e)^3)/f
```

mupad [B] time = 11.82, size = 69, normalized size = 0.99

$$\frac{b^{2}\tan\left(e+f\,x\right)^{3}}{3\,f}-\frac{\tan\left(e+f\,x\right)^{2}\,\left(a^{2}+2\,b\,a\right)+\frac{a^{2}}{3}}{f\tan\left(e+f\,x\right)^{3}}+\frac{b\tan\left(e+f\,x\right)\,\left(2\,a+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^4,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)^2*(2*a*b + a^2) + a^2/3)/(f*tan(
e + f*x)^3) + (b*tan(e + f*x)*(2*a + b))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^2 \csc^4\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x)**4, x)
```

3.54 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=93

$$-\frac{(a^2+4ab+b^2)\cot(e+fx)}{f}-\frac{a^2\cot^5(e+fx)}{5f}+\frac{2b(a+b)\tan(e+fx)}{f}-\frac{2a(a+b)\cot^3(e+fx)}{3f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -(a^2+4*a*b+b^2)*cot(f*x+e)/f-2/3*a*(a+b)*cot(f*x+e)^3/f-1/5*a^2*cot(f*x+e)
^5/f+2*b*(a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3663, 448}

$$-\frac{(a^2+4ab+b^2)\cot(e+fx)}{f}-\frac{a^2\cot^5(e+fx)}{5f}+\frac{2b(a+b)\tan(e+fx)}{f}-\frac{2a(a+b)\cot^3(e+fx)}{3f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a^2 + 4*a*b + b^2)*Cot[e + f*x])/f) - (2*a*(a + b)*Cot[e + f*x]^3)/(3*f
) - (a^2*Cot[e + f*x]^5)/(5*f) + (2*b*(a + b)*Tan[e + f*x])/f + (b^2*Tan[e
+ f*x]^3)/(3*f)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+bx^2)^2}{x^6}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(2b(a+b)+\frac{a^2}{x^6}+\frac{2a(a+b)}{x^4}+\frac{a^2+4ab+b^2}{x^2}+b^2x^2\right)dx, x, \tan(e+\right.}{f}\\ &= -\frac{(a^2+4ab+b^2)\cot(e+fx)}{f}-\frac{2a(a+b)\cot^3(e+fx)}{3f}-\frac{a^2\cot^5(e+}{5f}\end{aligned}$$

Mathematica [A] time = 0.81, size = 88, normalized size = 0.95

$$\frac{5b\tan(e+fx)\left(6a+b\sec^2(e+fx)+5b\right)-\cot(e+fx)\left(3a^2\csc^4(e+fx)+8a^2+2a(2a+5b)\csc^2(e+fx)+50a\right.}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-(Cot[e + f*x]*(8*a^2 + 50*a*b + 15*b^2 + 2*a*(2*a + 5*b)*Csc[e + f*x]^2 +
 3*a^2*Csc[e + f*x]^4)) + 5*b*(6*a + 5*b + b*Sec[e + f*x]^2)*Tan[e + f*x])/
(15*f)
```

fricas [A] time = 0.53, size = 137, normalized size = 1.47

$$-\frac{8\left(a^2+10\,ab+5\,b^2\right)\cos\left(fx+e\right)^8-20\left(a^2+10\,ab+5\,b^2\right)\cos\left(fx+e\right)^6+15\left(a^2+10\,ab+5\,b^2\right)\cos\left(fx+e\right.}{15\left(f\cos\left(fx+e\right)^7-2\,f\cos\left(fx+e\right)^5+f\cos\left(fx+e\right)^3\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^2 + 10*a*b + 5*b^2)*cos(f*x + e)^8 - 20*(a^2 + 10*a*b + 5*b^2)*
cos(f*x + e)^6 + 15*(a^2 + 10*a*b + 5*b^2)*cos(f*x + e)^4 - 10*(3*a*b + b^2
)*cos(f*x + e)^2 - 5*b^2)/((f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*cos(f
*x + e)^3)*sin(f*x + e))
```

giac [A] time = 5.95, size = 128, normalized size = 1.38

$$\frac{5\,b^2\tan\left(fx+e\right)^3+30\,ab\tan\left(fx+e\right)+30\,b^2\tan\left(fx+e\right)-\frac{15\,a^2\tan\left(fx+e\right)^4+60\,ab\tan\left(fx+e\right)^4+15\,b^2\tan\left(fx+e\right)^4+10\,a^2\,\mathrm{ta}}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(5*b^2*tan(f*x + e)^3 + 30*a*b*tan(f*x + e) + 30*b^2*tan(f*x + e) - (1
5*a^2*tan(f*x + e)^4 + 60*a*b*tan(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 + 10*a
^2*tan(f*x + e)^2 + 10*a*b*tan(f*x + e)^2 + 3*a^2)/tan(f*x + e)^5)/f
```

maple [A] time = 0.74, size = 136, normalized size = 1.46

$$\frac{a^2\left(-\frac{8}{15}-\frac{\left(\csc^4\left(fx+e\right)\right)}{5}-\frac{4\left(\csc^2\left(fx+e\right)\right)}{15}\right)\cot\left(fx+e\right)+2ab\left(-\frac{1}{3\sin\left(fx+e\right)^3\cos\left(fx+e\right)}+\frac{4}{3\sin\left(fx+e\right)\cos\left(fx+e\right)}-\frac{8\cot\left(fx+e\right)}{3}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-8/15-1/5*csc(f*x+e)^4-4/15*csc(f*x+e)^2)*cot(f*x+e)+2*a*b*(-1/3/
sin(f*x+e)^3/cos(f*x+e)+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e))+b^2*(1/3/
sin(f*x+e)/cos(f*x+e)^3+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e)))
```

maxima [A] time = 0.33, size = 88, normalized size = 0.95

$$\frac{5\,b^2\tan\left(fx+e\right)^3+30\left(ab+b^2\right)\tan\left(fx+e\right)-\frac{15\left(a^2+4\,ab+b^2\right)\tan\left(fx+e\right)^4+10\left(a^2+ab\right)\tan\left(fx+e\right)^2+3\,a^2}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(5*b^2*tan(f*x + e)^3 + 30*(a*b + b^2)*tan(f*x + e) - (15*(a^2 + 4*a*b
 + b^2)*tan(f*x + e)^4 + 10*(a^2 + a*b)*tan(f*x + e)^2 + 3*a^2)/tan(f*x + e
)^5)/f
```

mupad [B] time = 12.19, size = 90, normalized size = 0.97

$$\frac{b^2\tan\left(e+f\,x\right)^3}{3\,f}-\frac{\tan\left(e+f\,x\right)^4\,\left(a^2+4\,a\,b+b^2\right)+\frac{a^2}{5}+\tan\left(e+f\,x\right)^2\,\left(\frac{2\,a^2}{3}+\frac{2\,b\,a}{3}\right)}{f\tan\left(e+f\,x\right)^5}+\frac{2\,b\tan\left(e+f\,x\right)\,\left(a+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^6,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)^4*(4*a*b + a^2 + b^2) + a^2/5 +
tan(e + f*x)^2*((2*a*b)/3 + (2*a^2)/3))/(f*tan(e + f*x)^5) + (2*b*tan(e + f
*x)*(a + b))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.55 $\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$-\frac{a^2\cos(e+fx)}{f(a-b)^3}-\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{7/2}}-\frac{\cos^5(e+fx)}{5f(a-b)}+\frac{(2a-b)\cos^3(e+fx)}{3f(a-b)^2}$$

```
[Out] -a^2*cos(f*x+e)/(a-b)^3/f+1/3*(2*a-b)*cos(f*x+e)^3/(a-b)^2/f-1/5*cos(f*x+e)
^5/(a-b)/f-a^2*arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/(a-b)^(7/2)/f
```

Rubi [A] time = 0.18, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3664, 461, 205}

$$-\frac{a^2\cos(e+fx)}{f(a-b)^3}-\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{7/2}}-\frac{\cos^5(e+fx)}{5f(a-b)}+\frac{(2a-b)\cos^3(e+fx)}{3f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a - b)^(7/2)*f
)) - (a^2*Cos[e + f*x])/((a - b)^3*f) + ((2*a - b)*Cos[e + f*x]^3)/(3*(a -
b)^2*f) - Cos[e + f*x]^5/(5*(a - b)*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a-b+bx^2\right)}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)x^6} + \frac{-2a+b}{(a-b)^2x^4} + \frac{a^2}{(a-b)^3x^2} - \frac{a^2b}{(a-b)^3\left(a-b+bx^2\right)}\right)dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{a^2\cos(e+fx)}{(a-b)^3f} + \frac{(2a-b)\cos^3(e+fx)}{3(a-b)^2f} - \frac{\cos^5(e+fx)}{5(a-b)f} - \frac{\left(a^2b\right)\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx,\right.}{(a-b)^3f}$$

$$= -\frac{a^2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{7/2}f} - \frac{a^2\cos(e+fx)}{(a-b)^3f} + \frac{(2a-b)\cos^3(e+fx)}{3(a-b)^2f} - \frac{\cos^5(e+fx)}{5(a-b)f}$$

Mathematica [A] time = 3.08, size = 177, normalized size = 1.51

$$\frac{\sqrt{a-b}\,\cos(e+fx)\left(4\left(7a^2-9ab+2b^2\right)\cos(2(e+fx))-89a^2-3(a-b)^2\cos(4(e+fx))-42ab+11b^2\right)+120a^2}{120f(a-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (120*a^2*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]] +
 120*a^2*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]] +
 Sqrt[a - b]*Cos[e + f*x]*(-89*a^2 - 42*a*b + 11*b^2 + 4*(7*a^2 - 9*a*b + 2
*b^2)*Cos[2*(e + f*x)] - 3*(a - b)^2*Cos[4*(e + f*x)]))/(120*(a - b)^(7/2)*
f)
```

fricas [A] time = 0.48, size = 294, normalized size = 2.51

$$\left[-\frac{6\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^5-10\left(2\,a^2-3\,ab+b^2\right)\cos\left(fx+e\right)^3+15\,a^2\sqrt{-\frac{b}{a-b}}\log\left(-\frac{(a-b)\cos\left(fx+e\right)^2-2\,(a-b)}{(a-b)\cos\left(fx\right.}\right.}{30\left(a^3-3\,a^2b+3\,ab^2-b^3\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(6*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 10*(2*a^2 - 3*a*b + b^2)*cos
(f*x + e)^3 + 15*a^2*sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a -
 b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 30*a
^2*cos(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f), -1/15*(3*(a^2 - 2*a*b
 + b^2)*cos(f*x + e)^5 - 5*(2*a^2 - 3*a*b + b^2)*cos(f*x + e)^3 + 15*a^2*sq
rt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 15*a^2*cos(
f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)]
```

giac [B] time = 2.70, size = 377, normalized size = 3.22

$$-\frac{\frac{15\,a^2b\arctan\left(-\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)-b}{\sqrt{ab-b^2}\cos\left(fx+e\right)+\sqrt{ab-b^2}}\right)}{\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\sqrt{ab-b^2}} - \frac{2\left(8\,a^2+9\,ab-2\,b^2-\frac{40\,a^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{30\,ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{10\,b^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{80\,a^2\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{10\,b^2(}{(\cos}\right.}{\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\left(\frac{\cos\left(fx+e\right)-}{\cos\left(fx+e\right)+}\right.}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*a^2*b*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a*b - b
^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(
a*b - b^2)) - 2*(8*a^2 + 9*a*b - 2*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) - 30*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 10*b^2*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + 10*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 90*a*b*(cos(f*x
+ e) - 1)^3/(cos(f*x + e) + 1)^3 + 30*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e
) + 1)^3 + 15*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*((cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5))/f
```

maple [A] time = 0.52, size = 205, normalized size = 1.75

$$-\frac{\left(\cos^5\left(fx+e\right)\right)a^2}{5f\left(a-b\right)^3}+\frac{2\left(\cos^5\left(fx+e\right)\right)ab}{5f\left(a-b\right)^3}-\frac{\left(\cos^5\left(fx+e\right)\right)b^2}{5f\left(a-b\right)^3}+\frac{2\left(\cos^3\left(fx+e\right)\right)a^2}{3f\left(a-b\right)^3}-\frac{\left(\cos^3\left(fx+e\right)\right)ab}{f\left(a-b\right)^3}+\frac{(\cos}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/5/f/(a-b)^3*cos(f*x+e)^5*a^2+2/5/f/(a-b)^3*cos(f*x+e)^5*a*b-1/5/f/(a-b)^
3*cos(f*x+e)^5*b^2+2/3/f/(a-b)^3*cos(f*x+e)^3*a^2-1/f/(a-b)^3*cos(f*x+e)^3*
a*b+1/3/f/(a-b)^3*cos(f*x+e)^3*b^2-a^2*cos(f*x+e)/(a-b)^3/f+1/f*a^2*b/(a-b)
^3/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 14.41, size = 643, normalized size = 5.50

$$-\frac{\frac{2\left(8a^2+9ab-2b^2\right)}{15(a-b)\left(a^2-2ab+b^2\right)}+\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(8a^2+b^2\right)}{3(a-b)\left(a^2-2ab+b^2\right)}+\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(4a^2+3ab-b^2\right)}{3(a-b)\left(a^2-2ab+b^2\right)}+\frac{4b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(3a-b)}{(a-b)\left(a^2-2ab+b^2\right)}+\frac{2ab\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8}{(a-b)\left(a^2-2ab+b^2\right)}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}-\frac{a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2),x)
```

```
[Out] - ((2*(9*a*b + 8*a^2 - 2*b^2))/(15*(a - b)*(a^2 - 2*a*b + b^2)) + (4*tan(e/
2 + (f*x)/2)^4*(8*a^2 + b^2))/(3*(a - b)*(a^2 - 2*a*b + b^2)) + (4*tan(e/2
+ (f*x)/2)^2*(3*a*b + 4*a^2 - b^2))/(3*(a - b)*(a^2 - 2*a*b + b^2)) + (4*b*
tan(e/2 + (f*x)/2)^6*(3*a - b))/((a - b)*(a^2 - 2*a*b + b^2)) + (2*a*b*tan(
e/2 + (f*x)/2)^8)/((a - b)*(a^2 - 2*a*b + b^2)))/(f*(5*tan(e/2 + (f*x)/2)^2
 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)
^8 + tan(e/2 + (f*x)/2)^10 + 1)) - (a^2*b^(1/2)*atan(((tan(e/2 + (f*x)/2)^2
```

```
*((a*b^(1/2)*(16*a^10*b + 16*a^4*b^7 - 96*a^5*b^6 + 240*a^6*b^5 - 320*a^7*b
^4 + 240*a^8*b^3 - 96*a^9*b^2))/(2*(a - b)^(13/2)) + (a^3*b^(1/2)*(a - 2*b)
*(16*a^12 - 176*a^11*b + 32*a^2*b^10 - 304*a^3*b^9 + 1296*a^4*b^8 - 3264*a^
5*b^7 + 5376*a^6*b^6 - 6048*a^7*b^5 + 4704*a^8*b^4 - 2496*a^9*b^3 + 864*a^1
0*b^2))/(8*(a - b)^(21/2))) + (a^3*b^(1/2)*(a - 2*b)*(144*a^11*b - 16*a^12
+ 16*a^3*b^9 - 144*a^4*b^8 + 576*a^5*b^7 - 1344*a^6*b^6 + 2016*a^7*b^5 - 20
16*a^8*b^4 + 1344*a^9*b^3 - 576*a^10*b^2))/(8*(a - b)^(21/2)))*(a - b)^7)/(
4*a^12*b + 4*a^6*b^7 - 24*a^7*b^6 + 60*a^8*b^5 - 80*a^9*b^4 + 60*a^10*b^3 -
 24*a^11*b^2)))/(f*(a - b)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.56 $\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$\frac{\cos^3(e+fx)}{3f(a-b)}-\frac{a\cos(e+fx)}{f(a-b)^2}-\frac{a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

```
[Out] -a*cos(f*x+e)/(a-b)^2/f+1/3*cos(f*x+e)^3/(a-b)/f-a*arctan(sec(f*x+e)*b^(1/2
)/(a-b)^(1/2))*b^(1/2)/(a-b)^(5/2)/f
```

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3664, 453, 325, 205}

$$\frac{\cos^3(e+fx)}{3f(a-b)}-\frac{a\cos(e+fx)}{f(a-b)^2}-\frac{a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a - b)^(5/2)*f))
 - (a*Cos[e + f*x])/((a - b)^2*f) + Cos[e + f*x]^3/(3*(a - b)*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{3(a-b)f} + \frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{(a-b)f}$$

$$= -\frac{a\cos(e+fx)}{(a-b)^2f} + \frac{\cos^3(e+fx)}{3(a-b)f} - \frac{(ab)\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \sec(e+fx)\right)}{(a-b)^2f}$$

$$= -\frac{a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} - \frac{a\cos(e+fx)}{(a-b)^2f} + \frac{\cos^3(e+fx)}{3(a-b)f}$$

Mathematica [A] time = 0.66, size = 149, normalized size = 1.77

$$\frac{(a-b)\cos(e+fx)((a-b)\cos(2(e+fx))-5a-b)+6a\sqrt{b}\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+6a\sqrt{b}\sqrt{a-b}\,\text{ta}}{6f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (6*a*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sq
rt[b]] + 6*a*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x
)/2])/Sqrt[b]] + (a - b)*Cos[e + f*x]*(-5*a - b + (a - b)*Cos[2*(e + f*x)])
)/(6*(a - b)^3*f)
```

fricas [A] time = 0.55, size = 206, normalized size = 2.45

$$\left[\frac{2\,(a-b)\cos\left(fx+e\right)^3+3\,a\sqrt{-\frac{b}{a-b}}\,\log\left(\frac{(a-b)\cos\left(fx+e\right)^2+2\,(a-b)\sqrt{-\frac{b}{a-b}}\,\cos\left(fx+e\right)-b}{(a-b)\cos\left(fx+e\right)^2+b}\right)-6\,a\cos\left(fx+e\right)}{6\left(a^2-2\,ab+b^2\right)f}, \frac{(a-b)\cos\left(fx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a - b)*cos(f*x + e)^3 + 3*a*sqrt(-b/(a - b))*log(((a - b)*cos(f*x
+ e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)
^2 + b)) - 6*a*cos(f*x + e))/((a^2 - 2*a*b + b^2)*f), 1/3*((a - b)*cos(f*x
+ e)^3 - 3*a*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b
) - 3*a*cos(f*x + e))/((a^2 - 2*a*b + b^2)*f)]
```

giac [B] time = 1.38, size = 180, normalized size = 2.14

$$\frac{ab\arctan\left(\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)}{\sqrt{ab-b^2}}\right)}{\left(a^2-2\,ab+b^2\right)\sqrt{ab-b^2}\,f}+\frac{a^2f^5\cos\left(fx+e\right)^3-2\,abf^5\cos\left(fx+e\right)^3+b^2f^5\cos\left(fx+e\right)^3-3\,a^2f^5\cos\left(f\right.}{3\left(a^3f^6-3\,a^2bf^6+3\,ab^2f^6-b^3f^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] a*b*arctan((a*cos(f*x + e) - b*cos(f*x + e))/sqrt(a*b - b^2))/((a^2 - 2*a*b
 + b^2)*sqrt(a*b - b^2)*f) + 1/3*(a^2*f^5*cos(f*x + e)^3 - 2*a*b*f^5*cos(f*
x + e)^3 + b^2*f^5*cos(f*x + e)^3 - 3*a^2*f^5*cos(f*x + e) + 3*a*b*f^5*cos(
f*x + e))/(a^3*f^6 - 3*a^2*b*f^6 + 3*a*b^2*f^6 - b^3*f^6)
```

maple [A] time = 0.57, size = 107, normalized size = 1.27

$$\frac{a\left(\cos^{3}\left(fx+e\right)\right)}{3f\left(a-b\right)^{2}}-\frac{b\left(\cos^{3}\left(fx+e\right)\right)}{3f\left(a-b\right)^{2}}-\frac{a\cos\left(fx+e\right)}{\left(a-b\right)^{2}f}+\frac{ab\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{(a-b)b}}\right)}{f\left(a-b\right)^{2}\sqrt{\left(a-b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/3/f/(a-b)^2*a*cos(f*x+e)^3-1/3/f/(a-b)^2*b*cos(f*x+e)^3-a*cos(f*x+e)/(a-b
)^2/f+1/f*a*b/(a-b)^2/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/
2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 13.37, size = 382, normalized size = 4.55

$$-\frac{\frac{2\,(2\,a+b)}{3\,(a-b)^2}+\frac{4\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{(a-b)^2}+\frac{2\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{(a-b)^2}}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}-\frac{a\sqrt{b}\,\mathrm{atan}\left(\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{\sqrt{b}\,\left(8\,a^7\,b-32\,a^6\,b^2+48\,a^5\,b^3-32\,a^4\cdots}{(a-b)^{9/2}}\cdots\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2),x)
```

```
[Out] - ((2*(2*a + b))/(3*(a - b)^2) + (4*a*tan(e/2 + (f*x)/2)^2)/(a - b)^2 + (2*
b*tan(e/2 + (f*x)/2)^4)/(a - b)^2)/(f*(3*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 +
 (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 + 1)) - (a*b^(1/2)*atan(((tan(e/2 + (f*x
)/2)^2*((b^(1/2)*(8*a^7*b + 8*a^3*b^5 - 32*a^4*b^4 + 48*a^5*b^3 - 32*a^6*b^
2))/(a - b)^(9/2) - (a*b^(1/2)*(a - 2*b)*(128*a^8*b - 16*a^9 + 32*a^2*b^7 -
 208*a^3*b^6 + 576*a^4*b^5 - 880*a^5*b^4 + 800*a^6*b^3 - 432*a^7*b^2))/(8*(
a - b)^(15/2))) - (a*b^(1/2)*(a - 2*b)*(16*a^9 - 96*a^8*b + 16*a^3*b^6 - 96
*a^4*b^5 + 240*a^5*b^4 - 320*a^6*b^3 + 240*a^7*b^2))/(8*(a - b)^(15/2)))*(a
 - b)^5)/(4*a^8*b + 4*a^4*b^5 - 16*a^5*b^4 + 24*a^6*b^3 - 16*a^7*b^2)))/(f*
(a - b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.57 $\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{\cos(e+fx)}{f(a-b)}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

```
[Out] -cos(f*x+e)/(a-b)/f-arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/(a-b)^(3
/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3664, 325, 205}

$$-\frac{\cos(e+fx)}{f(a-b)}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a - b)^(3/2)*f)) -
 Cos[e + f*x]/((a - b)*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)}\,dx,x,\sec(e+fx)\right)}{f}\\
&= -\frac{\cos(e+fx)}{(a-b)f}-\frac{b\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx,x,\sec(e+fx)\right)}{(a-b)f}\\
&= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f}-\frac{\cos(e+fx)}{(a-b)f}
\end{aligned}$$

Mathematica [B] time = 0.26, size = 121, normalized size = 2.02

$$\frac{(b-a)\cos(e+fx)+\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b
]] + Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sq
rt[b]] + (-a + b)*Cos[e + f*x])/((a - b)^2*f)
```

fricas [A] time = 0.50, size = 158, normalized size = 2.63

$$\left[-\frac{\sqrt{-\frac{b}{a-b}}\log\left(-\frac{(a-b)\cos\left(fx+e\right)^2-2(a-b)\sqrt{-\frac{b}{a-b}}\cos\left(fx+e\right)-b}{(a-b)\cos\left(fx+e\right)^2+b}\right)+2\cos\left(fx+e\right)}{2(a-b)f},-\frac{\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\cos\left(fx+e\right)}{b}\right)\cdots}{(a-b)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a
 - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*cos(f*x + e))/((
a - b)*f), -(sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b
) + cos(f*x + e))/((a - b)*f)]
```

giac [A] time = 1.70, size = 81, normalized size = 1.35

$$-\frac{f\cos\left(fx+e\right)}{af^2-bf^2}+\frac{b\arctan\left(\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)}{\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}\,(a-b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -f*cos(f*x + e)/(a*f^2 - b*f^2) + b*arctan((a*cos(f*x + e) - b*cos(f*x + e)
)/sqrt(a*b - b^2))/(sqrt(a*b - b^2)*(a - b)*f)
```

maple [A] time = 0.38, size = 63, normalized size = 1.05

$$-\frac{\cos\left(fx+e\right)}{(a-b)\,f}+\frac{b\arctan\left(\frac{(a-b)\cos\left(fx+e\right)}{\sqrt{(a-b)b}}\right)}{f\,(a-b)\sqrt{(a-b)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -cos(f*x+e)/(a-b)/f+1/f*b/(a-b)/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a
-b)*b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 11.79, size = 112, normalized size = 1.87

$$\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{-a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^2+a\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-3\,a\,b+2\,b^2}{2\,\sqrt{b}\,(a-b)^{3/2}}\right)}{f\,(a-b)^{3/2}}-\frac{2\,\sqrt{a-b}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(a-b)^{3/2}+(a-b)^{3/2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (b^(1/2)*atan((a^2 - a^2*tan(e/2 + (f*x)/2)^2 - 3*a*b + 2*b^2 + a*b*tan(e/2
 + (f*x)/2)^2)/(2*b^(1/2)*(a - b)^(3/2))))/(f*(a - b)^(3/2)) - (2*(a - b)^(
1/2))/(f*(tan(e/2 + (f*x)/2)^2*(a - b)^(3/2) + (a - b)^(3/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(e+fx\right)}{a+b\tan^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*tan(e + f*x)**2), x)
```

3.58 $\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{af\sqrt{a-b}}-\frac{\tanh^{-1}(\cos(e+fx))}{af}$$

```
[Out] -arctanh(cos(f*x+e))/a/f-arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a/f
/(a-b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3664, 391, 207, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{af\sqrt{a-b}}-\frac{\tanh^{-1}(\cos(e+fx))}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a*Sqrt[a - b]*f)) -
 ArcTanh[Cos[e + f*x]]/(a*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{af} - \frac{b\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \sec(e+fx)\right)}{af}$$
$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}\,f} - \frac{\tanh^{-1}(\cos(e+fx))}{af}$$

Mathematica [B] time = 0.24, size = 144, normalized size = 2.40

$$\frac{\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right) + \sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right) - (a-b)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right.}{af(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b
]] + Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sq
rt[b]] - (a - b)*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))/(a*(a - b
)*f)
```

fricas [A] time = 0.54, size = 184, normalized size = 3.07

$$\left[\frac{\sqrt{-\frac{b}{a-b}}\log\left(\frac{(a-b)\cos\left(fx+e\right)^2+2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos\left(fx+e\right)-b}{(a-b)\cos\left(fx+e\right)^2+b}\right) - \log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right) + \log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{2\,af}, -\frac{2\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a -
 b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - log(1/2*cos(f*x + e)
 + 1/2) + log(-1/2*cos(f*x + e) + 1/2))/(a*f), -1/2*(2*sqrt(b/(a - b))*arct
an(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + log(1/2*cos(f*x + e) + 1/2) -
 log(-1/2*cos(f*x + e) + 1/2))/(a*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(1/4/a*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1))))-2*b/a*
1/4/sqrt(-b^2+a*b)*atan((-a*cos(f*x+exp(1))+b*cos(f*x+exp(1))+b)/(sqrt(-b^2
+a*b)*cos(f*x+exp(1))+sqrt(-b^2+a*b))))
```

maple [A] time = 0.56, size = 75, normalized size = 1.25

$$\frac{b \arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{(a-b)b}}\right)}{fa\sqrt{(a-b)\,b}}+\frac{\ln\left(-1+\cos\left(fx+e\right)\right)}{2fa}-\frac{\ln\left(1+\cos\left(fx+e\right)\right)}{2fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f/a*b/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))+1/2/f/a*ln
(-1+cos(f*x+e))-1/2/f/a*ln(1+cos(f*x+e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 11.86, size = 91, normalized size = 1.52

$$\frac{\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{af}-\frac{\sqrt{b}\ \mathrm{atan}\left(\frac{b-a\ \cos\left(e+fx\right)+b\ \cos\left(e+fx\right)}{2\ \sqrt{b}\ \cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\ \sqrt{a-b}}\right)}{af\ \sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)),x)
```

```
[Out] log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))/(a*f) - (b^(1/2)*atan((b - a*cos
(e + f*x) + b*cos(e + f*x))/(2*b^(1/2)*cos(e/2 + (f*x)/2)^2*(a - b)^(1/2)))
)/(a*f*(a - b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{a+b\tan^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*tan(e + f*x)**2), x)
```

3.59 $\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=89

$$-\frac{(a-2b)\tanh^{-1}(\cos(e+fx))}{2a^2f}-\frac{\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^2f}-\frac{\cot(e+fx)\csc(e+fx)}{2af}$$

```
[Out] -1/2*(a-2*b)*arctanh(cos(f*x+e))/a^2/f-1/2*cot(f*x+e)*csc(f*x+e)/a/f-arctan
(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)*b^(1/2)/a^2/f
```

Rubi [A] time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 471, 522, 207, 205}

$$-\frac{(a-2b)\tanh^{-1}(\cos(e+fx))}{2a^2f}-\frac{\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^2f}-\frac{\cot(e+fx)\csc(e+fx)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a^2*f))
 - ((a - 2*b)*ArcTanh[Cos[e + f*x]])/(2*a^2*f) - (Cot[e + f*x]*Csc[e + f*x]
)/(2*a*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
```

```
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\mathrm{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af} + \frac{\mathrm{Subst}\left(\int \frac{a-b-bx^2}{(-1+x^2)(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af} + \frac{(a-2b)\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{2a^2f} - \frac{((a-b)b)}{}$$

$$= -\frac{\sqrt{a-b}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^2f} - \frac{(a-2b)\tanh^{-1}(\cos(e+fx))}{2a^2f} - \frac{\cot(e+fx)\csc}{2af}$$

Mathematica [B] time = 0.65, size = 195, normalized size = 2.19

$$\frac{8\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right) + 8\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right) - a\csc^2\left(\frac{1}{2}(e+fx)\right) + a\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (8*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt
[b]] + 8*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2]
)/Sqrt[b]] - a*Csc[(e + f*x)/2]^2 - 4*a*Log[Cos[(e + f*x)/2]] + 8*b*Log[Cos
[(e + f*x)/2]] + 4*a*Log[Sin[(e + f*x)/2]] - 8*b*Log[Sin[(e + f*x)/2]] + a*
Sec[(e + f*x)/2]^2)/(8*a^2*f)
```

fricas [A] time = 0.58, size = 327, normalized size = 3.67

$$\left[\frac{2\sqrt{-ab+b^2}\left(\cos\left(fx+e\right)^2-1\right)\log\left(-\frac{(a-b)\cos\left(fx+e\right)^2+2\sqrt{-ab+b^2}\cos\left(fx+e\right)-b}{(a-b)\cos\left(fx+e\right)^2+b}\right) + 2a\cos\left(fx+e\right) - \left((a-2b)\cos\right.}{4\left(a^2f\cos\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a*b + b^2)*(cos(f*x + e)^2 - 1)*log(-((a - b)*cos(f*x + e)^2
+ 2*sqrt(-a*b + b^2)*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*a*
cos(f*x + e) - ((a - 2*b)*cos(f*x + e)^2 - a + 2*b)*log(1/2*cos(f*x + e) +
1/2) + ((a - 2*b)*cos(f*x + e)^2 - a + 2*b)*log(-1/2*cos(f*x + e) + 1/2))/(
a^2*f*cos(f*x + e)^2 - a^2*f), 1/4*(4*sqrt(a*b - b^2)*(cos(f*x + e)^2 - 1)*
arctan(sqrt(a*b - b^2)*cos(f*x + e)/b) + 2*a*cos(f*x + e) - ((a - 2*b)*cos(
f*x + e)^2 - a + 2*b)*log(1/2*cos(f*x + e) + 1/2) + ((a - 2*b)*cos(f*x + e)
^2 - a + 2*b)*log(-1/2*cos(f*x + e) + 1/2))/(a^2*f*cos(f*x + e)^2 - a^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*1/16/a+(-2*(1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-
a)*1/16/a^2/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))+(-2*a*b+2*b^2)*1/4/a^2/
sqrt(-b^2+a*b)*atan((-a*cos(f*x+exp(1))+b*cos(f*x+exp(1))+b)/(sqrt(-b^2+a*b
)*cos(f*x+exp(1))+sqrt(-b^2+a*b)))+(a-2*b)*1/8/a^2*ln(abs(1-cos(f*x+exp(1))
)/abs(1+cos(f*x+exp(1)))))
```

maple [B] time = 0.69, size = 189, normalized size = 2.12

$$\frac{b \arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{(a-b)b}}\right)}{fa\sqrt{(a-b)\,b}}-\frac{b^2 \arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{(a-b)b}}\right)}{f\,a^2\sqrt{(a-b)\,b}}+\frac{1}{4fa\left(-1+\cos\left(fx+e\right)\right)}+\frac{\ln\left(-1+\cos\left(fx+e\right)\right)}{4fa}-\frac{\ln\left(-1+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f/a*b/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))-1/f*b^2/a^
2/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))+1/4/f/a/(-1+cos(
f*x+e))+1/4/f/a*ln(-1+cos(f*x+e))-1/2/f/a^2*ln(-1+cos(f*x+e))*b+1/4/f/a/(1+
cos(f*x+e))-1/4/f/a*ln(1+cos(f*x+e))+1/2/f/a^2*ln(1+cos(f*x+e))*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 13.39, size = 591, normalized size = 6.64

$$-\frac{a\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2-2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)+2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)-\frac{1}{2}\right)+4\,\mathrm{atan}\left(\frac{2a^4b\cos\left(\frac{e}{2}+\frac{f}{2}\cdots\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)),x)
```

```
[Out] -(a*(cos(e/2 + (f*x)/2)^2 - 2*cos(e/2 + (f*x)/2)^2*log(sin(e/2 + (f*x)/2)/c
os(e/2 + (f*x)/2)) + 2*cos(e/2 + (f*x)/2)^4*log(sin(e/2 + (f*x)/2)/cos(e/2
+ (f*x)/2)) - 1/2) + 4*atan((6*b^5*cos(e/2 + (f*x)/2)^2 + 3*a*b^4 - a^4*b -
 6*a^2*b^3 + 4*a^3*b^2 + 20*a^2*b^3*cos(e/2 + (f*x)/2)^2 - 10*a^3*b^2*cos(e
/2 + (f*x)/2)^2 - 18*a*b^4*cos(e/2 + (f*x)/2)^2 + 2*a^4*b*cos(e/2 + (f*x)/2
)^2)/(6*cos(e/2 + (f*x)/2)^2*(a*b - b^2)^(5/2) - 2*a^2*cos(e/2 + (f*x)/2)^2
*(a*b - b^2)^(3/2)))*cos(e/2 + (f*x)/2)^2*(a*b - b^2)^(1/2) - 4*atan((6*b^5
*cos(e/2 + (f*x)/2)^2 + 3*a*b^4 - a^4*b - 6*a^2*b^3 + 4*a^3*b^2 + 20*a^2*b^
3*cos(e/2 + (f*x)/2)^2 - 10*a^3*b^2*cos(e/2 + (f*x)/2)^2 - 18*a*b^4*cos(e/2
```

```
 + (f*x)/2)^2 + 2*a^4*b*cos(e/2 + (f*x)/2)^2)/(6*cos(e/2 + (f*x)/2)^2*(a*b
- b^2)^(5/2) - 2*a^2*cos(e/2 + (f*x)/2)^2*(a*b - b^2)^(3/2)))*cos(e/2 + (f*
x)/2)^4*(a*b - b^2)^(1/2) + 4*b*cos(e/2 + (f*x)/2)^2*log(sin(e/2 + (f*x)/2)
/cos(e/2 + (f*x)/2)) - 4*b*cos(e/2 + (f*x)/2)^4*log(sin(e/2 + (f*x)/2)/cos(
e/2 + (f*x)/2)))/(4*a^2*f*cos(e/2 + (f*x)/2)^2 - 4*a^2*f*cos(e/2 + (f*x)/2)
^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*tan(e + f*x)**2), x)
```

3.60 $\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{b}\,(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^3f}-\frac{(5a-4b)\cot(e+fx)\csc(e+fx)}{8a^2f}-\frac{\left(3a^2-12ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8a^3f}-$$

```
[Out] -1/8*(3*a^2-12*a*b+8*b^2)*arctanh(cos(f*x+e))/a^3/f-1/8*(5*a-4*b)*cot(f*x+e
)*csc(f*x+e)/a^2/f-1/4*cot(f*x+e)^3*csc(f*x+e)/a/f-(a-b)^(3/2)*arctan(sec(f
*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^3/f
```

Rubi [A] time = 0.18, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3664, 470, 527, 522, 207, 205}

$$-\frac{\left(3a^2-12ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8a^3f}-\frac{(5a-4b)\cot(e+fx)\csc(e+fx)}{8a^2f}-\frac{\sqrt{b}\,(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^3f}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a^3*f
)) - ((3*a^2 - 12*a*b + 8*b^2)*ArcTanh[Cos[e + f*x]])/(8*a^3*f) - ((5*a - 4
*b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f) - (Cot[e + f*x]^3*Csc[e + f*x])/(4
*a*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\left(a-b+bx^2\right)}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af} - \frac{\text{Subst}\left(\int \frac{-a+b+(-4a+3b)x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)}\,dx, x, \sec(e+fx)\right)}{4af}$$

$$= -\frac{(5a-4b)\cot(e+fx)\csc(e+fx)}{8a^2f} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af} - \frac{\text{Subst}\left(\int \frac{-(3a-4b)(\cdots}{\left(-1+\cdots\right.}\right.}{\cdots}$$

$$= -\frac{(5a-4b)\cot(e+fx)\csc(e+fx)}{8a^2f} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af} - \frac{\left((a-b)^2b\right)\text{Subst}\cdots}{\cdots}$$

$$= -\frac{(a-b)^{3/2}\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^3f} - \frac{\left(3a^2-12ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8a^3f} - \frac{(5\cdots}{\cdots}$$

Mathematica [B] time = 6.26, size = 326, normalized size = 2.51

$$\frac{\sqrt{b}\,(a-b)^{3/2}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{a-b}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{a}\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{b}}\right)}{a^3f} + \frac{\sqrt{b}\,(a-b)^{3/2}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{a-b}\cos\left(\frac{1}{2}(e+f\cdots\right.\right.}{\sqrt{b}}\right.}{a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a - b)^(3/2)*Sqrt[b]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/
2] - Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(a^3*f) + ((a - b)^(3/2)*Sqrt[b]*
ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/2] + Sqrt[a]*Sin[(e + f
*x)/2]))/Sqrt[b]])/(a^3*f) + ((-3*a + 4*b)*Csc[(e + f*x)/2]^2)/(32*a^2*f) -
 Csc[(e + f*x)/2]^4/(64*a*f) + ((-3*a^2 + 12*a*b - 8*b^2)*Log[Cos[(e + f*x)
/2]])/(8*a^3*f) + ((3*a^2 - 12*a*b + 8*b^2)*Log[Sin[(e + f*x)/2]])/(8*a^3*f
) + ((3*a - 4*b)*Sec[(e + f*x)/2]^2)/(32*a^2*f) + Sec[(e + f*x)/2]^4/(64*a*
f)
```

fricas [B] time = 0.57, size = 630, normalized size = 4.85

$$\left[\frac{2\left(3a^2-4ab\right)\cos\left(fx+e\right)^3-8\left((a-b)\cos\left(fx+e\right)^4-2(a-b)\cos\left(fx+e\right)^2+a-b\right)\sqrt{-ab+b^2}\log\left(\frac{(a-b)\cos(\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a^2 - 4*a*b)*cos(f*x + e)^3 - 8*((a - b)*cos(f*x + e)^4 - 2*(a
- b)*cos(f*x + e)^2 + a - b)*sqrt(-a*b + b^2)*log(((a - b)*cos(f*x + e)^2 -
 2*sqrt(-a*b + b^2)*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - 2*(5*
a^2 - 4*a*b)*cos(f*x + e) - ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3
*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2)*log(1/2*cos
(f*x + e) + 1/2) + ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 12
*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2)*log(-1/2*cos(f*x + e
) + 1/2))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f), 1/16*(2*
(3*a^2 - 4*a*b)*cos(f*x + e)^3 + 16*((a - b)*cos(f*x + e)^4 - 2*(a - b)*cos
(f*x + e)^2 + a - b)*sqrt(a*b - b^2)*arctan(sqrt(a*b - b^2)*cos(f*x + e)/b)
 - 2*(5*a^2 - 4*a*b)*cos(f*x + e) - ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^
4 - 2*(3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2)*log
(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*
a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2)*log(-1/2*cos
(f*x + e) + 1/2))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+256*(1-cos(f
*x+exp(1)))/(1+cos(f*x+exp(1)))*a-256*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)
))*b)*1/4096/a^2+(-18*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2+72*((
1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a*b-48*((1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1))))^2*b^2-8*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2+8*(1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a*b-a^2)*1/128/a^3/((1-cos(f*x+exp(1)))/
(1+cos(f*x+exp(1))))^2+(-2*a^2*b+4*a*b^2-2*b^3)*1/4/a^3/sqrt(-b^2+a*b)*atan
((-a*cos(f*x+exp(1))+b*cos(f*x+exp(1))+b)/(sqrt(-b^2+a*b)*cos(f*x+exp(1))+s
qrt(-b^2+a*b)))+(3*a^2-12*a*b+8*b^2)*1/32/a^3*ln(abs(1-cos(f*x+exp(1)))/abs
(1+cos(f*x+exp(1)))))
```

maple [B] time = 0.60, size = 344, normalized size = 2.65

$$\frac{b\arctan\left(\frac{(a-b)\cos\left(fx+e\right)}{\sqrt{(a-b)b}}\right)}{fa\sqrt{(a-b)\,b}}-\frac{2b^2\arctan\left(\frac{(a-b)\cos\left(fx+e\right)}{\sqrt{(a-b)b}}\right)}{f\,a^2\sqrt{(a-b)\,b}}+\frac{b^3\arctan\left(\frac{(a-b)\cos\left(fx+e\right)}{\sqrt{(a-b)b}}\right)}{f\,a^3\sqrt{(a-b)\,b}}-\frac{1}{16fa\left(-1+\cos\left(fx+e\right)\right)^2}+\frac{\cdots}{16f\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f/a*b/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))-2/f*b^2/a^
2/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))+1/f*b^3/a^3/((a-
```

```
b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))-1/16/f/a/(-1+cos(f*x+e
))^2+3/16/f/a/(-1+cos(f*x+e))-1/4/f/a^2/(-1+cos(f*x+e))*b+3/16/f/a*ln(-1+co
s(f*x+e))-3/4/f/a^2*ln(-1+cos(f*x+e))*b+1/2/f/a^3*ln(-1+cos(f*x+e))*b^2+1/1
6/f/a/(1+cos(f*x+e))^2+3/16/f/a/(1+cos(f*x+e))-1/4/f/a^2/(1+cos(f*x+e))*b-3
/16/f/a*ln(1+cos(f*x+e))+3/4/f/a^2*ln(1+cos(f*x+e))*b-1/2/f/a^3*ln(1+cos(f*
x+e))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 14.72, size = 740, normalized size = 5.69

$$a^2\left(\frac{3\cos\left(3e+3fx\right)}{4}-\frac{11\cos\left(e+fx\right)}{4}+\frac{9\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{8}-\frac{3\cos\left(2e+2fx\right)\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{2}+\frac{3\cos\left(4e+4fx\right)\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{8}\right)+3\,b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)),x)
```

```
[Out] (a^2*((3*cos(3*e + 3*f*x))/4 - (11*cos(e + f*x))/4 + (9*log(sin(e/2 + (f*x)
/2)/cos(e/2 + (f*x)/2)))/8 - (3*cos(2*e + 2*f*x)*log(sin(e/2 + (f*x)/2)/cos
(e/2 + (f*x)/2)))/2 + (3*cos(4*e + 4*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2 +
(f*x)/2)))/8) + 3*b^2*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)) - a*(b*cos
(3*e + 3*f*x) - b*cos(e + f*x) + (9*b*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x
)/2)))/2 - 6*b*cos(2*e + 2*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))
+ (3*b*cos(4*e + 4*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/2) - 4*
b^2*cos(2*e + 2*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)) + b^2*cos(4
*e + 4*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)) + 3*b^(1/2)*atan((a^
4*cos(e + f*x) - a^3*b - 3*a*b^3 + b^4*cos(e + f*x) + b^4 + 3*a^2*b^2 + 6*a
^2*b^2*cos(e + f*x) - 4*a*b^3*cos(e + f*x) - 4*a^3*b*cos(e + f*x))/(2*b^(1/
2)*cos(e/2 + (f*x)/2)^2*(a - b)^(7/2)))*(a - b)^(3/2) - 4*b^(1/2)*atan((a^4
*cos(e + f*x) - a^3*b - 3*a*b^3 + b^4*cos(e + f*x) + b^4 + 3*a^2*b^2 + 6*a^
2*b^2*cos(e + f*x) - 4*a*b^3*cos(e + f*x) - 4*a^3*b*cos(e + f*x))/(2*b^(1/2
)*cos(e/2 + (f*x)/2)^2*(a - b)^(7/2)))*cos(2*e + 2*f*x)*(a - b)^(3/2) + b^(
1/2)*atan((a^4*cos(e + f*x) - a^3*b - 3*a*b^3 + b^4*cos(e + f*x) + b^4 + 3*
a^2*b^2 + 6*a^2*b^2*cos(e + f*x) - 4*a*b^3*cos(e + f*x) - 4*a^3*b*cos(e + f
*x))/(2*b^(1/2)*cos(e/2 + (f*x)/2)^2*(a - b)^(7/2)))*cos(4*e + 4*f*x)*(a -
b)^(3/2))/(3*a^3*f - 4*a^3*f*cos(2*e + 2*f*x) + a^3*f*cos(4*e + 4*f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5\left(e+fx\right)}{a+b\tan^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**5/(a + b*tan(e + f*x)**2), x)
```

3.61 $\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=178

$$-\frac{a^{5/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^4}-\frac{\left(11a^2-4ab+b^2\right)\sin(e+fx)\cos(e+fx)}{16f(a-b)^3}+\frac{x\left(5a^3+15a^2b-5ab^2+b^3\right)}{16(a-b)^4}+\frac{\sin^3(e+\dots}{6\dots}$$

```
[Out] 1/16*(5*a^3+15*a^2*b-5*a*b^2+b^3)*x/(a-b)^4-1/16*(11*a^2-4*a*b+b^2)*cos(f*x
+e)*sin(f*x+e)/(a-b)^3/f+1/8*(3*a-b)*cos(f*x+e)^3*sin(f*x+e)/(a-b)^2/f+1/6*
cos(f*x+e)^3*sin(f*x+e)^3/(a-b)/f-a^(5/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2)
)*b^(1/2)/(a-b)^4/f
```

Rubi [A] time = 0.29, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3663, 470, 578, 527, 522, 203, 205}

$$-\frac{\left(11a^2-4ab+b^2\right)\sin(e+fx)\cos(e+fx)}{16f(a-b)^3}+\frac{x\left(15a^2b+5a^3-5ab^2+b^3\right)}{16(a-b)^4}-\frac{a^{5/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^4}+\frac{\sin^3(e+\dots}{6\dots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*x)/(16*(a - b)^4) - (a^(5/2)*Sqrt[b]*Ar
cTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)^4*f) - ((11*a^2 - 4*a*b + b^
2)*Cos[e + f*x]*Sin[e + f*x])/(16*(a - b)^3*f) + ((3*a - b)*Cos[e + f*x]^3*
Sin[e + f*x])/(8*(a - b)^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*(a - b)*
f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6(a-b)f} - \frac{\text{Subst}\left(\int \frac{x^2(3a-3(2a-b)x^2)}{(1+x^2)^3(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{6(a-b)f}$$

$$= \frac{(3a-b)\cos^3(e+fx)\sin(e+fx)}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6(a-b)f} - \frac{\text{Subst}\left(\int \frac{3a(3a-b)-\dots}{(1+\dots}\right.\dots}{\dots}$$

$$= -\frac{(11a^2-4ab+b^2)\cos(e+fx)\sin(e+fx)}{16(a-b)^3f} + \frac{(3a-b)\cos^3(e+fx)\sin(e+fx)}{8(a-b)^2f} + \dots$$

$$= -\frac{(11a^2-4ab+b^2)\cos(e+fx)\sin(e+fx)}{16(a-b)^3f} + \frac{(3a-b)\cos^3(e+fx)\sin(e+fx)}{8(a-b)^2f} + \dots$$

$$= \frac{(5a^3+15a^2b-5ab^2+b^3)x}{16(a-b)^4} - \frac{a^{5/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^4f} - \frac{(11a^2-4ab+b^2)\cos\dots}{16(a-\dots}$$

Mathematica [A] time = 0.57, size = 140, normalized size = 0.79

$$-\frac{192a^{5/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) - 12(5a^3+15a^2b-5ab^2+b^3)(e+fx) + (a-b)^3\sin(6(e+fx)) - 3(3a-b)(\dots}{192f(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/192*(-12*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*(e + f*x) + 192*a^(5/2)*Sqrt
[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + 3*(a - b)*(5*a - b)*(3*a + b)*
Sin[2*(e + f*x)] - 3*(a - b)^2*(3*a - b)*Sin[4*(e + f*x)] + (a - b)^3*Sin[6
*(e + f*x)])/((a - b)^4*f)
```

fricas [A] time = 0.56, size = 521, normalized size = 2.93

$$\left[\frac{12\sqrt{-ab}\,a^2\log\left(\frac{\left(a^2+6ab+b^2\right)\cos\left(fx+e\right)^4-2\left(3ab+b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a+b\right)\cos\left(fx+e\right)^3-b\cos\left(fx+e\right)\right)\sqrt{-ab}\sin\left(fx+e\right)+b^2}{\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^4+2\left(ab-b^2\right)\cos\left(fx+e\right)^2+b^2}\right)+3\left(5a^3+1\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(12*sqrt(-a*b)*a^2*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b
 + b^2)*cos(f*x + e)^2 + 4*((a + b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-
a*b)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2
)*cos(f*x + e)^2 + b^2)) + 3*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*f*x - (8*(a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - 2*(13*a^3 - 33*a^2*b + 27*a*
b^2 - 7*b^3)*cos(f*x + e)^3 + 3*(11*a^3 - 15*a^2*b + 5*a*b^2 - b^3)*cos(f*x
 + e))*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f), 1/48*
(24*sqrt(a*b)*a^2*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*co
s(f*x + e)*sin(f*x + e))) + 3*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*f*x - (8*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - 2*(13*a^3 - 33*a^2*b + 27*a
*b^2 - 7*b^3)*cos(f*x + e)^3 + 3*(11*a^3 - 15*a^2*b + 5*a*b^2 - b^3)*cos(f*
x + e))*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f)]
```

giac [A] time = 2.09, size = 292, normalized size = 1.64

$$-\frac{\frac{48\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)a^3b}{\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\sqrt{ab}}-\frac{3\left(5a^3+15a^2b-5ab^2+b^3\right)\left(fx+e\right)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}+\frac{33a^2\tan\left(fx+e\right)^5-12ab\tan\left(fx+e\right)^5+3b^2\tan\left(fx+e\right)^5+40a\cdots}{\cdots}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/48*(48*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b)))*a^3*b/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sqrt(a*b)) - 3*(
5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*(f*x + e)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4) + (33*a^2*tan(f*x + e)^5 - 12*a*b*tan(f*x + e)^5 + 3*b^2*tan(f
*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 16*a*b*tan(f*x + e)^3 - 8*b^2*tan(f*x +
 e)^3 + 15*a^2*tan(f*x + e) + 12*a*b*tan(f*x + e) - 3*b^2*tan(f*x + e))/((a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*(tan(f*x + e)^2 + 1)^3))/f
```

maple [B] time = 0.52, size = 545, normalized size = 3.06

$$-\frac{b a^3\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{f\left(a-b\right)^4\sqrt{ab}}-\frac{11\left(\tan^5\left(fx+e\right)\right)a^3}{16f\left(a-b\right)^4\left(1+\tan^2\left(fx+e\right)\right)^3}+\frac{15\left(\tan^5\left(fx+e\right)\right)a^2b}{16f\left(a-b\right)^4\left(1+\tan^2\left(fx+e\right)\right)^3}-\frac{5\left(\tan^5\left(fx\cdots\right.\right.}{16f\left(a-b\right)^4\left(1+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f*b*a^3/(a-b)^4/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-11/16/f/(a-
b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)^5*a^3+15/16/f/(a-b)^4/(1+tan(f*x+e)^2)^3
*tan(f*x+e)^5*a^2*b-5/16/f/(a-b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)^5*b^2*a+1/
16/f/(a-b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)^5*b^3-5/6/f/(a-b)^4/(1+tan(f*x+e
)^2)^3*tan(f*x+e)^3*a^3+1/2/f/(a-b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)^3*a^2*b
+1/2/f/(a-b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)^3*b^2*a-1/6/f/(a-b)^4/(1+tan(f
*x+e)^2)^3*tan(f*x+e)^3*b^3-5/16/f/(a-b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)*a^
3+1/16/f/(a-b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)*a^2*b+5/16/f/(a-b)^4/(1+tan(
f*x+e)^2)^3*tan(f*x+e)*b^2*a-1/16/f/(a-b)^4/(1+tan(f*x+e)^2)^3*tan(f*x+e)*b
^3+5/16/f/(a-b)^4*arctan(tan(f*x+e))*a^3+15/16/f/(a-b)^4*arctan(tan(f*x+e))
*a^2*b-5/16/f/(a-b)^4*arctan(tan(f*x+e))*b^2*a+1/16/f/(a-b)^4*arctan(tan(f*
x+e))*b^3
```

maxima [A] time = 0.81, size = 305, normalized size = 1.71

$$-\frac{\frac{48\,a^{3}b\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4\,ab^{3}+b^{4}\right)\sqrt{ab}}-\frac{3\,\left(5\,a^{3}+15\,a^{2}b-5\,ab^{2}+b^{3}\right)\left(fx+e\right)}{a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4\,ab^{3}+b^{4}}+\frac{3\,\left(11\,a^{2}-4\,ab+b^{2}\right)\tan\left(fx+e\right)^{5}+8\,\left(5\,a^{2}+2\,ab-b^{2}\right)\tan\cdots}{\left(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}\right)\tan\left(fx+e\right)^{6}+3\,\left(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}\right)\tan\left(fx+e\right)^{4}\cdots}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/48*(48*a^3*b*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - 4*a^3*b + 6*a^2*b^
2 - 4*a*b^3 + b^4)*sqrt(a*b)) - 3*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*(f*x +
 e)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + (3*(11*a^2 - 4*a*b + b^2)
*tan(f*x + e)^5 + 8*(5*a^2 + 2*a*b - b^2)*tan(f*x + e)^3 + 3*(5*a^2 + 4*a*b
 - b^2)*tan(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*tan(f*x + e)^6 + 3*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*tan(f*x + e)^4 + a^3 - 3*a^2*b + 3*a*b^2 - b
^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*tan(f*x + e)^2))/f
```

mupad [B] time = 17.02, size = 4910, normalized size = 27.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (atan(-((((((3*a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4 + 32*a^4*b^9 - (77*a
^5*b^8)/2 + 14*a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5 + (107*a^9*b^4)/4 - 9*
a^10*b^3 + (5*a^11*b^2)/4)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84
*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) - (tan(e +
f*x)*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*(1024*b^11 - 7168*a*b^10 + 20
480*a^2*b^9 - 28672*a^3*b^8 + 14336*a^4*b^7 + 14336*a^5*b^6 - 28672*a^6*b^5
 + 20480*a^7*b^4 - 7168*a^8*b^3 + 1024*a^9*b^2))/(4096*(a^4 - 4*a^3*b - 4*a
*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^
3*b^3 + 15*a^4*b^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i))/(32*(a^4 -
4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (tan(e + f*x)*(b^9 - 10*a*b^8 + 55*
a^2*b^7 - 140*a^3*b^6 + 175*a^4*b^5 + 150*a^5*b^4 + 281*a^6*b^3))/(128*(a^6
 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a^2*b
*15i - a*b^2*5i + a^3*5i + b^3*1i)*1i)/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 +
 6*a^2*b^2)) - (((((3*a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4 + 32*a^4*b^9
- (77*a^5*b^8)/2 + 14*a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5 + (107*a^9*b^4)
/4 - 9*a^10*b^3 + (5*a^11*b^2)/4)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b
^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) + (t
an(e + f*x)*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*(1024*b^11 - 7168*a*b^
10 + 20480*a^2*b^9 - 28672*a^3*b^8 + 14336*a^4*b^7 + 14336*a^5*b^6 - 28672*
a^6*b^5 + 20480*a^7*b^4 - 7168*a^8*b^3 + 1024*a^9*b^2))/(4096*(a^4 - 4*a^3*
b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4
```

- 20*a^3*b^3 + 15*a^4*b^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i))/(32*
(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e + f*x)*(b^9 - 10*a*b^
8 + 55*a^2*b^7 - 140*a^3*b^6 + 175*a^4*b^5 + 150*a^5*b^4 + 281*a^6*b^3))/(1
28*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))
*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*1i)/(32*(a^4 - 4*a^3*b - 4*a*b^3
+ b^4 + 6*a^2*b^2)))/(((a^3*b^8)/128 - (9*a^4*b^7)/128 + (23*a^5*b^6)/64 -
(55*a^6*b^5)/64 + (145*a^7*b^4)/128 + (55*a^8*b^3)/128)/(9*a*b^8 - 9*a^8*b
+ a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*
b^3 + 36*a^7*b^2) + (((((3*a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4 + 32*a^4
*b^9 - (77*a^5*b^8)/2 + 14*a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5 + (107*a^9
*b^4)/4 - 9*a^10*b^3 + (5*a^11*b^2)/4)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*
a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2)
 - (tan(e + f*x)*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*(1024*b^11 - 7168
*a*b^10 + 20480*a^2*b^9 - 28672*a^3*b^8 + 14336*a^4*b^7 + 14336*a^5*b^6 - 2
8672*a^6*b^5 + 20480*a^7*b^4 - 7168*a^8*b^3 + 1024*a^9*b^2))/(4096*(a^4 - 4
*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2
*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i))
/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (tan(e + f*x)*(b^9 - 10
*a*b^8 + 55*a^2*b^7 - 140*a^3*b^6 + 175*a^4*b^5 + 150*a^5*b^4 + 281*a^6*b^3
))/(128*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b
^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i))/(32*(a^4 - 4*a^3*b - 4*a*b^
3 + b^4 + 6*a^2*b^2)) + (((((3*a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4 + 32
*a^4*b^9 - (77*a^5*b^8)/2 + 14*a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5 + (107
*a^9*b^4)/4 - 9*a^10*b^3 + (5*a^11*b^2)/4)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 -
 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*
b^2) + (tan(e + f*x)*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*(1024*b^11 -
7168*a*b^10 + 20480*a^2*b^9 - 28672*a^3*b^8 + 14336*a^4*b^7 + 14336*a^5*b^6
 - 28672*a^6*b^5 + 20480*a^7*b^4 - 7168*a^8*b^3 + 1024*a^9*b^2))/(4096*(a^4
 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15
*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*
1i))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e + f*x)*(b^9
- 10*a*b^8 + 55*a^2*b^7 - 140*a^3*b^6 + 175*a^4*b^5 + 150*a^5*b^4 + 281*a^6
b^3))/(128(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a
^4*b^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i))/(32*(a^4 - 4*a^3*b - 4*
a*b^3 + b^4 + 6*a^2*b^2))))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*1i)/(1
6*f*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - ((tan(e + f*x)*(4*a*b +
5*a^2 - b^2))/(16*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e + f*x)^5*(11*a^
2 - 4*a*b + b^2))/(16*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e + f*x)^3*(2
*a*b + 5*a^2 - b^2))/(6*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(f*(3*tan(e + f*x
)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) - (atan((((-a^5*b)^(1/2)*((((
3*a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4 + 32*a^4*b^9 - (77*a^5*b^8)/2 + 1
4*a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5 + (107*a^9*b^4)/4 - 9*a^10*b^3 + (5
*a^11*b^2)/4)/(2*(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 -
 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2)) - (tan(e + f*x)*(-a^
5*b)^(1/2)*(1024*b^11 - 7168*a*b^10 + 20480*a^2*b^9 - 28672*a^3*b^8 + 14336
*a^4*b^7 + 14336*a^5*b^6 - 28672*a^6*b^5 + 20480*a^7*b^4 - 7168*a^8*b^3 + 1
024*a^9*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5
*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(-a^5*b)^(1/2)
)/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (tan(e + f*x)*(b^9 - 10
*a*b^8 + 55*a^2*b^7 - 140*a^3*b^6 + 175*a^4*b^5 + 150*a^5*b^4 + 281*a^6*b^3
))/(256*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b
^2)))*1i)/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2) - ((-a^5*b)^(1/2)*(((
(3*a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4 + 32*a^4*b^9 - (77*a^5*b^8)/2 +
14*a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5 + (107*a^9*b^4)/4 - 9*a^10*b^3 + (
5*a^11*b^2)/4)/(2*(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6
- 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2)) + (tan(e + f*x)*(-a
^5*b)^(1/2)*(1024*b^11 - 7168*a*b^10 + 20480*a^2*b^9 - 28672*a^3*b^8 + 1433
6*a^4*b^7 + 14336*a^5*b^6 - 28672*a^6*b^5 + 20480*a^7*b^4 - 7168*a^8*b^3 +
1024*a^9*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^

```
5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(-a^5*b)^(1/2
))/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e + f*x)*(b^9 - 1
0*a*b^8 + 55*a^2*b^7 - 140*a^3*b^6 + 175*a^4*b^5 + 150*a^5*b^4 + 281*a^6*b^
3))/(256*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*
b^2)))*1i)/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))/(((a^3*b^8)/128 - (
9*a^4*b^7)/128 + (23*a^5*b^6)/64 - (55*a^6*b^5)/64 + (145*a^7*b^4)/128 + (5
5*a^8*b^3)/128)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 -
126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) + ((-a^5*b)^(1/2)*((((
3*a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4 + 32*a^4*b^9 - (77*a^5*b^8)/2 + 1
4*a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5 + (107*a^9*b^4)/4 - 9*a^10*b^3 + (5
*a^11*b^2)/4)/(2*(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 -
 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2)) - (tan(e + f*x)*(-a^
5*b)^(1/2)*(1024*b^11 - 7168*a*b^10 + 20480*a^2*b^9 - 28672*a^3*b^8 + 14336
*a^4*b^7 + 14336*a^5*b^6 - 28672*a^6*b^5 + 20480*a^7*b^4 - 7168*a^8*b^3 + 1
024*a^9*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5
*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(-a^5*b)^(1/2)
)/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (tan(e + f*x)*(b^9 - 10
*a*b^8 + 55*a^2*b^7 - 140*a^3*b^6 + 175*a^4*b^5 + 150*a^5*b^4 + 281*a^6*b^3
))/(256*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b
^2))))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2) + ((-a^5*b)^(1/2)*((((3*
a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4 + 32*a^4*b^9 - (77*a^5*b^8)/2 + 14*
a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5 + (107*a^9*b^4)/4 - 9*a^10*b^3 + (5*a
^11*b^2)/4)/(2*(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 1
26*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2)) + (tan(e + f*x)*(-a^5*
b)^(1/2)*(1024*b^11 - 7168*a*b^10 + 20480*a^2*b^9 - 28672*a^3*b^8 + 14336*a
^4*b^7 + 14336*a^5*b^6 - 28672*a^6*b^5 + 20480*a^7*b^4 - 7168*a^8*b^3 + 102
4*a^9*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b
 - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(-a^5*b)^(1/2))/
(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e + f*x)*(b^9 - 10*a
*b^8 + 55*a^2*b^7 - 140*a^3*b^6 + 175*a^4*b^5 + 150*a^5*b^4 + 281*a^6*b^3))
/(256*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2
))))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-a^5*b)^(1/2)*1i)/(f*(a
^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.62 $\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=129

$$-\frac{a^{3/2}\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^3}+\frac{x\left(3a^2+6ab-b^2\right)}{8(a-b)^3}+\frac{\sin(e+fx)\cos^3(e+fx)}{4f(a-b)}-\frac{(5a-b)\sin(e+fx)\cos(e+fx)}{8f(a-b)^2}$$

```
[Out] 1/8*(3*a^2+6*a*b-b^2)*x/(a-b)^3-1/8*(5*a-b)*cos(f*x+e)*sin(f*x+e)/(a-b)^2/f
+1/4*cos(f*x+e)^3*sin(f*x+e)/(a-b)/f-a^(3/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1
/2))*b^(1/2)/(a-b)^3/f
```

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3663, 470, 527, 522, 203, 205}

$$\frac{x\left(3a^2+6ab-b^2\right)}{8(a-b)^3}-\frac{a^{3/2}\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^3}+\frac{\sin(e+fx)\cos^3(e+fx)}{4f(a-b)}-\frac{(5a-b)\sin(e+fx)\cos(e+fx)}{8f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((3*a^2 + 6*a*b - b^2)*x)/(8*(a - b)^3) - (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*
Tan[e + f*x])/Sqrt[a]])/((a - b)^3*f) - ((5*a - b)*Cos[e + f*x]*Sin[e + f*x
])/(8*(a - b)^2*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*(a - b)*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f} - \frac{\text{Subst}\left(\int \frac{a+(-4a+b)x^2}{(1+x^2)^2(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= -\frac{(5a-b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f} + \frac{\text{Subst}\left(\int \frac{a(3a+b)-(5}{(1+x^2)(a}\right.}{8}$$

$$= -\frac{(5a-b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f} - \frac{(a^2b)\,\text{Subst}\left(\int \frac{1}{a+b}\right.}{(a}$$

$$= \frac{(3a^2+6ab-b^2)\,x}{8(a-b)^3} - \frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^3f} - \frac{(5a-b)\cos(e+fx)\sin(e+fx}{8(a-b)^2f}$$

Mathematica [A] time = 0.27, size = 99, normalized size = 0.77

$$\frac{-32a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) + 4\left(3a^2+6ab-b^2\right)(e+fx) + (a-b)^2\sin(4(e+fx)) - 8a(a-b)\sin(2(e+fx}{32f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (4*(3*a^2 + 6*a*b - b^2)*(e + f*x) - 32*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan
[e + f*x])/Sqrt[a]] - 8*a*(a - b)*Sin[2*(e + f*x)] + (a - b)^2*Sin[4*(e + f
*x)])/(32*(a - b)^3*f)
```

fricas [A] time = 0.50, size = 383, normalized size = 2.97

$$\left[\frac{\left(3a^2+6ab-b^2\right)fx - 2\sqrt{-ab}\,a\log\left(\frac{\left(a^2+6ab+b^2\right)\cos\left(fx+e\right)^4 - 2\left(3ab+b^2\right)\cos\left(fx+e\right)^2 - 4\left(\left(a+b\right)\cos\left(fx+e\right)^3 - b\cos\left(fx+e\right)\right)\sqrt{-ab}\,\text{si}}{\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^4 + 2\left(ab-b^2\right)\cos\left(fx+e\right)^2 + b^2}\right.}{8\left(a^3-3a^2b+3ab^2-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/8*((3*a^2 + 6*a*b - b^2)*f*x - 2*sqrt(-a*b)*a*log(((a^2 + 6*a*b + b^2)*c
os(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*((a + b)*cos(f*x + e)^3
- b*cos(f*x + e))*sqrt(-a*b)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f
*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)) + (2*(a^2 - 2*a*b + b^2)*c
os(f*x + e)^3 - (5*a^2 - 6*a*b + b^2)*cos(f*x + e))*sin(f*x + e))/((a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*f), 1/8*((3*a^2 + 6*a*b - b^2)*f*x + 4*sqrt(a*b)*a*
arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*cos(f*x + e)*sin(f*x
 + e))) + (2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 6*a*b + b^2)*cos
(f*x + e))*sin(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)]
```

giac [A] time = 2.44, size = 190, normalized size = 1.47

$$-\frac{\frac{8\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)a^2b}{(a^3-3a^2b+3ab^2-b^3)\sqrt{ab}}-\frac{(3a^2+6ab-b^2)(fx+e)}{a^3-3a^2b+3ab^2-b^3}+\frac{5a\tan(fx+e)^3-b\tan(fx+e)^3+3a\tan(fx+e)+b\tan(fx+e)}{(a^2-2ab+b^2)(\tan(fx+e)^2+1)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*a^2*b/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b)) - (3*a^2 + 6*a*b -
b^2)*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (5*a*tan(f*x + e)^3 - b*ta
n(f*x + e)^3 + 3*a*tan(f*x + e) + b*tan(f*x + e))/((a^2 - 2*a*b + b^2)*(tan
(f*x + e)^2 + 1)^2))/f
```

maple [B] time = 0.62, size = 304, normalized size = 2.36

$$-\frac{b\,a^2\arctan\left(\frac{\tan(fx+e)b}{\sqrt{ab}}\right)}{f(a-b)^3\sqrt{ab}}-\frac{5\left(\tan^3(fx+e)\right)a^2}{8f(a-b)^3\left(1+\tan^2(fx+e)\right)^2}+\frac{3\left(\tan^3(fx+e)\right)ab}{4f(a-b)^3\left(1+\tan^2(fx+e)\right)^2}-\frac{\left(\tan^3(fx+\right.}{8f(a-b)^3\left(1+\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f*b*a^2/(a-b)^3/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-5/8/f/(a-b)
^3/(1+tan(f*x+e)^2)^2*tan(f*x+e)^3*a^2+3/4/f/(a-b)^3/(1+tan(f*x+e)^2)^2*tan
(f*x+e)^3*a*b-1/8/f/(a-b)^3/(1+tan(f*x+e)^2)^2*tan(f*x+e)^3*b^2-3/8/f/(a-b)
^3/(1+tan(f*x+e)^2)^2*tan(f*x+e)*a^2+1/4/f/(a-b)^3/(1+tan(f*x+e)^2)^2*tan(f
*x+e)*a*b+1/8/f/(a-b)^3/(1+tan(f*x+e)^2)^2*tan(f*x+e)*b^2+3/8/f/(a-b)^3*arc
tan(tan(f*x+e))*a^2+3/4/f/(a-b)^3*arctan(tan(f*x+e))*a*b-1/8/f/(a-b)^3*arct
an(tan(f*x+e))*b^2
```

maxima [A] time = 0.85, size = 183, normalized size = 1.42

$$-\frac{\frac{8a^2b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3-3a^2b+3ab^2-b^3)\sqrt{ab}}-\frac{(3a^2+6ab-b^2)(fx+e)}{a^3-3a^2b+3ab^2-b^3}+\frac{(5a-b)\tan(fx+e)^3+(3a+b)\tan(fx+e)}{(a^2-2ab+b^2)\tan(fx+e)^4+2(a^2-2ab+b^2)\tan(fx+e)^2+a^2-2ab+b^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^2*b*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - 3*a^2*b + 3*a*b^2 -
b^3)*sqrt(a*b)) - (3*a^2 + 6*a*b - b^2)*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2
```

```
- b^3) + ((5*a - b)*tan(f*x + e)^3 + (3*a + b)*tan(f*x + e))/((a^2 - 2*a*b
+ b^2)*tan(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*tan(f*x + e)^2 + a^2 - 2*a*b
+ b^2))/f
```

mupad [B] time = 15.74, size = 3588, normalized size = 27.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (atan(((((tan(e + f*x)*(b^7 - 12*a*b^6 + 30*a^2*b^5 + 36*a^3*b^4 + 73*a^4*b
^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((32*a*b^9 - 96*a^
2*b^8 - 96*a^3*b^7 + 800*a^4*b^6 - 1440*a^5*b^5 + 1248*a^6*b^4 - 544*a^7*b^
3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^
3 + 15*a^4*b^2)) - (tan(e + f*x)*(-a^3*b)^(1/2)*(1280*a*b^8 - 256*b^9 - 230
4*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 256
*a^7*b^2))/(64*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(a^4 - 4*a^3*b - 4*a*b^3 + b
^4 + 6*a^2*b^2)))*(-a^3*b)^(1/2))/(2*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))*(-a^
3*b)^(1/2)*1i)/(2*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (((tan(e + f*x)*(b^7 -
 12*a*b^6 + 30*a^2*b^5 + 36*a^3*b^4 + 73*a^4*b^3))/(32*(a^4 - 4*a^3*b - 4*a
*b^3 + b^4 + 6*a^2*b^2)) - (((32*a*b^9 - 96*a^2*b^8 - 96*a^3*b^7 + 800*a^4*
b^6 - 1440*a^5*b^5 + 1248*a^6*b^4 - 544*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*
a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (tan(e + f
*x)*(-a^3*b)^(1/2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 12
80*a^4*b^5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(64*(3*a*b^2 - 3*a
^2*b + a^3 - b^3)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-a^3*b)^(1
/2))/(2*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))*(-a^3*b)^(1/2)*1i)/(2*(3*a*b^2 -
3*a^2*b + a^3 - b^3)))/((a^2*b^6 - 11*a^3*b^5 + 27*a^4*b^4 + 15*a^5*b^3)/(3
2*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) +
 (((tan(e + f*x)*(b^7 - 12*a*b^6 + 30*a^2*b^5 + 36*a^3*b^4 + 73*a^4*b^3))/(
32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((32*a*b^9 - 96*a^2*b^8
- 96*a^3*b^7 + 800*a^4*b^6 - 1440*a^5*b^5 + 1248*a^6*b^4 - 544*a^7*b^3 + 96
*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15
*a^4*b^2)) - (tan(e + f*x)*(-a^3*b)^(1/2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*
b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b
^2))/(64*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6
*a^2*b^2)))*(-a^3*b)^(1/2))/(2*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))*(-a^3*b)^(
1/2))/(2*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) - (((tan(e + f*x)*(b^7 - 12*a*b^6
 + 30*a^2*b^5 + 36*a^3*b^4 + 73*a^4*b^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^
4 + 6*a^2*b^2)) - (((32*a*b^9 - 96*a^2*b^8 - 96*a^3*b^7 + 800*a^4*b^6 - 144
0*a^5*b^5 + 1248*a^6*b^4 - 544*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6
*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (tan(e + f*x)*(-a^3
*b)^(1/2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^
5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(64*(3*a*b^2 - 3*a^2*b + a^
3 - b^3)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-a^3*b)^(1/2))/(2*(
3*a*b^2 - 3*a^2*b + a^3 - b^3)))*(-a^3*b)^(1/2))/(2*(3*a*b^2 - 3*a^2*b + a^
3 - b^3))))*(-a^3*b)^(1/2)*1i)/(f*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) - ((tan(
e + f*x)^3*(5*a - b))/(8*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)*(3*a + b))/(8
*(a^2 - 2*a*b + b^2)))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (atan(
((((tan(e + f*x)*(b^7 - 12*a*b^6 + 30*a^2*b^5 + 36*a^3*b^4 + 73*a^4*b^3))/(
32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((32*a*b^9 - 96*a^2*b^8
- 96*a^3*b^7 + 800*a^4*b^6 - 1440*a^5*b^5 + 1248*a^6*b^4 - 544*a^7*b^3 + 96
*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15
*a^4*b^2)) - (tan(e + f*x)*(6*a*b + 3*a^2 - b^2)*(1280*a*b^8 - 256*b^9 - 23
04*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 25
6*a^7*b^2))/(512*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)*(a^4 - 4*a^3*b - 4
*a*b^3 + b^4 + 6*a^2*b^2)))*(6*a*b + 3*a^2 - b^2))/(16*(a*b^2*3i - a^2*b*3i
 + a^3*1i - b^3*1i)))*(6*a*b + 3*a^2 - b^2)*1i)/(16*(a*b^2*3i - a^2*b*3i +
a^3*1i - b^3*1i)) + (((tan(e + f*x)*(b^7 - 12*a*b^6 + 30*a^2*b^5 + 36*a^3*b
```

```
^4 + 73*a^4*b^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (((32*
a*b^9 - 96*a^2*b^8 - 96*a^3*b^7 + 800*a^4*b^6 - 1440*a^5*b^5 + 1248*a^6*b^4
 - 544*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^
4 - 20*a^3*b^3 + 15*a^4*b^2)) + (tan(e + f*x)*(6*a*b + 3*a^2 - b^2)*(1280*a
*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4
+ 1280*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)
*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(6*a*b + 3*a^2 - b^2))/(16*(
a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))*(6*a*b + 3*a^2 - b^2)*1i)/(16*(a*b
^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))/((a^2*b^6 - 11*a^3*b^5 + 27*a^4*b^4 +
 15*a^5*b^3)/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 +
 15*a^4*b^2)) + (((tan(e + f*x)*(b^7 - 12*a*b^6 + 30*a^2*b^5 + 36*a^3*b^4 +
 73*a^4*b^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((32*a*b^
9 - 96*a^2*b^8 - 96*a^3*b^7 + 800*a^4*b^6 - 1440*a^5*b^5 + 1248*a^6*b^4 - 5
44*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 -
20*a^3*b^3 + 15*a^4*b^2)) - (tan(e + f*x)*(6*a*b + 3*a^2 - b^2)*(1280*a*b^8
 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4 + 12
80*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)*(a^
4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(6*a*b + 3*a^2 - b^2))/(16*(a*b^
2*3i - a^2*b*3i + a^3*1i - b^3*1i)))*(6*a*b + 3*a^2 - b^2))/(16*(a*b^2*3i -
 a^2*b*3i + a^3*1i - b^3*1i)) - (((tan(e + f*x)*(b^7 - 12*a*b^6 + 30*a^2*b^
5 + 36*a^3*b^4 + 73*a^4*b^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^
2)) - (((32*a*b^9 - 96*a^2*b^8 - 96*a^3*b^7 + 800*a^4*b^6 - 1440*a^5*b^5 +
1248*a^6*b^4 - 544*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6
 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (tan(e + f*x)*(6*a*b + 3*a^2 -
b^2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2
304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i - a^2*b*3i + a^3*
1i - b^3*1i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(6*a*b + 3*a^2 -
 b^2))/(16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))*(6*a*b + 3*a^2 - b^2))
/(16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i))))*(6*a*b + 3*a^2 - b^2)*1i)/(
8*f*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.63 $\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^2}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)}+\frac{x(a+b)}{2(a-b)^2}$$

```
[Out] 1/2*(a+b)*x/(a-b)^2-1/2*cos(f*x+e)*sin(f*x+e)/(a-b)/f-arctan(b^(1/2)*tan(f*
x+e)/a^(1/2))*a^(1/2)*b^(1/2)/(a-b)^2/f
```

Rubi [A] time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 471, 522, 203, 205}

$$-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^2}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)}+\frac{x(a+b)}{2(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a + b)*x)/(2*(a - b)^2) - (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a]])/((a - b)^2*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a - b)*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
```

```
&& IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f} + \frac{\text{Subst}\left(\int \frac{a-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f} - \frac{(ab)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{(a-b)^2f} + \frac{(a+b)\,\text{Subst}\left(\int \right.}{2}$$

$$= \frac{(a+b)x}{2(a-b)^2} - \frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^2f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f}$$

Mathematica [A] time = 0.16, size = 69, normalized size = 0.84

$$\frac{2(a+b)(e+fx)+(b-a)\sin(2(e+fx))-4\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{4f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (2*(a + b)*(e + f*x) - 4*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a]] + (-a + b)*Sin[2*(e + f*x)])/(4*(a - b)^2*f)
```

fricas [A] time = 0.48, size = 274, normalized size = 3.34

$$\left[\frac{2\,(a+b)fx-2\,(a-b)\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{-ab}\,\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2+4\left((a+b)\cos\left(f\right.\right.}{\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^4+2\left(ab-b^2\right)\cos}\right.}{4\left(a^2-2\,ab+b^2\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)*f*x - 2*(a - b)*cos(f*x + e)*sin(f*x + e) + sqrt(-a*b)*log(
((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*((
a + b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b)*sin(f*x + e) + b^2)/((a^
2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)))/((a
^2 - 2*a*b + b^2)*f), 1/2*((a + b)*f*x - (a - b)*cos(f*x + e)*sin(f*x + e)
+ sqrt(a*b)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*cos(f*x
+ e)*sin(f*x + e))))/((a^2 - 2*a*b + b^2)*f)]
```

giac [A] time = 2.88, size = 113, normalized size = 1.38

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)ab}{\left(a^2-2\,ab+b^2\right)\sqrt{ab}}-\frac{\left(fx+e\right)(a+b)}{a^2-2\,ab+b^2}+\frac{\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^2+1\right)(a-b)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*a*b/((a^2 - 2*a*b + b^2)*sqrt(a*b)) - (f*x + e)*(a + b)/(a^2 - 2*a*b
+ b^2) + tan(f*x + e)/((tan(f*x + e)^2 + 1)*(a - b)))/f
```

maple [A] time = 0.52, size = 137, normalized size = 1.67

$$-\frac{ba\arctan\left(\frac{\tan(fx+e)b}{\sqrt{ab}}\right)}{f(a-b)^2\sqrt{ab}}-\frac{\tan\left(fx+e\right)a}{2f(a-b)^2\left(1+\tan^2\left(fx+e\right)\right)}+\frac{\tan\left(fx+e\right)b}{2f(a-b)^2\left(1+\tan^2\left(fx+e\right)\right)}+\frac{\arctan\left(\tan\left(fx+e\right.\right.}{2f(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f*b*a/(a-b)^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/2/f/(a-b)^2
*tan(f*x+e)/(1+tan(f*x+e)^2)*a+1/2/f/(a-b)^2*tan(f*x+e)/(1+tan(f*x+e)^2)*b+
1/2/f/(a-b)^2*arctan(tan(f*x+e))*a+1/2/f/(a-b)^2*arctan(tan(f*x+e))*b
```

maxima [A] time = 0.96, size = 93, normalized size = 1.13

$$-\frac{\frac{2\,ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^2-2\,ab+b^2)\sqrt{ab}}-\frac{(fx+e)(a+b)}{a^2-2\,ab+b^2}+\frac{\tan(fx+e)}{(a-b)\tan(fx+e)^2+a-b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2 - 2*a*b + b^2)*sqrt(a*b)
) - (f*x + e)*(a + b)/(a^2 - 2*a*b + b^2) + tan(f*x + e)/((a - b)*tan(f*x +
 e)^2 + a - b))/f
```

mupad [B] time = 12.75, size = 190, normalized size = 2.32

$$\frac{b\,\sin\left(2\,e+2\,f\,x\right)-a\,\sin\left(2\,e+2\,f\,x\right)+2\,a\,\mathrm{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)+2\,b\,\mathrm{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)-\mathrm{atan}\left(\frac{b^3\,\sin(e+f\,x)\,\sqrt{-a\,b}\,1\mathrm{i}-}{\cos(e+f\,x)\,a}\right.}{4\,f\,a^2-8\,f\,a\,b+4\,f\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (b*sin(2*e + 2*f*x) - a*sin(2*e + 2*f*x) + 2*a*atan(sin(e + f*x)/cos(e + f*
x)) + 2*b*atan(sin(e + f*x)/cos(e + f*x)) - atan((b^3*sin(e + f*x)*(-a*b)^(
1/2)*1i - a*b^2*sin(e + f*x)*(-a*b)^(1/2)*2i + a^2*b*sin(e + f*x)*(-a*b)^(1
/2)*1i)/(a*b^3*cos(e + f*x) - 2*a^2*b^2*cos(e + f*x) + a^3*b*cos(e + f*x)))
*(-a*b)^(1/2)*4i)/(4*a^2*f + 4*b^2*f - 8*a*b*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.64 $\int \frac{1}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{a-b}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\,f(a-b)}$$

```
[Out] x/(a-b)-arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/(a-b)/f/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3660, 3675, 205}

$$\frac{x}{a-b}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\,f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-1),x]
```

```
[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a -
b)*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3660

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b\tan^2(e+fx)}\,dx &= \frac{x}{a-b}-\frac{b\int \frac{\sec^2(e+fx)}{a+b\tan^2(e+fx)}\,dx}{a-b}\\
&= \frac{x}{a-b}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\tan(e+fx)\right)}{(a-b)f}\\
&= \frac{x}{a-b}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)f}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 49, normalized size = 0.98

$$\frac{\tan^{-1}(\tan(e+fx))-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}}}{af-bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-1),x]
```

```
[Out] (ArcTan[Tan[e + f*x]] - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/Sq
rt[a])/(a*f - b*f)
```

fricas [A] time = 0.54, size = 182, normalized size = 3.64

$$\left[\frac{4\,fx-\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan\left(fx+e\right)^4-6\,ab\tan\left(fx+e\right)^2+a^2+4\left(ab\tan\left(fx+e\right)^3-a^2\tan\left(fx+e\right)\right)\sqrt{-\frac{b}{a}}}{b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2}\right)}{4\,(a-b)f},\ \frac{2\,fx-\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan\left(fx+e\right)^2-\right.}{2\,b\tan\left(fx\right.}\right.}{2\,(a-b)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x - sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a
^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e
)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a - b)*f), 1/2*(2*f*x - sqrt(b/a)*arc
tan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a - b)*f)]
```

giac [A] time = 1.95, size = 68, normalized size = 1.36

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,(a-b)}-\frac{fx+e}{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*
b/(sqrt(a*b)*(a - b)) - (f*x + e)/(a - b))/f
```

maple [A] time = 0.22, size = 52, normalized size = 1.04

$$-\frac{b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{f\,(a-b)\,\sqrt{ab}}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{f\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f*b/(a-b)/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+1/f/(a-b)*arctan(
tan(f*x+e))
```

maxima [A] time = 0.86, size = 48, normalized size = 0.96

$$-\frac{\frac{b\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a-b)}-\frac{fx+e}{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (f*x + e)/(a - b
))/f
```

mupad [B] time = 11.69, size = 948, normalized size = 18.96

$$-\frac{\operatorname{atan}\left(\frac{\frac{-4b^3\tan\left(e+fx\right)+\frac{\left(4b^4-8ab^3+4a^2b^2+\frac{\tan\left(e+fx\right)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1i}{2a-2b}\right)1i}{2a-2b}}{2a-2b}+\frac{-4b^3\tan\left(e+fx\right)+\frac{\left(8ab^3-4b^4-4a^2b^2+\frac{\tan\left(e+fx\right)\left(-8a^3b^2+8a^2b^3\cdots\right.}{2a-2b}\right.}{2a-2b}}{2a-2b}}{\frac{\left(-4b^3\tan\left(e+fx\right)+\frac{\left(4b^4-8ab^3+4a^2b^2+\frac{\tan\left(e+fx\right)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1i}{2a-2b}\right)1i}{2a-2b}\right)1i}{2a-2b}-\frac{\left(-4b^3\tan\left(e+fx\right)+\frac{\left(8ab^3-4b^4-4a^2b^2+\frac{\tan\left(e+fx\right)\left(-8a^3b^2+8a^2b^3\cdots\right.}{2a-2b}\right.}{2a-2b}\right.}{2a-2b}}\right)\cdots}{f\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (atan((((-a*b)^(1/2)*(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 +
 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^
3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))*1i)/(a*b - a^2) + ((-a*b)^(1/2)*
(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(4*a*b^3 - 2*b^4 - 2*a^2*b^2 + (tan(e +
 f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2
))))/(2*(a*b - a^2)))*1i)/(a*b - a^2))/(((-a*b)^(1/2)*(2*b^3*tan(e + f*x) -
 ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 + 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8
*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))
)/(a*b - a^2) - ((-a*b)^(1/2)*(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(4*a*b^3
- 2*b^4 - 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b
^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2))))/(a*b - a^2)))*(-a*b)^(
1/2)*1i)/(a*f*(a - b)) - atan(((((4*b^4 - 8*a*b^3 + 4*a^2*b^2 + (tan(e + f*
x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*
b) - 4*b^3*tan(e + f*x))/(2*a - 2*b) + (((8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (ta
n(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(
2*a - 2*b) - 4*b^3*tan(e + f*x))/(2*a - 2*b))/(((((4*b^4 - 8*a*b^3 + 4*a^2*
b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*
b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))*1i)/(2*a - 2*b) - ((((8*a*b^3 - 4
*b^4 - 4*a^2*b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*
1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))*1i)/(2*a - 2*b)))/(f
*(a - b))
```

sympy [A] time = 2.35, size = 280, normalized size = 5.60

$$\begin{cases} \frac{\tilde{\infty} x}{\tan^{2}{\left(e \right)}} & \text{for}\: a = 0 \wedge b = 0 \wedge f = 0 \\ \frac{- x - \frac{1}{f \tan{\left(e + f x \right)}}}{b} & \text{for}\: a = 0 \\ \frac{f x \tan^{2}{\left(e + f x \right)}}{2 b f \tan^{2}{\left(e + f x \right)} + 2 b f} + \frac{f x}{2 b f \tan^{2}{\left(e + f x \right)} + 2 b f} + \frac{\tan{\left(e + f x \right)}}{2 b f \tan^{2}{\left(e + f x \right)} + 2 b f} & \text{for}\: a = b \\ \frac{x}{a + b \tan^{2}{\left(e \right)}} & \text{for}\: f = 0 \\ \frac{x}{a} & \text{for}\: b = 0 \\ \frac{2 i \sqrt{a} f x \sqrt{\frac{1}{b}}}{2 i a^{\frac{3}{2}} f \sqrt{\frac{1}{b}} - 2 i \sqrt{a} b f \sqrt{\frac{1}{b}}} - \frac{\log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \tan{\left(e + f x \right)} \right)}}{2 i a^{\frac{3}{2}} f \sqrt{\frac{1}{b}} - 2 i \sqrt{a} b f \sqrt{\frac{1}{b}}} + \frac{\log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \tan{\left(e + f x \right)} \right)}}{2 i a^{\frac{3}{2}} f \sqrt{\frac{1}{b}} - 2 i \sqrt{a} b f \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x - 1/(f*ta
n(e + f*x)))/b, Eq(a, 0)), (f*x*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*
b*f) + f*x/(2*b*f*tan(e + f*x)**2 + 2*b*f) + tan(e + f*x)/(2*b*f*tan(e + f*
x)**2 + 2*b*f), Eq(a, b)), (x/(a + b*tan(e)**2), Eq(f, 0)), (x/a, Eq(b, 0))
, (2*I*sqrt(a)*f*x*sqrt(1/b)/(2*I*a**(3/2)*f*sqrt(1/b) - 2*I*sqrt(a)*b*f*sq
rt(1/b)) - log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*I*a**(3/2)*f*sqrt(1/
b) - 2*I*sqrt(a)*b*f*sqrt(1/b)) + log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(
2*I*a**(3/2)*f*sqrt(1/b) - 2*I*sqrt(a)*b*f*sqrt(1/b)), True))
```

3.65 $\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af}$$

```
[Out] -cot(f*x+e)/a/f-arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(3/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3663, 325, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(3/2)*f)) - Cot[e + f
*x]/(a*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)}\,dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{\cot(e+fx)}{af}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\tan(e+fx)\right)}{af}\\
&= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 48, normalized size = 1.00

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(3/2)*f)) - Cot[e + f
*x]/(a*f)
```

fricas [B] time = 0.47, size = 257, normalized size = 5.35

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6ab+b^2)\cos(fx+e)^4-2(3ab+b^2)\cos(fx+e)^2+4\left((a^2+ab)\cos(fx+e)^3-ab\cos(fx+e)\right)\sqrt{-\frac{b}{a}}\sin(fx+e)+b^2}{(a^2-2ab+b^2)\cos(fx+e)^4+2(ab-b^2)\cos(fx+e)^2+b^2}\right)\sin(fx+e)}{4af\sin(fx+e)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*
cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/
a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*
cos(f*x + e)^2 + b^2))*sin(f*x + e) - 4*cos(f*x + e))/(a*f*sin(f*x + e)), 1
/2*(sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x
+ e)*sin(f*x + e)))*sin(f*x + e) - 2*cos(f*x + e))/(a*f*sin(f*x + e))]
```

giac [A] time = 2.90, size = 62, normalized size = 1.29

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,a}+\frac{1}{a\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*
b/(sqrt(a*b)*a) + 1/(a*tan(f*x + e)))/f
```

maple [A] time = 0.58, size = 46, normalized size = 0.96

$$-\frac{b\arctan\left(\frac{\tan(fx+e)b}{\sqrt{ab}}\right)}{fa\sqrt{ab}}-\frac{1}{fa\tan(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f/a*b/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/f/a/tan(f*x+e)
```

maxima [A] time = 0.96, size = 42, normalized size = 0.88

$$-\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{1}{a\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a) + 1/(a*tan(f*x + e)))/f
```

mupad [B] time = 10.99, size = 40, normalized size = 0.83

$$-\frac{\cot\left(e+f\,x\right)}{a\,f}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)}{\sqrt{a}}\right)}{a^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)),x)
```

```
[Out] - cot(e + f*x)/(a*f) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x))/a^(1/2)))/(a^(3
/2)*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^2\left(e+fx\right)}{a+b\tan^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*tan(e + f*x)**2), x)
```

3.66 $\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{b}\,(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{(a-b)\cot(e+fx)}{a^2f}-\frac{\cot^3(e+fx)}{3af}$$

```
[Out] -(a-b)*cot(f*x+e)/a^2/f-1/3*cot(f*x+e)^3/a/f-(a-b)*arctan(b^(1/2)*tan(f*x+e
)/a^(1/2))*b^(1/2)/a^(5/2)/f
```

Rubi [A] time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3663, 453, 325, 205}

$$-\frac{\sqrt{b}\,(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{(a-b)\cot(e+fx)}{a^2f}-\frac{\cot^3(e+fx)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(5/2)*f)) - (
(a - b)*Cot[e + f*x])/(a^2*f) - Cot[e + f*x]^3/(3*a*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot^3(e+fx)}{3af} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{af}$$
$$= -\frac{(a-b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af} - \frac{((a-b)b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{a^2f}$$
$$= -\frac{(a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{(a-b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af}$$

Mathematica [A] time = 0.31, size = 73, normalized size = 0.96

$$\frac{3\sqrt{b}(b-a)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) - \sqrt{a}\cot(e+fx)\left(a\csc^2(e+fx)+2a-3b\right)}{3a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (3*Sqrt[b]*(-a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - Sqrt[a]*Cot[e
+ f*x]*(2*a - 3*b + a*Csc[e + f*x]^2))/(3*a^(5/2)*f)
```

fricas [B] time = 0.47, size = 373, normalized size = 4.91

$$\left[-\frac{4(2a-3b)\cos\left(fx+e\right)^3 + 3\left((a-b)\cos\left(fx+e\right)^2 - a + b\right)\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^2+6ab+b^2\right)\cos\left(fx+e\right)^4 - 2\left(3ab+b^2\right)\cos\left(fx+e\right)^2 -}{\left(a^2-2ab+b^2\right)\cos\left(fx+\right.}\right.}{12\left(a^2f\cos\left(fx+e\right)^2 - a^2f\right)\sin\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(2*a - 3*b)*cos(f*x + e)^3 + 3*((a - b)*cos(f*x + e)^2 - a + b)*s
qrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x
 + e)^2 - 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(
f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x
 + e)^2 + b^2))*sin(f*x + e) - 12*(a - b)*cos(f*x + e))/((a^2*f*cos(f*x + e
)^2 - a^2*f)*sin(f*x + e)), -1/6*(2*(2*a - 3*b)*cos(f*x + e)^3 - 3*((a - b)
*cos(f*x + e)^2 - a + b)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*
sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(a - b)*cos(f*x +
 e))/((a^2*f*cos(f*x + e)^2 - a^2*f)*sin(f*x + e))]
```

giac [A] time = 2.99, size = 97, normalized size = 1.28

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)\left(ab-b^2\right)}{\sqrt{ab}\,a^2} + \frac{3a\tan\left(fx+e\right)^2 - 3b\tan\left(fx+e\right)^2 + a}{a^2\tan\left(fx+e\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*(a*b - b^2)/(sqrt(a*b)*a^2) + (3*a*tan(f*x + e)^2 - 3*b*tan(f*x + e)^
2 + a)/(a^2*tan(f*x + e)^3))/f
```

maple [A] time = 0.58, size = 107, normalized size = 1.41

$$-\frac{b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{fa\sqrt{ab}}+\frac{b^{2}\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{f\,a^{2}\sqrt{ab}}-\frac{1}{3fa\tan\left(fx+e\right)^{3}}-\frac{1}{fa\tan\left(fx+e\right)}+\frac{b}{f\,a^{2}\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f/a*b/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+1/f*b^2/a^2/(a*b)^(1/
2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/3/f/a/tan(f*x+e)^3-1/f/a/tan(f*x+e)+1
/f/a^2/tan(f*x+e)*b
```

maxima [A] time = 0.88, size = 68, normalized size = 0.89

$$-\frac{\frac{3\left(ab-b^{2}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}+\frac{3\,(a-b)\tan\left(fx+e\right)^{2}+a}{a^{2}\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a*b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2) + (3*(
a - b)*tan(f*x + e)^2 + a)/(a^2*tan(f*x + e)^3))/f
```

mupad [B] time = 11.05, size = 67, normalized size = 0.88

$$-\frac{\frac{1}{3\,a}+\frac{\tan\left(e+f\,x\right)^{2}\,(a-b)}{a^{2}}}{f\tan\left(e+f\,x\right)^{3}}-\frac{\sqrt{b}\ \mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)}{\sqrt{a}}\right)\,(a-b)}{a^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)),x)
```

```
[Out] - (1/(3*a) + (tan(e + f*x)^2*(a - b))/a^2)/(f*tan(e + f*x)^3) - (b^(1/2)*at
an((b^(1/2)*tan(e + f*x))/a^(1/2))*(a - b))/(a^(5/2)*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{4}\left(e+fx\right)}{a+b\tan^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*tan(e + f*x)**2), x)
```

3.67 $\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{b}\,(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f}-\frac{(a-b)^2\cot(e+fx)}{a^3f}-\frac{(2a-b)\cot^3(e+fx)}{3a^2f}-\frac{\cot^5(e+fx)}{5af}$$

```
[Out] -(a-b)^2*cot(f*x+e)/a^3/f-1/3*(2*a-b)*cot(f*x+e)^3/a^2/f-1/5*cot(f*x+e)^5/a
/f-(a-b)^2*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(7/2)/f
```

Rubi [A] time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3663, 461, 205}

$$-\frac{\sqrt{b}\,(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f}-\frac{(2a-b)\cot^3(e+fx)}{3a^2f}-\frac{(a-b)^2\cot(e+fx)}{a^3f}-\frac{\cot^5(e+fx)}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*f)) -
 ((a - b)^2*Cot[e + f*x])/(a^3*f) - ((2*a - b)*Cot[e + f*x]^3)/(3*a^2*f) -
Cot[e + f*x]^5/(5*a*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^6} + \frac{2a-b}{a^2x^4} + \frac{(a-b)^2}{a^3x^2} - \frac{(a-b)^2b}{a^3(a+bx^2)}\right)dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a-b)^2\cot(e+fx)}{a^3f} - \frac{(2a-b)\cot^3(e+fx)}{3a^2f} - \frac{\cot^5(e+fx)}{5af} - \frac{\left((a-b)^2b\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{(a-b)^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f} - \frac{(a-b)^2\cot(e+fx)}{a^3f} - \frac{(2a-b)\cot^3(e+fx)}{3a^2f} -$$

Mathematica [A] time = 0.80, size = 103, normalized size = 0.98

$$\frac{-\sqrt{a}\cot(e+fx)\left(3a^2\csc^4(e+fx)+8a^2+a(4a-5b)\csc^2(e+fx)-25ab+15b^2\right)-15\sqrt{b}(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\,\text{t}}{}\right.}{15a^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-15*(a - b)^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - Sqrt[a]*Cot
[e + f*x]*(8*a^2 - 25*a*b + 15*b^2 + a*(4*a - 5*b)*Csc[e + f*x]^2 + 3*a^2*C
sc[e + f*x]^4))/(15*a^(7/2)*f)
```

fricas [B] time = 0.46, size = 543, normalized size = 5.17

$$\left[-\frac{4\left(8a^2-25ab+15b^2\right)\cos\left(fx+e\right)^5-20\left(4a^2-11ab+6b^2\right)\cos\left(fx+e\right)^3-15\left(\left(a^2-2ab+b^2\right)\cos\left(fx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(8*a^2 - 25*a*b + 15*b^2)*cos(f*x + e)^5 - 20*(4*a^2 - 11*a*b + 6
*b^2)*cos(f*x + e)^3 - 15*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 - 2*
a*b + b^2)*cos(f*x + e)^2 + a^2 - 2*a*b + b^2)*sqrt(-b/a)*log(((a^2 + 6*a*b
 + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b)*co
s(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a
*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x + e
) + 60*(a^2 - 2*a*b + b^2)*cos(f*x + e))/((a^3*f*cos(f*x + e)^4 - 2*a^3*f*c
os(f*x + e)^2 + a^3*f)*sin(f*x + e)), -1/30*(2*(8*a^2 - 25*a*b + 15*b^2)*co
s(f*x + e)^5 - 10*(4*a^2 - 11*a*b + 6*b^2)*cos(f*x + e)^3 - 15*((a^2 - 2*a*
b + b^2)*cos(f*x + e)^4 - 2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^2 + a^2 - 2*a*
b + b^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos
(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(a^2 - 2*a*b + b^2)*cos(f*x + e)
)/((a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f)*sin(f*x + e))]
```

giac [A] time = 1.90, size = 151, normalized size = 1.44

$$-\frac{\frac{15\left(a^2b-2ab^2+b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}\,a^3}+\frac{15a^2\tan(fx+e)^4-30ab\tan(fx+e)^4+15b^2\tan(fx+e)^4+10a^2\tan(fx+e)^2-5ab\,\text{t}}{a^3\tan(fx+e)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(a^2*b - 2*a*b^2 + b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + ar
ctan(b*tan(f*x + e)/sqrt(a*b)))/(sqrt(a*b)*a^3) + (15*a^2*tan(f*x + e)^4 -
30*a*b*tan(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 - 5*a
*b*tan(f*x + e)^2 + 3*a^2)/(a^3*tan(f*x + e)^5))/f
```

maple [B] time = 0.55, size = 191, normalized size = 1.82

$$-\frac{b\arctan\left(\frac{\tan(fx+e)b}{\sqrt{ab}}\right)}{fa\sqrt{ab}}+\frac{2b^2\arctan\left(\frac{\tan(fx+e)b}{\sqrt{ab}}\right)}{fa^2\sqrt{ab}}-\frac{b^3\arctan\left(\frac{\tan(fx+e)b}{\sqrt{ab}}\right)}{fa^3\sqrt{ab}}-\frac{1}{5fa\tan\left(fx+e\right)^5}-\frac{2}{3fa\tan\left(fx+e\right)^3}+\frac{}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f/a*b/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+2/f*b^2/a^2/(a*b)^(1/
2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/f*b^3/a^3/(a*b)^(1/2)*arctan(tan(f*x+
e)*b/(a*b)^(1/2))-1/5/f/a/tan(f*x+e)^5-2/3/f/a/tan(f*x+e)^3+1/3/f/a^2/tan(f
*x+e)^3*b-1/f/a/tan(f*x+e)+2/f/a^2/tan(f*x+e)*b-1/f/a^3/tan(f*x+e)*b^2
```

maxima [A] time = 0.80, size = 104, normalized size = 0.99

$$-\frac{\frac{15\left(a^2b-2\,ab^2+b^3\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}+\frac{15\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^4+5\left(2\,a^2-ab\right)\tan\left(fx+e\right)^2+3\,a^2}{a^3\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(a^2*b - 2*a*b^2 + b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*
b)*a^3) + (15*(a^2 - 2*a*b + b^2)*tan(f*x + e)^4 + 5*(2*a^2 - a*b)*tan(f*x
+ e)^2 + 3*a^2)/(a^3*tan(f*x + e)^5))/f
```

mupad [B] time = 11.39, size = 115, normalized size = 1.10

$$-\frac{\frac{1}{5\,a}+\frac{\tan\left(e+f\,x\right)^2\,(2\,a-b)}{3\,a^2}+\frac{\tan\left(e+f\,x\right)^4\,\left(a^2-2\,a\,b+b^2\right)}{a^3}}{f\,\tan\left(e+f\,x\right)^5}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)\,(a-b)^2}{\sqrt{a}\,\left(a^2-2\,a\,b+b^2\right)}\right)\,(a-b)^2}{a^{7/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)),x)
```

```
[Out] - (1/(5*a) + (tan(e + f*x)^2*(2*a - b))/(3*a^2) + (tan(e + f*x)^4*(a^2 - 2*
a*b + b^2))/a^3)/(f*tan(e + f*x)^5) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x)*(
a - b)^2)/(a^(1/2)*(a^2 - 2*a*b + b^2)))*(a - b)^2)/(a^(7/2)*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.68 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=204

$$-\frac{\left(5a^2+10ab-b^2\right)\cos(e+fx)}{5f(a-b)^4}-\frac{b\left(5a^2+2b^2\right)\sec(e+fx)}{10f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}+\frac{(10a-3b)\cos^3(e+fx)}{15f(a-b)^3}-\frac{\cos}{5f(a-b)\left(a+\right.}$$

```
[Out] -1/5*(5*a^2+10*a*b-b^2)*cos(f*x+e)/(a-b)^4/f+1/15*(10*a-3*b)*cos(f*x+e)^3/(
a-b)^3/f-1/5*cos(f*x+e)^5/(a-b)/f/(a-b+b*sec(f*x+e)^2)-1/10*b*(5*a^2+2*b^2)
*sec(f*x+e)/(a-b)^4/f/(a-b+b*sec(f*x+e)^2)-1/2*a*(3*a+4*b)*arctan(sec(f*x+e
)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/(a-b)^(9/2)/f
```

Rubi [A] time = 0.31, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 462, 456, 1261, 205}

$$-\frac{\left(5a^2+10ab-b^2\right)\cos(e+fx)}{5f(a-b)^4}-\frac{b\left(5a^2+2b^2\right)\sec(e+fx)}{10f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}+\frac{(10a-3b)\cos^3(e+fx)}{15f(a-b)^3}-\frac{\cos}{5f(a-b)\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(a*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*(a -
 b)^(9/2)*f) - ((5*a^2 + 10*a*b - b^2)*Cos[e + f*x])/(5*(a - b)^4*f) + ((10
*a - 3*b)*Cos[e + f*x]^3)/(15*(a - b)^3*f) - Cos[e + f*x]^5/(5*(a - b)*f*(a
 - b + b*Sec[e + f*x]^2)) - (b*(5*a^2 + 2*b^2)*Sec[e + f*x])/(10*(a - b)^4*
f*(a - b + b*Sec[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{-10a+3b+5(a-b)x^2}{x^4\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{5(a-b)f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\left(5a^2+2b^2\right)\sec(e+fx)}{10(a-b)^4 f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\,\text{Sub}}{}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\left(5a^2+2b^2\right)\sec(e+fx)}{10(a-b)^4 f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\,\text{Sub}}{}$$

$$= -\frac{\left(5a^2+10ab-b^2\right)\cos(e+fx)}{5(a-b)^4 f} + \frac{(10a-3b)\cos^3(e+fx)}{15(a-b)^3 f} - \frac{\cos^5(e+f}{5(a-b)f\left(a-b+b\,\text{s}\right.}$$

$$= -\frac{a\sqrt{b}(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{9/2}f} - \frac{\left(5a^2+10ab-b^2\right)\cos(e+fx)}{5(a-b)^4 f} + \frac{(10a-3b)}{15(a}$$

Mathematica [A] time = 3.83, size = 215, normalized size = 1.05

$$\frac{\frac{(a-b)(5(5a+3b)\cos(3(e+fx))+3(b-a)\cos(5(e+fx)))-30\cos(e+fx)\left(a^2\left(\frac{8b}{(a-b)\cos(2(e+fx))+a+b}+5\right)+18ab+b^2\right)}{(a-b)^4} + \frac{120a\sqrt{b}(3a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}\right.}{\sqrt{b}}\right.}{(a-b)^{9/2}}}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((120*a*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])
/Sqrt[b]])/(a - b)^(9/2) + (120*a*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] +
 Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(9/2) + (-30*Cos[e + f*x]*(18*
a*b + b^2 + a^2*(5 + (8*b)/(a + b + (a - b)*Cos[2*(e + f*x)]))) + (a - b)*(
5*(5*a + 3*b)*Cos[3*(e + f*x)] + 3*(-a + b)*Cos[5*(e + f*x)]))/(a - b)^4)/(
240*f)
```

fricas [A] time = 0.56, size = 593, normalized size = 2.91

$$\left[-\frac{12\left(a^3-3a^2b+3ab^2-b^3\right)\cos\left(fx+e\right)^7-4\left(10a^3-23a^2b+16ab^2-3b^3\right)\cos\left(fx+e\right)^5+20\left(3a^3+a^2b\right.}{60\left(\left(a^5-5a^4b+10a^3b^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(12*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 4*(10*a^3 - 23*
a^2*b + 16*a*b^2 - 3*b^3)*cos(f*x + e)^5 + 20*(3*a^3 + a^2*b - 4*a*b^2)*cos
(f*x + e)^3 - 15*(3*a^2*b + 4*a*b^2 + (3*a^3 + a^2*b - 4*a*b^2)*cos(f*x + e
)^2)*sqrt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a -
b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 30*(3*a^2*b + 4*a*b^2
)*cos(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*
f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f), -1/3
0*(6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 2*(10*a^3 - 23*a^2*b
+ 16*a*b^2 - 3*b^3)*cos(f*x + e)^5 + 10*(3*a^3 + a^2*b - 4*a*b^2)*cos(f*x +
 e)^3 + 15*(3*a^2*b + 4*a*b^2 + (3*a^3 + a^2*b - 4*a*b^2)*cos(f*x + e)^2)*s
qrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 15*(3*a^2*
b + 4*a*b^2)*cos(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*
b^4 - b^5)*f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^
5)*f)]
```

giac [B] time = 4.71, size = 561, normalized size = 2.75

$$-\frac{\frac{15\left(3a^2b+4ab^2\right)\arctan\left(-\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)-b}{\sqrt{ab-b^2}\cos\left(fx+e\right)+\sqrt{ab-b^2}}\right)}{\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\sqrt{ab-b^2}}+\frac{30\left(a^2b+\frac{a^2b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{2ab^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}\right)}{\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\left(a+\frac{2a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{4b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*(3*a^2*b + 4*a*b^2)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)
/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^4 - 4*a^3*b + 6*a^2*
b^2 - 4*a*b^3 + b^4)*sqrt(a*b - b^2)) + 30*(a^2*b + a^2*b*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) - 2*a*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/((a^4
 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2)) - 4*(8*a^2 + 34*a*b + 3*b^2 - 40*a^2*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) - 140*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 160*a*b*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2 + 30*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 - 180*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 30*a*b*(cos(f*x
 + e) - 1)^4/(cos(f*x + e) + 1)^4 + 15*b^2*(cos(f*x + e) - 1)^4/(cos(f*x +
e) + 1)^4)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*((cos(f*x + e) - 1)
/(cos(f*x + e) + 1) - 1)^5))/f
```

maple [B] time = 0.58, size = 388, normalized size = 1.90

$$-\frac{\left(\cos^5\left(fx+e\right)\right)a^2}{5f\left(a-b\right)^2\left(a^2-2ab+b^2\right)}+\frac{2\left(\cos^5\left(fx+e\right)\right)ab}{5f\left(a-b\right)^2\left(a^2-2ab+b^2\right)}-\frac{\left(\cos^5\left(fx+e\right)\right)b^2}{5f\left(a-b\right)^2\left(a^2-2ab+b^2\right)}+\frac{2\left(\cos^3\left(fx+e\right)\right)a^2}{3f\left(a-b\right)^2\left(a^2-2ab+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/5/f/(a-b)^2/(a^2-2*a*b+b^2)*cos(f*x+e)^5*a^2+2/5/f/(a-b)^2/(a^2-2*a*b+b^
2)*cos(f*x+e)^5*a*b-1/5/f/(a-b)^2/(a^2-2*a*b+b^2)*cos(f*x+e)^5*b^2+2/3/f/(a
-b)^2/(a^2-2*a*b+b^2)*cos(f*x+e)^3*a^2-2/3/f/(a-b)^2/(a^2-2*a*b+b^2)*cos(f*
x+e)^3*a*b-1/f/(a-b)^2/(a^2-2*a*b+b^2)*cos(f*x+e)*a^2-2/f/(a-b)^2/(a^2-2*a*
b+b^2)*cos(f*x+e)*a*b-1/2/f*a^2*b/(a-b)^4*cos(f*x+e)/(a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)+3/2/f*a^2*b/(a-b)^4/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a
-b)*b)^(1/2))+2/f*a*b^2/(a-b)^4/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a
-b)*b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 15.49, size = 1049, normalized size = 5.14

$$-\frac{\frac{16\,a^3+83\,a^2\,b+6\,a\,b^2}{15\,(a-b)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(32\,a^3-83\,a^2\,b+366\,a\,b^2\right)}{3\,(a-b)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(16\,a^3+223\,a^2\,b+1336\,a\,b^2\right)}{15\,(a-b)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}\,\left(6\,a^2\,b+11\right.}{(a-b)\,\left(a^3-3\,a^2\,b+3\,a\right.}}{f\,\left(a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{14}+(3\,a+4\,b)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{12}+(a+20\,b)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}+(40\,b-5\,a)\,\tan\left(\frac{e}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] - ((6*a*b^2 + 83*a^2*b + 16*a^3)/(15*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3
)) + (tan(e/2 + (f*x)/2)^8*(366*a*b^2 - 83*a^2*b + 32*a^3))/(3*(a - b)*(3*a
*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e/2 + (f*x)/2)^4*(1336*a*b^2 + 223*a^2*
b + 16*a^3))/(15*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (2*tan(e/2 + (f
*x)/2)^10*(11*a*b^2 + 6*a^2*b + 4*b^3))/((a - b)*(3*a*b^2 - 3*a^2*b + a^3 -
 b^3)) + (4*tan(e/2 + (f*x)/2)^6*(73*a*b^2 + 32*a^2*b - 12*a^3 + 12*b^3))/(
3*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (2*tan(e/2 + (f*x)/2)^2*(145*a
*b^2 + 134*a^2*b + 24*a^3 + 12*b^3))/(15*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 -
 b^3)) + (a*tan(e/2 + (f*x)/2)^12*(3*a*b + 4*b^2))/((a - b)*(3*a*b^2 - 3*a^
2*b + a^3 - b^3)))/(f*(a + tan(e/2 + (f*x)/2)^4*(a + 20*b) + tan(e/2 + (f*x
)/2)^10*(a + 20*b) + tan(e/2 + (f*x)/2)^2*(3*a + 4*b) + tan(e/2 + (f*x)/2)^
12*(3*a + 4*b) - tan(e/2 + (f*x)/2)^6*(5*a - 40*b) - tan(e/2 + (f*x)/2)^8*(
5*a - 40*b) + a*tan(e/2 + (f*x)/2)^14)) - (a*b^(1/2)*atan(((a - b)^9*(tan(e
/2 + (f*x)/2)^2*((b^(1/2)*(3*a + 4*b)*(24*a^12*b + 32*a^3*b^10 - 232*a^4*b^
9 + 704*a^5*b^8 - 1120*a^6*b^7 + 896*a^7*b^6 - 112*a^8*b^5 - 448*a^9*b^4 +
416*a^10*b^3 - 160*a^11*b^2))/(4*(a - b)^(17/2)) - (a*b^(1/2)*(a - 2*b)*(3*
a + 4*b)^2*(224*a^14*b - 16*a^15 + 32*a^2*b^13 - 400*a^3*b^12 + 2304*a^4*b^
11 - 8096*a^5*b^10 + 19360*a^6*b^9 - 33264*a^7*b^8 + 42240*a^8*b^7 - 40128*
a^9*b^6 + 28512*a^10*b^5 - 14960*a^11*b^4 + 5632*a^12*b^3 - 1440*a^13*b^2))
/(32*(a - b)^(27/2))) - (a*b^(1/2)*(a - 2*b)*(3*a + 4*b)^2*(16*a^15 - 192*a
^14*b + 16*a^3*b^12 - 192*a^4*b^11 + 1056*a^5*b^10 - 3520*a^6*b^9 + 7920*a^
7*b^8 - 12672*a^8*b^7 + 14784*a^9*b^6 - 12672*a^10*b^5 + 7920*a^11*b^4 - 35
20*a^12*b^3 + 1056*a^13*b^2))/(32*(a - b)^(27/2))))/(9*a^14*b + 16*a^4*b^11
 - 104*a^5*b^10 + 265*a^6*b^9 - 296*a^7*b^8 + 28*a^8*b^7 + 280*a^9*b^6 - 26
```

```
6*a^10*b^5 + 40*a^11*b^4 + 76*a^12*b^3 - 48*a^13*b^2))*(3*a + 4*b))/(2*f*(a
 - b)^(9/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.69 $\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=133

$$\frac{\cos^3(e+fx)}{3f(a-b)^2}-\frac{(a+b)\cos(e+fx)}{f(a-b)^3}-\frac{ab\sec(e+fx)}{2f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)}-\frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2f(a-b)^{7/2}}$$

```
[Out] -(a+b)*cos(f*x+e)/(a-b)^3/f+1/3*cos(f*x+e)^3/(a-b)^2/f-1/2*a*b*sec(f*x+e)/(
a-b)^3/f/(a-b+b*sec(f*x+e)^2)-1/2*(3*a+2*b)*arctan(sec(f*x+e)*b^(1/2)/(a-b)
^(1/2))*b^(1/2)/(a-b)^(7/2)/f
```

Rubi [A] time = 0.18, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3664, 456, 1261, 205}

$$\frac{\cos^3(e+fx)}{3f(a-b)^2}-\frac{(a+b)\cos(e+fx)}{f(a-b)^3}-\frac{ab\sec(e+fx)}{2f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)}-\frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2f(a-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*(a - b
)^(7/2)*f) - ((a + b)*Cos[e + f*x])/((a - b)^3*f) + Cos[e + f*x]^3/(3*(a -
b)^2*f) - (a*b*Sec[e + f*x])/(2*(a - b)^3*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{ab\sec(e+fx)}{2(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int \frac{\frac{2}{(a-b)b} - \frac{2ax^2}{(a-b)^2 b} + \frac{ax^4}{(a-b)^3}}{x^4\left(a-b+bx^2\right)}\,dx, x, \sec(e\right.}{2f}$$

$$= -\frac{ab\sec(e+fx)}{2(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int\left(\frac{2}{(a-b)^2 bx^4} + \frac{2(a+b)}{b(-a+b)^3x^2} + \frac{3}{(a-b)^3}\right.\right.}{2f}$$

$$= -\frac{(a+b)\cos(e+fx)}{(a-b)^3 f} + \frac{\cos^3(e+fx)}{3(a-b)^2 f} - \frac{ab\sec(e+fx)}{2(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)} - \frac{(b}{}$$

$$= -\frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{7/2}f} - \frac{(a+b)\cos(e+fx)}{(a-b)^3 f} + \frac{\cos^3(e+fx)}{3(a-b)^2 f} - \frac{}{2(a-}$$

Mathematica [A] time = 3.32, size = 182, normalized size = 1.37

$$\frac{-\frac{\cos(e+fx)\left(\frac{12ab}{(a-b)\cos(2(e+fx))+a+b}+9a+15b\right)+(b-a)\cos(3(e+fx))}{(a-b)^3} + \frac{6\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{7/2}} + \frac{6\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\right.}{}\right.}{(a-b)^{7/2}}}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((6*Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqr
t[b]])/(a - b)^(7/2) + (6*Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[a - b] + Sqrt[a]
*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(7/2) - (Cos[e + f*x]*(9*a + 15*b + (1
2*a*b)/(a + b + (a - b)*Cos[2*(e + f*x)])) + (-a + b)*Cos[3*(e + f*x)])/(a
- b)^3)/(12*f)
```

fricas [A] time = 0.57, size = 456, normalized size = 3.43

$$\left[\frac{4\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^5 - 4\left(3\,a^2-ab-2\,b^2\right)\cos\left(fx+e\right)^3 - 3\left(\left(3\,a^2-ab-2\,b^2\right)\cos\left(fx+e\right)^2 + 3\,a\right.}{12\left(\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)f\cos\left(fx+e\right)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 4*(3*a^2 - a*b - 2*b^2)*cos(f
*x + e)^3 - 3*((3*a^2 - a*b - 2*b^2)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a - b))*cos(f*
x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - 6*(3*a*b + 2*b^2)*cos(f*x + e))
/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*b - 3
*a^2*b^2 + 3*a*b^3 - b^4)*f), 1/6*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 2
*(3*a^2 - a*b - 2*b^2)*cos(f*x + e)^3 - 3*((3*a^2 - a*b - 2*b^2)*cos(f*x +
```

```
e)^2 + 3*a*b + 2*b^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f
*x + e)/b) - 3*(3*a*b + 2*b^2)*cos(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 -
4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f)]
```

giac [B] time = 2.53, size = 368, normalized size = 2.77

$$\frac{a^4 f^{11} \cos\left(fx+e\right)^3 - 4\,a^3 b f^{11} \cos\left(fx+e\right)^3 + 6\,a^2 b^2 f^{11} \cos\left(fx+e\right)^3 - 4\,a b^3 f^{11} \cos\left(fx+e\right)^3 + b^4 f^{11} \cos\left(fx+\right.}{3\left(a^6 f^{12} - 6\,a^5 b f^{12} + 15\,a^4 b^2 f^{12} - 20\,a^3 b^3 f^{12} +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(a^4*f^11*cos(f*x + e)^3 - 4*a^3*b*f^11*cos(f*x + e)^3 + 6*a^2*b^2*f^11
*cos(f*x + e)^3 - 4*a*b^3*f^11*cos(f*x + e)^3 + b^4*f^11*cos(f*x + e)^3 - 3
*a^4*f^11*cos(f*x + e) + 6*a^3*b*f^11*cos(f*x + e) - 6*a*b^3*f^11*cos(f*x +
 e) + 3*b^4*f^11*cos(f*x + e))/(a^6*f^12 - 6*a^5*b*f^12 + 15*a^4*b^2*f^12 -
 20*a^3*b^3*f^12 + 15*a^2*b^4*f^12 - 6*a*b^5*f^12 + b^6*f^12) - 1/2*a*b*cos
(f*x + e)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(a*cos(f*x + e)^2 - b*cos(f*x +
e)^2 + b)*f) + 1/2*(3*a*b + 2*b^2)*arctan((a*cos(f*x + e) - b*cos(f*x + e))
/sqrt(a*b - b^2))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b - b^2)*f)
```

maple [B] time = 0.56, size = 269, normalized size = 2.02

$$\frac{a\left(\cos^3\left(fx+e\right)\right)}{3f\,(a-b)\left(a^2-2ab+b^2\right)} - \frac{b\left(\cos^3\left(fx+e\right)\right)}{3f\,(a-b)\left(a^2-2ab+b^2\right)} - \frac{a\cos\left(fx+e\right)}{f\,(a-b)\left(a^2-2ab+b^2\right)} - \frac{\cos\left(fx+e\right)b}{f\,(a-b)\left(a^2-2ab+b^2\right)} - \frac{}{2f\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/3/f/(a-b)/(a^2-2*a*b+b^2)*a*cos(f*x+e)^3-1/3/f/(a-b)/(a^2-2*a*b+b^2)*b*co
s(f*x+e)^3-1/f/(a-b)/(a^2-2*a*b+b^2)*a*cos(f*x+e)-1/f/(a-b)/(a^2-2*a*b+b^2)
*cos(f*x+e)*b-1/2/f*b/(a-b)^3*a*cos(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)+3/2/f*b/(a-b)^3/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))*
a+1/f*b^2/(a-b)^3/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 14.72, size = 737, normalized size = 5.54

$$\sqrt{b}\,\mathrm{atan}\left(\frac{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{\sqrt{b}\,\left(3\,a+2\,b\right)\left(24\,a^9\,b-128\,a^8\,b^2+264\,a^7\,b^3-240\,a^6\,b^4+40\,a^5\,b^5+96\,a^4\,b^6-72\,a^3\,b^7+16\,a^2\,b^8\right)}{4\,a\,\left(a-b\right)^{13/2}}+\frac{\sqrt{b}\,\left(a-2\,b\right)\left(3\,a+2\,b\right)^2\left(16\,a^{12}-176\,a^{11}\,b+864\,a^{10}\right.}{}\right.\right.}{-9\,a^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (b^(1/2)*atan(((tan(e/2 + (f*x)/2)^2*((b^(1/2)*(3*a + 2*b)*(24*a^9*b + 16*a
^2*b^8 - 72*a^3*b^7 + 96*a^4*b^6 + 40*a^5*b^5 - 240*a^6*b^4 + 264*a^7*b^3 -
 128*a^8*b^2))/(4*a*(a - b)^(13/2)) + (b^(1/2)*(a - 2*b)*(3*a + 2*b)^2*(16*
a^12 - 176*a^11*b + 32*a^2*b^10 - 304*a^3*b^9 + 1296*a^4*b^8 - 3264*a^5*b^7
 + 5376*a^6*b^6 - 6048*a^7*b^5 + 4704*a^8*b^4 - 2496*a^9*b^3 + 864*a^10*b^2
))/(32*a*(a - b)^(21/2))) + (b^(1/2)*(a - 2*b)*(3*a + 2*b)^2*(144*a^11*b -
16*a^12 + 16*a^3*b^9 - 144*a^4*b^8 + 576*a^5*b^7 - 1344*a^6*b^6 + 2016*a^7*
b^5 - 2016*a^8*b^4 + 1344*a^9*b^3 - 576*a^10*b^2))/(32*a*(a - b)^(21/2)))*(
a - b)^7)/(12*a^3*b^8 - 4*a^2*b^9 - 9*a^10*b + 3*a^4*b^7 - 46*a^5*b^6 + 45*
a^6*b^5 + 24*a^7*b^4 - 67*a^8*b^3 + 42*a^9*b^2))*(3*a + 2*b))/(2*f*(a - b)^
(7/2)) - ((11*a*b + 4*a^2)/(3*(a - b)*(a^2 - 2*a*b + b^2)) + (tan(e/2 + (f*
x)/2)^8*(3*a*b + 2*b^2))/((a - b)*(a^2 - 2*a*b + b^2)) + (2*tan(e/2 + (f*x)
/2)^6*(2*a^2 - 3*a*b + 11*b^2))/((a - b)*(a^2 - 2*a*b + b^2)) + (2*tan(e/2
+ (f*x)/2)^2*(9*a*b + 2*a^2 + 19*b^2))/(3*(a - b)*(a^2 - 2*a*b + b^2)) + (2
*tan(e/2 + (f*x)/2)^4*(22*a*b - 10*a^2 + 33*b^2))/(3*(a - b)*(a^2 - 2*a*b +
 b^2)))/(f*(a + tan(e/2 + (f*x)/2)^2*(a + 4*b) + tan(e/2 + (f*x)/2)^8*(a +
4*b) - tan(e/2 + (f*x)/2)^4*(2*a - 12*b) - tan(e/2 + (f*x)/2)^6*(2*a - 12*b
) + a*tan(e/2 + (f*x)/2)^10))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.70 $\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{3\cos(e+fx)}{2f(a-b)^2}+\frac{\cos(e+fx)}{2f(a-b)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2f(a-b)^{5/2}}$$

```
[Out] -3/2*cos(f*x+e)/(a-b)^2/f+1/2*cos(f*x+e)/(a-b)/f/(a-b+b*sec(f*x+e)^2)-3/2*a
rctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/(a-b)^(5/2)/f
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3664, 290, 325, 205}

$$-\frac{3\cos(e+fx)}{2f(a-b)^2}+\frac{\cos(e+fx)}{2f(a-b)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*(a - b)^(5/2)*f)
 - (3*Cos[e + f*x])/(2*(a - b)^2*f) + Cos[e + f*x]/(2*(a - b)*f*(a - b + b*
Sec[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)}{2(a-b)f\left(a-b+b\sec^2(e+fx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)}\,dx, x, \sec(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{3\cos(e+fx)}{2(a-b)^2 f} + \frac{\cos(e+fx)}{2(a-b)f\left(a-b+b\sec^2(e+fx)\right)} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx,\right.}{2(a-b)^2 f}$$

$$= -\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{5/2} f} - \frac{3\cos(e+fx)}{2(a-b)^2 f} + \frac{\cos(e+fx)}{2(a-b)f\left(a-b+b\sec^2(e+fx)\right)}$$

Mathematica [**A**] time = 0.83, size = 146, normalized size = 1.45

$$\frac{\frac{2\cos(e+fx)\left(-\frac{b}{(a-b)\cos(2(e+fx))+a+b}-1\right)}{(a-b)^2} + \frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}} + \frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((3*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a -
b)^(5/2) + (3*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[
b]])/(a - b)^(5/2) + (2*Cos[e + f*x]*(-1 - b/(a + b + (a - b)*Cos[2*(e + f*
x)])))/(a - b)^2)/(2*f)
```

fricas [**A**] time = 0.48, size = 307, normalized size = 3.04

$$\left[-\frac{4(a-b)\cos\left(fx+e\right)^3 - 3\left((a-b)\cos\left(fx+e\right)^2+b\right)\sqrt{-\frac{b}{a-b}}\log\left(\frac{(a-b)\cos\left(fx+e\right)^2+2(a-b)\sqrt{-\frac{b}{a-b}}\cos\left(fx+e\right)-b}{(a-b)\cos\left(fx+e\right)^2+b}\right)+6}{4\left(\left(a^3-3a^2b+3ab^2-b^3\right)f\cos\left(fx+e\right)^2+\left(a^2b-2ab^2+b^3\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a - b)*cos(f*x + e)^3 - 3*((a - b)*cos(f*x + e)^2 + b)*sqrt(-b/(a
 - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e
) - b)/((a - b)*cos(f*x + e)^2 + b)) + 6*b*cos(f*x + e))/((a^3 - 3*a^2*b +
3*a*b^2 - b^3)*f*cos(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f), -1/2*(2*(a -
b)*cos(f*x + e)^3 + 3*((a - b)*cos(f*x + e)^2 + b)*sqrt(b/(a - b))*arctan(-
(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 3*b*cos(f*x + e))/((a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*f*cos(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f)]
```

giac [**A**] time = 2.89, size = 153, normalized size = 1.51

$$-\frac{f^3\cos\left(fx+e\right)}{a^2f^4-2abf^4+b^2f^4} + \frac{3b\arctan\left(\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)}{\sqrt{ab-b^2}}\right)}{2\left(a^2-2ab+b^2\right)\sqrt{ab-b^2}\,f} - \frac{b\cos\left(fx+e\right)}{2\left(a\cos\left(fx+e\right)^2-b\cos\left(fx+e\right)^2+b\right)\left(a^2-2ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -f^3*cos(f*x + e)/(a^2*f^4 - 2*a*b*f^4 + b^2*f^4) + 3/2*b*arctan((a*cos(f*x
 + e) - b*cos(f*x + e))/sqrt(a*b - b^2))/((a^2 - 2*a*b + b^2)*sqrt(a*b - b^
2)*f) - 1/2*b*cos(f*x + e)/((a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*(a^2
- 2*a*b + b^2)*f)
```

maple [A] time = 0.42, size = 114, normalized size = 1.13

$$-\frac{\cos\left(fx+e\right)}{f\left(a^2-2ab+b^2\right)}-\frac{b\cos\left(fx+e\right)}{2f\left(a-b\right)^2\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)}+\frac{3b\arctan\left(\frac{(a-b)\cos\left(fx+e\right)}{\sqrt{(a-b)b}}\right)}{2f\left(a-b\right)^2\sqrt{\left(a-b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/f/(a^2-2*a*b+b^2)*cos(f*x+e)-1/2/f*b/(a-b)^2*cos(f*x+e)/(a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)+3/2/f*b/(a-b)^2/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/(
(a-b)*b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 13.86, size = 436, normalized size = 4.32

$$-\frac{\frac{2\,a+b}{(a-b)^2}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(2\,a^2-a\,b+2\,b^2\right)}{a\,\left(a^2-2\,a\,b+b^2\right)}+\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(-2\,a^2+4\,a\,b+b^2\right)}{a\,(a-b)^2}}{f\,\left(a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+\left(4\,b-a\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\left(4\,b-a\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\right)}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{(a-b)^5\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{\sqrt{b}\,(18 \cdots}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] - ((2*a + b)/(a - b)^2 + (tan(e/2 + (f*x)/2)^4*(2*a^2 - a*b + 2*b^2))/(a*(a
^2 - 2*a*b + b^2)) + (2*tan(e/2 + (f*x)/2)^2*(4*a*b - 2*a^2 + b^2))/(a*(a -
 b)^2))/(f*(a - tan(e/2 + (f*x)/2)^2*(a - 4*b) - tan(e/2 + (f*x)/2)^4*(a -
4*b) + a*tan(e/2 + (f*x)/2)^6)) - (3*b^(1/2)*atan(((a - b)^5*(tan(e/2 + (f*
x)/2)^2*((b^(1/2)*(18*a^6*b + 18*a^2*b^5 - 72*a^3*b^4 + 108*a^4*b^3 - 72*a^
5*b^2))/(a*(a - b)^(9/2)) - (9*b^(1/2)*(a - 2*b)*(128*a^8*b - 16*a^9 + 32*a
^2*b^7 - 208*a^3*b^6 + 576*a^4*b^5 - 880*a^5*b^4 + 800*a^6*b^3 - 432*a^7*b^
2))/(32*a*(a - b)^(15/2))) - (9*b^(1/2)*(a - 2*b)*(16*a^9 - 96*a^8*b + 16*a
^3*b^6 - 96*a^4*b^5 + 240*a^5*b^4 - 320*a^6*b^3 + 240*a^7*b^2))/(32*a*(a -
b)^(15/2))))/(9*a^6*b + 9*a^2*b^5 - 36*a^3*b^4 + 54*a^4*b^3 - 36*a^5*b^2)))
/(2*f*(a - b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
[Out] Integral(sin(e + f*x)/(a + b*tan(e + f*x)**2)**2, x)
```

3.71 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{b}\,(3a-2b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^2f(a-b)^{3/2}}-\frac{\tanh^{-1}(\cos(e+fx))}{a^2f}-\frac{b\sec(e+fx)}{2af(a-b)\left(a+b\sec^2(e+fx)-b\right)}$$

```
[Out] -arctanh(cos(f*x+e))/a^2/f-1/2*b*sec(f*x+e)/a/(a-b)/f/(a-b+b*sec(f*x+e)^2)-
1/2*(3*a-2*b)*arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^2/(a-b)^(3/2
)/f
```

Rubi [A] time = 0.13, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3664, 414, 522, 207, 205}

$$-\frac{\sqrt{b}\,(3a-2b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^2f(a-b)^{3/2}}-\frac{\tanh^{-1}(\cos(e+fx))}{a^2f}-\frac{b\sec(e+fx)}{2af(a-b)\left(a+b\sec^2(e+fx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*a^2*(a
 - b)^(3/2)*f) - ArcTanh[Cos[e + f*x]]/(a^2*f) - (b*Sec[e + f*x])/(2*a*(a -
 b)*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
```

```
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{2a(a-b)f\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{\left(-1+x^2\right)\left(a-b+bx^2\right)}\,dx, x, \sec(e+$$

$$= -\frac{b\sec(e+fx)}{2a(a-b)f\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{a^2f} - \frac{((3$$

$$= -\frac{(3a-2b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^2(a-b)^{3/2}f} - \frac{\tanh^{-1}(\cos(e+fx))}{a^2f} - \frac{b\sec(e}{2a(a-b)f\left(a-b+\right.}$$

Mathematica [A] time = 0.90, size = 184, normalized size = 1.67

$$\frac{-\frac{2ab\cos(e+fx)}{(a-b)((a-b)\cos(2(e+fx))+a+b)} + \frac{\sqrt{b}(3a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{3/2}} + \frac{\sqrt{b}(3a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{3/2}} + 2\log\left(\sin\left(\right.\right.}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[
b]])/(a - b)^(3/2) + ((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan
[(e + f*x)/2])/Sqrt[b]])/(a - b)^(3/2) - (2*a*b*Cos[e + f*x])/((a - b)*(a +
 b + (a - b)*Cos[2*(e + f*x)])) - 2*Log[Cos[(e + f*x)/2]] + 2*Log[Sin[(e +
f*x)/2]])/(2*a^2*f)
```

fricas [B] time = 0.62, size = 470, normalized size = 4.27

$$\left[-\frac{2ab\cos\left(fx+e\right) - \left(\left(3a^2-5ab+2b^2\right)\cos\left(fx+e\right)^2 + 3ab-2b^2\right)\sqrt{-\frac{b}{a-b}}\log\left(\frac{(a-b)\cos\left(fx+e\right)^2+2(a-b)\sqrt{-\frac{b}{a-b}}\,\mathrm{c}}{(a-b)\cos\left(fx+e\right)^2+b}\right.}{4\left(\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*cos(f*x + e) - ((3*a^2 - 5*a*b + 2*b^2)*cos(f*x + e)^2 + 3*a*b
 - 2*b^2)*sqrt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/
(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*((a^2 - 2*a*b
+ b^2)*cos(f*x + e)^2 + a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) - 2*((a^2 -
2*a*b + b^2)*cos(f*x + e)^2 + a*b - b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^
4 - 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b - a^2*b^2)*f), -1/2*(a*b*c
os(f*x + e) + ((3*a^2 - 5*a*b + 2*b^2)*cos(f*x + e)^2 + 3*a*b - 2*b^2)*sqrt
```

```
(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + ((a^2 - 2*a*b
 + b^2)*cos(f*x + e)^2 + a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) - ((a^2 - 2
*a*b + b^2)*cos(f*x + e)^2 + a*b - b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^4
 - 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b - a^2*b^2)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(1/4/a^2*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1))))+(-3*
a*b+2*b^2)*1/4/(a^3-a^2*b)/sqrt(-b^2+a*b)*atan((-a*cos(f*x+exp(1))+b*cos(f*
x+exp(1))+b)/(sqrt(-b^2+a*b)*cos(f*x+exp(1))+sqrt(-b^2+a*b)))+((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1)))*a*b-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b
^2-a*b)/(2*a^3-2*a^2*b)/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-2*(1
-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1)))*b+a))
```

maple [A] time = 0.64, size = 179, normalized size = 1.63

$$-\frac{b\cos\left(fx+e\right)}{2fa\left(a-b\right)\left(a\left(\cos^{2}\left(fx+e\right)\right)-\left(\cos^{2}\left(fx+e\right)\right)b+b\right)}+\frac{3b\arctan\left(\frac{\left(a-b\right)\cos\left(fx+e\right)}{\sqrt{\left(a-b\right)b}}\right)}{2fa\left(a-b\right)\sqrt{\left(a-b\right)b}}-\frac{b^{2}\arctan\left(\frac{\left(a-b\right)\cos\left(fx+e\right)}{\sqrt{\left(a-b\right)b}}\right)}{f\,a^{2}\left(a-b\right)\sqrt{\left(a-b\right)b}}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b/a/(a-b)*cos(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+3/2/f*b/a/(a-
b)/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))-1/f*b^2/a^2/(a-
b)/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))+1/2/f/a^2*ln(-1
+cos(f*x+e))-1/2/f/a^2*ln(1+cos(f*x+e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 13.72, size = 1140, normalized size = 10.36

$$\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}-\frac{\frac{b}{a\,\left(a-b\right)}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(a\,b-2\,b^2\right)}{a^2\,\left(a-b\right)}}{f\,\left(a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+\left(4\,b-2\,a\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+a\right)}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{b^{3/2}\,{\left(3\,a-2\,b\right)}^3\,\left(2\,a^{10}-58\,\ldots\right)}{8\,a^6\,\ldots}\ldots\right)}{\ldots}\right)}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^2),x)
```

```
[Out] log(tan(e/2 + (f*x)/2))/(a^2*f) - (b/(a*(a - b)) - (tan(e/2 + (f*x)/2)^2*(a
*b - 2*b^2))/(a^2*(a - b)))/(f*(a - tan(e/2 + (f*x)/2)^2*(2*a - 4*b) + a*ta
n(e/2 + (f*x)/2)^4)) + (b^(1/2)*atan(((tan(e/2 + (f*x)/2)^2*((((b^(3/2)*(3*
a - 2*b)^3*(2*a^10 - 58*a^9*b + 96*a^4*b^6 - 432*a^5*b^5 + 772*a^6*b^4 - 68
6*a^7*b^3 + 306*a^8*b^2))/(8*a^6*(a - b)^(9/2)*(3*a^4*b - a^5 + a^2*b^3 - 3
*a^3*b^2)) + (2*b^(1/2)*(3*a - 2*b)*(108*a*b^5 + 9*a^5*b - 24*b^6 - 188*a^2
*b^4 + 158*a^3*b^3 - 63*a^4*b^2))/(a^2*(a - b)^(3/2)*(3*a^4*b - a^5 + a^2*b
^3 - 3*a^3*b^2)))*(768*a*b^4 - 259*a^4*b + 27*a^5 - 192*b^5 - 1164*a^2*b^3
+ 820*a^3*b^2))/(2*a^5*(a - b)^(9/2)*(36*a*b^2 - 39*a^2*b + 16*a^3 - 12*b^3
)) - (((8*(4*b^4 - 12*a*b^3 + 9*a^2*b^2))/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*
b^2) - (b*(3*a - 2*b)^2*(2*a^8 - 35*a^7*b + 96*a^2*b^6 - 432*a^3*b^5 + 746*
a^4*b^4 - 611*a^5*b^3 + 234*a^6*b^2))/(2*a^4*(a - b)^3*(3*a^4*b - a^5 + a^2
*b^3 - 3*a^3*b^2)))*(2*a^4 - 47*a^3*b - 240*a*b^3 + 96*b^4 + 186*a^2*b^2))/
(a^5*b^(1/2)*(a - b)^3*(36*a*b^2 - 39*a^2*b + 16*a^3 - 12*b^3))) + (((b^(1/
2)*(3*a - 2*b)*(12*a^5*b - 20*a^2*b^4 + 60*a^3*b^3 - 53*a^4*b^2))/(a^2*(a -
 b)^(3/2)*(a^5 - 2*a^4*b + a^3*b^2)) + (b^(3/2)*(3*a - 2*b)^3*(4*a^10 - 24*
a^9*b + 16*a^6*b^4 - 48*a^7*b^3 + 52*a^8*b^2))/(16*a^6*(a - b)^(9/2)*(a^5 -
 2*a^4*b + a^3*b^2)))*(768*a*b^4 - 259*a^4*b + 27*a^5 - 192*b^5 - 1164*a^2*
b^3 + 820*a^3*b^2))/(2*a^5*(a - b)^(9/2)*(36*a*b^2 - 39*a^2*b + 16*a^3 - 12
*b^3)) - (((4*(4*b^4 - 12*a*b^3 + 9*a^2*b^2))/(a^5 - 2*a^4*b + a^3*b^2) - (
b*(3*a - 2*b)^2*(4*a^8 - 36*a^7*b + 32*a^4*b^4 - 96*a^5*b^3 + 96*a^6*b^2))/
(4*a^4*(a - b)^3*(a^5 - 2*a^4*b + a^3*b^2)))*(2*a^4 - 47*a^3*b - 240*a*b^3
+ 96*b^4 + 186*a^2*b^2))/(a^5*b^(1/2)*(a - b)^3*(36*a*b^2 - 39*a^2*b + 16*a
^3 - 12*b^3)))*(4*a^7*(a - b)^(9/2) - 12*a^6*b*(a - b)^(9/2) - 4*a^4*b^3*(a
 - b)^(9/2) + 12*a^5*b^2*(a - b)^(9/2)))/(9*a^2*b - 12*a*b^2 + 4*b^3))*(3*a
 - 2*b))/(2*a^2*f*(a - b)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.72 $\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{(a-4b)\tanh^{-1}(\cos(e+fx))}{2a^3f}-\frac{\sqrt{b}\,(3a-4b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^3f\sqrt{a-b}}-\frac{b\sec(e+fx)}{a^2f\left(a+b\sec^2(e+fx)-b\right)}-\frac{\cot(e+fx)\,\mathrm{cs}}{2af\left(a+b\sec^2(e\right.}$$

```
[Out] -1/2*(a-4*b)*arctanh(cos(f*x+e))/a^3/f-1/2*cot(f*x+e)*csc(f*x+e)/a/f/(a-b+b
*sec(f*x+e)^2)-b*sec(f*x+e)/a^2/f/(a-b+b*sec(f*x+e)^2)-1/2*(3*a-4*b)*arctan
(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^3/f/(a-b)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3664, 471, 527, 522, 207, 205}

$$-\frac{b\sec(e+fx)}{a^2f\left(a+b\sec^2(e+fx)-b\right)}-\frac{(a-4b)\tanh^{-1}(\cos(e+fx))}{2a^3f}-\frac{\sqrt{b}\,(3a-4b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^3f\sqrt{a-b}}-\frac{\cot(e+fx)\,\mathrm{cs}}{2af\left(a+b\sec^2(e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*a^3*Sq
rt[a - b]*f) - ((a - 4*b)*ArcTanh[Cos[e + f*x]])/(2*a^3*f) - (Cot[e + f*x]*
Csc[e + f*x])/(2*a*f*(a - b + b*Sec[e + f*x]^2)) - (b*Sec[e + f*x])/(a^2*f*
(a - b + b*Sec[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{a-b-3bx^2}{\left(-1+x^2\right)\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\sec(e+fx)}{a^2f\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2(a-2b}{(-1+}\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\sec(e+fx)}{a^2f\left(a-b+b\sec^2(e+fx)\right)} + \frac{(a-4b)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{(3a-4b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^3\sqrt{a-b}\,f} - \frac{(a-4b)\tanh^{-1}(\cos(e+fx))}{2a^3f} - \frac{\cot(e+}{2af\left(a-b\right.}$$

Mathematica [B] time = 6.31, size = 325, normalized size = 2.21

$$\frac{(a-4b)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2a^3f} + \frac{(4b-a)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2a^3f} - \frac{\sqrt{b}(3a-4b)\sqrt{a-b}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{a-b}\cos\right.}{}\right.}{2a^3f(b-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*((3*a - 4*b)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]
*Cos[(e + f*x)/2] - Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(a^3*(-a + b)*f) -
 ((3*a - 4*b)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos
[(e + f*x)/2] + Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(2*a^3*(-a + b)*f) - (
b*Cos[e + f*x])/(a^2*f*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])) -
 Csc[(e + f*x)/2]^2/(8*a^2*f) + ((-a + 4*b)*Log[Cos[(e + f*x)/2]])/(2*a^3*f
) + ((a - 4*b)*Log[Sin[(e + f*x)/2]])/(2*a^3*f) + Sec[(e + f*x)/2]^2/(8*a^2
*f)
```

fricas [B] time = 0.56, size = 672, normalized size = 4.57

$$\left[\frac{2\left(a^2-2\,ab\right)\cos\left(fx+e\right)^3+4\,ab\cos\left(fx+e\right)-\left(\left(3\,a^2-7\,ab+4\,b^2\right)\cos\left(fx+e\right)^4-\left(3\,a^2-10\,ab+8\,b^2\right)\cos\left(f\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2 - 2*a*b)*cos(f*x + e)^3 + 4*a*b*cos(f*x + e) - ((3*a^2 - 7*a*b
 + 4*b^2)*cos(f*x + e)^4 - (3*a^2 - 10*a*b + 8*b^2)*cos(f*x + e)^2 - 3*a*b
+ 4*b^2)*sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/
(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - ((a^2 - 5*a*b +
4*b^2)*cos(f*x + e)^4 - (a^2 - 6*a*b + 8*b^2)*cos(f*x + e)^2 - a*b + 4*b^2)
*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 5*a*b + 4*b^2)*cos(f*x + e)^4 - (a^2
 - 6*a*b + 8*b^2)*cos(f*x + e)^2 - a*b + 4*b^2)*log(-1/2*cos(f*x + e) + 1/2
))/((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x +
e)^2), 1/4*(2*(a^2 - 2*a*b)*cos(f*x + e)^3 + 4*a*b*cos(f*x + e) - 2*((3*a^2
 - 7*a*b + 4*b^2)*cos(f*x + e)^4 - (3*a^2 - 10*a*b + 8*b^2)*cos(f*x + e)^2
- 3*a*b + 4*b^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x +
e)/b) - ((a^2 - 5*a*b + 4*b^2)*cos(f*x + e)^4 - (a^2 - 6*a*b + 8*b^2)*cos(f
*x + e)^2 - a*b + 4*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 5*a*b + 4*b^
2)*cos(f*x + e)^4 - (a^2 - 6*a*b + 8*b^2)*cos(f*x + e)^2 - a*b + 4*b^2)*log
(-1/2*cos(f*x + e) + 1/2))/((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4
 - 2*a^3*b)*f*cos(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*1/16/a^2+(-2*((1-cos(f
*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^2+8*((1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1))))^3*a*b+((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2-16*((1-cos(f*
x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1)))*a^2-28*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a*b-3*a^2)*1/48/a^3/(((
1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a-2*((1-cos(f*x+exp(1)))/(1+cos(f
*x+exp(1))))^2*a+4*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b+(1-cos(f*x
+exp(1)))/(1+cos(f*x+exp(1)))*a)+(a-4*b)*1/8/a^3*ln(abs(1-cos(f*x+exp(1)))/
abs(1+cos(f*x+exp(1))))+(-3*a*b+4*b^2)*1/4/a^3/sqrt(-b^2+a*b)*atan((-a*cos(
f*x+exp(1))+b*cos(f*x+exp(1))+b)/(sqrt(-b^2+a*b)*cos(f*x+exp(1))+sqrt(-b^2+
a*b))))
```

maple [A] time = 0.65, size = 229, normalized size = 1.56

$$-\frac{b\cos\left(fx+e\right)}{2f\,a^2\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)}+\frac{3b\arctan\left(\frac{(a-b)\cos\left(fx+e\right)}{\sqrt{(a-b)b}}\right)}{2f\,a^2\sqrt{(a-b)\,b}}-\frac{2b^2\arctan\left(\frac{(a-b)\cos\left(fx+e\right)}{\sqrt{(a-b)b}}\right)}{f\,a^3\sqrt{(a-b)\,b}}+\frac{}{4f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b/a^2*cos(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+3/2/f*b/a^2/((a-b
)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))-2/f*b^2/a^3/((a-b)*b)^(
1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))+1/4/f/a^2/(-1+cos(f*x+e))+1/4
/f/a^2*ln(-1+cos(f*x+e))-1/f/a^3*ln(-1+cos(f*x+e))*b+1/4/f/a^2/(1+cos(f*x+e
))-1/4/f/a^2*ln(1+cos(f*x+e))+1/f/a^3*ln(1+cos(f*x+e))*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 12.37, size = 917, normalized size = 6.24

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{8\,a^2\,f}-\frac{\frac{a}{2}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(a-6\,b)+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(a^2-8\,a\,b+16\,b^2\right)}{2\,a}}{f\,\left(4\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+4\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(16\,a^2\,b-8\,a^3\right)\right)}+\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right.\right.}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^2),x)
```

```
[Out] tan(e/2 + (f*x)/2)^2/(8*a^2*f) - (a/2 - tan(e/2 + (f*x)/2)^2*(a - 6*b) + (t
an(e/2 + (f*x)/2)^4*(a^2 - 8*a*b + 16*b^2))/(2*a))/(f*(4*a^3*tan(e/2 + (f*x
)/2)^2 + 4*a^3*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^4*(16*a^2*b - 8*a^
3))) + (log(tan(e/2 + (f*x)/2))*(a - 4*b))/(2*a^3*f) + (b^(1/2)*atan((2*a^2
*((b^(1/2)*(3*a - 4*b)*(12*a^6*b - 160*a^3*b^4 + 240*a^4*b^3 - 106*a^5*b^2)
)/(2*a^9*(a - b)^(1/2)) + (b^(3/2)*(3*a - 4*b)^3*(8*a^11 - 32*a^10*b + 32*a
^9*b^2))/(32*a^15*(a - b)^(3/2)))*(a - b)*(15*a^4 - 182*a^3*b - 864*a*b^3 +
 384*b^4 + 648*a^2*b^2))/((9*a^2*b - 24*a*b^2 + 16*b^3)*(72*a*b^2 - 27*a^2*
b + 4*a^3 - 48*b^3)) - (4*a^7*tan(e/2 + (f*x)/2)^2*(a - b)^(3/2)*((((4*(16*
b^4 - 24*a*b^3 + 9*a^2*b^2))/a^5 - (b*(3*a - 4*b)^2*(2*a^8 - 46*a^7*b + 384
*a^4*b^4 - 672*a^5*b^3 + 344*a^6*b^2))/(4*a^11*(a - b)))*(a^4 - 31*a^3*b -
336*a*b^3 + 192*b^4 + 180*a^2*b^2))/(b^(1/2)*(b*(27*a^7 + b*(48*a^5*b - 72*
a^6)) - 4*a^8)) + (((b^(1/2)*(3*a - 4*b)*(192*a*b^5 + 9*a^5*b - 384*a^2*b^4
 + 268*a^3*b^3 - 78*a^4*b^2))/(a^8*(a - b)^(1/2)) - (b^(3/2)*(3*a - 4*b)^3*
(104*a^9*b - 4*a^10 + 192*a^7*b^3 - 288*a^8*b^2))/(16*a^14*(a - b)^(3/2)))*
(15*a^4 - 182*a^3*b - 864*a*b^3 + 384*b^4 + 648*a^2*b^2))/(2*a^5*(a - b)^(1
/2)*(72*a*b^2 - 27*a^2*b + 4*a^3 - 48*b^3))))/(9*a^2*b - 24*a*b^2 + 16*b^3)
 + (4*a^7*(a - b)^(3/2)*((2*(112*a*b^4 - 64*b^5 - 60*a^2*b^3 + 9*a^3*b^2))/
a^6 + (b*(3*a - 4*b)^2*(56*a^8*b - 4*a^9 + 128*a^6*b^3 - 160*a^7*b^2))/(8*a
^12*(a - b)))*(a^4 - 31*a^3*b - 336*a*b^3 + 192*b^4 + 180*a^2*b^2))/(b^(1/2
)*(b*(27*a^7 + b*(48*a^5*b - 72*a^6)) - 4*a^8)*(9*a^2*b - 24*a*b^2 + 16*b^3
)))*(3*a - 4*b))/(2*a^3*f*(a - b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.73 $\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=210

$$-\frac{3\sqrt{b}\,(a-2b)\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^4f}-\frac{3b(3a-4b)\sec(e+fx)}{8a^3f\left(a+b\sec^2(e+fx)-b\right)}-\frac{(5a-6b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a+b\sec^2(e+fx)-b\right)}-\cdots$$

```
[Out] -3/8*(a^2-8*a*b+8*b^2)*arctanh(cos(f*x+e))/a^4/f-1/8*(5*a-6*b)*cot(f*x+e)*c
sc(f*x+e)/a^2/f/(a-b+b*sec(f*x+e)^2)-1/4*cot(f*x+e)^3*csc(f*x+e)/a/f/(a-b+b
*sec(f*x+e)^2)-3/8*(3*a-4*b)*b*sec(f*x+e)/a^3/f/(a-b+b*sec(f*x+e)^2)-3/2*(a
-2*b)*arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)*b^(1/2)/a^4/f
```

Rubi [A] time = 0.28, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3664, 470, 527, 522, 207, 205}

$$-\frac{3\left(a^2-8ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8a^4f}-\frac{3b(3a-4b)\sec(e+fx)}{8a^3f\left(a+b\sec^2(e+fx)-b\right)}-\frac{3\sqrt{b}\,(a-2b)\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^4f}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*(a - 2*b)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]
])/(2*a^4*f) - (3*(a^2 - 8*a*b + 8*b^2)*ArcTanh[Cos[e + f*x]])/(8*a^4*f) -
((5*a - 6*b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f*(a - b + b*Sec[e + f*x]^2)
) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*a*f*(a - b + b*Sec[e + f*x]^2)) - (3*(
3*a - 4*b)*b*Sec[e + f*x])/(8*a^3*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-a+b+(-4a+5b)x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{4af}$$

$$= -\frac{(5a-6b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-3(a\cdots}{(\cdots}\right.}{\cdots}$$

$$= -\frac{(5a-6b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)} - \frac{3(3a-4b)\cdots}{8a^3f\left(a-b+\cdots\right.}$$

$$= -\frac{(5a-6b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)} - \frac{3(3a-4b)\cdots}{8a^3f\left(a-b+\cdots\right.}$$

$$= -\frac{3(a-2b)\sqrt{a-b}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^4f} - \frac{3\left(a^2-8ab+8b^2\right)\tanh^{-1}(\cos(e+fx)\cdots}{8a^4f}$$

Mathematica [A] time = 6.36, size = 392, normalized size = 1.87

$$\frac{3\sqrt{b}(a-2b)\sqrt{a-b}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{a-b}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{a}\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{b}}\right)}{2a^4f} + \frac{3\sqrt{b}(a-2b)\sqrt{a-b}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\cdots\right.}{\cdots}\right.}{2a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (3*(a - 2*b)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[
(e + f*x)/2] - Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(2*a^4*f) + (3*(a - 2*b
```

```
)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/2
] + Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(2*a^4*f) + (-(a*b*Cos[e + f*x]) +
 b^2*Cos[e + f*x])/(a^3*f*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])
) + ((-3*a + 8*b)*Csc[(e + f*x)/2]^2)/(32*a^3*f) - Csc[(e + f*x)/2]^4/(64*a
^2*f) - (3*(a^2 - 8*a*b + 8*b^2)*Log[Cos[(e + f*x)/2]])/(8*a^4*f) + (3*(a^2
 - 8*a*b + 8*b^2)*Log[Sin[(e + f*x)/2]])/(8*a^4*f) + ((3*a - 8*b)*Sec[(e +
f*x)/2]^2)/(32*a^3*f) + Sec[(e + f*x)/2]^4/(64*a^2*f)
```

fricas [B] time = 0.59, size = 1052, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(a^3 - 5*a^2*b + 4*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 24*a^2*b + 2
4*a*b^2)*cos(f*x + e)^3 - 12*((a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^6 - (2*a^2
 - 7*a*b + 6*b^2)*cos(f*x + e)^4 + (a^2 - 5*a*b + 6*b^2)*cos(f*x + e)^2 + a
*b - 2*b^2)*sqrt(-a*b + b^2)*log(((a - b)*cos(f*x + e)^2 - 2*sqrt(-a*b + b^
2)*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - 6*(3*a^2*b - 4*a*b^2)*
cos(f*x + e) - 3*((a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 - (2*a^
3 - 19*a^2*b + 40*a*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 + 8*b^3
+ (a^3 - 11*a^2*b + 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e)
 + 1/2) + 3*((a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 - (2*a^3 - 1
9*a^2*b + 40*a*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 + 8*b^3 + (a^
3 - 11*a^2*b + 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1
/2))/((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*
x + e)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2), 1/16*(6*(a^3 - 5*a^2*b + 4*a*
b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 24*a^2*b + 24*a*b^2)*cos(f*x + e)^3 + 24*(
(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^6 - (2*a^2 - 7*a*b + 6*b^2)*cos(f*x + e)
^4 + (a^2 - 5*a*b + 6*b^2)*cos(f*x + e)^2 + a*b - 2*b^2)*sqrt(a*b - b^2)*ar
ctan(sqrt(a*b - b^2)*cos(f*x + e)/b) - 6*(3*a^2*b - 4*a*b^2)*cos(f*x + e) -
 3*((a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 - (2*a^3 - 19*a^2*b +
 40*a*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 + 8*b^3 + (a^3 - 11*a^
2*b + 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((
a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 - (2*a^3 - 19*a^2*b + 40*a
*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 + 8*b^3 + (a^3 - 11*a^2*b +
 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^5 - a
^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*x + e)^4 + (a^
5 - 3*a^4*b)*f*cos(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2*b-3*(1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1)))*a*b^2+2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*
b^3-a^2*b+a*b^2)*1/2/a^4/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-2*(
1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+e
xp(1)))*b+a)+(-18*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2+144*((1-c
os(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a*b-144*((1-cos(f*x+exp(1)))/(1+cos(
f*x+exp(1))))^2*b^2-8*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2+16*(1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a*b-a^2)*1/128/a^4/((1-cos(f*x+exp(1)))/(
1+cos(f*x+exp(1))))^2+(32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2+2
56*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2-512*(1-cos(f*x+exp(1)))/(1+c
```

```
os(f*x+exp(1)))*a*b)*1/4096/a^4+(-3*a^2*b+9*a*b^2-6*b^3)*1/4/a^4/sqrt(-b^2+
a*b)*atan((-a*cos(f*x+exp(1))+b*cos(f*x+exp(1))+b)/(sqrt(-b^2+a*b)*cos(f*x+
exp(1))+sqrt(-b^2+a*b)))+(3*a^2-24*a*b+24*b^2)*1/32/a^4*ln(abs(1-cos(f*x+ex
p(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [B] time = 0.57, size = 428, normalized size = 2.04

$$-\frac{b\cos\left(fx+e\right)}{2f\,a^{2}\left(a\left(\cos^{2}\left(fx+e\right)\right)-\left(\cos^{2}\left(fx+e\right)\right)b+b\right)}+\frac{b^{2}\cos\left(fx+e\right)}{2f\,a^{3}\left(a\left(\cos^{2}\left(fx+e\right)\right)-\left(\cos^{2}\left(fx+e\right)\right)b+b\right)}+\frac{3b\arctan(}{2f\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b/a^2*cos(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+1/2/f*b^2/a^3*cos
(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+3/2/f*b/a^2/((a-b)*b)^(1/2)*arcta
n((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))-9/2/f*b^2/a^3/((a-b)*b)^(1/2)*arctan((a
-b)*cos(f*x+e)/((a-b)*b)^(1/2))+3/f*b^3/a^4/((a-b)*b)^(1/2)*arctan((a-b)*co
s(f*x+e)/((a-b)*b)^(1/2))-1/16/f/a^2/(-1+cos(f*x+e))^2+3/16/f/a^2/(-1+cos(f
*x+e))-1/2/f/a^3/(-1+cos(f*x+e))*b+3/16/f/a^2*ln(-1+cos(f*x+e))-3/2/f/a^3*l
n(-1+cos(f*x+e))*b+3/2/f/a^4*ln(-1+cos(f*x+e))*b^2+1/16/f/a^2/(1+cos(f*x+e)
)^2+3/16/f/a^2/(1+cos(f*x+e))-1/2/f/a^3/(1+cos(f*x+e))*b-3/16/f/a^2*ln(1+co
s(f*x+e))+3/2/f/a^3*ln(1+cos(f*x+e))*b-3/2/f/a^4*ln(1+cos(f*x+e))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 12.31, size = 1113, normalized size = 5.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^2),x)
```

```
[Out] tan(e/2 + (f*x)/2)^4/(64*a^2*f) - (a^2/4 - tan(e/2 + (f*x)/2)^4*((15*a^2)/4
 - 32*a*b + 32*b^2) + (3*a*tan(e/2 + (f*x)/2)^2*(a - 2*b))/2 + (2*tan(e/2 +
 (f*x)/2)^6*(24*a*b^2 - 10*a^2*b + a^3 - 16*b^3))/a)/(f*(16*a^4*tan(e/2 + (
f*x)/2)^4 + 16*a^4*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^6*(64*a^3*b -
32*a^4))) + (tan(e/2 + (f*x)/2)^2*(a - 2*b))/(8*a^3*f) + (log(tan(e/2 + (f*
x)/2))*(3*a^2 - 24*a*b + 24*b^2))/(8*a^4*f) + (3*atan((8*a^10*tan(e/2 + (f*
x)/2)^2*((((756*a*b^6 - 216*b^7 - 1026*a^2*b^5 + 675*a^3*b^4 - 216*a^4*b^3
+ 27*a^5*b^2)/a^8 + (9*(a - 2*b)^2*(a*b - b^2)*(180*a^10*b - 6*a^11 + 2304*
a^6*b^5 - 5760*a^7*b^4 + 4944*a^8*b^3 - 1656*a^9*b^2))/(16*a^16))*(960*a*b^
4 - 38*a^4*b + a^5 - 384*b^5 - 840*a^2*b^3 + 300*a^3*b^2))/(2*a^5*(b*(a - b
))^(3/2)*(a^4 - 12*a^3*b - 96*a*b^3 + 48*b^4 + 60*a^2*b^2)) + (((27*(a - 2*
b)^3*(a*b - b^2)^(3/2)*(416*a^12*b - 16*a^13 + 768*a^10*b^3 - 1152*a^11*b^2
))/(64*a^20) - (3*(a - 2*b)*(a*b - b^2)^(1/2)*(27*a^8*b + 1728*a^2*b^7 - 60
48*a^3*b^6 + 8352*a^4*b^5 - 5760*a^5*b^4 + 2070*a^6*b^3 - 369*a^7*b^2))/(4*
a^12))*(4*a^4 - 60*a^3*b - 384*a*b^3 + 192*b^4 + 252*a^2*b^2))/(a^5*b*(144*
```

```
a*b^4 - 13*a^4*b + a^5 - 48*b^5 - 156*a^2*b^3 + 72*a^3*b^2))))/(27*a^2 - 10
8*a*b + 108*b^2) + (8*a^5*((27*(a - 2*b)^3*(a*b - b^2)^(3/2)*(32*a^14 - 128
*a^13*b + 128*a^12*b^2))/(128*a^21) + (3*(a - 2*b)*(a*b - b^2)^(1/2)*(36*a^
9*b - 1440*a^4*b^6 + 4320*a^5*b^5 - 4824*a^6*b^4 + 2448*a^7*b^3 - 540*a^8*b
^2))/(8*a^13))*(4*a^4 - 60*a^3*b - 384*a*b^3 + 192*b^4 + 252*a^2*b^2))/(b*(
27*a^2 - 108*a*b + 108*b^2)*(144*a*b^4 - 13*a^4*b + a^5 - 48*b^5 - 156*a^2*
b^3 + 72*a^3*b^2)) - (4*a^5*((864*b^8 - 3456*a*b^7 + 5508*a^2*b^6 - 4428*a^
3*b^5 + 1863*a^4*b^4 - 378*a^5*b^3 + 27*a^6*b^2)/(2*a^9) - (9*(a - 2*b)^2*(
a*b - b^2)*(12*a^12 - 240*a^11*b + 768*a^8*b^4 - 1536*a^9*b^3 + 1008*a^10*b
^2))/(32*a^17))*(960*a*b^4 - 38*a^4*b + a^5 - 384*b^5 - 840*a^2*b^3 + 300*a
^3*b^2))/((b*(a - b))^(3/2)*(27*a^2 - 108*a*b + 108*b^2)*(a^4 - 12*a^3*b -
96*a*b^3 + 48*b^4 + 60*a^2*b^2)))*(a - 2*b)*(a*b - b^2)^(1/2))/(2*a^4*f)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.74 $\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=196

$$\frac{3x\left(a^2+6ab+b^2\right)}{8(a-b)^4}-\frac{3\sqrt{a}\,\sqrt{b}\,(a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2f(a-b)^4}-\frac{3b(3a+b)\tan(e+fx)}{8f(a-b)^3\left(a+b\tan^2(e+fx)\right)}+\frac{\sin(e+fx)\cos^3(e+}{4f(a-b)\left(a+b\tan^2(e\right.}$$

```
[Out] 3/8*(a^2+6*a*b+b^2)*x/(a-b)^4-3/2*(a+b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*
a^(1/2)*b^(1/2)/(a-b)^4/f-1/8*(5*a+b)*cos(f*x+e)*sin(f*x+e)/(a-b)^2/f/(a+b*
tan(f*x+e)^2)+1/4*cos(f*x+e)^3*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)-3/8*b*
(3*a+b)*tan(f*x+e)/(a-b)^3/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.25, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 470, 527, 522, 203, 205}

$$\frac{3x\left(a^2+6ab+b^2\right)}{8(a-b)^4}-\frac{3\sqrt{a}\,\sqrt{b}\,(a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2f(a-b)^4}-\frac{3b(3a+b)\tan(e+fx)}{8f(a-b)^3\left(a+b\tan^2(e+fx)\right)}+\frac{\sin(e+fx)\cos^3(e+}{4f(a-b)\left(a+b\tan^2(e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (3*(a^2 + 6*a*b + b^2)*x)/(8*(a - b)^4) - (3*Sqrt[a]*Sqrt[b]*(a + b)*ArcTan
[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*(a - b)^4*f) - ((5*a + b)*Cos[e + f*x]
*Sin[e + f*x])/(8*(a - b)^2*f*(a + b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*Sin
[e + f*x])/(4*(a - b)*f*(a + b*Tan[e + f*x]^2)) - (3*b*(3*a + b)*Tan[e + f*
x])/(8*(a - b)^3*f*(a + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^3\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{a+(-4a-b)x^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= -\frac{(5a+b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2 f\left(a+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(5a+b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2 f\left(a+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{3b(3a}{8(a-b)^3 f}$$

$$= -\frac{(5a+b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2 f\left(a+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{3b(3a}{8(a-b)^3 f}$$

$$= \frac{3\left(a^2+6ab+b^2\right)x}{8(a-b)^4} - \frac{3\sqrt{a}\sqrt{b}\,(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^4 f} - \frac{(5a+b)\cos(e+fx}{8(a-b)^2 f\left(a+b\,\text{ta}\right.}$$

Mathematica [A] time = 1.65, size = 136, normalized size = 0.69

$$\frac{12\left(a^2+6ab+b^2\right)(e+fx)+(a-b)^2\sin(4(e+fx))-8(a+b)(a-b)\sin(2(e+fx))-48\sqrt{a}\sqrt{b}\,(a+b)\tan^{-1}\left(\right.}{32f(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (12*(a^2 + 6*a*b + b^2)*(e + f*x) - 48*Sqrt[a]*Sqrt[b]*(a + b)*ArcTan[(Sqrt
[b]*Tan[e + f*x])/Sqrt[a]] - 8*(a - b)*(a + b)*Sin[2*(e + f*x)] - (16*a*(a
```

```
- b)*b*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)]) + (a - b)^2*Sin
[4*(e + f*x)])/(32*(a - b)^4*f)
```

fricas [A] time = 0.64, size = 705, normalized size = 3.60

$$\left[\frac{3\left(a^3+5a^2b-5ab^2-b^3\right)fx\cos\left(fx+e\right)^2+3\left(a^2b+6ab^2+b^3\right)fx+3\left(\left(a^2-b^2\right)\cos\left(fx+e\right)^2+ab+b^2\right)\sqrt{-a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a^3 + 5*a^2*b - 5*a*b^2 - b^3)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 6*a
*b^2 + b^3)*f*x + 3*((a^2 - b^2)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a*b)*log
(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*(
(a + b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b)*sin(f*x + e) + b^2)/((a
^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)) + (
2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - (5*a^3 - 9*a^2*b + 3*a*b
^2 + b^3)*cos(f*x + e)^3 - 3*(3*a^2*b - 2*a*b^2 - b^3)*cos(f*x + e))*sin(f*
x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*
x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f), 1/8*(3*(a^3
+ 5*a^2*b - 5*a*b^2 - b^3)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 6*a*b^2 + b^3)*f
*x + 6*((a^2 - b^2)*cos(f*x + e)^2 + a*b + b^2)*sqrt(a*b)*arctan(1/2*((a +
b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*cos(f*x + e)*sin(f*x + e))) + (2*(a^3
 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - (5*a^3 - 9*a^2*b + 3*a*b^2 + b
^3)*cos(f*x + e)^3 - 3*(3*a^2*b - 2*a*b^2 - b^3)*cos(f*x + e))*sin(f*x + e)
)/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)
^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f)]
```

giac [A] time = 2.70, size = 266, normalized size = 1.36

$$\frac{\frac{3\left(a^2+6ab+b^2\right)\left(fx+e\right)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{4ab\tan\left(fx+e\right)}{\left(a^3-3a^2b+3ab^2-b^3\right)\left(b\tan\left(fx+e\right)^2+a\right)}-\frac{12\left(a^2b+ab^2\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\sqrt{ab}}-\frac{5a\tan\left(fx+e\right)^3+\cdots}{\left(a^3-\cdots\right.}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^2 + 6*a*b + b^2)*(f*x + e)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
 b^4) - 4*a*b*tan(f*x + e)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(b*tan(f*x + e)
^2 + a)) - 12*(a^2*b + a*b^2)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan
(b*tan(f*x + e)/sqrt(a*b)))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sq
rt(a*b)) - (5*a*tan(f*x + e)^3 + 3*b*tan(f*x + e)^3 + 3*a*tan(f*x + e) + 5*
b*tan(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(tan(f*x + e)^2 + 1)^2))/f
```

maple [B] time = 0.59, size = 411, normalized size = 2.10

$$-\frac{a^2b\tan\left(fx+e\right)}{2f\left(a-b\right)^4\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{a\,b^2\tan\left(fx+e\right)}{2f\left(a-b\right)^4\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{3a^2b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{2f\left(a-b\right)^4\sqrt{ab}}-\frac{3a\,b^2\arctan\left(\cdots\right.}{2f\left(a-b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*a^2*b/(a-b)^4*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2/f*a*b^2/(a-b)^4*tan(
f*x+e)/(a+b*tan(f*x+e)^2)-3/2/f*a^2*b/(a-b)^4/(a*b)^(1/2)*arctan(tan(f*x+e)
*b/(a*b)^(1/2))-3/2/f*a*b^2/(a-b)^4/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(
1/2))-5/8/f/(a-b)^4/(1+tan(f*x+e)^2)^2*tan(f*x+e)^3*a^2+1/4/f/(a-b)^4/(1+ta
n(f*x+e)^2)^2*tan(f*x+e)^3*a*b+3/8/f/(a-b)^4/(1+tan(f*x+e)^2)^2*tan(f*x+e)^
3*b^2-3/8/f/(a-b)^4/(1+tan(f*x+e)^2)^2*tan(f*x+e)*a^2+5/8/f/(a-b)^4/(1+tan(
f*x+e)^2)^2*tan(f*x+e)*b^2-1/4/f/(a-b)^4/(1+tan(f*x+e)^2)^2*tan(f*x+e)*a*b+
9/4/f/(a-b)^4*arctan(tan(f*x+e))*a*b+3/8/f/(a-b)^4*arctan(tan(f*x+e))*b^2+3
/8/f/(a-b)^4*arctan(tan(f*x+e))*a^2
```

maxima [A] time = 1.02, size = 312, normalized size = 1.59

$$\frac{\frac{3\left(a^{2}+6\,ab+b^{2}\right)\left(fx+e\right)}{a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4\,ab^{3}+b^{4}}-\frac{12\left(a^{2}b+ab^{2}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4\,ab^{3}+b^{4}\right)\sqrt{ab}}-\frac{3\left(3\,ab+b^{2}\right)\tan\left(fx+e\right)^{5}+\left(5\,a^{2}+14\,ab+5\,b^{2}\right)\mathrm{ta}}{\left(a^{3}b-3\,a^{2}b^{2}+3\,ab^{3}-b^{4}\right)\tan\left(fx+e\right)^{6}+\left(a^{4}-a^{3}b-3\,a^{2}b^{2}+5\,ab^{3}-2\,b^{4}\right)\tan\left(fx+e\right)^{4}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a^2 + 6*a*b + b^2)*(f*x + e)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
 b^4) - 12*(a^2*b + a*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - 4*a^3*b
 + 6*a^2*b^2 - 4*a*b^3 + b^4)*sqrt(a*b)) - (3*(3*a*b + b^2)*tan(f*x + e)^5
+ (5*a^2 + 14*a*b + 5*b^2)*tan(f*x + e)^3 + 3*(a^2 + 3*a*b)*tan(f*x + e))/(
(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*tan(f*x + e)^6 + (a^4 - a^3*b - 3*a^2*b
^2 + 5*a*b^3 - 2*b^4)*tan(f*x + e)^4 + a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3 +
(2*a^4 - 5*a^3*b + 3*a^2*b^2 + a*b^3 - b^4)*tan(f*x + e)^2))/f
```

mupad [B] time = 16.14, size = 4616, normalized size = 23.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (atan(((((tan(e + f*x)*(108*a*b^6 + 9*b^7 + 486*a^2*b^5 + 396*a^3*b^4 + 153
*a^4*b^3))/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 1
5*a^4*b^2)) - (3*(((9*a*b^11)/2 - (69*a^2*b^10)/2 + 114*a^3*b^9 - 210*a^4*b
^8 + 231*a^5*b^7 - 147*a^6*b^6 + 42*a^7*b^5 + 6*a^8*b^4 - (15*a^9*b^3)/2 +
(3*a^10*b^2)/2)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 -
126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) - (3*tan(e + f*x)*(a*b
*6i + a^2*1i + b^2*1i)*(256*b^11 - 1792*a*b^10 + 5120*a^2*b^9 - 7168*a^3*b^
8 + 3584*a^4*b^7 + 3584*a^5*b^6 - 7168*a^6*b^5 + 5120*a^7*b^4 - 1792*a^8*b^
3 + 256*a^9*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6
*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a*b*6i +
a^2*1i + b^2*1i))/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(a*b*6i
 + a^2*1i + b^2*1i)*3i)/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) +
(((tan(e + f*x)*(108*a*b^6 + 9*b^7 + 486*a^2*b^5 + 396*a^3*b^4 + 153*a^4*b^
3))/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b
^2)) + (3*(((9*a*b^11)/2 - (69*a^2*b^10)/2 + 114*a^3*b^9 - 210*a^4*b^8 + 23
1*a^5*b^7 - 147*a^6*b^6 + 42*a^7*b^5 + 6*a^8*b^4 - (15*a^9*b^3)/2 + (3*a^10
*b^2)/2)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4
*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) + (3*tan(e + f*x)*(a*b*6i + a
^2*1i + b^2*1i)*(256*b^11 - 1792*a*b^10 + 5120*a^2*b^9 - 7168*a^3*b^8 + 358
4*a^4*b^7 + 3584*a^5*b^6 - 7168*a^6*b^5 + 5120*a^7*b^4 - 1792*a^8*b^3 + 256
*a^9*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b
- 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a*b*6i + a^2*1i
+ b^2*1i))/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(a*b*6i + a^2*
1i + b^2*1i)*3i)/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))/(((27*a*
```

b^7)/64 + (135*a^2*b^6)/32 + (189*a^3*b^5)/16 + (297*a^4*b^4)/32 + (81*a^5*
b^3)/64)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4
*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) - (3*((tan(e + f*x)*(108*a*b^
6 + 9*b^7 + 486*a^2*b^5 + 396*a^3*b^4 + 153*a^4*b^3))/(32*(a^6 - 6*a^5*b -
6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) - (3*(((9*a*b^11)/2
- (69*a^2*b^10)/2 + 114*a^3*b^9 - 210*a^4*b^8 + 231*a^5*b^7 - 147*a^6*b^6 +
42*a^7*b^5 + 6*a^8*b^4 - (15*a^9*b^3)/2 + (3*a^10*b^2)/2)/(9*a*b^8 - 9*a^8
*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a
^6*b^3 + 36*a^7*b^2) - (3*tan(e + f*x)*(a*b*6i + a^2*1i + b^2*1i)*(256*b^11
- 1792*a*b^10 + 5120*a^2*b^9 - 7168*a^3*b^8 + 3584*a^4*b^7 + 3584*a^5*b^6
- 7168*a^6*b^5 + 5120*a^7*b^4 - 1792*a^8*b^3 + 256*a^9*b^2))/(512*(a^4 - 4*
a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*
b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a*b*6i + a^2*1i + b^2*1i))/(16*(a^4 - 4*a
^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(a*b*6i + a^2*1i + b^2*1i))/(16*(a^4 -
4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (3*((tan(e + f*x)*(108*a*b^6 + 9*b^
7 + 486*a^2*b^5 + 396*a^3*b^4 + 153*a^4*b^3))/(32*(a^6 - 6*a^5*b - 6*a*b^5
+ b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (3*(((9*a*b^11)/2 - (69*a^
2*b^10)/2 + 114*a^3*b^9 - 210*a^4*b^8 + 231*a^5*b^7 - 147*a^6*b^6 + 42*a^7*
b^5 + 6*a^8*b^4 - (15*a^9*b^3)/2 + (3*a^10*b^2)/2)/(9*a*b^8 - 9*a^8*b + a^9
- b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 +
36*a^7*b^2) + (3*tan(e + f*x)*(a*b*6i + a^2*1i + b^2*1i)*(256*b^11 - 1792*
a*b^10 + 5120*a^2*b^9 - 7168*a^3*b^8 + 3584*a^4*b^7 + 3584*a^5*b^6 - 7168*a
^6*b^5 + 5120*a^7*b^4 - 1792*a^8*b^3 + 256*a^9*b^2))/(512*(a^4 - 4*a^3*b -
4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20
*a^3*b^3 + 15*a^4*b^2)))*(a*b*6i + a^2*1i + b^2*1i))/(16*(a^4 - 4*a^3*b - 4
*a*b^3 + b^4 + 6*a^2*b^2)))*(a*b*6i + a^2*1i + b^2*1i))/(16*(a^4 - 4*a^3*b
- 4*a*b^3 + b^4 + 6*a^2*b^2))))*(a*b*6i + a^2*1i + b^2*1i)*3i)/(8*f*(a^4 -
4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - ((3*tan(e + f*x)^5*(3*a*b + b^2))/(
8*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e + f*x)^3*(14*a*b + 5*a^2 + 5*b^
2))/(8*(a - b)*(a^2 - 2*a*b + b^2)) + (3*tan(e + f*x)*(3*a*b + a^2))/(8*(a
- b)*(a^2 - 2*a*b + b^2)))/(f*(a + b*tan(e + f*x)^6 + tan(e + f*x)^2*(2*a +
b) + tan(e + f*x)^4*(a + 2*b))) + (atan((((-a*b)^(1/2)*(a + b)*((tan(e + f
x)(108*a*b^6 + 9*b^7 + 486*a^2*b^5 + 396*a^3*b^4 + 153*a^4*b^3))/(32*(a^6
- 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) - (3*((
(9*a*b^11)/2 - (69*a^2*b^10)/2 + 114*a^3*b^9 - 210*a^4*b^8 + 231*a^5*b^7 -
147*a^6*b^6 + 42*a^7*b^5 + 6*a^8*b^4 - (15*a^9*b^3)/2 + (3*a^10*b^2)/2)/(9*
a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a
^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) - (3*tan(e + f*x)*(-a*b)^(1/2)*(a + b)*(2
56*b^11 - 1792*a*b^10 + 5120*a^2*b^9 - 7168*a^3*b^8 + 3584*a^4*b^7 + 3584*a
^5*b^6 - 7168*a^6*b^5 + 5120*a^7*b^4 - 1792*a^8*b^3 + 256*a^9*b^2))/(128*(a
^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 +
15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(-a*b)^(1/2)*(a + b))/(4*(a^4 - 4*a
^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*3i)/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 +
6*a^2*b^2)) + ((-a*b)^(1/2)*(a + b)*((tan(e + f*x)*(108*a*b^6 + 9*b^7 + 48
6*a^2*b^5 + 396*a^3*b^4 + 153*a^4*b^3))/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^6
+ 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (3*(((9*a*b^11)/2 - (69*a^2*b^10
)/2 + 114*a^3*b^9 - 210*a^4*b^8 + 231*a^5*b^7 - 147*a^6*b^6 + 42*a^7*b^5 +
6*a^8*b^4 - (15*a^9*b^3)/2 + (3*a^10*b^2)/2)/(9*a*b^8 - 9*a^8*b + a^9 - b^9
- 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^
7*b^2) + (3*tan(e + f*x)*(-a*b)^(1/2)*(a + b)*(256*b^11 - 1792*a*b^10 + 512
0*a^2*b^9 - 7168*a^3*b^8 + 3584*a^4*b^7 + 3584*a^5*b^6 - 7168*a^6*b^5 + 512
0*a^7*b^4 - 1792*a^8*b^3 + 256*a^9*b^2))/(128*(a^4 - 4*a^3*b - 4*a*b^3 + b^
4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 1
5*a^4*b^2)))*(-a*b)^(1/2)*(a + b))/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^
2*b^2)))*3i)/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))/(((27*a*b^7)/
64 + (135*a^2*b^6)/32 + (189*a^3*b^5)/16 + (297*a^4*b^4)/32 + (81*a^5*b^3)/
64)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5
+ 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) - (3*(-a*b)^(1/2)*(a + b)*((tan(e
+ f*x)*(108*a*b^6 + 9*b^7 + 486*a^2*b^5 + 396*a^3*b^4 + 153*a^4*b^3))/(32*(

```
a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) - (3
*(((9*a*b^11)/2 - (69*a^2*b^10)/2 + 114*a^3*b^9 - 210*a^4*b^8 + 231*a^5*b^7
 - 147*a^6*b^6 + 42*a^7*b^5 + 6*a^8*b^4 - (15*a^9*b^3)/2 + (3*a^10*b^2)/2)/
(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 12
6*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) - (3*tan(e + f*x)*(-a*b)^(1/2)*(a + b)
*(256*b^11 - 1792*a*b^10 + 5120*a^2*b^9 - 7168*a^3*b^8 + 3584*a^4*b^7 + 358
4*a^5*b^6 - 7168*a^6*b^5 + 5120*a^7*b^4 - 1792*a^8*b^3 + 256*a^9*b^2))/(128
*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6
 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(-a*b)^(1/2)*(a + b))/(4*(a^4 -
4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 +
 6*a^2*b^2)) + (3*(-a*b)^(1/2)*(a + b)*((tan(e + f*x)*(108*a*b^6 + 9*b^7 +
486*a^2*b^5 + 396*a^3*b^4 + 153*a^4*b^3))/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^
6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (3*(((9*a*b^11)/2 - (69*a^2*b^
10)/2 + 114*a^3*b^9 - 210*a^4*b^8 + 231*a^5*b^7 - 147*a^6*b^6 + 42*a^7*b^5
+ 6*a^8*b^4 - (15*a^9*b^3)/2 + (3*a^10*b^2)/2)/(9*a*b^8 - 9*a^8*b + a^9 - b
^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*
a^7*b^2) + (3*tan(e + f*x)*(-a*b)^(1/2)*(a + b)*(256*b^11 - 1792*a*b^10 + 5
120*a^2*b^9 - 7168*a^3*b^8 + 3584*a^4*b^7 + 3584*a^5*b^6 - 7168*a^6*b^5 + 5
120*a^7*b^4 - 1792*a^8*b^3 + 256*a^9*b^2))/(128*(a^4 - 4*a^3*b - 4*a*b^3 +
b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 +
 15*a^4*b^2)))*(-a*b)^(1/2)*(a + b))/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*
a^2*b^2))))/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))*(-a*b)^(1/2)*
(a + b)*3i)/(2*f*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.75 $\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}\,f(a-b)^3}-\frac{b\tan(e+fx)}{f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x(a+3b)}{2(a-b)^3}$$

```
[Out] 1/2*(a+3*b)*x/(a-b)^3-1/2*(3*a+b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2
)/(a-b)^3/f/a^(1/2)-1/2*cos(f*x+e)*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)-b*
tan(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.16, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 471, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}\,f(a-b)^3}-\frac{b\tan(e+fx)}{f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x(a+3b)}{2(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((a + 3*b)*x)/(2*(a - b)^3) - (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/(2*Sqrt[a]*(a - b)^3*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a -
 b)*f*(a + b*Tan[e + f*x]^2)) - (b*Tan[e + f*x])/((a - b)^2*f*(a + b*Tan[e
+ f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{a-3bx^2}{\left(1+x^2\right)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a}{(1 \cdots}\right.}{\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{(a-b)^2f\left(a+b\tan^2(e+fx)\right)} - \frac{(b(3a+b))\,\text{S}\cdots}{\cdots}$$

$$= \frac{(a+3b)x}{2(a-b)^3} - \frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}\,(a-b)^3f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)} - \cdots$$

Mathematica [A] time = 1.79, size = 111, normalized size = 0.80

$$-\frac{-2(a+3b)(e+fx)+(a-b)\sin(2(e+fx))+\frac{2\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2b(a-b)\sin(2(e+fx))}{(a-b)\cos(2(e+fx))+a+b}}{4f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/4*(-2*(a + 3*b)*(e + f*x) + (2*Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a]])/Sqrt[a] + (a - b)*Sin[2*(e + f*x)] + (2*(a - b)*b*Sin[2*(e
 + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)]))/((a - b)^3*f)
```

fricas [A] time = 0.60, size = 568, normalized size = 4.12

$$\left[\frac{4\left(a^2+2ab-3b^2\right)fx\cos\left(fx+e\right)^2+4\left(ab+3b^2\right)fx-\left(\left(3a^2-2ab-b^2\right)\cos\left(fx+e\right)^2+3ab+b^2\right)\sqrt{-\frac{b}{a}}\,\text{lo}\cdots}{8\left(\left(a^4-4a^3b\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + 2*a*b - 3*b^2)*f*x*cos(f*x + e)^2 + 4*(a*b + 3*b^2)*f*x - ((
3*a^2 - 2*a*b - b^2)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(-b/a)*log(((a^2 + 6
*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*((a^2 + a*b
)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 -
 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)) - 4*((a
^2 - 2*a*b + b^2)*cos(f*x + e)^3 + 2*(a*b - b^2)*cos(f*x + e))*sin(f*x + e)
)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*b -
3*a^2*b^2 + 3*a*b^3 - b^4)*f), 1/4*(2*(a^2 + 2*a*b - 3*b^2)*f*x*cos(f*x + e
)^2 + 2*(a*b + 3*b^2)*f*x + ((3*a^2 - 2*a*b - b^2)*cos(f*x + e)^2 + 3*a*b +
 b^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*
x + e)*sin(f*x + e))) - 2*((a^2 - 2*a*b + b^2)*cos(f*x + e)^3 + 2*(a*b - b^
2)*cos(f*x + e))*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)
*f*cos(f*x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f)]
```

giac [A] time = 3.09, size = 195, normalized size = 1.41

$$\frac{\frac{(fx+e)(a+3b)}{a^3-3a^2b+3ab^2-b^3} - \frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(3ab+b^2)}{(a^3-3a^2b+3ab^2-b^3)\sqrt{ab}} - \frac{2b\tan(fx+e)^3+a\tan(fx+e)+b\tan(fx+e)}{\left(b\tan(fx+e)^4+a\tan(fx+e)^2+b\tan(fx+e)^2+a\right)(a^2-2ab+b^2)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a + 3*b)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (pi*floor((f*x +
 e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*(3*a*b + b^2)/((a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b)) - (2*b*tan(f*x + e)^3 + a*tan(f*x +
 e) + b*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan(f*x + e)^2 + b*tan(f*x + e
)^2 + a)*(a^2 - 2*a*b + b^2)))/f
```

maple [A] time = 0.51, size = 240, normalized size = 1.74

$$-\frac{b\tan(fx+e)\,a}{2f(a-b)^3\left(a+b\left(\tan^2(fx+e)\right)\right)}+\frac{b^2\tan(fx+e)}{2f(a-b)^3\left(a+b\left(\tan^2(fx+e)\right)\right)}-\frac{3b\arctan\left(\frac{\tan(fx+e)b}{\sqrt{ab}}\right)a}{2f(a-b)^3\sqrt{ab}}-\frac{b^2\arctan\left(\frac{\tan}{}\right.}{2f(a-b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f/(a-b)^3*b*tan(f*x+e)/(a+b*tan(f*x+e)^2)*a+1/2/f/(a-b)^3*b^2*tan(f*x+
e)/(a+b*tan(f*x+e)^2)-3/2/f/(a-b)^3*b/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)
^(1/2))*a-1/2/f/(a-b)^3*b^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/
2/f/(a-b)^3*tan(f*x+e)/(1+tan(f*x+e)^2)*a+1/2/f/(a-b)^3*tan(f*x+e)/(1+tan(f
*x+e)^2)*b+1/2/f/(a-b)^3*arctan(tan(f*x+e))*a+3/2/f/(a-b)^3*arctan(tan(f*x+
e))*b
```

maxima [A] time = 0.73, size = 185, normalized size = 1.34

$$\frac{\frac{(fx+e)(a+3b)}{a^3-3a^2b+3ab^2-b^3} - \frac{(3ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3-3a^2b+3ab^2-b^3)\sqrt{ab}} - \frac{2b\tan(fx+e)^3+(a+b)\tan(fx+e)}{(a^2b-2ab^2+b^3)\tan(fx+e)^4+a^3-2a^2b+ab^2+(a^3-a^2b-ab^2+b^3)\tan(fx+e)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((f*x + e)*(a + 3*b)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a*b + b^2)*ar
ctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b))
- (2*b*tan(f*x + e)^3 + (a + b)*tan(f*x + e))/((a^2*b - 2*a*b^2 + b^3)*tan(
f*x + e)^4 + a^3 - 2*a^2*b + a*b^2 + (a^3 - a^2*b - a*b^2 + b^3)*tan(f*x +
e)^2))/f
```

mupad [B] time = 14.94, size = 3301, normalized size = 23.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (atan((((-a*b)^(1/2)*((tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*b^3))/(a^4 - 4
*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2) + ((-a*b)^(1/2)*(3*a + b)*((10*a*b^8 -
2*b^9 - 18*a^2*b^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6*b^3 - 2*
a^7*b^2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*
b^2) - (tan(e + f*x)*(-a*b)^(1/2)*(3*a + b)*(40*a*b^8 - 8*b^9 - 72*a^2*b^7
+ 40*a^3*b^6 + 40*a^4*b^5 - 72*a^5*b^4 + 40*a^6*b^3 - 8*a^7*b^2))/(4*(a*b^3
 + 3*a^3*b - a^4 - 3*a^2*b^2)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))
)/(4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))*(3*a + b)*1i)/(4*(a*b^3 + 3*a^3*
b - a^4 - 3*a^2*b^2)) + ((-a*b)^(1/2)*((tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a
^2*b^3))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2) - ((-a*b)^(1/2)*(3*a +
 b)*((10*a*b^8 - 2*b^9 - 18*a^2*b^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4
+ 10*a^6*b^3 - 2*a^7*b^2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*
a^3*b^3 + 15*a^4*b^2) + (tan(e + f*x)*(-a*b)^(1/2)*(3*a + b)*(40*a*b^8 - 8*
b^9 - 72*a^2*b^7 + 40*a^3*b^6 + 40*a^4*b^5 - 72*a^5*b^4 + 40*a^6*b^3 - 8*a^
7*b^2))/(4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)*(a^4 - 4*a^3*b - 4*a*b^3 + b
^4 + 6*a^2*b^2))))/(4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))*(3*a + b)*1i)/(
4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))/((5*a*b^4 + (3*b^5)/2 + (3*a^2*b^3)
/2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)
- ((-a*b)^(1/2)*((tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*b^3))/(a^4 - 4*a^3*
b - 4*a*b^3 + b^4 + 6*a^2*b^2) + ((-a*b)^(1/2)*(3*a + b)*((10*a*b^8 - 2*b^9
 - 18*a^2*b^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6*b^3 - 2*a^7*b
^2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)
- (tan(e + f*x)*(-a*b)^(1/2)*(3*a + b)*(40*a*b^8 - 8*b^9 - 72*a^2*b^7 + 40*
a^3*b^6 + 40*a^4*b^5 - 72*a^5*b^4 + 40*a^6*b^3 - 8*a^7*b^2))/(4*(a*b^3 + 3*
a^3*b - a^4 - 3*a^2*b^2)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(4*
(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))*(3*a + b))/(4*(a*b^3 + 3*a^3*b - a^4
- 3*a^2*b^2)) + ((-a*b)^(1/2)*((tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*b^3))
/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2) - ((-a*b)^(1/2)*(3*a + b)*((10
*a*b^8 - 2*b^9 - 18*a^2*b^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6
*b^3 - 2*a^7*b^2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3
+ 15*a^4*b^2) + (tan(e + f*x)*(-a*b)^(1/2)*(3*a + b)*(40*a*b^8 - 8*b^9 - 72
*a^2*b^7 + 40*a^3*b^6 + 40*a^4*b^5 - 72*a^5*b^4 + 40*a^6*b^3 - 8*a^7*b^2))/
(4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a
^2*b^2))))/(4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))*(3*a + b))/(4*(a*b^3 +
3*a^3*b - a^4 - 3*a^2*b^2))))*(-a*b)^(1/2)*(3*a + b)*1i)/(2*f*(a*b^3 + 3*a^
3*b - a^4 - 3*a^2*b^2)) - (atan((((a + 3*b)*(((a + 3*b)*((10*a*b^8 - 2*b^9
- 18*a^2*b^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6*b^3 - 2*a^7*b^
2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2) -
 (tan(e + f*x)*(a + 3*b)*(40*a*b^8 - 8*b^9 - 72*a^2*b^7 + 40*a^3*b^6 + 40*a
^4*b^5 - 72*a^5*b^4 + 40*a^6*b^3 - 8*a^7*b^2))/(4*(a*b^2*3i - a^2*b*3i + a^
3*1i - b^3*1i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(4*(a*b^2*3i
- a^2*b*3i + a^3*1i - b^3*1i)) + (tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*b^3
))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*1i)/(4*(a*b^2*3i - a^2*b*3i
 + a^3*1i - b^3*1i)) - ((a + 3*b)*(((a + 3*b)*((10*a*b^8 - 2*b^9 - 18*a^2*b
^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6*b^3 - 2*a^7*b^2)/(a^6 -
6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2) + (tan(e +
f*x)*(a + 3*b)*(40*a*b^8 - 8*b^9 - 72*a^2*b^7 + 40*a^3*b^6 + 40*a^4*b^5 - 7
```

```
2*a^5*b^4 + 40*a^6*b^3 - 8*a^7*b^2))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3
*1i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(4*(a*b^2*3i - a^2*b*3i
 + a^3*1i - b^3*1i)) - (tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*b^3))/(a^4 -
4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*1i)/(4*(a*b^2*3i - a^2*b*3i + a^3*1i
- b^3*1i)))/(((a + 3*b)*(((a + 3*b)*((10*a*b^8 - 2*b^9 - 18*a^2*b^7 + 10*a^
3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6*b^3 - 2*a^7*b^2)/(a^6 - 6*a^5*b -
6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2) - (tan(e + f*x)*(a +
3*b)*(40*a*b^8 - 8*b^9 - 72*a^2*b^7 + 40*a^3*b^6 + 40*a^4*b^5 - 72*a^5*b^4
+ 40*a^6*b^3 - 8*a^7*b^2))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)*(a^4
- 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i
- b^3*1i)) + (tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*b^3))/(a^4 - 4*a^3*b -
4*a*b^3 + b^4 + 6*a^2*b^2)))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) -
(5*a*b^4 + (3*b^5)/2 + (3*a^2*b^3)/2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a
^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2) + ((a + 3*b)*(((a + 3*b)*((10*a*b^8 - 2*b
^9 - 18*a^2*b^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6*b^3 - 2*a^7
*b^2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2
) + (tan(e + f*x)*(a + 3*b)*(40*a*b^8 - 8*b^9 - 72*a^2*b^7 + 40*a^3*b^6 + 4
0*a^4*b^5 - 72*a^5*b^4 + 40*a^6*b^3 - 8*a^7*b^2))/(4*(a*b^2*3i - a^2*b*3i +
 a^3*1i - b^3*1i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(4*(a*b^2*
3i - a^2*b*3i + a^3*1i - b^3*1i)) - (tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*
b^3))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))/(4*(a*b^2*3i - a^2*b*3i
 + a^3*1i - b^3*1i))))*(a + 3*b)*1i)/(2*f*(a*b^2*3i - a^2*b*3i + a^3*1i - b
^3*1i)) - ((tan(e + f*x)*(a + b))/(2*(a^2 - 2*a*b + b^2)) + (b*tan(e + f*x)
^3)/(a^2 - 2*a*b + b^2))/(f*(a + tan(e + f*x)^2*(a + b) + b*tan(e + f*x)^4)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.76 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{b}\,(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}f(a-b)^2}-\frac{b\tan(e+fx)}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x}{(a-b)^2}$$

```
[Out] x/(a-b)^2-1/2*(3*a-b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(3/2)/(a
-b)^2/f-1/2*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3661, 414, 522, 203, 205}

$$-\frac{\sqrt{b}\,(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}f(a-b)^2}-\frac{b\tan(e+fx)}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-2),x]
```

```
[Out] x/(a - b)^2 - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2
*a^(3/2)*(a - b)^2*f) - (b*Tan[e + f*x])/(2*a*(a - b)*f*(a + b*Tan[e + f*x]
^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
```

```
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)^2 f} - \frac{((3a-b)b)}{}$$

$$= \frac{x}{(a-b)^2} - \frac{(3a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2 f} - \frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 1.11, size = 88, normalized size = 0.91

$$\frac{\frac{\sqrt{b}(b-3a)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b(b-a)\tan(e+fx)}{a\left(a+b\tan^2(e+fx)\right)} + 2\tan^{-1}(\tan(e+fx))}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-2),x]
```

```
[Out] (2*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a]])/a^(3/2) + (b*(-a + b)*Tan[e + f*x])/(a*(a + b*Tan[e + f*x]^2)))/
(2*(a - b)^2*f)
```

fricas [A] time = 0.50, size = 390, normalized size = 4.02

$$\left[\frac{8\,abfx\tan\left(fx+e\right)^2 + 8\,a^2fx - \left(\left(3\,ab-b^2\right)\tan\left(fx+e\right)^2 + 3\,a^2 - ab\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan\left(fx+e\right)^4 - 6\,ab\tan\left(fx+e\right)^2 + a^2 + 4\left(\right.}{b^2\tan\left(fx+e\right)^4 + 2\,ab}\right.}{8\left(\left(a^3b-2\,a^2b^2+ab^3\right)f\tan\left(fx+e\right)^2 + \left(a^4-2\,a^3b+a^2b^2\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*b*f*x*tan(f*x + e)^2 + 8*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2
+ 3*a^2 - a*b)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x +
e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a*b - b^2)*tan(f*x + e))/((a^3*b -
 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f), 1/4*(4
*a*b*f*x*tan(f*x + e)^2 + 4*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2 + 3*a^2
 - a*b)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x +
e))) - 2*(a*b - b^2)*tan(f*x + e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x +
 e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f)]
```

giac [A] time = 1.80, size = 127, normalized size = 1.31

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)\left(3\,ab-b^2\right)}{\left(a^3-2\,a^2b+ab^2\right)\sqrt{ab}}-\frac{2\left(fx+e\right)}{a^2-2\,ab+b^2}+\frac{b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)\left(a^2-ab\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
)))*(3*a*b - b^2)/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b)) - 2*(f*x + e)/(a^2 -
2*a*b + b^2) + b*tan(f*x + e)/((b*tan(f*x + e)^2 + a)*(a^2 - a*b)))/f
```

maple [A] time = 0.18, size = 160, normalized size = 1.65

$$-\frac{b\tan\left(fx+e\right)}{2\left(a-b\right)^2 f\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{b^2\tan\left(fx+e\right)}{2f\left(a-b\right)^2 a\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{3b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{2f\left(a-b\right)^2\sqrt{ab}}+\frac{b^2\arctan\left(\right.}{2f\left(a-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2*b*tan(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)+1/2/f*b^2/(a-b)^2/a*tan(f*x+
e)/(a+b*tan(f*x+e)^2)-3/2/f*b/(a-b)^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)
^(1/2))+1/2/f*b^2/(a-b)^2/a/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+1/
f/(a-b)^2*arctan(tan(f*x+e))
```

maxima [A] time = 0.58, size = 114, normalized size = 1.18

$$-\frac{\frac{b\tan\left(fx+e\right)}{a^3-a^2b+\left(a^2b-ab^2\right)\tan\left(fx+e\right)^2}+\frac{\left(3\,ab-b^2\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^3-2\,a^2b+ab^2\right)\sqrt{ab}}-\frac{2\left(fx+e\right)}{a^2-2\,ab+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*tan(f*x + e)/(a^3 - a^2*b + (a^2*b - a*b^2)*tan(f*x + e)^2) + (3*a*
b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b
)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2))/f
```

mupad [B] time = 13.53, size = 2489, normalized size = 25.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (2*atan((((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 -
4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(16*a^
2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2
*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2
) + (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2
)))/(2*a^2 - 4*a*b + 2*b^2) - ((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^
4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) +
 (tan(e + f*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*
b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(
```

```
2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^
4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^2))/((((((2*a*b^7 - 12*a^2*b^
6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 +
a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5
+ 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^
2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(b^5 - 6*a
*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^
2) + (((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a
^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(e + f*x)*(16*a^2*b
^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a
^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^
2) - (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2
*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - ((3*a*b^3)/2 - b^4/2)/(3*a^4*b - a^5 + a^
2*b^3 - 3*a^3*b^2))))/(f*(2*a^2 - 4*a*b + 2*b^2)) - (atan((((-a^3*b)^(1/2)*
((tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2))
- (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^
2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(-a^3*b)^(1/2)*(3*
a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16
*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b
)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a - b)*1i)/(4*(a^5 - 2
*a^4*b + a^3*b^2)) + ((-a^3*b)^(1/2)*((tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2
*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)) + (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5
 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^
2) + (tan(e + f*x)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a
^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2
)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b +
 a^3*b^2)))*(3*a - b)*1i)/(4*(a^5 - 2*a^4*b + a^3*b^2)))/(((3*a*b^3)/2 - b^
4/2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + ((-a^3*b)^(1/2)*((tan(e + f*x)
*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)) - (((2*a*b^7 -
 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b -
a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2
*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*
(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b)^(1/2)*(3*a -
 b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)
) - ((-a^3*b)^(1/2)*((tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 -
2*a^3*b + a^2*b^2)) + (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 1
8*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(e + f*x
)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b
^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b
+ a^3*b^2)))*(-a^3*b)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a
- b))/(4*(a^5 - 2*a^4*b + a^3*b^2))))*(-a^3*b)^(1/2)*(3*a - b)*1i)/(2*f*(a^
5 - 2*a^4*b + a^3*b^2)) - (b*tan(e + f*x))/(2*a*f*(a + b*tan(e + f*x)^2)*(a
 - b))
```

sympy [A] time = 28.54, size = 2322, normalized size = 23.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a**2, Eq(b,
 0)), ((x + 1/(f*tan(e + f*x)) - 1/(3*f*tan(e + f*x)**3))/b**2, Eq(a, 0)),
(3*f*x*tan(e + f*x)**4/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**
2 + 8*b**2*f) + 6*f*x*tan(e + f*x)**2/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f
*tan(e + f*x)**2 + 8*b**2*f) + 3*f*x/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*
tan(e + f*x)**2 + 8*b**2*f) + 3*tan(e + f*x)**3/(8*b**2*f*tan(e + f*x)**4 +
 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 5*tan(e + f*x)/(8*b**2*f*tan(e + f
*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f), Eq(a, b)), (x/(a + b*tan(e)
**2)**2, Eq(f, 0)), (4*I*a**(5/2)*f*x*sqrt(1/b)/(4*I*a**(9/2)*f*sqrt(1/b) +
```

```
 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) -
8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/
b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) + 4*I*a**(3/2)*b*f*x*sq
rt(1/b)*tan(e + f*x)**2/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1
/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt
(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f
*sqrt(1/b)*tan(e + f*x)**2) - 2*I*a**(3/2)*b*sqrt(1/b)*tan(e + f*x)/(4*I*a*
*(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7
/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**
(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) + 2
*I*sqrt(a)*b**2*sqrt(1/b)*tan(e + f*x)/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(
7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5
/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*
a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) - 3*a**2*log(-I*sqrt(a)*sqrt(1/b
) + tan(e + f*x))/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*ta
n(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*
tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(
1/b)*tan(e + f*x)**2) + 3*a**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I
*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a*
*(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*
a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2)
- 3*a*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(9
/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)
*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/
2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) + a*b*
log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a*
*(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**
(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*
I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) + 3*a*b*log(I*sqrt(a)*sqrt(1/b
) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*
b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b
**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3
/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) - a*b*log(I*sqrt(a)*sqrt(1/b) + tan(e
 + f*x))/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x
)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f
*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(
e + f*x)**2) + b**2*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**
2/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 -
8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2
+ 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x
)**2) - b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a
**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(
7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a*
*(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2), T
rue))
```

3.77 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2} dx$

Optimal. Leaf size=82

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{3\cot(e+fx)}{2a^2f}+\frac{\cot(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] -3/2*cot(f*x+e)/a^2/f-3/2*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(5/2
)/f+1/2*cot(f*x+e)/a/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3663, 290, 325, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{3\cot(e+fx)}{2a^2f}+\frac{\cot(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*a^(5/2)*f) - (3*Cot[
e + f*x])/(2*a^2*f) + Cot[e + f*x]/(2*a*f*(a + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{3\cot(e+fx)}{2a^2f} + \frac{\cot(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+\right.}{2a^2f}$$

$$= -\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f} - \frac{3\cot(e+fx)}{2a^2f} + \frac{\cot(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 0.67, size = 83, normalized size = 1.01

$$\frac{\sqrt{a}\left(-\frac{b\sin(2(e+fx))}{(a-b)\cos(2(e+fx))+a+b} - 2\cot(e+fx)\right) - 3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(-2*Cot[e + f*
x] - (b*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)])))/(2*a^(5/2)*f
)
```

fricas [B] time = 0.52, size = 373, normalized size = 4.55

$$\left[-\frac{4\,(2\,a-3\,b)\cos\left(fx+e\right)^3 - 3\left((a-b)\cos\left(fx+e\right)^2+b\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2+4}{\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right.}\right.}{8\left(a^2bf+\left(a^3-a^2b\right)f\cos\left(fx+e\right)^2\right)\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a - 3*b)*cos(f*x + e)^3 - 3*((a - b)*cos(f*x + e)^2 + b)*sqrt(-
b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)
^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x +
 e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)
^2 + b^2))*sin(f*x + e) + 12*b*cos(f*x + e))/((a^2*b*f + (a^3 - a^2*b)*f*co
s(f*x + e)^2)*sin(f*x + e)), -1/4*(2*(2*a - 3*b)*cos(f*x + e)^3 - 3*((a - b
)*cos(f*x + e)^2 + b)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqr
t(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 6*b*cos(f*x + e))/((a^
2*b*f + (a^3 - a^2*b)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

giac [A] time = 1.48, size = 93, normalized size = 1.13

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,a^2} + \frac{3\,b\tan\left(fx+e\right)^2+2\,a}{\left(b\tan\left(fx+e\right)^3+a\tan\left(fx+e\right)\right)a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*b/(sqrt(a*b)*a^2) + (3*b*tan(f*x + e)^2 + 2*a)/((b*tan(f*x + e)^3 + a
*tan(f*x + e))*a^2))/f
```

maple [A] time = 0.58, size = 75, normalized size = 0.91

$$-\frac{b\tan\left(fx+e\right)}{2f\,a^{2}\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)}-\frac{3b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{2f\,a^{2}\sqrt{ab}}-\frac{1}{f\,a^{2}\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)-3/2/f*b/a^2/(a*b)^(1/2)*arctan(t
an(f*x+e)*b/(a*b)^(1/2))-1/f/a^2/tan(f*x+e)
```

maxima [A] time = 0.93, size = 73, normalized size = 0.89

$$-\frac{\frac{3\,b\tan\left(fx+e\right)^{2}+2\,a}{a^{2}b\tan\left(fx+e\right)^{3}+a^{3}\tan\left(fx+e\right)}+\frac{3\,b\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*b*tan(f*x + e)^2 + 2*a)/(a^2*b*tan(f*x + e)^3 + a^3*tan(f*x + e))
+ 3*b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2))/f
```

mupad [B] time = 11.50, size = 70, normalized size = 0.85

$$-\frac{\frac{1}{a}+\frac{3\,b\tan\left(e+f\,x\right)^{2}}{2\,a^{2}}}{f\,\left(b\tan\left(e+f\,x\right)^{3}+a\tan\left(e+f\,x\right)\right)}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)}{\sqrt{a}}\right)}{2\,a^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^2),x)
```

```
[Out] - (1/a + (3*b*tan(e + f*x)^2)/(2*a^2))/(f*(a*tan(e + f*x) + b*tan(e + f*x)^
3)) - (3*b^(1/2)*atan((b^(1/2)*tan(e + f*x))/a^(1/2)))/(2*a^(5/2)*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.78 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=116

$$-\frac{\sqrt{b}\,(3a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{b(a-b)\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)}-\frac{(a-2b)\cot(e+fx)}{a^3f}-\frac{\cot^3(e+fx)}{3a^2f}$$

```
[Out] -(a-2*b)*cot(f*x+e)/a^3/f-1/3*cot(f*x+e)^3/a^2/f-1/2*(3*a-5*b)*arctan(b^(1/
2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(7/2)/f-1/2*(a-b)*b*tan(f*x+e)/a^3/f/(a+b*
tan(f*x+e)^2)
```

Rubi [A] time = 0.15, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3663, 456, 1261, 205}

$$-\frac{\sqrt{b}\,(3a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{b(a-b)\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)}-\frac{(a-2b)\cot(e+fx)}{a^3f}-\frac{\cot^3(e+fx)}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((3*a - 5*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*a^(7/2)*f)
 - ((a - 2*b)*Cot[e + f*x])/(a^3*f) - Cot[e + f*x]^3/(3*a^2*f) - ((a - b)*b
*Tan[e + f*x])/(2*a^3*f*(a + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a-b)b\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int \frac{-\frac{2}{ab}-\frac{2(a-b)x^2}{a^2b}+\frac{(a-b)x^4}{a^3}}{x^4\left(a+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= -\frac{(a-b)b\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int\left(-\frac{2}{a^2bx^4}-\frac{2(a-2b)}{a^3bx^2}+\frac{3a-5b}{a^3\left(a+bx^2\right)}\right)dx, x, \tan(e$$

$$= -\frac{(a-2b)\cot(e+fx)}{a^3f} - \frac{\cot^3(e+fx)}{3a^2f} - \frac{(a-b)b\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)} - \frac{((3a-5b)b)\,S}{}$$

$$= -\frac{(3a-5b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f} - \frac{(a-2b)\cot(e+fx)}{a^3f} - \frac{\cot^3(e+fx)}{3a^2f} - \frac{(a-}{2a^3f\,(a}$$

Mathematica [A] time = 0.99, size = 112, normalized size = 0.97

$$\frac{3\sqrt{b}(5b-3a)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)+\sqrt{a}\left(\frac{3b(b-a)\sin(2(e+fx))}{(a-b)\cos(2(e+fx))+a+b}-2\cot(e+fx)\left(a\csc^2(e+fx)+2a-6b\right)\right)}{6a^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (3*Sqrt[b]*(-3*a + 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(-
2*Cot[e + f*x]*(2*a - 6*b + a*Csc[e + f*x]^2) + (3*b*(-a + b)*Sin[2*(e + f*
x)])/(a + b + (a - b)*Cos[2*(e + f*x)])))/(6*a^(7/2)*f)
```

fricas [B] time = 0.65, size = 587, normalized size = 5.06

$$\left[-\frac{4\left(4a^2-19ab+15b^2\right)\cos\left(fx+e\right)^5-8\left(3a^2-14ab+15b^2\right)\cos\left(fx+e\right)^3+3\left(\left(3a^2-8ab+5b^2\right)\cos\left(fx-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(4*(4*a^2 - 19*a*b + 15*b^2)*cos(f*x + e)^5 - 8*(3*a^2 - 14*a*b + 15
*b^2)*cos(f*x + e)^3 + 3*((3*a^2 - 8*a*b + 5*b^2)*cos(f*x + e)^4 - (3*a^2 -
 11*a*b + 10*b^2)*cos(f*x + e)^2 - 3*a*b + 5*b^2)*sqrt(-b/a)*log(((a^2 + 6*
a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*((a^2 + a*b)
*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 -
2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x
+ e) - 12*(3*a*b - 5*b^2)*cos(f*x + e))/(((a^4 - a^3*b)*f*cos(f*x + e)^4 -
a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x + e)^2)*sin(f*x + e)), -1/12*(2*(4*a^2
- 19*a*b + 15*b^2)*cos(f*x + e)^5 - 4*(3*a^2 - 14*a*b + 15*b^2)*cos(f*x + e
```

```
)^3 - 3*((3*a^2 - 8*a*b + 5*b^2)*cos(f*x + e)^4 - (3*a^2 - 11*a*b + 10*b^2)
*cos(f*x + e)^2 - 3*a*b + 5*b^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)
^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(3*a*b -
5*b^2)*cos(f*x + e))/(((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4 - 2*
a^3*b)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

giac [A] time = 3.34, size = 142, normalized size = 1.22

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)\left(3\,ab-5\,b^{2}\right)}{\sqrt{ab}\,a^{3}}+\frac{3\left(ab\tan\left(fx+e\right)-b^{2}\tan\left(fx+e\right)\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)a^{3}}+\frac{2\left(3\,a\tan\left(fx+e\right)^{2}-6\,b\tan\left(fx+e\right)^{2}+a\right)}{a^{3}\tan\left(fx+e\right)^{3}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*(3*a*b - 5*b^2)/(sqrt(a*b)*a^3) + 3*(a*b*tan(f*x + e) - b^2*tan(f*x +
 e))/((b*tan(f*x + e)^2 + a)*a^3) + 2*(3*a*tan(f*x + e)^2 - 6*b*tan(f*x + e
)^2 + a)/(a^3*tan(f*x + e)^3))/f
```

maple [A] time = 0.62, size = 169, normalized size = 1.46

$$-\frac{b\tan\left(fx+e\right)}{2f\,a^{2}\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)}+\frac{b^{2}\tan\left(fx+e\right)}{2f\,a^{3}\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)}-\frac{3b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{2f\,a^{2}\sqrt{ab}}+\frac{5b^{2}\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{2f\,a^{3}\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2/f/a^3*b^2*tan(f*x+e)/(a+b*ta
n(f*x+e)^2)-3/2/f*b/a^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+5/2/f/
a^3*b^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/3/f/a^2/tan(f*x+e)^3
-1/f/a^2/tan(f*x+e)+2/f/a^3/tan(f*x+e)*b
```

maxima [A] time = 0.62, size = 115, normalized size = 0.99

$$-\frac{\frac{3\left(3\,ab-5\,b^{2}\right)\tan\left(fx+e\right)^{4}+2\left(3\,a^{2}-5\,ab\right)\tan\left(fx+e\right)^{2}+2\,a^{2}}{a^{3}b\tan\left(fx+e\right)^{5}+a^{4}\tan\left(fx+e\right)^{3}}+\frac{3\left(3\,ab-5\,b^{2}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{3}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*((3*(3*a*b - 5*b^2)*tan(f*x + e)^4 + 2*(3*a^2 - 5*a*b)*tan(f*x + e)^2
+ 2*a^2)/(a^3*b*tan(f*x + e)^5 + a^4*tan(f*x + e)^3) + 3*(3*a*b - 5*b^2)*ar
ctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3))/f
```

mupad [B] time = 11.37, size = 108, normalized size = 0.93

$$-\frac{\frac{1}{3\,a}+\frac{\tan\left(e+f\,x\right)^{2}\,\left(3\,a-5\,b\right)}{3\,a^{2}}+\frac{b\tan\left(e+f\,x\right)^{4}\,\left(3\,a-5\,b\right)}{2\,a^{3}}}{f\,\left(b\tan\left(e+f\,x\right)^{5}+a\tan\left(e+f\,x\right)^{3}\right)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)}{\sqrt{a}}\right)\,\left(3\,a-5\,b\right)}{2\,a^{7/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^2),x)
```

```
[Out] - (1/(3*a) + (tan(e + f*x)^2*(3*a - 5*b))/(3*a^2) + (b*tan(e + f*x)^4*(3*a
- 5*b))/(2*a^3))/(f*(a*tan(e + f*x)^3 + b*tan(e + f*x)^5)) - (b^(1/2)*atan(
(b^(1/2)*tan(e + f*x))/a^(1/2))*(3*a - 5*b))/(2*a^(7/2)*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.79 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2} dx$

Optimal. Leaf size=182

$$-\frac{\sqrt{b}(3a-7b)(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f}-\frac{(10a-7b)\cot^3(e+fx)}{15a^3f}-\frac{b\left(5a^2-10ab+7b^2\right)\tan(e+fx)}{10a^4f\left(a+b\tan^2(e+fx)\right)}-\frac{\left(5a^2-2\right.}{}$$

```
[Out] -1/5*(5*a^2-20*a*b+14*b^2)*cot(f*x+e)/a^4/f-1/15*(10*a-7*b)*cot(f*x+e)^3/a^
3/f-1/2*(3*a-7*b)*(a-b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(9/2)/
f-1/5*cot(f*x+e)^5/a/f/(a+b*tan(f*x+e)^2)-1/10*b*(5*a^2-10*a*b+7*b^2)*tan(f
*x+e)/a^4/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.22, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3663, 462, 456, 1261, 205}

$$-\frac{b\left(5a^2-10ab+7b^2\right)\tan(e+fx)}{10a^4f\left(a+b\tan^2(e+fx)\right)}-\frac{\left(5a^2-20ab+14b^2\right)\cot(e+fx)}{5a^4f}-\frac{\sqrt{b}(3a-7b)(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((3*a - 7*b)*(a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*a^
(9/2)*f) - ((5*a^2 - 20*a*b + 14*b^2)*Cot[e + f*x])/(5*a^4*f) - ((10*a - 7*
b)*Cot[e + f*x]^3)/(15*a^3*f) - Cot[e + f*x]^5/(5*a*f*(a + b*Tan[e + f*x]^2
)) - (b*(5*a^2 - 10*a*b + 7*b^2)*Tan[e + f*x])/(10*a^4*f*(a + b*Tan[e + f*x
]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{10a-7b+5ax^2}{x^4\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)} - \frac{b\left(5a^2-10ab+7b^2\right)\tan(e+fx)}{10a^4f\left(a+b\tan^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int \frac{2\left(\frac{7}{a}-\frac{10}{b}\right.}{}\right.}{}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)} - \frac{b\left(5a^2-10ab+7b^2\right)\tan(e+fx)}{10a^4f\left(a+b\tan^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int\left(-\frac{2(10}{a^2}\right.\right.}{}$$

$$= -\frac{\left(5a^2-20ab+14b^2\right)\cot(e+fx)}{5a^4f} - \frac{(10a-7b)\cot^3(e+fx)}{15a^3f} - \frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+\right.}$$

$$= -\frac{(3a-7b)(a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f} - \frac{\left(5a^2-20ab+14b^2\right)\cot(e+fx)}{5a^4f} - \frac{(10}{}$$

Mathematica [A] time = 1.90, size = 151, normalized size = 0.83

$$\frac{\sqrt{a}\left(-2\cot(e+fx)\left(3a^2\csc^4(e+fx)+8a^2+2a(2a-5b)\csc^2(e+fx)-50ab+45b^2\right)-\frac{15b(a-b)^2\sin(2(e+fx))}{(a-b)\cos(2(e+fx))+a+b}\right)-1}{30a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-15*Sqrt[b]*(3*a^2 - 10*a*b + 7*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
] + Sqrt[a]*(-2*Cot[e + f*x]*(8*a^2 - 50*a*b + 45*b^2 + 2*a*(2*a - 5*b)*Csc
[e + f*x]^2 + 3*a^2*Csc[e + f*x]^4) - (15*(a - b)^2*b*Sin[2*(e + f*x)])/(a
+ b + (a - b)*Cos[2*(e + f*x)])))/(30*a^(9/2)*f)
```

fricas [B] time = 0.58, size = 855, normalized size = 4.70

$$\left[-\frac{4\left(16\,a^{3}-131\,a^{2}b+220\,ab^{2}-105\,b^{3}\right)\cos\left(fx+e\right)^{7}-4\left(40\,a^{3}-321\,a^{2}b+590\,ab^{2}-315\,b^{3}\right)\cos\left(fx+e\right)^{5}+\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/120*(4*(16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 4*(4
0*a^3 - 321*a^2*b + 590*a*b^2 - 315*b^3)*cos(f*x + e)^5 + 20*(6*a^3 - 47*a^
2*b + 104*a*b^2 - 63*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 13*a^2*b + 17*a*b^2
 - 7*b^3)*cos(f*x + e)^6 - (6*a^3 - 29*a^2*b + 44*a*b^2 - 21*b^3)*cos(f*x +
 e)^4 + 3*a^2*b - 10*a*b^2 + 7*b^3 + (3*a^3 - 19*a^2*b + 37*a*b^2 - 21*b^3)
*cos(f*x + e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*
a*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e
))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(
a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(3*a^2*b - 10*a*b^2 + 7
*b^3)*cos(f*x + e))/(((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3
*a^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2)*sin(f*x + e)),
 -1/60*(2*(16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 2*(40
*a^3 - 321*a^2*b + 590*a*b^2 - 315*b^3)*cos(f*x + e)^5 + 10*(6*a^3 - 47*a^2
*b + 104*a*b^2 - 63*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 13*a^2*b + 17*a*b^2
- 7*b^3)*cos(f*x + e)^6 - (6*a^3 - 29*a^2*b + 44*a*b^2 - 21*b^3)*cos(f*x +
e)^4 + 3*a^2*b - 10*a*b^2 + 7*b^3 + (3*a^3 - 19*a^2*b + 37*a*b^2 - 21*b^3)*
cos(f*x + e)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)
/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(3*a^2*b - 10*a*b^2 + 7*b
^3)*cos(f*x + e))/(((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a
^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

giac [A] time = 3.07, size = 212, normalized size = 1.16

$$-\frac{\frac{15\left(3\,a^{2}b-10\,ab^{2}+7\,b^{3}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}\,a^{4}}+\frac{15\left(a^{2}b\tan\left(fx+e\right)-2\,ab^{2}\tan\left(fx+e\right)+b^{3}\tan\left(fx+e\right)\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)a^{4}}+\frac{2\left(15\,a^{2}\tan\left(fx+e\right)^{4}\cdots\right.}{\cdots}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*(3*a^2*b - 10*a*b^2 + 7*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
 + arctan(b*tan(f*x + e)/sqrt(a*b)))/(sqrt(a*b)*a^4) + 15*(a^2*b*tan(f*x +
e) - 2*a*b^2*tan(f*x + e) + b^3*tan(f*x + e))/((b*tan(f*x + e)^2 + a)*a^4)
+ 2*(15*a^2*tan(f*x + e)^4 - 60*a*b*tan(f*x + e)^4 + 45*b^2*tan(f*x + e)^4
+ 10*a^2*tan(f*x + e)^2 - 10*a*b*tan(f*x + e)^2 + 3*a^2)/(a^4*tan(f*x + e)^
5))/f
```

maple [A] time = 0.58, size = 281, normalized size = 1.54

$$-\frac{b\tan\left(fx+e\right)}{2f\,a^{2}\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)}+\frac{b^{2}\tan\left(fx+e\right)}{f\,a^{3}\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)}-\frac{b^{3}\tan\left(fx+e\right)}{2f\,a^{4}\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)}-\frac{3b\arctan\left(\frac{\tan\left(fx+e\right)\cdots}{\sqrt{ab}}\right)}{2f\,a^{2}\sqrt{ab}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/f/a^3*b^2*tan(f*x+e)/(a+b*tan(
f*x+e)^2)-1/2/f*b^3/a^4*tan(f*x+e)/(a+b*tan(f*x+e)^2)-3/2/f*b/a^2/(a*b)^(1/
2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+5/f/a^3*b^2/(a*b)^(1/2)*arctan(tan(f*x+
e)*b/(a*b)^(1/2))-7/2/f*b^3/a^4/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2)
)-1/5/f/a^2/tan(f*x+e)^5-2/3/f/a^2/tan(f*x+e)^3+2/3/f/a^3/tan(f*x+e)^3*b-1/
f/a^2/tan(f*x+e)+4/f/a^3/tan(f*x+e)*b-3/f/a^4/tan(f*x+e)*b^2
```

maxima [A] time = 0.79, size = 161, normalized size = 0.88

$$-\frac{\frac{15\left(3a^2b-10ab^2+7b^3\right)\tan\left(fx+e\right)^6+10\left(3a^3-10a^2b+7ab^2\right)\tan\left(fx+e\right)^4+6a^3+2\left(10a^3-7a^2b\right)\tan\left(fx+e\right)^2}{a^4b\tan\left(fx+e\right)^7+a^5\tan\left(fx+e\right)^5}+\frac{15\left(3a^2b-10ab^2+7b^3\right)\arctan\left(\frac{b\tan(f\dots}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}}{30f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/30*((15*(3*a^2*b - 10*a*b^2 + 7*b^3)*tan(f*x + e)^6 + 10*(3*a^3 - 10*a^2
*b + 7*a*b^2)*tan(f*x + e)^4 + 6*a^3 + 2*(10*a^3 - 7*a^2*b)*tan(f*x + e)^2)
/(a^4*b*tan(f*x + e)^7 + a^5*tan(f*x + e)^5) + 15*(3*a^2*b - 10*a*b^2 + 7*b
^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4))/f
```

mupad [B] time = 12.37, size = 178, normalized size = 0.98

$$-\frac{\frac{1}{5a}+\frac{\tan\left(e+fx\right)^4\left(3a^2-10ab+7b^2\right)}{3a^3}+\frac{\tan\left(e+fx\right)^2\left(10a-7b\right)}{15a^2}+\frac{b\tan\left(e+fx\right)^6\left(3a^2-10ab+7b^2\right)}{2a^4}}{f\left(b\tan\left(e+fx\right)^7+a\tan\left(e+fx\right)^5\right)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan\left(e+fx\right)\left(a-b\right)\left(3a-7b\right)}{\sqrt{a}\left(3a^2-10ab+7b^2\right)}\right)\dots}{2a^{9/2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^2),x)
```

```
[Out] - (1/(5*a) + (tan(e + f*x)^4*(3*a^2 - 10*a*b + 7*b^2))/(3*a^3) + (tan(e + f
*x)^2*(10*a - 7*b))/(15*a^2) + (b*tan(e + f*x)^6*(3*a^2 - 10*a*b + 7*b^2))/
(2*a^4))/(f*(a*tan(e + f*x)^5 + b*tan(e + f*x)^7)) - (b^(1/2)*atan((b^(1/2)
*tan(e + f*x)*(a - b)*(3*a - 7*b))/(a^(1/2)*(3*a^2 - 10*a*b + 7*b^2)))*(a -
 b)*(3*a - 7*b))/(2*a^(9/2)*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.80 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=264

$$-\frac{\left(5a^2+20ab+2b^2\right)\cos(e+fx)}{5f(a-b)^5}-\frac{b\left(35a^2+40ab+24b^2\right)\sec(e+fx)}{40f(a-b)^5\left(a+b\sec^2(e+fx)-b\right)}-\frac{b\left(5a^2+4b^2\right)\sec(e+fx)}{20f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)^2}$$

```
[Out] -1/5*(5*a^2+20*a*b+2*b^2)*cos(f*x+e)/(a-b)^5/f+1/15*(10*a-b)*cos(f*x+e)^3/(
a-b)^4/f-1/5*cos(f*x+e)^5/(a-b)/f/(a-b+b*sec(f*x+e)^2)^2-1/20*b*(5*a^2+4*b^
2)*sec(f*x+e)/(a-b)^4/f/(a-b+b*sec(f*x+e)^2)^2-1/40*b*(35*a^2+40*a*b+24*b^2
)*sec(f*x+e)/(a-b)^5/f/(a-b+b*sec(f*x+e)^2)-1/8*(15*a^2+40*a*b+8*b^2)*arcta
n(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/(a-b)^(11/2)/f
```

Rubi [A] time = 0.41, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3664, 462, 456, 1259, 1261, 205}

$$-\frac{\left(5a^2+20ab+2b^2\right)\cos(e+fx)}{5f(a-b)^5}-\frac{b\left(35a^2+40ab+24b^2\right)\sec(e+fx)}{40f(a-b)^5\left(a+b\sec^2(e+fx)-b\right)}-\frac{b\left(5a^2+4b^2\right)\sec(e+fx)}{20f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(Sqrt[b]*(15*a^2 + 40*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a -
b]])/(8*(a - b)^(11/2)*f) - ((5*a^2 + 20*a*b + 2*b^2)*Cos[e + f*x])/(5*(a -
 b)^5*f) + ((10*a - b)*Cos[e + f*x]^3)/(15*(a - b)^4*f) - Cos[e + f*x]^5/(5
*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^2) - (b*(5*a^2 + 4*b^2)*Sec[e + f*x])
/(20*(a - b)^4*f*(a - b + b*Sec[e + f*x]^2)^2) - (b*(35*a^2 + 40*a*b + 24*b
^2)*Sec[e + f*x])/(40*(a - b)^5*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-10a+b+5(a-b)x^2}{x^4\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{5(a-b)f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b\left(5a^2+4b^2\right)\sec(e+fx)}{20(a-b)^4 f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b\,\text{S}}{}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b\left(5a^2+4b^2\right)\sec(e+fx)}{20(a-b)^4 f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b(}{40(}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b\left(5a^2+4b^2\right)\sec(e+fx)}{20(a-b)^4 f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b(}{40(}$$

$$= -\frac{\left(5a^2+20ab+2b^2\right)\cos(e+fx)}{5(a-b)^5 f} + \frac{(10a-b)\cos^3(e+fx)}{15(a-b)^4 f} - \frac{\cos^5(e+}{5(a-b)f\left(a-b+b\,\text{s}\right.}$$

$$= -\frac{\sqrt{b}\left(15a^2+40ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{11/2}f} - \frac{\left(5a^2+20ab+2b^2\right)\cos(e+fx)}{5(a-b)^5 f} +$$

Mathematica [A] time = 5.81, size = 278, normalized size = 1.05

$$\frac{\frac{(a-b)(5(5a+7b)\cos(3(e+fx))+3(b-a)\cos(5(e+fx)))-30\cos(e+fx)\left(a^2\left(-\frac{8b^2}{((a-b)\cos(2(e+fx))+a+b)^2}+\frac{18b}{(a-b)\cos(2(e+fx))+a+b}+5\right)+16ab\left(\frac{b}{(a-b)\cos(2(e+fx))+\cdots}\right.\right.}{(a-b)^5}}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((30*Sqrt[b]*(15*a^2 + 40*a*b + 8*b^2)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e
 + f*x)/2])/Sqrt[b]])/(a - b)^(11/2) + (30*Sqrt[b]*(15*a^2 + 40*a*b + 8*b^2
)*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(11/2)
+ (-30*Cos[e + f*x]*(11*b^2 + 16*a*b*(2 + b/(a + b + (a - b)*Cos[2*(e + f*x
)])) + a^2*(5 - (8*b^2)/(a + b + (a - b)*Cos[2*(e + f*x)])^2 + (18*b)/(a +
b + (a - b)*Cos[2*(e + f*x)]))) + (a - b)*(5*(5*a + 7*b)*Cos[3*(e + f*x)] +
 3*(-a + b)*Cos[5*(e + f*x)]))/(a - b)^5)/(240*f)
```

fricas [B] time = 0.78, size = 1018, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(48*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^9 - 16
*(10*a^4 - 31*a^3*b + 33*a^2*b^2 - 13*a*b^3 + b^4)*cos(f*x + e)^7 + 16*(15*
a^4 + 10*a^3*b - 57*a^2*b^2 + 24*a*b^3 + 8*b^4)*cos(f*x + e)^5 + 50*(15*a^3
*b + 25*a^2*b^2 - 32*a*b^3 - 8*b^4)*cos(f*x + e)^3 + 15*((15*a^4 + 10*a^3*b
 - 57*a^2*b^2 + 24*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 + 40*a*b^3 +
8*b^4 + 2*(15*a^3*b + 25*a^2*b^2 - 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-
b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a - b))*cos(f*
x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 30*(15*a^2*b^2 + 40*a*b^3 + 8*b
^4)*cos(f*x + e))/((a^7 - 7*a^6*b + 21*a^5*b^2 - 35*a^4*b^3 + 35*a^3*b^4 -
21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e)^4 + 2*(a^6*b - 6*a^5*b^2 + 15*a^
4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^5*b^
2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*f), -1/120*(24*(a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^9 - 8*(10*a^4 - 31*a^
3*b + 33*a^2*b^2 - 13*a*b^3 + b^4)*cos(f*x + e)^7 + 8*(15*a^4 + 10*a^3*b -
57*a^2*b^2 + 24*a*b^3 + 8*b^4)*cos(f*x + e)^5 + 25*(15*a^3*b + 25*a^2*b^2 -
 32*a*b^3 - 8*b^4)*cos(f*x + e)^3 + 15*((15*a^4 + 10*a^3*b - 57*a^2*b^2 + 2
4*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 + 40*a*b^3 + 8*b^4 + 2*(15*a^3
*b + 25*a^2*b^2 - 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(b/(a - b))*arctan(
-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 15*(15*a^2*b^2 + 40*a*b^3 + 8*b^
4)*cos(f*x + e))/((a^7 - 7*a^6*b + 21*a^5*b^2 - 35*a^4*b^3 + 35*a^3*b^4 - 2
1*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e)^4 + 2*(a^6*b - 6*a^5*b^2 + 15*a^4
*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^5*b^2
 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*f)]
```

giac [B] time = 5.40, size = 885, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/120*(15*(15*a^2*b + 40*a*b^2 + 8*b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*
x + e) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^5 - 5*a^4
*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sqrt(a*b - b^2)) + 30*(9*a^3*
```

```
b + 6*a^2*b^2 + 27*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 32*a^2*b^2
*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 40*a*b^3*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) + 27*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 54*a^2*
b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 24*a*b^3*(cos(f*x + e) - 1)
^2/(cos(f*x + e) + 1)^2 + 48*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
+ 9*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 16*a^2*b^2*(cos(f*x +
 e) - 1)^3/(cos(f*x + e) + 1)^3 - 8*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e
) + 1)^3)/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(a + 2
*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2) - 16*(8*a^2 + 5
9*a*b + 23*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 250*a*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) - 70*b^2*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 320*a*b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 140*b^2*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 - 270*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 90*b^2*(c
os(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 45*a*b*(cos(f*x + e) - 1)^4/(cos(
f*x + e) + 1)^4 + 45*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^5 -
 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*((cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 1)^5))/f
```

maple [B] time = 0.64, size = 844, normalized size = 3.20

$$-\frac{\left(\cos^5\left(fx+e\right)\right)a^2}{5f\left(a^3-3a^2b+3b^2a-b^3\right)\left(a^2-2ab+b^2\right)}+\frac{2\left(\cos^5\left(fx+e\right)\right)ab}{5f\left(a^3-3a^2b+3b^2a-b^3\right)\left(a^2-2ab+b^2\right)}-\frac{\left(\cos^5\left(fx\right.\right.}{5f\left(a^3-3a^2b+3b^2a-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/5/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a^2-2*a*b+b^2)*cos(f*x+e)^5*a^2+2/5/f/(a^
3-3*a^2*b+3*a*b^2-b^3)/(a^2-2*a*b+b^2)*cos(f*x+e)^5*a*b-1/5/f/(a^3-3*a^2*b+
3*a*b^2-b^3)/(a^2-2*a*b+b^2)*cos(f*x+e)^5*b^2+2/3/f/(a^3-3*a^2*b+3*a*b^2-b^
3)/(a^2-2*a*b+b^2)*a^2*cos(f*x+e)^3-1/3/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a^2-2*
a*b+b^2)*cos(f*x+e)^3*a*b-1/3/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a^2-2*a*b+b^2)*c
os(f*x+e)^3*b^2-1/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a^2-2*a*b+b^2)*a^2*cos(f*x+e
)-4/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a^2-2*a*b+b^2)*a*cos(f*x+e)*b-1/f/(a^3-3*a
^2*b+3*a*b^2-b^3)/(a^2-2*a*b+b^2)*cos(f*x+e)*b^2-9/8/f*b/(a-b)^5/(a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3*a^3+1/8/f*b^2/(a-b)^5/(a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3*a^2+1/f*b^3/(a-b)^5/(a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)^2*cos(f*x+e)^3*a-7/8/f*b^2/(a-b)^5/(a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)^2*cos(f*x+e)*a^2-1/f*b^3/(a-b)^5/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^
2*cos(f*x+e)*a+15/8/f*b/(a-b)^5/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a
-b)*b)^(1/2))*a^2+5/f*b^2/(a-b)^5/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/(
(a-b)*b)^(1/2))*a+1/f*b^3/(a-b)^5/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/(
(a-b)*b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 16.33, size = 1536, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] (b^(1/2)*atan(((a - b)^11*(2*tan(e/2 + (f*x)/2)^2*((b^(1/2)*(40*a*b + 15*a^
2 + 8*b^2)*(640*a^3*b^12 - 128*a^2*b^13 - 240*a^14*b + 400*a^4*b^11 - 11040
*a^5*b^10 + 39120*a^6*b^9 - 73344*a^7*b^8 + 84000*a^8*b^7 - 58560*a^9*b^6 +
 20640*a^10*b^5 + 1280*a^11*b^4 - 4528*a^12*b^3 + 1760*a^13*b^2))/(16*a*(a
- b)^(21/2)) - (b^(1/2)*(a - 2*b)*(40*a*b + 15*a^2 + 8*b^2)^2*(128*a^18 - 2
176*a^17*b + 256*a^2*b^16 - 3968*a^3*b^15 + 28800*a^4*b^14 - 129920*a^5*b^1
3 + 407680*a^6*b^12 - 943488*a^7*b^11 + 1665664*a^8*b^10 - 2288000*a^9*b^9
+ 2471040*a^10*b^8 - 2104960*a^11*b^7 + 1409408*a^12*b^6 - 733824*a^13*b^5
+ 291200*a^14*b^4 - 85120*a^15*b^3 + 17280*a^16*b^2))/(512*a*(a - b)^(33/2)
)) - (b^(1/2)*(a - 2*b)*(40*a*b + 15*a^2 + 8*b^2)^2*(1920*a^17*b - 128*a^18
 + 128*a^3*b^15 - 1920*a^4*b^14 + 13440*a^5*b^13 - 58240*a^6*b^12 + 174720*
a^7*b^11 - 384384*a^8*b^10 + 640640*a^9*b^9 - 823680*a^10*b^8 + 823680*a^11
*b^7 - 640640*a^12*b^6 + 384384*a^13*b^5 - 174720*a^14*b^4 + 58240*a^15*b^3
 - 13440*a^16*b^2))/(256*a*(a - b)^(33/2))))/(225*a^16*b + 64*a^2*b^15 - 16
80*a^4*b^13 + 3920*a^5*b^12 + 7665*a^6*b^11 - 50778*a^7*b^10 + 104685*a^8*b
^9 - 111960*a^9*b^8 + 57330*a^10*b^7 + 2660*a^11*b^6 - 20286*a^12*b^5 + 924
0*a^13*b^4 - 35*a^14*b^3 - 1050*a^15*b^2))*(40*a*b + 15*a^2 + 8*b^2))/(8*f*
(a - b)^(11/2)) - ((607*a^3*b + 64*a^4 + 274*a^2*b^2)/(60*(a - b)*(a^4 - 4*
a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e/2 + (f*x)/2)^14*(128*a*b^3 + 1
5*a^3*b + 24*b^4 + 85*a^2*b^2))/(2*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 +
 6*a^2*b^2)) + (tan(e/2 + (f*x)/2)^12*(936*a*b^3 - 365*a^3*b + 64*a^4 + 936
*b^4 + 1075*a^2*b^2))/(6*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2
)) + (tan(e/2 + (f*x)/2)^10*(4268*a*b^3 + 921*a^3*b - 224*a^4 + 1872*b^4 -
1545*a^2*b^2))/(6*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (t
an(e/2 + (f*x)/2)^4*(6224*a*b^3 - 671*a^3*b - 128*a^4 + 1832*b^4 + 5973*a^2
*b^2))/(30*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e/2
+ (f*x)/2)^6*(20696*a*b^3 + 867*a^3*b - 448*a^4 + 6280*b^4 - 935*a^2*b^2))/
(30*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e/2 + (f*x)
/2)^8*(21740*a*b^3 - 4064*a^3*b + 1312*a^4 + 12560*b^4 + 1527*a^2*b^2))/(30
*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e/2 + (f*x)/2)
^2*(1036*a*b^3 + 447*a^3*b + 32*a^4 + 2265*a^2*b^2))/(30*(a - b)*(a^4 - 4*a
^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (b*tan(e/2 + (f*x)/2)^16*(8*a*b^2 + 40
*a^2*b + 15*a^3))/(4*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))/
(f*(a^2*tan(e/2 + (f*x)/2)^18 + tan(e/2 + (f*x)/2)^4*(24*a*b - 4*a^2 + 16*b
^2) + tan(e/2 + (f*x)/2)^14*(24*a*b - 4*a^2 + 16*b^2) + tan(e/2 + (f*x)/2)^
6*(8*a*b - 4*a^2 + 80*b^2) + tan(e/2 + (f*x)/2)^12*(8*a*b - 4*a^2 + 80*b^2)
 + tan(e/2 + (f*x)/2)^8*(6*a^2 - 40*a*b + 160*b^2) + tan(e/2 + (f*x)/2)^10*
(6*a^2 - 40*a*b + 160*b^2) + a^2 + tan(e/2 + (f*x)/2)^2*(8*a*b + a^2) + tan
(e/2 + (f*x)/2)^16*(8*a*b + a^2)))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.81 $\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=180

$$\frac{\cos^3(e+fx)}{3f(a-b)^3}-\frac{(a+2b)\cos(e+fx)}{f(a-b)^4}-\frac{b(7a+4b)\sec(e+fx)}{8f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}-\frac{ab\sec(e+fx)}{4f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^2}-5\cdots$$

```
[Out] -(a+2*b)*cos(f*x+e)/(a-b)^4/f+1/3*cos(f*x+e)^3/(a-b)^3/f-1/4*a*b*sec(f*x+e)
/(a-b)^3/f/(a-b+b*sec(f*x+e)^2)^2-1/8*b*(7*a+4*b)*sec(f*x+e)/(a-b)^4/f/(a-b
+b*sec(f*x+e)^2)-5/8*(3*a+4*b)*arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/
2)/(a-b)^(9/2)/f
```

Rubi [A] time = 0.25, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 456, 1259, 1261, 205}

$$\frac{\cos^3(e+fx)}{3f(a-b)^3}-\frac{(a+2b)\cos(e+fx)}{f(a-b)^4}-\frac{b(7a+4b)\sec(e+fx)}{8f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}-\frac{ab\sec(e+fx)}{4f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^2}-5\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-5*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(8*(a -
 b)^(9/2)*f) - ((a + 2*b)*Cos[e + f*x])/((a - b)^4*f) + Cos[e + f*x]^3/(3*(
a - b)^3*f) - (a*b*Sec[e + f*x])/(4*(a - b)^3*f*(a - b + b*Sec[e + f*x]^2)^
2) - (b*(7*a + 4*b)*Sec[e + f*x])/(8*(a - b)^4*f*(a - b + b*Sec[e + f*x]^2)
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{ab\sec(e+fx)}{4(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b\,\text{Subst}\left(\int \frac{\frac{4}{(a-b)b}-\frac{4ax^2}{(a-b)^2 b}+\frac{3ax^4}{(a-b)^3}}{x^4\left(a-b+bx^2\right)^2}\,dx, x, \sec(\right.}{4f}$$

$$= -\frac{ab\sec(e+fx)}{4(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b(7a+4b)\sec(e+fx)}{8(a-b)^4 f\left(a-b+b\sec^2(e+fx)\right)} - -$$

$$= -\frac{ab\sec(e+fx)}{4(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b(7a+4b)\sec(e+fx)}{8(a-b)^4 f\left(a-b+b\sec^2(e+fx)\right)} - -$$

$$= -\frac{(a+2b)\cos(e+fx)}{(a-b)^4 f} + \frac{\cos^3(e+fx)}{3(a-b)^3 f} - \frac{ab\sec(e+fx)}{4(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)^2} -$$

$$= -\frac{5\sqrt{b}(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{9/2} f} - \frac{(a+2b)\cos(e+fx)}{(a-b)^4 f} + \frac{\cos^3(e+fx)}{3(a-b)^3 f} - -$$

Mathematica [A] time = 5.95, size = 230, normalized size = 1.28

$$\frac{\frac{2\left(3\cos(e+fx)\left(a\left(\frac{4b^2}{((a-b)\cos(2(e+fx))+a+b)^2}-\frac{9b}{(a-b)\cos(2(e+fx))+a+b}-3\right)+b\left(-\frac{4b}{(a-b)\cos(2(e+fx))+a+b}-9\right)\right)+(a-b)\cos(3(e+fx))\right)}{(a-b)^4}+\frac{15\sqrt{b}(3a+4b)\tan^{-1}\left(\right.}{(a-b}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((15*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sq
rt[b]])/(a - b)^(9/2) + (15*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] + Sqrt[
a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(9/2) + (2*(3*Cos[e + f*x]*(a*(-3 +
(4*b^2)/(a + b + (a - b)*Cos[2*(e + f*x)])^2 - (9*b)/(a + b + (a - b)*Cos[2
```

```
*(e + f*x)])) + b*(-9 - (4*b)/(a + b + (a - b)*Cos[2*(e + f*x)]))) + (a - b
)*Cos[3*(e + f*x)]))/(a - b)^4)/(24*f)
```

fricas [B] time = 0.67, size = 775, normalized size = 4.31

$$\left[\frac{16\left(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)^{7}-16\left(3\,a^{3}-2\,a^{2}b-5\,ab^{2}+4\,b^{3}\right)\cos\left(fx+e\right)^{5}-50\left(3\,a^{2}b+ab^{2}-4\,b^{3}\right.}{48\left(\left(a^{6}-6\,a^{5}b+15\,a^{4}b^{2}-20\,a^{3}b^{3}+15\,a^{2}b^{4}-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 16*(3*a^3 - 2*a^
2*b - 5*a*b^2 + 4*b^3)*cos(f*x + e)^5 - 50*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*
x + e)^3 + 15*((3*a^3 - 2*a^2*b - 5*a*b^2 + 4*b^3)*cos(f*x + e)^4 + 3*a*b^2
 + 4*b^3 + 2*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*x + e)^2)*sqrt(-b/(a - b))*log
(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a
 - b)*cos(f*x + e)^2 + b)) - 30*(3*a*b^2 + 4*b^3)*cos(f*x + e))/((a^6 - 6*a
^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*f*cos(f*x + e)
^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*f*cos(
f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f), 1/24*(8*
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 8*(3*a^3 - 2*a^2*b - 5*a*b
^2 + 4*b^3)*cos(f*x + e)^5 - 25*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*x + e)^3 -
15*((3*a^3 - 2*a^2*b - 5*a*b^2 + 4*b^3)*cos(f*x + e)^4 + 3*a*b^2 + 4*b^3 +
2*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*x + e)^2)*sqrt(b/(a - b))*arctan(-(a - b)
*sqrt(b/(a - b))*cos(f*x + e)/b) - 15*(3*a*b^2 + 4*b^3)*cos(f*x + e))/((a^6
 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*f*cos(f*
x + e)^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*
f*cos(f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f)]
```

giac [B] time = 7.40, size = 563, normalized size = 3.13

$$\frac{a^{6}f^{17}\cos\left(fx+e\right)^{3}-6\,a^{5}bf^{17}\cos\left(fx+e\right)^{3}+15\,a^{4}b^{2}f^{17}\cos\left(fx+e\right)^{3}-20\,a^{3}b^{3}f^{17}\cos\left(fx+e\right)^{3}+15\,a^{2}b^{4}f^{17}\,\mathrm{c}}{3\left(a^{9}f^{18}-9\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/3*(a^6*f^17*cos(f*x + e)^3 - 6*a^5*b*f^17*cos(f*x + e)^3 + 15*a^4*b^2*f^1
7*cos(f*x + e)^3 - 20*a^3*b^3*f^17*cos(f*x + e)^3 + 15*a^2*b^4*f^17*cos(f*x
 + e)^3 - 6*a*b^5*f^17*cos(f*x + e)^3 + b^6*f^17*cos(f*x + e)^3 - 3*a^6*f^1
7*cos(f*x + e) + 9*a^5*b*f^17*cos(f*x + e) - 30*a^3*b^3*f^17*cos(f*x + e) +
 45*a^2*b^4*f^17*cos(f*x + e) - 27*a*b^5*f^17*cos(f*x + e) + 6*b^6*f^17*cos
(f*x + e))/(a^9*f^18 - 9*a^8*b*f^18 + 36*a^7*b^2*f^18 - 84*a^6*b^3*f^18 + 1
26*a^5*b^4*f^18 - 126*a^4*b^5*f^18 + 84*a^3*b^6*f^18 - 36*a^2*b^7*f^18 + 9*
a*b^8*f^18 - b^9*f^18) + 5/8*(3*a*b + 4*b^2)*arctan((a*cos(f*x + e) - b*cos
(f*x + e))/sqrt(a*b - b^2))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sq
rt(a*b - b^2)*f) - 1/8*(9*a^2*b*cos(f*x + e)^3/f - 5*a*b^2*cos(f*x + e)^3/f
 - 4*b^3*cos(f*x + e)^3/f + 7*a*b^2*cos(f*x + e)/f + 4*b^3*cos(f*x + e)/f)/
((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a*cos(f*x + e)^2 - b*cos(f*x
+ e)^2 + b)^2)
```

maple [B] time = 0.59, size = 504, normalized size = 2.80

$$\frac{a\left(\cos^{3}\left(fx+e\right)\right)}{3f\left(a^{3}-3a^{2}b+3b^{2}a-b^{3}\right)\left(a-b\right)}-\frac{b\left(\cos^{3}\left(fx+e\right)\right)}{3f\left(a^{3}-3a^{2}b+3b^{2}a-b^{3}\right)\left(a-b\right)}-\frac{a\cos\left(fx+e\right)}{f\left(a^{3}-3a^{2}b+3b^{2}a-b^{3}\right)\left(a-b\right)}-\frac{}{f\left(a^{3}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] 1/3/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a-b)*a*cos(f*x+e)^3-1/3/f/(a^3-3*a^2*b+3*a
*b^2-b^3)/(a-b)*b*cos(f*x+e)^3-1/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a-b)*a*cos(f*
x+e)-2/f/(a^3-3*a^2*b+3*a*b^2-b^3)/(a-b)*cos(f*x+e)*b-9/8/f*b/(a-b)^4/(a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)^2*a^2*cos(f*x+e)^3+5/8/f*b^2/(a-b)^4/(a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3*a+1/2/f*b^3/(a-b)^4/(a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3-7/8/f*b^2/(a-b)^4/(a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)^2*a*cos(f*x+e)-1/2/f*b^3/(a-b)^4/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)^2*cos(f*x+e)+15/8/f*b/(a-b)^4/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((
a-b)*b)^(1/2))*a+5/2/f*b^2/(a-b)^4/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/
((a-b)*b)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [**B**] time = 15.45, size = 1154, normalized size = 6.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] - ((6*a*b^2 + 83*a^2*b + 16*a^3)/(12*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3
)) + (tan(e/2 + (f*x)/2)^2*(299*a*b^2 - 8*a^3 + 24*b^3))/(6*(a - b)*(3*a*b^
2 - 3*a^2*b + a^3 - b^3)) + (5*a*tan(e/2 + (f*x)/2)^12*(3*a*b + 4*b^2))/(4*
(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e/2 + (f*x)/2)^10*(28*a*b^3
 - 32*a^3*b + 8*a^4 + 8*b^4 + 93*a^2*b^2))/(2*a*(a - b)*(3*a*b^2 - 3*a^2*b
+ a^3 - b^3)) + (tan(e/2 + (f*x)/2)^6*(546*a*b^3 - 144*a^3*b + 56*a^4 + 36*
b^4 + 31*a^2*b^2))/(3*a*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e/2
 + (f*x)/2)^4*(1208*a*b^3 + 71*a^3*b - 96*a^4 + 48*b^4 + 344*a^2*b^2))/(12*
a*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e/2 + (f*x)/2)^8*(1704*a*
b^3 + 569*a^3*b - 176*a^4 + 144*b^4 - 666*a^2*b^2))/(12*a*(a - b)*(3*a*b^2
- 3*a^2*b + a^3 - b^3)))/(f*(tan(e/2 + (f*x)/2)^2*(8*a*b - a^2) + tan(e/2 +
 (f*x)/2)^12*(8*a*b - a^2) + a^2*tan(e/2 + (f*x)/2)^14 + tan(e/2 + (f*x)/2)
^4*(8*a*b - 3*a^2 + 16*b^2) + tan(e/2 + (f*x)/2)^10*(8*a*b - 3*a^2 + 16*b^2
) + tan(e/2 + (f*x)/2)^6*(3*a^2 - 16*a*b + 48*b^2) + tan(e/2 + (f*x)/2)^8*(
3*a^2 - 16*a*b + 48*b^2) + a^2)) - (5*b^(1/2)*atan((2*(tan(e/2 + (f*x)/2)^2
*((5*b^(1/2)*(3*a + 4*b)*(240*a^11*b + 320*a^2*b^10 - 2320*a^3*b^9 + 7040*a
^4*b^8 - 11200*a^5*b^7 + 8960*a^6*b^6 - 1120*a^7*b^5 - 4480*a^8*b^4 + 4160*
a^9*b^3 - 1600*a^10*b^2))/(16*a*(a - b)^(17/2)) - (25*b^(1/2)*(a - 2*b)*(3*
a + 4*b)^2*(1792*a^14*b - 128*a^15 + 256*a^2*b^13 - 3200*a^3*b^12 + 18432*a
^4*b^11 - 64768*a^5*b^10 + 154880*a^6*b^9 - 266112*a^7*b^8 + 337920*a^8*b^7
 - 321024*a^9*b^6 + 228096*a^10*b^5 - 119680*a^11*b^4 + 45056*a^12*b^3 - 11
520*a^13*b^2))/(512*a*(a - b)^(27/2))) - (25*b^(1/2)*(a - 2*b)*(3*a + 4*b)^
2*(128*a^15 - 1536*a^14*b + 128*a^3*b^12 - 1536*a^4*b^11 + 8448*a^5*b^10 -
28160*a^6*b^9 + 63360*a^7*b^8 - 101376*a^8*b^7 + 118272*a^9*b^6 - 101376*a^
10*b^5 + 63360*a^11*b^4 - 28160*a^12*b^3 + 8448*a^13*b^2))/(512*a*(a - b)^(
27/2)))*(a - b)^9)/(225*a^12*b + 400*a^2*b^11 - 2600*a^3*b^10 + 6625*a^4*b^
```

```
9 - 7400*a^5*b^8 + 700*a^6*b^7 + 7000*a^7*b^6 - 6650*a^8*b^5 + 1000*a^9*b^4
 + 1900*a^10*b^3 - 1200*a^11*b^2))*(3*a + 4*b))/(8*f*(a - b)^(9/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.82 $\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{15\cos(e+fx)}{8f(a-b)^3}+\frac{5\cos(e+fx)}{8f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)}+\frac{\cos(e+fx)}{4f(a-b)\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8f(a-b)^{7/2}}$$

```
[Out] -15/8*cos(f*x+e)/(a-b)^3/f+1/4*cos(f*x+e)/(a-b)/f/(a-b+b*sec(f*x+e)^2)^2+5/
8*cos(f*x+e)/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)-15/8*arctan(sec(f*x+e)*b^(1/2)/
(a-b)^(1/2))*b^(1/2)/(a-b)^(7/2)/f
```

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3664, 290, 325, 205}

$$-\frac{15\cos(e+fx)}{8f(a-b)^3}+\frac{5\cos(e+fx)}{8f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)}+\frac{\cos(e+fx)}{4f(a-b)\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8f(a-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(8*(a - b)^(7/2)*f
) - (15*Cos[e + f*x])/(8*(a - b)^3*f) + Cos[e + f*x]/(4*(a - b)*f*(a - b +
b*Sec[e + f*x]^2)^2) + (5*Cos[e + f*x])/(8*(a - b)^2*f*(a - b + b*Sec[e + f
*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)}{4(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{4(a-b)f}$$

$$= \frac{\cos(e+fx)}{4(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{5\cos(e+fx)}{8(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)} + \frac{15\,\text{Sub}}{}$$

$$= -\frac{15\cos(e+fx)}{8(a-b)^3f} + \frac{\cos(e+fx)}{4(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{5\cos(e+fx)}{8(a-b)^2f\left(a-b+b\sec^2\right.}$$

$$= -\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{7/2}f} - \frac{15\cos(e+fx)}{8(a-b)^3f} + \frac{\cos(e+fx)}{4(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2}$$

Mathematica [A] time = 1.77, size = 170, normalized size = 1.23

$$\frac{\frac{2\cos(e+fx)\left(\frac{4b^2}{((a-b)\cos(2(e+fx))+a+b)^2}-\frac{9b}{(a-b)\cos(2(e+fx))+a+b}-4\right)}{(a-b)^3}+\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{7/2}}+\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{7/2}}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((15*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a -
 b)^(7/2) + (15*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqr
t[b]])/(a - b)^(7/2) + (2*Cos[e + f*x]*(-4 + (4*b^2)/(a + b + (a - b)*Cos[2
*(e + f*x)])^2 - (9*b)/(a + b + (a - b)*Cos[2*(e + f*x)])))/(a - b)^3)/(8*f
)
```

fricas [B] time = 0.61, size = 556, normalized size = 4.03

$$\left[-\frac{16\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^5+50\left(ab-b^2\right)\cos\left(fx+e\right)^3+30b^2\cos\left(fx+e\right)+15\left(\left(a^2-2ab+b^2\right)\cos\left(f\right.\right.}{16\left(\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5\right)f\cos\left(fx+e\right)^4+2\left(a^4b-4a^3b^2+6\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 + 50*(a*b - b^2)*cos(f*x + e)
^3 + 30*b^2*cos(f*x + e) + 15*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b
- b^2)*cos(f*x + e)^2 + b^2)*sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2
- 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)
))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e
)^4 + 2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos(f*x + e)^2 +
```

```
(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f), -1/8*(8*(a^2 - 2*a*b + b^2)*cos(f
*x + e)^5 + 25*(a*b - b^2)*cos(f*x + e)^3 + 15*b^2*cos(f*x + e) + 15*((a^2
- 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)*sqrt(b/
(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b))/((a^5 - 5*a^4*b +
 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)^4 + 2*(a^4*b - 4*a
^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos(f*x + e)^2 + (a^3*b^2 - 3*a^2*b^3
 + 3*a*b^4 - b^5)*f)]
```

giac [A] time = 4.68, size = 223, normalized size = 1.62

$$-\frac{f^5\cos\left(fx+e\right)}{a^3f^6-3\,a^2bf^6+3\,ab^2f^6-b^3f^6}+\frac{15\,b\arctan\left(\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)}{\sqrt{ab-b^2}}\right)}{8\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\sqrt{ab-b^2}\,f}-\frac{\frac{9\,ab\cos\left(fx+e\right)^3}{f}-\frac{9\,b^2\cos\left(f\right.}{f}}{8\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\left(a\cos\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -f^5*cos(f*x + e)/(a^3*f^6 - 3*a^2*b*f^6 + 3*a*b^2*f^6 - b^3*f^6) + 15/8*b*
arctan((a*cos(f*x + e) - b*cos(f*x + e))/sqrt(a*b - b^2))/((a^3 - 3*a^2*b +
 3*a*b^2 - b^3)*sqrt(a*b - b^2)*f) - 1/8*(9*a*b*cos(f*x + e)^3/f - 9*b^2*co
s(f*x + e)^3/f + 7*b^2*cos(f*x + e)/f)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(a*
cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^2)
```

maple [A] time = 0.47, size = 221, normalized size = 1.60

$$-\frac{\cos\left(fx+e\right)}{f\left(a^3-3a^2b+3b^2a-b^3\right)}-\frac{9ba\left(\cos^3\left(fx+e\right)\right)}{8f\left(a-b\right)^3\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)^2}+\frac{9b^2\left(\mathrm{c}\right.}{8f\left(a-b\right)^3\left(a\left(\cos^2\left(fx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/f/(a^3-3*a^2*b+3*a*b^2-b^3)*cos(f*x+e)-9/8/f*b/(a-b)^3/(a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)^2*a*cos(f*x+e)^3+9/8/f*b^2/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)^2*cos(f*x+e)^3-7/8/f*b^2/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)^2*cos(f*x+e)+15/8/f*b/(a-b)^3/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((
a-b)*b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 14.87, size = 780, normalized size = 5.65

$$-\frac{\frac{8\,a^2+9\,a\,b-2\,b^2}{4\,\left(a-b\right)\,\left(a^2-2\,a\,b+b^2\right)}-\frac{{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(16\,a^4-41\,a^3\,b+27\,a^2\,b^2-40\,a\,b^3+8\,b^4\right)}{2\,a^2\,\left(a-b\right)\,\left(a^2-2\,a\,b+b^2\right)}+\frac{{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(24\,a^4-64\,a^3\,b+53\,a^2\,b^2+40\,a\,b^3-8\,b^4\right)}{2\,a^2\,\left(a-b\right)\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\mathrm{tan}\left(\frac{e}{2}\right.}{}}{f\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(8\,a\,b-3\,a^2\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(8\,a\,b-3\,a^2\right)+a^2\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(2\,a^2-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] - ((9*a*b + 8*a^2 - 2*b^2)/(4*(a - b)*(a^2 - 2*a*b + b^2)) - (tan(e/2 + (f*
x)/2)^6*(16*a^4 - 41*a^3*b - 40*a*b^3 + 8*b^4 + 27*a^2*b^2))/(2*a^2*(a - b)
*(a^2 - 2*a*b + b^2)) + (tan(e/2 + (f*x)/2)^4*(40*a*b^3 - 64*a^3*b + 24*a^4
 - 8*b^4 + 53*a^2*b^2))/(2*a^2*(a - b)*(a^2 - 2*a*b + b^2)) + (tan(e/2 + (f
*x)/2)^8*(24*a*b^2 - 9*a^2*b + 8*a^3 - 8*b^3))/(4*a*(a - b)*(a^2 - 2*a*b +
b^2)) + (tan(e/2 + (f*x)/2)^2*(27*a*b^2 + 23*a^2*b - 16*a^3 - 4*b^3))/(2*a*
(a - b)*(a^2 - 2*a*b + b^2)))/(f*(tan(e/2 + (f*x)/2)^2*(8*a*b - 3*a^2) + ta
n(e/2 + (f*x)/2)^8*(8*a*b - 3*a^2) + a^2*tan(e/2 + (f*x)/2)^10 + tan(e/2 +
(f*x)/2)^4*(2*a^2 - 8*a*b + 16*b^2) + tan(e/2 + (f*x)/2)^6*(2*a^2 - 8*a*b +
 16*b^2) + a^2)) - (15*b^(1/2)*atan(((a - b)^7*(2*tan(e/2 + (f*x)/2)^2*((b^
(1/2)*(225*a^8*b + 225*a^2*b^7 - 1350*a^3*b^6 + 3375*a^4*b^5 - 4500*a^5*b^4
 + 3375*a^6*b^3 - 1350*a^7*b^2))/(a*(a - b)^(13/2)) + (225*b^(1/2)*(a - 2*b
)*(128*a^12 - 1408*a^11*b + 256*a^2*b^10 - 2432*a^3*b^9 + 10368*a^4*b^8 - 2
6112*a^5*b^7 + 43008*a^6*b^6 - 48384*a^7*b^5 + 37632*a^8*b^4 - 19968*a^9*b^
3 + 6912*a^10*b^2))/(512*a*(a - b)^(21/2))) + (225*b^(1/2)*(a - 2*b)*(1152*
a^11*b - 128*a^12 + 128*a^3*b^9 - 1152*a^4*b^8 + 4608*a^5*b^7 - 10752*a^6*b
^6 + 16128*a^7*b^5 - 16128*a^8*b^4 + 10752*a^9*b^3 - 4608*a^10*b^2))/(256*a
*(a - b)^(21/2))))/(225*a^8*b + 225*a^2*b^7 - 1350*a^3*b^6 + 3375*a^4*b^5 -
 4500*a^5*b^4 + 3375*a^6*b^3 - 1350*a^7*b^2)))/(8*f*(a - b)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.83 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=166

$$-\frac{\tanh^{-1}(\cos(e+fx))}{a^3f}-\frac{b(7a-4b)\sec(e+fx)}{8a^2f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)}-\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^3f(a-b)^{5/2}}-\frac{}{4}$$

```
[Out] -arctanh(cos(f*x+e))/a^3/f-1/4*b*sec(f*x+e)/a/(a-b)/f/(a-b+b*sec(f*x+e)^2)^
2-1/8*(7*a-4*b)*b*sec(f*x+e)/a^2/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)-1/8*(15*a^2
-20*a*b+8*b^2)*arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^3/(a-b)^(5/
2)/f
```

Rubi [A] time = 0.22, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3664, 414, 527, 522, 207, 205}

$$-\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^3f(a-b)^{5/2}}-\frac{b(7a-4b)\sec(e+fx)}{8a^2f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)}-\frac{\tanh^{-1}(\cos(e+fx))}{a^3f}-\frac{}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a -
b]])/(8*a^3*(a - b)^(5/2)*f) - ArcTanh[Cos[e + f*x]]/(a^3*f) - (b*Sec[e + f
*x])/(4*a*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^2) - ((7*a - 4*b)*b*Sec[e +
f*x])/(8*a^2*(a - b)^2*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{4a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-b-3bx^2}{\left(-1+x^2\right)\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+f\ldots\right.}{4a(a-b)f}$$

$$= -\frac{b\sec(e+fx)}{4a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7a-4b)b\sec(e+fx)}{8a^2(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)} + \ldots$$

$$= -\frac{b\sec(e+fx)}{4a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7a-4b)b\sec(e+fx)}{8a^2(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)} + \ldots$$

$$= -\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^3(a-b)^{5/2}f} - \frac{\tanh^{-1}(\cos(e+fx))}{a^3f} - \frac{\ldots}{4a(a-b)f\ldots}$$

Mathematica [A] time = 3.44, size = 247, normalized size = 1.49

$$\frac{\frac{8a^2b^2\cos(e+fx)}{(a-b)^2((a-b)\cos(2(e+fx))+a+b)^2} + \frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}} + \frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}} - \ldots}{8a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e +
f*x)/2])/Sqrt[b]])/(a - b)^(5/2) + (Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTa
n[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(5/2) + (8*a^2
*b^2*Cos[e + f*x])/((a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)])^2) - (2*a*
(9*a - 4*b)*b*Cos[e + f*x])/((a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)]))
- 8*Log[Cos[(e + f*x)/2]] + 8*Log[Sin[(e + f*x)/2]])/(8*a^3*f)
```

fricas [B] time = 0.79, size = 1050, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(9*a^3*b - 13*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^3 - ((15*a^4 - 50*a
^3*b + 63*a^2*b^2 - 36*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 20*a*b^
3 + 8*b^4 + 2*(15*a^3*b - 35*a^2*b^2 + 28*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sq
rt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos
(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*(7*a^2*b^2 - 4*a*b^3)*cos(
f*x + e) + 8*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^4 +
a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*cos(f*x + e
)^2)*log(1/2*cos(f*x + e) + 1/2) - 8*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
+ b^4)*cos(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*
a*b^3 - b^4)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 - 4*a^6*b
+ 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f
), -1/8*((9*a^3*b - 13*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^3 + ((15*a^4 - 50*a^
3*b + 63*a^2*b^2 - 36*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 20*a*b^3
 + 8*b^4 + 2*(15*a^3*b - 35*a^2*b^2 + 28*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqr
t(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + (7*a^2*b^2 -
 4*a*b^3)*cos(f*x + e) + 4*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos
(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^
4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) - 4*((a^4 - 4*a^3*b + 6*a^2*
b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b -
3*a^2*b^2 + 3*a*b^3 - b^4)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((
a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 + 2*(a^6*
b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^
3 + a^3*b^4)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(1/4/a^3*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1))))+(-15
*a^2*b+20*a*b^2-8*b^3)*1/4/(4*a^5-8*a^4*b+4*a^3*b^2)/sqrt(-b^2+a*b)*atan((-
a*cos(f*x+exp(1))+b*cos(f*x+exp(1))+b)/(sqrt(-b^2+a*b)*cos(f*x+exp(1))+sqrt
(-b^2+a*b)))+(9*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^3*b-28*((1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^2*b^2+16*((1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1))))^3*a*b^3-27*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^3*
b+90*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2*b^2-120*((1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1))))^2*a*b^3+48*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1))))^2*b^4+27*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^3*b-68*(1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1)))*a^2*b^2+32*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1)))*a*b^3-9*a^3*b+6*a^2*b^2)/(8*a^5-16*a^4*b+8*a^3*b^2)/(((1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1))))^2*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+4*
(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a)^2)
```

maple [B] time = 0.79, size = 408, normalized size = 2.46

$$-\frac{9b\left(\cos^3\left(fx+e\right)\right)}{8fa\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)^2(a-b)}+\frac{b^2\left(\cos^3\left(fx+e\right)\right)}{2fa^2\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)^2(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -9/8/f*b/a/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2/(a-b)*cos(f*x+e)^3+1/2/f*b^2
/a^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2/(a-b)*cos(f*x+e)^3-7/8/f*b^2/a/(a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2/(a^2-2*a*b+b^2)*cos(f*x+e)+1/2/f*b^3/a^2/(
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2/(a^2-2*a*b+b^2)*cos(f*x+e)+15/8/f*b/a/(a
^2-2*a*b+b^2)/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))-5/2/
f*b^2/a^2/(a^2-2*a*b+b^2)/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)
^(1/2))+1/f*b^3/a^3/(a^2-2*a*b+b^2)/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)
/((a-b)*b)^(1/2))+1/2/f/a^3*ln(-1+cos(f*x+e))-1/2/f/a^3*ln(1+cos(f*x+e))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [**B**] time = 16.27, size = 1844, normalized size = 11.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^3),x)
```

```
[Out] log(tan(e/2 + (f*x)/2))/(a^3*f) - ((3*(3*a*b - 2*b^2))/(4*a*(a^2 - 2*a*b +
b^2)) + (3*tan(e/2 + (f*x)/2)^4*(40*a*b^3 + 9*a^3*b - 16*b^4 - 30*a^2*b^2))
/(4*a^3*(a^2 - 2*a*b + b^2)) - (tan(e/2 + (f*x)/2)^6*(9*a^2*b - 28*a*b^2 +
16*b^3))/(4*a^2*(a^2 - 2*a*b + b^2)) - (tan(e/2 + (f*x)/2)^2*(27*a^2*b - 68
*a*b^2 + 32*b^3))/(4*a^2*(a^2 - 2*a*b + b^2)))/(f*(tan(e/2 + (f*x)/2)^2*(8*
a*b - 4*a^2) + tan(e/2 + (f*x)/2)^6*(8*a*b - 4*a^2) + a^2*tan(e/2 + (f*x)/2
)^8 + tan(e/2 + (f*x)/2)^4*(6*a^2 - 16*a*b + 16*b^2) + a^2)) + (b^(1/2)*ata
n(((tan(e/2 + (f*x)/2)^2*((((b^(3/2)*(15*a^2 - 20*a*b + 8*b^2)^3*(4096*a^15
*b - 128*a^16 + 6144*a^7*b^9 - 46080*a^8*b^8 + 150784*a^9*b^7 - 281216*a^10
*b^6 + 327168*a^11*b^5 - 243584*a^12*b^4 + 113920*a^13*b^3 - 31104*a^14*b^2
))/(32768*a^9*(a - b)^(15/2)*(a^11 - 6*a^10*b + a^5*b^6 - 6*a^6*b^5 + 15*a^
7*b^4 - 20*a^8*b^3 + 15*a^9*b^2)) - (b^(1/2)*(15*a^2 - 20*a*b + 8*b^2)*(153
6*a*b^9 + 720*a^9*b - 11520*a^2*b^8 + 37760*a^3*b^7 - 70400*a^4*b^6 + 81384
*a^5*b^5 - 59564*a^6*b^4 + 26864*a^7*b^3 - 6780*a^8*b^2))/(128*a^3*(a - b)^
(5/2)*(a^11 - 6*a^10*b + a^5*b^6 - 6*a^6*b^5 + 15*a^7*b^4 - 20*a^8*b^3 + 15
*a^9*b^2)))*(3072*a*b^4 - 1090*a^4*b + 111*a^5 - 768*b^5 - 4752*a^2*b^3 + 3
424*a^3*b^2))/(2*a^5*(a - b)^(13/2)*(960*a*b^4 - 1055*a^4*b + 256*a^5 - 192
*b^5 - 1920*a^2*b^3 + 1960*a^3*b^2)) + (((576*a*b^6 - 64*b^7 - 1920*a^2*b^5
 + 3160*a^3*b^4 - 2625*a^4*b^3 + 900*a^5*b^2)/(8*(a^11 - 6*a^10*b + a^5*b^6
 - 6*a^6*b^5 + 15*a^7*b^4 - 20*a^8*b^3 + 15*a^9*b^2)) + (b*(15*a^2 - 20*a*b
 + 8*b^2)^2*(2768*a^12*b - 128*a^13 + 6144*a^4*b^9 - 46080*a^5*b^8 + 150656
*a^6*b^7 - 279104*a^7*b^6 + 318672*a^8*b^5 - 228160*a^9*b^4 + 99424*a^10*b^
3 - 24192*a^11*b^2))/(2048*a^6*(a - b)^5*(a^11 - 6*a^10*b + a^5*b^6 - 6*a^6
*b^5 + 15*a^7*b^4 - 20*a^8*b^3 + 15*a^9*b^2)))*(1344*a*b^4 - 205*a^4*b + 8*
a^5 - 384*b^5 - 1752*a^2*b^3 + 980*a^3*b^2))/(a^5*b^(1/2)*(a - b)^6*(960*a*
b^4 - 1055*a^4*b + 256*a^5 - 192*b^5 - 1920*a^2*b^3 + 1960*a^3*b^2))) + (((
b^(1/2)*(15*a^2 - 20*a*b + 8*b^2)*(240*a^8*b - 320*a^3*b^6 + 1600*a^4*b^5 -
 3232*a^5*b^4 + 3208*a^6*b^3 - 1505*a^7*b^2))/(64*a^3*(a - b)^(5/2)*(a^10 -
```

```
 4*a^9*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^2)) + (b^(3/2)*(15*a^2 - 20*a*b +
8*b^2)^3*(64*a^15 - 512*a^14*b + 256*a^9*b^6 - 1280*a^10*b^5 + 2624*a^11*b^
4 - 2816*a^12*b^3 + 1664*a^13*b^2))/(16384*a^9*(a - b)^(15/2)*(a^10 - 4*a^9
*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^2)))*(3072*a*b^4 - 1090*a^4*b + 111*a^5
- 768*b^5 - 4752*a^2*b^3 + 3424*a^3*b^2))/(2*a^5*(a - b)^(13/2)*(960*a*b^4
- 1055*a^4*b + 256*a^5 - 192*b^5 - 1920*a^2*b^3 + 1960*a^3*b^2)) - (((64*b^
6 - 320*a*b^5 + 640*a^2*b^4 - 600*a^3*b^3 + 225*a^4*b^2)/(4*(a^10 - 4*a^9*b
 + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^2)) - (b*(15*a^2 - 20*a*b + 8*b^2)^2*(64*a
^12 - 752*a^11*b + 512*a^6*b^6 - 2560*a^7*b^5 + 5216*a^8*b^4 - 5424*a^9*b^3
 + 2944*a^10*b^2))/(1024*a^6*(a - b)^5*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^7*b^
3 + 6*a^8*b^2)))*(1344*a*b^4 - 205*a^4*b + 8*a^5 - 384*b^5 - 1752*a^2*b^3 +
 980*a^3*b^2))/(a^5*b^(1/2)*(a - b)^6*(960*a*b^4 - 1055*a^4*b + 256*a^5 - 1
92*b^5 - 1920*a^2*b^3 + 1960*a^3*b^2)))*(256*a^13*(a - b)^(15/2) - 1536*a^1
2*b*(a - b)^(15/2) + 256*a^7*b^6*(a - b)^(15/2) - 1536*a^8*b^5*(a - b)^(15/
2) + 3840*a^9*b^4*(a - b)^(15/2) - 5120*a^10*b^3*(a - b)^(15/2) + 3840*a^11
*b^2*(a - b)^(15/2)))/(225*a^4*b - 320*a*b^4 + 64*b^5 + 640*a^2*b^3 - 600*a
^3*b^2))*(15*a^2 - 20*a*b + 8*b^2))/(8*a^3*f*(a - b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.84 $\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=205

$$-\frac{(a-6b)\tanh^{-1}(\cos(e+fx))}{2a^4 f}-\frac{b(11a-12b)\sec(e+fx)}{8a^3 f(a-b)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3b\sec(e+fx)}{4a^2 f\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{\sqrt{b}\left(15a^2 -\right.}{}$$

```
[Out] -1/2*(a-6*b)*arctanh(cos(f*x+e))/a^4/f-1/2*cot(f*x+e)*csc(f*x+e)/a/f/(a-b+b
*sec(f*x+e)^2)^2-3/4*b*sec(f*x+e)/a^2/f/(a-b+b*sec(f*x+e)^2)^2-1/8*(11*a-12
*b)*b*sec(f*x+e)/a^3/(a-b)/f/(a-b+b*sec(f*x+e)^2)-1/8*(15*a^2-40*a*b+24*b^2
)*arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^4/(a-b)^(3/2)/f
```

Rubi [A] time = 0.29, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3664, 471, 527, 522, 207, 205}

$$-\frac{\sqrt{b}\left(15a^2-40ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^4 f(a-b)^{3/2}}-\frac{b(11a-12b)\sec(e+fx)}{8a^3 f(a-b)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3b\sec(e+fx)}{4a^2 f\left(a+b\sec^2(e+fx)-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(Sqrt[b]*(15*a^2 - 40*a*b + 24*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a -
 b]])/(8*a^4*(a - b)^(3/2)*f) - ((a - 6*b)*ArcTanh[Cos[e + f*x]])/(2*a^4*f)
 - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*(a - b + b*Sec[e + f*x]^2)^2) - (3*b*
Sec[e + f*x])/(4*a^2*f*(a - b + b*Sec[e + f*x]^2)^2) - ((11*a - 12*b)*b*Sec
[e + f*x])/(8*a^3*(a - b)*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a-b-5bx^2}{\left(-1+x^2\right)\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{3b\sec(e+fx)}{4a^2f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{2(2}{}\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{3b\sec(e+fx)}{4a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(11a-}{8a^3(a-b)f(}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{3b\sec(e+fx)}{4a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(11a-}{8a^3(a-b)f(}$$

$$= -\frac{\sqrt{b}\left(15a^2-40ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^4(a-b)^{3/2}f} - \frac{(a-6b)\tanh^{-1}(\cos(e+fx))}{2a^4f}$$

Mathematica [B] time = 6.55, size = 414, normalized size = 2.02

$$\frac{(a-6b)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2a^4f} + \frac{(6b-a)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2a^4f} + \frac{8b^2\cos(e+fx)-9ab\cos(e+fx)}{4a^3f(a-b)(a\cos(2(e+fx))+a-b\cos(2(e+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (Sqrt[a - b]*Sqrt[b]*(15*a^2 - 40*a*b + 24*b^2)*ArcTan[(Sec[(e + f*x)/2]*(S
qrt[a - b]*Cos[(e + f*x)/2] - Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(8*a^4*(
-a + b)^2*f) + (Sqrt[a - b]*Sqrt[b]*(15*a^2 - 40*a*b + 24*b^2)*ArcTan[(Sec[
```

```
(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/2] + Sqrt[a]*Sin[(e + f*x)/2]))/Sqr
t[b]])/(8*a^4*(-a + b)^2*f) + (b^2*Cos[e + f*x])/(a^2*(a - b)*f*(a + b + a*
Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])^2) + (-9*a*b*Cos[e + f*x] + 8*b^2*Co
s[e + f*x])/(4*a^3*(a - b)*f*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x
)])) - Csc[(e + f*x)/2]^2/(8*a^3*f) + ((-a + 6*b)*Log[Cos[(e + f*x)/2]])/(2
*a^4*f) + ((a - 6*b)*Log[Sin[(e + f*x)/2]])/(2*a^4*f) + Sec[(e + f*x)/2]^2/
(8*a^3*f)
```

fricas [B] time = 0.96, size = 1419, normalized size = 6.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*a^4 - 21*a^3*b + 29*a^2*b^2 - 12*a*b^3)*cos(f*x + e)^5 + 2*(17*
a^3*b - 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^3 - ((15*a^4 - 70*a^3*b + 119*a
^2*b^2 - 88*a*b^3 + 24*b^4)*cos(f*x + e)^6 - (15*a^4 - 100*a^3*b + 229*a^2*
b^2 - 216*a*b^3 + 72*b^4)*cos(f*x + e)^4 - 15*a^2*b^2 + 40*a*b^3 - 24*b^4 -
 (30*a^3*b - 125*a^2*b^2 + 168*a*b^3 - 72*b^4)*cos(f*x + e)^2)*sqrt(-b/(a -
 b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e)
 - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*(11*a^2*b^2 - 12*a*b^3)*cos(f*x + e
) - 4*((a^4 - 9*a^3*b + 21*a^2*b^2 - 19*a*b^3 + 6*b^4)*cos(f*x + e)^6 - (a^
4 - 11*a^3*b + 37*a^2*b^2 - 45*a*b^3 + 18*b^4)*cos(f*x + e)^4 - a^2*b^2 + 7
*a*b^3 - 6*b^4 - (2*a^3*b - 17*a^2*b^2 + 33*a*b^3 - 18*b^4)*cos(f*x + e)^2)
*log(1/2*cos(f*x + e) + 1/2) + 4*((a^4 - 9*a^3*b + 21*a^2*b^2 - 19*a*b^3 +
6*b^4)*cos(f*x + e)^6 - (a^4 - 11*a^3*b + 37*a^2*b^2 - 45*a*b^3 + 18*b^4)*c
os(f*x + e)^4 - a^2*b^2 + 7*a*b^3 - 6*b^4 - (2*a^3*b - 17*a^2*b^2 + 33*a*b^
3 - 18*b^4)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 - 3*a^6*b +
 3*a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6 - (a^7 - 5*a^6*b + 7*a^5*b^2 - 3*a^4
*b^3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*a^5*b^2 + 3*a^4*b^3)*f*cos(f*x + e)^2
 - (a^5*b^2 - a^4*b^3)*f), 1/8*((4*a^4 - 21*a^3*b + 29*a^2*b^2 - 12*a*b^3)*
cos(f*x + e)^5 + (17*a^3*b - 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^3 - ((15*a
^4 - 70*a^3*b + 119*a^2*b^2 - 88*a*b^3 + 24*b^4)*cos(f*x + e)^6 - (15*a^4 -
 100*a^3*b + 229*a^2*b^2 - 216*a*b^3 + 72*b^4)*cos(f*x + e)^4 - 15*a^2*b^2
+ 40*a*b^3 - 24*b^4 - (30*a^3*b - 125*a^2*b^2 + 168*a*b^3 - 72*b^4)*cos(f*x
 + e)^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) +
(11*a^2*b^2 - 12*a*b^3)*cos(f*x + e) - 2*((a^4 - 9*a^3*b + 21*a^2*b^2 - 19*
a*b^3 + 6*b^4)*cos(f*x + e)^6 - (a^4 - 11*a^3*b + 37*a^2*b^2 - 45*a*b^3 + 1
8*b^4)*cos(f*x + e)^4 - a^2*b^2 + 7*a*b^3 - 6*b^4 - (2*a^3*b - 17*a^2*b^2 +
 33*a*b^3 - 18*b^4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 2*((a^4 -
 9*a^3*b + 21*a^2*b^2 - 19*a*b^3 + 6*b^4)*cos(f*x + e)^6 - (a^4 - 11*a^3*b
+ 37*a^2*b^2 - 45*a*b^3 + 18*b^4)*cos(f*x + e)^4 - a^2*b^2 + 7*a*b^3 - 6*b^
4 - (2*a^3*b - 17*a^2*b^2 + 33*a*b^3 - 18*b^4)*cos(f*x + e)^2)*log(-1/2*cos
(f*x + e) + 1/2))/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6 -
 (a^7 - 5*a^6*b + 7*a^5*b^2 - 3*a^4*b^3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*a^
5*b^2 + 3*a^4*b^3)*f*cos(f*x + e)^2 - (a^5*b^2 - a^4*b^3)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+12*(1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1)))*b-a)*1/16/a^4/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp
(1)))+(9*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^3*b-32*((1-cos(f*x+e
```

```
xp(1)))/(1+cos(f*x+exp(1))))^3*a^2*b^2+24*((1-cos(f*x+exp(1)))/(1+cos(f*x+e
xp(1))))^3*a*b^3-27*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^3*b+102*(
(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2*b^2-152*((1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1))))^2*a*b^3+80*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^
2*b^4+27*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^3*b-80*(1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1)))*a^2*b^2+56*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a
*b^3-9*a^3*b+10*a^2*b^2)/(8*a^5-8*a^4*b)/(((1-cos(f*x+exp(1)))/(1+cos(f*x+e
xp(1))))^2*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1)))*b+a)^2+(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*1/16
/a^3+(a-6*b)*1/8/a^4*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1))))+(-15
*a^2*b+40*a*b^2-24*b^3)*1/4/(4*a^5-4*a^4*b)/sqrt(-b^2+a*b)*atan((-a*cos(f*x
+exp(1))+b*cos(f*x+exp(1))+b)/(sqrt(-b^2+a*b)*cos(f*x+exp(1))+sqrt(-b^2+a*b
))))
```

maple [B] time = 0.82, size = 435, normalized size = 2.12

$$-\frac{9b\left(\cos^3\left(fx+e\right)\right)}{8f\,a^2\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)^2}+\frac{b^2\left(\cos^3\left(fx+e\right)\right)}{f\,a^3\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)^2}-\frac{}{8f\,a^2\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -9/8/f*b/a^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3+1/f*b^2/a^3/(
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3-7/8/f*b^2/a^2/(a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)^2/(a-b)*cos(f*x+e)+1/f*b^3/a^3/(a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)^2/(a-b)*cos(f*x+e)+15/8/f*b/a^2/(a-b)/((a-b)*b)^(1/2)*arctan((a-
b)*cos(f*x+e)/((a-b)*b)^(1/2))-5/f*b^2/a^3/(a-b)/((a-b)*b)^(1/2)*arctan((a-
b)*cos(f*x+e)/((a-b)*b)^(1/2))+3/f*b^3/a^4/(a-b)/((a-b)*b)^(1/2)*arctan((a-
b)*cos(f*x+e)/((a-b)*b)^(1/2))+1/4/f/a^3/(-1+cos(f*x+e))+1/4/f/a^3*ln(-1+co
s(f*x+e))-3/2/f/a^4*ln(-1+cos(f*x+e))*b+1/4/f/a^3/(1+cos(f*x+e))-1/4/f/a^3*
ln(1+cos(f*x+e))+3/2/f/a^4*ln(1+cos(f*x+e))*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 13.22, size = 1652, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^3),x)
```

```
[Out] tan(e/2 + (f*x)/2)^2/(8*a^3*f) - (a^2/2 + (tan(e/2 + (f*x)/2)^4*(96*a*b^2 -
 38*a^2*b + 3*a^3 - 64*b^3))/(a - b) + (tan(e/2 + (f*x)/2)^8*(64*a*b^2 - 19
*a^2*b + a^3 - 48*b^3))/(2*(a - b)) - (tan(e/2 + (f*x)/2)^2*(14*a*b^2 - 15*
a^2*b + 2*a^3))/(a - b) - (tan(e/2 + (f*x)/2)^6*(2*a^4 - 33*a^3*b - 152*a*b
^3 + 80*b^4 + 106*a^2*b^2))/(a*(a - b)))/(f*(4*a^5*tan(e/2 + (f*x)/2)^2 + 4
*a^5*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^6*(24*a^5 - 64*a^4*b + 64*a
```

```
^3*b^2) + tan(e/2 + (f*x)/2)^4*(32*a^4*b - 16*a^5) + tan(e/2 + (f*x)/2)^8*(
32*a^4*b - 16*a^5))) + (log(tan(e/2 + (f*x)/2))*(a - 6*b))/(2*a^4*f) + (b^(
1/2)*atan(((tan(e/2 + (f*x)/2)^2*((((b^(3/2)*(15*a^2 - 40*a*b + 24*b^2)^3*(
128*a^16 - 3712*a^15*b + 6144*a^10*b^6 - 27648*a^11*b^5 + 49408*a^12*b^4 -
43904*a^13*b^3 + 19584*a^14*b^2))/(32768*a^12*(a - b)^(9/2)*(3*a^10*b - a^1
1 + a^8*b^3 - 3*a^9*b^2)) + (b^(1/2)*(15*a^2 - 40*a*b + 24*b^2)*(360*a^9*b
- 13824*a^2*b^8 + 66816*a^3*b^7 - 132864*a^4*b^6 + 139776*a^5*b^5 - 83240*a
^6*b^4 + 27836*a^7*b^3 - 4860*a^8*b^2))/(128*a^4*(a - b)^(3/2)*(3*a^10*b -
a^11 + a^8*b^3 - 3*a^9*b^2)))*(63*a^6 - 1013*a^5*b - 9600*a*b^5 + 2304*b^6
+ 15792*a^2*b^4 - 12888*a^3*b^3 + 5342*a^4*b^2))/(2*a^5*(a - b)^(9/2)*(5760
*a*b^4 - 735*a^4*b + 64*a^5 - 1728*b^5 - 6960*a^2*b^3 + 3600*a^3*b^2)) - ((
(6912*a*b^6 - 1728*b^7 - 10800*a^2*b^5 + 8240*a^3*b^4 - 3075*a^4*b^3 + 450*
a^5*b^2)/(8*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)) + (b*(15*a^2 - 40*a*b
+ 24*b^2)^2*(1936*a^12*b - 64*a^13 + 18432*a^6*b^7 - 86016*a^7*b^6 + 161664
*a^8*b^5 - 155008*a^9*b^4 + 78736*a^10*b^3 - 19680*a^11*b^2))/(2048*a^8*(a
- b)^3*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))*(3072*a*b^4 - 145*a^4*b +
4*a^5 - 1152*b^5 - 2856*a^2*b^3 + 1080*a^3*b^2))/(a^5*b^(1/2)*(a - b)^3*(57
60*a*b^4 - 735*a^4*b + 64*a^5 - 1728*b^5 - 6960*a^2*b^3 + 3600*a^3*b^2))) +
 (((b^(3/2)*(15*a^2 - 40*a*b + 24*b^2)^3*(128*a^16 - 768*a^15*b + 512*a^12*
b^4 - 1536*a^13*b^3 + 1664*a^14*b^2))/(32768*a^12*(a - b)^(9/2)*(a^11 - 2*a
^10*b + a^9*b^2)) + (b^(1/2)*(15*a^2 - 40*a*b + 24*b^2)*(240*a^9*b - 5760*a
^4*b^6 + 19200*a^5*b^5 - 23776*a^6*b^4 + 13344*a^7*b^3 - 3250*a^8*b^2))/(12
8*a^4*(a - b)^(3/2)*(a^11 - 2*a^10*b + a^9*b^2)))*(63*a^6 - 1013*a^5*b - 96
00*a*b^5 + 2304*b^6 + 15792*a^2*b^4 - 12888*a^3*b^3 + 5342*a^4*b^2))/(2*a^5
*(a - b)^(9/2)*(5760*a*b^4 - 735*a^4*b + 64*a^5 - 1728*b^5 - 6960*a^2*b^3 +
 3600*a^3*b^2)) - (((12096*a*b^6 - 3456*b^7 - 15840*a^2*b^5 + 9520*a^3*b^4
- 2550*a^4*b^3 + 225*a^5*b^2)/(8*(a^11 - 2*a^10*b + a^9*b^2)) + (b*(15*a^2
- 40*a*b + 24*b^2)^2*(1248*a^12*b - 64*a^13 + 3072*a^8*b^5 - 9728*a^9*b^4 +
 11328*a^10*b^3 - 5856*a^11*b^2))/(2048*a^8*(a - b)^3*(a^11 - 2*a^10*b + a^
9*b^2)))*(3072*a*b^4 - 145*a^4*b + 4*a^5 - 1152*b^5 - 2856*a^2*b^3 + 1080*a
^3*b^2))/(a^5*b^(1/2)*(a - b)^3*(5760*a*b^4 - 735*a^4*b + 64*a^5 - 1728*b^5
 - 6960*a^2*b^3 + 3600*a^3*b^2)))*(256*a^13*(a - b)^(9/2) - 768*a^12*b*(a -
 b)^(9/2) - 256*a^10*b^3*(a - b)^(9/2) + 768*a^11*b^2*(a - b)^(9/2)))/(225*
a^4*b - 1920*a*b^4 + 576*b^5 + 2320*a^2*b^3 - 1200*a^3*b^2))*(15*a^2 - 40*a
*b + 24*b^2))/(8*a^4*f*(a - b)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.85 $\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3} dx$

Optimal. Leaf size=259

$$-\frac{3b(a-2b)\sec(e+fx)}{2a^4f\left(a+b\sec^2(e+fx)-b\right)}-\frac{b(7a-12b)\sec(e+fx)}{8a^3f\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{3\left(a^2-12a\right.}{}$$

```
[Out] -3/8*(a^2-12*a*b+16*b^2)*arctanh(cos(f*x+e))/a^5/f-1/8*(5*a-8*b)*cot(f*x+e)
*csc(f*x+e)/a^2/f/(a-b+b*sec(f*x+e)^2)^2-1/4*cot(f*x+e)^3*csc(f*x+e)/a/f/(a
-b+b*sec(f*x+e)^2)^2-1/8*(7*a-12*b)*b*sec(f*x+e)/a^3/f/(a-b+b*sec(f*x+e)^2)
^2-3/2*(a-2*b)*b*sec(f*x+e)/a^4/f/(a-b+b*sec(f*x+e)^2)-3/8*(5*a^2-20*a*b+16
*b^2)*arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^5/f/(a-b)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3664, 470, 527, 522, 207, 205}

$$-\frac{3\left(a^2-12ab+16b^2\right)\tanh^{-1}(\cos(e+fx))}{8a^5f}-\frac{3\sqrt{b}\left(5a^2-20ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^5f\sqrt{a-b}}-\frac{3b(a-2b)\sec(}{2a^4f\left(a+b\sec^2(e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-3*Sqrt[b]*(5*a^2 - 20*a*b + 16*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a
- b]])/(8*a^5*Sqrt[a - b]*f) - (3*(a^2 - 12*a*b + 16*b^2)*ArcTanh[Cos[e + f
*x]])/(8*a^5*f) - ((5*a - 8*b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f*(a - b +
 b*Sec[e + f*x]^2)^2) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*a*f*(a - b + b*Sec
[e + f*x]^2)^2) - ((7*a - 12*b)*b*Sec[e + f*x])/(8*a^3*f*(a - b + b*Sec[e +
 f*x]^2)^2) - (3*(a - 2*b)*b*Sec[e + f*x])/(2*a^4*f*(a - b + b*Sec[e + f*x]
^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-a+b+(-4a+7b)x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^3}\,dx, x, \sec(e+fx)\right)}{4af}$$

$$= -\frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-(3 \cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7a-12b \cdots}{8a^3f\left(a-b+\cdots\right.}$$

$$= -\frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7a-12b \cdots}{8a^3f\left(a-b+\cdots\right.}$$

$$= -\frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7a-12b \cdots}{8a^3f\left(a-b+\cdots\right.}$$

$$= -\frac{3\sqrt{b}\left(5a^2-20ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^5\sqrt{a-b}\,f} - \frac{3\left(a^2-12ab+16b^2\right)\tanh^{-1}(\cos \cdots}{8a^5f}$$

Mathematica [A] time = 6.58, size = 468, normalized size = 1.81

$$-\frac{3\left(3ab\cos(e+fx)-4b^2\cos(e+fx)\right)}{4a^4f(a\cos(2(e+fx))+a-b\cos(2(e+fx))+b)}-\frac{3(a-4b)\csc^2\left(\frac{1}{2}(e+fx)\right)}{32a^4f}+\frac{3(a-4b)\sec^2\left(\frac{1}{2}(e+fx)\right)}{32a^4f}+\cdots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-3*Sqrt[a - b]*Sqrt[b]*(5*a^2 - 20*a*b + 16*b^2)*ArcTan[(Sec[(e + f*x)/2]*
(Sqrt[a - b]*Cos[(e + f*x)/2] - Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(8*a^5
*(-a + b)*f) - (3*Sqrt[a - b]*Sqrt[b]*(5*a^2 - 20*a*b + 16*b^2)*ArcTan[(Sec
[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/2] + Sqrt[a]*Sin[(e + f*x)/2]))/Sq
rt[b]])/(8*a^5*(-a + b)*f) + (b^2*Cos[e + f*x])/(a^3*f*(a + b + a*Cos[2*(e
+ f*x)] - b*Cos[2*(e + f*x)])^2) - (3*(3*a*b*Cos[e + f*x] - 4*b^2*Cos[e + f
*x]))/(4*a^4*f*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])) - (3*(a -
 4*b)*Csc[(e + f*x)/2]^2)/(32*a^4*f) - Csc[(e + f*x)/2]^4/(64*a^3*f) - (3*(
a^2 - 12*a*b + 16*b^2)*Log[Cos[(e + f*x)/2]])/(8*a^5*f) + (3*(a^2 - 12*a*b
+ 16*b^2)*Log[Sin[(e + f*x)/2]])/(8*a^5*f) + (3*(a - 4*b)*Sec[(e + f*x)/2]^
2)/(32*a^4*f) + Sec[(e + f*x)/2]^4/(64*a^3*f)
```

fricas [B] time = 0.89, size = 1693, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(a^4 - 9*a^3*b + 16*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 -
 46*a^3*b + 108*a^2*b^2 - 72*a*b^3)*cos(f*x + e)^5 - 2*(19*a^3*b - 72*a^2*b
^2 + 72*a*b^3)*cos(f*x + e)^3 + 3*((5*a^4 - 30*a^3*b + 61*a^2*b^2 - 52*a*b^
3 + 16*b^4)*cos(f*x + e)^8 - 2*(5*a^4 - 35*a^3*b + 86*a^2*b^2 - 88*a*b^3 +
32*b^4)*cos(f*x + e)^6 + (5*a^4 - 50*a^3*b + 166*a^2*b^2 - 216*a*b^3 + 96*b
^4)*cos(f*x + e)^4 + 5*a^2*b^2 - 20*a*b^3 + 16*b^4 + 2*(5*a^3*b - 30*a^2*b^
2 + 56*a*b^3 - 32*b^4)*cos(f*x + e)^2)*sqrt(-b/(a - b))*log(((a - b)*cos(f*
x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x +
e)^2 + b)) - 24*(a^2*b^2 - 2*a*b^3)*cos(f*x + e) - 3*((a^4 - 14*a^3*b + 41*
a^2*b^2 - 44*a*b^3 + 16*b^4)*cos(f*x + e)^8 - 2*(a^4 - 15*a^3*b + 54*a^2*b^
2 - 72*a*b^3 + 32*b^4)*cos(f*x + e)^6 + (a^4 - 18*a^3*b + 94*a^2*b^2 - 168*
a*b^3 + 96*b^4)*cos(f*x + e)^4 + a^2*b^2 - 12*a*b^3 + 16*b^4 + 2*(a^3*b - 1
4*a^2*b^2 + 40*a*b^3 - 32*b^4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2)
+ 3*((a^4 - 14*a^3*b + 41*a^2*b^2 - 44*a*b^3 + 16*b^4)*cos(f*x + e)^8 - 2*(
a^4 - 15*a^3*b + 54*a^2*b^2 - 72*a*b^3 + 32*b^4)*cos(f*x + e)^6 + (a^4 - 18
*a^3*b + 94*a^2*b^2 - 168*a*b^3 + 96*b^4)*cos(f*x + e)^4 + a^2*b^2 - 12*a*b
^3 + 16*b^4 + 2*(a^3*b - 14*a^2*b^2 + 40*a*b^3 - 32*b^4)*cos(f*x + e)^2)*lo
g(-1/2*cos(f*x + e) + 1/2))/((a^7 - 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^8 + a
^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^6 + (a^7 - 6*a^6*b
+ 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2)*f*cos(f*x + e)^2), 1/
16*(6*(a^4 - 9*a^3*b + 16*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 - 46
*a^3*b + 108*a^2*b^2 - 72*a*b^3)*cos(f*x + e)^5 - 2*(19*a^3*b - 72*a^2*b^2
+ 72*a*b^3)*cos(f*x + e)^3 - 6*((5*a^4 - 30*a^3*b + 61*a^2*b^2 - 52*a*b^3 +
 16*b^4)*cos(f*x + e)^8 - 2*(5*a^4 - 35*a^3*b + 86*a^2*b^2 - 88*a*b^3 + 32*
b^4)*cos(f*x + e)^6 + (5*a^4 - 50*a^3*b + 166*a^2*b^2 - 216*a*b^3 + 96*b^4)
*cos(f*x + e)^4 + 5*a^2*b^2 - 20*a*b^3 + 16*b^4 + 2*(5*a^3*b - 30*a^2*b^2 +
 56*a*b^3 - 32*b^4)*cos(f*x + e)^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/
(a - b))*cos(f*x + e)/b) - 24*(a^2*b^2 - 2*a*b^3)*cos(f*x + e) - 3*((a^4 -
14*a^3*b + 41*a^2*b^2 - 44*a*b^3 + 16*b^4)*cos(f*x + e)^8 - 2*(a^4 - 15*a^3
*b + 54*a^2*b^2 - 72*a*b^3 + 32*b^4)*cos(f*x + e)^6 + (a^4 - 18*a^3*b + 94*
```

```
a^2*b^2 - 168*a*b^3 + 96*b^4)*cos(f*x + e)^4 + a^2*b^2 - 12*a*b^3 + 16*b^4
+ 2*(a^3*b - 14*a^2*b^2 + 40*a*b^3 - 32*b^4)*cos(f*x + e)^2)*log(1/2*cos(f*
x + e) + 1/2) + 3*((a^4 - 14*a^3*b + 41*a^2*b^2 - 44*a*b^3 + 16*b^4)*cos(f*
x + e)^8 - 2*(a^4 - 15*a^3*b + 54*a^2*b^2 - 72*a*b^3 + 32*b^4)*cos(f*x + e)
^6 + (a^4 - 18*a^3*b + 94*a^2*b^2 - 168*a*b^3 + 96*b^4)*cos(f*x + e)^4 + a^
2*b^2 - 12*a*b^3 + 16*b^4 + 2*(a^3*b - 14*a^2*b^2 + 40*a*b^3 - 32*b^4)*cos(
f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 - 2*a^6*b + a^5*b^2)*f*cos(
f*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^6 + (
a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2)*f*cos(f
*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*((32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^3+256*(1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^3-768*(1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1)))*a^2*b)*1/4096/a^6+(-6*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^6*a^
4+72*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^6*a^3*b-96*((1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1))))^6*a^2*b^2+16*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1
))))^5*a^4-168*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*a^3*b+384*((1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*a^2*b^2-256*((1-cos(f*x+exp(1)))/(1+c
os(f*x+exp(1))))^5*a*b^3-5*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a^4-
64*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a^3*b+192*((1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1))))^4*a^2*b^2-256*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1
))))^4*a*b^3+256*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b^4-20*((1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^4+360*((1-cos(f*x+exp(1)))/(1+cos(f*
x+exp(1))))^3*a^3*b-1024*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^2*b^
2+896*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a*b^3+20*((1-cos(f*x+exp(
1)))/(1+cos(f*x+exp(1))))^2*a^4-216*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
))^2*a^3*b+304*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2*b^2-4*(1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^4+16*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1)))*a^3*b-a^4)*1/128/a^5/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a-2
*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+4*((1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1))))^2*b+(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a)^2+(3*a^2-36*
a*b+48*b^2)*1/32/a^5*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1))))+(-15
*a^2*b+60*a*b^2-48*b^3)*1/4/a^5*1/4/sqrt(-b^2+a*b)*atan((-a*cos(f*x+exp(1))
+b*cos(f*x+exp(1))+b)/(sqrt(-b^2+a*b)*cos(f*x+exp(1))+sqrt(-b^2+a*b))))
```

maple [B] time = 0.67, size = 560, normalized size = 2.16

$$-\frac{9b\left(\cos^3\left(fx+e\right)\right)}{8f\,a^2\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)^2}+\frac{21b^2\left(\cos^3\left(fx+e\right)\right)}{8f\,a^3\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)^2}-\frac{}{2f\,a^4\left(a\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -9/8/f*b/a^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3+21/8/f*b^2/a^
3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3-3/2/f*b^3/a^4/(a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)^2*cos(f*x+e)^3-7/8/f*b^2/a^3/(a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)^2*cos(f*x+e)+3/2/f*b^3/a^4/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*
cos(f*x+e)+15/8/f*b/a^3/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(
1/2))-15/2/f*b^2/a^4/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2
```

```
))+6/f*b^3/a^5/((a-b)*b)^(1/2)*arctan((a-b)*cos(f*x+e)/((a-b)*b)^(1/2))-1/1
6/f/a^3/(-1+cos(f*x+e))^2+3/16/f/a^3/(-1+cos(f*x+e))-3/4/f/a^4/(-1+cos(f*x+
e))*b+3/16/f/a^3*ln(-1+cos(f*x+e))-9/4/f/a^4*ln(-1+cos(f*x+e))*b+3/f/a^5*ln
(-1+cos(f*x+e))*b^2+1/16/f/a^3/(1+cos(f*x+e))^2+3/16/f/a^3/(1+cos(f*x+e))-3
/4/f/a^4/(1+cos(f*x+e))*b-3/16/f/a^3*ln(1+cos(f*x+e))+9/4/f/a^4*ln(1+cos(f*
x+e))*b-3/f/a^5*ln(1+cos(f*x+e))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 12.82, size = 1357, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^3),x)
```

```
[Out] tan(e/2 + (f*x)/2)^4/(64*a^3*f) + (tan(e/2 + (f*x)/2)^2*((3*(a - 2*b))/(16*
a^4) - 1/(16*a^3)))/f + (tan(e/2 + (f*x)/2)^4*(100*a*b^2 - 72*a^2*b + (13*a
^3)/2) - tan(e/2 + (f*x)/2)^10*(144*a*b^2 - 42*a^2*b + 2*a^3 - 128*b^3) - t
an(e/2 + (f*x)/2)^6*(496*a*b^2 - 174*a^2*b + 11*a^3 - 416*b^3) + tan(e/2 +
(f*x)/2)^2*(4*a^2*b - a^3) - a^3/4 + (tan(e/2 + (f*x)/2)^8*(31*a^4 - 592*a^
3*b - 2944*a*b^3 + 1792*b^4 + 2016*a^2*b^2))/(4*a))/(f*(16*a^6*tan(e/2 + (f
*x)/2)^4 + 16*a^6*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)^8*(96*a^6 - 25
6*a^5*b + 256*a^4*b^2) + tan(e/2 + (f*x)/2)^6*(128*a^5*b - 64*a^6) + tan(e/
2 + (f*x)/2)^10*(128*a^5*b - 64*a^6))) + (log(tan(e/2 + (f*x)/2))*(3*a^2 -
36*a*b + 48*b^2))/(8*a^5*f) + (3*b^(1/2)*atan((a^13*(a - b)^(3/2)*((256*((3
456*b^8 - 11232*a*b^7 + 14256*a^2*b^6 - 8910*a^3*b^5 + 2835*a^4*b^4 - (3375
*a^5*b^3)/8 + (675*a^6*b^2)/32)/a^12 - (9*b*(5*a^2 - 20*a*b + 16*b^2)^2*(19
2*a^14 - 4992*a^13*b + 24576*a^10*b^4 - 43008*a^11*b^3 + 24576*a^12*b^2))/(
8192*a^22*(a - b)))*(1728*a*b^4 - 45*a^4*b + a^5 - 768*b^5 - 1344*a^2*b^3 +
 420*a^3*b^2))/(b^(1/2)*(b*(b*(b*(1680*a^7 + b*(768*a^5*b - 1920*a^6)) - 60
0*a^8) + 75*a^9) - 4*a^10)) - 256*tan(e/2 + (f*x)/2)^2*((((4752*a*b^6 - 172
8*b^7 - 4860*a^2*b^5 + 2295*a^3*b^4 - (2025*a^4*b^3)/4 + (675*a^5*b^2)/16)/
a^11 + (9*b*(5*a^2 - 20*a*b + 16*b^2)^2*(3552*a^12*b - 96*a^13 + 73728*a^8*
b^5 - 165888*a^9*b^4 + 125952*a^10*b^3 - 36480*a^11*b^2))/(4096*a^21*(a - b
)))*(1728*a*b^4 - 45*a^4*b + a^5 - 768*b^5 - 1344*a^2*b^3 + 420*a^3*b^2))/(
b^(1/2)*(b*(b*(b*(1680*a^7 + b*(768*a^5*b - 1920*a^6)) - 600*a^8) + 75*a^9)
 - 4*a^10)) - (((b^(3/2)*(5*a^2 - 20*a*b + 16*b^2)^3*((351*a^15*b)/128 - (2
7*a^16)/256 + (81*a^13*b^3)/16 - (243*a^14*b^2)/32))/(a^26*(a - b)^(3/2)) -
 (3*b^(1/2)*(5*a^2 - 20*a*b + 16*b^2)*(540*a^9*b + 110592*a^3*b^7 - 331776*
a^4*b^6 + 389376*a^5*b^5 - 225792*a^6*b^4 + 67248*a^7*b^3 - 9720*a^8*b^2))/
(256*a^16*(a - b)^(1/2)))*(4224*a*b^4 - 330*a^4*b + 17*a^5 - 1536*b^5 - 422
4*a^2*b^3 + 1848*a^3*b^2))/(2*a^5*(a - b)^(1/2)*(1920*a*b^4 - 75*a^4*b + 4*
a^5 - 768*b^5 - 1680*a^2*b^3 + 600*a^3*b^2))) + (128*((b^(3/2)*(5*a^2 - 20*
a*b + 16*b^2)^3*((27*a^17)/256 - (27*a^16*b)/64 + (27*a^15*b^2)/64))/(a^27*
(a - b)^(3/2)) + (3*b^(1/2)*(5*a^2 - 20*a*b + 16*b^2)*(720*a^10*b - 92160*a
^5*b^6 + 230400*a^6*b^5 - 210816*a^7*b^4 + 85824*a^8*b^3 - 14760*a^9*b^2))/
(512*a^17*(a - b)^(1/2)))*(4224*a*b^4 - 330*a^4*b + 17*a^5 - 1536*b^5 - 422
4*a^2*b^3 + 1848*a^3*b^2))/(a^5*(a - b)^(1/2)*(1920*a*b^4 - 75*a^4*b + 4*a^
5 - 768*b^5 - 1680*a^2*b^3 + 600*a^3*b^2))))/(675*a^4*b - 17280*a*b^4 + 691
```

```
2*b^5 + 15120*a^2*b^3 - 5400*a^3*b^2))*(5*a^2 - 20*a*b + 16*b^2))/(8*a^5*f*
(a - b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.86 $\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=250

$$-\frac{3\sqrt{b}\left(5a^2+10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}\,f(a-b)^5}+\frac{3x\left(a^2+10ab+5b^2\right)}{8(a-b)^5}-\frac{3b(a+b)\tan(e+fx)}{2f(a-b)^4\left(a+b\tan^2(e+fx)\right)}-\frac{b(7}{8f(a-}$$

```
[Out] 3/8*(a^2+10*a*b+5*b^2)*x/(a-b)^5-3/8*(5*a^2+10*a*b+b^2)*arctan(b^(1/2)*tan(
f*x+e)/a^(1/2))*b^(1/2)/(a-b)^5/f/a^(1/2)-1/8*(5*a+3*b)*cos(f*x+e)*sin(f*x+
e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^2+1/4*cos(f*x+e)^3*sin(f*x+e)/(a-b)/f/(a+b*
tan(f*x+e)^2)^2-1/8*b*(7*a+5*b)*tan(f*x+e)/(a-b)^3/f/(a+b*tan(f*x+e)^2)^2-3
/2*b*(a+b)*tan(f*x+e)/(a-b)^4/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.33, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 470, 527, 522, 203, 205}

$$-\frac{3\sqrt{b}\left(5a^2+10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}\,f(a-b)^5}+\frac{3x\left(a^2+10ab+5b^2\right)}{8(a-b)^5}-\frac{3b(a+b)\tan(e+fx)}{2f(a-b)^4\left(a+b\tan^2(e+fx)\right)}-\frac{b(7}{8f(a-}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (3*(a^2 + 10*a*b + 5*b^2)*x)/(8*(a - b)^5) - (3*Sqrt[b]*(5*a^2 + 10*a*b + b
^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(8*Sqrt[a]*(a - b)^5*f) - ((5*a
 + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(8*(a - b)^2*f*(a + b*Tan[e + f*x]^2)^2)
 + (Cos[e + f*x]^3*Sin[e + f*x])/(4*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - (
b*(7*a + 5*b)*Tan[e + f*x])/(8*(a - b)^3*f*(a + b*Tan[e + f*x]^2)^2) - (3*b
*(a + b)*Tan[e + f*x])/(2*(a - b)^4*f*(a + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^3\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+(-4a-3b)x^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b(7a \cdots}{8(a-b)^3f \cdots}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b(7a \cdots}{8(a-b)^3f \cdots}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b(7a \cdots}{8(a-b)^3f \cdots}$$

$$= \frac{3\left(a^2+10ab+5b^2\right)x}{8(a-b)^5} - \frac{3\sqrt{b}\left(5a^2+10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}(a-b)^5f} - \frac{(5a+3b)\cos \cdots}{8(a-b)^2f \cdots}$$

Mathematica [A] time = 0.93, size = 194, normalized size = 0.78

$$\frac{12\left(a^2+10ab+5b^2\right)(e+fx) - \frac{12\sqrt{b}\left(5a^2+10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{16ab^2(a-b)\sin(2(e+fx))}{((a-b)\cos(2(e+fx))+a+b)^2} + (a-b)^2\sin(4(e+fx)) \cdots}{32f(a-b)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (12*(a^2 + 10*a*b + 5*b^2)*(e + f*x) - (12*Sqrt[b]*(5*a^2 + 10*a*b + b^2)*A
rcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/Sqrt[a] - 8*(a - b)*(a + 2*b)*Sin[2*
(e + f*x)] + (16*a*(a - b)*b^2*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e
+ f*x)])^2 - (4*(a - b)*b*(9*a + 5*b)*Sin[2*(e + f*x)])/(a + b + (a - b)*Co
s[2*(e + f*x)]) + (a - b)^2*Sin[4*(e + f*x)])/(32*(a - b)^5*f)
```

fricas [B] time = 0.97, size = 1191, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*(a^4 + 8*a^3*b - 14*a^2*b^2 + 5*b^4)*f*x*cos(f*x + e)^4 + 24*(a^3
*b + 9*a^2*b^2 - 5*a*b^3 - 5*b^4)*f*x*cos(f*x + e)^2 + 12*(a^2*b^2 + 10*a*b
^3 + 5*b^4)*f*x - 3*((5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x + e)^4 +
5*a^2*b^2 + 10*a*b^3 + b^4 + 2*(5*a^3*b + 5*a^2*b^2 - 9*a*b^3 - b^4)*cos(f*
x + e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b
^2)*cos(f*x + e)^2 - 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt
(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b
^2)*cos(f*x + e)^2 + b^2)) + 4*(2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^
4)*cos(f*x + e)^7 - (5*a^4 - 12*a^3*b + 6*a^2*b^2 + 4*a*b^3 - 3*b^4)*cos(f*
x + e)^5 - (19*a^3*b - 21*a^2*b^2 - 15*a*b^3 + 17*b^4)*cos(f*x + e)^3 - 12*
(a^2*b^2 - b^4)*cos(f*x + e))*sin(f*x + e))/((a^7 - 7*a^6*b + 21*a^5*b^2 -
35*a^4*b^3 + 35*a^3*b^4 - 21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e)^4 + 2*
(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)*
f*cos(f*x + e)^2 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6
 - b^7)*f), 1/16*(6*(a^4 + 8*a^3*b - 14*a^2*b^2 + 5*b^4)*f*x*cos(f*x + e)^4
 + 12*(a^3*b + 9*a^2*b^2 - 5*a*b^3 - 5*b^4)*f*x*cos(f*x + e)^2 + 6*(a^2*b^2
 + 10*a*b^3 + 5*b^4)*f*x + 3*((5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x
+ e)^4 + 5*a^2*b^2 + 10*a*b^3 + b^4 + 2*(5*a^3*b + 5*a^2*b^2 - 9*a*b^3 - b^
4)*cos(f*x + e)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b
/a)/(b*cos(f*x + e)*sin(f*x + e))) + 2*(2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*
b^3 + b^4)*cos(f*x + e)^7 - (5*a^4 - 12*a^3*b + 6*a^2*b^2 + 4*a*b^3 - 3*b^4
)*cos(f*x + e)^5 - (19*a^3*b - 21*a^2*b^2 - 15*a*b^3 + 17*b^4)*cos(f*x + e)
^3 - 12*(a^2*b^2 - b^4)*cos(f*x + e))*sin(f*x + e))/((a^7 - 7*a^6*b + 21*a^
5*b^2 - 35*a^4*b^3 + 35*a^3*b^4 - 21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e
)^4 + 2*(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6
 + b^7)*f*cos(f*x + e)^2 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 +
 5*a*b^6 - b^7)*f)]
```

giac [A] time = 3.12, size = 399, normalized size = 1.60

$$\frac{3\left(a^2+10\,ab+5\,b^2\right)\left(fx+e\right)}{a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5} - \frac{3\left(5\,a^2b+10\,ab^2+b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5\right)\sqrt{ab}} - \frac{12\,ab^2\tan\left(fx+e\right)^7+12\,b^3\tan\left(fx+e\right)^7+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^2 + 10*a*b + 5*b^2)*(f*x + e)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^
2*b^3 + 5*a*b^4 - b^5) - 3*(5*a^2*b + 10*a*b^2 + b^3)*(pi*floor((f*x + e)/p
i + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^5 - 5*a^4*b + 10*a^
3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sqrt(a*b)) - (12*a*b^2*tan(f*x + e)^7 +
 12*b^3*tan(f*x + e)^7 + 19*a^2*b*tan(f*x + e)^5 + 34*a*b^2*tan(f*x + e)^5
```

```
+ 19*b^3*tan(f*x + e)^5 + 5*a^3*tan(f*x + e)^3 + 31*a^2*b*tan(f*x + e)^3 +
31*a*b^2*tan(f*x + e)^3 + 5*b^3*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) + 18*a^
2*b*tan(f*x + e) + 3*a*b^2*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan(f*x + e
)^2 + b*tan(f*x + e)^2 + a)^2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)))
/f
```

maple [B] time = 0.63, size = 598, normalized size = 2.39

$$-\frac{7b^2\left(\tan^3\left(fx+e\right)\right)a^2}{8f\left(a-b\right)^5\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{b^3\left(\tan^3\left(fx+e\right)\right)a}{4f\left(a-b\right)^5\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{5b^4\left(\tan^3\left(fx+e\right)\right)}{8f\left(a-b\right)^5\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f*b^2/(a-b)^5/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3*a^2+1/4/f*b^3/(a-b)^5/
(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3*a+5/8/f*b^4/(a-b)^5/(a+b*tan(f*x+e)^2)^2*
tan(f*x+e)^3-9/8/f*b/(a-b)^5/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)*a^3+3/4/f*b^2/
(a-b)^5/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)*a^2+3/8/f*b^3/(a-b)^5/(a+b*tan(f*x+
e)^2)^2*tan(f*x+e)*a-15/8/f*b/(a-b)^5/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)
^(1/2))*a^2-15/4/f*b^2/(a-b)^5/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))
*a-3/8/f*b^3/(a-b)^5/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/4/f/(a-
b)^5/(1+tan(f*x+e)^2)^2*tan(f*x+e)^3*a*b+7/8/f/(a-b)^5/(1+tan(f*x+e)^2)^2*t
an(f*x+e)^3*b^2-5/8/f/(a-b)^5/(1+tan(f*x+e)^2)^2*tan(f*x+e)^3*a^2-3/8/f/(a-
b)^5/(1+tan(f*x+e)^2)^2*tan(f*x+e)*a^2-3/4/f/(a-b)^5/(1+tan(f*x+e)^2)^2*tan
(f*x+e)*a*b+9/8/f/(a-b)^5/(1+tan(f*x+e)^2)^2*tan(f*x+e)*b^2+15/4/f/(a-b)^5*
arctan(tan(f*x+e))*a*b+15/8/f/(a-b)^5*arctan(tan(f*x+e))*b^2+3/8/f/(a-b)^5*
arctan(tan(f*x+e))*a^2
```

maxima [A] time = 0.70, size = 460, normalized size = 1.84

$$\frac{\frac{3\left(a^2+10\,ab+5\,b^2\right)\left(fx+e\right)}{a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5}-\frac{3\left(5\,a^2b+10\,ab^2+b^3\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5\right)\sqrt{ab}}-\frac{12\left(ab^2+b^3\right)}{\left(a^4b^2-4\,a^3b^3+6\,a^2b^4-4\,ab^5+b^6\right)\tan\left(fx+e\right)^8+2\left(a^5b-3\,a^4b^2+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a^2 + 10*a*b + 5*b^2)*(f*x + e)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^
2*b^3 + 5*a*b^4 - b^5) - 3*(5*a^2*b + 10*a*b^2 + b^3)*arctan(b*tan(f*x + e)
/sqrt(a*b))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sqrt
(a*b)) - (12*(a*b^2 + b^3)*tan(f*x + e)^7 + (19*a^2*b + 34*a*b^2 + 19*b^3)*
tan(f*x + e)^5 + (5*a^3 + 31*a^2*b + 31*a*b^2 + 5*b^3)*tan(f*x + e)^3 + 3*(
a^3 + 6*a^2*b + a*b^2)*tan(f*x + e))/((a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*
a*b^5 + b^6)*tan(f*x + e)^8 + 2*(a^5*b - 3*a^4*b^2 + 2*a^3*b^3 + 2*a^2*b^4
- 3*a*b^5 + b^6)*tan(f*x + e)^6 + a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a
^2*b^4 + (a^6 - 9*a^4*b^2 + 16*a^3*b^3 - 9*a^2*b^4 + b^6)*tan(f*x + e)^4 +
2*(a^6 - 3*a^5*b + 2*a^4*b^2 + 2*a^3*b^3 - 3*a^2*b^4 + a*b^5)*tan(f*x + e)^
2))/f
```

mupad [B] time = 16.39, size = 5965, normalized size = 23.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] (atan(((((tan(e + f*x)*(540*a*b^6 + 117*b^7 + 990*a^2*b^5 + 540*a^3*b^4 + 1
17*a^4*b^3))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 +
 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) + (3*((6*a*b^13 - (3*b^14)/2 + 21*a
^2*b^12 - 210*a^3*b^11 + (1395*a^4*b^10)/2 - 1332*a^5*b^9 + 1638*a^6*b^8 -
1332*a^7*b^7 + (1395*a^8*b^6)/2 - 210*a^9*b^5 + 21*a^10*b^4 + 6*a^11*b^3 -
(3*a^12*b^2)/2)/(a^12 - 12*a^11*b - 12*a*b^11 + b^12 + 66*a^2*b^10 - 220*a^
3*b^9 + 495*a^4*b^8 - 792*a^5*b^7 + 924*a^6*b^6 - 792*a^7*b^5 + 495*a^8*b^4
 - 220*a^9*b^3 + 66*a^10*b^2) - (3*tan(e + f*x)*(10*a*b + a^2 + 5*b^2)*(115
2*a*b^12 - 128*b^13 - 4480*a^2*b^11 + 9600*a^3*b^10 - 11520*a^4*b^9 + 5376*
a^5*b^8 + 5376*a^6*b^7 - 11520*a^7*b^6 + 9600*a^8*b^5 - 4480*a^9*b^4 + 1152
*a^10*b^3 - 128*a^11*b^2))/(256*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^
2*b^3*10i + a^3*b^2*10i)*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a
^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(10*a*b + a^2 + 5*b^2))/(1
6*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^2*b^3*10i + a^3*b^2*10i)))*(10
*a*b + a^2 + 5*b^2)*3i)/(16*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^2*b^
3*10i + a^3*b^2*10i)) + (((tan(e + f*x)*(540*a*b^6 + 117*b^7 + 990*a^2*b^5
+ 540*a^3*b^4 + 117*a^4*b^3))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b
^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (3*((6*a*b^13 -
(3*b^14)/2 + 21*a^2*b^12 - 210*a^3*b^11 + (1395*a^4*b^10)/2 - 1332*a^5*b^9
+ 1638*a^6*b^8 - 1332*a^7*b^7 + (1395*a^8*b^6)/2 - 210*a^9*b^5 + 21*a^10*b^
4 + 6*a^11*b^3 - (3*a^12*b^2)/2)/(a^12 - 12*a^11*b - 12*a*b^11 + b^12 + 66*
a^2*b^10 - 220*a^3*b^9 + 495*a^4*b^8 - 792*a^5*b^7 + 924*a^6*b^6 - 792*a^7*
b^5 + 495*a^8*b^4 - 220*a^9*b^3 + 66*a^10*b^2) + (3*tan(e + f*x)*(10*a*b +
a^2 + 5*b^2)*(1152*a*b^12 - 128*b^13 - 4480*a^2*b^11 + 9600*a^3*b^10 - 1152
0*a^4*b^9 + 5376*a^5*b^8 + 5376*a^6*b^7 - 11520*a^7*b^6 + 9600*a^8*b^5 - 44
80*a^9*b^4 + 1152*a^10*b^3 - 128*a^11*b^2))/(256*(a*b^4*5i - a^4*b*5i + a^5
*1i - b^5*1i - a^2*b^3*10i + a^3*b^2*10i)*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 +
28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(10*a*b +
 a^2 + 5*b^2))/(16*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^2*b^3*10i + a
^3*b^2*10i)))*(10*a*b + a^2 + 5*b^2)*3i)/(16*(a*b^4*5i - a^4*b*5i + a^5*1i
- b^5*1i - a^2*b^3*10i + a^3*b^2*10i)))/(((1755*a*b^7)/64 + (135*b^8)/64 +
(2511*a^2*b^6)/32 + (2511*a^3*b^5)/32 + (1755*a^4*b^4)/64 + (135*a^5*b^3)/6
4)/(a^12 - 12*a^11*b - 12*a*b^11 + b^12 + 66*a^2*b^10 - 220*a^3*b^9 + 495*a
^4*b^8 - 792*a^5*b^7 + 924*a^6*b^6 - 792*a^7*b^5 + 495*a^8*b^4 - 220*a^9*b^
3 + 66*a^10*b^2) - (3*((tan(e + f*x)*(540*a*b^6 + 117*b^7 + 990*a^2*b^5 + 5
40*a^3*b^4 + 117*a^4*b^3))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6
- 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) + (3*((6*a*b^13 - (3*
b^14)/2 + 21*a^2*b^12 - 210*a^3*b^11 + (1395*a^4*b^10)/2 - 1332*a^5*b^9 + 1
638*a^6*b^8 - 1332*a^7*b^7 + (1395*a^8*b^6)/2 - 210*a^9*b^5 + 21*a^10*b^4 +
 6*a^11*b^3 - (3*a^12*b^2)/2)/(a^12 - 12*a^11*b - 12*a*b^11 + b^12 + 66*a^2
*b^10 - 220*a^3*b^9 + 495*a^4*b^8 - 792*a^5*b^7 + 924*a^6*b^6 - 792*a^7*b^5
 + 495*a^8*b^4 - 220*a^9*b^3 + 66*a^10*b^2) - (3*tan(e + f*x)*(10*a*b + a^2
 + 5*b^2)*(1152*a*b^12 - 128*b^13 - 4480*a^2*b^11 + 9600*a^3*b^10 - 11520*a
^4*b^9 + 5376*a^5*b^8 + 5376*a^6*b^7 - 11520*a^7*b^6 + 9600*a^8*b^5 - 4480*
a^9*b^4 + 1152*a^10*b^3 - 128*a^11*b^2))/(256*(a*b^4*5i - a^4*b*5i + a^5*1i
 - b^5*1i - a^2*b^3*10i + a^3*b^2*10i)*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*
a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(10*a*b + a^
2 + 5*b^2))/(16*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^2*b^3*10i + a^3*
b^2*10i)))*(10*a*b + a^2 + 5*b^2))/(16*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*
1i - a^2*b^3*10i + a^3*b^2*10i)) + (3*((tan(e + f*x)*(540*a*b^6 + 117*b^7 +
 990*a^2*b^5 + 540*a^3*b^4 + 117*a^4*b^3))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b
^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (3*
((6*a*b^13 - (3*b^14)/2 + 21*a^2*b^12 - 210*a^3*b^11 + (1395*a^4*b^10)/2 -
1332*a^5*b^9 + 1638*a^6*b^8 - 1332*a^7*b^7 + (1395*a^8*b^6)/2 - 210*a^9*b^5
 + 21*a^10*b^4 + 6*a^11*b^3 - (3*a^12*b^2)/2)/(a^12 - 12*a^11*b - 12*a*b^11
 + b^12 + 66*a^2*b^10 - 220*a^3*b^9 + 495*a^4*b^8 - 792*a^5*b^7 + 924*a^6*b
^6 - 792*a^7*b^5 + 495*a^8*b^4 - 220*a^9*b^3 + 66*a^10*b^2) + (3*tan(e + f*
x)*(10*a*b + a^2 + 5*b^2)*(1152*a*b^12 - 128*b^13 - 4480*a^2*b^11 + 9600*a^
3*b^10 - 11520*a^4*b^9 + 5376*a^5*b^8 + 5376*a^6*b^7 - 11520*a^7*b^6 + 9600
```

*a^8*b^5 - 4480*a^9*b^4 + 1152*a^10*b^3 - 128*a^11*b^2))/(256*(a*b^4*5i - a
^4*b*5i + a^5*1i - b^5*1i - a^2*b^3*10i + a^3*b^2*10i)*(a^8 - 8*a^7*b - 8*a
*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2
)))*(10*a*b + a^2 + 5*b^2))/(16*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^
2*b^3*10i + a^3*b^2*10i)))*(10*a*b + a^2 + 5*b^2))/(16*(a*b^4*5i - a^4*b*5i
 + a^5*1i - b^5*1i - a^2*b^3*10i + a^3*b^2*10i))))*(10*a*b + a^2 + 5*b^2)*3
i)/(8*f*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^2*b^3*10i + a^3*b^2*10i)
) - ((3*tan(e + f*x)*(a*b^2 + 6*a^2*b + a^3))/(8*(a^4 - 4*a^3*b - 4*a*b^3 +
 b^4 + 6*a^2*b^2)) + (tan(e + f*x)^5*(34*a*b^2 + 19*a^2*b + 19*b^3))/(8*(a^
4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e + f*x)^3*(31*a*b^2 + 31*
a^2*b + 5*a^3 + 5*b^3))/(8*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (
3*b*tan(e + f*x)^7*(a*b + b^2))/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b
^2)))/(f*(tan(e + f*x)^4*(4*a*b + a^2 + b^2) + a^2 + tan(e + f*x)^2*(2*a*b
 + 2*a^2) + tan(e + f*x)^6*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^8)) + (atan(((
(-a*b)^(1/2)*((tan(e + f*x)*(540*a*b^6 + 117*b^7 + 990*a^2*b^5 + 540*a^3*b^
4 + 117*a^4*b^3))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*
b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) + (3*(-a*b)^(1/2)*((6*a*b^13 -
 (3*b^14)/2 + 21*a^2*b^12 - 210*a^3*b^11 + (1395*a^4*b^10)/2 - 1332*a^5*b^9
 + 1638*a^6*b^8 - 1332*a^7*b^7 + (1395*a^8*b^6)/2 - 210*a^9*b^5 + 21*a^10*b
^4 + 6*a^11*b^3 - (3*a^12*b^2)/2)/(a^12 - 12*a^11*b - 12*a*b^11 + b^12 + 66
*a^2*b^10 - 220*a^3*b^9 + 495*a^4*b^8 - 792*a^5*b^7 + 924*a^6*b^6 - 792*a^7
*b^5 + 495*a^8*b^4 - 220*a^9*b^3 + 66*a^10*b^2) - (3*tan(e + f*x)*(-a*b)^(1
/2)*(10*a*b + 5*a^2 + b^2)*(1152*a*b^12 - 128*b^13 - 4480*a^2*b^11 + 9600*a
^3*b^10 - 11520*a^4*b^9 + 5376*a^5*b^8 + 5376*a^6*b^7 - 11520*a^7*b^6 + 960
0*a^8*b^5 - 4480*a^9*b^4 + 1152*a^10*b^3 - 128*a^11*b^2))/(256*(a*b^5 + 5*a
^5*b - a^6 - 5*a^2*b^4 + 10*a^3*b^3 - 10*a^4*b^2)*(a^8 - 8*a^7*b - 8*a*b^7
 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(
10*a*b + 5*a^2 + b^2))/(16*(a*b^5 + 5*a^5*b - a^6 - 5*a^2*b^4 + 10*a^3*b^3
 - 10*a^4*b^2)))*(10*a*b + 5*a^2 + b^2)*3i)/(16*(a*b^5 + 5*a^5*b - a^6 - 5*a
^2*b^4 + 10*a^3*b^3 - 10*a^4*b^2)) + ((-a*b)^(1/2)*((tan(e + f*x)*(540*a*b^
6 + 117*b^7 + 990*a^2*b^5 + 540*a^3*b^4 + 117*a^4*b^3))/(16*(a^8 - 8*a^7*b
 - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^
6*b^2)) - (3*(-a*b)^(1/2)*((6*a*b^13 - (3*b^14)/2 + 21*a^2*b^12 - 210*a^3*b
^11 + (1395*a^4*b^10)/2 - 1332*a^5*b^9 + 1638*a^6*b^8 - 1332*a^7*b^7 + (139
5*a^8*b^6)/2 - 210*a^9*b^5 + 21*a^10*b^4 + 6*a^11*b^3 - (3*a^12*b^2)/2)/(a^
12 - 12*a^11*b - 12*a*b^11 + b^12 + 66*a^2*b^10 - 220*a^3*b^9 + 495*a^4*b^8
 - 792*a^5*b^7 + 924*a^6*b^6 - 792*a^7*b^5 + 495*a^8*b^4 - 220*a^9*b^3 + 66
*a^10*b^2) + (3*tan(e + f*x)*(-a*b)^(1/2)*(10*a*b + 5*a^2 + b^2)*(1152*a*b^
12 - 128*b^13 - 4480*a^2*b^11 + 9600*a^3*b^10 - 11520*a^4*b^9 + 5376*a^5*b^
8 + 5376*a^6*b^7 - 11520*a^7*b^6 + 9600*a^8*b^5 - 4480*a^9*b^4 + 1152*a^10*
b^3 - 128*a^11*b^2))/(256*(a*b^5 + 5*a^5*b - a^6 - 5*a^2*b^4 + 10*a^3*b^3 -
 10*a^4*b^2)*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*
a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(10*a*b + 5*a^2 + b^2))/(16*(a*b^5 + 5
*a^5*b - a^6 - 5*a^2*b^4 + 10*a^3*b^3 - 10*a^4*b^2)))*(10*a*b + 5*a^2 + b^2
)*3i)/(16*(a*b^5 + 5*a^5*b - a^6 - 5*a^2*b^4 + 10*a^3*b^3 - 10*a^4*b^2)))/(
((1755*a*b^7)/64 + (135*b^8)/64 + (2511*a^2*b^6)/32 + (2511*a^3*b^5)/32 + (
1755*a^4*b^4)/64 + (135*a^5*b^3)/64)/(a^12 - 12*a^11*b - 12*a*b^11 + b^12 +
 66*a^2*b^10 - 220*a^3*b^9 + 495*a^4*b^8 - 792*a^5*b^7 + 924*a^6*b^6 - 792*
a^7*b^5 + 495*a^8*b^4 - 220*a^9*b^3 + 66*a^10*b^2) - (3*(-a*b)^(1/2)*((tan(
e + f*x)*(540*a*b^6 + 117*b^7 + 990*a^2*b^5 + 540*a^3*b^4 + 117*a^4*b^3))/(
16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 -
56*a^5*b^3 + 28*a^6*b^2)) + (3*(-a*b)^(1/2)*((6*a*b^13 - (3*b^14)/2 + 21*a^
2*b^12 - 210*a^3*b^11 + (1395*a^4*b^10)/2 - 1332*a^5*b^9 + 1638*a^6*b^8 - 1
332*a^7*b^7 + (1395*a^8*b^6)/2 - 210*a^9*b^5 + 21*a^10*b^4 + 6*a^11*b^3 - (
3*a^12*b^2)/2)/(a^12 - 12*a^11*b - 12*a*b^11 + b^12 + 66*a^2*b^10 - 220*a^3
*b^9 + 495*a^4*b^8 - 792*a^5*b^7 + 924*a^6*b^6 - 792*a^7*b^5 + 495*a^8*b^4
 - 220*a^9*b^3 + 66*a^10*b^2) - (3*tan(e + f*x)*(-a*b)^(1/2)*(10*a*b + 5*a^2
 + b^2)*(1152*a*b^12 - 128*b^13 - 4480*a^2*b^11 + 9600*a^3*b^10 - 11520*a^4
*b^9 + 5376*a^5*b^8 + 5376*a^6*b^7 - 11520*a^7*b^6 + 9600*a^8*b^5 - 4480*a^

```
9*b^4 + 1152*a^10*b^3 - 128*a^11*b^2))/(256*(a*b^5 + 5*a^5*b - a^6 - 5*a^2*
b^4 + 10*a^3*b^3 - 10*a^4*b^2)*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6
- 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(10*a*b + 5*a^2 + b^
2))/(16*(a*b^5 + 5*a^5*b - a^6 - 5*a^2*b^4 + 10*a^3*b^3 - 10*a^4*b^2)))*(10
*a*b + 5*a^2 + b^2))/(16*(a*b^5 + 5*a^5*b - a^6 - 5*a^2*b^4 + 10*a^3*b^3 -
10*a^4*b^2)) + (3*(-a*b)^(1/2)*((tan(e + f*x)*(540*a*b^6 + 117*b^7 + 990*a^
2*b^5 + 540*a^3*b^4 + 117*a^4*b^3))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28
*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (3*(-a*b)^
(1/2)*((6*a*b^13 - (3*b^14)/2 + 21*a^2*b^12 - 210*a^3*b^11 + (1395*a^4*b^10
)/2 - 1332*a^5*b^9 + 1638*a^6*b^8 - 1332*a^7*b^7 + (1395*a^8*b^6)/2 - 210*a
^9*b^5 + 21*a^10*b^4 + 6*a^11*b^3 - (3*a^12*b^2)/2)/(a^12 - 12*a^11*b - 12*
a*b^11 + b^12 + 66*a^2*b^10 - 220*a^3*b^9 + 495*a^4*b^8 - 792*a^5*b^7 + 924
*a^6*b^6 - 792*a^7*b^5 + 495*a^8*b^4 - 220*a^9*b^3 + 66*a^10*b^2) + (3*tan(
e + f*x)*(-a*b)^(1/2)*(10*a*b + 5*a^2 + b^2)*(1152*a*b^12 - 128*b^13 - 4480
*a^2*b^11 + 9600*a^3*b^10 - 11520*a^4*b^9 + 5376*a^5*b^8 + 5376*a^6*b^7 - 1
1520*a^7*b^6 + 9600*a^8*b^5 - 4480*a^9*b^4 + 1152*a^10*b^3 - 128*a^11*b^2))
/(256*(a*b^5 + 5*a^5*b - a^6 - 5*a^2*b^4 + 10*a^3*b^3 - 10*a^4*b^2)*(a^8 -
8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3
 + 28*a^6*b^2)))*(10*a*b + 5*a^2 + b^2))/(16*(a*b^5 + 5*a^5*b - a^6 - 5*a^2
*b^4 + 10*a^3*b^3 - 10*a^4*b^2)))*(10*a*b + 5*a^2 + b^2))/(16*(a*b^5 + 5*a^
5*b - a^6 - 5*a^2*b^4 + 10*a^3*b^3 - 10*a^4*b^2))))*(-a*b)^(1/2)*(10*a*b +
5*a^2 + b^2)*3i)/(8*f*(a*b^5 + 5*a^5*b - a^6 - 5*a^2*b^4 + 10*a^3*b^3 - 10*
a^4*b^2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.87 $\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=193

$$-\frac{\sqrt{b}\left(15a^2+10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}f(a-b)^4}-\frac{b(11a+b)\tan(e+fx)}{8af(a-b)^3\left(a+b\tan^2(e+fx)\right)}-\frac{3b\tan(e+fx)}{4f(a-b)^2\left(a+b\tan^2(e+fx)\right)^2}-$$

```
[Out] 1/2*(a+5*b)*x/(a-b)^4-1/8*(15*a^2+10*a*b-b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(
1/2))*b^(1/2)/a^(3/2)/(a-b)^4/f-1/2*cos(f*x+e)*sin(f*x+e)/(a-b)/f/(a+b*tan(
f*x+e)^2)^2-3/4*b*tan(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^2-1/8*b*(11*a+b)*
tan(f*x+e)/a/(a-b)^3/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.25, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 471, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2+10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}f(a-b)^4}-\frac{b(11a+b)\tan(e+fx)}{8af(a-b)^3\left(a+b\tan^2(e+fx)\right)}-\frac{3b\tan(e+fx)}{4f(a-b)^2\left(a+b\tan^2(e+fx)\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((a + 5*b)*x)/(2*(a - b)^4) - (Sqrt[b]*(15*a^2 + 10*a*b - b^2)*ArcTan[(Sqrt
[b]*Tan[e + f*x])/Sqrt[a]])/(8*a^(3/2)*(a - b)^4*f) - (Cos[e + f*x]*Sin[e +
 f*x])/(2*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - (3*b*Tan[e + f*x])/(4*(a -
b)^2*f*(a + b*Tan[e + f*x]^2)^2) - (b*(11*a + b)*Tan[e + f*x])/(8*a*(a - b)
^3*f*(a + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a-5bx^2}{\left(1+x^2\right)\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b(11}{8a(a-b}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b(11}{8a(a-b}$$

$$= \frac{(a+5b)x}{2(a-b)^4} - \frac{\sqrt{b}\left(15a^2+10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^4f} - \frac{\cos(e+fx)\sin(e}{2(a-b)f\left(a+b\tan^2(\right.}$$

Mathematica [A] time = 2.56, size = 164, normalized size = 0.85

$$\frac{\frac{\sqrt{b}\left(-15a^2-10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{4b^2(a-b)\sin(2(e+fx))}{((a-b)\cos(2(e+fx))+a+b)^2} + 4(a+5b)(e+fx) - 2(a-b)\sin(2(e+fx)) - \frac{b(a-b)(}{a((a-b)}}{8f(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (4*(a + 5*b)*(e + f*x) + (Sqrt[b]*(-15*a^2 - 10*a*b + b^2)*ArcTan[(Sqrt[b]*
Tan[e + f*x])/Sqrt[a]])/a^(3/2) - 2*(a - b)*Sin[2*(e + f*x)] + (4*(a - b)*b
^2*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)])^2 - ((a - b)*b*(9*a
```

```
 + b)*Sin[2*(e + f*x)])/(a*(a + b + (a - b)*Cos[2*(e + f*x)]))))/(8*(a - b)^
4*f)
```

fricas [B] time = 0.88, size = 1076, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(16*(a^4 + 3*a^3*b - 9*a^2*b^2 + 5*a*b^3)*f*x*cos(f*x + e)^4 + 32*(a^
3*b + 4*a^2*b^2 - 5*a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 5*a*b^3)*f*x
- ((15*a^4 - 20*a^3*b - 6*a^2*b^2 + 12*a*b^3 - b^4)*cos(f*x + e)^4 + 15*a^2
*b^2 + 10*a*b^3 - b^4 + 2*(15*a^3*b - 5*a^2*b^2 - 11*a*b^3 + b^4)*cos(f*x +
 e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)
*cos(f*x + e)^2 - 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b
/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)
*cos(f*x + e)^2 + b^2)) - 4*(4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*cos(f*x
+ e)^5 + (17*a^3*b - 33*a^2*b^2 + 15*a*b^3 + b^4)*cos(f*x + e)^3 + (11*a^2*
b^2 - 10*a*b^3 - b^4)*cos(f*x + e))*sin(f*x + e))/((a^7 - 6*a^6*b + 15*a^5*
b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*f*cos(f*x + e)^4 + 2*(a^
6*b - 5*a^5*b^2 + 10*a^4*b^3 - 10*a^3*b^4 + 5*a^2*b^5 - a*b^6)*f*cos(f*x +
e)^2 + (a^5*b^2 - 4*a^4*b^3 + 6*a^3*b^4 - 4*a^2*b^5 + a*b^6)*f), 1/16*(8*(a
^4 + 3*a^3*b - 9*a^2*b^2 + 5*a*b^3)*f*x*cos(f*x + e)^4 + 16*(a^3*b + 4*a^2*
b^2 - 5*a*b^3)*f*x*cos(f*x + e)^2 + 8*(a^2*b^2 + 5*a*b^3)*f*x + ((15*a^4 -
20*a^3*b - 6*a^2*b^2 + 12*a*b^3 - b^4)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b
^3 - b^4 + 2*(15*a^3*b - 5*a^2*b^2 - 11*a*b^3 + b^4)*cos(f*x + e)^2)*sqrt(b
/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f
*x + e))) - 2*(4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + (17*a
^3*b - 33*a^2*b^2 + 15*a*b^3 + b^4)*cos(f*x + e)^3 + (11*a^2*b^2 - 10*a*b^3
 - b^4)*cos(f*x + e))*sin(f*x + e))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b
^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*f*cos(f*x + e)^4 + 2*(a^6*b - 5*a^5*b^
2 + 10*a^4*b^3 - 10*a^3*b^4 + 5*a^2*b^5 - a*b^6)*f*cos(f*x + e)^2 + (a^5*b^
2 - 4*a^4*b^3 + 6*a^3*b^4 - 4*a^2*b^5 + a*b^6)*f)]
```

giac [A] time = 2.93, size = 282, normalized size = 1.46

$$\frac{\frac{4\left(fx+e\right)\left(a+5\,b\right)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4} - \frac{\left(15\,a^2b+10\,ab^2-b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4\right)\sqrt{ab}} - \frac{4\,\tan\left(fx+e\right)}{\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\left(\tan\left(fx+e\right)^2+1\right)} - \frac{7\,ab^2\tan\left(fx\right.}{}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*(f*x + e)*(a + 5*b)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (1
5*a^2*b + 10*a*b^2 - b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*t
an(f*x + e)/sqrt(a*b)))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sq
rt(a*b)) - 4*tan(f*x + e)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(tan(f*x + e)^2
+ 1)) - (7*a*b^2*tan(f*x + e)^3 + b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e)
 - a*b^2*tan(f*x + e))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*(b*tan(f*x + e)
^2 + a)^2))/f
```

maple [B] time = 0.56, size = 430, normalized size = 2.23

$$-\frac{7b^2a\left(\tan^3\left(fx+e\right)\right)}{8f\left(a-b\right)^4\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{3b^3\left(\tan^3\left(fx+e\right)\right)}{4f\left(a-b\right)^4\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{b^4\left(\tan^3\left(fx+e\right)\right)}{8f\left(a-b\right)^4\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f/(a-b)^4*b^2/(a+b*tan(f*x+e)^2)^2*a*tan(f*x+e)^3+3/4/f/(a-b)^4*b^3/(a
+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+1/8/f/(a-b)^4*b^4/(a+b*tan(f*x+e)^2)^2/a*ta
n(f*x+e)^3-9/8/f/(a-b)^4*b/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)*a^2+5/4/f/(a-b)^
4*b^2/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)*a-1/8/f/(a-b)^4*b^3/(a+b*tan(f*x+e)^2
)^2*tan(f*x+e)-15/8/f/(a-b)^4*b*a/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/
2))-5/4/f/(a-b)^4*b^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+1/8/f/(a
-b)^4*b^3/a/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/2/f/(a-b)^4*tan(
f*x+e)/(1+tan(f*x+e)^2)*a+1/2/f/(a-b)^4*tan(f*x+e)/(1+tan(f*x+e)^2)*b+1/2/f
/(a-b)^4*arctan(tan(f*x+e))*a+5/2/f/(a-b)^4*arctan(tan(f*x+e))*b
```

maxima [A] time = 0.64, size = 346, normalized size = 1.79

$$\frac{\frac{4\left(fx+e\right)(a+5\,b)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{\left(15\,a^2b+10\,ab^2-b^3\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4\right)\sqrt{ab}}-\frac{\left(11\,ab^2+b^3\right)\tan\left(fx+e\right)^5+\left(17\,a^2b+6\,\cdots\right.}{\left(a^4b^2-3\,a^3b^3+3\,a^2b^4-ab^5\right)\tan\left(fx+e\right)^6+a^6-3\,a^5b+3\,a^4b^2-a^3b^3+\left(2\,a^5b-\cdots\right.}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*(f*x + e)*(a + 5*b)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (1
5*a^2*b + 10*a*b^2 - b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5 - 4*a^4*b
+ 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sqrt(a*b)) - ((11*a*b^2 + b^3)*tan(f*x + e
)^5 + (17*a^2*b + 6*a*b^2 + b^3)*tan(f*x + e)^3 + (4*a^3 + 9*a^2*b - a*b^2)
*tan(f*x + e))/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*tan(f*x + e)^6 +
a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3 + (2*a^5*b - 5*a^4*b^2 + 3*a^3*b^3 + a^
2*b^4 - a*b^5)*tan(f*x + e)^4 + (a^6 - a^5*b - 3*a^4*b^2 + 5*a^3*b^3 - 2*a^
2*b^4)*tan(f*x + e)^2))/f
```

mupad [B] time = 16.49, size = 4997, normalized size = 25.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] - ((tan(e + f*x)^5*(11*a*b^2 + b^3))/(8*a*(3*a*b^2 - 3*a^2*b + a^3 - b^3))
+ (tan(e + f*x)*(9*a*b + 4*a^2 - b^2))/(8*(a - b)*(a^2 - 2*a*b + b^2)) + (b
*tan(e + f*x)^3*(6*a*b + 17*a^2 + b^2))/(8*a*(a - b)*(a^2 - 2*a*b + b^2)))/
(f*(tan(e + f*x)^2*(2*a*b + a^2) + tan(e + f*x)^4*(2*a*b + b^2) + a^2 + b^2
*tan(e + f*x)^6)) - (atan(((((tan(e + f*x)*(b^7 - 20*a*b^6 + 470*a^2*b^5 +
460*a^3*b^4 + 241*a^4*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a
^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) - ((((17*a^2*b^11)/2 - (a*b^12)/2 - 48*a
^3*b^10 + 138*a^4*b^9 - 231*a^5*b^8 + 231*a^6*b^7 - 126*a^7*b^6 + 18*a^8*b^
5 + (39*a^9*b^4)/2 - (23*a^10*b^3)/2 + 2*a^11*b^2)/(9*a^10*b - a^11 + a^2*b
^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a
^8*b^3 - 36*a^9*b^2) - (tan(e + f*x)*(a*1i + b*5i)*(256*a^2*b^11 - 1792*a^3
*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^
8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(128*(a^4 - 4*a^3*b -
 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b
^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a*1i + b*5i))/(4*(a^4 - 4*a^3*b - 4*a*b^3
+ b^4 + 6*a^2*b^2)))*(a*1i + b*5i)*1i)/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 +
6*a^2*b^2)) + (((tan(e + f*x)*(b^7 - 20*a*b^6 + 470*a^2*b^5 + 460*a^3*b^4 +
 241*a^4*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a
^5*b^3 + 15*a^6*b^2)) + ((((17*a^2*b^11)/2 - (a*b^12)/2 - 48*a^3*b^10 + 138
*a^4*b^9 - 231*a^5*b^8 + 231*a^6*b^7 - 126*a^7*b^6 + 18*a^8*b^5 + (39*a^9*b
```

```
^4)/2 - (23*a^10*b^3)/2 + 2*a^11*b^2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^
8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a
^9*b^2) + (tan(e + f*x)*(a*1i + b*5i)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*
a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*
a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(128*(a^4 - 4*a^3*b - 4*a*b^3 + b^
4 + 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b
^3 + 15*a^6*b^2)))*(a*1i + b*5i))/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2
*b^2)))*(a*1i + b*5i)*1i)/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))/
(((39*a^2*b^5)/4 - (5*b^7)/64 - (3*a*b^6)/32 + (475*a^3*b^4)/32 + (165*a^4*
b^3)/64)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 +
 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) - (((tan(e + f*x)*(b^
7 - 20*a*b^6 + 470*a^2*b^5 + 460*a^3*b^4 + 241*a^4*b^3))/(32*(a^8 - 6*a^7*b
 + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) - ((((17*a^
2*b^11)/2 - (a*b^12)/2 - 48*a^3*b^10 + 138*a^4*b^9 - 231*a^5*b^8 + 231*a^6*
b^7 - 126*a^7*b^6 + 18*a^8*b^5 + (39*a^9*b^4)/2 - (23*a^10*b^3)/2 + 2*a^11*
b^2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126
*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) - (tan(e + f*x)*(a*1i + b
*5i)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6
*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^1
1*b^2))/(128*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^8 - 6*a^7*b + a
^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a*1i + b*5i))
/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(a*1i + b*5i))/(4*(a^4 -
4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((tan(e + f*x)*(b^7 - 20*a*b^6 + 4
70*a^2*b^5 + 460*a^3*b^4 + 241*a^4*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a
^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) + ((((17*a^2*b^11)/2 - (a*b
^12)/2 - 48*a^3*b^10 + 138*a^4*b^9 - 231*a^5*b^8 + 231*a^6*b^7 - 126*a^7*b^
6 + 18*a^8*b^5 + (39*a^9*b^4)/2 - (23*a^10*b^3)/2 + 2*a^11*b^2)/(9*a^10*b -
 a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a
^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) + (tan(e + f*x)*(a*1i + b*5i)*(256*a^2*b^
11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*
b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(128*(a^
4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b
^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a*1i + b*5i))/(4*(a^4 - 4*a^3
*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(a*1i + b*5i))/(4*(a^4 - 4*a^3*b - 4*a*b^
3 + b^4 + 6*a^2*b^2))))*(a*1i + b*5i)*1i)/(2*f*(a^4 - 4*a^3*b - 4*a*b^3 + b
^4 + 6*a^2*b^2)) - (atan(((((tan(e + f*x)*(b^7 - 20*a*b^6 + 470*a^2*b^5 + 4
60*a^3*b^4 + 241*a^4*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^
4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) - ((-a^3*b)^(1/2)*(((17*a^2*b^11)/2 - (a*
b^12)/2 - 48*a^3*b^10 + 138*a^4*b^9 - 231*a^5*b^8 + 231*a^6*b^7 - 126*a^7*b
^6 + 18*a^8*b^5 + (39*a^9*b^4)/2 - (23*a^10*b^3)/2 + 2*a^11*b^2)/(9*a^10*b
- a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*
a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) - (tan(e + f*x)*(-a^3*b)^(1/2)*(10*a*b +
 15*a^2 - b^2)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8
+ 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3
 + 256*a^11*b^2))/(512*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)*(a
^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2))
)*(10*a*b + 15*a^2 - b^2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5
*b^2)))*(-a^3*b)^(1/2)*(10*a*b + 15*a^2 - b^2)*1i)/(16*(a^7 - 4*a^6*b + a^3
*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) + (((tan(e + f*x)*(b^7 - 20*a*b^6 + 470*a^2*
b^5 + 460*a^3*b^4 + 241*a^4*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5
+ 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) + ((-a^3*b)^(1/2)*(((17*a^2*b^11)/
2 - (a*b^12)/2 - 48*a^3*b^10 + 138*a^4*b^9 - 231*a^5*b^8 + 231*a^6*b^7 - 12
6*a^7*b^6 + 18*a^8*b^5 + (39*a^9*b^4)/2 - (23*a^10*b^3)/2 + 2*a^11*b^2)/(9*
a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5
 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) + (tan(e + f*x)*(-a^3*b)^(1/2)*(1
0*a*b + 15*a^2 - b^2)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a
^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a
^10*b^3 + 256*a^11*b^2))/(512*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*
b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^
```

```
6*b^2)))*(10*a*b + 15*a^2 - b^2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3
+ 6*a^5*b^2)))*(-a^3*b)^(1/2)*(10*a*b + 15*a^2 - b^2)*1i)/(16*(a^7 - 4*a^6*
b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))/(((39*a^2*b^5)/4 - (5*b^7)/64 - (3*a
*b^6)/32 + (475*a^3*b^4)/32 + (165*a^4*b^3)/64)/(9*a^10*b - a^11 + a^2*b^9
- 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*
b^3 - 36*a^9*b^2) - (((tan(e + f*x)*(b^7 - 20*a*b^6 + 470*a^2*b^5 + 460*a^3
*b^4 + 241*a^4*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4
- 20*a^5*b^3 + 15*a^6*b^2)) - ((-a^3*b)^(1/2)*(((17*a^2*b^11)/2 - (a*b^12)/
2 - 48*a^3*b^10 + 138*a^4*b^9 - 231*a^5*b^8 + 231*a^6*b^7 - 126*a^7*b^6 + 1
8*a^8*b^5 + (39*a^9*b^4)/2 - (23*a^10*b^3)/2 + 2*a^11*b^2)/(9*a^10*b - a^11
 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^
4 + 84*a^8*b^3 - 36*a^9*b^2) - (tan(e + f*x)*(-a^3*b)^(1/2)*(10*a*b + 15*a^
2 - b^2)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584
*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256
*a^11*b^2))/(512*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)*(a^8 - 6
*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(10*
a*b + 15*a^2 - b^2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2))
)*(-a^3*b)^(1/2)*(10*a*b + 15*a^2 - b^2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*
a^4*b^3 + 6*a^5*b^2)) + (((tan(e + f*x)*(b^7 - 20*a*b^6 + 470*a^2*b^5 + 460
*a^3*b^4 + 241*a^4*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*
b^4 - 20*a^5*b^3 + 15*a^6*b^2)) + ((-a^3*b)^(1/2)*(((17*a^2*b^11)/2 - (a*b^
12)/2 - 48*a^3*b^10 + 138*a^4*b^9 - 231*a^5*b^8 + 231*a^6*b^7 - 126*a^7*b^6
 + 18*a^8*b^5 + (39*a^9*b^4)/2 - (23*a^10*b^3)/2 + 2*a^11*b^2)/(9*a^10*b -
a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^
7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) + (tan(e + f*x)*(-a^3*b)^(1/2)*(10*a*b + 1
5*a^2 - b^2)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 +
3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 +
 256*a^11*b^2))/(512*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)*(a^8
 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*
(10*a*b + 15*a^2 - b^2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b
^2)))*(-a^3*b)^(1/2)*(10*a*b + 15*a^2 - b^2))/(16*(a^7 - 4*a^6*b + a^3*b^4
- 4*a^4*b^3 + 6*a^5*b^2))))*(-a^3*b)^(1/2)*(10*a*b + 15*a^2 - b^2)*1i)/(8*f
*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.88 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=150

$$-\frac{b(7a-3b)\tan(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}f(a-b)^3}-\frac{b\tan(e+fx)}{4af(a-b)\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] x/(a-b)^3-1/8*(15*a^2-10*a*b+3*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1
/2)/a^(5/2)/(a-b)^3/f-1/4*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8*(
7*a-3*b)*b*tan(f*x+e)/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3661, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}f(a-b)^3}-\frac{b(7a-3b)\tan(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{b\tan(e+fx)}{4af(a-b)\left(a+b\tan^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-3),x]
```

```
[Out] x/(a - b)^3 - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*f) - (b*Tan[e + f*x])/(4*a*(a - b)*f*(a
+ b*Tan[e + f*x]^2)^2) - ((7*a - 3*b)*b*Tan[e + f*x])/(8*a^2*(a - b)^2*f*(a
 + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-3b-3bx^2}{(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-3b)b\tan(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}}{}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-3b)b\tan(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}}{}$$

$$= \frac{x}{(a-b)^3} - \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3f} - \frac{b\tan(e+fx}{4a(a-b)f\left(a+b\tan^2\right.}$$

Mathematica [A] time = 1.97, size = 138, normalized size = 0.92

$$-\frac{\frac{b(7a-3b)(a-b)\tan(e+fx)}{a^2\left(a+b\tan^2(e+fx)\right)} + \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2b(a-b)^2\tan(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^2} - 8\tan^{-1}(\tan(e+fx))}{8f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-3),x]
```

```
[Out] -1/8*(-8*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(
Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Tan[e + f*x])/(a*(
a + b*Tan[e + f*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Tan[e + f*x])/(a^2*(a + b
*Tan[e + f*x]^2)))/((a - b)^3*f)
```

fricas [B] time = 0.70, size = 742, normalized size = 4.95

$$\left[\frac{32\,a^2b^2fx\tan\left(fx+e\right)^4+64\,a^3bfx\tan\left(fx+e\right)^2+32\,a^4fx-4\left(7\,a^2b^2-10\,ab^3+3\,b^4\right)\tan\left(fx+e\right)^3-\left(\left(15\,a^2b\right.\right.}{32\left(\left(a^5b^2-3\,a^4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*a^2*b^2*f*x*tan(f*x + e)^4 + 64*a^3*b*f*x*tan(f*x + e)^2 + 32*a^4
*f*x - 4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 10*
a*b^3 + 3*b^4)*tan(f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b
 - 10*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4
 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*s
qrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(9*a^3*b
- 14*a^2*b^2 + 5*a*b^3)*tan(f*x + e))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*ta
n(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/16*(16*a^2*b^2*f
*x*tan(f*x + e)^4 + 32*a^3*b*f*x*tan(f*x + e)^2 + 16*a^4*f*x - 2*(7*a^2*b^2
 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(
f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b - 10*a^2*b^2 + 3*a
*b^3)*tan(f*x + e)^2)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)
/(b*tan(f*x + e))) - 2*(9*a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(f*x + e))/((a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 -
 a^4*b^3)*f)]
```

giac [A] time = 1.41, size = 213, normalized size = 1.42

$$-\frac{\frac{\left(15\,a^2b-10\,ab^2+3\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^5-3\,a^4b+3\,a^3b^2-a^2b^3\right)\sqrt{ab}}-\frac{8\left(fx+e\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{7\,ab^2\tan\left(fx+e\right)^3-3\,b^3\tan\left(fx+e\right)^3+9\,a^2b\tan\left(fx+e\right)-5\,ab}{\left(a^4-2\,a^3b+a^2b^2\right)\left(b\tan\left(fx+e\right)^2+a\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sq
rt(a*b)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (7*a*b^2*tan(f*x +
 e)^3 - 3*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) - 5*a*b^2*tan(f*x + e))
/((a^4 - 2*a^3*b + a^2*b^2)*(b*tan(f*x + e)^2 + a)^2))/f
```

maple [B] time = 0.27, size = 350, normalized size = 2.33

$$-\frac{7b^2\left(\tan^3\left(fx+e\right)\right)}{8f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{5b^3\left(\tan^3\left(fx+e\right)\right)}{4f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2a}-\frac{3b^4\left(\tan^3\left(fx+e\right)\right)}{8f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f*b^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+5/4/f*b^3/(a-b)^3/(a+b
*tan(f*x+e)^2)^2/a*tan(f*x+e)^3-3/8/f*b^4/(a-b)^3/(a+b*tan(f*x+e)^2)^2/a^2*
tan(f*x+e)^3-9/8/f*b/(a-b)^3/(a+b*tan(f*x+e)^2)^2*a*tan(f*x+e)+7/4/f*b^2/(a
```

```
-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)-5/8/f*b^3/(a-b)^3/(a+b*tan(f*x+e)^2)^
2/a*tan(f*x+e)-15/8/f*b/(a-b)^3/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2)
)+5/4/f*b^2/(a-b)^3/a/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-3/8/f*b^
3/(a-b)^3/a^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+1/f/(a-b)^3*arct
an(tan(f*x+e))
```

maxima [A] time = 0.54, size = 227, normalized size = 1.51

$$-\frac{\frac{\left(15\,a^2b-10\,ab^2+3\,b^3\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^5-3\,a^4b+3\,a^3b^2-a^2b^3\right)\sqrt{ab}}+\frac{\left(7\,ab^2-3\,b^3\right)\tan\left(fx+e\right)^3+\left(9\,a^2b-5\,ab^2\right)\tan\left(fx+e\right)}{a^6-2\,a^5b+a^4b^2+\left(a^4b^2-2\,a^3b^3+a^2b^4\right)\tan\left(fx+e\right)^4+2\left(a^5b-2\,a^4b^2+a^3b^3\right)\tan\left(fx+e\right)^2}-\frac{8\left(fx+\right.}{a^3-3\,a^2b+3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5
- 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(a*b)) + ((7*a*b^2 - 3*b^3)*tan(f*x +
e)^3 + (9*a^2*b - 5*a*b^2)*tan(f*x + e))/(a^6 - 2*a^5*b + a^4*b^2 + (a^4*b^
2 - 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 + 2*(a^5*b - 2*a^4*b^2 + a^3*b^3)*t
an(f*x + e)^2) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

mupad [B] time = 14.93, size = 3901, normalized size = 26.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] (atan((((-a^5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300
*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b
^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^
6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^1
0 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2))
- (tan(e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 128
0*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 128
0*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8
 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10
*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b
 + 3*b^2)*1i)/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^5*b)^(1/2)*
((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3)
)/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((96*a^2*b^10 -
 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 +
5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 -
 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(e + f*x)*(-a^5*b
)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^
7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b
^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 -
4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a
^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2)*1i)/(16*(3*a^
7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))/((51*a*b^5 - 9*b^6 - 115*a^2*b^4 + 105*a
^3*b^3)/(32*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3
 + 15*a^8*b^2)) - ((-a^5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^
2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^
3 + 6*a^6*b^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^
7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^
2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15
*a^8*b^2)) - (tan(e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^
4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^
```

```
9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^
6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(
15*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a
^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^5*b
)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*
a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((96*a^
2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^
7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a
^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(e + f*x)
*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*b^8 + 230
4*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 25
6*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^
4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/
(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2))/(16*
(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2))))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b +
3*b^2)*1i)/(8*f*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) - ((tan(e + f*x)^3*(
7*a*b^2 - 3*b^3))/(8*a^2*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)*(9*a*b - 5*b^
2))/(8*a*(a^2 - 2*a*b + b^2)))/(f*(a^2 + b^2*tan(e + f*x)^4 + 2*a*b*tan(e +
 f*x)^2)) - (2*atan((((((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^
5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^1
0*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3
 + 15*a^8*b^2)) - (tan(e + f*x)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 -
 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2)
*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a
^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*
x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 -
 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 -
2*b^3) - ((((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760
*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(
a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2
)) + (tan(e + f*x)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^
6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2)*1i)/(32*(6*
a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a
^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (tan(e + f*x)*(9*b^7 -
60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a
^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))/((51
*a*b^5 - 9*b^6 - 115*a^2*b^4 + 105*a^3*b^3)/(32*(a^10 - 6*a^9*b + a^4*b^6 -
 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (((((96*a^2*b^10 - 80
0*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 528
0*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*
a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) - (tan(e + f*x)*(256*a^4*b
^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b
^4 - 1280*a^10*b^3 + 256*a^11*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b
^3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^
2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*
a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^
2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (((((96*a^2*b^10 - 800*a^3*b
^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b
^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5
 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(e + f*x)*(256*a^4*b^9 - 12
80*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 12
80*a^10*b^3 + 256*a^11*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^
8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2
*a^3 - 2*b^3) + (tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4
 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i
)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))))/(f*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*
b^3))
```

<u>**sympy**</u> **[A]** time = 138.50, size = 9629, normalized size = 64.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**6, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a**3, Eq(b,
 0)), ((-x - 1/(f*tan(e + f*x)) + 1/(3*f*tan(e + f*x)**3) - 1/(5*f*tan(e +
f*x)**5))/b**3, Eq(a, 0)), (15*f*x*tan(e + f*x)**6/(48*b**3*f*tan(e + f*x)*
*6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) +
 45*f*x*tan(e + f*x)**4/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x
)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 45*f*x*tan(e + f*x)**2/(48
*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f
*x)**2 + 48*b**3*f) + 15*f*x/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e
+ f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 15*tan(e + f*x)**5/(4
8*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e +
f*x)**2 + 48*b**3*f) + 40*tan(e + f*x)**3/(48*b**3*f*tan(e + f*x)**6 + 144*
b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 33*tan(e
 + f*x)/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*
f*tan(e + f*x)**2 + 48*b**3*f), Eq(a, b)), (x/(a + b*tan(e)**2)**3, Eq(f, 0
)), (16*I*a**(9/2)*f*x*sqrt(1/b)/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/
2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(
11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*ta
n(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt
(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16
*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)*
*4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*
sqrt(1/b)*tan(e + f*x)**4) + 32*I*a**(7/2)*b*f*x*sqrt(1/b)*tan(e + f*x)**2/
(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2
- 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*
x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b*
*2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(
9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48
*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/
b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) - 18*I
*a**(7/2)*b*sqrt(1/b)*tan(e + f*x)/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(1
3/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a*
*(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*
tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sq
rt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 -
16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x
)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*
f*sqrt(1/b)*tan(e + f*x)**4) + 16*I*a**(5/2)*b**2*f*x*sqrt(1/b)*tan(e + f*x
)**4/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x
)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e
 + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/
2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I
*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b)
 + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sq
rt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) -
 14*I*a**(5/2)*b**2*sqrt(1/b)*tan(e + f*x)**3/(16*I*a**(15/2)*f*sqrt(1/b) +
 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b
) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f
*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2
)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e +
 f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*
tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**
(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) + 28*I*a**(5/2)*b**2*sqrt(1/b)*tan(
```

```
e + f*x)/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e +
 f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*t
an(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**
(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 +
96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(
1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*
f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**
4) + 20*I*a**(3/2)*b**3*sqrt(1/b)*tan(e + f*x)**3/(16*I*a**(15/2)*f*sqrt(1/
b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt
(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b*
*2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**
(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan
(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1
/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I
*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) - 10*I*a**(3/2)*b**3*sqrt(1/b)*
tan(e + f*x)/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan
(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/
b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I
*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**
4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*s
qrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b
**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*
x)**4) - 6*I*sqrt(a)*b**4*sqrt(1/b)*tan(e + f*x)**3/(16*I*a**(15/2)*f*sqrt(
1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sq
rt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*
b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a
**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*t
an(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt
(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16
*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) - 15*a**4*log(-I*sqrt(a)*sqrt
(1/b) + tan(e + f*x))/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt
(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*
f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)*
*2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e
 + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)
*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a
**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*t
an(e + f*x)**4) + 15*a**4*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**
(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a*
*(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 9
6*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt
(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3
*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/
2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e
+ f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) - 30*a**3*b*log
(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16*I*a**(15/2)*f*sqr
t(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*
sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2
)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I
*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)
*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sq
rt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 -
16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) + 10*a**3*b*log(-I*sqrt(a)*
sqrt(1/b) + tan(e + f*x))/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*
sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b
**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f
*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*t
an(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(
9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32
```

```
*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/
b)*tan(e + f*x)**4) + 30*a**3*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan
(e + f*x)**2/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan
(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/
b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I
*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**
4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*s
qrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b
**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*
x)**4) - 10*a**3*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(15/2)*
f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)
*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**
(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) -
 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt
(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4
*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)*
*2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) - 15*a**2*b**2*log(-I*
sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(15/2)*f*sqrt(1/
b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt
(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b*
*2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**
(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan
(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1
/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I
*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) + 20*a**2*b**2*log(-I*sqrt(a)*s
qrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I
*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 1
6*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt
(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**
3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)
**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e
 + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)
*b**5*f*sqrt(1/b)*tan(e + f*x)**4) - 3*a**2*b**2*log(-I*sqrt(a)*sqrt(1/b) +
 tan(e + f*x))/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*t
an(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(
1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48
*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)
**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f
*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)
*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e +
f*x)**4) + 15*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x
)**4/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x
)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e
 + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/
2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I
*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b)
 + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sq
rt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) -
 20*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16*I
*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*
I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4
 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*
sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*
b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a*
*(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*ta
n(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) + 3*a**2*b*
*2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(15/2)*f*sqrt(1/b) + 32
*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) +
 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sq
```

```
rt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b
**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*
x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan
(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/
2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) + 10*a*b**3*log(-I*sqrt(a)*sqrt(1/b) +
 tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)
*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11
/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(
e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1
/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I
*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4
 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sq
rt(1/b)*tan(e + f*x)**4) - 6*a*b**3*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x)
)*tan(e + f*x)**2/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b
)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sq
rt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 +
 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f
*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**
3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7
/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e
 + f*x)**4) - 10*a*b**3*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x
)**4/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x
)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e
 + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/
2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I
*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b)
 + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sq
rt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) +
 6*a*b**3*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16*I*a**
(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a*
*(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 9
6*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt
(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3
*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/
2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e
+ f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) - 3*b**4*log(-I
*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(15/2)*f*sqrt(1
/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqr
t(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b
**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a*
*(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*ta
n(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(
1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*
I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) + 3*b**4*log(I*sqrt(a)*sqrt(1/
b) + tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(1
3/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a*
*(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*
tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sq
rt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 -
16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x
)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*
f*sqrt(1/b)*tan(e + f*x)**4), True))
```

3.89 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=112

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f}-\frac{15\cot(e+fx)}{8a^3f}+\frac{5\cot(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)}+\frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2}$$

```
[Out] -15/8*cot(f*x+e)/a^3/f-15/8*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(7
/2)/f+1/4*cot(f*x+e)/a/f/(a+b*tan(f*x+e)^2)^2+5/8*cot(f*x+e)/a^2/f/(a+b*tan
(f*x+e)^2)
```

Rubi [A] time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3663, 290, 325, 205}

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{5\cot(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)}-\frac{15\cot(e+fx)}{8a^3f}+\frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(8*a^(7/2)*f) - (15*Co
t[e + f*x])/(8*a^3*f) + Cot[e + f*x]/(4*a*f*(a + b*Tan[e + f*x]^2)^2) + (5*
Cot[e + f*x])/(8*a^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2} + \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)} + \frac{15\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)}\,dx,\right.}{8a^2f}$$

$$= -\frac{15\cot(e+fx)}{8a^3f} + \frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)} - \frac{(15b)\,\text{S}}{}$$

$$= -\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f} - \frac{15\cot(e+fx)}{8a^3f} + \frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2} + \frac{}{8a^2f\,(}$$

Mathematica [A] time = 0.99, size = 144, normalized size = 1.29

$$\frac{\frac{4a^{3/2}b^2\sin(2(e+fx))}{(a-b)((a-b)\cos(2(e+fx))+a+b)^2} - 15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) - \frac{\sqrt{a}\,b(9a-7b)\sin(2(e+fx))}{(a-b)((a-b)\cos(2(e+fx))+a+b)} - 8\sqrt{a}\cot(e+fx)}{8a^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - 8*Sqrt[a]*Cot[e + f*x
] + (4*a^(3/2)*b^2*Sin[2*(e + f*x)])/((a - b)*(a + b + (a - b)*Cos[2*(e + f
*x)])^2) - (Sqrt[a]*(9*a - 7*b)*b*Sin[2*(e + f*x)])/((a - b)*(a + b + (a -
b)*Cos[2*(e + f*x)])))/(8*a^(7/2)*f)
```

fricas [B] time = 0.85, size = 555, normalized size = 4.96

$$\left[-\frac{4\left(8a^2-25ab+15b^2\right)\cos\left(fx+e\right)^5+20\left(5ab-6b^2\right)\cos\left(fx+e\right)^3-15\left(\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^4+2\left(\right.\right.}{32\left(a^3b^2f+\left(a^5-2a^4b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(4*(8*a^2 - 25*a*b + 15*b^2)*cos(f*x + e)^5 + 20*(5*a*b - 6*b^2)*cos
(f*x + e)^3 - 15*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*
x + e)^2 + b^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a
*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e)
)*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a
*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*b^2*cos(f*x + e))/((a^3*
b^2*f + (a^5 - 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^4 + 2*(a^4*b - a^3*b^2)*f*
cos(f*x + e)^2)*sin(f*x + e)), -1/16*(2*(8*a^2 - 25*a*b + 15*b^2)*cos(f*x +
 e)^5 + 10*(5*a*b - 6*b^2)*cos(f*x + e)^3 - 15*((a^2 - 2*a*b + b^2)*cos(f*x
 + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)*sqrt(b/a)*arctan(1/2*((a + b)
```

```
*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e)
+ 30*b^2*cos(f*x + e))/((a^3*b^2*f + (a^5 - 2*a^4*b + a^3*b^2)*f*cos(f*x +
e)^4 + 2*(a^4*b - a^3*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

giac [A] time = 2.78, size = 109, normalized size = 0.97

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,a^3}+\frac{7\,b^2\tan(fx+e)^3+9\,ab\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)^2a^3}+\frac{8}{a^3\tan(fx+e)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b)))*b/(sqrt(a*b)*a^3) + (7*b^2*tan(f*x + e)^3 + 9*a*b*tan(f*x + e))/((b*
tan(f*x + e)^2 + a)^2*a^3) + 8/(a^3*tan(f*x + e)))/f
```

maple [A] time = 0.75, size = 108, normalized size = 0.96

$$-\frac{7b^2\left(\tan^3\left(fx+e\right)\right)}{8f\,a^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{9b\tan\left(fx+e\right)}{8f\,a^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{15b\arctan\left(\frac{\tan(fx+e)b}{\sqrt{ab}}\right)}{8f\,a^3\sqrt{ab}}-\frac{1}{f\,a^3\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f/a^3*b^2/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-9/8/f/a^2*b/(a+b*tan(f*x+e
)^2)^2*tan(f*x+e)-15/8/f/a^3*b/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))
-1/f/a^3/tan(f*x+e)
```

maxima [A] time = 0.58, size = 105, normalized size = 0.94

$$-\frac{\frac{15\,b^2\tan(fx+e)^4+25\,ab\tan(fx+e)^2+8\,a^2}{a^3b^2\tan(fx+e)^5+2\,a^4b\tan(fx+e)^3+a^5\tan(fx+e)}+\frac{15\,b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*b^2*tan(f*x + e)^4 + 25*a*b*tan(f*x + e)^2 + 8*a^2)/(a^3*b^2*tan(
f*x + e)^5 + 2*a^4*b*tan(f*x + e)^3 + a^5*tan(f*x + e)) + 15*b*arctan(b*tan
(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3))/f
```

mupad [B] time = 11.22, size = 102, normalized size = 0.91

$$-\frac{\frac{1}{a}+\frac{25\,b\tan(e+f\,x)^2}{8\,a^2}+\frac{15\,b^2\tan(e+f\,x)^4}{8\,a^3}}{f\left(a^2\tan\left(e+f\,x\right)+2\,a\,b\tan\left(e+f\,x\right)^3+b^2\tan\left(e+f\,x\right)^5\right)}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a}}\right)}{8\,a^{7/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^3),x)
```

```
[Out] - (1/a + (25*b*tan(e + f*x)^2)/(8*a^2) + (15*b^2*tan(e + f*x)^4)/(8*a^3))/(
f*(a^2*tan(e + f*x) + b^2*tan(e + f*x)^5 + 2*a*b*tan(e + f*x)^3)) - (15*b^(
1/2)*atan((b^(1/2)*tan(e + f*x))/a^(1/2)))/(8*a^(7/2)*f)
```

<u>sympy</u> [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.90 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=154

$$-\frac{5\sqrt{b}(3a-7b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f}-\frac{b(7a-11b)\tan(e+fx)}{8a^4f\left(a+b\tan^2(e+fx)\right)}-\frac{(a-3b)\cot(e+fx)}{a^4f}-\frac{b(a-b)\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx\right.}$$

```
[Out] -(a-3*b)*cot(f*x+e)/a^4/f-1/3*cot(f*x+e)^3/a^3/f-5/8*(3*a-7*b)*arctan(b^(1/
2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(9/2)/f-1/4*(a-b)*b*tan(f*x+e)/a^3/f/(a+b*
tan(f*x+e)^2)^2-1/8*(7*a-11*b)*b*tan(f*x+e)/a^4/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.21, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3663, 456, 1259, 1261, 205}

$$-\frac{5\sqrt{b}(3a-7b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f}-\frac{b(7a-11b)\tan(e+fx)}{8a^4f\left(a+b\tan^2(e+fx)\right)}-\frac{b(a-b)\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2}-\frac{(a-3b)\cot(e+}{a^4f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-5*(3*a - 7*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(8*a^(9/2)*
f) - ((a - 3*b)*Cot[e + f*x])/(a^4*f) - Cot[e + f*x]^3/(3*a^3*f) - ((a - b)
*b*Tan[e + f*x])/(4*a^3*f*(a + b*Tan[e + f*x]^2)^2) - ((7*a - 11*b)*b*Tan[e
 + f*x])/(8*a^4*f*(a + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a-b)b\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\,\text{Subst}\left(\int \frac{-\frac{4}{ab}-\frac{4(a-b)x^2}{a^2b}+\frac{3(a-b)x^4}{a^3}}{x^4\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4f}$$

$$= -\frac{(a-b)b\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-11b)b\tan(e+fx)}{8a^4f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-8ab-8(a-2b)bx}{x^4(a+}\right.}{}$$

$$= -\frac{(a-b)b\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-11b)b\tan(e+fx)}{8a^4f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int\left(-\frac{8b}{x^4}-\frac{8(a-3b}{ax^2}\right.\right.}{}$$

$$= -\frac{(a-3b)\cot(e+fx)}{a^4f} - \frac{\cot^3(e+fx)}{3a^3f} - \frac{(a-b)b\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-11b)b}{8a^4f\left(a+b\,t\right.}$$

$$= -\frac{5(3a-7b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f} - \frac{(a-3b)\cot(e+fx)}{a^4f} - \frac{\cot^3(e+fx)}{3a^3f} - \frac{(a}{4a^3f\,(}$$

Mathematica [A] time = 1.83, size = 146, normalized size = 0.95

$$\frac{\sqrt{a}\left(-\frac{3b\sin(2(e+fx))\left(\left(9a^2-20ab+11b^2\right)\cos(2(e+fx))+9a^2-6ab-11b^2\right)}{((a-b)\cos(2(e+fx))+a+b)^2} - 8\cot(e+fx)\left(a\csc^2(e+fx)+2a-9b\right)\right) + 15\sqrt{b}\,(7b-}{24a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (15*Sqrt[b]*(-3*a + 7*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(
-8*Cot[e + f*x]*(2*a - 9*b + a*Csc[e + f*x]^2) - (3*b*(9*a^2 - 6*a*b - 11*b
^2 + (9*a^2 - 20*a*b + 11*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a + b +
 (a - b)*Cos[2*(e + f*x)])^2))/(24*a^(9/2)*f)
```

fricas [B] time = 0.76, size = 857, normalized size = 5.56

$$\left[-\frac{4\left(16a^3-131a^2b+220ab^2-105b^3\right)\cos\left(fx+e\right)^7-4\left(24a^3-206a^2b+485ab^2-315b^3\right)\cos\left(fx+e\right)^5 -}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/96*(4*(16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 4*(24
*a^3 - 206*a^2*b + 485*a*b^2 - 315*b^3)*cos(f*x + e)^5 - 20*(15*a^2*b - 62*
a*b^2 + 63*b^3)*cos(f*x + e)^3 + 15*((3*a^3 - 13*a^2*b + 17*a*b^2 - 7*b^3)*
cos(f*x + e)^6 - (3*a^3 - 19*a^2*b + 37*a*b^2 - 21*b^3)*cos(f*x + e)^4 - 3*
a*b^2 + 7*b^3 - (6*a^2*b - 23*a*b^2 + 21*b^3)*cos(f*x + e)^2)*sqrt(-b/a)*lo
g(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*
((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b
^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^
2))*sin(f*x + e) - 60*(3*a*b^2 - 7*b^3)*cos(f*x + e))/(((a^6 - 2*a^5*b + a^
4*b^2)*f*cos(f*x + e)^6 - a^4*b^2*f - (a^6 - 4*a^5*b + 3*a^4*b^2)*f*cos(f*x
 + e)^4 - (2*a^5*b - 3*a^4*b^2)*f*cos(f*x + e)^2)*sin(f*x + e)), -1/48*(2*(
16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 2*(24*a^3 - 206*
a^2*b + 485*a*b^2 - 315*b^3)*cos(f*x + e)^5 - 10*(15*a^2*b - 62*a*b^2 + 63*
b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 13*a^2*b + 17*a*b^2 - 7*b^3)*cos(f*x + e
)^6 - (3*a^3 - 19*a^2*b + 37*a*b^2 - 21*b^3)*cos(f*x + e)^4 - 3*a*b^2 + 7*b
^3 - (6*a^2*b - 23*a*b^2 + 21*b^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan(1/2*((a
 + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x
+ e) - 30*(3*a*b^2 - 7*b^3)*cos(f*x + e))/(((a^6 - 2*a^5*b + a^4*b^2)*f*cos
(f*x + e)^6 - a^4*b^2*f - (a^6 - 4*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^4 - (2
*a^5*b - 3*a^4*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

giac [A] time = 2.17, size = 175, normalized size = 1.14

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)\left(3ab-7b^2\right)}{\sqrt{ab}\,a^4}+\frac{3\left(7ab^2\tan\left(fx+e\right)^3-11b^3\tan\left(fx+e\right)^3+9a^2b\tan\left(fx+e\right)-13ab^2\tan\left(fx+e\right)\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^2a^4}+\frac{8(3}{}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b)))*(3*a*b - 7*b^2)/(sqrt(a*b)*a^4) + 3*(7*a*b^2*tan(f*x + e)^3 - 11*b^
3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) - 13*a*b^2*tan(f*x + e))/((b*tan(f*
x + e)^2 + a)^2*a^4) + 8*(3*a*tan(f*x + e)^2 - 9*b*tan(f*x + e)^2 + a)/(a^4
*tan(f*x + e)^3))/f
```

maple [A] time = 0.79, size = 235, normalized size = 1.53

$$-\frac{7b^2\left(\tan^3\left(fx+e\right)\right)}{8fa^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{11b^3\left(\tan^3\left(fx+e\right)\right)}{8fa^4\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{9b\tan\left(fx+e\right)}{8fa^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{13b^2\,\mathrm{ta}}{8fa^3\left(a+b\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f/a^3*b^2/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+11/8/f/a^4*b^3/(a+b*tan(f*
x+e)^2)^2*tan(f*x+e)^3-9/8/f/a^2*b/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)+13/8/f/a
^3*b^2/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)-15/8/f/a^3*b/(a*b)^(1/2)*arctan(tan(
f*x+e)*b/(a*b)^(1/2))+35/8/f/a^4*b^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^
(1/2))-1/3/f/a^3/tan(f*x+e)^3-1/f/a^3/tan(f*x+e)+3/f/a^4/tan(f*x+e)*b
```

maxima [A] time = 0.70, size = 158, normalized size = 1.03

$$-\frac{\frac{15\left(3\,ab^2-7\,b^3\right)\tan\left(fx+e\right)^6+25\left(3\,a^2b-7\,ab^2\right)\tan\left(fx+e\right)^4+8\,a^3+8\left(3\,a^3-7\,a^2b\right)\tan\left(fx+e\right)^2}{a^4b^2\tan\left(fx+e\right)^7+2\,a^5b\tan\left(fx+e\right)^5+a^6\tan\left(fx+e\right)^3}+\frac{15\left(3\,ab-7\,b^2\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*((15*(3*a*b^2 - 7*b^3)*tan(f*x + e)^6 + 25*(3*a^2*b - 7*a*b^2)*tan(f*
x + e)^4 + 8*a^3 + 8*(3*a^3 - 7*a^2*b)*tan(f*x + e)^2)/(a^4*b^2*tan(f*x + e
)^7 + 2*a^5*b*tan(f*x + e)^5 + a^6*tan(f*x + e)^3) + 15*(3*a*b - 7*b^2)*arc
tan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4))/f
```

mupad [B] time = 12.28, size = 147, normalized size = 0.95

$$-\frac{\frac{1}{3\,a}+\frac{\tan\left(e+f\,x\right)^2\,\left(3\,a-7\,b\right)}{3\,a^2}+\frac{25\,b\tan\left(e+f\,x\right)^4\,\left(3\,a-7\,b\right)}{24\,a^3}+\frac{5\,b^2\tan\left(e+f\,x\right)^6\,\left(3\,a-7\,b\right)}{8\,a^4}}{f\,\left(a^2\tan\left(e+f\,x\right)^3+2\,a\,b\tan\left(e+f\,x\right)^5+b^2\tan\left(e+f\,x\right)^7\right)}-\frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(e+f\,x\right)}{\sqrt{a}}\right)\,\left(3\,a-7\,b\right)}{8\,a^{9/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^3),x)
```

```
[Out] - (1/(3*a) + (tan(e + f*x)^2*(3*a - 7*b))/(3*a^2) + (25*b*tan(e + f*x)^4*(3
*a - 7*b))/(24*a^3) + (5*b^2*tan(e + f*x)^6*(3*a - 7*b))/(8*a^4))/(f*(a^2*t
an(e + f*x)^3 + b^2*tan(e + f*x)^7 + 2*a*b*tan(e + f*x)^5)) - (5*b^(1/2)*at
an((b^(1/2)*tan(e + f*x))/a^(1/2))*(3*a - 7*b))/(8*a^(9/2)*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.91 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=231

$$-\frac{(10a-9b)\cot^3(e+fx)}{15a^4f}-\frac{\sqrt{b}\left(15a^2-70ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f}-\frac{b\left(35a^2-110ab+99b^2\right)\tan(e+fx)}{40a^5f\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] -1/5*(5*a^2-30*a*b+27*b^2)*cot(f*x+e)/a^5/f-1/15*(10*a-9*b)*cot(f*x+e)^3/a^
4/f-1/8*(15*a^2-70*a*b+63*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a
^(11/2)/f-1/5*cot(f*x+e)^5/a/f/(a+b*tan(f*x+e)^2)^2-1/20*b*(5*a^2-10*a*b+9*
b^2)*tan(f*x+e)/a^4/f/(a+b*tan(f*x+e)^2)^2-1/40*b*(35*a^2-110*a*b+99*b^2)*t
an(f*x+e)/a^5/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.30, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3663, 462, 456, 1259, 1261, 205}

$$-\frac{\sqrt{b}\left(15a^2-70ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f}-\frac{b\left(35a^2-110ab+99b^2\right)\tan(e+fx)}{40a^5f\left(a+b\tan^2(e+fx)\right)}-\frac{b\left(5a^2-10ab+9b^2\right)\tan(e+fx)}{20a^4f\left(a+b\tan^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(Sqrt[b]*(15*a^2 - 70*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]]
)/(8*a^(11/2)*f) - ((5*a^2 - 30*a*b + 27*b^2)*Cot[e + f*x])/(5*a^5*f) - ((1
0*a - 9*b)*Cot[e + f*x]^3)/(15*a^4*f) - Cot[e + f*x]^5/(5*a*f*(a + b*Tan[e
+ f*x]^2)^2) - (b*(5*a^2 - 10*a*b + 9*b^2)*Tan[e + f*x])/(20*a^4*f*(a + b*T
an[e + f*x]^2)^2) - (b*(35*a^2 - 110*a*b + 99*b^2)*Tan[e + f*x])/(40*a^5*f*
(a + b*Tan[e + f*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{10a-9b+5ax^2}{x^4\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\left(5a^2-10ab+9b^2\right)\tan(e+fx)}{20a^4f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\,\text{Subst}\left(\int \frac{4\left(\frac{9}{a}-\frac{1}{}\right.}{}\right.}{}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\left(5a^2-10ab+9b^2\right)\tan(e+fx)}{20a^4f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\left(35a^2-110ab\right.}{40a^5f\left(a+\right.}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\left(5a^2-10ab+9b^2\right)\tan(e+fx)}{20a^4f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\left(35a^2-110ab\right.}{40a^5f\left(a+\right.}$$

$$= -\frac{\left(5a^2-30ab+27b^2\right)\cot(e+fx)}{5a^5f} - \frac{(10a-9b)\cot^3(e+fx)}{15a^4f} - \frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+\right.}$$

$$= -\frac{\sqrt{b}\left(15a^2-70ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f} - \frac{\left(5a^2-30ab+27b^2\right)\cot(e+fx)}{5a^5f}$$

Mathematica [A] time = 1.68, size = 346, normalized size = 1.50

$$-960\sqrt{b}\left(15a^2-70ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)-\frac{2\sqrt{a}\cot(e+fx)\csc^4(e+fx)\left(-128a^4\cos(6(e+fx))+64a^4\cos(8(e+fx))+1600a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-960*Sqrt[b]*(15*a^2 - 70*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a]] - (2*Sqrt[a]*(1600*a^4 - 165*a^3*b + 637*a^2*b^2 - 28875*a*b^3 + 33075
*b^4 + 4*(416*a^4 - 447*a^3*b - 1400*a^2*b^2 + 13125*a*b^3 - 13230*b^4)*Cos
[2*(e + f*x)] - 4*(32*a^4 - 257*a^3*b - 2821*a^2*b^2 + 8925*a*b^3 - 6615*b^
4)*Cos[4*(e + f*x)] - 128*a^4*Cos[6*(e + f*x)] + 1788*a^3*b*Cos[6*(e + f*x)
] - 8800*a^2*b^2*Cos[6*(e + f*x)] + 14700*a*b^3*Cos[6*(e + f*x)] - 7560*b^4
*Cos[6*(e + f*x)] + 64*a^4*Cos[8*(e + f*x)] - 863*a^3*b*Cos[8*(e + f*x)] +
2479*a^2*b^2*Cos[8*(e + f*x)] - 2625*a*b^3*Cos[8*(e + f*x)] + 945*b^4*Cos[8
*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^4)/(a + b + (a - b)*Cos[2*(e + f*x)]
)^2)/(7680*a^(11/2)*f)
```

fricas [B] time = 0.58, size = 1199, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/480*(4*(64*a^4 - 863*a^3*b + 2479*a^2*b^2 - 2625*a*b^3 + 945*b^4)*cos(f
*x + e)^9 - 4*(160*a^4 - 2173*a^3*b + 7158*a^2*b^2 - 8925*a*b^3 + 3780*b^4)
*cos(f*x + e)^7 + 4*(120*a^4 - 1685*a^3*b + 7104*a^2*b^2 - 11025*a*b^3 + 56
70*b^4)*cos(f*x + e)^5 + 20*(75*a^3*b - 530*a^2*b^2 + 1155*a*b^3 - 756*b^4)
*cos(f*x + e)^3 - 15*((15*a^4 - 100*a^3*b + 218*a^2*b^2 - 196*a*b^3 + 63*b^
4)*cos(f*x + e)^8 - 2*(15*a^4 - 115*a^3*b + 303*a^2*b^2 - 329*a*b^3 + 126*b
^4)*cos(f*x + e)^6 + (15*a^4 - 160*a^3*b + 573*a^2*b^2 - 798*a*b^3 + 378*b^
4)*cos(f*x + e)^4 + 15*a^2*b^2 - 70*a*b^3 + 63*b^4 + 2*(15*a^3*b - 100*a^2*
b^2 + 203*a*b^3 - 126*b^4)*cos(f*x + e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b
^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*
x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b +
 b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x + e) +
60*(15*a^2*b^2 - 70*a*b^3 + 63*b^4)*cos(f*x + e))/(((a^7 - 2*a^6*b + a^5*b^
2)*f*cos(f*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2)*f*cos(f*x +
 e)^6 + (a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2
)*f*cos(f*x + e)^2)*sin(f*x + e)), -1/240*(2*(64*a^4 - 863*a^3*b + 2479*a^2
*b^2 - 2625*a*b^3 + 945*b^4)*cos(f*x + e)^9 - 2*(160*a^4 - 2173*a^3*b + 715
8*a^2*b^2 - 8925*a*b^3 + 3780*b^4)*cos(f*x + e)^7 + 2*(120*a^4 - 1685*a^3*b
 + 7104*a^2*b^2 - 11025*a*b^3 + 5670*b^4)*cos(f*x + e)^5 + 10*(75*a^3*b - 5
30*a^2*b^2 + 1155*a*b^3 - 756*b^4)*cos(f*x + e)^3 - 15*((15*a^4 - 100*a^3*b
 + 218*a^2*b^2 - 196*a*b^3 + 63*b^4)*cos(f*x + e)^8 - 2*(15*a^4 - 115*a^3*b
 + 303*a^2*b^2 - 329*a*b^3 + 126*b^4)*cos(f*x + e)^6 + (15*a^4 - 160*a^3*b
+ 573*a^2*b^2 - 798*a*b^3 + 378*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 70*a*b^3
 + 63*b^4 + 2*(15*a^3*b - 100*a^2*b^2 + 203*a*b^3 - 126*b^4)*cos(f*x + e)^2
)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x +
e)*sin(f*x + e)))*sin(f*x + e) + 30*(15*a^2*b^2 - 70*a*b^3 + 63*b^4)*cos(f*
x + e))/(((a^7 - 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^8 + a^5*b^2*f - 2*(a^7 -
 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^6 + (a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(
f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

giac [A] time = 4.13, size = 263, normalized size = 1.14

$$-\frac{\frac{15\left(15a^2b-70ab^2+63b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}\,a^5}+\frac{15\left(7a^2b^2\tan(fx+e)^3-22ab^3\tan(fx+e)^3+15b^4\tan(fx+e)^3+9a^3b\tan(fx+e\right.}{\left(b\tan(fx+e)^2+a\right)^2a^5}}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/120*(15*(15*a^2*b - 70*a*b^2 + 63*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))/(sqrt(a*b)*a^5) + 15*(7*a^2*b^2*tan
(f*x + e)^3 - 22*a*b^3*tan(f*x + e)^3 + 15*b^4*tan(f*x + e)^3 + 9*a^3*b*tan
(f*x + e) - 26*a^2*b^2*tan(f*x + e) + 17*a*b^3*tan(f*x + e))/((b*tan(f*x +
e)^2 + a)^2*a^5) + 8*(15*a^2*tan(f*x + e)^4 - 90*a*b*tan(f*x + e)^4 + 90*b^
2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 - 15*a*b*tan(f*x + e)^2 + 3*a^2)/(
a^5*tan(f*x + e)^5))/f
```

maple [A] time = 0.73, size = 380, normalized size = 1.65

$$-\frac{7b^2\left(\tan^3\left(fx+e\right)\right)}{8fa^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{11b^3\left(\tan^3\left(fx+e\right)\right)}{4fa^4\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{15b^4\left(\tan^3\left(fx+e\right)\right)}{8fa^5\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{9b\tan\left(f\right.}{8fa^2\left(a+b\left(\tan\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f/a^3*b^2/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+11/4/f/a^4*b^3/(a+b*tan(f*
x+e)^2)^2*tan(f*x+e)^3-15/8/f*b^4/a^5/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-9/8
/f/a^2*b/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)+13/4/f/a^3*b^2/(a+b*tan(f*x+e)^2)^
2*tan(f*x+e)-17/8/f*b^3/a^4/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)-15/8/f/a^3*b/(a
*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+35/4/f/a^4*b^2/(a*b)^(1/2)*arcta
n(tan(f*x+e)*b/(a*b)^(1/2))-63/8/f*b^3/a^5/(a*b)^(1/2)*arctan(tan(f*x+e)*b/
(a*b)^(1/2))-1/5/f/a^3/tan(f*x+e)^5+1/f/a^4/tan(f*x+e)^3*b-2/3/f/a^3/tan(f*
x+e)^3-1/f/a^3/tan(f*x+e)+6/f/a^4/tan(f*x+e)*b-6/f/a^5/tan(f*x+e)*b^2
```

maxima [A] time = 0.73, size = 212, normalized size = 0.92

$$-\frac{\frac{15\left(15a^2b^2-70ab^3+63b^4\right)\tan(fx+e)^8+25\left(15a^3b-70a^2b^2+63ab^3\right)\tan(fx+e)^6+8\left(15a^4-70a^3b+63a^2b^2\right)\tan(fx+e)^4+24a^4+8\left(10a^4-9a^3b\right)\tan(f\ldots}{a^5b^2\tan(fx+e)^9+2a^6b\tan(fx+e)^7+a^7\tan(fx+e)^5}}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/120*((15*(15*a^2*b^2 - 70*a*b^3 + 63*b^4)*tan(f*x + e)^8 + 25*(15*a^3*b
- 70*a^2*b^2 + 63*a*b^3)*tan(f*x + e)^6 + 8*(15*a^4 - 70*a^3*b + 63*a^2*b^2
)*tan(f*x + e)^4 + 24*a^4 + 8*(10*a^4 - 9*a^3*b)*tan(f*x + e)^2)/(a^5*b^2*t
an(f*x + e)^9 + 2*a^6*b*tan(f*x + e)^7 + a^7*tan(f*x + e)^5) + 15*(15*a^2*b
 - 70*a*b^2 + 63*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^5))/f
```

mupad [B] time = 13.31, size = 199, normalized size = 0.86

$$-\frac{\frac{1}{5a}+\frac{\tan(e+fx)^4\left(15a^2-70ab+63b^2\right)}{15a^3}+\frac{\tan(e+fx)^2\left(10a-9b\right)}{15a^2}+\frac{5b\tan(e+fx)^6\left(15a^2-70ab+63b^2\right)}{24a^4}+\frac{b^2\tan(e+fx)^8\left(15a^2-70ab+63b^2\right)}{8a^5}}{f\left(a^2\tan\left(e+fx\right)^5+2ab\tan\left(e+fx\right)^7+b^2\tan\left(e+fx\right)^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^3),x)
```

```
[Out] - (1/(5*a) + (tan(e + f*x)^4*(15*a^2 - 70*a*b + 63*b^2))/(15*a^3) + (tan(e
+ f*x)^2*(10*a - 9*b))/(15*a^2) + (5*b*tan(e + f*x)^6*(15*a^2 - 70*a*b + 63
*b^2))/(24*a^4) + (b^2*tan(e + f*x)^8*(15*a^2 - 70*a*b + 63*b^2))/(8*a^5))/
(f*(a^2*tan(e + f*x)^5 + b^2*tan(e + f*x)^9 + 2*a*b*tan(e + f*x)^7)) - (b^(
1/2)*atan((b^(1/2)*tan(e + f*x))/a^(1/2))*(15*a^2 - 70*a*b + 63*b^2))/(8*a^
(11/2)*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.92 $\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=161

$$-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{5f(a-b)}+\frac{2(5a-4b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{15f(a-b)^2}-\frac{\cos(e+fx)\sqrt{a+}}{}$$

```
[Out] 2/15*(5*a-4*b)*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(3/2)/(a-b)^2/f-1/5*cos(f*
x+e)^5*(a-b+b*sec(f*x+e)^2)^(3/2)/(a-b)/f+arctanh(sec(f*x+e)*b^(1/2)/(a-b+b
*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.17, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3664, 462, 451, 277, 217, 206}

$$-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{5f(a-b)}+\frac{2(5a-4b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{15f(a-b)^2}-\frac{\cos(e+fx)\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/f
- (Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/f + (2*(5*a - 4*b)*Cos[e +
f*x]^3*(a - b + b*Sec[e + f*x]^2)^(3/2))/(15*(a - b)^2*f) - (Cos[e + f*x]^5
*(a - b + b*Sec[e + f*x]^2)^(3/2))/(5*(a - b)*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
```

```
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\sqrt{a-b+bx^2}}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{5(a-b)f} + \frac{\text{Subst}\left(\int \frac{(-2(5a-4b)+5(a-\cdots}{x^4}\cdots\right)}{5\cdots}$$

$$= \frac{2(5a-4b)\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{15(a-b)^2f} - \frac{\cos^5(e+fx)\left(\cdots\right.}{\cdots}$$

$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{2(5a-4b)\cos^3(e+fx)\left(a-\cdots\right.}{15(a-\cdots}$$

$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{2(5a-4b)\cos^3(e+fx)\left(a-\cdots\right.}{15(a-\cdots}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f}\cdots$$

Mathematica [A] time = 3.29, size = 208, normalized size = 1.29

$$\frac{\cos(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\sqrt{(a-b)\cos(2(e+fx))+a+b}\left(4\left(7a^2-15ab+8\cdots\right.\right.\right.}{120\sqrt{2}f(a-b)^2\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(120*Sqrt[2]*(a - b)^2*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*C
os[2*(e + f*x)]]/(Sqrt[2]*Sqrt[b])] + Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]
]*(-89*a^2 + 254*a*b - 149*b^2 + 4*(7*a^2 - 15*a*b + 8*b^2)*Cos[2*(e + f*x)
] - 3*(a - b)^2*Cos[4*(e + f*x)]))*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*
Sec[e + f*x]^2])/(120*Sqrt[2]*(a - b)^2*f*Sqrt[a + b + (a - b)*Cos[2*(e + f
*x)]])
```

fricas [A] time = 0.66, size = 378, normalized size = 2.35

$$\left[\frac{15\left(a^2-2\,ab+b^2\right)\sqrt{b}\log\left(-\frac{(a-b)\cos\left(fx+e\right)^2+2\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+2b}{\cos\left(fx+e\right)^2}\right)-2\left(3\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^5}{30\left(a^2-2\,ab+b^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(a^2 - 2*a*b + b^2)*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt
(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/c
os(f*x + e)^2) - 2*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - (10*a^2 - 21*a*b
 + 11*b^2)*cos(f*x + e)^3 + (15*a^2 - 40*a*b + 23*b^2)*cos(f*x + e))*sqrt((
(a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 - 2*a*b + b^2)*f), -1/15
*(15*(a^2 - 2*a*b + b^2)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + (3*(a^2 - 2*a*b + b^2)*cos(f*x +
 e)^5 - (10*a^2 - 21*a*b + 11*b^2)*cos(f*x + e)^3 + (15*a^2 - 40*a*b + 23*b
^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2
- 2*a*b + b^2)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*32*(1/240*(15*b*(-sqrt(a)*tan((f
*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*
tan((f*x+exp(1))/2)^2+a))^9-165*sqrt(a)*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+s
qrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/
2)^2+a))^8-320*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/
2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^7+320*b^2*(-sq
rt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1
))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^7+540*a*b*(-sqrt(a)*tan((f*x+exp(1))/
2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+ex
p(1))/2)^2+a))^7-640*sqrt(a)*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan
((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^
6-2960*sqrt(a)*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/
2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^6+2940*sqrt(a)
*a*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((
f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^6+832*a^3*(-sqrt(a)*tan((f*x
+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*ta
n((f*x+exp(1))/2)^2+a))^5+2848*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*t
an((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a)
)^5-2464*a*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4
-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^5-1246*a^2*b*(-sqr
t(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1)
)/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^5-2880*a^5*(-sqrt(a)*tan((f*x+exp(1))/
2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+ex
p(1))/2)^2+a))+3840*b^5*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp
(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))+2560*sqrt
```

```
(a)*a^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*ta
n((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^4-23040*a*b^4*(-sqrt(a)*t
an((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2
+4*b*tan((f*x+exp(1))/2)^2+a))+46240*a^2*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)^
2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1
))/2)^2+a))-41760*a^3*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+e
xp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))+17735*a
^4*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((
f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))-6560*sqrt(a)*b^3*(-sqrt(a)*t
an((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2
+4*b*tan((f*x+exp(1))/2)^2+a))^4+16400*sqrt(a)*a*b^2*(-sqrt(a)*tan((f*x+exp
(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f
*x+exp(1))/2)^2+a))^4-11590*sqrt(a)*a^2*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+s
qrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/
2)^2+a))^4+320*a^4*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/
2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3+5120*b^4*(-s
qrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(
1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3-16320*a*b^3*(-sqrt(a)*tan((f*x+exp
(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f
*x+exp(1))/2)^2+a))^3+14720*a^2*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*
tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a
))^3-4500*a^3*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^
4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3-3200*sqrt(a)*a^
4*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x
+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2-3840*sqrt(a)*b^4*(-sqrt(a)*ta
n((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+
4*b*tan((f*x+exp(1))/2)^2+a))^2+18880*sqrt(a)*a*b^3*(-sqrt(a)*tan((f*x+exp(
1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*
x+exp(1))/2)^2+a))^2-27760*sqrt(a)*a^2*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+
sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))
/2)^2+a))^2+15980*sqrt(a)*a^3*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan(
(f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2
-768*sqrt(a)*a^5+3840*sqrt(a)*b^5-14080*sqrt(a)*a*b^4+20448*sqrt(a)*a^2*b^3
-14864*sqrt(a)*a^3*b^2+5379*sqrt(a)*a^4*b)/(-2*sqrt(a)*(-sqrt(a)*tan((f*x+e
xp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan(
(f*x+exp(1))/2)^2+a))+(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1
))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2-3*a+4*b)^
5+1/32*b*atan(1/2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a)+sqrt(a*tan((f*x+e
xp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))/sqrt(-b
))/sqrt(-b))*sign(tan((f*x+exp(1))/2)^2-1)
```

maple [B] time = 5.18, size = 7044, normalized size = 43.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x)
[Out] result too large to display
```

maxima [A] time = 0.81, size = 203, normalized size = 1.26

$$\frac{\frac{20\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3}{a-b} - 30\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right) - 15\sqrt{b}\log\left(\frac{\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)-\sqrt{b}}{\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+\sqrt{b}}\right) - \cdots}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/30*(20*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/(a - b) - 30*sqrt(
a - b + b/cos(f*x + e)^2)*cos(f*x + e) - 15*sqrt(b)*log((sqrt(a - b + b/cos
(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*
x + e) + sqrt(b))) - 2*(3*(a - b + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 -
 5*(a - b + b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3)/(a^2 - 2*a*b + b^2))/
f
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^5\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.93 $\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=113

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3f(a-b)}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}$$

```
[Out] 1/3*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(3/2)/(a-b)/f+arctanh(sec(f*x+e)*b^(1
/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(
1/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3664, 451, 277, 217, 206}

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3f(a-b)}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/f
- (Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/f + (Cos[e + f*x]^3*(a - b
+ b*Sec[e + f*x]^2)^(3/2))/(3*(a - b)*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
```

```
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)\sqrt{a-b+bx^2}}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{3(a-b)f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{x^2}\,dx, x, \sec\right.}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e\right.}{3(a-b)f}$$

$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e\right.}{3(a-b)f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} +$$

Mathematica [A] time = 1.06, size = 170, normalized size = 1.50

$$\frac{\cos(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\sqrt{(a-b)\cos(2(e+fx))+a+b}\,((a-b)\cos(2(e+fx)\right.}{6\sqrt{2}f(a-b)\sqrt{(a-b)\cos(2(e+fx))+a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(6*Sqrt[2]*(a - b)*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2
*(e + f*x)]]/(Sqrt[2]*Sqrt[b])] + Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*(-
5*a + 7*b + (a - b)*Cos[2*(e + f*x)]))*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x
)])*Sec[e + f*x]^2])/(6*Sqrt[2]*(a - b)*f*Sqrt[a + b + (a - b)*Cos[2*(e + f
*x)]])
```

fricas [A] time = 0.61, size = 277, normalized size = 2.45

$$\left[\frac{3\,(a-b)\sqrt{b}\,\log\left(-\frac{(a-b)\cos\left(fx+e\right)^2+2\,\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+2\,b}{\cos\left(fx+e\right)^2}\right)+2\left((a-b)\cos\left(fx+e\right)^3-(3\,a-4\,b)\cos\left(fx\right.\right.}{6\,(a-b)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a - b)*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a -
 b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2)
 + 2*((a - b)*cos(f*x + e)^3 - (3*a - 4*b)*cos(f*x + e))*sqrt(((a - b)*cos(
```

```
f*x + e)^2 + b)/cos(f*x + e)^2))/((a - b)*f), -1/3*(3*(a - b)*sqrt(-b)*arct
an(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/
b) - ((a - b)*cos(f*x + e)^3 - (3*a - 4*b)*cos(f*x + e))*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2))/((a - b)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)abs(f)*((1/3*a^2*f^4*sqrt(a*f^2*(-cos(f*x+exp(1))/f)^2-b*f^
2*(-cos(f*x+exp(1))/f)^2+b)*(a*f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+e
xp(1))/f)^2+b)*sign(cos(f*x+exp(1)))*sign(f)-a^3*f^4*sqrt(a*f^2*(-cos(f*x+e
xp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)*sign(cos(f*x+exp(1)))*sign(f)+1
/3*b^2*f^4*sqrt(a*f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b
)*(a*f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)*sign(cos(f*
x+exp(1)))*sign(f)+b^3*f^4*sqrt(a*f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(f*
x+exp(1))/f)^2+b)*sign(cos(f*x+exp(1)))*sign(f)-2/3*a*b*f^4*sqrt(a*f^2*(-co
s(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)*(a*f^2*(-cos(f*x+exp(1))
/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)*sign(cos(f*x+exp(1)))*sign(f)-3*a*b^2
*f^4*sqrt(a*f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)*sign
(cos(f*x+exp(1)))*sign(f)+3*a^2*b*f^4*sqrt(a*f^2*(-cos(f*x+exp(1))/f)^2-b*f
^2*(-cos(f*x+exp(1))/f)^2+b)*sign(cos(f*x+exp(1)))*sign(f))/(a^3*f^6-b^3*f^
6+3*a*b^2*f^6-3*a^2*b*f^6)-b*sign(cos(f*x+exp(1)))*sign(f)*atan(sqrt(a*f^2*
(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)/sqrt(-b))/sqrt(-b)/f
^2-(-3*a*b*atan(sqrt(b)/sqrt(-b))-3*a*sqrt(-b)*sqrt(b)+3*b^2*atan(sqrt(b)/s
qrt(-b))+4*b*sqrt(-b)*sqrt(b))/(3*a*f^2*sqrt(-b)-3*b*f^2*sqrt(-b))*sign(cos
(f*x+exp(1)))*sign(f))
```

maple [B] time = 1.02, size = 4296, normalized size = 38.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/6/f*(-1+cos(f*x+e))^2*(-3*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/
sin(f*x+e)^2/a^(1/2))*b^(9/2)*4^(1/2)+3*ln(-4*(-1+cos(f*x+e))*(a^(1/2)*cos(
f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*co
s(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(9/2)*4^(1/2)-4*((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)*b^(5/2)*a^(1/2)+8*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)*b^(3/2)*a^(3/2)-4*((a*cos(f*x+e)^2-co
s(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)*a^(5/2)+9*ln(-4*(-1+cos(f*x
+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^
2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)
-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(5/2)*4^(1/2)*a^2-9*a
rctanh(1/8*(-1+cos(f*x+e))*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(
f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)
```

```
*4^(1/2))*a^(5/2)*4^(1/2)*b^2+3*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)
+b)/sin(f*x+e)^2/a^(1/2))*b^(3/2)*4^(1/2)*a^3-3*ln(-4*(-1+cos(f*x+e))*(a^(1
/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x
+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(3/2)*4^(1/2)*a^3+9*arctanh(1/8
*(-1+cos(f*x+e))*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*
a^(3/2)*4^(1/2)*b^3-3*arctanh(1/8*(-1+cos(f*x+e))*(4^(1/2)*cos(f*x+e)-4^(1/
2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f
*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a^(1/2)*4^(1/2)*b^4-15*((a*cos(f*x+e)^2-co
s(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(5/2)*a^(3/2)*4^(1/2)+12*((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)*a^(5/2)*4^(1/2)-
3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*a^(7/2
)*4^(1/2)+2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)*b^(5
/2)*cos(f*x+e)^4*a^(1/2)-4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e)
)^2)^(3/2)*b^(3/2)*cos(f*x+e)^4*a^(3/2)+2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)*cos(f*x+e)^4*a^(5/2)+8*((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)*b^(5/2)*cos(f*x+e)^3*a^(1/2)-16*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)*b^(3/2)*cos(f*x+e)
^3*a^(3/2)+8*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)*b^(
1/2)*cos(f*x+e)^3*a^(5/2)-3*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/
sin(f*x+e)^2/a^(1/2))*b^(9/2)*cos(f*x+e)*4^(1/2)+3*ln(-4*(-1+cos(f*x+e))*(a
^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2
)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(
f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(9/2)*cos(f*x+e)*4^(1/2)+6*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)*b^(5/2)*cos(f*x+e
)^2*a^(1/2)-12*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)*b
^(3/2)*cos(f*x+e)^2*a^(3/2)+6*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x
+e))^2)^(3/2)*b^(1/2)*cos(f*x+e)^2*a^(5/2)-4*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(1+cos(f*x+e))^2)^(3/2)*b^(5/2)*cos(f*x+e)*a^(1/2)+8*((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)*b^(3/2)*cos(f*x+e)*a^(3/2)-4*((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)*cos(f*x+e)*a
^(5/2)+6*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(7/2)
*a^(1/2)*4^(1/2)+9*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e
)^2/a^(1/2))*b^(7/2)*4^(1/2)*a-9*ln(-4*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e
)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*4^(1/2)*a+3*arctanh(1/8*(-1+cos(f*x+e))*
(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*a^(7/2)*4^(1/2)*b
-9*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+
e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(
5/2)*4^(1/2)*a^2+6*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/
2)*b^(7/2)*cos(f*x+e)*a^(1/2)*4^(1/2)+9*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(
f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*co
s(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*cos(f*x+e)*4^(1/2)*a-9*ln(-4*(-1+
cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f
*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*
a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*cos(f*x+
e)*4^(1/2)*a+3*arctanh(1/8*(-1+cos(f*x+e))*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*co
s(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^
```

```
2)^(1/2)*b^(1/2)*4^(1/2))*cos(f*x+e)*a^(7/2)*4^(1/2)*b-9*ln(-2*(-1+cos(f*x+
e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2
)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-
a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(5/2)*cos(f*x+e)*4^(1/
2)*a^2+9*ln(-4*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+co
s(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2
))*b^(5/2)*cos(f*x+e)*4^(1/2)*a^2-9*arctanh(1/8*(-1+cos(f*x+e))*(4^(1/2)*co
s(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*cos(f*x+e)*a^(5/2)*4^(1/2)*b
^2+3*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*
x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b
^(3/2)*cos(f*x+e)*4^(1/2)*a^3-3*ln(-4*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)
+b)/sin(f*x+e)^2/a^(1/2))*b^(3/2)*cos(f*x+e)*4^(1/2)*a^3+9*arctanh(1/8*(-1+
cos(f*x+e))*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*cos(f
*x+e)*a^(3/2)*4^(1/2)*b^3-3*arctanh(1/8*(-1+cos(f*x+e))*(4^(1/2)*cos(f*x+e)
-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1
+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*cos(f*x+e)*a^(1/2)*4^(1/2)*b^4-18*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(5/2)*cos(f*x+e)
*a^(3/2)*4^(1/2)+18*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1
/2)*b^(3/2)*cos(f*x+e)*a^(5/2)*4^(1/2)-6*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)*a^(7/2)*4^(1/2))*cos(f*x+e)*((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)/sin(f*x+e)^4/(a-b)^3/a^(1/2)/b^(1/2)
```

maxima [A] time = 0.86, size = 132, normalized size = 1.17

$$\frac{\frac{2\left(a-b+\frac{b}{\cos\left(fx+e\right)^{2}}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{3}}{a-b}-6\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)-3\sqrt{b}\log\left(\frac{\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)-\sqrt{b}}{\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+\sqrt{b}}\right)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/(a - b) - 6*sqrt(a -
 b + b/cos(f*x + e)^2)*cos(f*x + e) - 3*sqrt(b)*log((sqrt(a - b + b/cos(f*x
 + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x +
e) + sqrt(b))))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^{3}\,\sqrt{b\,\tan\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.94 $\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}$$

```
[Out] arctanh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)
*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3664, 277, 217, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/f
- (Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\,dx, x, \sec(e\right.}{f}$$
$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+}{\sqrt{a-b+b\sec}}\right.}{f}$$
$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f}$$

Mathematica [A] time = 0.56, size = 140, normalized size = 1.94

$$-\frac{\sin(2(e+fx))\csc(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\sqrt{2}\sqrt{(a-b)\cos(2(e+fx))+a+b}-\right.}{4f\sqrt{(a-b)\cos(2(e+fx))+a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/4*((-2*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]/(Sqrt[2]*S
qrt[b])] + Sqrt[2]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])*Csc[e + f*x]*Sqr
t[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*Sin[2*(e + f*x)])/(f*S
qrt[a + b + (a - b)*Cos[2*(e + f*x)]])
```

fricas [A] time = 0.70, size = 203, normalized size = 2.82

$$\left[-\frac{2\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) - \sqrt{b}\log\left(-\frac{(a-b)\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)}{2f}, -\frac{\sqrt{-b}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) - s
qrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -(sqrt(-b)*a
rctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)/b) + sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)abs(f)*(-sqrt(a*f^2*(-c
os(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)*sign(cos(f*x+exp(1)))*s
ign(f)/f^2-b*sign(cos(f*x+exp(1)))*sign(f)*atan(sqrt(a*f^2*(-cos(f*x+exp(1)
)/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)/sqrt(-b))/sqrt(-b)/f^2-(-b*atan(sqrt
(b)/sqrt(-b))-sqrt(-b)*sqrt(b))/f^2/sqrt(-b)*sign(cos(f*x+exp(1)))*sign(f))

maple [B] time = 0.39, size = 144, normalized size = 2.00

$$\frac{\sqrt{\frac{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}{\cos\left(fx+e\right)^2}}\ \cos\left(fx+e\right)\left(\sqrt{b}\ \ln\left(\frac{2\sqrt{b}\ \sqrt{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}+2b}{\cos\left(fx+e\right)}\right)-\sqrt{a\left(\cos^2\left(fx+e\right)\right)\cdots}\right.}{f\sqrt{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x)

[Out] 1/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)*(b^(1
/2)*ln(2*(b^(1/2)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2)+b)/cos(f*x+e))-(a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2))/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1
/2)

maxima [A] time = 0.45, size = 97, normalized size = 1.35

$$-\frac{2\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\ \cos\left(fx+e\right)+\sqrt{b}\ \log\left(\frac{\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\ \cos\left(fx+e\right)-\sqrt{b}}{\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\ \cos\left(fx+e\right)+\sqrt{b}}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] -1/2*(2*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b)*log((sqrt(a -
 b + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)
^2)*cos(f*x + e) + sqrt(b))))/f

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)\,\sqrt{b\,{\tan\left(e+f\,x\right)}^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2),x)

[Out] int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\ \sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*sin(e + f*x), x)

3.95 $\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}$$

```
[Out] -arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh(s
ec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3664, 402, 217, 206, 377, 207}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/
f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]
])/f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
```

```
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\right.}{f}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{-1+ax^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \right.}{f}$$
$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}$$

Mathematica [B] time = 6.65, size = 295, normalized size = 3.51

$$\frac{\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\left(\tan^2\left(\frac{1}{2}(e+\right.\right.}{\sqrt{a\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-1\right)^2+}}\right.\right.}{2f\sqrt{\sec^4\left(\frac{1}{2}(e+fx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((2*Sqrt[b]*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))/Sqrt[4*b*Tan[(e + f*
x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] - Sqrt[a]*(ArcTanh[(a - (a - 2*b)
*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e
+ f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sq
rt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]))*Cos[e + f*x]
*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]
)/(2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])
```

fricas [A] time = 0.68, size = 514, normalized size = 6.12

$$\left[\frac{\sqrt{a}\log\left(-\frac{2\left((a-b)\cos(fx+e)^2-2\sqrt{a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+b\right)}{\cos(fx+e)^2-1}\right)+\sqrt{b}\log\left(-\frac{(a-b)\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\,c\right.}{\cos(fx+e)^2}}{2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1))
 + sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -1/2*(2*
sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
cos(f*x + e)/b) - sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt((
(a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x
+ e)^2 - 1)))/f, 1/2*(2*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + sqrt(b)*log(-((a - b)*cos(f*x + e
)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e) + 2*b)/cos(f*x + e)^2))/f, (sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) - sqrt(-b)*arctan(sqrt(-b)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
```

```
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assu
mes constant sign by intervals (correct if the argument is real):Check [abs
(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^2-1 were not checkedUn
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integratio
n of abs or sign assumes constant sign by intervals (correct if the argumen
t is real):Check [abs(t_nostep^2-1)]Warning, replacing 0 by ` u`, a substit
ution variable should perhaps be purged.Warning, replacing 0 by ` u`, a sub
stitution variable should perhaps be purged.Warning, replacing 0 by ` u`, a
 substitution variable should perhaps be purged.Warning, replacing 0 by ` u
`, a substitution variable should perhaps be purged.Warning, replacing 0 by
 ` u`, a substitution variable should perhaps be purged.Warning, replacing
0 by ` u`, a substitution variable should perhaps be purged.Warning, replac
ing 0 by ` u`, a substitution variable should perhaps be purged.Warning, re
placing 0 by ` u`, a substitution variable should perhaps be purged.Warning
, replacing 0 by ` u`, a substitution variable should perhaps be purged.War
ning, integration of abs or sign assumes constant sign by intervals (correc
t if the argument is real):Check [abs(t_nostep)]Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.Non
 regular value [0] was discarded and replaced randomly by 0=[-25]Warning, n
eed to choose a branch for the root of a polynomial with parameters. This m
ight be wrong.Non regular value [0] was discarded and replaced randomly by
0=[84]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.Non regular value [0] was discarded and repla
ced randomly by 0=[-99]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.Non regular value [0] was di
scarded and replaced randomly by 0=[-54]Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.Non regular
 value [0] was discarded and replaced randomly by 0=[-56]Warning, need to c
hoose a branch for the root of a polynomial with parameters. This might be
wrong.Non regular value [0] was discarded and replaced randomly by 0=[-32]E
valuation time: 1.39index.cc index_m operator + Error: Bad Argument Value
```

maple [B] time = 1.19, size = 721, normalized size = 8.58

$$-\frac{\sqrt{\frac{a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b}{\cos(fx+e)^2}}\ \sqrt{4}\ \cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)\left(2\sqrt{a}\ \operatorname{arctanh}\left(\frac{(-1+\cos(fx+e))(\sqrt{4}\ \cos(fx+e)-}{8\sin(fx+e)^2\sqrt{\frac{a(\cos^2(fx}{(1}}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*4^(1/2)*cos(f
*x+e)*(-1+cos(f*x+e))*(2*a^(1/2)*arctanh(1/8*(-1+cos(f*x+e))*(4^(1/2)*cos(f
*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*b+2*ln(-2*(-1+cos(f*x+e))*(a^(1
/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x
```

```
+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(3/2)-2*b^(3/2)*ln(-4*(-1+cos(f
*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e)
)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/
2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))-ln(-2*(-1+cos(f*x+e))
*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(
1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*c
os(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*a*b^(1/2)-a*ln(-4*(a^(1/2)*
cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-
b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*b^(1/2))/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)/a^(1/2)/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan\left(fx+e\right)^2 + a} \csc\left(fx+e\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b \tan\left(e+f\,x\right)^2 + a}}{\sin\left(e+f\,x\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x),x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2\left(e+fx\right)} \csc\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x), x)
```

3.96 $\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=127

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2\sqrt{a}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

```
[Out] -1/2*(a+b)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/f/a^(1/2)
+arctanh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*cot(f
*x+e)*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3664, 467, 523, 217, 206, 377, 207}

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2\sqrt{a}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((a + b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/(
2*Sqrt[a]*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/f - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])
/(2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
```

```
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a-b+bx^2}}{\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a-b+2}{\left(-1+x^2\right)\sqrt{a}}\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx}}\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx\right.}{}$$

$$= -\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2\sqrt{a}\,f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} - $$

Mathematica [B] time = 3.74, size = 460, normalized size = 3.62

$$-\frac{\cot(e+fx)\csc(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\sqrt{2}\sqrt{a}\sqrt{\sec^4\left(\frac{1}{2}(e+fx)\right)((a-b)\cos(2(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/4*(Cot[e + f*x]*Csc[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec
[e + f*x]^2]*(Sqrt[2]*Sqrt[a]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(
e + f*x)/2]^4] - 16*Sqrt[a]*Sqrt[b]*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^
2))/Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]]*Sin[(e +
f*x)/2]^2 + 4*(a + b)*ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*S
qrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]*Sin[(e + f*x)
/2]^2 + 4*a*ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*T
```

```
an[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]*Sin[(e + f*x)/2]^2 + 4
*b*ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f
*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]*Sin[(e + f*x)/2]^2))/(Sqrt[a]*f
*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])
```

fricas [A] time = 1.38, size = 849, normalized size = 6.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2
*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a
 + b)/(cos(f*x + e)^2 - 1)) + 2*(a*cos(f*x + e)^2 - a)*sqrt(b)*log(-((a - b
)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^2 - a*f), 1/2*((
(a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + a*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + (a*cos(f*x + e)^2 - a)*sqrt(b)*lo
g(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^2 - a*
f), -1/4*(4*(a*cos(f*x + e)^2 - a)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) - 2*a*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) - ((a + b)*cos(f*x + e)^2 - a
- b)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)))
/(a*f*cos(f*x + e)^2 - a*f), 1/2*(((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)
*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e)/a) - 2*(a*cos(f*x + e)^2 - a)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + a*sqrt(((a - b)*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(a*f*cos(f*x + e)^2 - a*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
```

```
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assu
mes constant sign by intervals (correct if the argument is real):Check [abs
(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^2-1 were not checkedUn
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Warning, integration of abs or sign assumes constant sign by intervals
 (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time
: 1.22Error: Bad Argument Type
```

maple [B] time = 1.24, size = 2075, normalized size = 16.34

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/8/f*(-1+cos(f*x+e))*(4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)^2*b^(3/2)*a^(1/2)*4^(1/2)-4*((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*b^(1/2)*a^(3/2)*4^(1/2)+3*ln(-
2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1
+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^
(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)
^2*b^(3/2)*4^(1/2)*a-ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*
x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*cos(f*
x+e)^2*b^(3/2)*4^(1/2)*a-4*ln(-4*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/s
in(f*x+e)^2/a^(1/2))*cos(f*x+e)^2*b^(3/2)*4^(1/2)*a+4*arctanh(1/8*(-1+cos(f
*x+e))*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*cos(f*x+e)
```

```
^2*a^(3/2)*4^(1/2)*b+8*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)
^(3/2)*cos(f*x+e)^2*b^(1/2)*a^(1/2)+2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*b^(1/2)*a^(3/2)*4^(1/2)-ln(-2*(-1+cos(f*x+
e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2
)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-
a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)^2*b^(1/2)*4^(
1/2)*a^2-ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+co
s(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/
2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*cos(f*x+e)^2*b^(1/
2)*4^(1/2)*a^2+16*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2
)*cos(f*x+e)*b^(1/2)*a^(1/2)-4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*
x+e))^2)^(1/2)*b^(3/2)*a^(1/2)*4^(1/2)+2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*a^(3/2)*4^(1/2)-3*ln(-2*(-1+cos(f*x+e))*(a
^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2
)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(
f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(3/2)*4^(1/2)*a+ln(-4*(a^(1/
2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+
e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*b^(3/2)*4^(1/2)*a+4*ln(-4*(-1+cos(f*x+e
))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)
^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a
*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(3/2)*4^(1/2)*a-4*arcta
nh(1/8*(-1+cos(f*x+e))*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+
e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(
1/2))*a^(3/2)*4^(1/2)*b+8*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))
^2)^(3/2)*b^(1/2)*a^(1/2)+ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/si
n(f*x+e)^2/a^(1/2))*b^(1/2)*4^(1/2)*a^2+ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+
cos(f*x+e)))*b^(1/2)*4^(1/2)*a^2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/cos(f*x+e)^2)^(1/2)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^
2)^(1/2)/sin(f*x+e)^4/a^(3/2)/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan\left(fx+e\right)^{2} + a} \csc\left(fx+e\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b \tan\left(e+f\,x\right)^{2}+a}}{\sin\left(e+f\,x\right)^{3}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^3,x)
[Out] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tan^{2}\left(e + fx\right)} \csc^{3}\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**3, x)
```

3.97 $\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=187

$$-\frac{(3a^2+6ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{3/2}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{4f}$$

```
[Out] -1/8*(3*a^2+6*a*b-b^2)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2
))/a^(3/2)/f+arctanh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)
/f-1/8*(3*a+b)*cot(f*x+e)*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/a/f-1/4*cot
(f*x+e)*csc(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.23, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3664, 467, 578, 523, 217, 206, 377, 207}

$$-\frac{(3a^2+6ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{3/2}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((3*a^2 + 6*a*b - b^2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/(8*a^(3/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a
 - b + b*Sec[e + f*x]^2]])/f - ((3*a + b)*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a
- b + b*Sec[e + f*x]^2])/(8*a*f) - (Cot[e + f*x]*Csc[e + f*x]^3*Sqrt[a - b
+ b*Sec[e + f*x]^2])/(4*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
```

```
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a-b+bx^2}}{\left(-1+x^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2\left(3(a-\right.}{\left(-1+x^2\right)^2}\right.}{}$$

$$= -\frac{(3a+b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8af} - \frac{\cot(e+fx}{}$$

$$= -\frac{(3a+b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8af} - \frac{\cot(e+fx}{}$$

$$= -\frac{(3a+b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8af} - \frac{\cot(e+fx}{}$$

$$= -\frac{\left(3a^2+6ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+}{\sqrt{a-b+b\sec^2(}\right.}{f}$$

Mathematica [B] time = 6.70, size = 1059, normalized size = 5.66

$$\frac{\sqrt{\frac{\cos(2(e+fx))a+a+b-b\cos(2(e+fx))}{\cos(2(e+fx))+1}}\left(\frac{(-3a\cos(e+fx)-b\cos(e+fx))\csc^2(e+fx)}{8a}-\frac{1}{4}\cot(e+fx)\csc^3(e+fx)\right)}{f}+\frac{(3a^2-2ba-b^2)\left(2\sqrt{a}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x
)])]*(((-3*a*Cos[e + f*x] - b*Cos[e + f*x])*Csc[e + f*x]^2)/(8*a) - (Cot[e
+ f*x]*Csc[e + f*x]^3)/4))/f + (-1/4*((3*a^2 + 14*a*b - b^2)*(2*Sqrt[a]*Arc
Tanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))/Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1
 + Tan[(e + f*x)/2]^2)^2]] - Sqrt[b]*(ArcTanh[(a - a*Tan[(e + f*x)/2]^2 + 2
*b*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(
e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*
Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]))*(1 + Cos[e
 + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a + b + (a
 - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(-1 + Tan[(e + f*x)/2]^2)*(
1 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x
)/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])/(Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a -
b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*
x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]) + ((3*a^2 - 2*a*b - b^2)*(2*Sqrt[
a]*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))/Sqrt[4*b*Tan[(e + f*x)/2]^2 +
 a*(-1 + Tan[(e + f*x)/2]^2)^2]] + Sqrt[b]*(ArcTanh[(a - a*Tan[(e + f*x)/2]
^2 + 2*b*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 +
 Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sq
rt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]))*(1 +
 Cos[e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a +
b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(-1 + Tan[(e + f*x)/2
]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e
 + f*x)/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])/(4*Sqrt[a]*Sqrt[b]*Sqrt[a + b
 + (a - b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan
[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]))/(8*a*f)
```

fricas [A] time = 2.02, size = 1273, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 6*a*b - b^2)*cos
(f*x + e)^2 + 3*a^2 + 6*a*b - b^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 +
 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
 a + b)/(cos(f*x + e)^2 - 1)) - 8*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^
2 + a^2)*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*((
3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(((a - b)*cos
(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x +
 e)^2 + a^2*f), 1/8*(((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 6*a
*b - b^2)*cos(f*x + e)^2 + 3*a^2 + 6*a*b - b^2)*sqrt(-a)*arctan(sqrt(-a)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + 4*(a^2*co
s(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(b)*log(-((a - b)*cos(f*x +
e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) + 2*b)/cos(f*x + e)^2) + ((3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)
```

```
*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f*co
s(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f), -1/16*(16*(a^2*cos(f*x + e)
^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + ((3*a^2 + 6*a*b - b^2)*c
os(f*x + e)^4 - 2*(3*a^2 + 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 + 6*a*b - b^
2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 + 2*sqrt(a)*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) -
2*((3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(((a - b)
*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f
*x + e)^2 + a^2*f), 1/8*(((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 +
 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 + 6*a*b - b^2)*sqrt(-a)*arctan(sqrt(-a
)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) - 8*(a^
2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(-b)*arctan(sqrt(-b)*sqr
t(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + ((3*a^2 +
a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 +
a^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
```

```
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assu
mes constant sign by intervals (correct if the argument is real):Check [abs
(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^2-1 were not checkedUn
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, int
egration of abs or sign assumes constant sign by intervals (correct if the
argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.54Error: Bad
Argument Type
```

maple [B] time = 1.34, size = 5378, normalized size = 28.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan\left(fx+e\right)^{2}+a} \csc\left(fx+e\right)^{5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b \tan\left(e+f x\right)^{2}+a}}{\sin\left(e+f x\right)^{5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^5,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^{2}\left(e+fx\right)} \csc^{5}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**5, x)
```

3.98 $\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=189

$$\frac{(3a^2-12ab+8b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{3/2}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e}{4f}$$

```
[Out] 1/8*(3*a^2-12*a*b+8*b^2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(
1/2))/(a-b)^(3/2)/f+arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^
(1/2)/f-1/8*(3*a-4*b)*cos(f*x+e)*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(a-b)/
f-1/4*cos(f*x+e)*sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.24, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3663, 467, 578, 523, 217, 206, 377, 203}

$$\frac{(3a^2-12ab+8b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{3/2}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((3*a^2 - 12*a*b + 8*b^2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[
e + f*x]^2]])/(8*(a - b)^(3/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a + b*Tan[e + f*x]^2]])/f - ((3*a - 4*b)*Cos[e + f*x]*Sin[e + f*x]*Sq
rt[a + b*Tan[e + f*x]^2])/(8*(a - b)*f) - (Cos[e + f*x]*Sin[e + f*x]^3*Sqrt
[a + b*Tan[e + f*x]^2])/(4*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
```

```
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{\left(1+x^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2(3a+4}{\left(1+x^2\right)^2\sqrt{}}\right.}{}$$

$$= -\frac{(3a-4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)f} - \frac{\cos(e+f}{}$$

$$= -\frac{(3a-4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)f} - \frac{\cos(e+f}{}$$

$$= -\frac{(3a-4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)f} - \frac{\cos(e+f}{}$$

$$= \frac{\left(3a^2-12ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{3/2}f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+}{\sqrt{a+b\tan^2(e}}\right.}{f}$$

Mathematica [C] time = 3.80, size = 330, normalized size = 1.75

$$\frac{\sin(2(e+fx))\sec^2(e+fx)\left(-\left((a-b)\left(6\left(a^2-3ab+2b^2\right)\cos(2(e+fx))+7a^2-(a-b)^2\cos(4(e+fx))-11b^2\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-((a - b)*(7*a^2 - 11*b^2 + 6*(a^2 - 3*a*b + 2*b^2)*Cos[2*(e + f*x)] - (a
 - b)^2*Cos[4*(e + f*x)])) + 2*Sqrt[2]*a*(3*a^2 - 7*a*b + 4*b^2)*Sqrt[((a +
 b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a
 + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - 2*Sqrt[2
]*a*(3*a^2 - 12*a*b + 8*b^2)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e
 + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)
])/(32*Sqrt[2]*(a - b)^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e +
f*x]^2])
```

fricas [B] time = 13.42, size = 2068, normalized size = 10.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*((3*a^2 - 12*a*b + 8*b^2)*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2
*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a
*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3
 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*
b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 -
 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b +
 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)
*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-
a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 16
*(a^2 - 2*a*b + b^2)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*
(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
+ 8*b^2)/cos(f*x + e)^4) + 8*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2
 - 11*a*b + 6*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/((a^2 - 2*a*b + b^2)*f), -1/64*(32*(a^2 - 2*a*b + b^2
)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b
)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x +
 e)^2 + b^2)*sin(f*x + e))) - (3*a^2 - 12*a*b + 8*b^2)*sqrt(-a + b)*log(128
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*
a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b
+ 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2
*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 1
6*b^4)*cos(f*x + e)^2 - 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^
7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2
*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^
3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e)) - 8*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 11*
a*b + 6*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e))/((a^2 - 2*a*b + b^2)*f), 1/32*((3*a^2 - 12*a*b + 8*b^2)*sqr
t(a - b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b
 + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*
```

```
b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*
b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 8*(a^2 - 2*a*b + b^2)*sqrt(b)
*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2
 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4
*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 11*a*b + 6*b^2)*cos(f*x +
 e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2
- 2*a*b + b^2)*f), 1/32*((3*a^2 - 12*a*b + 8*b^2)*sqrt(a - b)*arctan(-1/4*(
8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)
^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e
)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x +
 e)^2)*sin(f*x + e))) - 16*(a^2 - 2*a*b + b^2)*sqrt(-b)*arctan(1/2*((a - 2*
b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e))) +
4*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 11*a*b + 6*b^2)*cos(f*x
+ e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2
 - 2*a*b + b^2)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

maple [C] time = 1.99, size = 2498, normalized size = 13.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/8/f*(2*cos(f*x+e)^5*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^2-4*cos(f
*x+e)^5*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b+2*cos(f*x+e)^5*((2*I*
(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2+6*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2
)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))
/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos
(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*
((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^
(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)
*b^(1/2)+a-2*b)/a)^(1/2))*a^2*sin(f*x+e)-24*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1
/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e
))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*c
os(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e)
)*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*
b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/
2)*b^(1/2)+a-2*b)/a)^(1/2))*a*b*sin(f*x+e)+16*2^(1/2)*((I*cos(f*x+e)*(a-b)^
(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x
+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a
*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+
e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2
)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(
1/2)*b^(1/2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)+16*2^(1/2)*((I*cos(f*x+e)*(a-b
)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f
*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)
```

```
-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*
x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/
2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^
(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*a*b*sin(f*x+e)-16*2^(1/2)*((I*cos(f*x+e)*(a-
b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(
f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2
)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f
*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1
/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)
^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)-3*2^(1/2)*((I*cos(f*x+e)*(a-
b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(
f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2
)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*
x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2
)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^2*sin(f*
x+e)+4*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*c
os(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(
1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+
e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^
(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*
a*b+8*b^2)/a^2)^(1/2))*a*b*sin(f*x+e)-2*cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1
/2)+a-2*b)/a)^(1/2)*a^2+4*cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^
(1/2)*a*b-2*cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2-5*co
s(f*x+e)^3*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^2+13*cos(f*x+e)^3*((
2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b-8*cos(f*x+e)^3*((2*I*(a-b)^(1/2
)*b^(1/2)+a-2*b)/a)^(1/2)*b^2+5*cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*
b)/a)^(1/2)*a^2-13*cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a
*b+8*cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2-5*cos(f*x+e
)*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b+6*cos(f*x+e)*((2*I*(a-b)^(1
/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2+5*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)
*a*b-6*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2)*cos(f*x+e)*sin(f*x+e)
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(-1+cos(f*x+e))/(a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/(a
-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\,\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^4\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\,\sin^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*sin(e + f*x)**4, x)
```

3.99 $\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=128

$$\frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f\sqrt{a-b}}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

```
[Out] 1/2*(a-2*b)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)
^(1/2)+arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*c
os(f*x+e)*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3663, 467, 523, 217, 206, 377, 203}

$$\frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f\sqrt{a-b}}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((a - 2*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(
2*Sqrt[a - b]*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/f - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*
f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
```

```
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a+2bx^2}{\left(1+x^2\right)\sqrt{a+ \cdots$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-2b)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-2b)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{a-b}\,f} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \cdots$$

Mathematica [C] time = 3.84, size = 273, normalized size = 2.13

$$-\frac{\sin(2(e+fx))\sec^2(e+fx)\left((a-b)((a-b)\cos(2(e+fx))+a+b)+\sqrt{2}\,a(b-a)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a \cdots}{b}}\right.\cdots}{4\sqrt{2}\,f(a-b)\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/4*(((a - b)*(a + b + (a - b)*Cos[2*(e + f*x)]) + Sqrt[2]*a*(-a + b)*Sqrt
[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sq
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + Sq
rt[2]*a*(a - 2*b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/
b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])
```

```
*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(Sqrt[2]
*(a - b)*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

fricas [B] time = 1.37, size = 1847, normalized size = 14.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e)*sin(f*x + e) - (a - 2*b)*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*
b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*
b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3
+ 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b
^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 -
8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b +
5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*
cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a
 + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 4*(
a - b)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*
cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f
*x + e)^4))/((a - b)*f), -1/16*(8*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + 8*(a - b)*sqrt(-b)*arctan(1/2*
((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x +
 e))) - (a - 2*b)*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*
cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos
(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4
- 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*(a^3 -
3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b
^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^
3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((
a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a - b)*f), -1/8
*(4*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*
sin(f*x + e) - sqrt(a - b)*(a - 2*b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos
(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2
)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2
*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) - 2
*(a - b)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2
)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos
(f*x + e)^4))/((a - b)*f), -1/8*(4*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - sqrt(a - b)*(a - 2*b)*arctan(
-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*
x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f
*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos
(f*x + e)^2)*sin(f*x + e))) + 4*(a - b)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/((a - b)
*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan\left(fx+e\right)^{2} + a} \sin\left(fx+e\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

maple [C] time = 0.78, size = 1342, normalized size = 10.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f*(-4*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)
+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-
b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(
f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b
)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2
)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*b*sin(
f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a
*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)
^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*
x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/
a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)
*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*a*sin(f
*x+e)+4*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*
cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^
(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x
+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a
)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*
b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*b*sin(f*
x+e)+2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos
(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/
2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e)
)/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1
/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*
b+8*b^2)/a^2)^(1/2))*a*sin(f*x+e)+cos(f*x+e)^3*((2*I*(a-b)^(1/2)*b^(1/2)+a-
2*b)/a)^(1/2)*a-cos(f*x+e)^3*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b-co
s(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a+cos(f*x+e)^2*((2*I*(
a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b+cos(f*x+e)*((2*I*(a-b)^(1/2)*b^(1/2)+a
-2*b)/a)^(1/2)*b-((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b)*cos(f*x+e)*si
n(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(-1+cos(f*x
+e))/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^
(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan\left(fx+e\right)^2 + a}\, \sin\left(fx+e\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^2\, \sqrt{b\tan\left(e+f\,x\right)^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2\left(e + fx\right)}\,\sin^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*sin(e + f*x)**2, x)
```

3.100 $\int \sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f+arcta
nh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3661, 402, 217, 206, 377, 203}

$$\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
```

```
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan(e+\right.}{f}$$
$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+f}{\sqrt{a+b\tan^2(}\right.}{f}$$
$$= \frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

Mathematica [C] time = 0.89, size = 203, normalized size = 2.39

$$\frac{-i\sqrt{a-b}\,\log\left(-\frac{4i\left(\sqrt{a-b}\,\sqrt{a+b\tan^2(e+fx)}+a-ib\tan(e+fx)\right)}{(a-b)^{3/2}(\tan(e+fx)+i)}\right)+i\sqrt{a-b}\,\log\left(\frac{4i\left(\sqrt{a-b}\,\sqrt{a+b\tan^2(e+fx)}+a+ib\tan(e+fx)\right)}{(a-b)^{3/2}(\tan(e+fx)-i)}\right)+2\sqrt{b}\,\text{lo}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[a - b]*Log[((-4*I)*(a - I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a +
b*Tan[e + f*x]^2]))/((a - b)^(3/2)*(I + Tan[e + f*x]))] + I*Sqrt[a - b]*Log
[((4*I)*(a + I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a + b*Tan[e + f*x]^2]))/((
a - b)^(3/2)*(-I + Tan[e + f*x]))] + 2*Sqrt[b]*Log[b*Tan[e + f*x] + Sqrt[b]
*Sqrt[a + b*Tan[e + f*x]^2]])/(2*f)
```

fricas [A] time = 0.55, size = 410, normalized size = 4.82

$$\left[\frac{\sqrt{b}\,\log\left(2\,b\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{b}\,\tan\left(fx+e\right)+a\right)+\sqrt{-a+b}\,\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2+2\sqrt{b}}{\tan}\right.}{2\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)
*tan(f*x + e) + a) + sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)))/
f, 1/2*(2*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f
*x + e))) + sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*s
```

```
qrt(b)*tan(f*x + e) + a))/f, -1/2*(2*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2
+ a)*sqrt(-b)/(b*tan(f*x + e))) - sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)
^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
 e)^2 + 1)))/f, (sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b
)*tan(f*x + e))) - sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*t
an(f*x + e))))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan\left(fx+e\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [B] time = 0.25, size = 169, normalized size = 1.99

$$\frac{\sqrt{b}\,\ln\left(\tan\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}\right)}{f}-\frac{\sqrt{b^{4}\left(a-b\right)}\,\arctan\left(\frac{(a-b)b^{2}\tan\left(fx+e\right)}{\sqrt{b^{4}(a-b)}\,\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}\right)}{fb\left(a-b\right)}+\frac{a\sqrt{b^{4}\left(a-\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*b^(1/2)*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*(b^4*(a-b))
^(1/2)/b/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*
tan(f*x+e))+1/f*a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^
(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b \tan\left(e+fx\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2), x)
```

3.101 $\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=66

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)*(
a+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3663, 277, 217, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f - (C
ot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan(e+\right.}{f}$$
$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+}{\sqrt{a+b\tan^2}}\right.}{f}$$
$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Mathematica [C] time = 2.18, size = 156, normalized size = 2.36

$$-\frac{\tan(e+fx)\left(\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)-\sqrt{2}\,b\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{(a}}{}\right.\right.\right.}{\sqrt{2}\,f\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2 - Sqrt[2]*b*Sqrt[((a
+ b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((
a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Tan[e +
f*x])/(Sqrt[2]*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]))
```

fricas [B] time = 0.56, size = 331, normalized size = 5.02

$$\left[\frac{\sqrt{b}\log\left(\frac{\left(a^2-8ab+8b^2\right)\cos\left(fx+e\right)^4+8\left(ab-2b^2\right)\cos\left(fx+e\right)^2+4\left((a-2b)\cos\left(fx+e\right)^3+2b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin\left(fx+e\right)+8b^2}{\cos\left(fx+e\right)^4}\right)}{4f\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*c
os(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqr
t(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*
x + e)^4)*sin(f*x + e) - 4*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e))/(f*sin(f*x + e)), -1/2*(sqrt(-b)*arctan(1/2*((a - 2*b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x +
 e) + 2*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*
sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\,\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

maple [C] time = 4.52, size = 1215, normalized size = 18.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)*(-2^
(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)
-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1
/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1
/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin
(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2
)/a^2)^(1/2))*b*sin(f*x+e)*cos(f*x+e)+2*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*
b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a
)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f
*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((
2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/
2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^
(1/2)+a-2*b)/a)^(1/2))*b*sin(f*x+e)*cos(f*x+e)-sin(f*x+e)*EllipticF((-1+cos
(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(
1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*2^(1/2)
*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*co
s(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I
*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*b
+2*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f
*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)
*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/
a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/
2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2
)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*b*sin(f*x+e)+c
os(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a-cos(f*x+e)^2*((2*I*
(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b+((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(
1/2)*b)/sin(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/((2*I*(a-b)^(1/2)*b^(1
/2)+a-2*b)/a)^(1/2)
```

maxima [A] time = 0.63, size = 47, normalized size = 0.71

$$\frac{\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) - sqrt(b*tan(f*x + e)^2 + a)/tan
(f*x + e))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^2,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**2, x)
```

3.102 $\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)*(
a+b*tan(f*x+e)^2)^(1/2)/f-1/3*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2)/a/f
```

Rubi [A] time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3663, 451, 277, 217, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f - (C
ot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f - (Cot[e + f*x]^3*(a + b*Tan[e +
f*x]^2)^(3/2))/(3*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)\sqrt{a+bx^2}}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \tan(\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx\right.}{3af}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx\right.}{3af}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\text{co}}{}$$

Mathematica [C] time = 4.41, size = 204, normalized size = 2.04

$$-\frac{\tan(e+fx)\left(\csc^4(e+fx)\left(4\left(a^2-3ab-b^2\right)\cos(2(e+fx))+\left(-2a^2+ab+b^2\right)\cos(4(e+fx))+6a^2+11ab\right.\right.}{12\sqrt{2}\,af\sqrt{\sec^2(e+fx)((a-b)\cos(2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/12*(((6*a^2 + 11*a*b + 3*b^2 + 4*(a^2 - 3*a*b - b^2)*Cos[2*(e + f*x)] +
(-2*a^2 + a*b + b^2)*Cos[4*(e + f*x)])*Csc[e + f*x]^4 - 12*Sqrt[2]*a*b*Sqrt
[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sq
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Tan
[e + f*x])/(Sqrt[2]*a*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x
]^2])
```

fricas [B] time = 0.68, size = 435, normalized size = 4.35

$$\left[\frac{3\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{b}\log\left(\frac{\left(a^2-8ab+8b^2\right)\cos\left(fx+e\right)^4+8\left(ab-2b^2\right)\cos\left(fx+e\right)^2+4\left((a-2b)\cos\left(fx+e\right)^3+2b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{(a-b)}{c}}}{\cos\left(fx+e\right)^4}\right.}{12\left(af\cos\left(fx+e\right)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*cos(f*x + e)^2 - a)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x
+ e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b
*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*si
n(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + b)*cos(f*x + e
)^3 - (3*a + b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2))/((a*f*cos(f*x + e)^2 - a*f)*sin(f*x + e)), -1/6*(3*(a*cos(f*x + e)^2
- a)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt
(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*
x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) + 2*((2*a + b)*cos(f*x + e)^3 -
 (3*a + b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
/((a*f*cos(f*x + e)^2 - a*f)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan\left(fx+e\right)^{2}+a}\,\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

maple [C] time = 1.71, size = 2444, normalized size = 24.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f*(6*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-
2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(
1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*2^(
1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-
b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/
2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/
2)*cos(f*x+e)^3*a*b-3*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*(a-b)^
(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x
+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a
*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e
))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b
^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b+6*sin(f*x
+e)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/si
n(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2
*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*2^(1/2)*((I*cos(f*x
+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/
(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)
*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*cos(f*x+e)^2*
a*b-3*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)
/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^
2-8*a*b+8*b^2)/a^2)^(1/2))*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-
b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(
I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f
*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*cos(f*x+e)^2*a*b-6*sin(f*x+e)*EllipticPi((
-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*
(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2
*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*
b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a
)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f
```

```
*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*cos(f*x+e)*a*b+3*sin(f*x+e)*E
llipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+
e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2
)^(1/2))*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a
*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)
^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*
x+e))/a)^(1/2)*cos(f*x+e)*a*b-6*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-
I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*
(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*
cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b
)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b
)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-
2*b)/a)^(1/2))*a*b*sin(f*x+e)+3*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-
I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*
(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*
cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)
^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-
b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b*sin(f*x+e)+2*cos(f*x+e)
^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^2-cos(f*x+e)^4*((2*I*(a-b)^(
1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b-cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2
*b)/a)^(1/2)*b^2-3*cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a
^2+4*cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b+2*cos(f*x+e
)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2-3*((2*I*(a-b)^(1/2)*b^(1/
2)+a-2*b)/a)^(1/2)*a*b-((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2)*cos(f
*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/sin(f*x+e)^3/a/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2
)
```

maxima [A] time = 0.50, size = 76, normalized size = 0.76

$$\frac{3\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)-\frac{3\sqrt{b\tan\left(fx+e\right)^2+a}}{\tan\left(fx+e\right)}-\frac{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}{a\tan\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) - 3*sqrt(b*tan(f*x + e)^2
+ a)/tan(f*x + e) - (b*tan(f*x + e)^2 + a)^(3/2)/(a*tan(f*x + e)^3))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^4,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\,\csc^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**4, x)
```

3.103 $\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=141

$$-\frac{2(5a-b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{15a^2f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/}}{5af}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)*(
a+b*tan(f*x+e)^2)^(1/2)/f-2/15*(5*a-b)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2
)/a^2/f-1/5*cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2)/a/f
```

Rubi [A] time = 0.13, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3663, 462, 451, 277, 217, 206}

$$-\frac{2(5a-b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{15a^2f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/}}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f - (C
ot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f - (2*(5*a - b)*Cot[e + f*x]^3*(a
+ b*Tan[e + f*x]^2)^(3/2))/(15*a^2*f) - (Cot[e + f*x]^5*(a + b*Tan[e + f*x]
^2)^(3/2))/(5*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2\sqrt{a+bx^2}}{x^6}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{5af} + \frac{\text{Subst}\left(\int \frac{\left(2(5a-b)+5ax^2\right)\sqrt{a+bx}}{x^4}\right.}{5af}$$

$$= -\frac{2(5a-b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{15a^2f} - \frac{\cot^5(e+fx)\left(a+\right.}{5a}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{2(5a-b)\cot^3(e+fx)\left(a+b\,\text{ta}\right.}{15a^2f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{2(5a-b)\cot^3(e+fx)\left(a+b\,\text{ta}\right.}{15a^2f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - 2($$

Mathematica [C] time = 3.50, size = 287, normalized size = 2.04

$$-\frac{\tan(e+fx)\left(\csc^6(e+fx)\left(8a^3\cos(6(e+fx))+80a^3+a^2b\cos(6(e+fx))+198a^2b+\left(40a^3-241a^2b-149\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/240*(((80*a^3 + 198*a^2*b + 98*a*b^2 - 20*b^3 + (40*a^3 - 241*a^2*b - 14
9*a*b^2 + 30*b^3)*Cos[2*(e + f*x)] + (-32*a^3 + 42*a^2*b + 62*a*b^2 - 12*b^
3)*Cos[4*(e + f*x)] + 8*a^3*Cos[6*(e + f*x)] + a^2*b*Cos[6*(e + f*x)] - 11*
a*b^2*Cos[6*(e + f*x)] + 2*b^3*Cos[6*(e + f*x)])*Csc[e + f*x]^6 - 240*Sqrt[
2]*a^2*b*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Ellipt
icF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt
```

```
[2]], 1])*Tan[e + f*x])/(Sqrt[2]*a^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x
)])*Sec[e + f*x]^2])
```

fricas [B] time = 1.37, size = 587, normalized size = 4.16

$$\left[\frac{15\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2+a^2\right)\sqrt{b}\,\log\left(\frac{\left(a^2-8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4+8\left(ab-2\,b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-2\,b\right)\cos\left(fx+e\right)^3\right.}{\cos\left(fx+e\right)^4}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(b)*log(((a
^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a
 - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e
) - 4*((8*a^2 + 9*a*b - 2*b^2)*cos(f*x + e)^5 - (20*a^2 + 19*a*b - 4*b^2)*c
os(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/((a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x +
e)^2 + a^2*f)*sin(f*x + e)), -1/30*(15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x
+ e)^2 + a^2)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^
2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) + 2*((8*a^2 + 9*a*b -
2*b^2)*cos(f*x + e)^5 - (20*a^2 + 19*a*b - 4*b^2)*cos(f*x + e)^3 + (15*a^2
+ 10*a*b - 2*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/((a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f)*sin(f*x +
e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\,\csc\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^6, x)
```

maple [C] time = 1.33, size = 3769, normalized size = 26.73

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15/f*(-15*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/
2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(
a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+co
s(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*
b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+
a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^5*a^2*b-15*2^(1/2)*((I*c
os(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+
e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)
```

```
^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*Ellipti
cF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8
*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2
))*a^2*b*sin(f*x+e)+30*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(
1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*co
s(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e
)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b
^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*
I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(
1/2))*a^2*b*sin(f*x+e)+15*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^2*b+1
0*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b^2-2*((2*I*(a-b)^(1/2)*b^(1/
2)+a-2*b)/a)^(1/2)*b^3+30*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b
)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I
*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*
x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2
)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-
(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a
)^(1/2))*sin(f*x+e)*cos(f*x+e)^4*a^2*b-15*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2
)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))
/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos
(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*(
(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/
2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)*cos(f*
x+e)^4*a^2*b-60*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^
(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e
)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1
+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+
a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)
^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*s
in(f*x+e)*cos(f*x+e)^3*a^2*b+30*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-
I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*
(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*
cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)
^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-
b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^3*a^2
*b-60*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*co
s(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1
/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e
))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^
(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(
1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*
cos(f*x+e)^2*a^2*b+30*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1
/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos
(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)
-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(
1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b
^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^2*b+30*2^(1
/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b
*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2
)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2
)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(
f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b
)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*cos(f*x+e)
*a^2*b-15*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+
a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b
)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f
*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/
a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2
-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)*cos(f*x+e)*a^2*b+30*2^(1/2)*((I*cos(f*
```

```
x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)
/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2
)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((
-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*
(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2
*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*cos(f*x+e)^5*a^2*b-6*cos
(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^3+6*cos(f*x+e)^2*((2*
I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^3+8*cos(f*x+e)^6*((2*I*(a-b)^(1/2)*
b^(1/2)+a-2*b)/a)^(1/2)*a^3-20*cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b
)/a)^(1/2)*a^3+15*cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^
3+2*cos(f*x+e)^6*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^3+cos(f*x+e)^6
*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^2*b-11*cos(f*x+e)^6*((2*I*(a-b
)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b^2+9*cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1
/2)+a-2*b)/a)^(1/2)*a^2*b+32*cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/
a)^(1/2)*a*b^2-25*cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^
2*b-31*cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b^2)*cos(f*
x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/sin(f*x+e)^5/a^2/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/
2)
```

maxima [A] time = 0.49, size = 131, normalized size = 0.93

$$\frac{15\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)-\frac{15\sqrt{b\tan\left(fx+e\right)^2+a}}{\tan\left(fx+e\right)}-\frac{10\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}{a\tan\left(fx+e\right)^3}+\frac{2\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}b}{a^2\tan\left(fx+e\right)^3}-\frac{3\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}{a\tan\left(fx+e\right)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) - 15*sqrt(b*tan(f*x + e)
^2 + a)/tan(f*x + e) - 10*(b*tan(f*x + e)^2 + a)^(3/2)/(a*tan(f*x + e)^3) +
 2*(b*tan(f*x + e)^2 + a)^(3/2)*b/(a^2*tan(f*x + e)^3) - 3*(b*tan(f*x + e)^
2 + a)^(3/2)/(a*tan(f*x + e)^5))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\tan\left(e+fx\right)^2+a}}{\sin\left(e+fx\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^6,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\csc^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**6, x)
```

3.104 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=227

$$\frac{b(3a-7b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f(a-b)}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{5f(a-b)}+\frac{2\cos^3(e+fx)\left(a+\right.}{3f(\ }$$

```
[Out] -1/3*(3*a-7*b)*cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(3/2)/(a-b)/f+2/3*cos(f*x+e)
^3*(a-b+b*sec(f*x+e)^2)^(5/2)/(a-b)/f-1/5*cos(f*x+e)^5*(a-b+b*sec(f*x+e)^2)
^(5/2)/(a-b)/f+1/2*(3*a-7*b)*arctanh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2
)^(1/2))*b^(1/2)/f+1/2*(3*a-7*b)*b*sec(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a
-b)/f
```

Rubi [A] time = 0.22, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3664, 462, 453, 277, 195, 217, 206}

$$\frac{b(3a-7b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f(a-b)}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{5f(a-b)}+\frac{2\cos^3(e+fx)\left(a+\right.}{3f(\ }$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((3*a - 7*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e +
f*x]^2]])/(2*f) + ((3*a - 7*b)*b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2
])/(2*(a - b)*f) - ((3*a - 7*b)*Cos[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/
2))/(3*(a - b)*f) + (2*Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(5/2))/(3*
(a - b)*f) - (Cos[e + f*x]^5*(a - b + b*Sec[e + f*x]^2)^(5/2))/(5*(a - b)*f
)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^{3/2}}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{5/2}}{5(a-b)f} + \frac{\text{Subst}\left(\int \frac{\left(-10(a-b)+5(a-b)\ldots}{x^4}\ldots\right.}{5\ldots}$$

$$= \frac{2\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{5/2}}{3(a-b)f} - \frac{\cos^5(e+fx)\left(a-b+b\,\text{s}\ldots\right.}{5(a-b)f}$$

$$= -\frac{(3a-7b)\cos(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{3(a-b)f} + \frac{2\cos^3(e+fx)\left(\ldots\right.}{\ldots}$$

$$= \frac{(3a-7b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f} - \frac{(3a-7b)\cos(e+f\ldots}{\ldots}$$

$$= \frac{(3a-7b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f} - \frac{(3a-7b)\cos(e+f\ldots}{\ldots}$$

$$= \frac{(3a-7b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-7b)b\sec(e+fx)\sqrt{a\ldots}}{2(a-b)\ldots}$$

Mathematica [A] time = 5.57, size = 233, normalized size = 1.03

$$\frac{\cos(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(2\sqrt{(a-b)\cos(2(e+fx))+a+b}\left(4\left(7a^2-20ab+13b\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(120*
Sqrt[2]*Sqrt[b]*(3*a^2 - 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b + (a - b)*Cos[2
*(e + f*x)]]/(Sqrt[2]*Sqrt[b])] + 2*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*
(-89*a^2 + 474*a*b - 409*b^2 + 4*(7*a^2 - 20*a*b + 13*b^2)*Cos[2*(e + f*x)]
 - 3*(a - b)^2*Cos[4*(e + f*x)] + 60*(a - b)*b*Sec[e + f*x]^2)))/(240*Sqrt[
2]*(a - b)*f*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
```

fricas [A] time = 0.91, size = 426, normalized size = 1.88

$$\left[-\frac{15\left(3a^{2}-10ab+7b^{2}\right)\sqrt{b}\cos\left(fx+e\right)\log\left(-\frac{(a-b)\cos\left(fx+e\right)^{2}-2\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+2b}{\cos\left(fx+e\right)^{2}}\right)+2\left(6\left(a^{2}-2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(3*a^2 - 10*a*b + 7*b^2)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(
f*x + e)^2 - 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*co
s(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(6*(a^2 - 2*a*b + b^2)*cos(f*x + e)^6
 - 4*(5*a^2 - 13*a*b + 8*b^2)*cos(f*x + e)^4 + 2*(15*a^2 - 70*a*b + 58*b^2)
*cos(f*x + e)^2 - 15*a*b + 15*b^2)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*
x + e)^2))/((a - b)*f*cos(f*x + e)), -1/30*(15*(3*a^2 - 10*a*b + 7*b^2)*sqr
t(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e)/b)*cos(f*x + e) + (6*(a^2 - 2*a*b + b^2)*cos(f*x + e)^6 - 4*(5*a^
2 - 13*a*b + 8*b^2)*cos(f*x + e)^4 + 2*(15*a^2 - 70*a*b + 58*b^2)*cos(f*x +
 e)^2 - 15*a*b + 15*b^2)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
/((a - b)*f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
```

)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)2/f*2*(1/2*(-4*sqrt(a)*b^3*sign(tan((f*x+exp(1))/2)^2-1)-b^2*(-sq
rt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1
))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3*sign(tan((f*x+exp(1))/2)^2-1)-4*b^3
*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+
exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))*sign(tan((f*x+exp(1))/2)^2-1)+3*
sqrt(a)*a*b^2*sign(tan((f*x+exp(1))/2)^2-1)-sqrt(a)*a^2*b*sign(tan((f*x+exp
(1))/2)^2-1)-a*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)
^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3*sign(tan((f*x+
exp(1))/2)^2-1)+9*a*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp
(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))*sign(tan(
(f*x+exp(1))/2)^2-1)-3*a^2*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*
x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))*sign
(tan((f*x+exp(1))/2)^2-1)+5*sqrt(a)*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqr
t(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)
^2+a))^2*sign(tan((f*x+exp(1))/2)^2-1)-3*sqrt(a)*a*b*(-sqrt(a)*tan((f*x+exp
(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f
*x+exp(1))/2)^2+a))^2*sign(tan((f*x+exp(1))/2)^2-1))/(-2*sqrt(a)*(-sqrt(a)*
tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^
2+4*b*tan((f*x+exp(1))/2)^2+a))-(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan(
(f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2
-a+4*b)^2-1/15*(768*sqrt(a)*a^6*sign(tan((f*x+exp(1))/2)^2-1)+320*a^3*(-sqr
t(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1)
)/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^7*sign(tan((f*x+exp(1))/2)^2-1)-832*a^
4*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x
+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^5*sign(tan((f*x+exp(1))/2)^2-1)
-320*a^5*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*t
an((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3*sign(tan((f*x+exp(1))/
2)^2-1)+2880*a^6*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)
^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))*sign(tan((f*x+ex
p(1))/2)^2-1)+11520*sqrt(a)*b^6*sign(tan((f*x+exp(1))/2)^2-1)+45*b^2*(-sqrt
(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))
/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^9*sign(tan((f*x+exp(1))/2)^2-1)+880*b^3
*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+
exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^7*sign(tan((f*x+exp(1))/2)^2-1)+
6368*b^4*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*t
an((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^5*sign(tan((f*x+exp(1))/
2)^2-1)+14080*b^5*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2
)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3*sign(tan((f*x

+exp(1))/2)^2-1)+11520*b^6*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))*sign(tan((f*x+exp(1))/2)^2-1)-47360*sqrt(a)*a*b^5*sign(tan((f*x+exp(1))/2)^2-1)+79968*sqrt(a)*a^2*b^4*sign(tan((f*x+exp(1))/2)^2-1)-70512*sqrt(a)*a^3*b^3*sign(tan((f*x+exp(1))/2)^2-1)+640*sqrt(a)*a^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^6*sign(tan((f*x+exp(1))/2)^2-1)+33893*sqrt(a)*a^4*b^2*sign(tan((f*x+exp(1))/2)^2-1)-2560*sqrt(a)*a^4*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^4*sign(tan((f*x+exp(1))/2)^2-1)-8262*sqrt(a)*a^5*b*sign(tan((f*x+exp(1))/2)^2-1)+3200*sqrt(a)*a^5*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2*sign(tan((f*x+exp(1))/2)^2-1)-30*a*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^9*sign(tan((f*x+exp(1))/2)^2-1)+180*a*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^7*sign(tan((f*x+exp(1))/2)^2-1)-8912*a*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^5*sign(tan((f*x+exp(1))/2)^2-1)-48960*a*b^4*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3*sign(tan((f*x+exp(1))/2)^2-1)-72960*a*b^5*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))*sign(tan((f*x+exp(1))/2)^2-1)-1080*a^2*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^7*sign(tan((f*x+exp(1))/2)^2-1)+198*a^2*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^5*sign(tan((f*x+exp(1))/2)^2-1)+57680*a^2*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3*sign(tan((f*x+exp(1))/2)^2-1)+164960*a^2*b^4*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))*sign(tan((f*x+exp(1))/2)^2-1)+3388*a^3*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^5*sign(tan((f*x+exp(1))/2)^2-1)-28060*a^3*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3*sign(tan((f*x+exp(1))/2)^2-1)-180720*a^3*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))*sign(tan((f*x+exp(1))/2)^2-1)+5160*a^4*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3*sign(tan((f*x+exp(1))/2)^2-1)+102645*a^4*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))*sign(tan((f*x+exp(1))/2)^2-1)-28430*a^5*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))*sign(tan((f*x+exp(1))/2)^2-1)-435*sqrt(a)*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^8*sign(tan((f*x+exp(1))/2)^2-1)-6160*sqrt(a)*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^6*sign(tan((f*x+exp(1))/2)^2-1)-17440*sqrt(a)*b^4*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^4*sign(tan((f*x+exp(1))/2)^2-1)-11520*sqrt(a)*b^5*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2*sign(tan((f*x+exp(1))/2)^2-1)+330*sqrt(a)*a*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^8*sign(tan((f*x+exp(1))/2)^2-1)+10020*sqrt(a)*a*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^6*sign(tan((f*x+exp(1))/2)^2-1)+49840*sqrt(a)*a*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x

```
+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^4*sign(tan((f*x+exp(1))/2)^2-1)
+59840*sqrt(a)*a*b^4*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1)
)/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2*sign(tan((
f*x+exp(1))/2)^2-1)-4920*sqrt(a)*a^2*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt
(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^
2+a))^6*sign(tan((f*x+exp(1))/2)^2-1)-49290*sqrt(a)*a^2*b^2*(-sqrt(a)*tan((
f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b
*tan((f*x+exp(1))/2)^2+a))^4*sign(tan((f*x+exp(1))/2)^2-1)-104240*sqrt(a)*a
^2*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan
((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2*sign(tan((f*x+exp(1))/2)
^2-1)+19660*sqrt(a)*a^3*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+e
xp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^4*sign(
tan((f*x+exp(1))/2)^2-1)+80820*sqrt(a)*a^3*b^2*(-sqrt(a)*tan((f*x+exp(1))/2
)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp
(1))/2)^2+a))^2*sign(tan((f*x+exp(1))/2)^2-1)-27800*sqrt(a)*a^4*b*(-sqrt(a)
*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)
^2+4*b*tan((f*x+exp(1))/2)^2+a))^2*sign(tan((f*x+exp(1))/2)^2-1))/(-2*sqrt(
a)*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*
x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))+(-sqrt(a)*tan((f*x+exp(1))/2)^
2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1
))/2)^2+a))^2-3*a+4*b)^5+1/4*(-7*b^2*sign(tan((f*x+exp(1))/2)^2-1)+3*a*b*si
gn(tan((f*x+exp(1))/2)^2-1))*atan(1/2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(
a)+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(
1))/2)^2+a))/sqrt(-b))/sqrt(-b))
```

maple [B] time = 1.18, size = 2399, normalized size = 10.57

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/60/f*(-1+cos(f*x+e))^3*(-32*cos(f*x+e)^4*a^(5/2)*((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(5/2)+30*cos(f*x+e)^3*a^(9/2)*b^(1/2)*
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)-140*cos(f*x+e)^3
*a^(7/2)*b^(3/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)
+116*cos(f*x+e)^3*a^(5/2)*b^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos
(f*x+e))^2)^(1/2)+30*a^(9/2)*b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2-140*a^(7/2)*b^(3/2)*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2+116*a^(5/2)*b^(5/2)*((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2-15*cos(
f*x+e)*a^(7/2)*b^(3/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)
^(1/2)+15*cos(f*x+e)*a^(5/2)*b^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+
cos(f*x+e))^2)^(1/2)-30*cos(f*x+e)^2*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x
+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+
e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f
*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*a^2+30*cos(f*x+e)^2*ln(-4*(-1+cos(f*
x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))
^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2
)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*a^2+45*cos(f*x
+e)^2*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f
*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*
b^(5/2)*a^3-45*cos(f*x+e)^2*ln(-4*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/
sin(f*x+e)^2/a^(1/2))*b^(5/2)*a^3+6*cos(f*x+e)^7*a^(9/2)*((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)-12*cos(f*x+e)^7*a^(7/2)*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)+6*cos(f*x+
e)^7*a^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(
```

```
5/2)+6*cos(f*x+e)^6*a^(9/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e
))^2)^(1/2)*b^(1/2)-12*cos(f*x+e)^6*a^(7/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)+6*cos(f*x+e)^6*a^(5/2)*((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(5/2)-20*cos(f*x+e)^5*a^(9/2)*
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)+52*cos(f
*x+e)^5*a^(7/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*
b^(3/2)-32*cos(f*x+e)^5*a^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f
*x+e))^2)^(1/2)*b^(5/2)-15*cos(f*x+e)^2*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(
f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*co
s(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(9/2)*a+15*cos(f*x+e)^2*ln(-4*(-1+cos(f
*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e)
)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/
2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(9/2)*a+150*cos(f*x
+e)^2*a^(7/2)*arctanh(1/8*(-1+cos(f*x+e))*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos
(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2
)^(1/2)*b^(1/2)*4^(1/2))*b^2-105*cos(f*x+e)^2*a^(5/2)*arctanh(1/8*(-1+cos(f
*x+e))*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*b^3-45*cos
(f*x+e)^2*a^(9/2)*arctanh(1/8*(-1+cos(f*x+e))*(4^(1/2)*cos(f*x+e)-4^(1/2)-2
*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e
))^2)^(1/2)*b^(1/2)*4^(1/2))*b-20*cos(f*x+e)^4*a^(9/2)*((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)+52*cos(f*x+e)^4*a^(7/2)*((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)-15*a^(7/2)*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)+15*a^(5/2
)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(5/2))*cos(f
*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*4^(1/2)/((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)/sin(f*x+e)^6/(a-b)/a^(
5/2)/b^(1/2)
```

maxima [A] time = 0.81, size = 330, normalized size = 1.45

$$-\frac{\frac{12\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{5}{2}}\cos\left(fx+e\right)^5}{a-b} - 40\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3 + 60\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\,(a-b)\cos\left(fx+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/60*(12*(a - b + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5/(a - b) - 40*(a -
 b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 + 60*sqrt(a - b + b/cos(f*x + e
)^2)*(a - b)*cos(f*x + e) - 120*sqrt(a - b + b/cos(f*x + e)^2)*b*cos(f*x +
e) - 60*b^(3/2)*log((sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))
/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) - 30*(a*b - b^2)*
sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/((a - b + b/cos(f*x + e)^2)*cos
(f*x + e)^2 - b) + 45*(a*b - b^2)*log((sqrt(a - b + b/cos(f*x + e)^2)*cos(f
*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b)))
/sqrt(b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(e+f\,x\right)^5\,{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.105 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=186

$$\frac{b(3a-5b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f(a-b)}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{3f(a-b)}-\frac{(3a-5b)\cos(e+fx}{3}$$

```
[Out] -1/3*(3*a-5*b)*cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(3/2)/(a-b)/f+1/3*cos(f*x+e)
^3*(a-b+b*sec(f*x+e)^2)^(5/2)/(a-b)/f+1/2*(3*a-5*b)*arctanh(sec(f*x+e)*b^(1
/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*(3*a-5*b)*b*sec(f*x+e)*(a-b+b
*sec(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A] time = 0.16, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3664, 453, 277, 195, 217, 206}

$$\frac{b(3a-5b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f(a-b)}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{3f(a-b)}-\frac{(3a-5b)\cos(e+fx}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((3*a - 5*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e +
f*x]^2]])/(2*f) + ((3*a - 5*b)*b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2
])/(2*(a - b)*f) - ((3*a - 5*b)*Cos[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/
2))/(3*(a - b)*f) + (Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(5/2))/(3*(a
 - b)*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a-b+bx^2)^{3/2}}{x^4}dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{5/2}}{3(a-b)f} + \frac{(3a-5b)\,\text{Subst}\left(\int \frac{(a-b+b}{x^2}\right.}{3(a-}$$

$$= -\frac{(3a-5b)\cos(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{3(a-b)f} + \frac{\cos^3(e+fx)\left(a\right.}{}$$

$$= \frac{(3a-5b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f} - \frac{(3a-5b)\cos(e+f}{}$$

$$= \frac{(3a-5b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f} - \frac{(3a-5b)\cos(e+f}{}$$

$$= \frac{(3a-5b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-5b)b\sec(e+fx)\sqrt{a}}{2(a-b)}$$

Mathematica [**A**] time = 1.89, size = 188, normalized size = 1.01

$$\frac{\sec(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\sqrt{(a-b)\cos(2(e+fx))+a+b}(-8(a-3b)\cos(2(e+\right.}{24\sqrt{2}f\sqrt{(a-b)\cos(2(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((12*Sqrt[2]*(3*a - 5*b)*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2*(e + f*
x)]]/(Sqrt[2]*Sqrt[b])]*Cos[e + f*x]^2 + Sqrt[a + b + (a - b)*Cos[2*(e + f*
x)]]*(-9*a + 37*b - 8*(a - 3*b)*Cos[2*(e + f*x)] + (a - b)*Cos[4*(e + f*x)]
))*Sec[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(2
4*Sqrt[2]*f*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
```

fricas [A] time = 0.82, size = 307, normalized size = 1.65

$$\left[-\frac{3\,(3\,a-5\,b)\sqrt{b}\,\cos\left(fx+e\right)\log\left(-\frac{(a-b)\cos\left(fx+e\right)^{2}-2\,\sqrt{b}\,\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\,\cos\left(fx+e\right)+2\,b}{\cos\left(fx+e\right)^{2}}\right)-2\left(2\,(a-b)\cos\left(fx+e\right.\right.}{12\,f\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(3*a - 5*b)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(f*x + e)^2 - 2
*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2
*b)/cos(f*x + e)^2) - 2*(2*(a - b)*cos(f*x + e)^4 - 2*(3*a - 7*b)*cos(f*x +
 e)^2 + 3*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x
+ e)), -1/6*(3*(3*a - 5*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) - (2*(a - b)*cos(f*
x + e)^4 - 2*(3*a - 7*b)*cos(f*x + e)^2 + 3*b)*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)abs(f)*(-1/2*(b^2*sqrt(a*f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+exp(
1))/f)^2+b)*sign(cos(f*x+exp(1)))*sign(f)-a*b*sqrt(a*f^2*(-cos(f*x+exp(1))/
f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)*sign(cos(f*x+exp(1)))*sign(f))/f^2/(a*
f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2)+(1/3*f^4*sqrt(a*f^
2*(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)*(a*f^2*(-cos(f*x+e
xp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)*sign(cos(f*x+exp(1)))*sign(f)-a
*f^4*sqrt(a*f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)*sign
(cos(f*x+exp(1)))*sign(f)+2*b*f^4*sqrt(a*f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*(
-cos(f*x+exp(1))/f)^2+b)*sign(cos(f*x+exp(1)))*sign(f))/f^6-1/2*(-5*b^2*sig
n(cos(f*x+exp(1)))*sign(f)+3*a*b*sign(cos(f*x+exp(1)))*sign(f))*atan(sqrt(a
*f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)/sqrt(-b))/sqrt(
-b)/f^2)
```

maple [B] time = 0.80, size = 1104, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/12/f*(-1+cos(f*x+e))^3*(2*cos(f*x+e)^5*a^(7/2)*((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)-2*cos(f*x+e)^5*a^(5/2)*((a*cos(f*
```

```
x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)+2*cos(f*x+e)^4*a^(
7/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)-2*c
os(f*x+e)^4*a^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1
/2)*b^(3/2)-6*cos(f*x+e)^3*a^(7/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+co
s(f*x+e))^2)^(1/2)*b^(1/2)+14*cos(f*x+e)^3*a^(5/2)*((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)+9*cos(f*x+e)^2*a^(7/2)*arctanh(1
/8*(-1+cos(f*x+e))*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2
/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2)
)*b-6*cos(f*x+e)^2*a^(7/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e)
)^2)^(1/2)*b^(1/2)+14*cos(f*x+e)^2*a^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)-6*cos(f*x+e)^2*ln(-2*(-1+cos(f*x+e))*(a^
(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)
+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f
*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*a+6*cos(f*x+e)^2*ln(-4*
(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+c
os(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1
/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*a-15
*cos(f*x+e)^2*a^(5/2)*arctanh(1/8*(-1+cos(f*x+e))*(4^(1/2)*cos(f*x+e)-4^(1/
2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f
*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*b^2+3*cos(f*x+e)*a^(5/2)*((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)+3*a^(5/2)*((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2))*cos(f*x+e)*4^(1/2)*((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)/sin(f*x+e)^6/a^(5/2)/b^(1/2)
```

maxima [A] time = 0.98, size = 296, normalized size = 1.59

$$\frac{4\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3-12\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\,(a-b)\cos\left(fx+e\right)+12\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\,b\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 12*sqrt(a - b + b
/cos(f*x + e)^2)*(a - b)*cos(f*x + e) + 12*sqrt(a - b + b/cos(f*x + e)^2)*b
*cos(f*x + e) + 6*b^(3/2)*log((sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)
- sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) + 6*(a*
b - b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/((a - b + b/cos(f*x +
e)^2)*cos(f*x + e)^2 - b) - 9*(a*b - b^2)*log((sqrt(a - b + b/cos(f*x + e)^
2)*cos(f*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + s
qrt(b)))/sqrt(b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^3\,{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.106 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=113

$$\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f}-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{f}+\frac{3\sqrt{b}\,(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+f}{\sqrt{a+b\sec^2(e+f}}\right.}{2f}$$

[Out] -cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(3/2)/f+3/2*(a-b)*arctanh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+3/2*b*sec(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f

Rubi [A] time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3664, 277, 195, 217, 206}

$$\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f}-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{f}+\frac{3\sqrt{b}\,(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+f}{\sqrt{a+b\sec^2(e+f}}\right.}{2f}$$

Antiderivative was successfully verified.

[In] Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]

[Out] (3*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/(2*f) + (3*b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(2*f) - (Cos[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/2))/f

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
```

```
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{x^2}dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{f} + \frac{(3b)\,\text{Subst}\left(\int \sqrt{a-b+\right.}$$

$$= \frac{3b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} - \frac{\cos(e+fx)\left(a-b+b\,\text{se}\right.}{f}$$

$$= \frac{3b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} - \frac{\cos(e+fx)\left(a-b+b\,\text{se}\right.}{f}$$

$$= \frac{3(a-b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{3b\sec(e+fx)\sqrt{a-b+b}}{2f}$$

Mathematica [A] time = 1.33, size = 170, normalized size = 1.50

$$\frac{\sec(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(6\sqrt{2}\sqrt{b}(a-b)\cos^2(e+fx)\tanh^{-1}\left(\frac{\sqrt{(a-b)\cos(2(e+f}}{\sqrt{2}\sqrt{b}}\right.\right.}{4\sqrt{2}f\sqrt{(a-b)\cos(2(e+fx))+a+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((6*Sqrt[2]*(a - b)*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]/
(Sqrt[2]*Sqrt[b])]*Cos[e + f*x]^2 - 2*(a - 2*b + (a - b)*Cos[2*(e + f*x)])*
Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])*Sec[e + f*x]*Sqrt[(a + b + (a - b)*
Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(4*Sqrt[2]*f*Sqrt[a + b + (a - b)*Cos[2*
(e + f*x)]])
```

fricas [A] time = 1.00, size = 268, normalized size = 2.37

$$\left[-\frac{3(a-b)\sqrt{b}\cos(fx+e)\log\left(-\frac{(a-b)\cos(fx+e)^2-2\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)+2\left(2(a-b)\cos(fx+e)^2-\right.}{4f\cos(fx+e)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(a - b)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(f*x + e)^2 - 2*sqrt
(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/c
os(f*x + e)^2) + 2*(2*(a - b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e
```

```
)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/2*(3*(a - b)*sqrt(-b)*arctan
(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)
*cos(f*x + e) + (2*(a - b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)abs(f)*((-a*sqrt(a*f^2*(-cos(f*x+exp(1))/
f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)*sign(cos(f*x+exp(1)))*sign(f)+b*sqrt(a
*f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)*sign(cos(f*x+ex
p(1)))*sign(f))/f^2-1/2*(b^2*sqrt(a*f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(
f*x+exp(1))/f)^2+b)*sign(cos(f*x+exp(1)))*sign(f)-a*b*sqrt(a*f^2*(-cos(f*x+
exp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)*sign(cos(f*x+exp(1)))*sign(f))
/f^2/(a*f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2)-1/2*(-3*b^
2*sign(cos(f*x+exp(1)))*sign(f)+3*a*b*sign(cos(f*x+exp(1)))*sign(f))*atan(s
qrt(a*f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*(-cos(f*x+exp(1))/f)^2+b)/sqrt(-b))/
sqrt(-b)/f^2)
```

maple [B] time = 0.32, size = 357, normalized size = 3.16

$$\frac{\left(\frac{a\left(\cos^{2}\left(fx+e\right)\right)-\left(\cos^{2}\left(fx+e\right)\right)b+b}{\cos\left(fx+e\right)^{2}}\right)^{\frac{3}{2}}\cos\left(fx+e\right)\left(-\left(a\left(\cos^{2}\left(fx+e\right)\right)-\left(\cos^{2}\left(fx+e\right)\right)b+b\right)^{\frac{3}{2}}\left(\cos^{2}\left(fx+e\right)\right)a+\left(a\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)*(-(
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(3/2)*cos(f*x+e)^2*a+(a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)^(3/2)*cos(f*x+e)^2*b+3*b^(3/2)*ln(2*(b^(1/2)*(a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)^(1/2)+b)/cos(f*x+e))*cos(f*x+e)^2*a-3*b^(5/2)*ln(2*(b^(1/2
)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2)+b)/cos(f*x+e))*cos(f*x+e)^2+(a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)^(5/2)-3*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2
)*cos(f*x+e)^2*a*b+3*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2)*cos(f*x+e)^2*b
^2)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(3/2)/b
```

maxima [A] time = 0.85, size = 176, normalized size = 1.56

$$-\frac{4\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^{2}}}\,(a-b)\cos\left(fx+e\right)-\frac{2\left(ab-b^{2}\right)\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{\left(a-b+\frac{b}{\cos\left(fx+e\right)^{2}}\right)\cos\left(fx+e\right)^{2}-b}+\frac{3\left(ab-b^{2}\right)\log\left(\frac{\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)-\sqrt{b}}{\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+\sqrt{b}}\right.}{\sqrt{b}}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(a - b + b/cos(f*x + e)^2)*(a - b)*cos(f*x + e) - 2*(a*b - b^2)
*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/((a - b + b/cos(f*x + e)^2)*co
s(f*x + e)^2 - b) + 3*(a*b - b^2)*log((sqrt(a - b + b/cos(f*x + e)^2)*cos(f
*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b)))
/sqrt(b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)\,\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*sin(e + f*x), x)
```

3.107 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}+\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f}+\frac{\sqrt{b}\,(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}$$

```
[Out] -a^(3/2)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/f+1/2*(3*a-
b)*arctanh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*b*s
ec(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3664, 416, 523, 217, 206, 377, 207}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}+\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f}+\frac{\sqrt{b}\,(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/
f) + ((3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/(2*f) + (b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{-1+x^2}dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{2a^2-3ab+b^2+(3a-b)\ldots}{\left(-1+x^2\right)\sqrt{a-b+bx^2}}\ldots\right)}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)\sqrt{a-b+b\ldots}}\ldots\right)}{f}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{-1+ax^2}dx, x, \ldots\right)}{f}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} + \frac{(3a-b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+\ldots)}{\sqrt{a-b+b\sec^2(\ldots)}}\right)}{2f}$$

Mathematica [B] time = 5.19, size = 418, normalized size = 3.29

$$\frac{\sec^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-4a^{3/2}\cos^2(e+fx)\tanh^{-1}\left(\frac{\ldots}{\sqrt{a}\sqrt{\ldots}}\right)\ldots\right)}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sec[(e + f*x)/2]^2*(-4*Sqrt[b]*(-3*a + b)*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f
*x)/2]^2))/Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]]*Co
s[e + f*x]^2 - 4*a^(3/2)*ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a
]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]*Cos[e + f*
x]^2 - 4*a^(3/2)*ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[
4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]*Cos[e + f*x]^2 +
Sqrt[2]*b*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4] + Sqr
t[2]*b*Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2
]^4])*Sec[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
/(8*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])
```

fricas [A] time = 1.52, size = 747, normalized size = 5.88

$$\left[\frac{2\,a^{\frac{3}{2}}\cos\left(fx+e\right)\log\left(-\frac{2\left((a-b)\cos\left(fx+e\right)^{2}-2\,\sqrt{a}\,\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\,\cos\left(fx+e\right)+a+b\right)}{\cos\left(fx+e\right)^{2}-1}\right)-(3\,a-b)\sqrt{b}\,\cos\left(fx+e\right)\log\left(-\frac{(a-b}{\cdots}\right.}{4\,f\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a^(3/2)*cos(f*x + e)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqr
t(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f
*x + e)^2 - 1)) - (3*a - b)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(f*x + e)
^2 - 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e) + 2*b)/cos(f*x + e)^2) + 2*b*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(f*cos(f*x + e)), -1/2*((3*a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a
 - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) - a^
(3/2)*cos(f*x + e)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2
- 1)) - b*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e
)), 1/4*(4*sqrt(-a)*a*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)/a)*cos(f*x + e) - (3*a - b)*sqrt(b)*cos(f*x + e)*
log(-((a - b)*cos(f*x + e)^2 - 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*b*sqrt(((a - b)*cos
(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), 1/2*(2*sqrt(-a)*a*arcta
n(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a
)*cos(f*x + e) - (3*a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) + b*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
```

```
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^2-1 were
not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
```

```
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Warning, integration of abs or sign assumes constant
 sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-
1)]Evaluation time: 2.04Error: Bad Argument Type
```

maple [B] time = 0.97, size = 1249, normalized size = 9.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/4/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*4^(1/2)*cos(f
*x+e)*(-1+cos(f*x+e))^3*(4*cos(f*x+e)^2*ln(-4*(-1+cos(f*x+e))*(a^(1/2)*cos(
f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*co
s(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*a-4*cos(f*x+e)^2*ln(-2*(-1+cos(f*
x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))
^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2
)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*a+3*cos(f*x+e)
^2*a^(7/2)*arctanh(1/8*(-1+cos(f*x+e))*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*
x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(
1/2)*b^(1/2)*4^(1/2))*b+cos(f*x+e)*a^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)-6*cos(f*x+e)^2*ln(-4*(-1+cos(f*x+e))*(a^
(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)
+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f
*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(5/2)*a^2+6*cos(f*x+e)^2*ln(-
2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1
+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^
(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(5/2)*a^
2-cos(f*x+e)^2*a^(5/2)*arctanh(1/8*(-1+cos(f*x+e))*(4^(1/2)*cos(f*x+e)-4^(1
/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(
f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*b^2+a^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)-cos(f*x+e)^2*ln(-2*(-1+cos(f*x+e))*(a
^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2
)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(
f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(1/2)*a^4-cos(f*x+e)^2*ln(-4
*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(
1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*c
os(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*b^(1/2)*a^4)/sin(f*x+e)^6/((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)/a^(5/2)/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x),x)
[Out] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*csc(e + f*x), x)
```

3.108 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}-\frac{\sqrt{a}\,(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

```
[Out] -1/2*cot(f*x+e)*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(3/2)/f-1/2*(a+3*b)*arctanh
(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*a^(1/2)/f+1/2*(3*a+b)*arcta
nh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+b*sec(f*x+e)*(a
-b+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.21, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3664, 467, 528, 523, 217, 206, 377, 207}

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}-\frac{\sqrt{a}\,(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[a]*(a + 3*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f
*x]^2]])/(2*f) + (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a -
 b + b*Sec[e + f*x]^2]])/(2*f) + (b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x
]^2])/f - (Cot[e + f*x]*Csc[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/2))/(2*f
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
```

```
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a-b+bx^2\right)^{3/2}}{\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{2f} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(\cdots\right.}{2\cdots}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(\cdots\right.}{2\cdots}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(\cdots\right.}{2\cdots}$$

$$= -\frac{\sqrt{a}\,(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\cdots\right.}{2f}$$

Mathematica [B] time = 6.77, size = 1022, normalized size = 6.12

$$\frac{\sqrt{\frac{\cos(2(e+fx))a+a+b-b\cos(2(e+fx))}{\cos(2(e+fx))+1}}\left(\frac{1}{2}b\sec(e+fx)-\frac{1}{2}a\cot(e+fx)\csc(e+fx)\right)}{f}+\frac{(a^2-b^2)\left(2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\left(\tan^2\left(\frac{1}{2}(e+f\right.\right.}{\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(\tan\right.}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x
)])]*(-1/2*(a*Cot[e + f*x]*Csc[e + f*x]) + (b*Sec[e + f*x])/2))/f + (-1/4*(
(a^2 + 6*a*b + b^2)*(2*Sqrt[a]*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))/S
qrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] - Sqrt[b]*(Arc
Tanh[(a - a*Tan[(e + f*x)/2]^2 + 2*b*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*
Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-
1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan
[(e + f*x)/2]^2)^2])]))*(1 + Cos[e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 +
 Cos[e + f*x])^2]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f
*x)])]*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(e
+ f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])/(
Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(e +
f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])
 + ((a^2 - b^2)*(2*Sqrt[a]*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))/Sqrt[
4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] + Sqrt[b]*(ArcTanh
[(a - a*Tan[(e + f*x)/2]^2 + 2*b*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[
(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 +
Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e
+ f*x)/2]^2)^2])]))*(1 + Cos[e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos
[e + f*x])^2]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)]
)]*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(e + f*
x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])/(4*Sq
rt[a]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(e + f*
x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]))/
(2*f)
```

fricas [A] time = 1.97, size = 994, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos(f*x + e))*sqrt(a)*log(-2*((
a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) + ((3*a + b)*cos(f*x +
 e)^3 - (3*a + b)*cos(f*x + e))*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sq
rt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)
/cos(f*x + e)^2) + 2*((a + b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), -1/4*(2*((3*
a + b)*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt(-b)*arctan(sqrt(-b)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) - ((a + 3*b
)*cos(f*x + e)^3 - (a + 3*b)*cos(f*x + e))*sqrt(a)*log(-2*((a - b)*cos(f*x
+ e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e) + a + b)/(cos(f*x + e)^2 - 1)) - 2*((a + b)*cos(f*x + e)^2 - b)*sqrt
(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*
x + e)), 1/4*(2*((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos(f*x + e))*sqrt(-a
```

```
)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e)/a) + ((3*a + b)*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt(b)*log(
-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*((a + b)*cos(f*x + e)^2
 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3
- f*cos(f*x + e)), 1/2*(((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos(f*x + e))
*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*cos(f*x + e)/a) - ((3*a + b)*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt
(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)/b) + ((a + b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b
)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
```

to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^2-1 were
not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of abs
or sign assumes constant sign by intervals (correct if the argument is real
):Check [abs(t_nostep^2-1)]Evaluation time: 2.5Unable to divide, perhaps du
e to rounding error%%%{%%{[-16384,0]:[1,0,%%%{-1,[1]%%%}]%%},[6,9]%%%}+%%%{
%%{[%%%{81920,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,8]%%%}+%%%{%%{[%%%{-163
840,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,7]%%%}+%%%{%%{[%%%{163840,[3]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[6,6]%%%}+%%%{%%{[%%%{-81920,[4]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[6,5]%%%}+%%%{%%{[%%%{16384,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[6,4]%%%}+%%%{%%%{-98304,[1]%%%},[5,9]%%%}+%%%{%%%{491520,[2]%%%},[5,8
]%%%}+%%%{%%%{-983040,[3]%%%},[5,7]%%%}+%%%{%%%{983040,[4]%%%},[5,6]%%%}+%%
%{%%%{-491520,[5]%%%},[5,5]%%%}+%%%{%%%{98304,[6]%%%},[5,4]%%%}+%%%{%%{[196
608,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,10]%%%}+%%%{%%{[%%%{-1228800,[1]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[4,9]%%%}+%%%{%%{[%%%{3194880,[2]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[4,8]%%%}+%%%{%%{[%%%{-4423680,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]
%%},[4,7]%%%}+%%%{%%{[%%%{3440640,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,6]%

%%}+%%%{%%{[%%%{-1425408,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,5]%%%}+%%%{%
%{[%%%{245760,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,4]%%%}+%%%{%%%{786432,[
1]%%%},[3,10]%%%}+%%%{%%%{-4259840,[2]%%%},[3,9]%%%}+%%%{%%%{9502720,[3]%%%
},[3,8]%%%}+%%%{%%%{-11141120,[4]%%%},[3,7]%%%}+%%%{%%%{7208960,[5]%%%},[3,
6]%%%}+%%%{%%%{-2424832,[6]%%%},[3,5]%%%}+%%%{%%%{327680,[7]%%%},[3,4]%%%}+
%%%{%%{[-786432,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,11]%%%}+%%%{%%{[%%%{5111808,[
1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,10]%%%}+%%%{%%{[%%%{-14008320,[2]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[2,9]%%%}+%%%{%%{[%%%{20889600,[3]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[2,8]%%%}+%%%{%%{[%%%{-18186240,[4]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[2,7]%%%}+%%%{%%{[%%%{9142272,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[2,6]%%%}+%%%{%%{[%%%{-2408448,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,5]%%%}
+%%%{%%{[%%%{245760,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,4]%%%}+%%%{%%%{-1
572864,[1]%%%},[1,11]%%%}+%%%{%%%{8650752,[2]%%%},[1,10]%%%}+%%%{%%%{-19759
104,[3]%%%},[1,9]%%%}+%%%{%%%{24084480,[4]%%%},[1,8]%%%}+%%%{%%%{-16711680,
[5]%%%},[1,7]%%%}+%%%{%%%{6488064,[6]%%%},[1,6]%%%}+%%%{%%%{-1277952,[7]%%%
},[1,5]%%%}+%%%{%%%{98304,[8]%%%},[1,4]%%%}+%%%{%%{[1048576,0]:[1,0,%%%{-1,
[1]%%%}]%%},[0,12]%%%}+%%%{%%{[%%%{-6029312,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]
%%},[0,11]%%%}+%%%{%%{[%%%{14614528,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,1
0]%%%}+%%%{%%{[%%%{-19349504,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,9]%%%}+%
%%{%%{[%%%{15155200,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,8]%%%}+%%%{%%{[%%
%{-7110656,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,7]%%%}+%%%{%%{[%%%{1933312
,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,6]%%%}+%%%{%%{[%%%{-278528,[7]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[0,5]%%%}+%%%{%%{[%%%{16384,[8]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[0,4]%%%} / %%%{%%{poly1[%%%{-1,[1]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[6,0]%%%}+%%%{%%%{-6,[2]%%%},[5,0]%%%}+%%%{%%{[%%%{12,[1]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[4,1]%%%}+%%%{%%{poly1[%%%{-15,[2]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[4,0]%%%}+%%%{%%%{48,[2]%%%},[3,1]%%%}+%%%{%%%{-20,[3]%%%},[3,0]
%%%}+%%%{%%{[%%%{-48,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2]%%%}+%%%{%%{[%
%%{72,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,1]%%%}+%%%{%%{poly1[%%%{-15,[3]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0]%%%}+%%%{%%%{-96,[2]%%%},[1,2]%%%}+%%%
{%%%{48,[3]%%%},[1,1]%%%}+%%%{%%%{-6,[4]%%%},[1,0]%%%}+%%%{%%{[%%%{64,[1]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,3]%%%}+%%%{%%{poly1[%%%{-48,[2]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[0,2]%%%}+%%%{%%{[%%%{12,[3]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[0,1]%%%}+%%%{%%{poly1[%%%{-1,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0
]%%%} Error: Bad Argument Value

maple [B] time = 0.99, size = 2904, normalized size = 17.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/8/f*(-1+cos(f*x+e))^2*(6*cos(f*x+e)^3*a^(7/2)*arctanh(1/8*(-1+cos(f*x+e)
)*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*b+10*cos(f*x+e)
^3*ln(-4*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+
e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(
7/2)*a-10*cos(f*x+e)^3*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f
*x+e)^2/a^(1/2))*b^(7/2)*a+2*cos(f*x+e)^3*a^(5/2)*arctanh(1/8*(-1+cos(f*x+e
))*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*b^2-18*cos(f*x
+e)^3*ln(-4*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f
*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*
b^(5/2)*a^2+18*cos(f*x+e)^3*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(
```

```
f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/
sin(f*x+e)^2/a^(1/2))*b^(5/2)*a^2-6*cos(f*x+e)^2*a^(7/2)*arctanh(1/8*(-1+co
s(f*x+e))*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*b+2*cos
(f*x+e)^2*a^(7/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2
)*b^(1/2)+2*cos(f*x+e)^2*a^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(
f*x+e))^2)^(1/2)*b^(3/2)-10*cos(f*x+e)^2*ln(-4*(-1+cos(f*x+e))*(a^(1/2)*cos
(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*c
os(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*a+10*cos(f*x+e)^2*ln(-2*(-1+cos(
f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e
))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1
/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*a-3*cos(f*x+
e)^3*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*
x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b
^(3/2)*a^3-3*cos(f*x+e)^3*ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+c
os(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*b
^(3/2)*a^3-2*cos(f*x+e)^2*a^(5/2)*arctanh(1/8*(-1+cos(f*x+e))*(4^(1/2)*cos(
f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*4^(1/2))*b^2+18*cos(f*x+e)^2*ln(-4*(-1+
cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f
*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*
a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(5/2)*a^2-18*c
os(f*x+e)^2*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1
+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(
1/2))*b^(5/2)*a^2-cos(f*x+e)^3*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+
b)/sin(f*x+e)^2/a^(1/2))*b^(1/2)*a^4-cos(f*x+e)^3*ln(-4*(a^(1/2)*cos(f*x+e)
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+
e)+b)/(-1+cos(f*x+e)))*b^(1/2)*a^4+3*cos(f*x+e)^2*ln(-2*(-1+cos(f*x+e))*(a^
(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)
+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f
*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(3/2)*a^3+3*cos(f*x+e)^2*ln(-
4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^
(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*
cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*b^(3/2)*a^3-2*a^(5/2)*((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)+cos(f*x+e)^2*ln(-
2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1
+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^
(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*b^(1/2)*a^
4+cos(f*x+e)^2*ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^
2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*b^(1/2)*a^4)
*cos(f*x+e)*4^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)/sin(f*x+e)^6/a^(
5/2)/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan\left(e+fx\right)^2+a\right)^{3/2}}{\sin\left(e+fx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^3,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.109 $\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=223

$$-\frac{3\left(a^2+6ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8\sqrt{a}f}+\frac{3(a+3b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{8f}-\frac{3(a+b)\csc^2(e+f}{}$$

```
[Out] -1/4*cot(f*x+e)*csc(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(3/2)/f-3/8*(a^2+6*a*b+b^
2)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/f/a^(1/2)+3/2*(a+
b)*arctanh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+3/8*(a+
3*b)*sec(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f-3/8*(a+b)*csc(f*x+e)^2*sec(f*x
+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.36, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3664, 467, 577, 582, 523, 217, 206, 377, 207}

$$-\frac{3\left(a^2+6ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8\sqrt{a}f}+\frac{3(a+3b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{8f}-\frac{3(a+b)\csc^2(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a^2 + 6*a*b + b^2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/(8*Sqrt[a]*f) + (3*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Sec[e + f*
x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/(2*f) + (3*(a + 3*b)*Sec[e + f*x]*Sqrt
[a - b + b*Sec[e + f*x]^2])/(8*f) - (3*(a + b)*Csc[e + f*x]^2*Sec[e + f*x]*
Sqrt[a - b + b*Sec[e + f*x]^2])/(8*f) - (Cot[e + f*x]*Csc[e + f*x]^3*(a - b
 + b*Sec[e + f*x]^2)^(3/2))/(4*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{x^4(a-b+bx^2)^{3/2}}{(-1+x^2)^3}dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2 \cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{3(a+b)\csc^2(e+fx)\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8f} - \frac{\cot(e+\cdots}{\cdots}$$

$$= \frac{3(a+3b)\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8f} - \frac{3(a+b)\csc^2(e+fx\cdots}{\cdots}$$

$$= \frac{3(a+3b)\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8f} - \frac{3(a+b)\csc^2(e+fx\cdots}{\cdots}$$

$$= \frac{3(a+3b)\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8f} - \frac{3(a+b)\csc^2(e+fx\cdots}{\cdots}$$

$$= -\frac{3\left(a^2+6ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3\sqrt{b}(a+b)\tanh^{-1}\left(\cdots\right.}{2f\cdots}$$

Mathematica [A] time = 5.24, size = 409, normalized size = 1.83

$$\frac{\cos(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-\frac{3\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\cos^2(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}\left(\left(a^2+6ab+b^2\right)\left(\tanh^{-1}\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-2*C
sc[e + f*x]^2*(3*a + 5*b + 2*a*Csc[e + f*x]^2) + 8*b*Sec[e + f*x]^2 - (3*(-
8*Sqrt[a]*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))/Sqrt[4
*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] + (a^2 + 6*a*b + b^
2)*(ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f
*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[(e
 + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)
/2]^2)^2])]))*Sec[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]^2*Sec[(e + f*x)/2]^4])/(
Sqrt[a]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(
-1 + Tan[(e + f*x)/2]^2)^2])))/(16*Sqrt[2]*f)
```

fricas [A] time = 2.06, size = 1365, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^2 + 6*a*b + b^2)*cos(f*x + e)^5 - 2*(a^2 + 6*a*b + b^2)*cos(f*
x + e)^3 + (a^2 + 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*log(-2*((a - b)*cos(f*
x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) + 12*((a^2 + a*b)*cos(f*x + e)^5 -
2*(a^2 + a*b)*cos(f*x + e)^3 + (a^2 + a*b)*cos(f*x + e))*sqrt(b)*log(-((a -
 b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(3*(a^2 + 3*a*b)*cos(f*x + e)
^4 - (5*a^2 + 13*a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt(((a - b)*cos(f*x + e)^2
+ b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(
f*x + e)), 1/8*(3*((a^2 + 6*a*b + b^2)*cos(f*x + e)^5 - 2*(a^2 + 6*a*b + b^
2)*cos(f*x + e)^3 + (a^2 + 6*a*b + b^2)*cos(f*x + e))*sqrt(-a)*arctan(sqrt(
-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + 6*(
(a^2 + a*b)*cos(f*x + e)^5 - 2*(a^2 + a*b)*cos(f*x + e)^3 + (a^2 + a*b)*cos
(f*x + e))*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + (3
*(a^2 + 3*a*b)*cos(f*x + e)^4 - (5*a^2 + 13*a*b)*cos(f*x + e)^2 + 4*a*b)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^5 - 2*a*
f*cos(f*x + e)^3 + a*f*cos(f*x + e)), -1/16*(24*((a^2 + a*b)*cos(f*x + e)^5
 - 2*(a^2 + a*b)*cos(f*x + e)^3 + (a^2 + a*b)*cos(f*x + e))*sqrt(-b)*arctan
(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)
 - 3*((a^2 + 6*a*b + b^2)*cos(f*x + e)^5 - 2*(a^2 + 6*a*b + b^2)*cos(f*x +
e)^3 + (a^2 + 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*log(-2*((a - b)*cos(f*x +
e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) + a + b)/(cos(f*x + e)^2 - 1)) - 2*(3*(a^2 + 3*a*b)*cos(f*x + e)^4 - (
5*a^2 + 13*a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2))/(a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f*x +
e)), 1/8*(3*((a^2 + 6*a*b + b^2)*cos(f*x + e)^5 - 2*(a^2 + 6*a*b + b^2)*cos
(f*x + e)^3 + (a^2 + 6*a*b + b^2)*cos(f*x + e))*sqrt(-a)*arctan(sqrt(-a)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) - 12*((a^2
+ a*b)*cos(f*x + e)^5 - 2*(a^2 + a*b)*cos(f*x + e)^3 + (a^2 + a*b)*cos(f*x
+ e))*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e)/b) + (3*(a^2 + 3*a*b)*cos(f*x + e)^4 - (5*a^2 + 13*a*b)*
cos(f*x + e)^2 + 4*a*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
```

```
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^2-1 were
not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
```

```
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes c
onstant sign by intervals (correct if the argument is real):Check [abs(t_no
step^2-1)]Evaluation time: 3.02Unable to divide, perhaps due to rounding er
ror%%%{262144,[12,12]%%%}+%%%{%%%{-1572864,[1]%%%},[12,11]%%%}+%%%{%%%{3932
160,[2]%%%},[12,10]%%%}+%%%{%%%{-5242880,[3]%%%},[12,9]%%%}+%%%{%%%{3932160
,[4]%%%},[12,8]%%%}+%%%{%%%{-1572864,[5]%%%},[12,7]%%%}+%%%{%%%{262144,[6]%
%%},[12,6]%%%}+%%%{%%{[1572864,0]:[1,0,%%%{-1,[1]%%%}]%%},[11,12]%%%}+%%%{%
%{[%%%{-9437184,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,11]%%%}+%%%{%%{[%%%{
23592960,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,10]%%%}+%%%{%%{[%%%{-314572
80,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,9]%%%}+%%%{%%{[%%%{23592960,[4]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,8]%%%}+%%%{%%{[%%%{-9437184,[5]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[11,7]%%%}+%%%{%%{[%%%{1572864,[6]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[11,6]%%%}+%%%{-3145728,[10,13]%%%}+%%%{%%%{22020096,[1]%%%},[
10,12]%%%}+%%%{%%%{-66060288,[2]%%%},[10,11]%%%}+%%%{%%%{110100480,[3]%%%},
[10,10]%%%}+%%%{%%%{-110100480,[4]%%%},[10,9]%%%}+%%%{%%%{66060288,[5]%%%},
[10,8]%%%}+%%%{%%%{-22020096,[6]%%%},[10,7]%%%}+%%%{%%%{3145728,[7]%%%},[10
,6]%%%}+%%%{%%{[-12582912,0]:[1,0,%%%{-1,[1]%%%}]%%},[9,13]%%%}+%%%{%%{[%%%
{76021760,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9,12]%%%}+%%%{%%{[%%%{-191889
408,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9,11]%%%}+%%%{%%{[%%%{259522560,[3]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9,10]%%%}+%%%{%%{[%%%{-199229440,[4]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[9,9]%%%}+%%%{%%{[%%%{83361792,[5]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[9,8]%%%}+%%%{%%{[%%%{-15728640,[6]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[9,7]%%%}+%%%{%%{[%%%{524288,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9
,6]%%%}+%%%{12582912,[8,14]%%%}+%%%{%%%{-84934656,[1]%%%},[8,13]%%%}+%%%{%%
%{238288896,[2]%%%},[8,12]%%%}+%%%{%%%{-350748672,[3]%%%},[8,11]%%%}+%%%{%%
%{271319040,[4]%%%},[8,10]%%%}+%%%{%%%{-75497472,[5]%%%},[8,9]%%%}+%%%{%%%{
-36962304,[6]%%%},[8,8]%%%}+%%%{%%%{33030144,[7]%%%},[8,7]%%%}+%%%{%%%{-707
7888,[8]%%%},[8,6]%%%}+%%%{%%{[25165824,0]:[1,0,%%%{-1,[1]%%%}]%%},[7,14]%%
%}+%%%{%%{[%%%{-125829120,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,13]%%%}+%%%
{%%{[%%%{217055232,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,12]%%%}+%%%{%%{[%%
%{-69206016,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,11]%%%}+%%%{%%{[%%%{-2673
86880,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,10]%%%}+%%%{%%{[%%%{415236096,[
5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,9]%%%}+%%%{%%{[%%%{-267386880,[6]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[7,8]%%%}+%%%{%%{[%%%{81788928,[7]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[7,7]%%%}+%%%{%%{[%%%{-9437184,[8]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[7,6]%%%}+%%%{-16777216,[6,15]%%%}+%%%{%%%{75497472,[1]%%%},[6,14]
%%%}+%%%{%%%{-56623104,[2]%%%},[6,13]%%%}+%%%{%%%{-306184192,[3]%%%},[6,12]
%%%}+%%%{%%%{912261120,[4]%%%},[6,11]%%%}+%%%{%%%{-1157627904,[5]%%%},[6,10
]%%%}+%%%{%%%{794820608,[6]%%%},[6,9]%%%}+%%%{%%%{-289406976,[7]%%%},[6,8]%
%%}+%%%{%%%{44040192,[8]%%%},[6,7]%%%}+%%%{%%{[%%%{-75497472,[1]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[5,14]%%%}+%%%{%%{[%%%{452984832,[2]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[5,13]%%%}+%%%{%%{[%%%{-1123024896,[3]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[5,12]%%%}+%%%{%%{[%%%{1453326336,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[5,11]%%%}+%%%{%%{[%%%{-990904320,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,
10]%%%}+%%%{%%{[%%%{264241152,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,9]%%%}+
%%%{%%{[%%%{66060288,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,8]%%%}+%%%{%%{[%
%%{-56623104,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,7]%%%}+%%%{%%{[%%%{94371
84,[9]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,6]%%%}+%%%{%%%{50331648,[1]%%%},[4
```

```
,15]%%%}+%%%{%%%{-301989888,[2]%%%},[4,14]%%%}+%%%{%%%{710934528,[3]%%%},[4
,13]%%%}+%%%{%%%{-735313920,[4]%%%},[4,12]%%%}+%%%{%%%{51904512,[5]%%%},[4,
11]%%%}+%%%{%%%{684982272,[6]%%%},[4,10]%%%}+%%%{%%%{-751828992,[7]%%%},[4,
9]%%%}+%%%{%%%{370409472,[8]%%%},[4,8]%%%}+%%%{%%%{-86507520,[9]%%%},[4,7]%
%%}+%%%{%%%{7077888,[10]%%%},[4,6]%%%}+%%%{%%{[%%%{75497472,[2]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[3,14]%%%}+%%%{%%{[%%%{-478150656,[3]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[3,13]%%%}+%%%{%%{[%%%{1282932736,[4]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[3,12]%%%}+%%%{%%{[%%%{-1884291072,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[3,11]%%%}+%%%{%%{[%%%{1627914240,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,
10]%%%}+%%%{%%{[%%%{-819986432,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,9]%%%}
+%%%{%%{[%%%{218628096,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,8]%%%}+%%%{%%{
[%%%{-22020096,[9]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,7]%%%}+%%%{%%{[%%%{-52
4288,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,6]%%%}+%%%{%%%{-50331648,[2]%%%
},[2,15]%%%}+%%%{%%%{327155712,[3]%%%},[2,14]%%%}+%%%{%%%{-896532480,[4]%%%
},[2,13]%%%}+%%%{%%%{1324351488,[5]%%%},[2,12]%%%}+%%%{%%%{-1097859072,[6]%
%%},[2,11]%%%}+%%%{%%%{443547648,[7]%%%},[2,10]%%%}+%%%{%%%{3145728,[8]%%%}
,[2,9]%%%}+%%%{%%%{-78643200,[9]%%%},[2,8]%%%}+%%%{%%%{28311552,[10]%%%},[2
,7]%%%}+%%%{%%%{-3145728,[11]%%%},[2,6]%%%}+%%%{%%{[%%%{-25165824,[3]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[1,14]%%%}+%%%{%%{[%%%{163577856,[4]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[1,13]%%%}+%%%{%%{[%%%{-454557696,[5]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[1,12]%%%}+%%%{%%{[%%%{701497344,[6]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[1,11]%%%}+%%%{%%{[%%%{-652738560,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[1,10]%%%}+%%%{%%{[%%%{371195904,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,9]%%
%}+%%%{%%{[%%%{-124256256,[9]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,8]%%%}+%%%{
%%{[%%%{22020096,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,7]%%%}+%%%{%%{[%%%{
-1572864,[11]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,6]%%%}+%%%{%%%{16777216,[3]
%%%},[0,15]%%%}+%%%{%%%{-113246208,[4]%%%},[0,14]%%%}+%%%{%%%{330301440,[5]
%%%},[0,13]%%%}+%%%{%%%{-543424512,[6]%%%},[0,12]%%%}+%%%{%%%{552075264,[7]
%%%},[0,11]%%%}+%%%{%%%{-356253696,[8]%%%},[0,10]%%%}+%%%{%%%{144703488,[9]
%%%},[0,9]%%%}+%%%{%%%{-35389440,[10]%%%},[0,8]%%%}+%%%{%%%{4718592,[11]%%%
},[0,7]%%%}+%%%{%%%{-262144,[12]%%%},[0,6]%%%} / %%%{%%{poly1[%%%{-1,[1]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[12,0]%%%}+%%%{%%%{-6,[2]%%%},[11,0]%%%}+%%%{%
%{poly1[%%%{12,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,1]%%%}+%%%{%%{poly1[%
%%{-12,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,0]%%%}+%%%{%%%{48,[2]%%%},[9,
1]%%%}+%%%{%%%{-2,[3]%%%},[9,0]%%%}+%%%{%%{poly1[%%%{-48,[1]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[8,2]%%%}+%%%{%%{poly1[%%%{36,[2]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[8,1]%%%}+%%%{%%{poly1[%%%{27,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[8,
0]%%%}+%%%{%%%{-96,[2]%%%},[7,2]%%%}+%%%{%%%{-96,[3]%%%},[7,1]%%%}+%%%{%%%{
36,[4]%%%},[7,0]%%%}+%%%{%%{poly1[%%%{64,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[6,3]%%%}+%%%{%%{poly1[%%%{96,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,2]%%%}
+%%%{%%{poly1[%%%{-168,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,1]%%%}+%%%{%%%
{288,[3]%%%},[5,2]%%%}+%%%{%%%{-36,[5]%%%},[5,0]%%%}+%%%{%%{poly1[%%%{-192,
[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3]%%%}+%%%{%%{poly1[%%%{168,[4]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[4,1]%%%}+%%%{%%{poly1[%%%{-27,[5]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[4,0]%%%}+%%%{%%%{-288,[4]%%%},[3,2]%%%}+%%%{%%%{96,[5]%%%
},[3,1]%%%}+%%%{%%%{2,[6]%%%},[3,0]%%%}+%%%{%%{poly1[%%%{192,[3]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[2,3]%%%}+%%%{%%{poly1[%%%{-96,[4]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[2,2]%%%}+%%%{%%{poly1[%%%{-36,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[2,1]%%%}+%%%{%%{poly1[%%%{12,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0]%%
%}+%%%{%%%{96,[5]%%%},[1,2]%%%}+%%%{%%%{-48,[6]%%%},[1,1]%%%}+%%%{%%%{6,[7]
%%%},[1,0]%%%}+%%%{%%{poly1[%%%{-64,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,3
]%%%}+%%%{%%{poly1[%%%{48,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2]%%%}+%%%{
%%{poly1[%%%{-12,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,1]%%%}+%%%{%%{poly1[
%%%{1,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0]%%%} Error: Bad Argument Valu
e
```

maple [B] time = 1.34, size = 6194, normalized size = 27.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^5,x)
[Out] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.110 $\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=222

$$\frac{3\left(a^2-8ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f\sqrt{a-b}}-\frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}+\frac{3(a-2b)\sin^2(e+fx)\tan}{}$$

```
[Out] 3/8*(a^2-8*a*b+8*b^2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2
))/f/(a-b)^(1/2)+3/2*(a-2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^
(1/2))*b^(1/2)/f-3/8*(a-4*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+3/8*(a-2
*b)*sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-1/4*cos(f*x+e)*sin(f
*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A] time = 0.32, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3663, 467, 577, 582, 523, 217, 206, 377, 203}

$$\frac{3\left(a^2-8ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f\sqrt{a-b}}-\frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}+\frac{3(a-2b)\sin^2(e+fx)\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a^2 - 8*a*b + 8*b^2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(8*Sqrt[a - b]*f) + (3*(a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[
e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2*f) - (3*(a - 4*b)*Tan[e + f*x]*Sq
rt[a + b*Tan[e + f*x]^2])/(8*f) + (3*(a - 2*b)*Sin[e + f*x]^2*Tan[e + f*x]*
Sqrt[a + b*Tan[e + f*x]^2])/(8*f) - (Cos[e + f*x]*Sin[e + f*x]^3*(a + b*Tan
[e + f*x]^2)^(3/2))/(4*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b}}{(}\right.}{}$$

$$= \frac{3(a-2b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} - \cos(e+fx$$

$$= -\frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{3(a-2b)\sin^2(e+fx)}{}$$

$$= -\frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{3(a-2b)\sin^2(e+fx)}{}$$

$$= -\frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{3(a-2b)\sin^2(e+fx)}{}$$

$$= \frac{3\left(a^2-8ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8\sqrt{a-b}\,f} + \frac{3(a-2b)\sqrt{b}\,\tanh^{-1}\left(\right.}{2f}$$

Mathematica [C] time = 5.01, size = 278, normalized size = 1.25

$$\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\frac{3a\sin(2(e+fx))\csc^2(e+fx)\left(\left(a^2-5ab+4b^2\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right.\right.\right.}{\sqrt{2}\,b(a-b)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos}{}}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*((3*a*Csc[e + f*x]
^2*((a^2 - 5*a*b + 4*b^2)*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e
+ f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - (a^2 - 8*a*b + 8*b^2)*EllipticPi
[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]/Sqrt[2]], 1])*Sin[2*(e + f*x)])/(Sqrt[2]*(a - b)*b*Sqrt[((a + b + (a
 - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]) + ((-8*a + 18*b)*Sin[2*(e + f*x
)] + (a - b)*Sin[4*(e + f*x)] + 16*b*Tan[e + f*x])/4))/(8*Sqrt[2]*f)
```

fricas [B] time = 120.24, size = 2163, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2 - 8*a*b + 8*b^2)*sqrt(-a + b)*cos(f*x + e)*log(128*(a^4 - 4*
a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*
```

```
a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b
^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256
*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos
(f*x + e)^2 + 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^
3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*
b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x
 + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e)) + 24*(a^2 - 3*a*b + 2*b^2)*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b
+ 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 - 4*((a - 2*b)*cos
(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 8*(2*(a^2 - 2*a*b +
b^2)*cos(f*x + e)^4 - 5*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + 4*a*b - 4*b^
2)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a - b)
*f*cos(f*x + e)), -1/64*(48*(a^2 - 3*a*b + 2*b^2)*sqrt(-b)*arctan(1/2*((a -
 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))
*cos(f*x + e) + 3*(a^2 - 8*a*b + 8*b^2)*sqrt(-a + b)*cos(f*x + e)*log(128*(
a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^
3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b +
77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b
^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*
b^4)*cos(f*x + e)^2 + 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7
- 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b
 + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)
*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e)) - 8*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 5*(a^2 - 3*a*b
 + 2*b^2)*cos(f*x + e)^2 + 4*a*b - 4*b^2)*sqrt(((a - b)*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/((a - b)*f*cos(f*x + e)), 1/32*(3*(a^2 - 8*a
*b + 8*b^2)*sqrt(a - b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 -
 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*x + e
))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 -
 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 -
 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - 1
2*(a^2 - 3*a*b + 2*b^2)*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b + 8*b^2)*cos
(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 - 4*((a - 2*b)*cos(f*x + e)^3
+ 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*(2*(a^2 - 2*a*b + b^2)*cos(f*x
 + e)^4 - 5*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + 4*a*b - 4*b^2)*sqrt(((a
- b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a - b)*f*cos(f*x +
 e)), 1/32*(3*(a^2 - 8*a*b + 8*b^2)*sqrt(a - b)*arctan(-1/4*(8*(a^2 - 2*a*b
 + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*
a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b +
3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x
 + e)))*cos(f*x + e) - 24*(a^2 - 3*a*b + 2*b^2)*sqrt(-b)*arctan(1/2*((a - 2
*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*c
os(f*x + e) + 4*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 5*(a^2 - 3*a*b + 2*
b^2)*cos(f*x + e)^2 + 4*a*b - 4*b^2)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/((a - b)*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan \left(fx+e\right)^2+a\right)^{\frac{3}{2}} \sin \left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

maple [C] time = 1.17, size = 2630, normalized size = 11.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/8/f*(-5*cos(f*x+e)^5*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^2+24*sin
(f*x+e)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2
)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)
-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*2^(1/2)*((I*cos
(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)
+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(
1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*cos(f*x+e
)^2*a*b+12*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a
-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)
*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-
I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*
(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*
cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*cos(f*x+e)^2*a*b-4*((2*I*(a-b)^(1/2)*
b^(1/2)+a-2*b)/a)^(1/2)*b^2-12*cos(f*x+e)^5*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b
)/a)^(1/2)*b^2-4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^7*a*b
+4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^6*a*b+17*cos(f*x+e)
^5*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b-17*cos(f*x+e)^4*((2*I*(a-b
)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b-cos(f*x+e)^3*((2*I*(a-b)^(1/2)*b^(1/2)+
a-2*b)/a)^(1/2)*a*b+cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*
a*b+5*cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^2+12*cos(f*x
+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2+6*cos(f*x+e)^3*((2*I*(a
-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2-6*cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1
/2)+a-2*b)/a)^(1/2)*b^2+4*cos(f*x+e)*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1
/2)*b^2+2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^7*a^2-2*((2*
I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^6*a^2+2*b^2*((2*I*(a-b)^(1
/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^7-2*b^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-
2*b)/a)^(1/2)*cos(f*x+e)^6-3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)
*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+
cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^
(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+co
s(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^
(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^2+6*
cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^
(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*c
os(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+
e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*
b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(
2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)
^(1/2))*a^2+48*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b
^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)
^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*
x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2
*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/
2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^
(1/2)+a-2*b)/a)^(1/2))*b^2-48*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e
)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1
+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b
^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+
cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-
b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*
(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*b^2-48*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)
*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*co
```

```
s(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I
*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*E
llipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x
+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/
a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*a*b)*cos(f*x+e)*((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*sin(f*x+e)/(-1+cos(f*x+e))/
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \sin \left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin \left(e+f\,x\right)^4\,{\left(b\,\tan \left(e+f\,x\right)^2+a\right)}^{3/2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.111 $\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=165

$$\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-4b)\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{\sqrt{b}\,(3a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}$$

```
[Out] 1/2*(a-4*b)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(
1/2)/f+1/2*(3*a-4*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b
^(1/2)/f+b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-1/2*cos(f*x+e)*sin(f*x+e)*
(a+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A] time = 0.20, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3663, 467, 528, 523, 217, 206, 377, 203}

$$\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-4b)\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{\sqrt{b}\,(3a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - 4*b)*Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e +
 f*x]^2]])/(2*f) + ((3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a + b*Tan[e + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2]
)/f - (Cos[e + f*x]*Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2))/(2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
```

```
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{2f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+\ldots}}{\ldots}\right.}{\ldots}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx)\left(a+\ldots\right.}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx)\left(a+\ldots\right.}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx)\left(a+\ldots\right.}{2f}$$

$$= \frac{(a-4b)\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{(3a-4b)\sqrt{b}\,\tanh^{-1}\left(\ldots\right.}{2f}$$

Mathematica [C] time = 5.59, size = 324, normalized size = 1.96

$$-\frac{\sin(2(e+fx))\tan(e+fx)\sec^2(e+fx)\left(\csc(e+fx)\sec(e+fx)\left(3a^2+4(a-b)^2\cos(2(e+fx))+(a-b)^2\cos(4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/16*(Sec[e + f*x]^2*(-4*Sqrt[2]*a*(a - 2*b)*Cot[e + f*x]*Sqrt[((a + b + (
a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b +
 (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + 4*Sqrt[2]*a*(a
 - 4*b)*Cot[e + f*x]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^
2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x
)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + (3*a^2 - 6*a*b - 5*b^2 + 4*(a - b)^2*
Cos[2*(e + f*x)] + (a - b)^2*Cos[4*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x])*S
in[2*(e + f*x)]*Tan[e + f*x])/(Sqrt[2]*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f
*x)])*Sec[e + f*x]^2])
```

fricas [B] time = 8.00, size = 1931, normalized size = 11.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((a - 4*b)*sqrt(-a + b)*cos(f*x + e)*log(128*(a^4 - 4*a^3*b + 6*a^2*
b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*
b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3
+ 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b
^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 +
8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b +
5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*
cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a
 + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 2*(
3*a - 4*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 +
 8*(a*b - 2*b^2)*cos(f*x + e)^2 - 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x
 + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
e) + 8*b^2)/cos(f*x + e)^4) + 8*((a - b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), -1/16*(
4*(3*a - 4*b)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^
2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + (a - 4*b)*sqrt(-a +
b)*cos(f*x + e)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x
 + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6
+ 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a
^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34
*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*(a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e
)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a
^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*((a - b)*cos(f*x + e)^2 - b)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x
+ e)), 1/8*(sqrt(a - b)*(a - 4*b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*
x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*c
os(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/
((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^
3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*
```

```
x + e) - (3*a - 4*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*
x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 - 4*((a - 2*b)*cos(f*x + e)^3 + 2
*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((a - b)*cos(f*x + e)^2 - b)*sqrt
(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)
), 1/8*(sqrt(a - b)*(a - 4*b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x +
e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f
*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 -
(a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x +
e) - 2*(3*a - 4*b)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(
f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b
 - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 4*((a - b)*cos(
f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

maple [C] time = 0.77, size = 2261, normalized size = 13.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f*(cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-
I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*
(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*
cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)
^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-
b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^2-2*sin(f*x+e)*EllipticF(
(-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*
(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*
2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+
e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^
(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^
(1/2)*cos(f*x+e)^2*a*b-2*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-
b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(
f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2
)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f
*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(
1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b
)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*a^2+10*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I
*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*
x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-
b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*Ellip
ticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),
-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(
1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*a*b-8*cos(f*x+e)^2*sin(f*x+
e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f
*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)
```

```
*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/
a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/
2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/
2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*b^2-6*sin(f*x
+e)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/si
n(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2
*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*2^(1/2)*((I*cos(f*x
+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/
(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)
*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*cos(f*x+e)^2*
a*b+8*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*
(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-
2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*co
s(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^
(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*
a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*
b)/a)^(1/2))*b^2+cos(f*x+e)^5*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^2
-2*cos(f*x+e)^5*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b+cos(f*x+e)^5*
((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2-cos(f*x+e)^4*((2*I*(a-b)^(1/2
)*b^(1/2)+a-2*b)/a)^(1/2)*a^2+2*cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*
b)/a)^(1/2)*a*b-cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2-
cos(f*x+e)*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2+((2*I*(a-b)^(1/2)*
b^(1/2)+a-2*b)/a)^(1/2)*b^2)*cos(f*x+e)*sin(f*x+e)*((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/(-1+cos(f*x+e))/(a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)^2/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^2\,{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\sin^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*sin(e + f*x)**2, x)
```

3.112 $\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=125

$$\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}$$

```
[Out] (a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/2*(
3*a-2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2
*b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3661, 416, 523, 217, 206, 377, 203}

$$\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{\sqrt{b}\,(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f + ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}dx=\frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1+x^2}dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{\text{Subst}\left(\int \frac{a(2a-b)+(3a-2b)bx^2}{\left(1+x^2\right)\sqrt{a+bx^2}}dx,x,\tan(e+fx)\right)}{2f}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}dx,x,\tan(e+\ldots$$

$$=\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}dx,x,\frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e\ldots}}\right.}{f}$$

$$=\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a-2b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan\ldots}{}$$

Mathematica [C] time = 1.47, size = 233, normalized size = 1.86

$$\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}-i(a-b)^{3/2}\log\left(-\frac{4i\left(\sqrt{a-b}\sqrt{a+b\tan^2(e+fx)}+a-ib\tan(e+fx)\right)}{(a-b)^{5/2}(\tan(e+fx)+i)}\right)+i(a-b)^{3/2}\log\left(\frac{4i\left(\sqrt{a-\ldots}\right.}{}\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-I)*(a - b)^(3/2)*Log[((-4*I)*(a - I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a
+ b*Tan[e + f*x]^2]))/((a - b)^(5/2)*(I + Tan[e + f*x]))] + I*(a - b)^(3/2)
*Log[((4*I)*(a + I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a + b*Tan[e + f*x]^2])
)/((a - b)^(5/2)*(-I + Tan[e + f*x]))] + (3*a - 2*b)*Sqrt[b]*Log[b*Tan[e +
f*x] + Sqrt[b]*Sqrt[a + b*Tan[e + f*x]^2]] + b*Tan[e + f*x]*Sqrt[a + b*Tan[
e + f*x]^2])/(2*f)
```

fricas [A] time = 0.88, size = 537, normalized size = 4.30

$$\left[-\frac{(3a-2b)\sqrt{b}\log\left(2b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{b}\tan\left(fx+e\right)+a\right)+2(a-b)\sqrt{-a+b}\log\left(\right.}{4f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(a - b)*sqrt(-a + b)*log(-((a - 2*b)*ta
n(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/
(tan(f*x + e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, -1/
2*((3*a - 2*b)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f
*x + e))) - (-a + b)^(3/2)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - sqrt(b
*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/4*(4*(a - b)^(3/2)*arctan(-sqrt(b
*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a - 2*b)*sqrt(b)*log(
2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a)
 + 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/2*(2*(a - b)^(3/2)*arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a - 2*b)*s
qrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + sqrt
(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2), x)
```

maple [B] time = 0.26, size = 297, normalized size = 2.38

$$\frac{b\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\,\tan\left(fx+e\right)}{2f}+\frac{3\sqrt{b}\,a\ln\left(\tan\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\right)}{2f}-\frac{b^{\frac{3}{2}}\ln\left(\tan\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2*b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+3/2/f*b^(1/2)*a*ln(tan(f*x+e)*b
^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*b^(3/2)*ln(tan(f*x+e)*b^(1/2)+(a+b*tan
(f*x+e)^2)^(1/2))+1/f*(b^4*(a-b))^(1/2)/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^
(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-2/f*a/b*(b^4*(a-b))^(1/2)/(a-b)*
arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/f
*a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*ta
n(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2), x)
```

3.113 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=100

$$\frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{f}$$

```
[Out] 3/2*a*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+3/2*b*
(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3663, 277, 195, 217, 206}

$$\frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2
*f) + (3*b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f) - (Cot[e + f*x]*(
a + b*Tan[e + f*x]^2)^(3/2))/f
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
```

```
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{f} + \frac{(3b)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x\right.}{f}$$

$$= \frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{\cot(e+fx)\left(a+b\tan^2(e+fx\right.}{f}$$

$$= \frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{\cot(e+fx)\left(a+b\tan^2(e+fx\right.}{f}$$

$$= \frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Mathematica [C] time = 2.68, size = 220, normalized size = 2.20

$$\frac{\csc(e+fx)\sec^3(e+fx)\left(-4\left(2a^2+b^2\right)\cos(2(e+fx)) - 2a^2\cos(4(e+fx)) - 6a^2 + ab\cos(4(e+fx)) + 3\sqrt{2}\,ab\,s\right.}{8\sqrt{2}\,f\sqrt{\sec^2(e+fx)((a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Csc[e + f*x]*Sec[e + f*x]^3*(-6*a^2 - a*b + 3*b^2 - 4*(2*a^2 + b^2)*Cos[2*
(e + f*x)] - 2*a^2*Cos[4*(e + f*x)] + a*b*Cos[4*(e + f*x)] + b^2*Cos[4*(e +
 f*x)] + 3*Sqrt[2]*a*b*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x
]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f
*x]^2)/b]/Sqrt[2]], 1]*Sin[2*(e + f*x)]^2))/(8*Sqrt[2]*f*Sqrt[(a + b + (a -
 b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

fricas [B] time = 0.76, size = 387, normalized size = 3.87

$$\left[\frac{3a\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{\left(a^2-8ab+8b^2\right)\cos\left(fx+e\right)^4+8\left(ab-2b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-2b\right)\cos\left(fx+e\right)^3+2b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{\cos\left(fx+e\right)^4}\right.}{8f\cos\left(fx+e\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 +
8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x
+ e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + b)*cos(f*x + e)^2 - b)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)*sin(f*x
+ e)), -1/4*(3*a*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b -
 b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e) + 2*((
2*a + b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2))/(f*cos(f*x + e)*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \csc \left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

maple [C] time = 0.99, size = 1355, normalized size = 13.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)*(-
3*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b
)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I
*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*
x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)
*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/
2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b+6*sin(f*x+e)*EllipticPi((-1+c
os(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b
)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(
a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1
/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1
/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e
)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*cos(f*x+e)^3*a*b-3*sin(f*x+e)*Ell
ipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e)
,((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^
(1/2))*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*c
os(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(
1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+
e))/a)^(1/2)*cos(f*x+e)^2*a*b+6*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))*((2*I
*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+
a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/
2)+a-2*b)/a)^(1/2))*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2
)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f
*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b
)/(1+cos(f*x+e))/a)^(1/2)*cos(f*x+e)^2*a*b+2*cos(f*x+e)^4*((2*I*(a-b)^(1/2)
*b^(1/2)+a-2*b)/a)^(1/2)*a^2-cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/
a)^(1/2)*a*b-cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2+cos
(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b+2*cos(f*x+e)^2*((2*
I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2-((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/
a)^(1/2)*b^2)/sin(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2/((2*I*(a-b)^(1
/2)*b^(1/2)+a-2*b)/a)^(1/2)
```

maxima [A] time = 0.67, size = 73, normalized size = 0.73

$$\frac{3\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)+3\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,b\tan\left(fx+e\right)-\frac{2\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}{\tan\left(fx+e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) + 3*sqrt(b*tan(f*x + e)^
2 + a)*b*tan(f*x + e) - 2*(b*tan(f*x + e)^2 + a)^(3/2)/tan(f*x + e))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan\left(e+f\,x\right)^{2}+a\right)^{3/2}}{\sin\left(e+f\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^2,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.114 $\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=162

$$\frac{b(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}+\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right.}{3af}$$

```
[Out] 1/2*(3*a+2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/
f+1/2*b*(3*a+2*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/a/f-1/3*(3*a+2*b)*cot
(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/a/f-1/3*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(5
/2)/a/f
```

Rubi [A] time = 0.14, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3663, 453, 277, 195, 217, 206}

$$\frac{b(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}+\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right.}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]
^2]])/(2*f) + (b*(3*a + 2*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*a*
f) - ((3*a + 2*b)*Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2))/(3*a*f) - (Cot
[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(5/2))/(3*a*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+bx^2)^{3/2}}{x^4}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3af} + \frac{(3a+2b)\,\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^2}\right.}{3af}$$

$$= -\frac{(3a+2b)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af} - \frac{\cot^3(e+fx)\left(a+b\right.}{3af}$$

$$= \frac{b(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{(3a+2b)\cot(e+fx)}{3}$$

$$= \frac{b(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{(3a+2b)\cot(e+fx)}{3}$$

$$= \frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+2b)\tan(e+fx)\sqrt{a+}}{2af}$$

Mathematica [C] time = 1.93, size = 177, normalized size = 1.09

$$\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-4(a+2b)\cot(e+fx) + \frac{3\sqrt{2}(3a+2b)\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+}{b}}}{\sqrt{2}}\right.\right.}{\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}}\right.}{6\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-4*(a + 2*b)*Cot[
e + f*x] - 2*a*Cot[e + f*x]*Csc[e + f*x]^2 + (3*Sqrt[2]*(3*a + 2*b)*Cot[e +
 f*x]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x
```

```
]^2)/b]/Sqrt[2]], 1])/Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b] + 3*b*Tan[e + f*x]))/(6*Sqrt[2]*f)
```

fricas [A] time = 1.59, size = 497, normalized size = 3.07

$$\left[\frac{3\left((3a+2b)\cos\left(fx+e\right)^{3}-(3a+2b)\cos\left(fx+e\right)\right)\sqrt{b}\,\log\left(\frac{\left(a^{2}-8ab+8b^{2}\right)\cos\left(fx+e\right)^{4}+8\left(ab-2b^{2}\right)\cos\left(fx+e\right)^{2}+4\left((a-2b\right.}{\mathrm{co}}\right.}{24\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((3*a + 2*b)*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt(b)*lo
g(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 +
4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*
x + e) - 4*((4*a + 11*b)*cos(f*x + e)^4 - 2*(3*a + 7*b)*cos(f*x + e)^2 + 3*
b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^3 -
f*cos(f*x + e))*sin(f*x + e)), -1/12*(3*((3*a + 2*b)*cos(f*x + e)^3 - (3*a
+ 2*b)*cos(f*x + e))*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*co
s(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a
*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) + 2*((4*a + 11*
b)*cos(f*x + e)^4 - 2*(3*a + 7*b)*cos(f*x + e)^2 + 3*b)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^3 - f*cos(f*x + e))*sin(f*
x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

maple [C] time = 1.31, size = 4594, normalized size = 28.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/6/f*(-18*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+
a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)
^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*2
^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e
)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(
1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(
1/2)*cos(f*x+e)^3*a*b-9*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/
2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/
2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)
+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a
```

-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^4*a*b+18*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^5*a*b-9*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^5*a*b+18*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^4*a*b+9*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b-18*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*cos(f*x+e)^2*a*b+9*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*cos(f*x+e)^2*a*b+3*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2+7*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^6*a*b+12*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^5*b^2-6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^5*b^2+12*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^4*b^2-6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-

```
I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*
EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x
+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^
2)^(1/2))*cos(f*x+e)^4*b^2-12*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)
*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/
a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(
f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*(
(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a-b)^(1/2)*b^(1
/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b
^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^3*b^2+6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e
)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1
+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b
^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+c
os(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)
^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f
*x+e)^3*b^2+6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)
^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*
cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x
+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*
b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2
)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^2*b^2-4*cos(f*x+e)^4*((
2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b-3*cos(f*x+e)^2*((2*I*(a-b)^(1/2
)*b^(1/2)+a-2*b)/a)^(1/2)*a*b-6*cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*
b)/a)^(1/2)*a^2+25*cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b
^2-17*cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2+4*((2*I*(a
-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^6*a^2-11*b^2*((2*I*(a-b)^(1/2)
*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^6-12*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((
I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f
*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a
-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*Elli
pticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e)
,1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(
1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*b^2)*cos(f*x+e)*((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)^2/sin(f*x+e)^3/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)
```

maxima [A] time = 0.51, size = 175, normalized size = 1.08

$$\frac{9\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)+6\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)+9\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,b\tan\left(fx+e\right)+\frac{6\,\sqrt{b\tan\left(fx+e\right)^{2}+\cdots}}{a}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(9*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) + 6*b^(3/2)*arcsinh(b*ta
n(f*x + e)/sqrt(a*b)) + 9*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e) + 6*sqr
t(b*tan(f*x + e)^2 + a)*b^2*tan(f*x + e)/a - 6*(b*tan(f*x + e)^2 + a)^(3/2)
/tan(f*x + e) - 4*(b*tan(f*x + e)^2 + a)^(3/2)*b/(a*tan(f*x + e)) - 2*(b*ta
n(f*x + e)^2 + a)^(5/2)/(a*tan(f*x + e)^3))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{3/2}}{{\sin\left(e+f\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^4,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.115 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=196

$$\frac{b(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}+\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e\right.}{5af}$$

```
[Out] 1/2*(3*a+4*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/
f+1/2*b*(3*a+4*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/a/f-1/3*(3*a+4*b)*cot
(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/a/f-2/3*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(5
/2)/a/f-1/5*cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(5/2)/a/f
```

Rubi [A] time = 0.17, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3663, 462, 453, 277, 195, 217, 206}

$$\frac{b(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}+\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e\right.}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]
^2]])/(2*f) + (b*(3*a + 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*a*
f) - ((3*a + 4*b)*Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2))/(3*a*f) - (2*C
ot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(5/2))/(3*a*f) - (Cot[e + f*x]^5*(a +
b*Tan[e + f*x]^2)^(5/2))/(5*a*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2\left(a+bx^2\right)^{3/2}}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5af} + \frac{\text{Subst}\left(\int \frac{\left(10a+5ax^2\right)\left(a+bx^2\right)^{3/2}}{x^4}\right.}{5af}$$

$$= -\frac{2\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3af} - \frac{\cot^5(e+fx)\left(a+b\tan^2(e\right.}{5af}$$

$$= -\frac{(3a+4b)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af} - \frac{2\cot^3(e+fx)\left(a+\right.}{3a}$$

$$= \frac{b(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{(3a+4b)\cot(e+fx)}{3}$$

$$= \frac{b(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{(3a+4b)\cot(e+fx)}{3}$$

$$= \frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+4b)\tan(e+fx)\sqrt{a+}}{2af}$$

Mathematica [C] time = 2.17, size = 213, normalized size = 1.09

$$\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-\frac{2(8a^2+34ab+3b^2)\cot(e+fx)}{a}-4(2a+3b)\cot(e+fx)\csc^2(e+fx)-\right.}{30\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*((-2*(8*a^2 + 34*a
*b + 3*b^2)*Cot[e + f*x])/a - 4*(2*a + 3*b)*Cot[e + f*x]*Csc[e + f*x]^2 - 6
*a*Cot[e + f*x]*Csc[e + f*x]^4 + (15*Sqrt[2]*(3*a + 4*b)*Cot[e + f*x]*Ellip
ticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqr
t[2]], 1])/Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b] + 15
*b*Tan[e + f*x]))/(30*Sqrt[2]*f)
```

fricas [A] time = 5.22, size = 655, normalized size = 3.34

$$\left[\frac{15\left(\left(3a^2+4ab\right)\cos\left(fx+e\right)^5-2\left(3a^2+4ab\right)\cos\left(fx+e\right)^3+\left(3a^2+4ab\right)\cos\left(fx+e\right)\right)\sqrt{b}\log\left(\frac{\left(a^2-8ab+8b\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*((3*a^2 + 4*a*b)*cos(f*x + e)^5 - 2*(3*a^2 + 4*a*b)*cos(f*x + e)
^3 + (3*a^2 + 4*a*b)*cos(f*x + e))*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f
*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 +
2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((16*a^2 + 83*a*b +
 6*b^2)*cos(f*x + e)^6 - (40*a^2 + 193*a*b + 12*b^2)*cos(f*x + e)^4 + (30*a
^2 + 125*a*b + 6*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/((a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*co
s(f*x + e))*sin(f*x + e)), -1/60*(15*((3*a^2 + 4*a*b)*cos(f*x + e)^5 - 2*(3
*a^2 + 4*a*b)*cos(f*x + e)^3 + (3*a^2 + 4*a*b)*cos(f*x + e))*sqrt(-b)*arcta
n(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin
(f*x + e)))*sin(f*x + e) + 2*((16*a^2 + 83*a*b + 6*b^2)*cos(f*x + e)^6 - (4
0*a^2 + 193*a*b + 12*b^2)*cos(f*x + e)^4 + (30*a^2 + 125*a*b + 6*b^2)*cos(f
*x + e)^2 - 15*a*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a*
f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f*x + e))*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^6, x)
```

maple [C] time = 1.83, size = 6988, normalized size = 35.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.69, size = 202, normalized size = 1.03

$$\frac{45\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)+60\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)+45\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,b\tan\left(fx+e\right)+\frac{60\,\sqrt{b\tan\left(fx+e\right)^{2}}}{a}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/30*(45*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) + 60*b^(3/2)*arcsinh(b
*tan(f*x + e)/sqrt(a*b)) + 45*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e) + 6
0*sqrt(b*tan(f*x + e)^2 + a)*b^2*tan(f*x + e)/a - 30*(b*tan(f*x + e)^2 + a)
^(3/2)/tan(f*x + e) - 40*(b*tan(f*x + e)^2 + a)^(3/2)*b/(a*tan(f*x + e)) -
20*(b*tan(f*x + e)^2 + a)^(5/2)/(a*tan(f*x + e)^3) - 6*(b*tan(f*x + e)^2 +
a)^(5/2)/(a*tan(f*x + e)^5))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^6,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.116 $\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=144

$$-\frac{\left(15a^2-10ab+3b^2\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{15f(a-b)^3}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{5f(a-b)}+\frac{2(5a-3b)}{}$$

```
[Out] -1/15*(15*a^2-10*a*b+3*b^2)*cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^3/f
+2/15*(5*a-3*b)*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^2/f-1/5*cos(f
*x+e)^5*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A] time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3664, 462, 453, 264}

$$-\frac{\left(15a^2-10ab+3b^2\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{15f(a-b)^3}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{5f(a-b)}+\frac{2(5a-3b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((15*a^2 - 10*a*b + 3*b^2)*Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(1
5*(a - b)^3*f) + (2*(5*a - 3*b)*Cos[e + f*x]^3*Sqrt[a - b + b*Sec[e + f*x]^
2])/(15*(a - b)^2*f) - (Cos[e + f*x]^5*Sqrt[a - b + b*Sec[e + f*x]^2])/(5*(
a - b)*f)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{5(a-b)f} + \frac{\text{Subst}\left(\int \frac{-2(5a-3b)+5(a-b)x^2}{x^4\sqrt{a-b+bx^2}}\,dx, x, \sec(e+f\right.}{5(a-b)f}$$

$$= \frac{2(5a-3b)\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{15(a-b)^2f} - \frac{\cos^5(e+fx)\sqrt{a-b+b\sec^2(e+}}{5(a-b)f}$$

$$= -\frac{\left(15a^2-10ab+3b^2\right)\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{15(a-b)^3f} + \frac{2(5a-3b)\cos^3(e+f}{15(}$$

Mathematica [A] time = 2.14, size = 112, normalized size = 0.78

$$\frac{\cos(e+fx)\left(4\left(7a^2-10ab+3b^2\right)\cos(2(e+fx))-89a^2-3(a-b)^2\cos(4(e+fx))+34ab-9b^2\right)\sqrt{\sec^2(e+fx)(}}{120\sqrt{2}f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(-89*a^2 + 34*a*b - 9*b^2 + 4*(7*a^2 - 10*a*b + 3*b^2)*Cos[2*
(e + f*x)] - 3*(a - b)^2*Cos[4*(e + f*x)])*Sqrt[(a + b + (a - b)*Cos[2*(e +
 f*x)])*Sec[e + f*x]^2])/(120*Sqrt[2]*(a - b)^3*f)
```

fricas [A] time = 0.85, size = 124, normalized size = 0.86

$$-\frac{\left(3\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^5-2\left(5a^2-8ab+3b^2\right)\cos\left(fx+e\right)^3+\left(15a^2-10ab+3b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(}{}}}{15\left(a^3-3a^2b+3ab^2-b^3\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 2*(5*a^2 - 8*a*b + 3*b^2)*cos
(f*x + e)^3 + (15*a^2 - 10*a*b + 3*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2/15*(-320*a*(-sqrt(a)*tan((f*x+
exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan
((f*x+exp(1))/2)^2+a))^7-640*sqrt(a)*a*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt
(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^
2+a))^6+960*sqrt(a)*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1
```

))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^6+832*a^2*(
-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+ex
p(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^5-768*b^2*(-sqrt(a)*tan((f*x+exp(1
))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x
+exp(1))/2)^2+a))^5+896*a*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x
+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^5-288
0*a^4*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan(
(f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))+2560*sqrt(a)*a^2*(-sqrt(a)*
tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^
2+4*b*tan((f*x+exp(1))/2)^2+a))^4-3840*a^2*b^2*(-sqrt(a)*tan((f*x+exp(1))/2
)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp
(1))/2)^2+a))+7040*a^3*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+ex
p(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))-3520*sqr
t(a)*a*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*t
an((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^4+320*a^3*(-sqrt(a)*tan(
(f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*
b*tan((f*x+exp(1))/2)^2+a))^3+2560*a*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sq
rt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2
)^2+a))^3-3840*a^2*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1)
)/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3-3200*sqrt(
a)*a^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan
((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2+4160*sqrt(a)*a^2*b*(-sqr
t(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1)
)/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2-768*sqrt(a)*a^4-2048*sqrt(a)*a^2*b^2
+2496*sqrt(a)*a^3*b)/(-2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan
((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))+
(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+e
xp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2-3*a+4*b)^5/sign(tan((f*x+exp(1)
)/2)^2-1)

maple [A] time = 1.09, size = 169, normalized size = 1.17

$$-\frac{\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)\left(3\left(\cos^4\left(fx+e\right)\right)a^2-6\left(\cos^4\left(fx+e\right)\right)ab+3\left(\cos^4\left(fx+e\right)\right)b^2\right.}{15f\sqrt{\frac{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}{\cos\left(fx+e\right)^2}}\ \cos}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x)

[Out] -1/15/f*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(3*cos(f*x+e)^4*a^2-6*cos(f*x+e)^
4*a*b+3*cos(f*x+e)^4*b^2-10*a^2*cos(f*x+e)^2+16*cos(f*x+e)^2*a*b-6*b^2*cos(
f*x+e)^2+15*a^2-10*a*b+3*b^2)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)
^2)^(1/2)/cos(f*x+e)/(a-b)^3

maxima [A] time = 0.49, size = 214, normalized size = 1.49

$$-\frac{\frac{15\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a-b}+\frac{3\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{5}{2}}\cos\left(fx+e\right)^5-10\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}b\cos\left(fx+e\right)^3+15\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\,b^2\cos\left(fx+e\right)}{a^3-3a^2b+3ab^2-b^3}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] -1/15*(15*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a - b) + (3*(a - b +
 b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 10*(a - b + b/cos(f*x + e)^2)^(3/
2)*b*cos(f*x + e)^3 + 15*sqrt(a - b + b/cos(f*x + e)^2)*b^2*cos(f*x + e))/(
a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 10*((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f

```
*x + e)^3 - 3*sqrt(a - b + b/cos(f*x + e)^2)*b*cos(f*x + e))/(a^2 - 2*a*b +
 b^2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^5}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.117 $\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=88

$$\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3f(a-b)}-\frac{(3a-b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3f(a-b)^2}$$

```
[Out] -1/3*(3*a-b)*cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^2/f+1/3*cos(f*x+e)
^3*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3664, 453, 264}

$$\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3f(a-b)}-\frac{(3a-b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((3*a - b)*Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(3*(a - b)^2*f) +
(Cos[e + f*x]^3*Sqrt[a - b + b*Sec[e + f*x]^2])/(3*(a - b)*f)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{-1+x^2}{x^4\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)f} + \frac{(3a-b)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a-b+bx^2}}\,dx, x, \sec(e+\right.}{3(a-b)f}$$

$$= -\frac{(3a-b)\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)^2 f} + \frac{\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+f}}{3(a-b)f}$$

Mathematica [A] time = 1.49, size = 74, normalized size = 0.84

$$\frac{\cos(e+fx)((a-b)\cos(2(e+fx))-5a+b)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}{6\sqrt{2}f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(-5*a + b + (a - b)*Cos[2*(e + f*x)])*Sqrt[(a + b + (a - b)*C
os[2*(e + f*x)])*Sec[e + f*x]^2])/(6*Sqrt[2]*(a - b)^2*f)
```

fricas [A] time = 0.59, size = 75, normalized size = 0.85

$$\frac{\left((a-b)\cos\left(fx+e\right)^3-(3a-b)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left(a^2-2ab+b^2\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a - b)*cos(f*x + e)^3 - (3*a - b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a^2 - 2*a*b + b^2)*f)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.87, size = 104, normalized size = 1.18

$$\frac{\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b-3a+b\right)}{3f\sqrt{\frac{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b-3*a+
b)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(a-b)^
2
```

maxima [A] time = 0.35, size = 106, normalized size = 1.20

$$-\frac{\frac{3\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a-b}-\frac{\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-3\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^2-2\,ab+b^2}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a - b) - ((a - b + b/c
os(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 3*sqrt(a - b + b/cos(f*x + e)^2)*b*co
s(f*x + e))/(a^2 - 2*a*b + b^2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^3}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.118 $\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=37

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f(a-b)}$$

```
[Out] -cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3664, 264}

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/((a - b)*f))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\ &= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{(a-b)f}\end{aligned}$$

Mathematica [A] time = 0.60, size = 52, normalized size = 1.41

$$\frac{\cos(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}{\sqrt{2}\,f(b-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqr
t[2]*(-a + b)*f)
```

fricas [A] time = 0.62, size = 45, normalized size = 1.22

$$-\frac{\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{(a-b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)sqrt(b)/(a*abs(f)-b*abs
(f))*sign(cos(f*x+exp(1)))*sign(f)+sqrt(a*f^2*(-cos(f*x+exp(1))/f)^2-b*f^2*
(-cos(f*x+exp(1))/f)^2+b)/(-a*abs(f)*sign(cos(f*x+exp(1)))*sign(f)+b*abs(f)
*sign(cos(f*x+exp(1)))*sign(f))
```

maple [B] time = 0.37, size = 78, normalized size = 2.11

$$-\frac{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}{f\sqrt{\frac{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)*(a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(a-b)
```

maxima [A] time = 0.62, size = 35, normalized size = 0.95

$$-\frac{\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{(a-b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin\left(e+f\,x\right)}{\sqrt{b\,\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(sin(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

3.119 $\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{\sqrt{a}f}$$

```
[Out] -arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/f/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3664, 377, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]/(Sqrt[a]*f
))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{-1+ax^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{\sqrt{a}f}
\end{aligned}$$

Mathematica [B] time = 2.66, size = 226, normalized size = 5.38

$$-\frac{\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\tanh^{-1}\left(\frac{a-(a-2b)\tan^2\left(\frac{1}{2}(e+fx\right.}{\sqrt{a}\sqrt{a\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-1\right)^2+4b\tan}}\right.\right.}{2\sqrt{a}f\sqrt{\sec^4\left(\frac{1}{2}(e+fx)\right)((a-b)\cos(2(e+fx))+a+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/2*((ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e
+ f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan
[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f
*x)/2]^2)^2])])*Cos[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2
*(e + f*x)])*Sec[e + f*x]^2])/(Sqrt[a]*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f
*x)])*Sec[(e + f*x)/2]^4])
```

fricas [A] time = 0.76, size = 134, normalized size = 3.19

$$\left[\frac{\log\left(-\frac{2\left((a-b)\cos\left(fx+e\right)^2-2\sqrt{a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+a+b\right)}{\cos\left(fx+e\right)^2-1}\right)}{2\sqrt{a}f}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a}\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1))/(sqrt(a)
*f), sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)/a)/(a*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
```

```
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant
sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1
)]Discontinuities at zeroes of t_nostep^2-1 were not checkedWarning, integr
ation of abs or sign assumes constant sign by intervals (correct if the arg
ument is real):Check [abs(t_nostep^2-1)]Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
was done assuming [a,b]=[85,53]Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-33,71]Discontinuities at zeroes of t_nostep^2-1 were not c
heckedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, in
tegration of abs or sign assumes constant sign by intervals (correct if the
 argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 2.43Error: Bad
 Argument Type
```

maple [B] time = 1.22, size = 351, normalized size = 8.36

$$\frac{\sqrt{\frac{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}{\left(1+\cos\left(fx+e\right)\right)^2}}\left(\ln\left(-\frac{2\left(-1+\cos\left(fx+e\right)\right)\left(\sqrt{a}\,\cos\left(fx+e\right)\sqrt{\frac{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}{\left(1+\cos\left(fx+e\right)\right)^2}}+\sqrt{\frac{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}{\left(1+\cos\left(fx+e\right)\right)^2}}\,\sqrt{a}\right.}{\sin\left(fx+e\right)^2\sqrt{a}}\right.\right.}{2f\sqrt{\frac{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\right.}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*(ln(-2*(-1
+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(
f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)
*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))+ln(-4*(a^(1/2)*
cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-
b*cos(f*x+e)+b)/(-1+cos(f*x+e))))*sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*tan(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

3.120 $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=91

$$-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{3/2}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2af}$$

```
[Out] -1/2*(a-b)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(3/2)/f
-1/2*cot(f*x+e)*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3664, 471, 12, 377, 207}

$$-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{3/2}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((a - b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/(
2*a^(3/2)*f) - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(
2*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
```

```
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2af} + \frac{\text{Subst}\left(\int \frac{a-b}{\left(-1+x^2\right)\sqrt{a-b+bx^2}}\,dx, x, \right.}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2af} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)\sqrt{a-b+bx}}\right.}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2af} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-1+ax^2}\,dx, x, -\right.}{2af}$$

$$= -\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+f}}{2af}$$

Mathematica [B] time = 3.20, size = 303, normalized size = 3.33

$$-\frac{\cot(e+fx)\csc(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\sqrt{2}\sqrt{a}\sqrt{\sec^4\left(\frac{1}{2}(e+fx)\right)((a-b)\cos(2(\right.}}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/4*(Cot[e + f*x]*Csc[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec
[e + f*x]^2]*(Sqrt[2]*Sqrt[a]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(
e + f*x)/2]^4] + 4*(a - b)*ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt
[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]*Sin[(e +
 f*x)/2]^2 + 4*(a - b)*ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]
*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]*Sin[(e + f*
x)/2]^2))/(a^(3/2)*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/
2]^4])
```

fricas [A] time = 0.63, size = 284, normalized size = 3.12

$$\left[\frac{2a\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right) - \left((a-b)\cos\left(fx+e\right)^2 - a + b\right)\sqrt{a}\log\left(-\frac{2\left((a-b)\cos\left(fx+e\right)^2 + 2\sqrt{a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+}{\cos\left(fx+e\right)^2}}\right.}{\cos\left(fx+e\right)^2-1}\right.}{4\left(a^2f\cos\left(fx+e\right)^2 - a^2f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) -
((a - b)*cos(f*x + e)^2 - a + b)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 + 2
*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a
 + b)/(cos(f*x + e)^2 - 1)))/(a^2*f*cos(f*x + e)^2 - a^2*f), 1/2*(((a - b)*
cos(f*x + e)^2 - a + b)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + a*sqrt(((a - b)*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*cos(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
```

```
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant
sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1
)]Discontinuities at zeroes of t_nostep^2-1 were not checkedWarning, integr
ation of abs or sign assumes constant sign by intervals (correct if the arg
ument is real):Check [abs(t_nostep^2-1)]Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
was done assuming [a,b]=[85,53]Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-33,71]Discontinuities at zeroes of t_nostep^2-1 were not c
heckedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Warning, integration of abs or sign assumes constant sign by i
ntervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluat
ion time: 3.42Unable to divide, perhaps due to rounding error%%%{1,[4,0]%%%
}+%%%{%%%{-2,[1]%%%},[2,0]%%%}+%%%{%%%{1,[2]%%%},[0,0]%%%} / %%%{%%%{1,[1]%
%%},[4,0]%%%}+%%%{%%%{-2,[2]%%%},[2,0]%%%}+%%%{%%%{1,[3]%%%},[0,0]%%%} Erro
r: Bad Argument Value
```

maple [B] time = 1.20, size = 2801, normalized size = 30.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4/f*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1
+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(
f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)
*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)^3*a^
2-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(
f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e
))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1
/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)^3*a*b+((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(a^(1/2)*cos(
f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*co
s(f*x+e)+b)/(-1+cos(f*x+e)))*cos(f*x+e)^3*a^2-((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-co
s(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)
))*cos(f*x+e)^3*a*b-2*cos(f*x+e)^2*a^(5/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*co
```

```
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/s
in(f*x+e)^2/a^(1/2))*cos(f*x+e)^2*a^2-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1
+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*
x+e)^2/a^(1/2))*cos(f*x+e)^2*a*b+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(
f*x+e))^2)^(1/2)*ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e)
)^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*cos(f*x+e)
^2*a^2-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(a^
(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)
+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f
*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*cos(f*x+e)^2*a*b+2*cos(f*x+e)^2*a^(3
/2)*b-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+
cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f
*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*
a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)*a^2+(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x
+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^
2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)
-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)*a*b-((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(a^(1/2)*cos(f*x+e
)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x
+e)+b)/(-1+cos(f*x+e)))*cos(f*x+e)*a^2+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(
1+cos(f*x+e))^2)^(1/2)*ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(
f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*cos(
f*x+e)*a*b-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-2
*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+
cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(
1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*a^2+((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(
a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/
2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos
(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*a*b-((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*
x+e)))*a^2+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4
*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(
1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*c
os(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*a*b-2*a^(3/2)*b)*sin(f*x+e)^2/(-
1+cos(f*x+e))^2/cos(f*x+e)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)
^(1/2)/(1+cos(f*x+e))^2/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{3}}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^3\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(csc(e + f*x)**3/sqrt(a + b*tan(e + f*x)**2), x)
```

3.121 $\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=143

$$-\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{5/2}f}-\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{8a^2f}-\frac{\cot^3(e+fx)}{}$$

```
[Out] -3/8*(a-b)^2*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(5/2)
/f-1/8*(5*a-3*b)*cot(f*x+e)*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/a^2/f-1/4
*cot(f*x+e)^3*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.16, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3664, 470, 527, 12, 377, 207}

$$-\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{5/2}f}-\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{8a^2f}-\frac{\cot^3(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-3*(a - b)^2*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]
])/(8*a^(5/2)*f) - ((5*a - 3*b)*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a - b + b*Se
c[e + f*x]^2])/(8*a^2*f) - (Cot[e + f*x]^3*Csc[e + f*x]*Sqrt[a - b + b*Sec[
e + f*x]^2])/(4*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{4af} - \frac{\text{Subst}\left(\int \frac{-a+b-2(2a-b)x^2}{\left(-1+x^2\right)^2\sqrt{a-b+bx^2}}\,dx,\right.}{4af}$$

$$= -\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8a^2f} - \frac{\cot^3(e+fx)\csc(e+}{}$$

$$= -\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8a^2f} - \frac{\cot^3(e+fx)\csc(e+}{}$$

$$= -\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8a^2f} - \frac{\cot^3(e+fx)\csc(e+}{}$$

$$= -\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{5/2}f} - \frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b}}{8a^2f}$$

Mathematica [A] time = 4.76, size = 278, normalized size = 1.94

$$\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-\sqrt{2}\sqrt{a}\cot(e+fx)\csc(e+fx)\left(2a\csc^2(e+fx)+3a-3b\right)-\frac{3(a}{}\right.}{16a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-(Sqrt[2]*Sqrt[a]*Cot[e + f*x]*Csc[e + f*x]*(3*a - 3*b + 2*a*Csc[e + f*x]
^2)) - (3*(a - b)^2*(ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sq
rt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b
```

```
 + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-
1 + Tan[(e + f*x)/2]^2)^2])])*Cos[e + f*x]*Sec[(e + f*x)/2]^2)/Sqrt[(a + b
+ (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])*Sqrt[(a + b + (a - b)*Cos[
2*(e + f*x)])*Sec[e + f*x]^2])/(16*a^(5/2)*f)
```

fricas [A] time = 0.65, size = 437, normalized size = 3.06

$$\left[\frac{3\left(\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^4-2\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^2+a^2-2\,ab+b^2\right)\sqrt{a}\,\log\left(-\frac{2\left((a-b)\cos\left(fx+e\right)\right.}{}\right.}{16\left(a^3f\cos\left(fx+e\right)^4-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 - 2*a*b + b^2)*cos(f*
x + e)^2 + a^2 - 2*a*b + b^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sq
rt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a +
b)/(cos(f*x + e)^2 - 1)) + 2*(3*(a^2 - a*b)*cos(f*x + e)^3 - (5*a^2 - 3*a*b
)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^3*f*c
os(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f), 1/8*(3*((a^2 - 2*a*b + b^2
)*cos(f*x + e)^4 - 2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^2 + a^2 - 2*a*b + b^2
)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e)/a) + (3*(a^2 - a*b)*cos(f*x + e)^3 - (5*a^2 - 3*a*b)*cos(f*x
 + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^3*f*cos(f*x +
e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f/64*(2*
(1/4*tan((f*x+exp(1))/2)^2/a-1/8*(-18*a+12*b)/a^2)*sqrt(a*tan((f*x+exp(1))/
2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a)+2*(-(-6*a^3*(-s
qrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(
1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))-10*a*b^2*(-sqrt(a)*tan((f*x+exp(1))/
2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+ex
p(1))/2)^2+a))+16*a^2*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp
(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))+4*a^2*(-s
qrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(
1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3+6*b^2*(-sqrt(a)*tan((f*x+exp(1))/2
)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp
(1))/2)^2+a))^3-12*a*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(
1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3+3*sqrt(a
)*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan(
(f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2-5*sqrt(a)*a^3+4*sqrt(a)*a
^2*b)/a^2/(-(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*
a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2+a)^2-(3*sqrt(a)*a^2
```

```
+3*sqrt(a)*b^2-6*sqrt(a)*a*b)*ln(abs(a*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt
(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^
2+a))+sqrt(a)*a-2*sqrt(a)*b))/a^3+1/2*(-12*a^2-12*b^2+24*a*b)*atan((-sqrt(a
)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2
)^2+4*b*tan((f*x+exp(1))/2)^2+a))/sqrt(-a))/a^2/sqrt(-a)))/sign(tan((f*x+ex
p(1))/2)^2-1)
```

maple [B] time = 1.42, size = 6334, normalized size = 44.29

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^5\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{5}\left(e+fx\right)}{\sqrt{a+b\tan^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(csc(e + f*x)**5/sqrt(a + b*tan(e + f*x)**2), x)
```

3.122 $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=146

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{5/2}}+\frac{\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f(a-b)}-\frac{(5a-2b)\sin(e+fx)\cos(e+fx)}{8f(a-b)^2}$$

```
[Out] 3/8*a^2*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)
/f-1/8*(5*a-2*b)*cos(f*x+e)*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(a-b)^2/f+1
/4*cos(f*x+e)^3*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A] time = 0.17, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3663, 470, 527, 12, 377, 203}

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{5/2}}+\frac{\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f(a-b)}-\frac{(5a-2b)\sin(e+fx)\cos(e+fx)}{8f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (3*a^2*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(8*(a
 - b)^(5/2)*f) - ((5*a - 2*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Tan[e +
f*x]^2])/(8*(a - b)^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Tan[e +
f*x]^2])/(4*(a - b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4(a-b)f} - \frac{\text{Subst}\left(\int \frac{a-2(2a-b)x^2}{(1+x^2)^2\sqrt{a+bx^2}}\,dx, x, \tan(e\right.}{4(a-b)f}$$

$$= -\frac{(5a-2b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)^2 f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-}$$

$$= -\frac{(5a-2b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)^2 f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-}$$

$$= -\frac{(5a-2b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)^2 f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-}$$

$$= \frac{3a^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{5/2}f} - \frac{(5a-2b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)^2 f}$$

Mathematica [C] time = 4.24, size = 314, normalized size = 2.15

$$-\frac{\sin(2(e+fx))\sec^2(e+fx)\left(6\sqrt{2}\,a^3\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\,\Pi\left(-\frac{b}{a-b};\sin^{-1}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/32*(((a - b)*(7*a^2 + 8*a*b - 3*b^2 + 2*(3*a^2 - 5*a*b + 2*b^2)*Cos[2*(e
 + f*x)] - (a - b)^2*Cos[4*(e + f*x)]) + 6*Sqrt[2]*a^2*(-a + b)*Sqrt[((a +
b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a
```

```
+ b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + 6*Sqrt[2]
*a^3*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticPi
[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(Sqrt[2]*(a - b)^3*f*
Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

fricas [B] time = 4.69, size = 788, normalized size = 5.40

$$\left[-\frac{3\,a^2\sqrt{-a+b}\,\log\left(128\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)\cos\left(fx+e\right)^8-256\left(a^4-5\,a^3b+9\,a^2b^2-7\,ab^3+2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*a^2*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(
f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x
 + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11
*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*(a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*
cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 -
(a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a -
b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*(a^2 - 2*a*b +
b^2)*cos(f*x + e)^3 - (5*a^2 - 7*a*b + 2*b^2)*cos(f*x + e))*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2
- b^3)*f), 1/32*(3*sqrt(a - b)*a^2*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f
*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*
cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b
^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(
2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 7*a*b + 2*b^2)*cos(f*x + e)
)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^4}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [C] time = 1.08, size = 1169, normalized size = 8.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/8/f*sin(f*x+e)*(2*cos(f*x+e)^5*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*
a^2-4*cos(f*x+e)^5*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b+2*cos(f*x+
e)^5*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2-2*cos(f*x+e)^4*((2*I*(a-
b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^2+4*cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/
2)+a-2*b)/a)^(1/2)*a*b-2*cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(
1/2)*b^2-3*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)
+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-
b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(
f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)
/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^
2-8*a*b+8*b^2)/a^2)^(1/2))*a^2*sin(f*x+e)+6*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1
/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e
))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*c
os(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e)
)*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*
b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/
2)*b^(1/2)+a-2*b)/a)^(1/2))*a^2*sin(f*x+e)-5*cos(f*x+e)^3*((2*I*(a-b)^(1/2)
*b^(1/2)+a-2*b)/a)^(1/2)*a^2+9*cos(f*x+e)^3*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b
)/a)^(1/2)*a*b-4*cos(f*x+e)^3*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2
+5*cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^2-9*cos(f*x+e)^
2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b+4*cos(f*x+e)^2*((2*I*(a-b)^
(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2-5*cos(f*x+e)*((2*I*(a-b)^(1/2)*b^(1/2)+a-
2*b)/a)^(1/2)*a*b+2*cos(f*x+e)*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^
2+5*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b-2*((2*I*(a-b)^(1/2)*b^(1/
2)+a-2*b)/a)^(1/2)*b^2)/(-1+cos(f*x+e))/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
cos(f*x+e)^2)^(1/2)/cos(f*x+e)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/(a
-b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^4}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+fx\right)^4}{\sqrt{b\tan\left(e+fx\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

3.123 $\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=93

$$\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{3/2}}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f(a-b)}$$

```
[Out] 1/2*a*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f
-1/2*cos(f*x+e)*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A] time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3663, 471, 12, 377, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{3/2}}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (a*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2*(a - b
)^(3/2)*f) - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*(a -
 b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)f} + \frac{\text{Subst}\left(\int \frac{a}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)f} + \frac{a\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)f} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(}{\sqrt{a+b\,\text{ta}}}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)f}$$

Mathematica [C] time = 3.26, size = 270, normalized size = 2.90

$$-\frac{\sin(2(e+fx))\sec^2(e+fx)\left(\sqrt{2}\,a^2\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\,\Pi\left(-\frac{b}{a-b};\sin^{-1}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\right|1}{4\sqrt{2}\,f(a-b)^2\sqrt{\sec^2(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/4*(((a - b)*(a + b + (a - b)*Cos[2*(e + f*x)]) + Sqrt[2]*a*(-a + b)*Sqrt
[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sq
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + Sq
rt[2]*a^2*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Ellip
ticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(Sqrt[2]*(a - b)
^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

fricas [B] time = 0.70, size = 696, normalized size = 7.48

$$\left[-\frac{8\,(a-b)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)\sin\left(fx+e\right)-a\sqrt{-a+b}\,\log\left(128\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)\text{co}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e)*sin(f*x + e) - a*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*
b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4
)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*
(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*(a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2
- 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x
+ e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^2 - 2*a*
b + b^2)*f), -1/8*(4*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e)*sin(f*x + e) - sqrt(a - b)*a*arctan(-1/4*(8*(a^2 - 2*a*b +
 b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*
b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*
a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x +
 e))))/((a^2 - 2*a*b + b^2)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [C] time = 0.84, size = 795, normalized size = 8.55

$$-\frac{\sin\left(fx+e\right)\left(\sqrt{2}\,\sqrt{\frac{i\cos\left(fx+e\right)\sqrt{a-b}\,\sqrt{b}-i\sqrt{a-b}\,\sqrt{b}+a\cos\left(fx+e\right)-b\cos\left(fx+e\right)+b}{\left(1+\cos\left(fx+e\right)\right)a}}\,\sqrt{-\frac{2\left(i\cos\left(fx+e\right)\sqrt{a-b}\,\sqrt{b}-i\sqrt{a-b}\,\sqrt{b}-a\cos\left(fx\right.\right.}{\left(1+\cos\left(fx+e\right)\right)a}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f*sin(f*x+e)*(2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)
*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*
x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)
/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2
)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1
/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*(a
-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos
(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/
2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(
f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^
(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-
b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*a*sin(f*x+e)+cos(f*x+e)^3*((2*I*(a-b)^(1/
2)*b^(1/2)+a-2*b)/a)^(1/2)*a-cos(f*x+e)^3*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/
a)^(1/2)*b-cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a+cos(f*x
```

```
+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b+cos(f*x+e)*((2*I*(a-b)^(1
/2)*b^(1/2)+a-2*b)/a)^(1/2)*b-((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b)/
(-1+cos(f*x+e))/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/cos(
f*x+e)/(a-b)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(sin(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

3.124 $\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3661, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f}$$

Mathematica [A] time = 0.08, size = 46, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f)
```

fricas [A] time = 0.46, size = 125, normalized size = 2.72

$$\left[-\frac{\sqrt{-a+b}\,\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2-2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\,\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^2+1}\right)}{2\,(a-b)f}, \frac{\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{a-b}\,\tan\left(fx+e\right)}\right)}{\sqrt{a-b}\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1))/((a - b)*f), arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e)))/(sqrt(a - b)*f)
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [A] time = 0.35, size = 67, normalized size = 1.46

$$\frac{\sqrt{b^{4}\left(a-b\right)}\ \arctan\left(\frac{\left(a-b\right)b^{2}\tan\left(fx+e\right)}{\sqrt{b^{4}\left(a-b\right)}\ \sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}\right)}{f\,b^{2}\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan
(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 12.36, size = 40, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\tan\left(e+f\,x\right)\,\sqrt{a-b}}{\sqrt{b\,{\tan\left(e+f\,x\right)}^2+a}}\right)}{f\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] atan((tan(e + f*x)*(a - b)^(1/2))/(a + b*tan(e + f*x)^2)^(1/2))/(f*(a - b)^
(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*tan(e + f*x)**2), x)
```

3.125 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=30

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

```
[Out] -cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.07, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3663, 264}

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a*f))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}\end{aligned}$$

Mathematica [A] time = 0.25, size = 49, normalized size = 1.63

$$-\frac{\cot(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}{\sqrt{2}af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(S
qrt[2]*a*f))
```

fricas [A] time = 0.45, size = 49, normalized size = 1.63

$$-\frac{\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{af\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*f*sin(f*
x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{2}}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [A] time = 1.04, size = 57, normalized size = 1.90

$$-\frac{\sqrt{\frac{a\left(\cos^{2}\left(fx+e\right)\right)-\left(\cos^{2}\left(fx+e\right)\right)b+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{f\sin\left(fx+e\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/sin(
f*x+e)/a
```

maxima [A] time = 0.37, size = 30, normalized size = 1.00

$$-\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{af\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*tan(f*x + e)^2 + a)/(a*f*tan(f*x + e))
```

mupad [B] time = 12.86, size = 36, normalized size = 1.20

$$-\frac{\cot\left(e+f\,x\right)\,\sqrt{a+\frac{b\,\sin\left(e+f\,x\right)^{2}}{\cos\left(e+f\,x\right)^{2}}}}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2)),x)
```

```
[Out] -(cot(e + f*x)*(a + (b*sin(e + f*x)^2)/cos(e + f*x)^2)^(1/2))/(a*f)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

3.126 $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=74

$$-\frac{(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

```
[Out] -1/3*(3*a-2*b)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/3*cot(f*x+e)^3*(
a+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3663, 453, 264}

$$-\frac{(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((3*a - 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^2*f) - (Cot[e +
 f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a*f)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} + \frac{(3a-2b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx, x, \tan(e+fx\right.}{3af}$$

$$= -\frac{(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

Mathematica [A] time = 0.42, size = 68, normalized size = 0.92

$$-\frac{\cot(e+fx)\left(a\csc^2(e+fx)+2a-2b\right)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}{3\sqrt{2}\,a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/3*(Cot[e + f*x]*(2*a - 2*b + a*Csc[e + f*x]^2)*Sqrt[(a + b + (a - b)*Cos
[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqrt[2]*a^2*f)
```

fricas [A] time = 0.64, size = 90, normalized size = 1.22

$$-\frac{\left(2\,(a-b)\cos\left(fx+e\right)^3-(3\,a-2\,b)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a - b)*cos(f*x + e)^3 - (3*a - 2*b)*cos(f*x + e))*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2*f*cos(f*x + e)^2 - a^2*f)*sin(f*x +
 e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^4}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [A] time = 1.08, size = 86, normalized size = 1.16

$$\frac{\left(2a\left(\cos^2\left(fx+e\right)\right)-2\left(\cos^2\left(fx+e\right)\right)b-3a+2b\right)\sqrt{\frac{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}{\cos\left(fx+e\right)^2}}\;\cos\left(fx+e\right)}{3f\sin\left(fx+e\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f*(2*a*cos(f*x+e)^2-2*cos(f*x+e)^2*b-3*a+2*b)*((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/sin(f*x+e)^3/a^2
```

maxima [A] time = 0.78, size = 87, normalized size = 1.18

$$-\frac{\frac{3\sqrt{b\tan\left(fx+e\right)^2+a}}{a\tan\left(fx+e\right)}-\frac{2\sqrt{b\tan\left(fx+e\right)^2+a}\,b}{a^2\tan\left(fx+e\right)}+\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{a\tan\left(fx+e\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(b*tan(f*x + e)^2 + a)/(a*tan(f*x + e)) - 2*sqrt(b*tan(f*x + e)
^2 + a)*b/(a^2*tan(f*x + e)) + sqrt(b*tan(f*x + e)^2 + a)/(a*tan(f*x + e)^3
))/f
```

mupad [B] time = 18.82, size = 145, normalized size = 1.96

$$-\frac{2\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{a+\frac{b\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)^2}{\left(e^{e\,2i+f\,x\,2i}+1\right)^2}}\left(a\,1i-b\,1i-a\,e^{e\,2i+f\,x\,2i}\,4i+a\,e^{e\,4i+f\,x\,4i}\,1i+b\,e^{e\,2i+f\,x\,2i}\,2i-b\,e^{e\,4i+f\,x}\cdots\right.}{3\,a^2\,f\left(e^{e\,2i+f\,x\,2i}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2)),x)
```

```
[Out] -(2*(exp(e*2i + f*x*2i) + 1)*(a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e
*2i + f*x*2i) + 1)^2)^(1/2)*(a*1i - b*1i - a*exp(e*2i + f*x*2i)*4i + a*exp(
e*4i + f*x*4i)*1i + b*exp(e*2i + f*x*2i)*2i - b*exp(e*4i + f*x*4i)*1i))/(3*
a^2*f*(exp(e*2i + f*x*2i) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

3.127 $\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{2(5a-2b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{\left(15a^2-20ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f}-\frac{\cot^5(e+f\ldots}{}$$

```
[Out] -1/15*(15*a^2-20*a*b+8*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^3/f-2/15*
(5*a-2*b)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/5*cot(f*x+e)^5*(a+b
*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3663, 462, 453, 264}

$$-\frac{\left(15a^2-20ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f}-\frac{2(5a-2b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{\cot^5(e+f\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((15*a^2 - 20*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(15*a^
3*f) - (2*(5*a - 2*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a^2*f)
 - (Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*a*f)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5af} + \frac{\text{Subst}\left(\int \frac{2(5a-2b)+5ax^2}{x^4\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= -\frac{2(5a-2b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+f}{5af}$$

$$= -\frac{\left(15a^2-20ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} - \frac{2(5a-2b)\cot^3(e+fx}{15a}$$

Mathematica [A] time = 1.91, size = 90, normalized size = 0.73

$$-\frac{\cot(e+fx)\left(3a^2\csc^4(e+fx)+4a(a-b)\csc^2(e+fx)+8(a-b)^2\right)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a}}{15\sqrt{2}\,a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/15*(Cot[e + f*x]*(8*(a - b)^2 + 4*a*(a - b)*Csc[e + f*x]^2 + 3*a^2*Csc[e
 + f*x]^4)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqrt[2
]*a^3*f)
```

fricas [A] time = 1.11, size = 141, normalized size = 1.15

$$-\frac{\left(8\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^5-4\left(5\,a^2-9\,ab+4\,b^2\right)\cos\left(fx+e\right)^3+\left(15\,a^2-20\,ab+8\,b^2\right)\cos\left(fx+e\right)\right)}{15\left(a^3f\cos\left(fx+e\right)^4-2\,a^3f\cos\left(fx+e\right)^2+a^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 4*(5*a^2 - 9*a*b + 4*b^2)*cos
(f*x + e)^3 + (15*a^2 - 20*a*b + 8*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^
2 + a^3*f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^6}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [A] time = 1.33, size = 148, normalized size = 1.20

$$-\frac{\left(8\left(\cos^4\left(fx+e\right)\right)a^2-16\left(\cos^4\left(fx+e\right)\right)ab+8\left(\cos^4\left(fx+e\right)\right)b^2-20a^2\left(\cos^2\left(fx+e\right)\right)+36\left(\cos^2\left(fx+e\right)\right)}{15f\sin\left(fx+e\right)^5a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15/f*(8*cos(f*x+e)^4*a^2-16*cos(f*x+e)^4*a*b+8*cos(f*x+e)^4*b^2-20*a^2*c
os(f*x+e)^2+36*cos(f*x+e)^2*a*b-16*b^2*cos(f*x+e)^2+15*a^2-20*a*b+8*b^2)*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/sin(f*x+e)^
5/a^3
```

maxima [A] time = 0.78, size = 173, normalized size = 1.41

$$-\frac{\frac{15\sqrt{b\tan\left(fx+e\right)^2+a}}{a\tan\left(fx+e\right)}-\frac{20\sqrt{b\tan\left(fx+e\right)^2+a}\,b}{a^2\tan\left(fx+e\right)}+\frac{8\sqrt{b\tan\left(fx+e\right)^2+a}\,b^2}{a^3\tan\left(fx+e\right)}+\frac{10\sqrt{b\tan\left(fx+e\right)^2+a}}{a\tan\left(fx+e\right)^3}-\frac{4\sqrt{b\tan\left(fx+e\right)^2+a}\,b}{a^2\tan\left(fx+e\right)^3}+\frac{3\sqrt{b\tan\left(fx+e\right.}}{a\tan\left(fx+e\right)}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(b*tan(f*x + e)^2 + a)/(a*tan(f*x + e)) - 20*sqrt(b*tan(f*x +
 e)^2 + a)*b/(a^2*tan(f*x + e)) + 8*sqrt(b*tan(f*x + e)^2 + a)*b^2/(a^3*tan
(f*x + e)) + 10*sqrt(b*tan(f*x + e)^2 + a)/(a*tan(f*x + e)^3) - 4*sqrt(b*ta
n(f*x + e)^2 + a)*b/(a^2*tan(f*x + e)^3) + 3*sqrt(b*tan(f*x + e)^2 + a)/(a*
tan(f*x + e)^5))/f
```

mupad [B] time = 22.11, size = 761, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2)),x)
```

```
[Out] ((((a - b)*(32*a*b - 64*a^2 + 32*b^2))/(120*a^3*f*(a*1i - b*1i)) - ((a - b)
*(64*a^2 - 96*a*b + 32*b^2))/(120*a^3*f*(a*1i - b*1i)))*(a + (b*(exp(e*2i +
 f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)*(2*exp(e*2i + f*x*2i
) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) - 1)^2*(exp(e*2i + f*x*2i
) + 1)) + ((a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)
^2)^(1/2)*((2*(3*a - 3*b))/(3*a*f*(a*1i - b*1i)) + ((3*a - 3*b)*(96*a - 64*
b))/(240*a^2*f*(a*1i - b*1i)) + ((3*a - 3*b)*(256*a + 64*b))/(240*a^2*f*(a*
1i - b*1i)))*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i +
f*x*2i) - 1)^4*(exp(e*2i + f*x*2i) + 1)) + ((((a - b)*(32*a - 16*b))/(30*a^
2*f*(a*1i - b*1i)) + ((a - b)*(32*a + 48*b))/(30*a^2*f*(a*1i - b*1i)))*(a +
 (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)*(2*ex
p(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) - 1)^3*(ex
p(e*2i + f*x*2i) + 1)) - ((a - b)^2*(a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)
/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4
i) + 1)*8i)/(15*a^3*f*(exp(e*2i + f*x*2i) - 1)*(exp(e*2i + f*x*2i) + 1)) +
(8*(2*a - 2*b)*(a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i)
+ 1)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/(5*a*f*(exp(
e*2i + f*x*2i) - 1)^5*(exp(e*2i + f*x*2i) + 1)*(a*1i - b*1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(csc(e + f*x)**6/sqrt(a + b*tan(e + f*x)**2), x)
```

3.128 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=199

$$-\frac{2b\left(15a^2+10ab-b^2\right)\sec(e+fx)}{15f(a-b)^4\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\left(15a^2+10ab-b^2\right)\cos(e+fx)}{15f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cos^5(e+fx)}{5f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}+$$

```
[Out] -1/15*(15*a^2+10*a*b-b^2)*cos(f*x+e)/(a-b)^3/f/(a-b+b*sec(f*x+e)^2)^(1/2)+2
/15*(5*a-2*b)*cos(f*x+e)^3/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)^(1/2)-1/5*cos(f*x
+e)^5/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(1/2)-2/15*b*(15*a^2+10*a*b-b^2)*sec(f*x
+e)/(a-b)^4/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3664, 462, 453, 271, 191}

$$-\frac{2b\left(15a^2+10ab-b^2\right)\sec(e+fx)}{15f(a-b)^4\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\left(15a^2+10ab-b^2\right)\cos(e+fx)}{15f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cos^5(e+fx)}{5f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((15*a^2 + 10*a*b - b^2)*Cos[e + f*x])/(15*(a - b)^3*f*Sqrt[a - b + b*Sec[
e + f*x]^2]) + (2*(5*a - 2*b)*Cos[e + f*x]^3)/(15*(a - b)^2*f*Sqrt[a - b +
b*Sec[e + f*x]^2]) - Cos[e + f*x]^5/(5*(a - b)*f*Sqrt[a - b + b*Sec[e + f*x
]^2]) - (2*b*(15*a^2 + 10*a*b - b^2)*Sec[e + f*x])/(15*(a - b)^4*f*Sqrt[a -
 b + b*Sec[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{-2(5a-2b)+5(a-b)x^2}{x^4\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+\right.}{5(a-b)f}$$

$$= \frac{2(5a-2b)\cos^3(e+fx)}{15(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cos^5(e+fx)}{5(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{(15}{}$$

$$= -\frac{\left(15a^2+10ab-b^2\right)\cos(e+fx)}{15(a-b)^3f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{2(5a-2b)\cos^3(e+fx)}{15(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} -$$

$$= -\frac{\left(15a^2+10ab-b^2\right)\cos(e+fx)}{15(a-b)^3f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{2(5a-2b)\cos^3(e+fx)}{15(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} -$$

Mathematica [A] time = 1.94, size = 186, normalized size = 0.93

$$-\frac{\sec(e+fx)\left(3a^3\cos(6(e+fx))+150a^3-9a^2b\cos(6(e+fx))+1078a^2b+\left(125a^3+169a^2b-329ab^2+35b\right.\right.}{240\sqrt{2}f(a-b)^4\sqrt{\sec^2(e+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/240*((150*a^3 + 1078*a^2*b + 338*a*b^2 - 30*b^3 + (125*a^3 + 169*a^2*b -
 329*a*b^2 + 35*b^3)*Cos[2*(e + f*x)] - 2*(a - b)^2*(11*a + b)*Cos[4*(e + f
*x)] + 3*a^3*Cos[6*(e + f*x)] - 9*a^2*b*Cos[6*(e + f*x)] + 9*a*b^2*Cos[6*(e
 + f*x)] - 3*b^3*Cos[6*(e + f*x)])*Sec[e + f*x])/(Sqrt[2]*(a - b)^4*f*Sqrt[
(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

fricas [A] time = 0.65, size = 233, normalized size = 1.17

$$-\frac{\left(3\left(a^3-3a^2b+3ab^2-b^3\right)\cos\left(fx+e\right)^7-2\left(5a^3-12a^2b+9ab^2-2b^3\right)\cos\left(fx+e\right)^5+\left(15a^3-5a^2b-1\right.\right.}{15\left(\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5\right)f\cos\left(fx+e\right)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 2*(5*a^3 - 12*a^2
*b + 9*a*b^2 - 2*b^3)*cos(f*x + e)^5 + (15*a^3 - 5*a^2*b - 11*a*b^2 + b^3)*
cos(f*x + e)^3 + 2*(15*a^2*b + 10*a*b^2 - b^3)*cos(f*x + e))*sqrt(((a - b)*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b
^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a
*b^4 + b^5)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/
f*(2*(-(-4*a^2*b^5*sign(tan((f*x+exp(1))/2)^2-1)+12*a^3*b^4*sign(tan((f*x+e
xp(1))/2)^2-1)-12*a^4*b^3*sign(tan((f*x+exp(1))/2)^2-1)+4*a^5*b^2*sign(tan(
(f*x+exp(1))/2)^2-1))/(16*b^8-112*a*b^7+336*a^2*b^6-560*a^3*b^5+560*a^4*b^4
-336*a^5*b^3+112*a^6*b^2-16*a^7*b)-tan((f*x+exp(1))/2)^2*(-4*a^2*b^5*sign(t
an((f*x+exp(1))/2)^2-1)+12*a^3*b^4*sign(tan((f*x+exp(1))/2)^2-1)-12*a^4*b^3
*sign(tan((f*x+exp(1))/2)^2-1)+4*a^5*b^2*sign(tan((f*x+exp(1))/2)^2-1))/(16
*b^8-112*a*b^7+336*a^2*b^6-560*a^3*b^5+560*a^4*b^4-336*a^5*b^3+112*a^6*b^2-
16*a^7*b))/sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((
f*x+exp(1))/2)^2+a)+2*(15*a*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f
*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^9+6
0*sqrt(a)*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-
2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^8-165*sqrt(a)*a*b*(
-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+ex
p(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^8+320*a^3*(-sqrt(a)*tan((f*x+exp(1
))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x
+exp(1))/2)^2+a))^7-80*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+
exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^7+480*
a*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan(
(f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^7-420*a^2*b*(-sqrt(a)*tan((
f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b
*tan((f*x+exp(1))/2)^2+a))^7+640*sqrt(a)*a^3*(-sqrt(a)*tan((f*x+exp(1))/2)^
2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1
))/2)^2+a))^6-160*sqrt(a)*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f
*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^6-9
00*sqrt(a)*a^2*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)
^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^6-832*a^4*(-sqrt
(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))
/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^5+128*b^4*(-sqrt(a)*tan((f*x+exp(1))/2)
^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(
1))/2)^2+a))^5-272*a*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+ex
p(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^5+1728*a
^2*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan
((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^5-542*a^3*b*(-sqrt(a)*tan(
(f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*
b*tan((f*x+exp(1))/2)^2+a))^5+2880*a^6*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt
(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^
2+a))-2560*sqrt(a)*a^4*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(
1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^4+7680*a*b
```

```
^5*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*
x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))-31360*a^2*b^4*(-sqrt(a)*tan((f
*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*
tan((f*x+exp(1))/2)^2+a))+39120*a^3*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqr
t(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)
^2+a))-13440*a^4*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1)
)/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))-4985*a^5*b*(
-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+ex
p(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))+2240*sqrt(a)*b^4*(-sqrt(a)*tan((f*
x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*t
an((f*x+exp(1))/2)^2+a))^4-8480*sqrt(a)*a*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)
^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(
1))/2)^2+a))^4+5880*sqrt(a)*a^2*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*
tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a
))^4+3130*sqrt(a)*a^3*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp
(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^4-320*a^5
*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+
exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3-1280*b^5*(-sqrt(a)*tan((f*x+ex
p(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((
f*x+exp(1))/2)^2+a))^3+7680*a*b^4*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*ta
n((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))
^3-8560*a^2*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^
4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3-2080*a^3*b^2*(-
sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp
(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3+4140*a^4*b*(-sqrt(a)*tan((f*x+exp
(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f
*x+exp(1))/2)^2+a))^3+3200*sqrt(a)*a^5*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt
(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^
2+a))^2-8320*sqrt(a)*a*b^4*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+
exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2+2320
0*sqrt(a)*a^2*b^3*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2
)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2-15840*sqrt(a)
*a^3*b^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*t
an((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2-1940*sqrt(a)*a^4*b*(-s
qrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(
1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2+768*sqrt(a)*a^6+6400*sqrt(a)*a*b^5
-17472*sqrt(a)*a^2*b^4+15648*sqrt(a)*a^3*b^3-3412*sqrt(a)*a^4*b^2-1917*sqrt
(a)*a^5*b)/(-2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(
1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))+(-sqrt(a)*
tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^
2+4*b*tan((f*x+exp(1))/2)^2+a))^2-3*a+4*b)^5/(-15*a^3*sign(tan((f*x+exp(1))
/2)^2-1)+15*b^3*sign(tan((f*x+exp(1))/2)^2-1)-45*a*b^2*sign(tan((f*x+exp(1)
)/2)^2-1)+45*a^2*b*sign(tan((f*x+exp(1))/2)^2-1)))
```

maple [B] time = 7.05, size = 67748, normalized size = 340.44

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x)

[Out] result too large to display
```

maxima [B] time = 0.51, size = 389, normalized size = 1.95

$$-\frac{\frac{15\,b^3}{\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}+\frac{15\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^2-2\,ab+b^2}+\frac{3\left(\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{5}{2}}\cos\left(fx+e\right)^5-5\left(a-b+\cdots\right.\right.}{a^4-\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^3/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sqrt(a - b + b/c
os(f*x + e)^2)*cos(f*x + e)) + 15*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x +
e)/(a^2 - 2*a*b + b^2) + 3*((a - b + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5
 - 5*(a - b + b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a - b + b/
cos(f*x + e)^2)*b^2*cos(f*x + e))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^
4) - 10*((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a - b + b
/cos(f*x + e)^2)*b*cos(f*x + e))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 30*b^2/(
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)
) + 15*b/((a^2 - 2*a*b + b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)))
/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^5}{{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.129 $\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{2b(3a+b)\sec(e+fx)}{3f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}+\frac{\cos^3(e+fx)}{3f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{(3a+b)\cos(e+fx)}{3f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}$$

```
[Out] -1/3*(3*a+b)*cos(f*x+e)/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)^(1/2)+1/3*cos(f*x+e)
^3/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(1/2)-2/3*b*(3*a+b)*sec(f*x+e)/(a-b)^3/f/(a
-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3664, 453, 271, 191}

$$-\frac{2b(3a+b)\sec(e+fx)}{3f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}+\frac{\cos^3(e+fx)}{3f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{(3a+b)\cos(e+fx)}{3f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((3*a + b)*Cos[e + f*x])/(3*(a - b)^2*f*Sqrt[a - b + b*Sec[e + f*x]^2]) +
Cos[e + f*x]^3/(3*(a - b)*f*Sqrt[a - b + b*Sec[e + f*x]^2]) - (2*b*(3*a + b
)*Sec[e + f*x])/(3*(a - b)^3*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{3(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{(3a+b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+ \right.}{3(a-b)f}$$

$$= -\frac{(3a+b)\cos(e+fx)}{3(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(2b(3}{}$$

$$= -\frac{(3a+b)\cos(e+fx)}{3(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{}{3(a-}$$

Mathematica [A] time = 1.19, size = 106, normalized size = 0.81

$$-\frac{\sec(e+fx)\left(8\left(a^2-b^2\right)\cos(2(e+fx))+9a^2-(a-b)^2\cos(4(e+fx))+46ab+9b^2\right)}{12\sqrt{2}\,f(a-b)^3\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/12*((9*a^2 + 46*a*b + 9*b^2 + 8*(a^2 - b^2)*Cos[2*(e + f*x)] - (a - b)^2
*Cos[4*(e + f*x)])*Sec[e + f*x])/(Sqrt[2]*(a - b)^3*f*Sqrt[(a + b + (a - b)
*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

fricas [A] time = 0.65, size = 158, normalized size = 1.21

$$\frac{\left(\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^5-\left(3\,a^2-2\,ab-b^2\right)\cos\left(fx+e\right)^3-2\left(3\,ab+b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left(\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)f\cos\left(fx+e\right)^2+\left(a^3b-3\,a^2b^2+3\,ab^3-b^4\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - (3*a^2 - 2*a*b - b^2)*cos(f*x + e
)^3 - 2*(3*a*b + b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 +
(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^3}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^3/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

maple [B] time = 1.99, size = 14991, normalized size = 114.44

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.76, size = 216, normalized size = 1.65

$$-\frac{\frac{3\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^2-2\,ab+b^2}-\frac{\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3-6\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\,b\cos\left(fx+e\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{3\,b^2}{\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^2 - 2*a*b + b^2) - (
(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a - b + b/cos(f*x
+ e)^2)*b*cos(f*x + e))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 3*b^2/((a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)) + 3*b/((
a^2 - 2*a*b + b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^3}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.130 $\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{2b\sec(e+fx)}{f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cos(e+fx)}{f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}$$

```
[Out] -cos(f*x+e)/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(1/2)-2*b*sec(f*x+e)/(a-b)^2/f/(a-
b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3664, 271, 191}

$$-\frac{2b\sec(e+fx)}{f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cos(e+fx)}{f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cos[e + f*x]/((a - b)*f*Sqrt[a - b + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f
*x])/((a - b)^2*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^{3/2}}\,dx,x,\sec(e+fx)\right)}{f}\\
&= -\frac{\cos(e+fx)}{(a-b)f\sqrt{a-b+b\sec^2(e+fx)}}-\frac{(2b)\operatorname{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^{3/2}}\,dx,x,\sec(e+fx)\right)}{(a-b)f}\\
&= -\frac{\cos(e+fx)}{(a-b)f\sqrt{a-b+b\sec^2(e+fx)}}-\frac{2b\sec(e+fx)}{(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 1.61, size = 72, normalized size = 0.95

$$-\frac{\sec(e+fx)((a-b)\cos(2(e+fx))+a+3b)}{\sqrt{2}f(a-b)^2\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a + 3*b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x])/(Sqrt[2]*(a - b)^2*f*
Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]))
```

fricas [A] time = 0.57, size = 104, normalized size = 1.37

$$-\frac{\left((a-b)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{\left(a^3-3\,a^2b+3\,ab^2-b^3\right)f\cos\left(fx+e\right)^2+\left(a^2b-2\,ab^2+b^3\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cos(f*x + e)^2 + (a^
2*b - 2*a*b^2 + b^3)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(2*(-(-8*b^3*sign(tan((f*x+exp(1))/
2)^2-1)+8*a*b^2*sign(tan((f*x+exp(1))/2)^2-1))/(64*b^4-192*a*b^3+192*a^2*b^
2-64*a^3*b)-tan((f*x+exp(1))/2)^2*(-8*b^3*sign(tan((f*x+exp(1))/2)^2-1)+8*a
*b^2*sign(tan((f*x+exp(1))/2)^2-1))/(64*b^4-192*a*b^3+192*a^2*b^2-64*a^3*b)
)/sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1
))/2)^2+a)+2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a)+sqrt(a*tan((f*x+exp(1)
)/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))/(-2*a*sign(t
an((f*x+exp(1))/2)^2-1)+2*b*sign(tan((f*x+exp(1))/2)^2-1))/(-2*sqrt(a)*(-sq
rt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1
))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))+(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(
a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2
+a))^2-3*a+4*b))
```

maple [A] time = 0.35, size = 103, normalized size = 1.36

$$-\frac{\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+2b\right)}{f\left(\frac{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3\left(a-b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+2*b)/
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/cos(f*x+e)^3/(a-b)^2
```

maxima [A] time = 0.63, size = 83, normalized size = 1.09

$$-\frac{\frac{\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^2-2\,ab+b^2}+\frac{b}{\left(a^2-2\,ab+b^2\right)\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^2 - 2*a*b + b^2) + b/((a^2
 - 2*a*b + b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)}{{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{\left(e + fx \right)}}{\left(a + b \tan^{2}{\left(e + fx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.131 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{3/2}f}-\frac{b\sec(e+fx)}{af(a-b)\sqrt{a+b\sec^2(e+fx)-b}}$$

```
[Out] -arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(3/2)/f-b*sec(f*x
+e)/a/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3664, 382, 377, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{3/2}f}-\frac{b\sec(e+fx)}{af(a-b)\sqrt{a+b\sec^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]/(a^(3/2)*f
)) - (b*Sec[e + f*x])/(a*(a - b)*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{a(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{af}$$

$$= -\frac{b\sec(e+fx)}{a(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-1+ax^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{af}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\sec(e+fx)}{a(a-b)f\sqrt{a-b+b\sec^2(e+fx)}}$$

Mathematica [B] time = 5.77, size = 333, normalized size = 3.96

$$-\frac{\cos(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\sqrt{2}\sqrt{a}\,b(\cos(e+fx)+1)\sqrt{\sec^4\left(\frac{1}{2}\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(Cos[e + f*x]*Sec[(e + f*x)/2]^6*((a - b)*ArcTanh[(a - (a - 2*b)*Tan[(
e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)
/2]^2)^2])]*(a + b + (a - b)*Cos[2*(e + f*x)]) + (a - b)*ArcTanh[(2*b + a*(
-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Ta
n[(e + f*x)/2]^2)^2])]*(a + b + (a - b)*Cos[2*(e + f*x)]) + Sqrt[2]*Sqrt[a]
*b*(1 + Cos[e + f*x])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)
/2]^4])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(a^(3/2)*(
a - b)*f*((a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4)^(3/2))
```

fricas [B] time = 0.75, size = 355, normalized size = 4.23

$$\left[-\frac{2\,ab\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right) - \left(\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^2+ab-b^2\right)\sqrt{a}\log\left(-\frac{2\left((a-b)\cos\left(fx+e\right)^2-2\sqrt{a}\right.}{\mathrm{co}}\right.}{2\left(\left(a^4-2\,a^3b+a^2b^2\right)f\cos\left(fx+e\right)^2+\left(a^3b-a^2b^2\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
 - ((a^2 - 2*a*b + b^2)*cos(f*x + e)^2 + a*b - b^2)*sqrt(a)*log(-2*((a - b)
*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)))/((a^4 - 2*a^3*b + a^2*b^2)*
```

```
f*cos(f*x + e)^2 + (a^3*b - a^2*b^2)*f), -(a*b*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e) - ((a^2 - 2*a*b + b^2)*cos(f*x + e)^2 +
a*b - b^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e)/a))/((a^4 - 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (
a^3*b - a^2*b^2)*f)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
```

```
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assu
mes constant sign by intervals (correct if the argument is real):Check [abs
(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^2-1 were not checkedWa
rning, integration of abs or sign assumes constant sign by intervals (corre
ct if the argument is real):Check [abs(t_nostep^2-1)]Warning, need to choos
e a branch for the root of a polynomial with parameters. This might be wron
g.The choice was done assuming [a,b]=[85,53]Warning, need to choose a branc
h for the root of a polynomial with parameters. This might be wrong.The cho
ice was done assuming [a,b]=[-33,71]Discontinuities at zeroes of t_nostep^2
-1 were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Warning, integration of abs or sign assumes constant sign by intervals (c
orrect if the argument is real):Check [abs(t_nostep^2-1)]Warning, replacing
 0 by ` u`, a substitution variable should perhaps be purged.Warning, repla
cing 0 by ` u`, a substitution variable should perhaps be purged.Warning, r
eplacing 0 by ` u`, a substitution variable should perhaps be purged.Evalua
tion time: 3.85Error: Bad Argument Type
```

maple [B] time = 1.19, size = 3491, normalized size = 41.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f*(2*a^(5/2)*cos(f*x+e)^2*b+cos(f*x+e)^3*ln(-2*(-1+cos(f*x+e))*(a^(1/2
)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e
)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(1+cos(f*x+e))^2)^(1/2)*a^3-2*cos(f*x+e)^3*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*c
os(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b
*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+
cos(f*x+e))^2)^(1/2)*a^2*b+cos(f*x+e)^3*ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(
f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*co
s(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos
(f*x+e))^2)^(1/2)*a*b^2+cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+
cos(f*x+e))^2)^(1/2)*ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*
x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*a^3-2*
cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(
-4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)
^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a
*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*a^2*b+cos(f*x+e)^3*((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(a^(1/2)*cos(f*x+e)*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)
+b)/(-1+cos(f*x+e)))*a*b^2-2*a^(3/2)*cos(f*x+e)^2*b^2+cos(f*x+e)^2*ln(-2*(-
```

```
1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos
(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2
)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^3-2*cos(f*x+e)^2*ln(-2*(-1+c
os(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*
x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a
^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^2*b+cos(f*x+e)^2*ln(-2*(-1+cos(
f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e
))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1
/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a*b^2+cos(f*x+e)^2*((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-
1+cos(f*x+e)))*a^3-2*cos(f*x+e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos
(f*x+e))^2)^(1/2)*ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e
))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*a^2*b+cos
(f*x+e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*
(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1
/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*co
s(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*a*b^2+cos(f*x+e)*ln(-2*(-1+cos(f*
x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))
^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2
)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^2*b-cos(f*x+e)*ln(-2*(-1+cos(f*x+e))
*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(
1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*c
os(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a*b^2+cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x
+e)))*a^2*b-cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)
^(1/2)*ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(
f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)
*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*a*b^2+2*b^2*a^(3/2)+
ln(-2*(-1+cos(f*x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))
^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^2*b-ln(-2*(-1+cos(f*
x+e))*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))
^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2
)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a*b^2+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(a^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+
cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*
a^2*b-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(a^(
1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*
x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*a*b^2)/cos(f*x+e)^3/((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/a^(5/2)/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)\,{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2)),x)
[Out] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc{\left(e + f x \right)}}{\left(a + b \tan^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Integral(csc(e + f*x)/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.132 $\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{5/2}f}-\frac{3b\sec(e+fx)}{2a^2f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b\sec^2(e+fx)-b}}$$

```
[Out] -1/2*(a-3*b)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(5/2)
/f-1/2*cot(f*x+e)*csc(f*x+e)/a/f/(a-b+b*sec(f*x+e)^2)^(1/2)-3/2*b*sec(f*x+e
)/a^2/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3664, 471, 527, 12, 377, 207}

$$-\frac{3b\sec(e+fx)}{2a^2f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{5/2}f}-\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b\sec^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a - 3*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])
/(2*a^(5/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*Sqrt[a - b + b*Sec[e +
f*x]^2]) - (3*b*Sec[e + f*x])/(2*a^2*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-b-2bx^2}{\left(-1+x^2\right)\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2a^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{(a-3}{(-1+x^2}\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2a^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{(a-3b)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2a^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{(a-3b)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{3b\,\text{s}}{2a^2f\sqrt{a-b}}$$

Mathematica [B] time = 4.61, size = 308, normalized size = 2.43

$$-\frac{\frac{(a-3b)\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\tanh^{-1}\left(\frac{a-(a-2b)\tan^2\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a}\sqrt{a\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-1\right)^2+4b\tan^2\left(\frac{1}{2}(e+fx)\right)}}\right)+\tanh^{-1}\left(\frac{a(\tan}{\sqrt{a}\sqrt{a(\tan^2(\frac{1}{2}}}\right.\right.}{2a^{5/2}\sqrt{\sec^4\left(\frac{1}{2}(e+fx)\right)((a-b)\cos(2(e+fx))+a+b)}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(((a + 3*b + (a - 3*b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2*Sec[e + f*x])/
(Sqrt[2]*a^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]) + ((a
 - 3*b)*(ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(
e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + T
an[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e +
```

```
 f*x)/2]^2)^2])])*Cos[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos
[2*(e + f*x)])*Sec[e + f*x]^2])/(2*a^(5/2)*Sqrt[(a + b + (a - b)*Cos[2*(e +
 f*x)])*Sec[(e + f*x)/2]^4]))/f
```

fricas [A] time = 0.77, size = 455, normalized size = 3.58

$$\left[-\frac{\left(\left(a^2-4\,ab+3\,b^2\right)\cos\left(fx+e\right)^4-\left(a^2-5\,ab+6\,b^2\right)\cos\left(fx+e\right)^2-ab+3\,b^2\right)\sqrt{a}\,\log\left(-\frac{2\left((a-b)\cos\left(fx+e\right)^2+2\right.}{}\right.}{4\left(\left(a^4-a^3b\right)f\cos\left(fx+e\right)^4-a^3bf\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^4 - (a^2 - 5*a*b + 6*b^2)*cos(f*
x + e)^2 - a*b + 3*b^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 + 2*sqrt(a)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(co
s(f*x + e)^2 - 1)) - 2*((a^2 - 3*a*b)*cos(f*x + e)^3 + 3*a*b*cos(f*x + e))*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 - a^3*b)*f*cos(f*x
 + e)^4 - a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x + e)^2), 1/2*(((a^2 - 4*a*b +
 3*b^2)*cos(f*x + e)^4 - (a^2 - 5*a*b + 6*b^2)*cos(f*x + e)^2 - a*b + 3*b^2
)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e)/a) + ((a^2 - 3*a*b)*cos(f*x + e)^3 + 3*a*b*cos(f*x + e))*sqr
t(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 - a^3*b)*f*cos(f*x +
e)^4 - a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
```

```
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assu
mes constant sign by intervals (correct if the argument is real):Check [abs
(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^2-1 were not checkedWa
rning, integration of abs or sign assumes constant sign by intervals (corre
ct if the argument is real):Check [abs(t_nostep^2-1)]Warning, need to choos
e a branch for the root of a polynomial with parameters. This might be wron
g.The choice was done assuming [a,b]=[85,53]Warning, need to choose a branc
h for the root of a polynomial with parameters. This might be wrong.The cho
ice was done assuming [a,b]=[-33,71]Discontinuities at zeroes of t_nostep^2
-1 were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Warning, integration of abs or sign assumes const
ant sign by intervals (correct if the argument is real):Check [abs(t_nostep
^2-1)]Evaluation time: 5.61index.cc index_m i_lex_is_greater Error: Bad Arg
ument Value
```

maple [B] time = 1.19, size = 5633, normalized size = 44.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^3\,{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2)),x)
[Out] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}{\left(e + f x \right)}}{\left(a + b \tan^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Integral(csc(e + f*x)**3/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.133 $\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$-\frac{3(a-5b)(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{7/2}f}-\frac{b(13a-15b)\sec(e+fx)}{8a^3f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{c}{4a}$$

```
[Out] -3/8*(a-5*b)*(a-b)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/a
^(7/2)/f-5/8*(a-b)*cot(f*x+e)*csc(f*x+e)/a^2/f/(a-b+b*sec(f*x+e)^2)^(1/2)-1
/4*cot(f*x+e)^3*csc(f*x+e)/a/f/(a-b+b*sec(f*x+e)^2)^(1/2)-1/8*(13*a-15*b)*b
*sec(f*x+e)/a^3/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3664, 470, 527, 12, 377, 207}

$$-\frac{b(13a-15b)\sec(e+fx)}{8a^3f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{3(a-5b)(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{7/2}f}-\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{c}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a - 5*b)*(a - b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e +
 f*x]^2]])/(8*a^(7/2)*f) - (5*(a - b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f*S
qrt[a - b + b*Sec[e + f*x]^2]) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*a*f*Sqrt[
a - b + b*Sec[e + f*x]^2]) - ((13*a - 15*b)*b*Sec[e + f*x])/(8*a^3*f*Sqrt[a
 - b + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-a+b-4(a-b)x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{4af}$$

$$= -\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(13a-1}{8a^3f\sqrt{a-}}$$

$$= -\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(13a-1}{8a^3f\sqrt{a-}}$$

$$= -\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(13a-1}{8a^3f\sqrt{a-}}$$

$$= -\frac{3(a-5b)(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{7/2}f} - \frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}}$$

Mathematica [A] time = 5.09, size = 350, normalized size = 1.87

$$\frac{\frac{\csc^4(e+fx)\sec(e+fx)\left(\left(-8a^2+52ab-60b^2\right)\cos(2(e+fx))+(a-b)(3(a-5b)\cos(4(e+fx))-11a-45b)\right)}{4\sqrt{2}a^3\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}} - \frac{3(a-5b)(a-b)\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec^2}}{}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((((-8*a^2 + 52*a*b - 60*b^2)*Cos[2*(e + f*x)] + (a - b)*(-11*a - 45*b + 3*
(a - 5*b)*Cos[4*(e + f*x)]))*Csc[e + f*x]^4*Sec[e + f*x])/(4*Sqrt[2]*a^3*Sq
rt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]) - (3*(a - 5*b)*(a -
b)*(ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f
*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[(e
 + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)
/2]^2)^2])])*Cos[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e
 + f*x)])*Sec[e + f*x]^2])/(2*a^(7/2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)
])*Sec[(e + f*x)/2]^4]))/(8*f)
```

fricas [B] time = 1.12, size = 705, normalized size = 3.77

$$\left[\frac{3\left(\left(a^3-7a^2b+11ab^2-5b^3\right)\cos\left(fx+e\right)^6-\left(2a^3-15a^2b+28ab^2-15b^3\right)\cos\left(fx+e\right)^4+a^2b-6ab^2+5b^3 \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^3 - 7*a^2*b + 11*a*b^2 - 5*b^3)*cos(f*x + e)^6 - (2*a^3 - 15*a
^2*b + 28*a*b^2 - 15*b^3)*cos(f*x + e)^4 + a^2*b - 6*a*b^2 + 5*b^3 + (a^3 -
 9*a^2*b + 23*a*b^2 - 15*b^3)*cos(f*x + e)^2)*sqrt(a)*log(-2*((a - b)*cos(f
*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) + 2*(3*(a^3 - 6*a^2*b + 5*a*b^2)*c
os(f*x + e)^5 - (5*a^3 - 31*a^2*b + 30*a*b^2)*cos(f*x + e)^3 - (13*a^2*b -
15*a*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*x + e
)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2), 1/8*(3*((a^3 - 7*a^2*b + 11*a*b^2
 - 5*b^3)*cos(f*x + e)^6 - (2*a^3 - 15*a^2*b + 28*a*b^2 - 15*b^3)*cos(f*x +
e)^4 + a^2*b - 6*a*b^2 + 5*b^3 + (a^3 - 9*a^2*b + 23*a*b^2 - 15*b^3)*cos(f*
x + e)^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e)/a) + (3*(a^3 - 6*a^2*b + 5*a*b^2)*cos(f*x + e)^5 - (
5*a^3 - 31*a^2*b + 30*a*b^2)*cos(f*x + e)^3 - (13*a^2*b - 15*a*b^2)*cos(f*x
 + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^5 - a^4*b)*f*
cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^
4*b)*f*cos(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
```

```
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*(tan((f*x+exp
(1))/2)^2*(tan((f*x+exp(1))/2)^2*(-tan((f*x+exp(1))/2)^2*(-536870912*a^7*b^
2+536870912*a^8*b)/(137438953472*a^8*b^2*sign(tan((f*x+exp(1))/2)^2-1)-1374
38953472*a^9*b*sign(tan((f*x+exp(1))/2)^2-1))-(5368709120*a^6*b^3-912680550
4*a^7*b^2+3758096384*a^8*b)/(137438953472*a^8*b^2*sign(tan((f*x+exp(1))/2)^
2-1)-137438953472*a^9*b*sign(tan((f*x+exp(1))/2)^2-1)))-(64424509440*a^5*b^
4-133143986176*a^6*b^3+77846282240*a^7*b^2-9126805504*a^8*b)/(137438953472*
a^8*b^2*sign(tan((f*x+exp(1))/2)^2-1)-137438953472*a^9*b*sign(tan((f*x+exp(
1))/2)^2-1)))-(34359738368*a^5*b^4-61203283968*a^6*b^3+22011707392*a^7*b^2+
4831838208*a^8*b)/(137438953472*a^8*b^2*sign(tan((f*x+exp(1))/2)^2-1)-13743
8953472*a^9*b*sign(tan((f*x+exp(1))/2)^2-1)))/sqrt(a*tan((f*x+exp(1))/2)^4-
2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a)+2*(-1/64*(-6*a^3*(-s
qrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(
1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))-18*a*b^2*(-sqrt(a)*tan((f*x+exp(1))/
2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+ex
p(1))/2)^2+a))+20*a^2*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp
(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))+4*a^2*(-s
qrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(
1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3+14*b^2*(-sqrt(a)*tan((f*x+exp(1))/
2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+ex
p(1))/2)^2+a))^3-16*a*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp
(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3+3*sqrt(
a)*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan
((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2-4*sqrt(a)*a*b*(-sqrt(a)*
tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^
2+4*b*tan((f*x+exp(1))/2)^2+a))^2-5*sqrt(a)*a^3+8*sqrt(a)*a^2*b)/a^3/(-(-sq
rt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1
))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2+a)^2/sign(tan((f*x+exp(1))/2)^2-1)-
1/32*(3*a^2+15*b^2-18*a*b)*atan((-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan(
(f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))/s
qrt(-a))/a^3/sqrt(-a)/sign(tan((f*x+exp(1))/2)^2-1)-1/64*(3*sqrt(a)*a^2+15*
sqrt(a)*b^2-18*sqrt(a)*a*b)*ln(abs(a*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a
*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+
a))+sqrt(a)*a-2*sqrt(a)*b))/a^4/sign(tan((f*x+exp(1))/2)^2-1)))
```

maple [B] time = 1.25, size = 10582, normalized size = 56.59

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Integral(csc(e + f*x)**5/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.134 $\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$-\frac{b(13a+2b)\tan(e+fx)}{8f(a-b)^3\sqrt{a+b\tan^2(e+fx)}}+\frac{3a(a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{7/2}}+\frac{\sin(e+fx)\cos^3(e+fx)}{4f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{5a\sin}{8f(a-}$$

```
[Out] 3/8*a*(a+4*b)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)
^(7/2)/f-5/8*a*cos(f*x+e)*sin(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1/4
*cos(f*x+e)^3*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-1/8*b*(13*a+2*b)*
tan(f*x+e)/(a-b)^3/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3663, 470, 527, 12, 377, 203}

$$\frac{3a(a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{7/2}}-\frac{b(13a+2b)\tan(e+fx)}{8f(a-b)^3\sqrt{a+b\tan^2(e+fx)}}+\frac{\sin(e+fx)\cos^3(e+fx)}{4f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{5a\sin(}{8f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a*(a + 4*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/(8*(a - b)^(7/2)*f) - (5*a*Cos[e + f*x]*Sin[e + f*x])/(8*(a - b)^2*f*Sqr
t[a + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*(a - b)*f*Sqrt[
a + b*Tan[e + f*x]^2]) - (b*(13*a + 2*b)*Tan[e + f*x])/(8*(a - b)^3*f*Sqrt[
a + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
```

```
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^3\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{a-4ax^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= -\frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a(}{(1}\right.}{}$$

$$= -\frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{b(13a+}{8(a-b)^3f\sqrt{}}$$

$$= -\frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{b(13a+}{8(a-b)^3f\sqrt{}}$$

$$= -\frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{b(13a+}{8(a-b)^3f\sqrt{}}$$

$$= \frac{3a(a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{7/2}f} - \frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e}{4(a-b)f}$$

Mathematica [C] time = 3.59, size = 325, normalized size = 1.74

$$\sin(2(e+fx))\sec^2(e+fx)\left(-(a-b)\left(\left(6a^2-2ab-4b^2\right)\cos(2(e+fx))+7a^2-(a-b)^2\cos(4(e+fx))+48ab\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-((a - b)*(7*a^2 + 48*a*b + 5*b^2 + (6*a^2 - 2*a*b - 4*b^2)*Cos[2*(e + f*
x)] - (a - b)^2*Cos[4*(e + f*x)])) + 6*Sqrt[2]*a*(a^2 + 3*a*b - 4*b^2)*Sqrt
[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sq
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - 6*
Sqrt[2]*a^2*(a + 4*b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*
x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(32*
Sqrt[2]*(a - b)^4*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]
)
```

fricas [B] time = 128.62, size = 1046, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/64*(3*(a^2*b + 4*a*b^2 + (a^3 + 3*a^2*b - 4*a*b^2)*cos(f*x + e)^2)*sqrt(
-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8
- 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*
a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*
a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2
 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*
(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 2
4*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*co
s(f*x + e)^5 - 5*(a^3 - 2*a^2*b + a*b^2)*cos(f*x + e)^3 - (13*a^2*b - 11*a*
b^2 - 2*b^3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*
f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f), 1/32
*(3*(a^2*b + 4*a*b^2 + (a^3 + 3*a^2*b - 4*a*b^2)*cos(f*x + e)^2)*sqrt(a - b
)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^
2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((
a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b
^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4
*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)
*cos(f*x + e)^5 - 5*(a^3 - 2*a^2*b + a*b^2)*cos(f*x + e)^3 - (13*a^2*b - 11
*a*b^2 - 2*b^3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^
5)*f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 16.16, size = 6027, normalized size = 32.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^4}{\left(b\,\tan\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{4}\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.135 $\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{3b\tan(e+fx)}{2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{5/2}}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] 1/2*(a+2*b)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(
5/2)/f-1/2*cos(f*x+e)*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-3/2*b*tan
(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3663, 471, 527, 12, 377, 203}

$$\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{5/2}}-\frac{3b\tan(e+fx)}{2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(
2*(a - b)^(5/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a - b)*f*Sqrt[a + b*Ta
n[e + f*x]^2]) - (3*b*Tan[e + f*x])/(2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^
2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-2bx^2}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{(1}\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{(a+2b)\,\text{Sul}}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{(a+2b)\,\text{Sul}}{}$$

$$= \frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{5/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{3b\tan}{2(a-b)^2f\sqrt{a}}$$

Mathematica [C] time = 2.92, size = 282, normalized size = 2.10

$$-\frac{\sin(2(e+fx))\sec^2(e+fx)\left(-\sqrt{2}\left(a^2+ab-2b^2\right)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc}{b}}}{\sqrt{2}}\right.\right.\right.}{4\sqrt{2}f(a-b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*(((a - b)*(a + 5*b + (a - b)*Cos[2*(e + f*x)]) - Sqrt[2]*(a^2 + a*b -
2*b^2)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Elliptic
F[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2
]], 1] + Sqrt[2]*a*(a + 2*b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e
 + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)
])/(Sqrt[2]*(a - b)^3*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x
]^2])
```

fricas [B] time = 6.27, size = 908, normalized size = 6.78

$$\left[-\frac{\left(\left(a^2+ab-2\,b^2\right)\cos\left(fx+e\right)^2+ab+2\,b^2\right)\sqrt{-a+b}\,\log\left(128\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)\cos\left(fx+e\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((a^2 + a*b - 2*b^2)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(-a + b)*log
(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4
- 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^
3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160
*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3
 + 16*b^4)*cos(f*x + e)^2 + 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x +
 e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29
*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 1
6*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)) + 8*((a^2 - 2*a*b + b^2)*cos(f*x + e)^3 + 3*(a*b - b
^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3
*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f), 1/8*(((a^2 + a*b - 2*b^2)*cos(f*x + e)^
2 + a*b + 2*b^2)*sqrt(a - b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e
)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*
x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (
a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) - 4*((a^2 -
 2*a*b + b^2)*cos(f*x + e)^3 + 3*(a*b - b^2)*cos(f*x + e))*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2
 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f)
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 5.32, size = 1614, normalized size = 12.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*
b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a
)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f
*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2
*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)
-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*sin(f*x+e)-2*2^(1
/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b
*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2
)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2
)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(
f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*
b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*a*sin(f*x+e)+cos(f*x
+e)*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b-((2*I*(a-b)^(1/2)*b^(1/2)+a
-2*b)/a)^(1/2)*b)*sin(f*x+e)/(-1+cos(f*x+e))/((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/cos(f*x+e)^2)^(3/2)/cos(f*x+e)^3/a/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)
^(1/2)/(a-b)+1/4/f*(cos(2*f*x+2*e)^2*(a-b)^(3/2)*a^3*b-2*(a-b)^(3/2)*cos(2*
f*x+2*e)^2*a^2*b^2+(a-b)^(3/2)*cos(2*f*x+2*e)^2*a*b^3+2*(a-b)^(3/2)*cos(2*f
*x+2*e)*a^2*b^2+2*(a-b)^(3/2)*cos(2*f*x+2*e)*a*b^3-4*(a-b)^(3/2)*cos(2*f*x+
2*e)*b^4-4*(b^4*(a-b))^(1/2)*arctan((-1+cos(2*f*x+2*e))/((a*cos(2*f*x+2*e)-
b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)*b^2/(b
^4*(a-b))^(1/2))*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(c
os(2*f*x+2*e)+1))^(1/2)*a*(a-b)^(3/2)+2*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)
+a+b)/(cos(2*f*x+2*e)+1))^(1/2)*arctan((-1+cos(2*f*x+2*e))/((a*cos(2*f*x+2*
e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/
2))*sin(2*f*x+2*e)*a^4*b-8*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*
f*x+2*e)+1))^(1/2)*arctan((-1+cos(2*f*x+2*e))/((a*cos(2*f*x+2*e)-b*cos(2*f*
x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*sin(2*f*x
+2*e)*a^3*b^2+10*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1
))^(1/2)*arctan((-1+cos(2*f*x+2*e))/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b
)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*sin(2*f*x+2*e)*a^2*
b^3-4*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)*ar
ctan((-1+cos(2*f*x+2*e))/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*
x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*sin(2*f*x+2*e)*a*b^4-(a-b)^(3/
2)*a^3*b-3*(a-b)^(3/2)*a*b^3+4*(a-b)^(3/2)*b^4)/sin(2*f*x+2*e)/((a*cos(2*f*
x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/(a-b)^(9/2)/a/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.136 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\tan(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*tan
(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3661, 382, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\tan(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) - (b*Tan[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [C] time = 7.46, size = 214, normalized size = 2.52

$$\frac{4\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(\frac{15\left(3a+2b\tan^2(e+fx)\right)\left(a\sqrt{\frac{(a-b)\sin^2(2(e+fx))\left(a+b\tan^2(e+fx)\right)}{a^2}}-2\sin^{-1}\left(\sqrt{\frac{(a-b)\sin^2(e\cdots}{a}}\right.\right.}{\left(\frac{(a-b)\sin^2(2(e+fx))\left(a+b\tan^2(e+fx)\right)}{a^2}\right)^{3/2}}\cdots}{15a^4f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] (4*Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2]*(a*(a - b)*Hyperg
eometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^2 + (15*(3*
a + 2*b*Tan[e + f*x]^2)*(-2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*(a*Cos
[e + f*x]^2 + b*Sin[e + f*x]^2) + a*Sqrt[((a - b)*Sin[2*(e + f*x)]^2*(a + b
*Tan[e + f*x]^2))/a^2]))/(((a - b)*Sin[2*(e + f*x)]^2*(a + b*Tan[e + f*x]^2
))/a^2)^(3/2)))/(15*a^4*f)
```

fricas [A] time = 0.48, size = 310, normalized size = 3.65

$$\left[\frac{\left(ab\tan\left(fx+e\right)^2+a^2\right)\sqrt{-a+b}\,\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\,\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^2+1}\right)-2\,\sqrt{b\tan\left(fx+e\right)^2\cdots}}{2\left(\left(a^3b-2\,a^2b^2+ab^3\right)f\tan\left(fx+e\right)^2+\left(a^4-2\,a^3b+a^2b^2\right)f\right)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^
2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x + e))/((a^3*b
 - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f), ((a*
b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqr
t(a - b)*tan(f*x + e))) - sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x +
```

```
e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^
2)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(-3/2), x)
```

maple [A] time = 0.29, size = 104, normalized size = 1.22

$$-\frac{b\tan\left(fx+e\right)}{a\left(a-b\right)f\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}+\frac{\sqrt{b^{4}\left(a-b\right)}\,\arctan\left(\frac{(a-b)b^{2}\tan\left(fx+e\right)}{\sqrt{b^{4}(a-b)}\,\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}\right)}{f\left(a-b\right)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)^2*(b^4*(a-b))^(1
/2)/b^2*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x
+e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(-3/2), x)
```

3.137 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2b\tan(e+fx)}{a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot(e+fx)}{af\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -cot(f*x+e)/a/f/(a+b*tan(f*x+e)^2)^(1/2)-2*b*tan(f*x+e)/a^2/f/(a+b*tan(f*x+
e)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3663, 271, 191}

$$-\frac{2b\tan(e+fx)}{a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot(e+fx)}{af\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cot[e + f*x]/(a*f*Sqrt[a + b*Tan[e + f*x]^2])) - (2*b*Tan[e + f*x])/(a^2*
f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)}{af\sqrt{a+b\tan^2(e+fx)}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= -\frac{\cot(e+fx)}{af\sqrt{a+b\tan^2(e+fx)}} - \frac{2b\tan(e+fx)}{a^2f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [A] time = 0.75, size = 74, normalized size = 1.19

$$-\frac{\csc(e+fx)\sec(e+fx)((a-2b)\cos(2(e+fx))+a+2b)}{\sqrt{2}\,a^2f\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a + 2*b + (a - 2*b)*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x])/(Sqrt[
2]*a^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]))
```

fricas [A] time = 0.75, size = 90, normalized size = 1.45

$$-\frac{\left((a-2\,b)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{\left(a^2bf+\left(a^3-a^2b\right)f\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a^2*b*f + (a^3 - a^2*b)*f*cos(f*x + e)^2)*sin(f*x +
 e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 0.94, size = 109, normalized size = 1.76

$$-\frac{\left(a\left(\cos^2\left(fx+e\right)\right)-2\left(\cos^2\left(fx+e\right)\right)b+2b\right)\left(\frac{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\left(\cos^3\left(fx+e\right)\right)}{f\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)^2\sin\left(fx+e\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(a*cos(f*x+e)^2-2*cos(f*x+e)^2*b+2
*b)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^3/sin
(f*x+e)/a^2
```

maxima [A] time = 0.54, size = 58, normalized size = 0.94

$$-\frac{\frac{2\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a}\,a^2}+\frac{1}{\sqrt{b\tan\left(fx+e\right)^2+a}\,a\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(2*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^2) + 1/(sqrt(b*tan(f*x + e
)^2 + a)*a*tan(f*x + e)))/f
```

mupad [B] time = 18.30, size = 2978, normalized size = 48.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2)),x)
```

```
[Out] ((a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)*
(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*(exp(e*2i + f*x*2i)*(((a +
3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))
/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2
)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (3*
(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a
- b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
))/(a - b) + (3*(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i)) - (3*((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*
b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a
*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a +
2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a
 + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a
*1i - b*1i)) + ((a - b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i))))/(a - b) + (3*(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b
)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)
^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b
)*(a + 2*b)^2)/(a*b - a^2))*(a - b))/(4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) + (3*((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b))/(8*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))) - exp(e*4i + f*x*4i)*(((a + 3*b)*((
((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
 - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
 b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (3*(a - b)^4*(a + 2*b)
)/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^3*(a + 2*b)*(a
 + 3*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a +
3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))
/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2
)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (3*
(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a
- b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
```

```
))/(a - b) + (3*(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i)) - (3*((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*
b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a
*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a +
2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a
 + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a
*1i - b*1i)) + ((a - b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^4*(a + 2*b))/(4*f*(a*b^2 - a^2*b)*
(a*b - a^2)*(a*1i - b*1i)) + (3*((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)
^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b
)*(a + 2*b)^2)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) - (3*((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b))/(8*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i))) + ((a + 3*b)*((((((a - b)*(a - 2*b) - (a +
 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) - (3*(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) + ((a - b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (3*(a - b)^4*(a + 2*b))/(8*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (3*((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b))/(4*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^3*(a + 2*b)*(a + 3*b))
/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/((exp(e*2i + f*x*2i) + 1
)*(b - a - exp(e*2i + f*x*2i)*(a + 3*b) + exp(e*4i + f*x*4i)*(a + 3*b) + ex
p(e*6i + f*x*6i)*(a - b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.138 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2b(3a-4b)\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{(3a-4b)\cot(e+fx)}{3a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -1/3*(3*a-4*b)*cot(f*x+e)/a^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*cot(f*x+e)^3/a
/f/(a+b*tan(f*x+e)^2)^(1/2)-2/3*(3*a-4*b)*b*tan(f*x+e)/a^3/f/(a+b*tan(f*x+e
)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3663, 453, 271, 191}

$$-\frac{2b(3a-4b)\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{(3a-4b)\cot(e+fx)}{3a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((3*a - 4*b)*Cot[e + f*x])/(3*a^2*f*Sqrt[a + b*Tan[e + f*x]^2]) - Cot[e +
f*x]^3/(3*a*f*Sqrt[a + b*Tan[e + f*x]^2]) - (2*(3*a - 4*b)*b*Tan[e + f*x])/
(3*a^3*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3af}$$

$$= -\frac{(3a-4b)\cot(e+fx)}{3a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}} - \frac{(2(3a-4b)b)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(3a-4b)\cot(e+fx)}{3a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}} - \frac{2(3a-4b)b\tan(e+f}{3a^3f\sqrt{a+b\tan^2(e+f}}$$

Mathematica [A] time = 0.93, size = 119, normalized size = 1.04

$$\frac{\csc^3(e+fx)\sec(e+fx)\left(-2\left(a^2-6ab+8b^2\right)\cos(2(e+fx))+\left(a^2-5ab+4b^2\right)\cos(4(e+fx))-3a^2-7ab+12b\right.}{6\sqrt{2}\,a^3f\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-3*a^2 - 7*a*b + 12*b^2 - 2*(a^2 - 6*a*b + 8*b^2)*Cos[2*(e + f*x)] + (a^2
 - 5*a*b + 4*b^2)*Cos[4*(e + f*x)])*Csc[e + f*x]^3*Sec[e + f*x])/(6*Sqrt[2]
*a^3*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

fricas [A] time = 3.41, size = 155, normalized size = 1.36

$$-\frac{\left(2\left(a^2-5\,ab+4\,b^2\right)\cos\left(fx+e\right)^5-\left(3\,a^2-16\,ab+16\,b^2\right)\cos\left(fx+e\right)^3-2\left(3\,ab-4\,b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos}{\cos(}}}{3\left(\left(a^4-a^3b\right)f\cos\left(fx+e\right)^4-a^3bf-\left(a^4-2\,a^3b\right)f\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 - 5*a*b + 4*b^2)*cos(f*x + e)^5 - (3*a^2 - 16*a*b + 16*b^2)*co
s(f*x + e)^3 - 2*(3*a*b - 4*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/(((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4 - 2
*a^3*b)*f*cos(f*x + e)^2)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.08, size = 170, normalized size = 1.49

$$\frac{\left(2\left(\cos^4\left(fx+e\right)\right)a^2-10\left(\cos^4\left(fx+e\right)\right)ab+8\left(\cos^4\left(fx+e\right)\right)b^2-3a^2\left(\cos^2\left(fx+e\right)\right)+16\left(\cos^2\left(fx+e\right)\right)}{3f\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(2*cos(f*x+e)^4*a^2-10*cos(f*x+e)
^4*a*b+8*cos(f*x+e)^4*b^2-3*a^2*cos(f*x+e)^2+16*cos(f*x+e)^2*a*b-16*b^2*cos
(f*x+e)^2-6*a*b+8*b^2)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(
f*x+e)^2)^(3/2)/sin(f*x+e)^3/a^3
```

maxima [A] time = 0.70, size = 141, normalized size = 1.24

$$-\frac{\frac{6\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a}\,a^2}-\frac{8\,b^2\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a}\,a^3}+\frac{3}{\sqrt{b\tan\left(fx+e\right)^2+a}\,a\tan\left(fx+e\right)}-\frac{4\,b}{\sqrt{b\tan\left(fx+e\right)^2+a}\,a^2\tan\left(fx+e\right)}+\frac{1}{\sqrt{b\tan\left(fx+e\right)^2+a}\,a\,t}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(6*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^2) - 8*b^2*tan(f*x + e
)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 3/(sqrt(b*tan(f*x + e)^2 + a)*a*tan(f*
x + e)) - 4*b/(sqrt(b*tan(f*x + e)^2 + a)*a^2*tan(f*x + e)) + 1/(sqrt(b*tan
(f*x + e)^2 + a)*a*tan(f*x + e)^3))/f
```

mupad [B] time = 35.94, size = 269040, normalized size = 2360.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2)),x)
```

```
[Out] ((a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)*
(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*(((a + 3*b)*(((a + 3*b)*(((
a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
 b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
- b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a
^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a
 + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a
^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a
 - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
- b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a
 + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*
(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
 + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2
))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
```

```
*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8
*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
 - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*
(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(
a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*
((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + ((
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
 - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*
(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
 - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*
(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^
2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^
5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2
*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((
(((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
 - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a
 - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
 b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b
))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)
^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^
2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a
^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 -
 a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a
+ b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(
a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a
 - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a
*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
 - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 -
 a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*
a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((
a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2
*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((
((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
 b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b
)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (
(a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*(
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*
(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a
 - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)
*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4
*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)
```

^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^
2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b
)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2
b)(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)
^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b
)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*
((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + ((
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
 - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*
(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
 - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*
(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) -
(a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(
3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b)
+ (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3
f(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a
+ 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4
f(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b
))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a -
 b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (
(a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a -
2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
 - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
 a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2
b)(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b)
)/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a +
 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2
)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(
a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2
b) - (a - b)(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072
*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (
a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 -
 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
- ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(
a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)

```
*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 1
82*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))
/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)
*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(
a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7
*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i))))/(a - b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)
^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)
^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a +
2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (
((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
 - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
 b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((
a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 3
6*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)
^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i)) + ((a - b)^4*(a + 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*
a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a -
 b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(
a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64
*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a -
b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*
(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b
)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*
1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a
^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a +
 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*
(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a
+ b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a
+ 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/
(a - b) + ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(
a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(64*a^3*b - 7
2*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*
a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2
*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
 b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(64*a*b^3 - 184*a^3*b
+ 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b)
)/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^
2)/(a*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2
*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a
 + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b
```

)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a -

```
b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2
*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*
(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b
)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a
 - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a +
 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
 + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
+ 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(5
6*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*
a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*
(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a
+ 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a
 + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b
)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 -
 a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a
 - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(
a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
 + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))
/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2
)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(
a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)
*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2
)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*
(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(
a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/
(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)
*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a -
b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a
^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*
b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a +
 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b
) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*
(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/
(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*
b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a -
 b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b)
 - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2
 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768
*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) -
 (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
```

```
 b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024
*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)
*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b)
)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a +
2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2
*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 18
2*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*
b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b
))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a
+ 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^
2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*
b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((((a - b)*
(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(307
2*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b
)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b
))/(a*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2
*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(
a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2
*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*
b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(64*a^3*b -
72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(288*a*b^3 + 888*a^3*b + 1348
*a^4 + 480*b^4 - 380*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))
/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b
) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a*b^3 - 184
*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i))))/(a - b) - exp(e*4i + f*x*4i)*(((a + 3*b)*(((a + 3*b
)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
(((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*
(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b)
- (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
- b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a
*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3
072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)
*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b
) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) -
```

```
(a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 -
 a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a
^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (
a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a
^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3
*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a
 - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) +
 ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a
- 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a
*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b
- a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
 - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a
 - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)
*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*
b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a
- b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
 b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*
b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b
))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a
+ 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^
2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*
b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(
a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)
^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 -
 a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) -
(a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2)
)*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
 + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
(((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*
(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)))
)/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
 - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(
a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3
 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)
*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
 b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2
*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b
) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*
```

```
(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4
+ 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a -
 b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4
+ 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b)
 - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a
^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((
a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^
5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2
*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a
 - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
- b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2
*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
 (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a +
2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a
 - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b
)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*
b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (
((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(
a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*
(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2
*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*
(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3
072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)
*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)
*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*
(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a -
b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9
*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) +
((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
 - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(
a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*((
(a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
 a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
- a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (
(a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) -
(a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2)
)*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b
)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
```

```
*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^
2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a
^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1
i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 -
 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) -
 (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2
*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b
)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
(((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*
(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b
)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b
)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a +
 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
(((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
- b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((
a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2
*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
 a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*
a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*
a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4
*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(
a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/
(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a
*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(
a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(102
4*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((a + 2*b)^
3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*
b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
 b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
- b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a +
2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a
- b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(240*a*b^3 +
152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(
a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
```

```
 + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 9
6*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b)
- ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
 + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
+ 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a -
b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*
b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2
*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2
*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a - b)^8*(a + 2*b))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*
(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))
/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*
b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^
2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^
4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a
- b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*
(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*
(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)
^4*(a + 2*b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b)
 - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
 - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(240*a*b^3 + 152
*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3
*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2
*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*
(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b
)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*
b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
 (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))
*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)
^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*
b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a -
b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a
+ 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
 (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + ((
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
 - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (
(a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4
+ 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((
a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*
(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
```

```
*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2
 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((((
(a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)
^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((
a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3
072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(
9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a
- b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*
(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*
(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a
*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(
a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
 + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
(((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*
(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))
/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2
*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b
) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b -
a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^
2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)
*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)
*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3
*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/
(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)
/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a -
 b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(
a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/
(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)
*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a -
b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*
a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a
+ 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)
*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a
 + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a
 + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))
/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2
))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 +
 (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
```

```
 + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
+ 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((((
(a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)
^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((
a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a +
2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
 + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a +
 b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((
a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a
*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a
*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3
072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8
*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a -
b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b)
- (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^
2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
+ (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2)
)*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
)))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*
(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(10
24*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)
^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*
b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
- b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2
*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
 (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2
*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^
4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a
 - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
- b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^
4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768
*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((
((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
 b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b
```

)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (
(a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i))))/(a - b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*(
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*
(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a
- b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)
*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4
*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2
b)^3 + (((a - b)(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2
*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a
+ 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) +
((a - b)^4*(a + 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b
^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(
a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2
b) - (a - b)(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*
b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a
- 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*
a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a +
3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(
a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)))
)/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*
(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b)
)/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a -
2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))
/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b
)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a -
b) + ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2
b) - (a - b)(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b
^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b
^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3
+ (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*
(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(
a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(64*a*b^3 - 184*a^3*b + 114
8*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*
b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a
b - a^2))(a - b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*
b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(
a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/
(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a -
b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*

```
(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b
)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*
1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a
^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a +
 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*
(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a
+ b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a
+ 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/
(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 +
(((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
+ 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((
a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^
5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2
*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a
 - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
- b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2
*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a +
b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a
 + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)
^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*
b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*
(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b
)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) -
 (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2
))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
 (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))
*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a
 + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2
 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*
a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*
a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) +
((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)
*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b)
```

)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1
i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b
)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2
b)(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3
*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*
(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b
))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a -
2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b
))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*
b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(
a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/
(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*
b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b -
 a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((
a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b
 - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*
b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
- b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2
*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a +
 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*
(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
 - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^
2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3
+ (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b)
)*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (
a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3
*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) -
 (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a
 - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a +
 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a -
 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*
b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*
b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*
(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^

```
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2
)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4
 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)
*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b)
)*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1
024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b
)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2
*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
 - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(
a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((((a - b)*(a -
2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4
*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a
- 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a
*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a - 2
*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
 (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 8
4*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 -
 a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*
b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(288*a*b^3 + 888*a^3*b + 1348*a^4
+ 480*b^4 - 380*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768
*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (
a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a*b^3 - 184*a^3*
b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b
)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3
072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(
a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)
^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2
)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(
1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*
b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a +
 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 -
 a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(
a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 -
 a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a +
```

```
2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(
a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a +
 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a
 - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (
((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
 - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
 b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a
 - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b
)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*
b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (
((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(
a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*
(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2
*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*
(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) -
(a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)
*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b)
)*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36
*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^
7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30
*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
+ (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*((((((a -
b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*
(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b
)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*
1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3*b)
*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*
b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a
*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
+ ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(
a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)
*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)
*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(30
72*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a
 + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^
7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*
(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4
*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4
*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a
 - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a +
b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
```

2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^

```
4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*
b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a
- b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) +
((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a -
 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*
b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
 a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
- a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*
b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
(a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))
/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2
*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a
- b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)
^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b
)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + ((
(a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*
(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(
a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*
(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
- a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(
3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b
)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*
a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + (
(a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2
*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*
(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))
/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b
))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)
^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)
*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)
^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^
2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)
*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*
b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a
```

```
- b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
 b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96
*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a +
2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a
 + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 -
 a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2
*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
 (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a +
2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a
 - b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(7
68*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b)
 - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
 - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(10
24*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a -
b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*
b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*
b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a
+ 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a +
 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 -
182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a +
2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
 + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a +
 b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((
a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a
*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a
*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3
072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a - b)^8
*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a +
 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 +
30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
- ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a
 - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(
a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b
 - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*
b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) + ((a - b)^4*(a + 2*b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4
 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (
((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
 - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
 b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*
((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + ((
```

```
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
 - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*
(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
 - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*
(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) -
(a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(
3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b)
+ (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a
+ 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4
*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b
))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a -
 b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (
(a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a -
2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
 - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
 a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b
) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*
(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/
(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*
b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a -
 b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^
2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)
*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)
*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((
a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
+ 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(
a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
 + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(
56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*(
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3
*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)
*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a
 + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((
a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
 b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
- b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a
```

```
^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a
 + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a
^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a
 - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
- b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a
 + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*
(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
 + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2
))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8
*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
 - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*
(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(
a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*
((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + ((
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
 - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*
(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
 - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*
(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^
2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^
5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2
*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((
(((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
- b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a
- b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
 b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b
))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)
^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^
2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a
^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 -
 a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a
+ b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(
a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a
- 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a
*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
- b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 -
 a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*
a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((a + 2*b)^3
+ (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b)
)*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b
```

```
)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*
b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a -
b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(240*a*b^3 + 15
2*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
+ 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*
a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) -
((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)
*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b)
)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1
i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b
)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2
*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2
*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a -
 b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)
^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 +
 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a -
b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 +
 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) - ((a - b)^4*(a + 2*b)*(288*a*b^3 + 888*a^3*b + 1348*a^4 + 480*b^4 - 380
*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a +
2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a
 + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 -
 a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
- b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 -
352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
)))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(
a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2
*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072
*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a +
 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*
(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(
a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(102
4*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^
```

```
3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*
b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
 b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
- b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b
))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a +
 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)
*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*
b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a -
b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((((
(a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)
^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a -
b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2
*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*
(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b
)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a
 - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a +
 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
 + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
+ 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a
+ 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2
- a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
 - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(
a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^
2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(
a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
+ ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(
a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^
4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (
((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a -
 b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a
*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*
(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(307
2*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a
+ 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7
*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
)))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (
a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3
*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) +
 ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
+ 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b
```

```
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b
)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b
))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*
1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2
*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*
b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^
3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a +
b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a -
 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b)
 - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*
b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3
*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2
*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*
(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b
)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*
b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
 (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))
*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)
^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*
b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a -
b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a
+ 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
 (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + ((
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
 - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (
(a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4
+ 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((
a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*
(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2
 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((((
(a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)
^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((
a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3
072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(
9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a
- b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*
(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*
(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a
```

```
*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(
a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
 + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
(((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*
(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))
/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2
*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b
) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b -
a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^
2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)
*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)
*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((a + 2*b)^3 + (((
a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
+ 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(
a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
 + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)
*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(5
6*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*
b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(240*a*b^3 + 152*a^3
*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b
))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)
^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b
^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a +
 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b)
)/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^
2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*
1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a -
 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*
b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) +
 ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(
a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(
a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/
(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*
b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b -
 a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(56*
a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a
- b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a
+ b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a
 - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a +
4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^4*(a +
```

```
 2*b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a +
 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b
- 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a
*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a
 + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2
 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*
a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*
a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) +
((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)
*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b)
)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1
i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b
)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3
*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*
(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b
))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a -
2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b
))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*
b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*
(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(
a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(
a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)
*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b
)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
(((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*
(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b
)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b
)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a +
 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
(((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
- b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((
a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 +
 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a
 + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2
 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(
a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
```

(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^ 5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2 *b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(30 72*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9 *a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7 *(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a* b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a *b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a *b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/ (1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2* b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^ 2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a ^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b* 1i))))/(a - b) + ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2 *(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)* (a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)* (a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f* (a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(64*a*b^3 - 184* a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)* (a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b)) *(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2)) *(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2 *b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f* (a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(240*a*b^ 3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(2404*a^3*b - 196*a*b^3 + 401*a^4 - 363*b^4 + 698*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^ 2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^ 2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(288*a*b^3 + 888*a^3*b + 1348*a^4 + 480*b^4 - 380*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(

```
(((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
- b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(
a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
- ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(
a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (
a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3
*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) +
 (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a
 - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a +
 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a -
b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((
a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a
*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2
*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
- a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a
^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b)
 - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
 - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(
a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(
3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b
)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a -
b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2
*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*
(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b
)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a
 - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a +
 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
 + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
+ 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(5
6*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*
a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*
(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a
+ 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a
 + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b
)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
```

```
a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a
- 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(
a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
+ ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))
/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2
)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(
a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)
*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2
)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*
(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(
a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/
(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)
*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a -
b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a
^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*
b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b
) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*
(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/
(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*
b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a -
b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2
- a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768
*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) -
(a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024
*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)
*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b)
)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a +
2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2
*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 18
2*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*
b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b
))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a
+ 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^
2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*
b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(
a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2
```

b)^3 + (((a - b)(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(288*a*b^3 + 888*a^3*b + 1348*a^4 + 480*b^4 - 380*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(48*a*b^3 - 1904*a^3*b + 399*a^4 - 145*b^4 + 66*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(2404*a^3*b - 196*a*b^3 + 401*a^4 - 363*b^4 + 698*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2

```
*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*
b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(
768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b
) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1
024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(
a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b)
- (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(
a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b
)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a
^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b
)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b
 - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)
^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)
^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a -
b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/
(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)
/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*
b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
 (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))
*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)
^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b)
 - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
 - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(
a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(
3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b
)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a -
b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2
- a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*
a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) -
(a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*
a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a +
3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((
a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
+ ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a
 - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(
a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b
 - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*
b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a -
2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4
*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b
))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a +
 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(
```

```
a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
 b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2
*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 +
(((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a
*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)
*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^
2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)
*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(
3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b
)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2
)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4
*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a -
 b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(
a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(
9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) +
 ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
+ 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b
)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b
))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*
1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2
*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*
b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*
b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b
)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^
2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a +
2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((
a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 +
96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a
+ 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2
 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a -
 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b)
 - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^
4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*
b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a
+ 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/
(a - b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a +
2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a
*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/
(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*
b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
```

```
(a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(
1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a
- b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a +
2*b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*
a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a +
3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(
a + 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a
 + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3
- 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a
+ 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2
- a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a
^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a
 + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (
((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a -
 b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2
*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*
(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/
(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a - b)
^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a
 + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4
+ 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) - ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*
(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))
/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a
*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a
*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(
a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i)) + ((a - b)^4*(a + 2*b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b
^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) +
 (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + ((
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
 - b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b
)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
(((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*
(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b)
- (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
- b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a
*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3
072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)
*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b
) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) -
 (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 -
 a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a
^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (
a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a
```

^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3
*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a
 - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) +
 ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a
- 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a
b - a^2))(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b
- a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
 - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2
*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
 (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a +
2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a
 - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b
)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*
b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4
f(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (
((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(
a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*
(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2
*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*
(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3
072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)
*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)
((a - b)(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*
(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a -
b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9
*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) +
((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
 - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(
a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*((
(a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
 a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
- a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (
(a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a
b - a^2)(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) -
(a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2)
)*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b
)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
1i)) - (((((a - b)(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^
2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a
^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1
i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 -

8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) -
(a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((((a - b)*(a - 2*b) - (a + 2
b)^2)(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b
)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
(((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*
(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b
)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b
)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a +
2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
(((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
- b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((
a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
- b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2
*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2
- a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*
a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*
a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4
*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4
f(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(
a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/
(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a
*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(
a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(102
4*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((a + 2*b)^
3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*
b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
- b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a +
2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a
- b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(240*a*b^3 +
152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
b - a^2)(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(
a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
+ 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 9
6*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b)
- ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
+ 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
+ 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(

a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(288*a*b^3 + 888*a^3*b + 1348*a^4 + 480*b^4 - 380*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a

- b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)
^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b
)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2
b)(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + ((
(a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*
(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(
a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (
a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*
(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
(a - b)(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*
a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7
*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*
b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) +
 (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a
 - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*
(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b
)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3
072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(
a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)
^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) -
 (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
 b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*
b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*
(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b)
 + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(
a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
 + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a
*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*
(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a
*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a -
 b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2
*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a
^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*
a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a
 - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a
 + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a
 - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a +

```
4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a +
 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)
^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b
)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a +
3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
 + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2
))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a -
b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a +
2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a
- b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(
a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
 - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
(((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*
(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) +
 ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*
(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^
4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (
((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a -
 b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*
b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2
)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((
(((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
- b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a
+ 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/
(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)
*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((
a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^
4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b
)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*
1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/
(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)
/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
+ (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2)
)*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b
))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a +
 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a
*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2
*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
- a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b
)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*
b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^
```

```
2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((a + 2*b)^3 + (
((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(
a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*
(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2
*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*
(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^
2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*
(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(240*a*b^3 + 152*a
^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2
*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2
*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a
 + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b
)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 -
 a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a
 - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(
a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
 + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))
/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2
)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(
a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)
*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a - b)^8*(a + 2*b))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a -
2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
 - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(5
6*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*
a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*
(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a
+ 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^4*(a
 + 2*b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a
 + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*
b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) -
(a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(
3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b)
+ ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
 + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
+ 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*
```

```
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a -
b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*
b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2
*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2
*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a
^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a -
 b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a +
 b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a
- 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b
) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4
*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a +
3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^
2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)
*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)
*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3
*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
+ (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2)
)*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b
)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*
1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2
*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a -
 b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a
 + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
- (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(
a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) +
((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4
 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((
(a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)
*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b)
)*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((
((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
 b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b
)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (
(a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*(
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*
(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a
 - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)
*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4
*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)
^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(
```

a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/
(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
 (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))
(a - b)(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2
b)(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b)
)/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a +
2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*
b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b -
 a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i))))/(a - b) + ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)
^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b
)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2
b)(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + ((
(a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(64*a*b^3 - 18
4*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2
)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) -
 (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2
))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(240*a*
b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2
b)(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(2404*a^3*b - 196*a*b
^3 + 401*a^4 - 363*b^4 + 698*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b
)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*
b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a
 - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(288*a*b^
3 + 888*a^3*b + 1348*a^4 + 480*b^4 - 380*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2
b)(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*(((a + 3*b)
((((((a - b)(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(
a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7
*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) -
 (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
 b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*
b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*
(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b)
 + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) -
(a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^
3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a
 + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)

```
*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^
4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*
b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a
- b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) +
((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a -
 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*
b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
 a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
- a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*
b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
(a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))
/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2
*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a
- b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)
^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b
)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + ((
(a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*
(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(
a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*
(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
- a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(
3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b
)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*
a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + (
(a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2
*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*
(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))
/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b
))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)
^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)
*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)
^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^
2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)
*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*
```

b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a
- b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96
*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a +
2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2
*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
(a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a +
2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a
- b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(7
68*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b)
- (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
- b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(10
24*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a -
b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*
b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*
b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a
+ 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a +
2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 -
182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a +
2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4
f(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a +
b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((
a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a
*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a
*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3
072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b
)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3
072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a -
b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2
*b))/(a*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a
^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)
^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3
b))(a + 2*b))/(a*b - a^2))*(a - b)*(2404*a^3*b - 196*a*b^3 + 401*a^4 - 36
3*b^4 + 698*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
+ (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(
a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b
)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a
*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/
(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*
b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(288*a*b^3 + 8

88*a^3*b + 1348*a^4 + 480*b^4 - 380*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)
^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)
^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((
a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*
(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)
(a - b)^2(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b)
)*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(48*a*b^3 - 1904*a^3*b + 399*a^
4 - 145*b^4 + 66*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*
1i)) - ((a - b)^6*(a + 2*b)*(29*a^2 + 3*b^2))/(384*a^4*f*(a*b^2 - a^2*b)*(a
b - a^2)(a*1i - b*1i))) - exp(e*2i + f*x*2i)*(((a + 3*b)*(((a + 3*b)*(((a
 + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
 b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2
b)(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a
+ 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^
2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a
^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a
- b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
 b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b
- a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a
+ 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(
a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6
*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
+ (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2)
)*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
 + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*
(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b
)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*
b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
 (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a
 - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(
a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)
^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3
b))(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a
*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(
((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
- b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(
a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
- ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(
a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2
)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*
b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((
((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
 b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b
)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a -
 b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b)
)/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^
2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*
1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2
) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^

```
2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a +
 b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a
 + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a -
 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*
b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
 b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
- b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a
^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a
 + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b
)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 -
 a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((((
(a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)
^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((
a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3
072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(
9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a
- b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*
(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*
(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2
*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)
^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^
2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)
*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b
)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(
(((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
- b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(
a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
- ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(
a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (
a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3
*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) +
 (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a
 - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a +
 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*
```

```
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a -
b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((
a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a
*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2
*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
- a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a
^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)
^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))
/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a +
2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a
 + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)
*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b)
)*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2
 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*
a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a
 - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*
a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a
 + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 -
8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) -
 ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*
(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 18
2*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/
(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*
((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + ((
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
 - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*
(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
 - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*
(a*1i - b*1i))))/(a - b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^
2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^
5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a + 2
*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((
(((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
- b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a
 + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36
*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^
7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) + ((a - b)^4*(a + 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a
^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a -
b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*
a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2
*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b
```

)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a

```
- b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a
+ 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a
+ 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2
- a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*
a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*
a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) +
(((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a
- b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2
*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a +
2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))
/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b
)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a
- b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*
b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b -
a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^
2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a
^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*
1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b)
- (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
- b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a
*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3
072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)
*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b
) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*
(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(
a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(
a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)
*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a
- b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a +
2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b
- a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
+ 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*
a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56
*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a
- b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a
+ b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(
a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2
*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a +
4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a
+ 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b
))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)
^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 -
```

```
a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a
- 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a
*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
+ ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*
(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/
(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)
/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a
*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*
(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)
*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b)
)*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(
a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
 + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*
(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b
)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*
1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^
2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b
)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b)
 - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
 - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(
a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(
3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b
)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a -
b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2
- a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*
a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) -
(a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*
a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*
(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))
/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))
/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2
*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*
b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182
*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2
*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b)
)/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a +
 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2
)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((((a - b)*(
a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2
*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072
```

*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)
*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b)
)/(a*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*
b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a
 - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b
 - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
b - a^2)(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(64*a^3*b - 7
2*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(288*a*b^3 + 888*a^3*b + 1348*
a^4 + 480*b^4 - 380*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/
(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b)
 - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
 - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a*b^3 - 184*
a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a +
3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
 + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2
))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a -
b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
1i)) - ((a - b)^7(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*
b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
(a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))
/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2
b)(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a
 - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b
) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(76
8*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b)
 - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
 - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(102
4*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a
 + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) -
((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*
(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))
/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a
*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a
b - a^2) + (((a - b)(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(
a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a
 + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2
 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a
^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a
 + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (
((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a -
 b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2
b)^2)(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
 + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*

```
(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/
(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a -
 b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(
a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/
(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b
) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*
a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*
1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^
3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a +
4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a +
 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2
) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^
2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a -
 b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a
 - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)
*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*
b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a
- b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
 b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*
b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b
))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a
+ 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^
2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*
b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(
a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)
^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 -
 a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) -
(a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2)
)*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
 + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
(((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*
(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)))
)/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a
+ 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2
- a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a
^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a
 + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (
(a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2
*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2
```

```
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*
(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))
/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b
))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)
^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)
*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*
b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(
a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)
*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b)
)/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2
*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
 (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b)
)/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b
)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a
*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(
a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
 + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*
(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b
)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b
) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b -
a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a
 - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b
- a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
 - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
- a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
 b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
- b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2
*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*
a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a +
3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(
a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
- ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2
))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 +
 (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a
 + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2
 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*
a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*
a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) -
(((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a
- b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2
*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a +
2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
 + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a -
2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
 - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(307
```

```
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(
240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)
*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b)
)*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4
+ 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
)))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*
(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(10
24*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)
^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*
b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
- b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a - b)^8*(a + 2*
b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3
+ (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b)
)*(a + 2*b))/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 +
 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a -
b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b)
- (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^
2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
+ (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2)
)*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) + ((a - b)^4*(a + 2*b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a
^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((((a -
b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(240
*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a
- b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*
(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)
/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b
)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a -
 b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2
*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b)
)/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a +
 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2
)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a
 + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^
4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (
```

```
a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))
*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
+ (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(
a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))
/(a - b) - ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a +
 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 -
 a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a
+ b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((
a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2
*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
- a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(
a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/
(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b)
)/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^
2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b
)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b
 - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*
(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))
/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*
b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
(a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))
/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)
*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*
b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a
*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
+ ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(
a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)
*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b)
 - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a
^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a
- b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
- b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
 a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
- a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
 b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2
*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b
)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a
^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3
*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a
 + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
 ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 +
(((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(
```

a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b

```
b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768
*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((
((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
 b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b
)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (
(a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a -
 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b)
 - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^
4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*
b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a
+ 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/
(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
 b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
- b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a
^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 +
 (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(
a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b
)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)
^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b
)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b
)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a
*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/
(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*
b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^
2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^
4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a
- b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*
(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*
(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b)
+ ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*
(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(10
24*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*
(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b
 - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*
b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) +
 ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*
(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b)
- (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^
2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a -
 b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
 b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
 a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4
 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b)
 - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^
```

4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a +

```
2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
+ 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a +
 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 -
 a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a - b)^8*(a + 2*b))/(3072*
a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b
)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b
))/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^
2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^4*(a +
2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*
(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b
)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b
)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((
a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^
3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a -
b)^4*(a + 2*b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(
3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((((a - b)*(a - 2*
b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
(a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(240*a*b^3 + 1
52*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a -
2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4
*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b
))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a +
 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(
a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
 b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2
*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 +
(((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a
*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)
*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^
2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)
*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(
3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b
)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2
)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4
*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a -
 b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(
a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(
9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) +
 ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(
a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(102
4*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(
((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b
- a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
```

```
 - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) +
((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) -
 (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2
))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a -
b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a
^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*
1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4
- 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4
*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a +
2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
 + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*
b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(
a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/
(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a -
b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*
(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b
)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*
1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a
^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a +
 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*
(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a
+ b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a
+ 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/
(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 +
(((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
+ 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a -
2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4
*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b
))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a +
 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(
a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2
*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*
b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(
768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b
) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1
```

```
024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(
a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b)
- (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(
a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b
)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a
^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b
)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b
 - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)
^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)
^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a -
b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/
(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)
/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((a + 2*b)^3 + (((a - b)*(a - 2
*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
- a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (
(a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*
b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a
*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*
a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*
1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4
- 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*
1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2
) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^
2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a +
3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
 + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2
))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a -
b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a +
2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a
- b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(
a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
 - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
(((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*
(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) +
 ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i))))/(a - b) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
+ 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(240*a*b^3 + 1
52*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(307
```

```
2*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a
+ 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7
*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
) + ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b
^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a
 + 2*b)*(288*a*b^3 + 888*a^3*b + 1348*a^4 + 480*b^4 - 380*a^2*b^2))/(3072*a
^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)
*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b)
)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/
(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)
/(a*b - a^2))*(a - b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*
b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a
+ 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b
)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*
b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a +
 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2
) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^
2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a -
 b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a +
 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a
 - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*
(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
 (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))
*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
+ ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(
a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)
*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b
^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
(((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a
- b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a
*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^
2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*
b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*
b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((
((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
 - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
 b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a
 + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
 ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)
*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)
/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b
)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((((
(a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)
^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a -
b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))
/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2
)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1
i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
 + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2
```

))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a +
b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a
+ 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a -
2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
 - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i))))/(a - b) + ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
 b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2
b)(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a
+ 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*
b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
(a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))
/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2
b)(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a
- b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)
^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b
)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2
b)(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + ((
(a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*
(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(
a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*
(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
- a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(
3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b
)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*
a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + (
(a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2
*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*
(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))
/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b
))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)
^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)
*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)
^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3
b))(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^

```
2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)
*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*
b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a
- b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
 b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96
*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a +
2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a
 + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 -
 a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2
*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
 (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a +
2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a
 - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*
b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b
))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a +
 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b)
)/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^
2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*
1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a -
 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*
b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) +
 ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(
a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(
a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/
(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b -
84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1
024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b)
- (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
- b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(7
68*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*((
(a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2
*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((
(((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
- b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a
+ 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/
(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)
*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((
a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b
```

```
)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^
4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b
)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*
1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a
^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*
b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a +
 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b
)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2
*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b
)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3
072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(
a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)
^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)
*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^
4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a +
4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(
a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2
*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a +
 b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(
a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a
^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((
((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
 b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b
)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(6
4*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a
 + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*
(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)))
)/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((a + 2*b)^3 + (((a - b)
*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b)
)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1
i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b
)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a -
 b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(56*a^3*
b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(240*a*b^3 + 152*a^3*b - 4
94*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*
b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a
*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(
```

```
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*
(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b
 - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*
b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) +
 ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*
(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b)
- (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^
2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a -
 b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
 b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
 a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i))))/(a - b) + ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (
a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))
*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(56*a^3*b
- 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(
a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/
(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b
) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/
(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*
(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^
2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*
b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b - 494*
a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2
*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b)
)/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a +
3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))
/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2
)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1
i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a -
2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
 - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) +
((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a
*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(
a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 8
4*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
- b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a +
 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(10
24*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) -
 (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
 b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(76
8*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((
```

```
a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2
*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((
((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
 b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b
)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (
(a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*(
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*
(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a
 - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)
*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4
*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)
^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^
2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b
)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2
*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)
^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b
)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*
((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + ((
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
 - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*
(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
 - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*
(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) -
(a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(
3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b)
+ (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a
+ 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4
*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b
))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a -
 b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (
(a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a -
2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
 - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
 a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2
*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*
```

```
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b)
)/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a +
 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2
)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(
a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2
*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072
*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (
a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 -
 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
- ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(
a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)
*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 1
82*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))
/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)
*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(
a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7
*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i))))/(a - b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)
^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)
^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a +
2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (
((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
 - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
 b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((
a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 3
6*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)
^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i)) + ((a - b)^4*(a + 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*
a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a -
 b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(
a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64
*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a -
b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*
(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b
)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*
1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a
^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a +
 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*
(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a
+ b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a
+ 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/
(a - b) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
```

```
a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
 a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((
(a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b - 494*
a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a
^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2
- a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a
^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a
 + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*
(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(288*a*
b^3 + 888*a^3*b + 1348*a^4 + 480*b^4 - 380*a^2*b^2))/(3072*a^4*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a
 + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*
(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
 + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(
a - b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4
*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3
*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2
*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*
(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b
)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*
b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
 (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))
*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)
^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*
b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a -
b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a
+ 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
 (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + ((
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
 - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (
(a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4
+ 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((
a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*
(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2
 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((((
(a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)
^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((
a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3
072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(
9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a
- b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*
```

```
(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*
(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a
*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(
a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
 + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
(((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*
(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))
/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2
*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b
) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b -
a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^
2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)
*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)
*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3
*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/
(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)
/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a -
 b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(
a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/
(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)
*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a -
b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*
a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a
+ 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)
*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a
 + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a
 + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))
/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2
))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 +
 (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
 + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
+ 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((((
(a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)
^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((
a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a +
2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
 + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4
```

```
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a +
 b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((
a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a
*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a
*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3
072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8
*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a -
b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b)
- (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^
2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
+ (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2)
)*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
)))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*
(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(10
24*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)
^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*
b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
- b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2
*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
 (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2
*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^
4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a
 - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
- b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^
4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768
*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((
((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
 b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b
)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (
(a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i))))/(a - b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*(
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*
(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a
 - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)
*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4
*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2
*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2
```

```
*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a
 + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) +
 ((a - b)^4*(a + 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b
^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(
a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2
*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*
b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a
 - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*
a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a +
3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(
a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)))
)/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*
(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b)
)/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a -
2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))
/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b
)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a -
b) + ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2
*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b
^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b
^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3
 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*
(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(
a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(64*a*b^3 - 184*a^3*b + 114
8*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*
b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a
*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*
b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(
a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/
(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a -
b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*
(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b
)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*
1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a
^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a +
 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*
(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a
+ b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a
+ 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/
(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2)
```

```
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 +
(((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
+ 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((
a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^
5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2
*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a
 - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
- b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2
*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a +
b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a
 + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)
^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*
b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*
(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b
)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) -
 (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2
))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
 (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))
*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(
a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(102
4*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^
3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*
b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
 b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
- b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*
b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
(a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*
b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4
 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a
- b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
 b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b
- a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4
 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*
1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b
```

```
) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*
a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((((
(a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)
^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((
a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3
072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a -
 b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2
*b))/(a*b - a^2))*(a - b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*
a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6
*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
+ (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2)
)*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
 - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) -
(a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b
^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((
a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i)) - ((a - b)^4*(a + 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4
 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((
a - b)^4*(a + 2*b)*(2404*a^3*b - 196*a*b^3 + 401*a^4 - 363*b^4 + 698*a^2*b^
2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a
 - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b
 - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
 + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
+ 2*b)^2)/(a*b - a^2))*(a - b)*(288*a*b^3 + 888*a^3*b + 1348*a^4 + 480*b^4
- 380*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a -
 b) - ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)
^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))
/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3
*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/
(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)
/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2
*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
- a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (
(a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*
b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a
*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84
*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
 b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a +
2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(102
4*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) -
(a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768
```

```
*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a
 + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b
)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 -
 a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((((
(a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)
^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((
a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3
072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(
9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a
- b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*
(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*
(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2
*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)
^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^
2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)
*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b
)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*
(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(307
2*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a
+ 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7
*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
)))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(
a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*
b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a
- 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b
) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b
))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2
*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a
- b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2
) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^
2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((
a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*
(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a
^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*
a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a +
 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a
 - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/
(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2
*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
```

```
- a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((a + 2*b)^3 + (((a - b)*(
a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/
(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b)
)/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^
2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(56*a^3*b
- 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(240*a*b^3 + 152*a^3*b - 494
*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b
- a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
 - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*((
(a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
 a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
- a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (
(a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) -
(a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2)
)*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b
)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^
2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a
^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1
i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a
+ 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(
a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*
b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b -
 a^2))*(a - b)*(288*a*b^3 + 888*a^3*b + 1348*a^4 + 480*b^4 - 380*a^2*b^2))/
(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)
*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)
*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*
(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a -
b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9
*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^4
*(a + 2*b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(4
8*a*b^3 - 1904*a^3*b + 399*a^4 - 145*b^4 + 66*a^2*b^2))/(3072*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*
(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b - 494*a^4
 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1
i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
 + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2
))*(a - b)*(2404*a^3*b - 196*a*b^3 + 401*a^4 - 363*b^4 + 698*a^2*b^2))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3
+ (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b)
)*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*
```

```
(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(64*a*b^3
- 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b
)*(a + 2*b)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b
)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b
)*(48*a*b^3 - 1904*a^3*b + 399*a^4 - 145*b^4 + 66*a^2*b^2))/(3072*a^4*f*(a*
b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(
768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b
) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1
024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b)
- (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
- b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(29*a^2 + 3*b^2
))/(384*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((a + 2*b)^3 + ((
(a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 + 2*b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a
 + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^
4*(a + 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(2
404*a^3*b - 196*a*b^3 + 401*a^4 - 363*b^4 + 698*a^2*b^2))/(3072*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2
)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4
 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
+ (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2)
)*(a - b)*(288*a*b^3 + 888*a^3*b + 1348*a^4 + 480*b^4 - 380*a^2*b^2))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))) - ((a + 3*b)*(((a + 3*b)*(
((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b
- a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
 - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) +
((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(
a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2
*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072
*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a +
 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*
(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b
))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a -
 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b)
 - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b)
)/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*
b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a -
 b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
 b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3
072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a
 - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a +
 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2
*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b
)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*
b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b)
 - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
 - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(
```

```
a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(
3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b
)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a -
b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2
- a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*
a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) -
(a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*
a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a +
3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((
(((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
- b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a
 - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b
 - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 7
2*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
- a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(
9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) +
 ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b
 - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*
b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) +
 ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*
(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(
((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
 - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(
a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*((
(a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
 a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
- a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (
(a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) -
(a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2)
)*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b
)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^
2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a
^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1
i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 -
 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) -
 (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2
*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b
)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
```

(((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*
(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b
)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b
)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a +
 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
(((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
- b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((
a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2
*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
 a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*
a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*
a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4
*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4
f(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(
a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/
(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a
*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(
a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(102
4*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(
((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((
((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
 - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
 b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a
 + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
 ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)
((a - b)(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)
/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b
)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((((
(a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)
^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a -
b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
1i))))/(a - b) - ((a - b)^8(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*
a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2
b)^3 + (((a - b)(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
 b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b

```
)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
(((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*
(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b)
- (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
- b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a
*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3
072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)
*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b
) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) -
 (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 -
 a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a
^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (
a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a
^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3
*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a
 - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) +
 ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a
- 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a
*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) + ((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b
- a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
 - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2
*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a +
b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a
 + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)
^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*
b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)
*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(30
72*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) -
 (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
 b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4
 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 -
 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*
(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b
))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*
b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) +
 (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))
*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)
^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
```

```
2)*(a*1i - b*1i))))/(a - b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*
b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a -
b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a
+ 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) +
 (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + ((
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
 - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (
((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a -
 b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 +
 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a -
b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b
*1i)) + ((a - b)^4*(a + 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 4
8*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a
 - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)
*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(
64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a
 - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*
(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)
/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b
)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a -
 b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a -
 b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072
*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a
 + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b
)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*
a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(
a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)))
)/(a - b) + ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(
a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)
*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(64*a^3*b -
 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 3
6*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a +
 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 -
 a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
- b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(64*a*b^3 - 184*a^3*
b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b
)^2)/(a*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a
^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - (
(a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2
*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((
(((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
- b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a
+ 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) -
((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*
```

```
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/
(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)
*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((
a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^
4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b
)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*
1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a
^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*
b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a +
 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b
)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2
*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)
*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(
a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7
*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) -
 (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
 b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*
b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*
(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b)
 + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) -
(a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^
3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a
 + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^
4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*
b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a
- b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) +
((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a -
 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*
b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
 a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b
- a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*
b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
(a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))
/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2
*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a
- b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)
^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b
)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b
)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + ((
(a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
 + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*
```

```
(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(
a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*
(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
- a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(
3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b
)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*
a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + (
(a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2
*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
- b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*
(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))
/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b
))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)
^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)
*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)
^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i)) + ((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^
2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)
*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*
b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a
- b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
 b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96
*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a +
2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a
 + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 -
 a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2
*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) -
 (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b)
)/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a +
2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a
 - b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*
b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(7
68*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b)
 - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a
 - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(10
24*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a -
b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*
b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*
b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a
+ 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a +
 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 -
```

182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(288*a*b^3 + 888*a^3*b + 1348*a^4 + 480*b^4 - 380*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a -

```
b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*
b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)
*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))
/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2
)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) -
(a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(
3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b)
+ (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a
+ 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4
*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b
))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((
a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^
5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a -
2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b -
84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
- a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a
*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a
^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a
 - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a
 + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((
a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
 a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((
a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
 - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((
((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
 b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b
)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (
(a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a +
 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 -
 a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a
+ b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((
(a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b
)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*
b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(
a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(
3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^
8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a -
 b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^
```

```
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b)
 - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a
^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1
i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
 + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2
))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) -
(a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(
3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b)
+ ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
 + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a
+ 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*
1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(
a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a -
b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*
b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2
*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2
*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a
^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a
^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a -
 b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a +
 b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a
- 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b
) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4
*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a +
3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))
/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2
)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1
i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a -
2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
 - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) +
((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a
*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a
*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(
a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(
a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*
(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
+ 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a
 + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^
7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*
b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^
3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*
```

```
b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) -
 (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
 b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*
b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(30
72*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*
(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b)
 - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) -
(a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^
3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a
 + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)
*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^
4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a
 - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(
a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(
3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b
)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)
^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a
^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*
1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^
2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*
b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b
)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a
*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/
(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2
*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a
+ 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*
(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2
- a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a
^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a - b)^8*(a +
 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)
^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3
*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^
4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a
 - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*
b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b -
 a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^
2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a
^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*
1i)) + ((a - b)^4*(a + 2*b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 15
6*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a
 - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)
*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(
240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((a + 3*b)*(((((
(a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)
^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((
a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
```

```
 - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a +
2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a
 + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a +
 b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((
a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a
*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a
*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3
072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8
*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a -
b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2
 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b)
- (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^
2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
+ (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2)
)*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
)))/(a - b) - ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (
a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3
*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) +
 ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a
+ 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a
*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b
)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b
))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*
1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b -
a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2
*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*
b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^
3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a -
b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a +
b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a -
 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b)
 - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*
b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3
*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/
(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)
/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i
)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2
)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2
*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2
*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
- a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (
(a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
```

```
1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*
b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a
*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i
 - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*
b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
 - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(
a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
+ 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a
+ 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7
*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b
^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3
 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b
)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*((((((a - b)*(a - 2*b) -
(a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(
3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b)
+ (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a +
 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a
 - b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 352*b^4 + 156*a^2*b^2))/(3072*a^4*
f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*
b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a
 - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b
) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b
))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (((a + 2*b)^3 + (
((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(
a + 2*b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3
072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(
a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)
^4*(a + 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(
2404*a^3*b - 196*a*b^3 + 401*a^4 - 363*b^4 + 698*a^2*b^2))/(3072*a^4*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^
2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*
b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64*a^3*b - 72*a*b^3 - 182*a^
4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1
i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
 + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2
))*(a - b)*(288*a*b^3 + 888*a^3*b + 1348*a^4 + 480*b^4 - 380*a^2*b^2))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)*(((
a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2
*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
- a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((
((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a -
 b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b
)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a +
 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (
(a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
```

```
i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*(
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*
(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a
 - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)
*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4
*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)
^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^
2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b
)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2
*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)
^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b
)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*
b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*(((a + 3*b)*
((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + ((
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
 - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*
(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
 - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*
(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) -
(a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a -
b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b
^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(307
2*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(
3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b)
+ (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3
*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a
+ 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4
*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b
))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a -
 b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (
(a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(
a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a -
2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b
 - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b -
a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
 a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2
*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*
(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b)
)/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a +
 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a
 + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^
2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2
)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b
))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^
2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072
*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - (((((a - b)*(
a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2
```

```
*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072
*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (
a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 -
 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
- ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(
a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)
*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a^3*b - 72*a*b^3 - 1
82*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1
i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^3*(9*a + 4*b))
/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) - ((a + 3*b)
*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (
((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(
a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7
*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i))))/(a - b) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)
^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)
^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^6*(a +
2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (
((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
 - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
 b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((
a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b
)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 3
6*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)
^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i)) + ((a - b)^4*(a + 2*b)*(240*a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*
a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a -
 b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(
a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(64
*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^
2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a -
b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
 + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(
3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*
(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b
)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*
1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b
)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^5)/(3072*a
^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2*b)*(9*a +
 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*
(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a +
2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a
+ b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a
+ 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/
(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*
1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(
a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(64*a^3*b - 7
2*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)*(288*
a*b^3 + 888*a^3*b + 1348*a^4 + 480*b^4 - 380*a^2*b^2))/(3072*a^4*f*(a*b^2 -
 a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4
- 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i
)) - (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4
*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a
```

```
- 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a
*b - a^2))*(a - b)^3*(29*a^2 + 3*b^2))/(384*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)
*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))
/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2
)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*
(a*1i - b*1i)) - ((a - b)^4*(a + 2*b)*(64*a*b^3 - 184*a^3*b + 1148*a^4 - 35
2*b^4 + 156*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)
) + ((a - b)^4*(a + 2*b)*(48*a*b^3 - 1904*a^3*b + 399*a^4 - 145*b^4 + 66*a^
2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b
)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(240*
a*b^3 + 152*a^3*b - 494*a^4 - 138*b^4 + 48*a^2*b^2))/(3072*a^4*f*(a*b^2 - a
^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a -
 b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)*(2404*a^3*b - 196*a*b^3 + 401*a^4 - 3
63*b^4 + 698*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
))/((exp(e*2i + f*x*2i) - 1)*(exp(e*2i + f*x*2i) + 1)*(a - b + exp(e*2i + f
*x*2i)*(2*a + 2*b) + exp(e*4i + f*x*4i)*(a - b))) - ((a + (b*(exp(e*2i + f*
x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)*(2*exp(e*2i + f*x*2i) +
 exp(e*4i + f*x*4i) + 1)*(((a - b)*1i)/(24*a^3*f) + ((5*a - 3*b)*1i)/(24*a^
3*f) + ((11*a - 3*b)*1i)/(24*a^3*f) + ((8*a + 8*b)*1i)/(24*a^3*f) + ((15*a
- b)*1i)/(24*a^3*f)))/((exp(e*2i + f*x*2i) - 1)*(exp(e*2i + f*x*2i) + 1)) -
 ((a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)
*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*(((a - b)^2*(a^2 - 2*a*b +
 b^2)*25i)/(192*a^4*f*(a*1i - b*1i)^2) + ((a - b)*(a^2 - 2*a*b + b^2)*1i)/(
32*a^3*f*(a*1i - b*1i)^2) + ((5*a + b)*(a^2 - 2*a*b + b^2)*1i)/(48*a^3*f*(a
*1i - b*1i)^2) - ((a^2 - 2*a*b + b^2)*(4*a*b - 15*a^2 + 3*b^2)*1i)/(192*a^4
*f*(a*1i - b*1i)^2) + ((a^2 - 2*a*b + b^2)*(54*a*b + 39*a^2 - 21*b^2)*1i)/(
192*a^4*f*(a*1i - b*1i)^2) - ((a^2 - 2*a*b + b^2)*(48*a*b - 73*a^2 + 73*b^2
)*1i)/(192*a^4*f*(a*1i - b*1i)^2) + ((a^2 - 2*a*b + b^2)*(50*a*b + 78*a^2 +
 72*b^2)*1i)/(192*a^4*f*(a*1i - b*1i)^2)))/((exp(e*2i + f*x*2i) - 1)^2*(exp
(e*2i + f*x*2i) + 1)) - ((a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i
+ f*x*2i) + 1)^2)^(1/2)*(a^2 - 2*a*b + b^2)*(2*exp(e*2i + f*x*2i) + exp(e*4
i + f*x*4i) + 1)*4i)/(3*a^2*f*(exp(e*2i + f*x*2i) - 1)^3*(exp(e*2i + f*x*2i
) + 1)*(a*1i - b*1i)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.139 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{2b\left(15a^2-40ab+24b^2\right)\tan(e+fx)}{15a^4f\sqrt{a+b\tan^2(e+fx)}}-\frac{\left(15a^2-40ab+24b^2\right)\cot(e+fx)}{15a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot}{5af\sqrt{a+}}$$

```
[Out] -1/15*(15*a^2-40*a*b+24*b^2)*cot(f*x+e)/a^3/f/(a+b*tan(f*x+e)^2)^(1/2)-2/15
*(5*a-3*b)*cot(f*x+e)^3/a^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/5*cot(f*x+e)^5/a/f
/(a+b*tan(f*x+e)^2)^(1/2)-2/15*b*(15*a^2-40*a*b+24*b^2)*tan(f*x+e)/a^4/f/(a
+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3663, 462, 453, 271, 191}

$$-\frac{2b\left(15a^2-40ab+24b^2\right)\tan(e+fx)}{15a^4f\sqrt{a+b\tan^2(e+fx)}}-\frac{\left(15a^2-40ab+24b^2\right)\cot(e+fx)}{15a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot}{5af\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((15*a^2 - 40*a*b + 24*b^2)*Cot[e + f*x])/(15*a^3*f*Sqrt[a + b*Tan[e + f*x
]^2]) - (2*(5*a - 3*b)*Cot[e + f*x]^3)/(15*a^2*f*Sqrt[a + b*Tan[e + f*x]^2]
) - Cot[e + f*x]^5/(5*a*f*Sqrt[a + b*Tan[e + f*x]^2]) - (2*b*(15*a^2 - 40*a
*b + 24*b^2)*Tan[e + f*x])/(15*a^4*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{2(5a-3b)+5ax^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= -\frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^5(e+fx)}{5af\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(-15a^2+8(5a-3b\right.}{}$$

$$= -\frac{\left(15a^2-8(5a-3b)b\right)\cot(e+fx)}{15a^3f\sqrt{a+b\tan^2(e+fx)}} - \frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^5(}{5af\sqrt{a+b}}$$

$$= -\frac{\left(15a^2-8(5a-3b)b\right)\cot(e+fx)}{15a^3f\sqrt{a+b\tan^2(e+fx)}} - \frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^5(}{5af\sqrt{a+b}}$$

Mathematica [A] time = 1.46, size = 135, normalized size = 0.79

$$-\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\cot(e+fx)\left(3a^2\csc^4(e+fx)+8a^2+a(4a-9b)\csc^2(e+fx\right.\right.}{15\sqrt{2}\,a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(Cot[e + f*x
]*(8*a^2 - 41*a*b + 33*b^2 + a*(4*a - 9*b)*Csc[e + f*x]^2 + 3*a^2*Csc[e + f
*x]^4) + (15*(a - b)^2*b*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)
])))/(Sqrt[2]*a^4*f)
```

fricas [A] time = 31.86, size = 232, normalized size = 1.36

$$-\frac{\left(8\left(a^3-8a^2b+13ab^2-6b^3\right)\cos\left(fx+e\right)^7-4\left(5a^3-41a^2b+72ab^2-36b^3\right)\cos\left(fx+e\right)^5+\left(15a^3-130\right.\right.}{15\left(\left(a^5-a^4b\right)f\cos\left(fx+e\right)^6+a^4bf-\left(2a^5-3a^4b\right)f\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^3 - 8*a^2*b + 13*a*b^2 - 6*b^3)*cos(f*x + e)^7 - 4*(5*a^3 - 41*
a^2*b + 72*a*b^2 - 36*b^3)*cos(f*x + e)^5 + (15*a^3 - 130*a^2*b + 264*a*b^2
 - 144*b^3)*cos(f*x + e)^3 + 2*(15*a^2*b - 40*a*b^2 + 24*b^3)*cos(f*x + e))
*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^5 - a^4*b)*f*cos(f*
x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^4*b)*f
*cos(f*x + e)^2)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{6}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.10, size = 264, normalized size = 1.54

$$-\frac{\left(8\left(\cos^{6}\left(fx+e\right)\right)a^{3}-64\left(\cos^{6}\left(fx+e\right)\right)a^{2}b+104\left(\cos^{6}\left(fx+e\right)\right)a\,b^{2}-48\left(\cos^{6}\left(fx+e\right)\right)b^{3}-20\left(\cos^{4}\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(8*cos(f*x+e)^6*a^3-64*cos(f*x+
e)^6*a^2*b+104*cos(f*x+e)^6*a*b^2-48*cos(f*x+e)^6*b^3-20*cos(f*x+e)^4*a^3+1
64*cos(f*x+e)^4*a^2*b-288*cos(f*x+e)^4*a*b^2+144*cos(f*x+e)^4*b^3+15*cos(f*
x+e)^2*a^3-130*a^2*cos(f*x+e)^2*b+264*cos(f*x+e)^2*a*b^2-144*cos(f*x+e)^2*b
^3+30*a^2*b-80*b^2*a+48*b^3)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^
2)^(3/2)*cos(f*x+e)^3/sin(f*x+e)^5/a^4
```

maxima [A] time = 0.75, size = 255, normalized size = 1.49

$$-\frac{\frac{30\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a^{2}}-\frac{80\,b^{2}\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a^{3}}+\frac{48\,b^{3}\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a^{4}}+\frac{15}{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a\tan\left(fx+e\right)}-\frac{40\,b}{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a^{2}\tan\left(fx+e\right)}+}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(30*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^2) - 80*b^2*tan(f*x
+ e)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 48*b^3*tan(f*x + e)/(sqrt(b*tan(f*x
 + e)^2 + a)*a^4) + 15/(sqrt(b*tan(f*x + e)^2 + a)*a*tan(f*x + e)) - 40*b/(
sqrt(b*tan(f*x + e)^2 + a)*a^2*tan(f*x + e)) + 24*b^2/(sqrt(b*tan(f*x + e)^
2 + a)*a^3*tan(f*x + e)) + 10/(sqrt(b*tan(f*x + e)^2 + a)*a*tan(f*x + e)^3)
 - 6*b/(sqrt(b*tan(f*x + e)^2 + a)*a^2*tan(f*x + e)^3) + 3/(sqrt(b*tan(f*x
+ e)^2 + a)*a*tan(f*x + e)^5))/f
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**6/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.140 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{8b\left(5a^2+10ab+b^2\right)\sec(e+fx)}{15f(a-b)^5\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b\left(5a^2+10ab+b^2\right)\sec(e+fx)}{15f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\left(5a^2+10ab+b^2\right)\cos(e+fx)}{5f(a-b)^3\left(a+b\sec^2(e+fx)-b\right.}$$

```
[Out] -1/5*(5*a^2+10*a*b+b^2)*cos(f*x+e)/(a-b)^3/f/(a-b+b*sec(f*x+e)^2)^(3/2)+2/1
5*(5*a-b)*cos(f*x+e)^3/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)^(3/2)-1/5*cos(f*x+e)^
5/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(3/2)-4/15*b*(5*a^2+10*a*b+b^2)*sec(f*x+e)/(
a-b)^4/f/(a-b+b*sec(f*x+e)^2)^(3/2)-8/15*b*(5*a^2+10*a*b+b^2)*sec(f*x+e)/(a
-b)^5/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3664, 462, 453, 271, 192, 191}

$$-\frac{8b\left(5a^2+10ab+b^2\right)\sec(e+fx)}{15f(a-b)^5\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b\left(5a^2+10ab+b^2\right)\sec(e+fx)}{15f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\left(5a^2+10ab+b^2\right)\cos(e+fx)}{5f(a-b)^3\left(a+b\sec^2(e+fx)-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((5*a^2 + 10*a*b + b^2)*Cos[e + f*x])/(5*(a - b)^3*f*(a - b + b*Sec[e + f*
x]^2)^(3/2)) + (2*(5*a - b)*Cos[e + f*x]^3)/(15*(a - b)^2*f*(a - b + b*Sec[
e + f*x]^2)^(3/2)) - Cos[e + f*x]^5/(5*(a - b)*f*(a - b + b*Sec[e + f*x]^2)
^(3/2)) - (4*b*(5*a^2 + 10*a*b + b^2)*Sec[e + f*x])/(15*(a - b)^4*f*(a - b
+ b*Sec[e + f*x]^2)^(3/2)) - (8*b*(5*a^2 + 10*a*b + b^2)*Sec[e + f*x])/(15*
(a - b)^5*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-2(5a-b)+5(a-b)x^2}{x^4\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e\right.}{5(a-b)f}$$

$$= \frac{2(5a-b)\cos^3(e+fx)}{15(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}}$$

$$= -\frac{\left(5a^2+10ab+b^2\right)\cos(e+fx)}{5(a-b)^3f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{2(5a-b)\cos^3(e+fx)}{15(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3}}$$

$$= -\frac{\left(5a^2+10ab+b^2\right)\cos(e+fx)}{5(a-b)^3f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{2(5a-b)\cos^3(e+fx)}{15(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3}}$$

$$= -\frac{\left(5a^2+10ab+b^2\right)\cos(e+fx)}{5(a-b)^3f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{2(5a-b)\cos^3(e+fx)}{15(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3}}$$

Mathematica [A] time = 2.43, size = 294, normalized size = 1.19

$$-\frac{\cos(e+fx)\left(-16a^4\cos(6(e+fx))+3a^4\cos(8(e+fx))+425a^4+32a^3b\cos(6(e+fx))-12a^3b\cos(8(e+f\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/480*(Cos[e + f*x]*(425*a^4 + 4700*a^3*b + 6134*a^2*b^2 + 4700*a*b^3 + 42
5*b^4 + 48*(11*a^4 + 106*a^3*b - 106*a*b^3 - 11*b^4)*Cos[2*(e + f*x)] + 12*
```

```
(a - b)^2*(7*a^2 + 50*a*b + 7*b^2)*Cos[4*(e + f*x)] - 16*a^4*Cos[6*(e + f*x
)] + 32*a^3*b*Cos[6*(e + f*x)] - 32*a*b^3*Cos[6*(e + f*x)] + 16*b^4*Cos[6*(
e + f*x)] + 3*a^4*Cos[8*(e + f*x)] - 12*a^3*b*Cos[8*(e + f*x)] + 18*a^2*b^2
*Cos[8*(e + f*x)] - 12*a*b^3*Cos[8*(e + f*x)] + 3*b^4*Cos[8*(e + f*x)])*Sqr
t[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqrt[2]*(a - b)^5*f*
(a + b + (a - b)*Cos[2*(e + f*x)])^2)
```

fricas [A] time = 0.88, size = 370, normalized size = 1.49

$$-\frac{\left(3\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\cos\left(fx+e\right)^9-2\left(5a^4-16a^3b+18a^2b^2-8ab^3+b^4\right)\cos\left(fx+e\right)^7+3\left(5\right.\right.}{15\left(\left(a^7-7a^6b+21a^5b^2-35a^4b^3+35a^3b^4-21a^2b^5+7ab^6-b^7\right)f\cos\left(fx+e\right)^4+2\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^9 - 2*(5*
a^4 - 16*a^3*b + 18*a^2*b^2 - 8*a*b^3 + b^4)*cos(f*x + e)^7 + 3*(5*a^4 - 14
*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x + e)^5 + 12*(5*a^3*b + 5*a^2*b^2 - 9*a*b^
3 - b^4)*cos(f*x + e)^3 + 8*(5*a^2*b^2 + 10*a*b^3 + b^4)*cos(f*x + e))*sqrt
(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^7 - 7*a^6*b + 21*a^5*b^2
- 35*a^4*b^3 + 35*a^3*b^4 - 21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e)^4 +
2*(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7
)*f*cos(f*x + e)^2 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b
^6 - b^7)*f)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 4.52, size = 391, normalized size = 1.58

$$\frac{(a-b)^2\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)\left(3\left(\cos^8\left(fx+e\right)\right)a^4-12\left(\cos^8\left(fx+e\right)\right)a^3b+18\left(\cos^8\left(fx-\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/30/f*(a-b)^2*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(3*cos(f*x+e)^8*a^4-12*cos
(f*x+e)^8*a^3*b+18*cos(f*x+e)^8*a^2*b^2-12*cos(f*x+e)^8*a*b^3+3*cos(f*x+e)^
8*b^4-10*cos(f*x+e)^6*a^4+32*cos(f*x+e)^6*a^3*b-36*cos(f*x+e)^6*a^2*b^2+16*
cos(f*x+e)^6*a*b^3-2*cos(f*x+e)^6*b^4+15*cos(f*x+e)^4*a^4-42*a^2*b^2*cos(f*
x+e)^4+24*cos(f*x+e)^4*a*b^3+3*cos(f*x+e)^4*b^4+60*cos(f*x+e)^2*a^3*b+60*co
s(f*x+e)^2*a^2*b^2-108*cos(f*x+e)^2*a*b^3-12*cos(f*x+e)^2*b^4+40*a^2*b^2+80
*a*b^3+8*b^4)*4^(1/2)*a^7/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^
(5/2)/cos(f*x+e)^5/((-(a-b)*b)^(1/2)+a-b)^7/((-(a-b)*b)^(1/2)-a+b)^7
```

maxima [B] time = 0.72, size = 534, normalized size = 2.15

$$\frac{15\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3-3a^2b+3ab^2-b^3}+\frac{3\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5-20\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}b\cos(fx+e)^3+90\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}b^2\cos(fx+e)}{a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5}$$

$$-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^3 - 3*a^2*b + 3*a*
b^2 - b^3) + (3*(a - b + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 20*(a - b
 + b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 90*sqrt(a - b + b/cos(f*x + e
)^2)*b^2*cos(f*x + e))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
 b^5) - 10*((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a - b
+ b/cos(f*x + e)^2)*b*cos(f*x + e))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
b^4) + 5*(12*(a - b + b/cos(f*x + e)^2)*b^3*cos(f*x + e)^2 - b^4)/((a^5 - 5
*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(a - b + b/cos(f*x + e)^2
)^(3/2)*cos(f*x + e)^3) + 10*(9*(a - b + b/cos(f*x + e)^2)*b^2*cos(f*x + e)
^2 - b^3)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a - b + b/cos(f*x +
 e)^2)^(3/2)*cos(f*x + e)^3) + 5*(6*(a - b + b/cos(f*x + e)^2)*b*cos(f*x +
e)^2 - b^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(a - b + b/cos(f*x + e)^2)^(3/
2)*cos(f*x + e)^3))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^5}{{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.141 $\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=168

$$-\frac{8b(a+b)\sec(e+fx)}{3f(a-b)^4\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b(a+b)\sec(e+fx)}{3f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^{3/2}}+\frac{\cos^3(e+fx)}{3f(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

```
[Out] -(a+b)*cos(f*x+e)/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)^(3/2)+1/3*cos(f*x+e)^3/(a-
b)/f/(a-b+b*sec(f*x+e)^2)^(3/2)-4/3*b*(a+b)*sec(f*x+e)/(a-b)^3/f/(a-b+b*sec
(f*x+e)^2)^(3/2)-8/3*b*(a+b)*sec(f*x+e)/(a-b)^4/f/(a-b+b*sec(f*x+e)^2)^(1/2
)
```

Rubi [A] time = 0.16, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3664, 453, 271, 192, 191}

$$-\frac{8b(a+b)\sec(e+fx)}{3f(a-b)^4\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b(a+b)\sec(e+fx)}{3f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^{3/2}}+\frac{\cos^3(e+fx)}{3f(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(((a + b)*Cos[e + f*x])/((a - b)^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2))) +
Cos[e + f*x]^3/(3*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (4*b*(a + b
)*Sec[e + f*x])/(3*(a - b)^3*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (8*b*(a
+ b)*Sec[e + f*x])/(3*(a - b)^4*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{3(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^{5/2}}\,dx, x, \text{sec}\right.}{(a-b)f}$$

$$= -\frac{(a+b)\cos(e+fx)}{(a-b)^2 f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}}$$

$$= -\frac{(a+b)\cos(e+fx)}{(a-b)^2 f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}}$$

$$= -\frac{(a+b)\cos(e+fx)}{(a-b)^2 f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 1.55, size = 205, normalized size = 1.22

$$-\frac{\cos(e+fx)\left(a^3(-\cos(6(e+fx)))+26a^3+3a^2b\cos(6(e+fx))+186a^2b+3\left(11a^3+63a^2b-31ab^2-43b^3\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/24*(Cos[e + f*x]*(26*a^3 + 186*a^2*b + 190*a*b^2 + 110*b^3 + 3*(11*a^3 +
 63*a^2*b - 31*a*b^2 - 43*b^3)*Cos[2*(e + f*x)] + 6*(a - b)^2*(a + 3*b)*Cos
[4*(e + f*x)] - a^3*Cos[6*(e + f*x)] + 3*a^2*b*Cos[6*(e + f*x)] - 3*a*b^2*C
os[6*(e + f*x)] + b^3*Cos[6*(e + f*x)])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*
x)])*Sec[e + f*x]^2])/(Sqrt[2]*(a - b)^4*f*(a + b + (a - b)*Cos[2*(e + f*x)
])^2)
```

fricas [A] time = 0.73, size = 270, normalized size = 1.61

$$\frac{\left(\left(a^3-3a^2b+3ab^2-b^3\right)\cos\left(fx+e\right)^7-3\left(a^3-a^2b-ab^2+b^3\right)\cos\left(fx+e\right)^5-12\left(a^2b-b^3\right)\cos\right.}{3\left(\left(a^6-6a^5b+15a^4b^2-20a^3b^3+15a^2b^4-6ab^5+b^6\right)f\cos\left(fx+e\right)^4+2\left(a^5b-5a^4b^2+10a^3b^3-10a^2b^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 3*(a^3 - a^2*b - a*b^
2 + b^3)*cos(f*x + e)^5 - 12*(a^2*b - b^3)*cos(f*x + e)^3 - 8*(a*b^2 + b^3)
```

```
*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^6 - 6*
a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*f*cos(f*x + e
)^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*f*cos
(f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{3}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^3/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

maple [A] time = 1.50, size = 262, normalized size = 1.56

$$-\frac{(a-b)\left(a\left(\cos^{2}\left(fx+e\right)\right)-\left(\cos^{2}\left(fx+e\right)\right)b+b\right)\left(\left(\cos^{6}\left(fx+e\right)\right)a^{3}-3\left(\cos^{6}\left(fx+e\right)\right)a^{2}b+3\left(\cos^{6}\left(fx+e\right)\right.\right.}{6f\left(\frac{a(\cos}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/6/f*(a-b)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(cos(f*x+e)^6*a^3-3*cos(f*x+
e)^6*a^2*b+3*cos(f*x+e)^6*a*b^2-cos(f*x+e)^6*b^3-3*cos(f*x+e)^4*a^3+3*cos(f
*x+e)^4*a^2*b+3*cos(f*x+e)^4*a*b^2-3*cos(f*x+e)^4*b^3-12*a^2*cos(f*x+e)^2*b
+12*cos(f*x+e)^2*b^3-8*b^2*a-8*b^3)*4^(1/2)*a^5/((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5/((-(a-b)*b)^(1/2)+a-b)^5/((-(a-b)*
b)^(1/2)-a+b)^5
```

maxima [A] time = 1.05, size = 308, normalized size = 1.83

$$-\frac{\frac{3\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{a^{3}-3a^{2}b+3ab^{2}-b^{3}}-\frac{\left(a-b+\frac{b}{\cos\left(fx+e\right)^{2}}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{3}-9\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^{2}}}\,b\cos\left(fx+e\right)}{a^{4}-4a^{3}b+6a^{2}b^{2}-4ab^{3}+b^{4}}+\frac{9\left(a-b+\frac{b}{\cos\left(fx+e\right)^{2}}\right)b^{2}\cos\left(fx+e\right.}{\left(a^{4}-4a^{3}b+6a^{2}b^{2}-4ab^{3}+b^{4}\right)\left(a-b+\frac{b}{\cos\left(fx+\right.}\right.}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^
2 - b^3) - ((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a - b
+ b/cos(f*x + e)^2)*b*cos(f*x + e))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
b^4) + (9*(a - b + b/cos(f*x + e)^2)*b^2*cos(f*x + e)^2 - b^3)/((a^4 - 4*a^
3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x +
 e)^3) + (6*(a - b + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(e+f\,x\right)}^3}{{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

3.142 $\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{8b\sec(e+fx)}{3f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b\sec(e+fx)}{3f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\cos(e+fx)}{f(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

```
[Out] -cos(f*x+e)/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(3/2)-4/3*b*sec(f*x+e)/(a-b)^2/f/(
a-b+b*sec(f*x+e)^2)^(3/2)-8/3*b*sec(f*x+e)/(a-b)^3/f/(a-b+b*sec(f*x+e)^2)^(
1/2)
```

Rubi [A] time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3664, 271, 192, 191}

$$-\frac{8b\sec(e+fx)}{3f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b\sec(e+fx)}{3f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\cos(e+fx)}{f(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cos[e + f*x]/((a - b)*f*(a - b + b*Sec[e + f*x]^2)^(3/2))) - (4*b*Sec[e +
 f*x])/(3*(a - b)^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (8*b*Sec[e + f*x]
)/(3*(a - b)^3*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)}{(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+\right.}{(a-b)f}$$

$$= -\frac{\cos(e+fx)}{(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b\sec(e+fx)}{3(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} -$$

$$= -\frac{\cos(e+fx)}{(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b\sec(e+fx)}{3(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} -$$

Mathematica [A] time = 1.27, size = 124, normalized size = 1.05

$$-\frac{\cos(e+fx)\left(12\left(a^2+2ab-3b^2\right)\cos(2(e+fx))+3(a-b)^2\cos(4(e+fx))+(3a+5b)^2\right)\sqrt{\sec^2(e+fx)((a-}}{6\sqrt{2}f(a-b)^3((a-b)\cos(2(e+fx))+a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/6*(Cos[e + f*x]*((3*a + 5*b)^2 + 12*(a^2 + 2*a*b - 3*b^2)*Cos[2*(e + f*x
)] + 3*(a - b)^2*Cos[4*(e + f*x)])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*
Sec[e + f*x]^2])/(Sqrt[2]*(a - b)^3*f*(a + b + (a - b)*Cos[2*(e + f*x)])^2)
```

fricas [A] time = 0.54, size = 202, normalized size = 1.71

$$-\frac{\left(3\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^5+12\left(ab-b^2\right)\cos\left(fx+e\right)^3+8b^2\cos\left(fx+e\right)\right)\sqrt{}}{3\left(\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5\right)f\cos\left(fx+e\right)^4+2\left(a^4b-4a^3b^2+6a^2b^3-4ab^4+b^5\right)f\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 + 12*(a*b - b^2)*cos(f*x + e)^3
+ 8*b^2*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a
^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)^4 +
2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos(f*x + e)^2 + (a^3*b
^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
```

```
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(2*(tan(
(f*x+exp(1))/2)^2*(tan((f*x+exp(1))/2)^2*(-tan((f*x+exp(1))/2)^2*(-1536*b^1
1*sign(tan((f*x+exp(1))/2)^2-1)+19968*a*b^10*sign(tan((f*x+exp(1))/2)^2-1)-
96768*a^2*b^9*sign(tan((f*x+exp(1))/2)^2-1)+247296*a^3*b^8*sign(tan((f*x+ex
p(1))/2)^2-1)-376320*a^4*b^7*sign(tan((f*x+exp(1))/2)^2-1)+354816*a^5*b^6*s
ign(tan((f*x+exp(1))/2)^2-1)-204288*a^6*b^5*sign(tan((f*x+exp(1))/2)^2-1)+6
6048*a^7*b^4*sign(tan((f*x+exp(1))/2)^2-1)-9216*a^8*b^3*sign(tan((f*x+exp(1
))/2)^2-1))/(36864*b^12-368640*a*b^11+1658880*a^2*b^10-4423680*a^3*b^9+7741
440*a^4*b^8-9289728*a^5*b^7+7741440*a^6*b^6-4423680*a^7*b^5+1658880*a^8*b^4
-368640*a^9*b^3+36864*a^10*b^2)-(32256*b^11*sign(tan((f*x+exp(1))/2)^2-1)-2
35008*a*b^10*sign(tan((f*x+exp(1))/2)^2-1)+741888*a^2*b^9*sign(tan((f*x+exp
(1))/2)^2-1)-1322496*a^3*b^8*sign(tan((f*x+exp(1))/2)^2-1)+1451520*a^4*b^7*
sign(tan((f*x+exp(1))/2)^2-1)-999936*a^5*b^6*sign(tan((f*x+exp(1))/2)^2-1)+
419328*a^6*b^5*sign(tan((f*x+exp(1))/2)^2-1)-96768*a^7*b^4*sign(tan((f*x+ex
p(1))/2)^2-1)+9216*a^8*b^3*sign(tan((f*x+exp(1))/2)^2-1))/(36864*b^12-36864
0*a*b^11+1658880*a^2*b^10-4423680*a^3*b^9+7741440*a^4*b^8-9289728*a^5*b^7+7
741440*a^6*b^6-4423680*a^7*b^5+1658880*a^8*b^4-368640*a^9*b^3+36864*a^10*b^
2))-(32256*b^11*sign(tan((f*x+exp(1))/2)^2-1)-235008*a*b^10*sign(tan((f*x+e
xp(1))/2)^2-1)+741888*a^2*b^9*sign(tan((f*x+exp(1))/2)^2-1)-1322496*a^3*b^8
*sign(tan((f*x+exp(1))/2)^2-1)+1451520*a^4*b^7*sign(tan((f*x+exp(1))/2)^2-1
)-999936*a^5*b^6*sign(tan((f*x+exp(1))/2)^2-1)+419328*a^6*b^5*sign(tan((f*x
+exp(1))/2)^2-1)-96768*a^7*b^4*sign(tan((f*x+exp(1))/2)^2-1)+9216*a^8*b^3*s
ign(tan((f*x+exp(1))/2)^2-1))/(36864*b^12-368640*a*b^11+1658880*a^2*b^10-44
23680*a^3*b^9+7741440*a^4*b^8-9289728*a^5*b^7+7741440*a^6*b^6-4423680*a^7*b
^5+1658880*a^8*b^4-368640*a^9*b^3+36864*a^10*b^2))-(-1536*b^11*sign(tan((f*
x+exp(1))/2)^2-1)+19968*a*b^10*sign(tan((f*x+exp(1))/2)^2-1)-96768*a^2*b^9*
sign(tan((f*x+exp(1))/2)^2-1)+247296*a^3*b^8*sign(tan((f*x+exp(1))/2)^2-1)-
376320*a^4*b^7*sign(tan((f*x+exp(1))/2)^2-1)+354816*a^5*b^6*sign(tan((f*x+e
xp(1))/2)^2-1)-204288*a^6*b^5*sign(tan((f*x+exp(1))/2)^2-1)+66048*a^7*b^4*s
ign(tan((f*x+exp(1))/2)^2-1)-9216*a^8*b^3*sign(tan((f*x+exp(1))/2)^2-1))/(3
6864*b^12-368640*a*b^11+1658880*a^2*b^10-4423680*a^3*b^9+7741440*a^4*b^8-92
89728*a^5*b^7+7741440*a^6*b^6-4423680*a^7*b^5+1658880*a^8*b^4-368640*a^9*b^
3+36864*a^10*b^2))/sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4
*b*tan((f*x+exp(1))/2)^2+a)/(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2
)^2+4*b*tan((f*x+exp(1))/2)^2+a)+2*(sqrt(a)*tan((f*x+exp(1))/2)^2-sqrt(a)-s
qrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/
2)^2+a))/(2*a^2*sign(tan((f*x+exp(1))/2)^2-1)+2*b^2*sign(tan((f*x+exp(1))/2
)^2-1)-4*a*b*sign(tan((f*x+exp(1))/2)^2-1))/(-2*sqrt(a)*(-sqrt(a)*tan((f*x+
exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan
```

```
((f*x+exp(1))/2)^2+a))+(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(
1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2-3*a+4*b)
)
```

maple [A] time = 0.34, size = 147, normalized size = 1.25

$$-\frac{\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)\left(3\left(\cos^4\left(fx+e\right)\right)a^2-6\left(\cos^4\left(fx+e\right)\right)ab+3\left(\cos^4\left(fx+e\right)\right)b^2}{3f\left(\frac{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}{\cos\left(fx+e\right)^2}\right)^{\frac{5}{2}}\cos\left(fx+e\right)^5\left(a-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(3*cos(f*x+e)^4*a^2-6*cos(f*x+e)^4
*a*b+3*cos(f*x+e)^4*b^2+12*cos(f*x+e)^2*a*b-12*b^2*cos(f*x+e)^2+8*b^2)/((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5/(a-b)^3
```

maxima [A] time = 0.47, size = 135, normalized size = 1.14

$$-\frac{\frac{3\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^3-3a^2b+3ab^2-b^3}+\frac{6\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)b\cos\left(fx+e\right)^2-b^2}{\left(a^3-3a^2b+3ab^2-b^3\right)\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^
2 - b^3) + (6*(a - b + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)}{{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{\left(e + f x \right)}}{\left(a + b \tan^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.143 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{5/2}f}-\frac{b(5a-3b)\sec(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}-\frac{b\sec(e+fx)}{3af(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

```
[Out] -arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*sec
(f*x+e)/a/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(3/2)-1/3*(5*a-3*b)*b*sec(f*x+e)/a^2
/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3664, 414, 527, 12, 377, 207}

$$-\frac{b(5a-3b)\sec(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{5/2}f}-\frac{b\sec(e+fx)}{3af(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]/(a^(5/2)*f
)) - (b*Sec[e + f*x])/(3*a*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - ((
5*a - 3*b)*b*Sec[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a - b + b*Sec[e + f*x]^2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-b-2bx^2}{\left(-1+x^2\right)\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec\right.}{3a(a-b)f}$$

$$= -\frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\sec(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}}$$

$$= -\frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\sec(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}}$$

$$= -\frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\sec(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{3a^2(a-}$$

Mathematica [B] time = 5.46, size = 305, normalized size = 2.24

$$\frac{\cos(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-\frac{2\sqrt{2}\sqrt{a}b\left(3\left(2a^2-3ab+b^2\right)\cos(2(e+fx))+6a^2+ab-3b^2\right)}{(a-b)^2((a-b)\cos(2(e+fx))+a+b)^2} - \frac{3\sec^2(}{}\right.}{6a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cos[e + f*x]*((-2*Sqrt[2]*Sqrt[a]*b*(6*a^2 + a*b - 3*b^2 + 3*(2*a^2 - 3*a*
b + b^2)*Cos[2*(e + f*x)]))/((a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)])^2
) - (3*(ArcTanh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e
 + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Ta
n[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e +
f*x)/2]^2)^2])])*Sec[(e + f*x)/2]^2)/Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)]
)*Sec[(e + f*x)/2]^4])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]
^2])/(6*a^(5/2)*f)
```

fricas [B] time = 0.62, size = 696, normalized size = 5.12

$$\left[\frac{3\left(\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)\cos\left(fx+e\right)^4+a^2b^2-2\,ab^3+b^4+2\left(a^3b-3\,a^2b^2+3\,ab^3-b^4\right)\cos\left(fx+\right.}{6\left(\left(a^7-4\,a^6b+6\,a^5b^2-4\,a^4b^3+a^3b^4\right)f\,\mathrm{c}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^4 + a^2*b
^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*cos(f*x + e)^2)*
sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) - 2*(
3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 - 3*a*b^3)*cos(
f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 - 4*a^6*
b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^
2 + 3*a^4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)
*f), 1/3*(3*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^4 + a
^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*cos(f*x + e)
^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e)/a) - (3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*
a^2*b^2 - 3*a*b^3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2))/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^
4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2
 - 2*a^4*b^3 + a^3*b^4)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
```

```
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^2-1 were
not checkedWarning, integration of abs or sign assumes constant sign by int
ervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b]=[85,53]Warning, need to c
hoose a branch for the root of a polynomial with parameters. This might be
wrong.The choice was done assuming [a,b]=[-33,71]Discontinuities at zeroes
of t_nostep^2-1 were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2
```

```
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning
, integration of abs or sign assumes constant sign by intervals (correct if
 the argument is real):Check [abs(t_nostep^2-1)]Warning, replacing 0 by ` u
`, a substitution variable should perhaps be purged.Warning, replacing 0 by
 ` u`, a substitution variable should perhaps be purged.Warning, replacing
0 by ` u`, a substitution variable should perhaps be purged.Evaluation time
: 6.3Error: Bad Argument Type
```

maple [B] time = 2.46, size = 27448, normalized size = 201.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)\,\left(b\tan\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(5/2)),x)
[Out] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
[Out] Integral(csc(e + f*x)/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.144 $\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{(a-5b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{7/2}f}-\frac{b(13a-15b)\sec(e+fx)}{6a^3f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{5b\sec(e+fx)}{6a^2f\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{c}{2af}$$

```
[Out] -1/2*(a-5*b)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(7/2)
/f-1/2*cot(f*x+e)*csc(f*x+e)/a/f/(a-b+b*sec(f*x+e)^2)^(3/2)-5/6*b*sec(f*x+e
)/a^2/f/(a-b+b*sec(f*x+e)^2)^(3/2)-1/6*(13*a-15*b)*b*sec(f*x+e)/a^3/(a-b)/f
/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3664, 471, 527, 12, 377, 207}

$$-\frac{b(13a-15b)\sec(e+fx)}{6a^3f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{5b\sec(e+fx)}{6a^2f\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{(a-5b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{7/2}f}-\frac{c}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a - 5*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])
/(2*a^(7/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*(a - b + b*Sec[e + f*x]
^2)^(3/2)) - (5*b*Sec[e + f*x])/(6*a^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2))
- ((13*a - 15*b)*b*Sec[e + f*x])/(6*a^3*(a - b)*f*Sqrt[a - b + b*Sec[e + f*
x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a-b-4bx^2}{\left(-1+x^2\right)\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+f\right.}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(1}{6a^3(a-}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(1}{6a^3(a-}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(1}{6a^3(a-}$$

$$= -\frac{(a-5b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{7/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{6a^2f\left(\right.}$$

Mathematica [B] time = 4.66, size = 385, normalized size = 2.18

$$\frac{\sqrt{\frac{(a-b)\cos(2(e+fx))+a+b}{\cos(2(e+fx))+1}}\left(8ab^2\cos(e+fx)-24b(a-b)\cos(e+fx)((a-b)\cos(2(e+fx))+a+b)-3(a-b)\cot(e+fx)\csc(e+fx)((a-b)\cos(2(e+fx))+a+b)^2\right)}{3a^3(a-b)((a-b)\cos(2(e+fx))+a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ((Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(8*a*b^2*
Cos[e + f*x] - 24*(a - b)*b*Cos[e + f*x]*(a + b + (a - b)*Cos[2*(e + f*x)])
 - 3*(a - b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Cot[e + f*x]*Csc[e + f*x]
))/(3*a^3*(a - b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2) - ((a - 5*b)*(ArcTa
nh[(a - (a - 2*b)*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2
+ a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2
]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]
)])*Cos[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])
*Sec[e + f*x]^2])/(2*a^(7/2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e
 + f*x)/2]^4]))/(2*f)
```

fricas [B] time = 0.70, size = 889, normalized size = 5.02

$$\left[-\frac{3\left(\left(a^4-8\,a^3b+18\,a^2b^2-16\,ab^3+5\,b^4\right)\cos\left(fx+e\right)^6-\left(a^4-10\,a^3b+32\,a^2b^2-38\,ab^3+15\,b^4\right)\cos\left(fx+e\right)^4\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((a^4 - 8*a^3*b + 18*a^2*b^2 - 16*a*b^3 + 5*b^4)*cos(f*x + e)^6 -
 (a^4 - 10*a^3*b + 32*a^2*b^2 - 38*a*b^3 + 15*b^4)*cos(f*x + e)^4 - a^2*b^2
 + 6*a*b^3 - 5*b^4 - (2*a^3*b - 15*a^2*b^2 + 28*a*b^3 - 15*b^4)*cos(f*x + e
)^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 + 2*sqrt(a)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1))
- 2*(3*(a^4 - 7*a^3*b + 11*a^2*b^2 - 5*a*b^3)*cos(f*x + e)^5 + 2*(9*a^3*b -
 23*a^2*b^2 + 15*a*b^3)*cos(f*x + e)^3 + (13*a^2*b^2 - 15*a*b^3)*cos(f*x +
e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 - 3*a^6*b + 3*
a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6 - (a^7 - 5*a^6*b + 7*a^5*b^2 - 3*a^4*b^
3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*a^5*b^2 + 3*a^4*b^3)*f*cos(f*x + e)^2 -
(a^5*b^2 - a^4*b^3)*f), 1/6*(3*((a^4 - 8*a^3*b + 18*a^2*b^2 - 16*a*b^3 + 5*
b^4)*cos(f*x + e)^6 - (a^4 - 10*a^3*b + 32*a^2*b^2 - 38*a*b^3 + 15*b^4)*cos
(f*x + e)^4 - a^2*b^2 + 6*a*b^3 - 5*b^4 - (2*a^3*b - 15*a^2*b^2 + 28*a*b^3
- 15*b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + (3*(a^4 - 7*a^3*b + 11*a^2*b^2
- 5*a*b^3)*cos(f*x + e)^5 + 2*(9*a^3*b - 23*a^2*b^2 + 15*a*b^3)*cos(f*x + e
)^3 + (13*a^2*b^2 - 15*a*b^3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6
 - (a^7 - 5*a^6*b + 7*a^5*b^2 - 3*a^4*b^3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*
a^5*b^2 + 3*a^4*b^3)*f*cos(f*x + e)^2 - (a^5*b^2 - a^4*b^3)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
```

```
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs
```

```
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^2-1 were
not checkedWarning, integration of abs or sign assumes constant sign by int
ervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b]=[85,53]Warning, need to c
hoose a branch for the root of a polynomial with parameters. This might be
wrong.The choice was done assuming [a,b]=[-33,71]Discontinuities at zeroes
of t_nostep^2-1 were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign
by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Eva
luation time: 9.68index.cc index_m i_lex_is_greater Error: Bad Argument Val
ue
```

maple [B] time = 3.84, size = 38486, normalized size = 217.44

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^3\,{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}{\left(e+fx \right)}}{\left(a+b\tan^{2}{\left(e+fx \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.145 $\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{5b(11a-21b)\sec(e+fx)}{24a^4f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{b(23a-35b)\sec(e+fx)}{24a^3f\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\left(3a^2-30\right.}{}$$

```
[Out] -1/8*(3*a^2-30*a*b+35*b^2)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^
(1/2))/a^(9/2)/f-1/8*(5*a-7*b)*cot(f*x+e)*csc(f*x+e)/a^2/f/(a-b+b*sec(f*x+e
)^2)^(3/2)-1/4*cot(f*x+e)^3*csc(f*x+e)/a/f/(a-b+b*sec(f*x+e)^2)^(3/2)-1/24*
(23*a-35*b)*b*sec(f*x+e)/a^3/f/(a-b+b*sec(f*x+e)^2)^(3/2)-5/24*(11*a-21*b)*
b*sec(f*x+e)/a^4/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3664, 470, 527, 12, 377, 207}

$$-\frac{\left(3a^2-30ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{9/2}f}-\frac{5b(11a-21b)\sec(e+fx)}{24a^4f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{b(23a-35b)\sec(e+fx)}{24a^3f\left(a+b\sec^2(e+fx)-b\right)^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((3*a^2 - 30*a*b + 35*b^2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*S
ec[e + f*x]^2]])/(8*a^(9/2)*f) - ((5*a - 7*b)*Cot[e + f*x]*Csc[e + f*x])/(8
*a^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (Cot[e + f*x]^3*Csc[e + f*x])/(4
*a*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - ((23*a - 35*b)*b*Sec[e + f*x])/(24
*a^3*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (5*(11*a - 21*b)*b*Sec[e + f*x])
/(24*a^4*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
```

```
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a+b-2(2a-3b)x^2}{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+\cdots\right.}{4af}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23\cdots}{24a^3f\cdots}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23\cdots}{24a^3f\cdots}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23\cdots}{24a^3f\cdots}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23\cdots}{24a^3f\cdots}$$

$$= -\frac{\left(3a^2-30ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{9/2}f} - \frac{(5a-7b)\cot(e+fx)\csc(e\ldots}{8a^2f\left(a-b+b\sec^2(e+f\ldots\right.}$$

Mathematica [B] time = 6.98, size = 1142, normalized size = 4.82

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x
)])]*((4*b^2*Cos[e + f*x])/(3*a^3*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e
+ f*x)])^2) - (2*(2*a*b*Cos[e + f*x] - 3*b^2*Cos[e + f*x]))/(a^4*(a + b + a
*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])) + ((-3*a*Cos[e + f*x] + 11*b*Cos[e
 + f*x])*Csc[e + f*x]^2)/(8*a^4) - (Cot[e + f*x]*Csc[e + f*x]^3)/(4*a^3)))/
f + ((3*a^2 - 30*a*b + 35*b^2)*(-1/4*((2*Sqrt[a]*ArcTanh[(Sqrt[b]*(1 + Tan[
(e + f*x)/2]^2))/Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^
2]] - Sqrt[b]*(ArcTanh[(a - a*Tan[(e + f*x)/2]^2 + 2*b*Tan[(e + f*x)/2]^2)/
(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + A
rcTanh[(2*b + a*(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/
2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])]))*(1 + Cos[e + f*x])*Sqrt[(1 + Cos[
2*(e + f*x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])
/(1 + Cos[2*(e + f*x)])]*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)
*Sqrt[(4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + Tan[(e
+ f*x)/2]^2)^2])/(Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*Sq
rt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(
e + f*x)/2]^2)^2]) + ((2*Sqrt[a]*ArcTanh[(Sqrt[b]*(1 + Tan[(e + f*x)/2]^2))
/Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] + Sqrt[b]*(A
rcTanh[(a - a*Tan[(e + f*x)/2]^2 + 2*b*Tan[(e + f*x)/2]^2)/(Sqrt[a]*Sqrt[4*
b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])] + ArcTanh[(2*b + a*
(-1 + Tan[(e + f*x)/2]^2))/(Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + T
an[(e + f*x)/2]^2)^2])]))*(1 + Cos[e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1
 + Cos[e + f*x])^2]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e +
 f*x)])]*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(
e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])
/(4*Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(
e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)
^2])))/(8*a^4*f)
```

fricas [B] time = 0.80, size = 1037, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((3*a^4 - 36*a^3*b + 98*a^2*b^2 - 100*a*b^3 + 35*b^4)*cos(f*x + e)
^8 - 2*(3*a^4 - 39*a^3*b + 131*a^2*b^2 - 165*a*b^3 + 70*b^4)*cos(f*x + e)^6
 + (3*a^4 - 48*a^3*b + 233*a^2*b^2 - 390*a*b^3 + 210*b^4)*cos(f*x + e)^4 +
3*a^2*b^2 - 30*a*b^3 + 35*b^4 + 2*(3*a^3*b - 36*a^2*b^2 + 95*a*b^3 - 70*b^4
)*cos(f*x + e)^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt((
(a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x
+ e)^2 - 1)) + 2*(3*(3*a^4 - 33*a^3*b + 65*a^2*b^2 - 35*a*b^3)*cos(f*x + e)
^7 - (15*a^4 - 177*a^3*b + 445*a^2*b^2 - 315*a*b^3)*cos(f*x + e)^5 - (78*a^
3*b - 305*a^2*b^2 + 315*a*b^3)*cos(f*x + e)^3 - 5*(11*a^2*b^2 - 21*a*b^3)*c
os(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 - 2*a
^6*b + a^5*b^2)*f*cos(f*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2
)*f*cos(f*x + e)^6 + (a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*
b - 2*a^5*b^2)*f*cos(f*x + e)^2), 1/24*(3*((3*a^4 - 36*a^3*b + 98*a^2*b^2 -
 100*a*b^3 + 35*b^4)*cos(f*x + e)^8 - 2*(3*a^4 - 39*a^3*b + 131*a^2*b^2 - 1
65*a*b^3 + 70*b^4)*cos(f*x + e)^6 + (3*a^4 - 48*a^3*b + 233*a^2*b^2 - 390*a
*b^3 + 210*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 30*a*b^3 + 35*b^4 + 2*(3*a^3*b
```

```
 - 36*a^2*b^2 + 95*a*b^3 - 70*b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-a)
*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + (3*(3*
a^4 - 33*a^3*b + 65*a^2*b^2 - 35*a*b^3)*cos(f*x + e)^7 - (15*a^4 - 177*a^3*
b + 445*a^2*b^2 - 315*a*b^3)*cos(f*x + e)^5 - (78*a^3*b - 305*a^2*b^2 + 315
*a*b^3)*cos(f*x + e)^3 - 5*(11*a^2*b^2 - 21*a*b^3)*cos(f*x + e))*sqrt(((a -
 b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 - 2*a^6*b + a^5*b^2)*f*cos(f
*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^6 + (a
^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2)*f*cos(f*
x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*(tan((f*x+exp(1))/2)^2*
(tan((f*x+exp(1))/2)^2*(tan((f*x+exp(1))/2)^2*(tan((f*x+exp(1))/2)^2*(-tan(
(f*x+exp(1))/2)^2*(39582418599936*a^15*b^4-79164837199872*a^16*b^3+39582418
599936*a^17*b^2)/(-10133099161583616*a^16*b^4*sign(tan((f*x+exp(1))/2)^2-1)
+20266198323167232*a^17*b^3*sign(tan((f*x+exp(1))/2)^2-1)-10133099161583616
*a^18*b^2*sign(tan((f*x+exp(1))/2)^2-1))-(-554153860399104*a^14*b^5+1306219
813797888*a^15*b^4-949978046398464*a^16*b^3+197912092999680*a^17*b^2)/(-101
33099161583616*a^16*b^4*sign(tan((f*x+exp(1))/2)^2-1)+20266198323167232*a^1
7*b^3*sign(tan((f*x+exp(1))/2)^2-1)-10133099161583616*a^18*b^2*sign(tan((f*
x+exp(1))/2)^2-1)))-(-14777436277309440*a^13*b^6+40321290413801472*a^14*b^5
-37497744553672704*a^15*b^4+13141362975178752*a^16*b^3-1187472557998080*a^1
7*b^2)/(-10133099161583616*a^16*b^4*sign(tan((f*x+exp(1))/2)^2-1)+202661983
23167232*a^17*b^3*sign(tan((f*x+exp(1))/2)^2-1)-10133099161583616*a^18*b^2*
sign(tan((f*x+exp(1))/2)^2-1)))-(-44332308831928320*a^12*b^7+13363024519338
3936*a^13*b^6-149938201656557568*a^14*b^5+78294023990673408*a^15*b^4-196328
79625568256*a^16*b^3+1979120929996800*a^17*b^2)/(-10133099161583616*a^16*b^
4*sign(tan((f*x+exp(1))/2)^2-1)+20266198323167232*a^17*b^3*sign(tan((f*x+ex
p(1))/2)^2-1)-10133099161583616*a^18*b^2*sign(tan((f*x+exp(1))/2)^2-1)))-(-
30399297484750848*a^12*b^7+79164837199872000*a^13*b^6-65865144550293504*a^1
4*b^5+14447582788976640*a^15*b^4+4037406697193472*a^16*b^3-1385384650997760
*a^17*b^2)/(-10133099161583616*a^16*b^4*sign(tan((f*x+exp(1))/2)^2-1)+20266
198323167232*a^17*b^3*sign(tan((f*x+exp(1))/2)^2-1)-10133099161583616*a^18*
b^2*sign(tan((f*x+exp(1))/2)^2-1)))-(-8444249301319680*a^13*b^6+21084234974
232576*a^14*b^5-16479480277106688*a^15*b^4+3483252836794368*a^16*b^3+356241
767399424*a^17*b^2)/(-10133099161583616*a^16*b^4*sign(tan((f*x+exp(1))/2)^2
-1)+20266198323167232*a^17*b^3*sign(tan((f*x+exp(1))/2)^2-1)-10133099161583
616*a^18*b^2*sign(tan((f*x+exp(1))/2)^2-1)))/sqrt(a*tan((f*x+exp(1))/2)^4-2
*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a)/(a*tan((f*x+exp(1))/2
```

```
)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a)+2*(-1/64*(-6*a^3
*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+
exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))-26*a*b^2*(-sqrt(a)*tan((f*x+exp(
1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*
x+exp(1))/2)^2+a))+24*a^2*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x
+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))+4*a^2
*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+
exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3+22*b^2*(-sqrt(a)*tan((f*x+exp(
1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*
x+exp(1))/2)^2+a))^3-20*a*b*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x
+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^3+3*s
qrt(a)*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a
*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2-8*sqrt(a)*a*b*(-sqrt
(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))
/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2-5*sqrt(a)*a^3+12*sqrt(a)*a^2*b)/a^4/(
-(-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+
exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+a))^2+a)^2/sign(tan((f*x+exp(1))/2)^
2-1)-1/32*(3*a^2+35*b^2-30*a*b)*atan((-sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a
*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/2)^2+
a))/sqrt(-a))/a^4/sqrt(-a)/sign(tan((f*x+exp(1))/2)^2-1)-1/64*(3*sqrt(a)*a^
2+35*sqrt(a)*b^2-30*sqrt(a)*a*b)*ln(abs(a*(-sqrt(a)*tan((f*x+exp(1))/2)^2+s
qrt(a*tan((f*x+exp(1))/2)^4-2*a*tan((f*x+exp(1))/2)^2+4*b*tan((f*x+exp(1))/
2)^2+a))+sqrt(a)*a-2*sqrt(a)*b))/a^5/sign(tan((f*x+exp(1))/2)^2-1)))
```

maple [B] time = 7.12, size = 49917, normalized size = 210.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x)
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(5/2)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
[Out] Integral(csc(e + f*x)**5/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.146 $\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=246

$$\frac{\left(3a^2+24ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{9/2}}-\frac{5b(11a+10b)\tan(e+fx)}{24f(a-b)^4\sqrt{a+b\tan^2(e+fx)}}-\frac{b(23a+12b)\tan(e+fx)}{24f(a-b)^3\,(a+b\tan^2(e+fx)}$$

```
[Out] 1/8*(3*a^2+24*a*b+8*b^2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(
1/2))/(a-b)^(9/2)/f-5/24*b*(11*a+10*b)*tan(f*x+e)/(a-b)^4/f/(a+b*tan(f*x+e)
^2)^(1/2)-1/8*(5*a+2*b)*cos(f*x+e)*sin(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^
(3/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-1/24*b*(
23*a+12*b)*tan(f*x+e)/(a-b)^3/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.33, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3663, 470, 527, 12, 377, 203}

$$\frac{\left(3a^2+24ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{9/2}}-\frac{5b(11a+10b)\tan(e+fx)}{24f(a-b)^4\sqrt{a+b\tan^2(e+fx)}}-\frac{b(23a+12b)\tan(e+fx)}{24f(a-b)^3\,(a+b\tan^2(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*a^2 + 24*a*b + 8*b^2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[
e + f*x]^2]])/(8*(a - b)^(9/2)*f) - ((5*a + 2*b)*Cos[e + f*x]*Sin[e + f*x])
/(8*(a - b)^2*f*(a + b*Tan[e + f*x]^2)^(3/2)) + (Cos[e + f*x]^3*Sin[e + f*x
])/(4*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (b*(23*a + 12*b)*Tan[e + f*
x])/(24*(a - b)^3*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (5*b*(11*a + 10*b)*Tan[
e + f*x])/(24*(a - b)^4*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
```

```
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^3\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a-2(2a+b)x^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Sub}}{}$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{24(a}$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{24(a}$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{24(a}$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{24(a}$$

$$= \frac{\left(3a^2+24ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{9/2}f} - \frac{(5a+2b)\cos(e+fx)\sin(e+fx}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/}}$$

Mathematica [C] time = 5.67, size = 378, normalized size = 1.54

$$-\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-3\sqrt{2}\,ab\left(3a^2+24ab+8b^2\right)\sin(2(e+fx))\sin^2(e+fx)\left(\frac{\csc^2}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/96*(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-3*Sqrt[2]*
a*b*(3*a^2 + 24*a*b + 8*b^2)*(((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f
*x]^2)/b)^(3/2)*(2*(a - b)*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e
 + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - 2*a*EllipticPi[-(b/(a - b)), Ar
cSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]],
1])*Sin[e + f*x]^2*Sin[2*(e + f*x)] - a*(a - b)*(64*a*b^2*Sin[2*(e + f*x)]
- 64*b*(3*a + 2*b)*(a + b + (a - b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)] - 6*
(4*a + 7*b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Sin[2*(e + f*x)] + 3*(a -
b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Sin[4*(e + f*x)])))/(Sqrt[2]*a*(a -
 b)^5*f*(a + b + (a - b)*Cos[2*(e + f*x)])^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 15.66, size = 7943, normalized size = 32.29

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sin\left(e+f\,x\right)}^4}{{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{4}{\left(e+fx\right)}}{\left(a+b\tan^{2}{\left(e+fx\right)}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
[Out] Integral(sin(e + f*x)**4/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.147 $\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{b(13a+2b)\tan(e+fx)}{6af(a-b)^3\sqrt{a+b\tan^2(e+fx)}}-\frac{5b\tan(e+fx)}{6f(a-b)^2\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{(a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{7/2}}-\frac{\mathrm{s}}{2f(a}$$

```
[Out] 1/2*(a+4*b)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(
7/2)/f-1/6*b*(13*a+2*b)*tan(f*x+e)/a/(a-b)^3/f/(a+b*tan(f*x+e)^2)^(1/2)-1/2
*cos(f*x+e)*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-5/6*b*tan(f*x+e)/(a
-b)^2/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.21, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3663, 471, 527, 12, 377, 203}

$$\frac{(a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{7/2}}-\frac{b(13a+2b)\tan(e+fx)}{6af(a-b)^3\sqrt{a+b\tan^2(e+fx)}}-\frac{5b\tan(e+fx)}{6f(a-b)^2\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\mathrm{sir}}{2f(a-}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 4*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(
2*(a - b)^(7/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a - b)*f*(a + b*Tan[e
+ f*x]^2)^(3/2)) - (5*b*Tan[e + f*x])/(6*(a - b)^2*f*(a + b*Tan[e + f*x]^2)
^(3/2)) - (b*(13*a + 2*b)*Tan[e + f*x])/(6*a*(a - b)^3*f*Sqrt[a + b*Tan[e +
 f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a-4bx^2}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(1}{6a(a-}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(1}{6a(a-}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(1}{6a(a-}$$

$$= \frac{(a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{7/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b}{6(a-b)^2f\left(\right.}$$

Mathematica [C] time = 4.62, size = 309, normalized size = 1.71

$$-\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(-(a-b)\sin(2(e+fx))\left(8ab^2-4b(6a+b)((a-b)\cos(2(e+f\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/12*(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-((a - b)*(
8*a*b^2 - 4*b*(6*a + b)*(a + b + (a - b)*Cos[2*(e + f*x)]) - 3*a*(a + b + (
a - b)*Cos[2*(e + f*x)])^2)*Sin[2*(e + f*x)]) - (3*a*b*(a + 4*b)*(((a + b +
 (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)^(3/2)*(2*(a - b)*EllipticF[Ar
cSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]],
1] - 2*a*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e +
f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sin[e + f*x]^2*Sin[2*(e + f*x)])/Sq
rt[2]))/(Sqrt[2]*a*(a - b)^4*f*(a + b + (a - b)*Cos[2*(e + f*x)])^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

maple [B] time = 6.09, size = 2511, normalized size = 13.87

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f*(-1+cos(2*f*x+2*e))*(3*(b^4*(a-b))^(1/2)*arctan((-1+cos(2*f*x+2*e))/(
(a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x
+2*e)*(a-b)*b^2/(b^4*(a-b))^(1/2))*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)
/(cos(2*f*x+2*e)+1))^(3/2)*sin(2*f*x+2*e)*cos(2*f*x+2*e)*a^2+3*(b^4*(a-b))^
(1/2)*arctan((-1+cos(2*f*x+2*e))/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(
cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)*b^2/(b^4*(a-b))^(1/2))*sin(2*
f*x+2*e)*a^2*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(
3/2)-6*cos(2*f*x+2*e)^2*a^3*b^3+14*cos(2*f*x+2*e)^2*a^2*b^4-10*cos(2*f*x+2*
e)^2*a*b^5+2*cos(2*f*x+2*e)^2*b^6-10*cos(2*f*x+2*e)*a^2*b^4+14*cos(2*f*x+2*
```

```
e)*a*b^5-4*cos(2*f*x+2*e)*b^6+6*a^3*b^3-4*b^4*a^2-4*a*b^5+2*b^6)/sin(2*f*x+
2*e)^3/(a-b)^3/a^2/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)
+1))^(3/2)/b^2+1/12/f*(-1+cos(2*f*x+2*e))*(-52*(a-b)^(3/2)*cos(2*f*x+2*e)^2
*a*b^5-3*(a-b)^(3/2)*cos(2*f*x+2*e)^3*a^5*b+9*(a-b)^(3/2)*cos(2*f*x+2*e)^3*
a^4*b^2-3*(a-b)^(3/2)*cos(2*f*x+2*e)^2*a^5*b-3*(a-b)^(3/2)*cos(2*f*x+2*e)^2
*a^4*b^2+24*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*
f*x+2*e)+1))^(3/2)*(a-b)^(3/2)*arctan((-1+cos(2*f*x+2*e))/((a*cos(2*f*x+2*e
)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)*b^2/
(b^4*(a-b))^(1/2))*cos(2*f*x+2*e)*(b^4*(a-b))^(1/2)*a^2+33*(a-b)^(3/2)*a^3*
b^3-19*(a-b)^(3/2)*a^2*b^4-28*(a-b)^(3/2)*a*b^5-3*(a-b)^(3/2)*cos(2*f*x+2*e
)*a^3*b^3-55*(a-b)^(3/2)*cos(2*f*x+2*e)*a^2*b^4+80*(a-b)^(3/2)*cos(2*f*x+2*
e)*a*b^5-9*(a-b)^(3/2)*a^4*b^2*cos(2*f*x+2*e)+3*(a-b)^(3/2)*a^5*b*cos(2*f*x
+2*e)-9*(a-b)^(3/2)*cos(2*f*x+2*e)^3*a^3*b^3+3*(a-b)^(3/2)*cos(2*f*x+2*e)^3
*a^2*b^4-21*(a-b)^(3/2)*cos(2*f*x+2*e)^2*a^3*b^3+71*(a-b)^(3/2)*cos(2*f*x+2
*e)^2*a^2*b^4-16*(a-b)^(3/2)*cos(2*f*x+2*e)*b^6+8*(a-b)^(3/2)*cos(2*f*x+2*e
)^2*b^6-6*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*
x+2*e)+1))^(3/2)*arctan((-1+cos(2*f*x+2*e))/((a*cos(2*f*x+2*e)-b*cos(2*f*x+
2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*a^5*b+24*si
n(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(
3/2)*arctan((-1+cos(2*f*x+2*e))/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(c
os(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*a^4*b^2-30*sin(2*f*x+2*
e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arcta
n((-1+cos(2*f*x+2*e))/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2
*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*a^3*b^3+12*sin(2*f*x+2*e)*((a*cos
(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arctan((-1+cos(
2*f*x+2*e))/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1
/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*a^2*b^4+3*(a-b)^(3/2)*a^5*b+3*(a-b)^(3/2)*a
^4*b^2+8*(a-b)^(3/2)*b^6-6*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*
e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arctan((-1+cos(2*f*x+2*e))/((a*cos(2*f*x+
2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)^(
1/2))*cos(2*f*x+2*e)*a^5*b+24*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x
+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arctan((-1+cos(2*f*x+2*e))/((a*cos(2*f
*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b
)^(1/2))*cos(2*f*x+2*e)*a^4*b^2-30*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(
2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arctan((-1+cos(2*f*x+2*e))/((a*co
s(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)
*(a-b)^(1/2))*cos(2*f*x+2*e)*a^3*b^3+12*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b
*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arctan((-1+cos(2*f*x+2*e))/(
(a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x
+2*e)*(a-b)^(1/2))*cos(2*f*x+2*e)*a^2*b^4+24*(b^4*(a-b))^(1/2)*arctan((-1+c
os(2*f*x+2*e))/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))
^(1/2)/sin(2*f*x+2*e)*(a-b)*b^2/(b^4*(a-b))^(1/2))*sin(2*f*x+2*e)*a^2*((a*c
os(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*(a-b)^(3/2))/
(a-b)^(11/2)/sin(2*f*x+2*e)^3/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos
(2*f*x+2*e)+1))^(3/2)/a^2/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{{\left(b\tan\left(e+f\,x\right)^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}{\left(e + f x \right)}}{\left(a + b \tan^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
[Out] Integral(sin(e + f*x)**2/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.148 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{b(5a-2b)\tan(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{b\tan(e+fx)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3*(
5*a-2*b)*b*tan(f*x+e)/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+
e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3661, 414, 527, 12, 377, 203}

$$-\frac{b(5a-2b)\tan(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{b\tan(e+fx)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (b*Tan[e + f*x])/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - ((5*
a - 2*b)*b*Tan[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-2b-2bx^2}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+f \cdots\right.}{3a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \cdots$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \cdots$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \cdots$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b \cdots}{3a^2(a-b)^2f\sqrt{a \cdots}}$$

Mathematica [C] time = 9.46, size = 1331, normalized size = 9.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(1575*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]] -
 (3150*(a - b)*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2)/a +
 (1575*(a - b)^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4)/a
^2 + (2100*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/a - (
4200*(a - b)*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2*Tan[
```

```
e + f*x]^2)/a^2 + (2100*(a - b)^2*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]
]*Sin[e + f*x]^4*Tan[e + f*x]^2)/a^3 + (840*b^2*ArcSin[Sqrt[((a - b)*Sin[e
+ f*x]^2)/a]]*Tan[e + f*x]^4)/a^2 - (1680*(a - b)*b^2*ArcSin[Sqrt[((a - b)*
Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2*Tan[e + f*x]^4)/a^3 + (840*(a - b)^2*b^2
*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4*Tan[e + f*x]^4)/a^
4 + 2100*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan
[e + f*x]^2))/a] + 96*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)
/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e +
f*x]^2))/a] + 24*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*
x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan
[e + f*x]^2))/a] + (2800*b*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Tan[e + f*x]^
2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a + (168*b*Hypergeometri
c2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7
/2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a + (48
*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*(((a
- b)*Sin[e + f*x]^2)/a)^(7/2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Ta
n[e + f*x]^2))/a])/a + (1120*b^2*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Tan[e +
 f*x]^4*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a^2 + (72*b^2*Hype
rgeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]
^2)/a)^(7/2)*Tan[e + f*x]^4*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]
)/a^2 + (24*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x
]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Tan[e + f*x]^4*Sqrt[(Cos[e + f*x
]^2*(a + b*Tan[e + f*x]^2))/a])/a^2 - 1575*Sqrt[((a - b)*Cos[e + f*x]^2*Sin
[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2] - (2100*b*Tan[e + f*x]^2*Sqrt[((a
- b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2])/a - (840*b
^2*Tan[e + f*x]^4*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e
+ f*x]^2))/a^2])/a^2))/(315*a^2*f*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[a
 + b*Tan[e + f*x]^2]*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]*(1 + (
b*Tan[e + f*x]^2)/a))
```

fricas [B] time = 1.94, size = 561, normalized size = 4.19

$$\left[-\frac{3\left(a^2b^2\tan\left(fx+e\right)^4+2a^3b\tan\left(fx+e\right)^2+a^4\right)\sqrt{-a+b}\,\log\left(-\frac{(a-2b)\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\,\tan\left(fx+e\right)-}{\tan\left(fx+e\right)^2+1}\right.}{6\left(\left(a^5b^2-3a^4b^3+3a^3b^4-a^2b^5\right)f\tan\left(fx+e\right)^4+2\left(a^6b-3a^5b^2+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a^2*b^2*tan(f*x + e)^4 + 2*a^3*b*tan(f*x + e)^2 + a^4)*sqrt(-a +
b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*((5*a^2*b^2 - 7*a*b^3 + 2*b^
4)*tan(f*x + e)^3 + 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*ta
n(f*x + e)^2 + a))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x +
 e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7
 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/3*(3*(a^2*b^2*tan(f*x + e)^4 + 2*a^
3*b*tan(f*x + e)^2 + a^4)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(s
qrt(a - b)*tan(f*x + e))) - ((5*a^2*b^2 - 7*a*b^3 + 2*b^4)*tan(f*x + e)^3 +
 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/
((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b -
3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*
b^2 - a^4*b^3)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^(-5/2), x)
```

maple [A] time = 0.33, size = 176, normalized size = 1.31

$$-\frac{b\tan\left(fx+e\right)}{3a\left(a-b\right)f\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}}}-\frac{2b\tan\left(fx+e\right)}{3f\left(a-b\right)a^{2}\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}-\frac{b\tan\left(fx+e\right)}{f\left(a-b\right)^{2}a\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(5/2),x)
[Out] -1/3*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-2/3/f*b/(a-b)/a^2*tan(
f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)-1/f*b/(a-b)^2*tan(f*x+e)/a/(a+b*tan(f*x+e)^
2)^(1/2)+1/f/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan((a-b)*b^2/(b^4*(a-b))^(1/
2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^(5/2),x)
[Out] int(1/(a + b*tan(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(5/2),x)
[Out] Integral((a + b*tan(e + f*x)**2)**(-5/2), x)
```

3.149 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{8b\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b\tan(e+fx)}{3a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

```
[Out] -8/3*b*tan(f*x+e)/a^3/f/(a+b*tan(f*x+e)^2)^(1/2)-cot(f*x+e)/a/f/(a+b*tan(f*
x+e)^2)^(3/2)-4/3*b*tan(f*x+e)/a^2/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3663, 271, 192, 191}

$$-\frac{8b\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b\tan(e+fx)}{3a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cot[e + f*x]/(a*f*(a + b*Tan[e + f*x]^2)^(3/2))) - (4*b*Tan[e + f*x])/(3*
a^2*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (8*b*Tan[e + f*x])/(3*a^3*f*Sqrt[a +
b*Tan[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= -\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(8b)\,\text{Subst}\left(\int \frac{}{(a\cdots}\right.}{}$$

$$= -\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{8b\tan(e+\cdots}{3a^3f\sqrt{a+b\tan^2\cdots}}$$

Mathematica [A] time = 1.05, size = 133, normalized size = 1.37

$$-\frac{\cot(e+fx)\left(4\left(3a^2-8b^2\right)\cos(2(e+fx))+\left(3a^2-12ab+8b^2\right)\cos(4(e+fx))+3\left(3a^2+4ab+8b^2\right)\right)\sqrt{\sec^2(\cdots}}{6\sqrt{2}\,a^3f((a-b)\cos(2(e+fx))+a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/6*((3*(3*a^2 + 4*a*b + 8*b^2) + 4*(3*a^2 - 8*b^2)*Cos[2*(e + f*x)] + (3*
a^2 - 12*a*b + 8*b^2)*Cos[4*(e + f*x)])*Cot[e + f*x]*Sqrt[(a + b + (a - b)*
Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqrt[2]*a^3*f*(a + b + (a - b)*Cos[2*(e
 + f*x)])^2)
```

fricas [A] time = 10.47, size = 156, normalized size = 1.61

$$-\frac{\left(\left(3\,a^2-12\,ab+8\,b^2\right)\cos\left(fx+e\right)^5+4\left(3\,ab-4\,b^2\right)\cos\left(fx+e\right)^3+8\,b^2\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left(a^3b^2f+\left(a^5-2\,a^4b+a^3b^2\right)f\cos\left(fx+e\right)^4+2\left(a^4b-a^3b^2\right)f\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^5 + 4*(3*a*b - 4*b^2)*cos(f*x +
 e)^3 + 8*b^2*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)/((a^3*b^2*f + (a^5 - 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^4 + 2*(a^4*b - a^
3*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

maple [A] time = 1.12, size = 153, normalized size = 1.58

$$-\frac{\left(3\left(\cos^4\left(fx+e\right)\right)a^2-12\left(\cos^4\left(fx+e\right)\right)ab+8\left(\cos^4\left(fx+e\right)\right)b^2+12\left(\cos^2\left(fx+e\right)\right)ab-16b^2\left(\cos^2\left(fx+e\right.\right.}{3f\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)^4\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^4*(3*cos(f*x+e)^4*a^2-12*cos(f*x+e
)^4*a*b+8*cos(f*x+e)^4*b^2+12*cos(f*x+e)^2*a*b-16*b^2*cos(f*x+e)^2+8*b^2)*c
os(f*x+e)^5*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(5/2)/sin(f*x+
e)/a^3
```

maxima [A] time = 0.79, size = 85, normalized size = 0.88

$$-\frac{\frac{8\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a}\,a^3}+\frac{4\,b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a^2}+\frac{3}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a\tan\left(fx+e\right)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 4*b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a)^(3/2)*a^2) + 3/((b*tan(f*x + e)^2 + a)^(3/2)*a*tan(
f*x + e)))/f
```

mupad [B] time = 27.44, size = 324, normalized size = 3.34

$$-\frac{\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{a+\frac{b\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)^2}{\left(e^{e\,2i+f\,x\,2i}+1\right)^2}}\;\left(-a\,b\,12i+a^2\,3i+b^2\,8i+a^2\,e^{e\,2i+f\,x\,2i}\,12i+a^2\,e^{e\,4i+f\,x\,4i}\,18i+a^2\,e^{e\,6i+f\,x\,6i}\right.}{3\,a^3\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)\,\left(a-b+2\,a\,e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(5/2)),x)
```

```
[Out] -((exp(e*2i + f*x*2i) + 1)*(a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2
i + f*x*2i) + 1)^2)^(1/2)*(a^2*3i - a*b*12i + b^2*8i + a^2*exp(e*2i + f*x*2
i)*12i + a^2*exp(e*4i + f*x*4i)*18i + a^2*exp(e*6i + f*x*6i)*12i + a^2*exp(
e*8i + f*x*8i)*3i - b^2*exp(e*2i + f*x*2i)*32i + b^2*exp(e*4i + f*x*4i)*48i
 - b^2*exp(e*6i + f*x*6i)*32i + b^2*exp(e*8i + f*x*8i)*8i + a*b*exp(e*4i +
f*x*4i)*24i - a*b*exp(e*8i + f*x*8i)*12i))/(3*a^3*f*(exp(e*2i + f*x*2i) - 1
)*(a - b + 2*a*exp(e*2i + f*x*2i) + a*exp(e*4i + f*x*4i) + 2*b*exp(e*2i + f
*x*2i) - b*exp(e*4i + f*x*4i))^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^2\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.150 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{8b(a-2b)\tan(e+fx)}{3a^4f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b(a-2b)\tan(e+fx)}{3a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx\right.}$$

```
[Out] -8/3*(a-2*b)*b*tan(f*x+e)/a^4/f/(a+b*tan(f*x+e)^2)^(1/2)-(a-2*b)*cot(f*x+e)
/a^2/f/(a+b*tan(f*x+e)^2)^(3/2)-1/3*cot(f*x+e)^3/a/f/(a+b*tan(f*x+e)^2)^(3/
2)-4/3*(a-2*b)*b*tan(f*x+e)/a^3/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.15, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3663, 453, 271, 192, 191}

$$-\frac{8b(a-2b)\tan(e+fx)}{3a^4f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b(a-2b)\tan(e+fx)}{3a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(((a - 2*b)*Cot[e + f*x])/(a^2*f*(a + b*Tan[e + f*x]^2)^(3/2))) - Cot[e +
f*x]^3/(3*a*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (4*(a - 2*b)*b*Tan[e + f*x])/
(3*a^3*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (8*(a - 2*b)*b*Tan[e + f*x])/(3*a^
4*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx,x,\tan(e+fx)\right)}{af}$$

$$= -\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(4(a-2b)b)\,\text{Subst}\Big(}{}$$

$$= -\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{4(a-2b)b\tan(e}{3a^3f\left(a+b\tan^2(e+\right.}$$

$$= -\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{4(a-2b)b\tan(e}{3a^3f\left(a+b\tan^2(e+\right.}$$

Mathematica [A] time = 1.17, size = 140, normalized size = 0.96

$$\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\frac{2b\sin(2(e+fx))\left(\left(-3a^2+7ab-4b^2\right)\cos(2(e+fx))-3a^2+2ab+4b^2\right)}{((a-b)\cos(2(e+fx))+a+b)^2} - \cot(e+fx)\,(a\right.}{3\sqrt{2}\,a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-(Cot[e + f*x]*(2
*a - 8*b + a*Csc[e + f*x]^2)) + (2*b*(-3*a^2 + 2*a*b + 4*b^2 + (-3*a^2 + 7*
a*b - 4*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e
+ f*x)])^2))/(3*Sqrt[2]*a^4*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

maple [A] time = 1.31, size = 245, normalized size = 1.68

$$\frac{\left(2\left(\cos^{6}\left(fx+e\right)\right)a^{3}-18\left(\cos^{6}\left(fx+e\right)\right)a^{2}b+32\left(\cos^{6}\left(fx+e\right)\right)a\,b^{2}-16\left(\cos^{6}\left(fx+e\right)\right)b^{3}-3\left(\cos^{4}\left(fx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^4*(2*cos(f*x+e)^6*a^3-18*cos(f*x+e)
^6*a^2*b+32*cos(f*x+e)^6*a*b^2-16*cos(f*x+e)^6*b^3-3*cos(f*x+e)^4*a^3+30*co
s(f*x+e)^4*a^2*b-72*cos(f*x+e)^4*a*b^2+48*cos(f*x+e)^4*b^3-12*a^2*cos(f*x+e
)^2*b+48*cos(f*x+e)^2*a*b^2-48*cos(f*x+e)^2*b^3-8*b^2*a+16*b^3)*cos(f*x+e)^
5*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(5/2)/sin(f*x+e)^3/a^4
```

maxima [A] time = 0.78, size = 195, normalized size = 1.34

$$-\frac{\frac{8\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a^{3}}+\frac{4\,b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a^{2}}-\frac{16\,b^{2}\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a^{4}}-\frac{8\,b^{2}\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a^{3}}+\frac{3}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a\tan\left(fx+e\right)}-\frac{}{\left(b\,\mathrm{ta}\right.}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 4*b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a)^(3/2)*a^2) - 16*b^2*tan(f*x + e)/(sqrt(b*tan(f*x +
e)^2 + a)*a^4) - 8*b^2*tan(f*x + e)/((b*tan(f*x + e)^2 + a)^(3/2)*a^3) + 3/
((b*tan(f*x + e)^2 + a)^(3/2)*a*tan(f*x + e)) - 6*b/((b*tan(f*x + e)^2 + a)
^(3/2)*a^2*tan(f*x + e)) + 1/((b*tan(f*x + e)^2 + a)^(3/2)*a*tan(f*x + e)^3
))/f
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(5/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{4}\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.151 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{8b\left(5a^2-20ab+16b^2\right)\tan(e+fx)}{15a^5f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b\left(5a^2-20ab+16b^2\right)\tan(e+fx)}{15a^4f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\left(5a^2-\right.}{5a^3}$$

```
[Out] -8/15*b*(5*a^2-20*a*b+16*b^2)*tan(f*x+e)/a^5/f/(a+b*tan(f*x+e)^2)^(1/2)-1/5
*(5*a^2-20*a*b+16*b^2)*cot(f*x+e)/a^3/f/(a+b*tan(f*x+e)^2)^(3/2)-2/15*(5*a-
4*b)*cot(f*x+e)^3/a^2/f/(a+b*tan(f*x+e)^2)^(3/2)-1/5*cot(f*x+e)^5/a/f/(a+b*
tan(f*x+e)^2)^(3/2)-4/15*b*(5*a^2-20*a*b+16*b^2)*tan(f*x+e)/a^4/f/(a+b*tan(
f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.23, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3663, 462, 453, 271, 192, 191}

$$-\frac{8b\left(5a^2-20ab+16b^2\right)\tan(e+fx)}{15a^5f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b\left(5a^2-20ab+16b^2\right)\tan(e+fx)}{15a^4f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\left(5a^2-20ab+16b^2\right)\cot(e+fx)}{5a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((5*a^2 - 20*a*b + 16*b^2)*Cot[e + f*x])/(5*a^3*f*(a + b*Tan[e + f*x]^2)^(
3/2)) - (2*(5*a - 4*b)*Cot[e + f*x]^3)/(15*a^2*f*(a + b*Tan[e + f*x]^2)^(3/
2)) - Cot[e + f*x]^5/(5*a*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (4*b*(5*a^2 - 2
0*a*b + 16*b^2)*Tan[e + f*x])/(15*a^4*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (8*
b*(5*a^2 - 20*a*b + 16*b^2)*Tan[e + f*x])/(15*a^5*f*Sqrt[a + b*Tan[e + f*x]
^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{2(5a-4b)+5ax^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= -\frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\left(-15a^2+12(5\right.}{}$$

$$= -\frac{\left(5a^2-4(5a-4b)b\right)\cot(e+fx)}{5a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{co}}{5af\left(a+\right.}$$

$$= -\frac{\left(5a^2-4(5a-4b)b\right)\cot(e+fx)}{5a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{co}}{5af\left(a+\right.}$$

$$= -\frac{\left(5a^2-4(5a-4b)b\right)\cot(e+fx)}{5a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{co}}{5af\left(a+\right.}$$

Mathematica [A] time = 2.34, size = 174, normalized size = 0.79

$$\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(\frac{5b(b-a)\sin(2(e+fx))\left(\left(6a^2-17ab+11b^2\right)\cos(2(e+fx))+6a^2-7ab-11b^2\right)}{((a-b)\cos(2(e+fx))+a+b)^2} - \cot(e\right.}{15\sqrt{2}\,a^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-(Cot[e + f*x]*(8
*a^2 - 66*a*b + 73*b^2 + 2*a*(2*a - 7*b)*Csc[e + f*x]^2 + 3*a^2*Csc[e + f*x
]^4)) + (5*b*(-a + b)*(6*a^2 - 7*a*b - 11*b^2 + (6*a^2 - 17*a*b + 11*b^2)*C
os[2*(e + f*x)])*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)])^2))/(
15*Sqrt[2]*a^5*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{6}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

maple [A] time = 1.10, size = 371, normalized size = 1.69

$$-\frac{\left(8\left(\cos^{8}\left(fx+e\right)\right)a^{4}-112\left(\cos^{8}\left(fx+e\right)\right)a^{3}b+328\left(\cos^{8}\left(fx+e\right)\right)a^{2}b^{2}-352\left(\cos^{8}\left(fx+e\right)\right)a\,b^{3}+128\left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/15/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^4*(8*cos(f*x+e)^8*a^4-112*cos(f*x
+e)^8*a^3*b+328*cos(f*x+e)^8*a^2*b^2-352*cos(f*x+e)^8*a*b^3+128*cos(f*x+e)^
8*b^4-20*cos(f*x+e)^6*a^4+292*cos(f*x+e)^6*a^3*b-976*cos(f*x+e)^6*a^2*b^2+1
216*cos(f*x+e)^6*a*b^3-512*cos(f*x+e)^6*b^4+15*cos(f*x+e)^4*a^4-240*cos(f*x
+e)^4*a^3*b+1008*a^2*b^2*cos(f*x+e)^4-1536*cos(f*x+e)^4*a*b^3+768*cos(f*x+e
)^4*b^4+60*cos(f*x+e)^2*a^3*b-400*cos(f*x+e)^2*a^2*b^2+832*cos(f*x+e)^2*a*b
^3-512*cos(f*x+e)^2*b^4+40*a^2*b^2-160*a*b^3+128*b^4)*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/cos(f*x+e)^2)^(5/2)*cos(f*x+e)^5/sin(f*x+e)^5/a^5
```

maxima [A] time = 0.73, size = 337, normalized size = 1.54

$$-\frac{\frac{40\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a^{3}}+\frac{20\,b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a^{2}}-\frac{160\,b^{2}\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a^{4}}-\frac{80\,b^{2}\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a^{3}}+\frac{128\,b^{3}\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a^{5}}+\frac{64\,b^{3}\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/15*(40*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 20*b*tan(f*x +
e)/((b*tan(f*x + e)^2 + a)^(3/2)*a^2) - 160*b^2*tan(f*x + e)/(sqrt(b*tan(f*
x + e)^2 + a)*a^4) - 80*b^2*tan(f*x + e)/((b*tan(f*x + e)^2 + a)^(3/2)*a^3)
```

```
 + 128*b^3*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^5) + 64*b^3*tan(f*x +
 e)/((b*tan(f*x + e)^2 + a)^(3/2)*a^4) + 15/((b*tan(f*x + e)^2 + a)^(3/2)*a
*tan(f*x + e)) - 60*b/((b*tan(f*x + e)^2 + a)^(3/2)*a^2*tan(f*x + e)) + 48*
b^2/((b*tan(f*x + e)^2 + a)^(3/2)*a^3*tan(f*x + e)) + 10/((b*tan(f*x + e)^2
 + a)^(3/2)*a*tan(f*x + e)^3) - 8*b/((b*tan(f*x + e)^2 + a)^(3/2)*a^2*tan(f
*x + e)^3) + 3/((b*tan(f*x + e)^2 + a)^(3/2)*a*tan(f*x + e)^5))/f
```

mupad **[F(-1)]** time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^(5/2)),x)
[Out] \text{Hanged}
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{6}\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**(5/2),x)
[Out] Integral(csc(e + f*x)**6/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.152 $\int (d\sin(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=92

$$\frac{\tan(e+fx)\cos^2(e+fx)^{p+\frac{1}{2}}\left(b\tan^2(e+fx)\right)^p(d\sin(e+fx))^m\,{}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(m+2p+1);\frac{1}{2}(m+2p+3);\sin^2\right.}{f(m+2p+1)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+p)*hypergeom([1/2+p, 1/2+1/2*m+p],[3/2+1/2*m+p],sin(f*x
+e)^2)*(d*sin(f*x+e))^m*tan(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1+m+2*p)
```

Rubi [A] time = 0.15, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3658, 2602, 2577}

$$\frac{\tan(e+fx)\cos^2(e+fx)^{p+\frac{1}{2}}\left(b\tan^2(e+fx)\right)^p(d\sin(e+fx))^m\,{}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(m+2p+1);\frac{1}{2}(m+2p+3);\sin^2\right.}{f(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^(1/2 + p)*Hypergeometric2F1[(1 + 2*p)/2, (1 + m + 2*p)/2,
 (3 + m + 2*p)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^m*Tan[e + f*x]*(b*Tan[e
+ f*x]^2)^p)/(f*(1 + m + 2*p))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (d\sin(e+fx))^m\left(b\tan^2(e+fx)\right)^p\,dx &= \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int (d\sin(e+fx))^m\tan^{2p}(e+fx)\,dx\\ &= \left(d\cos^{2p}(e+fx)\sin(e+fx)(d\sin(e+fx))^{-1-2p}\left(b\tan^2(e+fx)\right)^p\right)\int \\ &= \frac{\cos^2(e+fx)^{\frac{1}{2}+p}\,{}_2F_1\left(\frac{1}{2}(1+2p),\frac{1}{2}(1+m+2p);\frac{1}{2}(3+m+2p);\sin^2(e+\right.}{f(1+m+2p)}\end{aligned}$$

Mathematica [C] time = 2.13, size = 292, normalized size = 3.17

$$\frac{(m+2p+3)\sin(e+fx)\left(b\tan\right.}{f(m+2p+1)\left((m+2p+3)F_1\left(\frac{m}{2}+p+\frac{1}{2};2p,m+1;\frac{m}{2}+p+\frac{3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\tan\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((3 + m + 2*p)*AppellF1[1/2 + m/2 + p, 2*p, 1 + m, 3/2 + m/2 + p, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(b*Tan[e +
f*x]^2)^p)/(f*(1 + m + 2*p)*((3 + m + 2*p)*AppellF1[1/2 + m/2 + p, 2*p, 1 +
 m, 3/2 + m/2 + p, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((1 + m)*Ap
pellF1[3/2 + m/2 + p, 2*p, 2 + m, 5/2 + m/2 + p, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] - 2*p*AppellF1[3/2 + m/2 + p, 1 + 2*p, 1 + m, 5/2 + m/2 + p,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2\right)^p\left(d\sin\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p*(d*sin(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2\right)^p\left(d\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*sin(f*x + e))^m, x)
```

maple [F] time = 4.96, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)\right)^m\left(b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2\right)^p\left(d\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*sin(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(d\,\sin\left(e+f\,x\right)\right)}^m\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
```

```
[Out] int((d*sin(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^2\left(e+fx\right)\right)^p \left(d\sin\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**p*(d*sin(e + f*x))**m, x)
```

3.153 $\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=121

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+2}{2},-p;\frac{m+3}{2};-t\right.}{f(m+1)}$$

[Out] AppellF1(1/2+1/2*m,1+1/2*m,-p,3/2+1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(sec(f*x+e)^2)^(1/2*m)*(d*sin(f*x+e))^m*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/(1+m)/((1+b*tan(f*x+e)^2/a)^p)

Rubi [A] time = 0.14, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3667, 511, 510}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+2}{2},-p;\frac{m+3}{2};-t\right.}{f(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]

[Out] (AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(Sec[e + f*x]^2)^(m/2)*(d*Sin[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + m)*(1 + (b*Tan[e + f*x]^2)/a)^p)

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3667

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(ff*(d*Sin[e + f*x])^m*(Sec[e + f*x]^2)^(m/2))/(f*Tan[e + f*x]^m), Subst[Int[((ff*x)^m*(a + b*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && !IntegerQ[m]
```

Rubi steps

$$\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \frac{\left(\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m \tan^{-m}(e+fx)\right) \text{Subst}\left(\int x^m \left(1+\right.\right.}{f}$$

$$= \frac{\left(\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m \tan^{-m}(e+fx) \left(a+b\tan^2(e+fx\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1+m}{2}; \frac{2+m}{2}, -p; \frac{3+m}{2}; -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a}\right) \sec^2(e+fx)^{m/}}{f(1}$$

Mathematica [B] time = 2.43, size = 275, normalized size = 2.27

$$\frac{a(m+3)\sin(e+fx)\cos(e+fx)(d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)}{f(m+1)\left(\tan^2(e+fx)\left(2bpF_1\left(\frac{m+3}{2}; \frac{m+2}{2}, 1-p; \frac{m+5}{2}; -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a}\right) - a(m+2)F_1\left(\frac{m+3}{2}; \frac{m+4}{2}, -p; \frac{m}{}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (a*(3 + m)*AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/a)]*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(a + b
*Tan[e + f*x]^2)^p)/(f*(1 + m)*(a*(3 + m)*AppellF1[(1 + m)/2, (2 + m)/2, -p
, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[(3
 + m)/2, (2 + m)/2, 1 - p, (5 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/a)] - a*(2 + m)*AppellF1[(3 + m)/2, (4 + m)/2, -p, (5 + m)/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2))
```

fricas [F] time = 1.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p \left(d\sin\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p \left(d\sin\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

maple [F] time = 2.14, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^m \left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^m\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(a + b*tan(e + f*x)^2)^p,x)
[Out] int((d*sin(e + f*x))^m*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

3.154 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=208

$$-\frac{\left(15a^2-20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2}\right.}{15f(a-b)^2}$$

```
[Out] 1/15*(-2*b*p+10*a-7*b)*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(1+p)/(a-b)^2/f-1/
5*cos(f*x+e)^5*(a-b+b*sec(f*x+e)^2)^(1+p)/(a-b)/f-1/15*(15*a^2-20*a*b*(1+p)
+4*b^2*(p^2+3*p+2))*cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/(
a-b))*(a-b+b*sec(f*x+e)^2)^p/(a-b)^2/f/((1+b*sec(f*x+e)^2/(a-b))^p)
```

Rubi [A] time = 0.22, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3664, 462, 453, 365, 364}

$$-\frac{\left(15a^2-20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2}\right.}{15f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((10*a - 7*b - 2*b*p)*Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(1
5*(a - b)^2*f) - (Cos[e + f*x]^5*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(5*(a
- b)*f) - ((15*a^2 - 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Cos[e + f*x]*H
ypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a - b))]*(a - b + b*S
ec[e + f*x]^2)^p)/(15*(a - b)^2*f*(1 + (b*Sec[e + f*x]^2)/(a - b))^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 0]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a-b+bx^2\right)^p}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{5(a-b)f} + \frac{\text{Subst}\left(\int \frac{\left(-10a+b(7+2p)\right.}{}\right.}{}$$

$$= \frac{(10a-7b-2bp)\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{15(a-b)^2f} - \frac{\cos^5(e}{}$$

$$= \frac{(10a-7b-2bp)\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{15(a-b)^2f} - \frac{\cos^5(e}{}$$

$$= \frac{(10a-7b-2bp)\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{15(a-b)^2f} - \frac{\cos^5(e}{}$$

Mathematica [A] time = 7.84, size = 283, normalized size = 1.36

$$-\frac{2^{p+3}\sin^4(e+fx)\cos(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\left(15a^2-20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2}\right.\right.}{15f(a-b)^2\left(-2^{p+2}\cos(2(e+fx))\left(\frac{a+b\tan^2(e+fx)}{a-b}\right)^p+2^p\cos(4\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -1/15*(2^(3 + p)*Cos[e + f*x]*Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p*((15*
a^2 - 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Hypergeometric2F1[-1/2, -p, 1
/2, -((b*Sec[e + f*x]^2)/(a - b))] + ((a + b + (a - b)*Cos[2*(e + f*x)])*(-
17*a + b*(11 + 4*p) + 3*(a - b)*Cos[2*(e + f*x)])*((a + b*Tan[e + f*x]^2)/(
a - b))^p)/4))/((a - b)^2*f*(3*(((a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e +
 f*x]^2)/(a - b))^p - 2^(2 + p)*Cos[2*(e + f*x)]*((a + b*Tan[e + f*x]^2)/(a
 - b))^p + 2^p*Cos[4*(e + f*x)]*((a + b*Tan[e + f*x]^2)/(a - b))^p))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\left(b\tan\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*tan(f*x + e)^2 + a)^p*s
in(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

maple [F] time = 5.20, size = 0, normalized size = 0.00

$$\int \left(\sin^5\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\sin\left(e+f\,x\right)}^5\,{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.155 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=140

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{p+1}}{3f(a-b)}-\frac{(3a-2b(p+1))\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+\right.}{3f(a-b)}$$

```
[Out] 1/3*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(1+p)/(a-b)/f-1/3*(3*a-2*b*(1+p))*cos
(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/(a-b))*(a-b+b*sec(f*x+e)
^2)^p/(a-b)/f/((1+b*sec(f*x+e)^2/(a-b))^p)
```

Rubi [A] time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3664, 453, 365, 364}

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{p+1}}{3f(a-b)}-\frac{(3a-2b(p+1))\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+\right.}{3f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(3*(a - b)*f) - ((3*a -
 2*b*(1 + p))*Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*
x]^2)/(a - b))]*(a - b + b*Sec[e + f*x]^2)^p)/(3*(a - b)*f*(1 + (b*Sec[e +
f*x]^2)/(a - b))^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a-b+bx^2)^p}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{3(a-b)f} + \frac{(3a-2b(1+p))\,\text{Subst}\Big(\int}{3(a}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{3(a-b)f} + \frac{\Big((3a-2b(1+p))\left(a-b+\right.}{}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{3(a-b)f} - \frac{(3a-2b(1+p))\cos(e+f}{}$$

Mathematica [A] time = 4.11, size = 184, normalized size = 1.31

$$\frac{\sin(e+fx)\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left((2b(p+1)-3a)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)+\left(\frac{a+b\tan^2(e+fx)}{a-b}\right)^p(a\,\text{c}\right.}{f\left(3a\sec^2(e+fx)\left(\frac{a+b\sec^2(e+fx)-b}{a-b}\right)^p-3(a-b)\left(\frac{a+b\tan^2(e+fx)}{a-b}\right)^{p+1}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Sin[e + f*x]*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p*((-3*a + 2*b*(1 + p))*H
ypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a - b))] + (a*Cos[e +
 f*x]^2 + b*Sin[e + f*x]^2)*((a + b*Tan[e + f*x]^2)/(a - b))^p))/(f*(3*a*Se
c[e + f*x]^2*((a - b + b*Sec[e + f*x]^2)/(a - b))^p - 3*(a - b)*((a + b*Tan
[e + f*x]^2)/(a - b))^(1 + p)))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^2-1\right)\left(b\tan\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

maple [F] time = 4.46, size = 0, normalized size = 0.00

$$\int \left(\sin^3\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^{3}\,\left(b\tan\left(e+f\,x\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.156 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=79

$$-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)}{f}$$

```
[Out] -cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/(a-b))*(a-b+b*sec(f*
x+e)^2)^p/f/((1+b*sec(f*x+e)^2/(a-b))^p)
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3664, 365, 364}

$$-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a -
b))]*(a - b + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/(a - b))^p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^p}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a-b+b\sec^2(e+fx)\right)^p \left(1+\frac{b\sec^2(e+fx)}{a-b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a-b}\right)^p}{x^2}\,dx,\right.}{f}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)\left(a-b+b\sec^2(e+fx)\right)^p\,(}{f}$$

Mathematica [A] time = 0.91, size = 80, normalized size = 1.01

$$-\frac{\cos(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{a+b\sec^2(e+fx)-b}{a-b}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a - b))]*(a + b*Tan[e + f*x]^2)^p)/(f*((a - b + b*Sec[e + f*x]^2)/(a - b))^p))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

maple [F] time = 0.91, size = 0, normalized size = 0.00

$$\int \sin\left(fx+e\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)\,\left(b\tan\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.157 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)}{f}$$

```
[Out] -AppellF1(1/2,1,-p,3/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/(a-b))*sec(f*x+e)*(a-b+
b*sec(f*x+e)^2)^p/f/((1+b*sec(f*x+e)^2/(a-b))^p)
```

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3664, 430, 429}

$$-\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/(a - b))]
*Sec[e + f*x]*(a - b + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/(a -
 b))^p))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^p}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a-b+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a-b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a-b}\right)^p}{-1+x^2}\,dx, x, \text{s}\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)\sec(e+fx)\left(a-b+b\sec^2(\right.}{f}$$

Mathematica [B] time = 15.12, size = 1215, normalized size = 13.81

$$\frac{}{2f\left(bp\sec^2(e+fx)\tan(e+fx)\left(\frac{2F_1\left(-p-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2}-p;-\cot^2(e+fx),-\frac{a\cot^2(e+fx)}{b}\right)\left(\frac{a\cot^2(e+fx)}{b}+1\right)^{-p}\sqrt{\sec^2(e+fx)}}{(2p+1)\sqrt{\csc^2(e+fx)}}-F_1\left(1;\frac{1}{2},-p;2;\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^(2*p)*((2*AppellF1[-1/2 - p, -1/2, -p,
 1/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2])/(
(1 + 2*p)*(1 + (a*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (AppellF1[1,
 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/(1 +
 (b*Tan[e + f*x]^2)/a)^p))/(2*f*(b*p*Sec[e + f*x]^2*Tan[e + f*x]*(a + b*Tan
[e + f*x]^2)^(-1 + p)*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*
x]^2, -((a*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + (a*Cot
[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (AppellF1[1, 1/2, -p, 2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/a
)^p) + ((a + b*Tan[e + f*x]^2)^p*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p,
-Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Sqrt[Sec[e + f*x]^2]
)/((1 + 2*p)*(1 + (a*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) + (4*a*p*Ap
pellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b
)]*Cot[e + f*x]*(1 + (a*Cot[e + f*x]^2)/b)^(-1 - p)*Sqrt[Csc[e + f*x]^2]*Sq
rt[Sec[e + f*x]^2])/(b*(1 + 2*p)) + (2*((-2*a*(-1/2 - p)*p*AppellF1[1/2 - p
, -1/2, 1 - p, 3/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Cot[e + f
*x]*Csc[e + f*x]^2)/(b*(1/2 - p)) - ((-1/2 - p)*AppellF1[1/2 - p, 1/2, -p,
3/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x
]^2)/(1/2 - p))*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + (a*Cot[e + f*x]^2)/b)
^p*Sqrt[Csc[e + f*x]^2]) + (2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e
+ f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2]*Tan[e + f*x])/((1 +
 2*p)*(1 + (a*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) + (2*b*p*AppellF1[
1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan
[e + f*x]^3*(1 + (b*Tan[e + f*x]^2)/a)^(-1 - p))/a - (2*AppellF1[1, 1/2, -p
, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])
/(1 + (b*Tan[e + f*x]^2)/a)^p - (Tan[e + f*x]^2*((b*p*AppellF1[2, 1/2, 1 -
p, 3, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x]
)/a - (AppellF1[2, 3/2, -p, 3, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Se
c[e + f*x]^2*Tan[e + f*x])/2))/(1 + (b*Tan[e + f*x]^2)/a)^p))/2))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

maple [F] time = 1.11, size = 0, normalized size = 0.00

$$\int\csc\left(fx+e\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\tan\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x),x)
[Out] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

3.158 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)}{3f}$$

```
[Out] 1/3*AppellF1(3/2,2,-p,5/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/(a-b))*sec(f*x+e)^3*
(a-b+b*sec(f*x+e)^2)^p/f/((1+b*sec(f*x+e)^2/(a-b))^p)
```

Rubi [A] time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3664, 511, 510}

$$\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 2, -p, 5/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/(a - b))]*S
ec[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^p)/(3*f*(1 + (b*Sec[e + f*x]^2)/(a
 - b))^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{x^2(a-b+bx^2)^p}{(-1+x^2)^2}dx,x,\sec(e+fx)\right)}{f}$$

$$=\frac{\left(\left(a-b+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a-b}\right)^{-p}\right)\text{Subst}\left(\int\frac{x^2\left(1+\frac{bx^2}{a-b}\right)^p}{(-1+x^2)^2}\,c\right.}{f}$$

$$=\frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)\sec^3(e+fx)\left(a-b+b\,\text{se}\right.}{3f}$$

Mathematica [B] time = 20.74, size = 252, normalized size = 2.74

$$\frac{b(2p-3)\cot(e+fx)\csc(e+fx)\left(a+b\tan^2(e+fx)\right)^pF_1\left(\frac{1}{2}-\right.}{f(2p-1)\left(b(2p-3)F_1\left(\frac{1}{2}-p;-\frac{1}{2},-p;\frac{3}{2}-p;-\cot^2(e+fx),-\frac{a\cot^2(e+fx)}{b}\right)-\cot^2(e+fx)\left(2apF_1\left(\frac{3}{2}-p;-\frac{1}{2},1-\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (b*(-3 + 2*p)*AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -((a*Co
t[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(-
1 + 2*p)*(b*(-3 + 2*p)*AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2
, -((a*Cot[e + f*x]^2)/b)] - (2*a*p*AppellF1[3/2 - p, -1/2, 1 - p, 5/2 - p,
 -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)] + b*AppellF1[3/2 - p, 1/2, -p, 5
/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)])*Cot[e + f*x]^2))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

maple [F] time = 1.12, size = 0, normalized size = 0.00

$$\int\left(\csc^3\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^3,x)
[Out] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

3.159 $\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

```
[Out] 1/3*AppellF1(3/2,2,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)^3*(a+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3663, 511, 510}

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^p}{(1+x^2)^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{(1+x^2)^2}dx, x, \text{ta}\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^3(e+fx)\left(a+b\tan^2(e\right.}{3f}$$

Mathematica [C] time = 18.78, size = 3698, normalized size = 44.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*Cos[e + f*x]^3*Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p*(AppellF1[1/2, 2,
 -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]/(-3*a*AppellF1[1/2, 2,
-p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2
, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2,
 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (
AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e +
 f*x]^2)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f
*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[
e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
 f*x]^2])*Tan[e + f*x]^2))*(-1/4*(Cos[2*(e + f*x)]^3*(a + b*Tan[e + f*x]^2)
^p) + (I/4)*Sin[2*(e + f*x)]*(a + b*Tan[e + f*x]^2)^p + (Sin[2*(e + f*x)]^2
*(a + b*Tan[e + f*x]^2)^p)/2 - (I/4)*Sin[2*(e + f*x)]^3*(a + b*Tan[e + f*x]
^2)^p + Cos[2*(e + f*x)]^2*((a + b*Tan[e + f*x]^2)^p/2 - (I/4)*Sin[2*(e + f
*x)]*(a + b*Tan[e + f*x]^2)^p) + Cos[2*(e + f*x)]*(-1/4*(a + b*Tan[e + f*x]
^2)^p - (Sin[2*(e + f*x)]^2*(a + b*Tan[e + f*x]^2)^p)/4)))/(f*(6*a*b*p*Sin[
e + f*x]^2*(a + b*Tan[e + f*x]^2)^(-1 + p)*(AppellF1[1/2, 2, -p, 3/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]/(-3*a*AppellF1[1/2, 2, -p, 3/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2, 3, -p, 5/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (AppellF1[1/2, -
p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*a*A
ppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p
*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a
*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e
 + f*x]^2)) + 3*a*Cos[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p*(AppellF1[1/2, 2,
 -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]/(-3*a*AppellF1[1/2, 2,
-p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2
, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2,
 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (
AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e +
 f*x]^2)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f
*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[
e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
 f*x]^2])*Tan[e + f*x]^2)) - 9*a*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e
 + f*x]^2)^p*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/a)]/(-3*a*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)] - 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f
```

```
*x]^2)/a)] - 2*a*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/a)])*Tan[e + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*
Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2,
-((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((
b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + 3*a*Cos[e + f*x]
^3*Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p*(((2*b*p*AppellF1[3/2, 2, 1 - p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/
(3*a) - (4*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
a)]*Sec[e + f*x]^2*Tan[e + f*x])/3)/(-3*a*AppellF1[1/2, 2, -p, 3/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2, 3, -p, 5/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (2*AppellF1[1/2,
-p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
+ f*x])/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*
x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e
 + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
f*x]^2])*Tan[e + f*x]^2) + (Sec[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1,
5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])
/(3*a) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x
]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b
*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (AppellF1[1/2, 2,
-p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(-4*(b*p*AppellF1[3/2, 2
, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2,
 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Sec[e + f*x]^2*Tan[
e + f*x] - 3*a*((2*b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (4*AppellF1[3/2, 3,
 -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e +
f*x])/3) - 2*Tan[e + f*x]^2*(b*p*((-6*b*(1 - p)*AppellF1[5/2, 2, 2 - p, 7/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(5
*a) - (12*AppellF1[5/2, 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/5) - 2*a*((6*b*p*AppellF1[5/2, 3, 1 - p,
 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x]
)/(5*a) - (18*AppellF1[5/2, 4, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(-3*a*AppellF1[1/2, 2, -p, 3/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2, 1 - p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2, 3, -p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)^2 - (AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*
(4*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]
^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]
)*Sec[e + f*x]^2*Tan[e + f*x] + 3*a*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -(
(b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a)
- (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/2, 1
- p, 2, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)
/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a)) - a*((6*b*p*Appel
lF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f
*x]^2*Tan[e + f*x])/(5*a) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]
^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*a*AppellF1[1
/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1
[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1
[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2
)^2)))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^{2}-1\right)\left(b\tan\left(fx+e\right)^{2}+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e)^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

maple [F] time = 4.82, size = 0, normalized size = 0.00

$$\int \left(\sin^{2}\left(fx+e\right)\right)\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^{2}\left(b\tan\left(e+f\,x\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

3.160 $\int \left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=78

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)*(a+b*tan(
f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^p\,dx=\frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(1+\right.}{f}$$

Mathematica [B] time = 0.59, size = 192, normalized size = 2.46

$$\frac{3a\sin(2(e+fx))\left(a+b\tan^2(e+fx)\right)^pF_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2\right.}{4f\tan^2(e+fx)\left(bpF_1\left(\frac{3}{2};1-p,1;\frac{5}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)-aF_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Si
n[2*(e + f*x)]*(a + b*Tan[e + f*x]^2)^p)/(6*a*f*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 4*f*(b*p*AppellF1[3/2, 1 - p, 1,
 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p, x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan \left(fx+e\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan \left(e+f\,x\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p,x)
[Out] int((a + b*tan(e + f*x)^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^{2}\left(e+fx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**p,x)
[Out] Integral((a + b*tan(e + f*x)**2)**p, x)
```

3.161 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=68

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

```
[Out] -cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/a)*(a+b*tan(f*x+e)^2
)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3663, 365, 364}

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*(
a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx, x, \right.}{f}$$

$$= -\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)\left(a+b\tan^2(e+fx)\right)^p\left(1 \right.}{f}$$

Mathematica [A] time = 0.72, size = 68, normalized size = 1.00

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*(
a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int \left(\csc^2\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^2,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.162 $\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=120

$$-\frac{(3a-b(1-2p))\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)}{3af}-\frac{\cot^3(e+f}{}$$

```
[Out] -1/3*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1+p)/a/f-1/3*(3*a-b*(1-2*p))*cot(f*x+
e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/a)*(a+b*tan(f*x+e)^2)^p/a/f/(
(1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.12, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3663, 453, 365, 364}

$$-\frac{(3a-b(1-2p))\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)}{3af}-\frac{\cot^3(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -(Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(1 + p))/(3*a*f) - ((3*a - b*(1 - 2
*p))*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]
*(a + b*Tan[e + f*x]^2)^p)/(3*a*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{(1+x^2)(a+bx^2)^p}{x^4}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{3af}-\frac{(-3a-b(-3+2(1+p)))\,\text{Sub}}{}$$

$$=-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{3af}-\frac{\left((-3a-b(-3+2(1+p)))\left(a\right.\right.}{}$$

$$=-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{3af}-\frac{(3a-b(1-2p))\cot(e+fx)\,{}_2}{}$$

Mathematica [A] time = 1.58, size = 111, normalized size = 0.92

$$\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(-\left((3a+b(2p-1))\tan^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx}{a}\right.\right.\right.}{3af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p*(-a - b*Tan[e + f*x]^2 - ((3*a + b
*(-1 + 2*p))*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*Tan[
e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/a)^p))/(3*a*f)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int\left(\csc^4\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan\left(e+f\,x\right)^{2}+a\right)^{p}}{\sin\left(e+f\,x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^4,x)
[Out] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

3.163 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=180

$$-\frac{\left(15a^2-b(1-2p)(10a-b(3-2p))\right)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e\cdot}{a}\right.}{15a^2f}$$

```
[Out] -1/15*(10*a-b*(3-2*p))*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1+p)/a^2/f-1/5*cot(
f*x+e)^5*(a+b*tan(f*x+e)^2)^(1+p)/a/f-1/15*(15*a^2-b*(10*a-b*(3-2*p))*(1-2*
p))*cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/a)*(a+b*tan(f*x+e
)^2)^p/a^2/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.20, antiderivative size = 176, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3663, 462, 453, 365, 364}

$$-\frac{\left(15-\frac{b(1-2p)(10a-b(3-2p))}{a^2}\right)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)}{15f}-\frac{(10a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((10*a - b*(3 - 2*p))*Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(1 + p))/(15*a
^2*f) - (Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(1 + p))/(5*a*f) - ((15 - (b
*(10*a - b*(3 - 2*p))*(1 - 2*p))/a^2)*Cot[e + f*x]*Hypergeometric2F1[-1/2,
-p, 1/2, -((b*Tan[e + f*x]^2)/a)]*(a + b*Tan[e + f*x]^2)^p)/(15*f*(1 + (b*T
an[e + f*x]^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+bx^2)^p}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{5af} + \frac{\text{Subst}\left(\int \frac{(10a-b(3-2p)+5ax^2)}{x^4}\right.}{5}$$

$$= -\frac{(10a-b(3-2p))\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{15a^2f} - \cot^5(e+$$

$$= -\frac{(10a-b(3-2p))\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{15a^2f} - \cot^5(e+$$

$$= -\frac{(10a-b(3-2p))\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{15a^2f} - \cot^5(e+$$

Mathematica [A] time = 1.34, size = 141, normalized size = 0.78

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\left(15\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)+3\cot^4(e+fx)\,{}_2F_1\left(-\frac{5}{2}\right.\right.}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -1/15*(Cot[e + f*x]*(3*Cot[e + f*x]^4*Hypergeometric2F1[-5/2, -p, -3/2, -((
b*Tan[e + f*x]^2)/a)] + 10*Cot[e + f*x]^2*Hypergeometric2F1[-3/2, -p, -1/2,
 -((b*Tan[e + f*x]^2)/a)] + 15*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e
+ f*x]^2)/a)])*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int \left(\csc^{6}\left(fx+e\right)\right)\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan\left(e+f\,x\right)^{2}+a\right)^{p}}{\sin\left(e+f\,x\right)^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^6,x)
[Out] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

3.164 $\int (d\sin(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=98

$$\frac{\tan(e+fx)(d\sin(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(np+1)} \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(\frac{1}{2}(np+1), \frac{1}{2}(m+np+1); \frac{1}{2}(m+np\right.}{f(m+np+1)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+1/2)*hypergeom([1/2*n*p+1/2, 1/2*n*p+1/2*m+1/2],[1/
2*n*p+1/2*m+3/2],sin(f*x+e)^2)*(d*sin(f*x+e))^m*tan(f*x+e)*(b*(c*tan(f*x+e)
)^n)^p/f/(n*p+m+1)
```

Rubi [A] time = 0.18, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3659, 2602, 2577}

$$\frac{\tan(e+fx)(d\sin(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(np+1)} \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(\frac{1}{2}(np+1), \frac{1}{2}(m+np+1); \frac{1}{2}(m+np\right.}{f(m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (1 + m + n*p
)/2, (3 + m + n*p)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^m*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\sin(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (d\sin(e+fx))^m(c\tan(e$$
$$= \left(d\cos^{np}(e+fx)\sin(e+fx)(d\sin(e+fx))^{-1-np}\left(b(c\tan(e+fx))^n\right)^{\cdot}\right.$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(1+m+np);\frac{1}{2}(3+m+np);\mathrm{s}\right.}{f(1+m+}$$

Mathematica [C] time = 2.03, size = 295, normalized size = 3.01

$$\frac{(m+np+3)\sin(e+fx)(}{f(m+np+1)\left((m+np+3)F_1\left(\frac{1}{2}(m+np+1);np,m+1;\frac{1}{2}(m+np+3);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((3 + m + n*p)*AppellF1[(1 + m + n*p)/2, n*p, 1 + m, (3 + m + n*p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(b*(c*T
an[e + f*x])^n)^p)/(f*(1 + m + n*p)*((3 + m + n*p)*AppellF1[(1 + m + n*p)/2
, n*p, 1 + m, (3 + m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2
*((1 + m)*AppellF1[(3 + m + n*p)/2, n*p, 2 + m, (5 + m + n*p)/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*p*AppellF1[(3 + m + n*p)/2, 1 + n*p, 1 +
 m, (5 + m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x
)/2]^2))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \left(d\sin\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*sin(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \left(d\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*sin(f*x + e))^m, x)
```

maple [F] time = 7.97, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^m \left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n} b\right)^{p} \left(d\sin\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*sin(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^m\,{\left(b\,{\left(c\,\tan\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int((d*sin(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \left(c \tan{\left(e + f x \right)}\right)^{n}\right)^{p} \left(d \sin{\left(e + f x \right)}\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*sin(e + f*x))**m, x)
```

3.165 $\int \sin^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=63

$$\frac{\tan^3(e+fx)\,{}_2F_1\left(2,\frac{1}{2}(np+3);\frac{1}{2}(np+5);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}$$

```
[Out] hypergeom([2, 1/2*n*p+3/2],[1/2*n*p+5/2],-tan(f*x+e)^2)*tan(f*x+e)^3*(b*(c*
tan(f*x+e))^n)^p/f/(n*p+3)
```

Rubi [A] time = 0.11, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3659, 2591, 364}

$$\frac{\tan^3(e+fx)\,{}_2F_1\left(2,\frac{1}{2}(np+3);\frac{1}{2}(np+5);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[2, (3 + n*p)/2, (5 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sin^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sin^2(e+fx)(c\tan(e+fx)) \\
&= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\operatorname{Subst}\left(\int \frac{x^{2+np}}{\left(c^2+x^2\right)^2}\,dx,x,c\tan(\right.}{f} \\
&= \frac{{}_2F_1\left(2,\frac{1}{2}(3+np);\frac{1}{2}(5+np);-\tan^2(e+fx)\right)\tan^3(e+fx)\left(b(c\tan(e+\right.}{f(3+np)}
\end{aligned}$$

Mathematica [C] time = 2.47, size = 517, normalized size = 8.21

$$\frac{}{f(np+1)\left(2(np+3)\cos^2\left(\frac{1}{2}(e+fx)\right)F_1\left(\frac{1}{2}(np+1);np,2;\frac{1}{2}(np+3);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (8*(6 + 2*n*p)*(AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] - AppellF1[(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*x)/2]^5*Sin[(e + f*x)/2]^3*
(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p)*(2*(3 + n*p)*AppellF1[(1 + n*p)/2, n
*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/
2]^2 - 2*(3 + n*p)*AppellF1[(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(2*AppellF1[(3 + n*p)/2,
 n*p, 3, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 3*AppellF1
[(3 + n*p)/2, n*p, 4, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + n*p*(-AppellF1[(3 + n*p)/2, 1 + n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + AppellF1[(3 + n*p)/2, 1 + n*p, 3, (5 + n*p)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*(-1 + Cos[e + f*x])))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^2-1\right)\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*((c*tan(f*x + e))^n*b)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \sin\left(fx+e\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^2, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(\sin^2\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \sin\left(fx+e\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(e+f\,x\right)^2\,{\left(b\,{\left(c\,\tan\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(sin(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \left(c \tan{\left(e + f x \right)}\right)^{n}\right)^{p} \sin^{2}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sin(e + f*x)**2, x)
```

3.166 $\int \left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=61

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*(c*ta
n(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{np}\,dx\\ &= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{np}}{c^2+x^2}\,dx,x,c\tan(e+fx)\right)}{f}\\ &= \frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.97

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f + f*n*p)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p, x)
```

3.167 $\int \csc^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=33

$$-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

```
[Out] -cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+1)
```

Rubi [A] time = 0.10, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = $\{3659, 2591, 30\}$

$$-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p)))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc^2(e+fx)(c\tan(e+fx))\\ &= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int x^{-2+np}\,dx,x,c\tan(e\right.}{f}\\ &= -\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 31, normalized size = 0.94

$$\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
[Out] (Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-1 + n*p))
```

fricas [A] time = 0.45, size = 51, normalized size = 1.55

$$\frac{\cos\left(fx+e\right)e^{\left(np\log\left(\frac{c\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)+p\log\left(b\right)\right)}}{\left(fnp-f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] cos(f*x + e)*e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))/((f*n*p -
f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^2, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int\left(\csc^{2}\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
[Out] int(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.77, size = 37, normalized size = 1.12

$$\frac{b^{p}c^{np}\left(\tan\left(fx+e\right)^{n}\right)^{p}}{\left(np-1\right)f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] b^p*c^(n*p)*(tan(f*x + e)^n)^p/((n*p - 1)*f*tan(f*x + e))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}}{\sin\left(e+fx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^2,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\csc^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*csc(e + f*x)**2, x)
```

3.168 $\int \csc^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=69

$$-\frac{\cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

```
[Out] -cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+1)-cot(f*x+e)^3*(b*(c*tan(f*x+e)
)^n)^p/f/(-n*p+3)
```

Rubi [A] time = 0.12, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3659, 2591, 14}

$$-\frac{\cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p))) - (Cot[e + f*x]^3*
(b*(c*Tan[e + f*x])^n)^p)/(f*(3 - n*p))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \csc^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc^4(e+fx)(c\tan(e+f$$

$$=\frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int x^{-4+np}\left(c^2+x^2\right)\right.}{f}$$

$$=\frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int\left(c^2x^{-4+np}+x^{-2+}\right.\right.}{f}$$

$$=-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}-\frac{\cot^3(e+fx)\left(b(c\tan(e+fx))^n\right.}{f(3-np)}$$

Mathematica [A] time = 0.16, size = 59, normalized size = 0.86

$$\frac{\cot(e+fx)\csc^2(e+fx)(\cos(2(e+fx))+np-2)\left(b(c\tan(e+fx))^n\right)^p}{f(np-3)(np-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((-2 + n*p + Cos[2*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2*(b*(c*Tan[e + f*
x])^n)^p)/(f*(-3 + n*p)*(-1 + n*p))
```

fricas [A] time = 0.43, size = 104, normalized size = 1.51

$$\frac{\left(2\cos\left(fx+e\right)^3+\left(np-3\right)\cos\left(fx+e\right)\right)e^{\left(np\log\left(\frac{c\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)+p\log\left(b\right)\right)}}{\left(fn^2p^2-4\,fnp-\left(fn^2p^2-4\,fnp+3\,f\right)\cos\left(fx+e\right)^2+3\,f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] (2*cos(f*x + e)^3 + (n*p - 3)*cos(f*x + e))*e^(n*p*log(c*sin(f*x + e)/cos(f
*x + e)) + p*log(b))/((f*n^2*p^2 - 4*f*n*p - (f*n^2*p^2 - 4*f*n*p + 3*f)*co
s(f*x + e)^2 + 3*f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^4, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int\left(\csc^4\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.37, size = 73, normalized size = 1.06

$$\frac{\frac{b^p c^{np}\left(\tan\left(fx+e\right)^n\right)^p}{\left(np-1\right)\tan\left(fx+e\right)}+\frac{b^p c^{np}\left(\tan\left(fx+e\right)^n\right)^p}{\left(np-3\right)\tan\left(fx+e\right)^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] (b^p*c^(n*p)*(tan(f*x + e)^n)^p/((n*p - 1)*tan(f*x + e)) + b^p*c^(n*p)*(tan
(f*x + e)^n)^p/((n*p - 3)*tan(f*x + e)^3))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\left(c\tan\left(e+f\,x\right)\right)^n\right)^p}{\sin\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^4,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

3.169 $\int \csc^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=104

$$-\frac{\cot^5(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(5-np)}-\frac{2\cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

```
[Out] -cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+1)-2*cot(f*x+e)^3*(b*(c*tan(f*x+
e))^n)^p/f/(-n*p+3)-cot(f*x+e)^5*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+5)
```

Rubi [A] time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3659, 2591, 270}

$$-\frac{\cot^5(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(5-np)}-\frac{2\cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p))) - (2*Cot[e + f*x]^
3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 - n*p)) - (Cot[e + f*x]^5*(b*(c*Tan[e + f
*x])^n)^p)/(f*(5 - n*p))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \csc^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc^6(e+fx)(c\tan(e+f$$

$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int x^{-6+np}\left(c^2+x^2\right)\right.}{f}$$

$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int\left(c^4x^{-6+np}+2c^2\right.\right.}{f}$$

$$= -\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)} - \frac{2\cot^3(e+fx)\left(b(c\tan(e+fx))\right.}{f(3-np)}$$

Mathematica [A] time = 0.30, size = 89, normalized size = 0.86

$$\frac{\cot(e+fx)\csc^4(e+fx)\left(2(np-3)\cos(2(e+fx))+\cos(4(e+fx))+n^2p^2-6np+8\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np-5)(np-3)(np-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((8 - 6*n*p + n^2*p^2 + 2*(-3 + n*p)*Cos[2*(e + f*x)] + Cos[4*(e + f*x)])*C
ot[e + f*x]*Csc[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p)/(f*(-5 + n*p)*(-3 + n*
p)*(-1 + n*p))
```

fricas [A] time = 0.44, size = 180, normalized size = 1.73

$$\frac{\left(8\cos\left(fx+e\right)^5+4\left(np-5\right)\cos\left(fx+e\right)^3+\left(n^2p^2-8np+15\right)\cos\left(fx+e\right)\right)e^{\left(np\log\right.}}{\left(fn^3p^3-9fn^2p^2+\left(fn^3p^3-9fn^2p^2+23fnp-15f\right)\cos\left(fx+e\right)^4+23fnp-2\left(fn^3p^3-9fn^2p^2+23fn\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] (8*cos(f*x + e)^5 + 4*(n*p - 5)*cos(f*x + e)^3 + (n^2*p^2 - 8*n*p + 15)*cos
(f*x + e))*e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))/((f*n^3*p^3
- 9*f*n^2*p^2 + (f*n^3*p^3 - 9*f*n^2*p^2 + 23*f*n*p - 15*f)*cos(f*x + e)^4
+ 23*f*n*p - 2*(f*n^3*p^3 - 9*f*n^2*p^2 + 23*f*n*p - 15*f)*cos(f*x + e)^2 -
 15*f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\csc\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^6, x)
```

maple [C] time = 26.46, size = 171293, normalized size = 1647.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.52, size = 108, normalized size = 1.04

$$\frac{\frac{b^p c^{np} \left(\tan\left(fx+e\right)^n\right)^p}{(np-1)\tan\left(fx+e\right)} + \frac{2\,b^p c^{np} \left(\tan\left(fx+e\right)^n\right)^p}{(np-3)\tan\left(fx+e\right)^3} + \frac{b^p c^{np} \left(\tan\left(fx+e\right)^n\right)^p}{(np-5)\tan\left(fx+e\right)^5}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] (b^p*c^(n*p)*(tan(f*x + e)^n)^p/((n*p - 1)*tan(f*x + e)) + 2*b^p*c^(n*p)*(t
an(f*x + e)^n)^p/((n*p - 3)*tan(f*x + e)^3) + b^p*c^(n*p)*(tan(f*x + e)^n)^
p/((n*p - 5)*tan(f*x + e)^5))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\left(c\tan\left(e+f\,x\right)\right)^n\right)^p}{\sin\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^6,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

3.170 $\int \sin^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=93

$$\frac{\sin^3(e+fx)\tan(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+4);\frac{1}{2}(np+6);\sin^2(e+fx)\right)\left(b(c\tan(e+f$$
$$f(np+4)$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+1/2)*hypergeom([1/2*n*p+2, 1/2*n*p+1/2],[1/2*n*p+3]
,sin(f*x+e)^2)*sin(f*x+e)^3*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+4)
```

Rubi [A] time = 0.14, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3659, 2602, 2577}

$$\frac{\sin^3(e+fx)\tan(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+4);\frac{1}{2}(np+6);\sin^2(e+fx)\right)\left(b(c\tan(e+f$$
$$f(np+4)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (4 + n*p)/2,
 (6 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*Tan[e + f*x]*(b*(c*Tan[e + f*x
])^n)^p)/(f*(4 + n*p))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sin^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sin^3(e+fx)(c\tan(e+f$$
$$= \left(\cos^{np}(e+fx)\sin^{-np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^{-np}(e+fx$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(4+np);\frac{1}{2}(6+np);\sin^2(e+fx$$
$$f(4+np)$$

Mathematica [C] time = 2.84, size = 506, normalized size = 5.44

$$\frac{f(np+2)\left(2(np+4)\cos^2\left(\frac{1}{2}(e+fx)\right)F_1\left(\frac{np}{2}+1;np,3;\frac{np}{2}+2;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2(np+4)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (4*(4 + n*p)*(AppellF1[1 + (n*p)/2, n*p, 3, 2 + (n*p)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] - AppellF1[1 + (n*p)/2, n*p, 4, 2 + (n*p)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*Sin[
e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 + n*p)*(2*(4 + n*p)*AppellF1[1 +
 (n*p)/2, n*p, 3, 2 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos
[(e + f*x)/2]^2 - 2*(4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 4, 2 + (n*p)/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(3*AppellF1[2
 + (n*p)/2, n*p, 4, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] -
 4*AppellF1[2 + (n*p)/2, n*p, 5, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + n*p*(-AppellF1[2 + (n*p)/2, 1 + n*p, 3, 3 + (n*p)/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[2 + (n*p)/2, 1 + n*p, 4, 3 + (n
*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*(-1 + Cos[e + f*x])))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^2-1\right)\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\sin\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^3, x)
```

maple [F] time = 33.92, size = 0, normalized size = 0.00

$$\int\left(\sin^3\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\sin\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^3\,{\left(b\,{\left(c\,\tan\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int(sin(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Timed out
```

3.171 $\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=91

$$\frac{\sin(e+fx)\tan(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right.}{f(np+2)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+1/2)*hypergeom([1/2*n*p+1, 1/2*n*p+1/2],[1/2*n*p+2]
,sin(f*x+e)^2)*sin(f*x+e)*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+2)
```

Rubi [A] time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3659, 2602, 2577}

$$\frac{\sin(e+fx)\tan(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right.}{f(np+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (2 + n*p)/2,
 (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*Tan[e + f*x]*(b*(c*Tan[e + f*x])
^n)^p)/(f*(2 + n*p))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sin(e+fx)(c\tan(e+fx))^{np}\\ &= \left(\cos^{np}(e+fx)\sin^{-np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^{-np}(e+fx)\sin\\ &= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(2+np);\frac{1}{2}(4+np);\sin^2(e+fx)\right)\text{s}}{f(2+np)}\end{aligned}$$

Mathematica [C] time = 1.31, size = 284, normalized size = 3.12

$$\frac{8(np+4)\sin^2\left(\frac{1}{2}(e+fx)\right)\cos^4\left(\frac{1}{2}(e+fx)\right.}{f(np+2)\left(2(np+4)\cos^2\left(\frac{1}{2}(e+fx)\right)F_1\left(\frac{np}{2}+1;np,2;\frac{np}{2}+2;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2(\cos(\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
[Out] (8*(4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 2, 2 + (n*p)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^4*Sin[(e + f*x)/2]^2*(b*(c*Tan[e + f
*x])^n)^p)/(f*(2 + n*p)*(2*(4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 2, 2 + (n*p
)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(2*App
ellF1[2 + (n*p)/2, n*p, 3, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] - n*p*AppellF1[2 + (n*p)/2, 1 + n*p, 2, 3 + (n*p)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x])))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \sin\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \sin\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

maple [F] time = 21.64, size = 0, normalized size = 0.00

$$\int \sin\left(fx+e\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
[Out] int(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \sin\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)\,{\left(b\,{\left(c\,\tan\left(e+f\,x\right)\right)}^n\right)}^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(sin(e + f*x)*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sin(e + f*x), x)
```

3.172 $\int \csc(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=81

$$\frac{\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+2);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+1/2)*hypergeom([1/2*n*p, 1/2*n*p+1/2],[1/2*n*p+1],s
in(f*x+e)^2)*sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/n/p
```

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3659, 2601, 2577}

$$\frac{\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+2);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(n*p)/2, (1 + n*p)/2, (2
+ n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*n*p)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \csc(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc(e+fx)(c\tan(e+fx)\\
&= \left(\cos^{np}(e+fx)\sin^{-np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^{-np}(e+fx)\\
&= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(1+np);\frac{1}{2}(2+np);\sin^2(e+fx)\right)\sec(e}{fnp}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 77, normalized size = 0.95

$$\frac{{}_2F_1\left(\frac{np}{2},np;\frac{np}{2}+1;\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^{np}\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[(n*p)/2, n*p, 1 + (n*p)/2, Tan[(e + f*x)/2]^2]*(Cos[e +
f*x]*Sec[(e + f*x)/2]^2)^(n*p)*(b*(c*Tan[e + f*x])^n)^p)/(f*n*p)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*csc(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e), x)
```

maple [F] time = 25.66, size = 0, normalized size = 0.00

$$\int\csc\left(fx+e\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p}{\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x),x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*csc(e + f*x), x)
```

3.173 $\int \csc^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=92

$$-\frac{\csc^2(e+fx)\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{1}{2}(np-2),\frac{1}{2}(np+1);\frac{np}{2};\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(2-np)}$$

```
[Out] -(cos(f*x+e)^2)^(1/2*n*p+1/2)*csc(f*x+e)^2*hypergeom([1/2*n*p-1, 1/2*n*p+1/
2],[1/2*n*p],sin(f*x+e)^2)*sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+2)
```

Rubi [A] time = 0.15, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3659, 2601, 2577}

$$-\frac{\csc^2(e+fx)\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{1}{2}(np-2),\frac{1}{2}(np+1);\frac{np}{2};\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(2-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -(((Cos[e + f*x]^2)^((1 + n*p)/2)*Csc[e + f*x]^2*Hypergeometric2F1[(-2 + n*
p)/2, (1 + n*p)/2, (n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(b*(c*Tan[e + f*x]
)^n)^p)/(f*(2 - n*p)))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \csc^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc^3(e+fx)(c\tan(e+f$$
$$= \left(\cos^{np}(e+fx)\sin^{-np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^{-np}(e+fx$$
$$= -\frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\csc^2(e+fx)\,{}_2F_1\left(\frac{1}{2}(-2+np),\frac{1}{2}(1+np);\frac{np}{2};\sin\right.}{f(2-np)}$$

Mathematica [C] time = 16.28, size = 1399, normalized size = 15.21

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[(e + f*x)/2]^2*Hypergeometric2F1[n*p, -1 + (n*p)/2, (n*p)/2, Tan[(e +
f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(n*p)*(b*(c*Tan[e + f*x])^n)^p
)/(f*(-8 + 4*n*p)) + ((4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 1, 2 + (n*p)/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]^2*(b*(c*Tan[e + f*x])
^n)^p)/(8*f*(2 + n*p)*((4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 1, 2 + (n*p)/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + AppellF1[2 +
 (n*p)/2, n*p, 2, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(-1
 + Cos[e + f*x]) + 2*n*p*AppellF1[2 + (n*p)/2, 1 + n*p, 1, 3 + (n*p)/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2)) + (Hypergeometri
c2F1[n*p, 1 + (n*p)/2, 2 + (n*p)/2, Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(
e + f*x)/2]^2)^(n*p)*Tan[(e + f*x)/2]^2*(b*(c*Tan[e + f*x])^n)^p)/(f*(8 + 4
*n*p)) + (Cot[(e + f*x)/2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(n*p)*((2 + n*
p)*Hypergeometric2F1[(n*p)/2, n*p, 1 + (n*p)/2, Tan[(e + f*x)/2]^2] - n*p*A
ppellF1[1 + (n*p)/2, n*p, 1, 2 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Tan[(e + f*x)/2]^2)*Tan[e + f*x]^(n*p)*(b*(c*Tan[e + f*x])^n)^p)/(8
*f*n*p*(2 + n*p)*(((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + n*p)*(-(Sec[(e +
 f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
)*((2 + n*p)*Hypergeometric2F1[(n*p)/2, n*p, 1 + (n*p)/2, Tan[(e + f*x)/2]^
2] - n*p*AppellF1[1 + (n*p)/2, n*p, 1, 2 + (n*p)/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*Tan[e + f*x]^(n*p))/(2*(2 + n*p)) + (
(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(n*p)*(-(n*p*AppellF1[1 + (n*p)/2, n*p, 1
, 2 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2]) - n*p*Tan[(e + f*x)/2]^2*(-(((1 + (n*p)/2)*AppellF1[2 + (
n*p)/2, n*p, 2, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + (n*p)/2)) + (n*p*(1 + (n*p)/2)*AppellF
1[2 + (n*p)/2, 1 + n*p, 1, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + (n*p)/2)) + (n*p*(2 + n*p)*
Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*(-Hypergeometric2F1[(n*p)/2, n*p, 1 + (n*
p)/2, Tan[(e + f*x)/2]^2] + (1 - Tan[(e + f*x)/2]^2)^(-(n*p))))/2)*Tan[e +
f*x]^(n*p))/(2*n*p*(2 + n*p)) + ((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(n*p)*Se
c[e + f*x]^2*((2 + n*p)*Hypergeometric2F1[(n*p)/2, n*p, 1 + (n*p)/2, Tan[(e
 + f*x)/2]^2] - n*p*AppellF1[1 + (n*p)/2, n*p, 1, 2 + (n*p)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*Tan[e + f*x]^(-1 + n*p))/(
2*(2 + n*p))))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \csc\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c \tan \left(f x + e\right)\right)^{n} b\right)^{p} \csc \left(f x + e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^3, x)
```

maple [F] time = 21.94, size = 0, normalized size = 0.00

$$\int \left(\csc^{3}\left(f x + e\right)\right) \left(b \left(c \tan \left(f x + e\right)\right)^{n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
[Out] int(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c \tan \left(f x + e\right)\right)^{n} b\right)^{p} \csc \left(f x + e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b \left(c \tan \left(e + f\, x\right)\right)^{n}\right)^{p}}{\sin \left(e + f\, x\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^3,x)
[Out] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \left(c \tan \left(e + f x\right)\right)^{n}\right)^{p} \csc^{3}\left(e + f x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*csc(e + f*x)**3, x)
```

3.174 $\int (d\sin(e+fx))^m \left(a+b\tan^n(e+fx)\right)^p \, dx$

Optimal. Leaf size=28

$$\text{Int}\left((d\sin(e+fx))^m \left(a+b\tan^n(e+fx)\right)^p, x\right)$$

```
[Out] Unintegrable((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d\sin(e+fx))^m \left(a+b\tan^n(e+fx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p,x]
[Out] Defer[Int][(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p, x]
```

Rubi steps

$$\int (d\sin(e+fx))^m \left(a+b\tan^n(e+fx)\right)^p \, dx = \int (d\sin(e+fx))^m \left(a+b\tan^n(e+fx)\right)^p \, dx$$

Mathematica [A] time = 2.80, size = 0, normalized size = 0.00

$$\int (d\sin(e+fx))^m \left(a+b\tan^n(e+fx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p,x]
[Out] Integrate[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^n + a\right)^p \left(d\sin\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^n + a)^p*(d*sin(f*x + e))^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^n + a\right)^p \left(d\sin\left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^n + a)^p*(d*sin(f*x + e))^m, x)
```

maple [A] time = 7.66, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^{m}\left(a+b\left(\tan^{n}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x)
[Out] int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{n}+a\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^n + a)^p*(d*sin(f*x + e))^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^m\,{\left(a+b\,{\mathrm{tan}\left(e+f\,x\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(a + b*tan(e + f*x)^n)^p,x)
[Out] int((d*sin(e + f*x))^m*(a + b*tan(e + f*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*tan(f*x+e)**n)**p,x)
[Out] Timed out
```

3.175 $\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=99

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(-m+2p+1)}\,{}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(-m+2p+1);\frac{1}{2}(2p+3\right.}{f(2p+1)}$$

```
[Out] (d*cos(f*x+e))^m*(cos(f*x+e)^2)^(1/2-1/2*m+p)*hypergeom([1/2+p, 1/2-1/2*m+p
],[3/2+p],sin(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1+2*p)
```

Rubi [A] time = 0.14, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3658, 2603, 2617}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(-m+2p+1)}\,{}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(-m+2p+1);\frac{1}{2}(2p+3\right.}{f(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((d*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + 2*p)/2)*Hypergeometric2F1[(1
 + 2*p)/2, (1 - m + 2*p)/2, (3 + 2*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*Ta
n[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Rule 2603

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cos[e + f*x])^FracPart[m]*(Sec[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sec[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right) \int (d\cos(e+fx))^m \tan^{2p}(e+fx)\, dx$$
$$= \left((d\cos(e+fx))^m \left(\frac{\sec(e+fx)}{d}\right)^m \tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right) \int$$
$$= \frac{(d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(1-m+2p)} \, {}_2F_1\left(\frac{1}{2}(1+2p), \frac{1}{2}(1-m+2p);\right.}{f(1+2p)}$$

Mathematica [A] time = 0.52, size = 81, normalized size = 0.82

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}\left(b\tan^2(e+fx)\right)^p (d\cos(e+fx))^m \, {}_2F_1\left(\frac{m}{2}+1, p+\frac{1}{2}; p+\frac{3}{2}; -\tan^2(e+fx)\right)}{f(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((d*Cos[e + f*x])^m*Hypergeometric2F1[1 + m/2, 1/2 + p, 3/2 + p, -Tan[e + f
*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(b*Tan[e + f*x]^2)^p)/(f*(1 + 2*
p))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\tan\left(fx+e\right)^2\right)^p \left(d\cos\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p*(d*cos(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\cos\left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*cos(f*x + e))^m, x)
```

maple [F] time = 4.90, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(fx+e\right)\right)^m \left(b\left(\tan^2\left(fx+e\right)\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\cos\left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2)^p*(d*cos(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\,\cos\left(e+f\,x\right)\right)^m\,\left(b\,\tan\left(e+f\,x\right)^2\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
[Out] int((d*cos(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^2\left(e+fx\right)\right)^p \left(d\cos\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
[Out] Integral((b*tan(e + f*x)**2)**p*(d*cos(e + f*x))**m, x)
```

3.176 $\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=108

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} F_1\left(\frac{1}{2};\frac{m+2}{2},-p;\frac{3}{2};-\tan^2(e+\right.}{f}$$

```
[Out] AppellF1(1/2,1+1/2*m,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(d*cos(f*x+e))
^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e
)^2/a)^p)
```

Rubi [A] time = 0.14, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3669, 3679, 430, 429}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} F_1\left(\frac{1}{2};\frac{m+2}{2},-p;\frac{3}{2};-\tan^2(e+\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)
]*(d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(a + b*Tan[e + f*x
]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(d*Cos[e + f*x])^FracPart[m]*(Sec
[e + f*x]/d)^FracPart[m], Int[(a + b*(c*Tan[e + f*x])^n)^p/(Sec[e + f*x]/d)
^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !IntegerQ[m]
```

Rule 3679

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(f
f*(d*Sec[e + f*x])^m)/(f*(Sec[e + f*x]^2)^(m/2)), Subst[Int[(1 + ff^2*x^2)^
(m/2 - 1)*(a + b*ff^2*x^2)^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \left((d\cos(e+fx))^m \left(\frac{\sec(e+fx)}{d}\right)^m\right) \int \left(\frac{\sec(e+fx)}{d}\right)^{-m} \left(a+b\,\mathrm{t}\right.$$

$$= \frac{\left((d\cos(e+fx))^m \sec^2(e+fx)^{m/2}\right) \mathrm{Subst}\left(\int \left(1+x^2\right)^{-1-\frac{m}{2}} \left(a+\right.\right.}{f}$$

$$= \frac{\left((d\cos(e+fx))^m \sec^2(e+fx)^{m/2} \left(a+b\tan^2(e+fx)\right)^p \left(1+\frac{b\,\mathrm{t}}{}\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2}; \frac{2+m}{2}, -p; \frac{3}{2}; -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a}\right) (d\cos(e+fx))^m}{}$$

Mathematica [**B**] time = 16.88, size = 2033, normalized size = 18.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*(d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(-1 - m/2)*Tan[e + f*x]*(a + b*Ta
n[e + f*x]^2)^(2*p))/(f*(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m
)/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)*((
6*a*b*p*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/a)]*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(-1 + p))/((Sec[e + f*x]^2)^
(m/2)*(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m)/2, -p, 5/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)) + (3*a*AppellF1[1/2
, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(a + b*Tan[
e + f*x]^2)^p)/((Sec[e + f*x]^2)^(m/2)*(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)
/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*Appel
lF1[3/2, (4 + m)/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan
[e + f*x]^2)) + (6*a*(-1 - m/2)*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(Sec[e + f*x]^2)^(-1 - m/2)*Tan[e + f*x]^2
*(a + b*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4
+ m)/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)
 + (3*a*(Sec[e + f*x]^2)^(-1 - m/2)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, (2 +
 m)/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2
*Tan[e + f*x])/(3*a) - ((2 + m)*AppellF1[3/2, 1 + (2 + m)/2, -p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/3)*(a + b
*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m)/2,
 -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) - (3*a
*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)
]*(Sec[e + f*x]^2)^(-1 - m/2)*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p*(2*(2*b
*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m)/2, -p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/a)])*Sec[e + f*x]^2*Tan[e + f*x] + 3*a*((2*b*p*AppellF1[3/2
```

```
, (2 + m)/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e +
f*x]^2*Tan[e + f*x])/(3*a) - ((2 + m)*AppellF1[3/2, 1 + (2 + m)/2, -p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/3)
+ Tan[e + f*x]^2*(2*b*p*((-6*b*(1 - p)*AppellF1[5/2, (2 + m)/2, 2 - p, 7/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(5*
a) - (3*(2 + m)*AppellF1[5/2, 1 + (2 + m)/2, 1 - p, 7/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/5) - a*(2 + m)*((6*b*p
*AppellF1[5/2, (4 + m)/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a) - (3*(4 + m)*AppellF1[5/2, 1 + (4 +
 m)/2, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Ta
n[e + f*x])/5))))/(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m)/2, -
p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)^2))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\left(d\cos\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\left(d\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

maple [F] time = 2.81, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(fx+e\right)\right)^{m}\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\left(d\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(d\,\cos\left(e+f\,x\right)\right)}^m\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x)^2)^p,x)
[Out] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

3.177 $\int (d\cos(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=101

$$\frac{\tan(e+fx)(d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(-m+np+1)} \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(\frac{1}{2}(np+1), \frac{1}{2}(-m+np+1); \frac{1}{2}(np+\right.}{f(np+1)}$$

```
[Out] (d*cos(f*x+e))^m*(cos(f*x+e)^2)^(1/2*n*p-1/2*m+1/2)*hypergeom([1/2*n*p+1/2,
 1/2*n*p-1/2*m+1/2],[1/2*n*p+3/2],sin(f*x+e)^2)*tan(f*x+e)*(b*(c*tan(f*x+e)
)^n)^p/f/(n*p+1)
```

Rubi [A] time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3659, 2603, 2617}

$$\frac{\tan(e+fx)(d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(-m+np+1)} \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(\frac{1}{2}(np+1), \frac{1}{2}(-m+np+1); \frac{1}{2}(np+\right.}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((d*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + n*p)/2)*Hypergeometric2F1[(1
 + n*p)/2, (1 - m + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 2603

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cos[e + f*x])^FracPart[m]*(Sec[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sec[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\cos(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (d\cos(e+fx))^m(c\,\mathrm{ta}$$

$$= \left((d\cos(e+fx))^m\left(\frac{\sec(e+fx)}{d}\right)^m (c\tan(e+fx))^{-np}\left(b(c\tan(e+\right.\right.$$

$$= \frac{(d\cos(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(1-m+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(1-m+\right.}{f(1+}$$

Mathematica [A] time = 0.53, size = 91, normalized size = 0.90

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\,{}_2F_1\left(\frac{m+2}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+f\right.}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((d*Cos[e + f*x])^m*Hypergeometric2F1[(2 + m)/2, (1 + n*p)/2, (3 + n*p)/2,
-Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)
^p)/(f*(1 + n*p))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\left(d\cos\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*cos(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\left(d\cos\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*cos(f*x + e))^m, x)
```

maple [F] time = 1.40, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(fx+e\right)\right)^m\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\left(d\cos\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*cos(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\,\cos\left(e+f\,x\right)\right)^m\,\left(b\,\left(c\,\tan\left(e+f\,x\right)\right)^n\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int((d*cos(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p \left(d\cos\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*cos(e + f*x))**m, x)
```

3.178 $\int (d\cos(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=57

$$(d\cos(e+fx))^m\left(\frac{\sec(e+fx)}{d}\right)^m \text{Int}\left(\left(\frac{\sec(e+fx)}{d}\right)^{-m}\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] (d*cos(f*x+e))^m*(sec(f*x+e)/d)^m*Unintegrable((a+b*(c*tan(f*x+e))^n)^p/((s
ec(f*x+e)/d)^m),x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d\cos(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] (d*Cos[e + f*x])^m*(Sec[e + f*x]/d)^m*Defer[Int][(a + b*(c*Tan[e + f*x])^n)
^p/(Sec[e + f*x]/d)^m, x]
```

Rubi steps

$$\int (d\cos(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx = \left((d\cos(e+fx))^m\left(\frac{\sec(e+fx)}{d}\right)^m\right)\int \left(\frac{\sec(e+fx)}{d}\right)^{-m}(a+$$

Mathematica [A] time = 2.78, size = 0, normalized size = 0.00

$$\int (d\cos(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Integrate[(d*Cos[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p\left(d\cos\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*cos(f*x + e))^m, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 7.98, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(fx+e\right)\right)^{m}\left(a+b\left(c\tan\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
[Out] int((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\left(d\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cos(f*x + e))^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(d\,\cos\left(e+f\,x\right)\right)}^m\,{\left(a+b\,{\left(c\,\tan\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p,x)
[Out] int((d*cos(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
[Out] Timed out
```

3.179 $\int \left(a+a\tan^2(c+dx)\right)^4 \, dx$

Optimal. Leaf size=65

$$\frac{a^4\tan^7(c+dx)}{7d}+\frac{3a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^3(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}$$

```
[Out] a^4*tan(d*x+c)/d+a^4*tan(d*x+c)^3/d+3/5*a^4*tan(d*x+c)^5/d+1/7*a^4*tan(d*x+
c)^7/d
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3657, 12, 3767}

$$\frac{a^4\tan^7(c+dx)}{7d}+\frac{3a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^3(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^4,x]
```

```
[Out] (a^4*Tan[c + d*x])/d + (a^4*Tan[c + d*x]^3)/d + (3*a^4*Tan[c + d*x]^5)/(5*d
) + (a^4*Tan[c + d*x]^7)/(7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+a\tan^2(c+dx)\right)^4 \, dx &= \int a^4\sec^8(c+dx)\,dx \\
&= a^4\int \sec^8(c+dx)\,dx \\
&= -\frac{a^4\,\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\,dx, x, -\tan(c+dx)\right)}{d} \\
&= \frac{a^4\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{d}+\frac{3a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 46, normalized size = 0.71

$$\frac{a^4\left(\frac{1}{7}\tan^7(c+dx)+\frac{3}{5}\tan^5(c+dx)+\tan^3(c+dx)+\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^4,x]
```

```
[Out] (a^4*(Tan[c + d*x] + Tan[c + d*x]^3 + (3*Tan[c + d*x]^5)/5 + Tan[c + d*x]^7
/7))/d
```

fricas [A] time = 0.40, size = 56, normalized size = 0.86

$$\frac{5\,a^4\tan\left(dx+c\right)^7+21\,a^4\tan\left(dx+c\right)^5+35\,a^4\tan\left(dx+c\right)^3+35\,a^4\tan\left(dx+c\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(5*a^4*tan(d*x + c)^7 + 21*a^4*tan(d*x + c)^5 + 35*a^4*tan(d*x + c)^3
+ 35*a^4*tan(d*x + c))/d
```

giac [B] time = 20.74, size = 519, normalized size = 7.98

$$-\frac{35\,a^4\tan\left(dx\right)^7\tan(c)^6+35\,a^4\tan\left(dx\right)^6\tan(c)^7+35\,a^4\tan\left(dx\right)^7\tan(c)^4-105\,a^4\tan\left(dx\right)^6\tan(c)^5-105\,a^4\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] -1/35*(35*a^4*tan(d*x)^7*tan(c)^6 + 35*a^4*tan(d*x)^6*tan(c)^7 + 35*a^4*tan
(d*x)^7*tan(c)^4 - 105*a^4*tan(d*x)^6*tan(c)^5 - 105*a^4*tan(d*x)^5*tan(c)^
6 + 35*a^4*tan(d*x)^4*tan(c)^7 + 21*a^4*tan(d*x)^7*tan(c)^2 - 35*a^4*tan(d*
x)^6*tan(c)^3 + 315*a^4*tan(d*x)^5*tan(c)^4 + 315*a^4*tan(d*x)^4*tan(c)^5 -
 35*a^4*tan(d*x)^3*tan(c)^6 + 21*a^4*tan(d*x)^2*tan(c)^7 + 5*a^4*tan(d*x)^7
 - 7*a^4*tan(d*x)^6*tan(c) + 105*a^4*tan(d*x)^5*tan(c)^2 - 315*a^4*tan(d*x)
^4*tan(c)^3 - 315*a^4*tan(d*x)^3*tan(c)^4 + 105*a^4*tan(d*x)^2*tan(c)^5 - 7
*a^4*tan(d*x)*tan(c)^6 + 5*a^4*tan(c)^7 + 21*a^4*tan(d*x)^5 - 35*a^4*tan(d*
x)^4*tan(c) + 315*a^4*tan(d*x)^3*tan(c)^2 + 315*a^4*tan(d*x)^2*tan(c)^3 - 3
5*a^4*tan(d*x)*tan(c)^4 + 21*a^4*tan(c)^5 + 35*a^4*tan(d*x)^3 - 105*a^4*tan
(d*x)^2*tan(c) - 105*a^4*tan(d*x)*tan(c)^2 + 35*a^4*tan(c)^3 + 35*a^4*tan(d
*x) + 35*a^4*tan(c))/(d*tan(d*x)^7*tan(c)^7 - 7*d*tan(d*x)^6*tan(c)^6 + 21*
d*tan(d*x)^5*tan(c)^5 - 35*d*tan(d*x)^4*tan(c)^4 + 35*d*tan(d*x)^3*tan(c)^3
 - 21*d*tan(d*x)^2*tan(c)^2 + 7*d*tan(d*x)*tan(c) - d)
```

maple [A] time = 0.02, size = 43, normalized size = 0.66

$$\frac{a^4\left(\frac{\left(\tan^7(dx+c)\right)}{7}+\frac{3\left(\tan^5(dx+c)\right)}{5}+\tan^3\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^4,x)
```

```
[Out] 1/d*a^4*(1/7*tan(d*x+c)^7+3/5*tan(d*x+c)^5+tan(d*x+c)^3+tan(d*x+c))
```

maxima [B] time = 0.47, size = 157, normalized size = 2.42

$$a^4x+\frac{\left(15\,\tan\left(dx+c\right)^7-21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+105\,dx+105\,c-105\,\tan\left(dx+c\right)\right)a^4}{105\,d}+\frac{4\left(3\,\tan\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 1/105*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 +
105*d*x + 105*c - 105*tan(d*x + c))*a^4/d + 4/15*(3*tan(d*x + c)^5 - 5*tan(
d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^4/d + 2*(tan(d*x + c)^3 + 3
*d*x + 3*c - 3*tan(d*x + c))*a^4/d - 4*(d*x + c - tan(d*x + c))*a^4/d
```

mupad [B] time = 11.52, size = 53, normalized size = 0.82

$$\frac{\frac{a^4\tan(c+d\,x)^7}{7}+\frac{3\,a^4\tan(c+d\,x)^5}{5}+a^4\tan\left(c+d\,x\right)^3+a^4\tan\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)^2)^4,x)
```

```
[Out] (a^4*tan(c + d*x) + a^4*tan(c + d*x)^3 + (3*a^4*tan(c + d*x)^5)/5 + (a^4*ta
n(c + d*x)^7)/7)/d
```

sympy [A] time = 1.06, size = 68, normalized size = 1.05

$$\begin{cases} \frac{a^4\tan^7\left(c+dx\right)}{7d}+\frac{3a^4\tan^5\left(c+dx\right)}{5d}+\frac{a^4\tan^3\left(c+dx\right)}{d}+\frac{a^4\tan\left(c+dx\right)}{d} & \text{for } d\neq 0 \\ x\left(a\tan^2\left(c\right)+a\right)^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**4,x)
```

```
[Out] Piecewise((a**4*tan(c + d*x)**7/(7*d) + 3*a**4*tan(c + d*x)**5/(5*d) + a**4
*tan(c + d*x)**3/d + a**4*tan(c + d*x)/d, Ne(d, 0)), (x*(a*tan(c)**2 + a)**
4, True))
```

3.180 $\int \left(a+a\tan^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=50

$$\frac{a^3\tan^5(c+dx)}{5d}+\frac{2a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan(c+dx)}{d}$$

```
[Out] a^3*tan(d*x+c)/d+2/3*a^3*tan(d*x+c)^3/d+1/5*a^3*tan(d*x+c)^5/d
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3657, 12, 3767}

$$\frac{a^3\tan^5(c+dx)}{5d}+\frac{2a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tan[c + d*x])/d + (2*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/
(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+a\tan^2(c+dx)\right)^3\,dx &= \int a^3\sec^6(c+dx)\,dx\\
&= a^3\int \sec^6(c+dx)\,dx\\
&= -\frac{a^3\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(c+dx)\right)}{d}\\
&= \frac{a^3\tan(c+dx)}{d}+\frac{2a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 38, normalized size = 0.76

$$\frac{a^3\left(\frac{1}{5}\tan^5(c+dx)+\frac{2}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^3,x]
```

```
[Out] (a^3*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d
```

fricas [A] time = 0.39, size = 43, normalized size = 0.86

$$\frac{3 a^3 \tan (dx + c)^5 + 10 a^3 \tan (dx + c)^3 + 15 a^3 \tan (dx + c)}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(3*a^3*tan(d*x + c)^5 + 10*a^3*tan(d*x + c)^3 + 15*a^3*tan(d*x + c))/d
```

giac [B] time = 7.95, size = 297, normalized size = 5.94

$$-\frac{15 a^3 \tan (dx)^5 \tan(c)^4 + 15 a^3 \tan (dx)^4 \tan(c)^5 + 10 a^3 \tan (dx)^5 \tan(c)^2 - 30 a^3 \tan (dx)^4 \tan(c)^3 - 30 a^3 \tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*a^3*tan(d*x)^5*tan(c)^4 + 15*a^3*tan(d*x)^4*tan(c)^5 + 10*a^3*tan
(d*x)^5*tan(c)^2 - 30*a^3*tan(d*x)^4*tan(c)^3 - 30*a^3*tan(d*x)^3*tan(c)^4
+ 10*a^3*tan(d*x)^2*tan(c)^5 + 3*a^3*tan(d*x)^5 - 5*a^3*tan(d*x)^4*tan(c) +
 60*a^3*tan(d*x)^3*tan(c)^2 + 60*a^3*tan(d*x)^2*tan(c)^3 - 5*a^3*tan(d*x)*t
an(c)^4 + 3*a^3*tan(c)^5 + 10*a^3*tan(d*x)^3 - 30*a^3*tan(d*x)^2*tan(c) - 3
0*a^3*tan(d*x)*tan(c)^2 + 10*a^3*tan(c)^3 + 15*a^3*tan(d*x) + 15*a^3*tan(c)
)/(d*tan(d*x)^5*tan(c)^5 - 5*d*tan(d*x)^4*tan(c)^4 + 10*d*tan(d*x)^3*tan(c)
^3 - 10*d*tan(d*x)^2*tan(c)^2 + 5*d*tan(d*x)*tan(c) - d)
```

maple [A] time = 0.03, size = 35, normalized size = 0.70

$$\frac{a^3 \left(\frac{\left(\tan^5(dx+c)\right)}{5} + \frac{2\left(\tan^3(dx+c)\right)}{3} + \tan (dx + c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^3,x)
```

```
[Out] 1/d*a^3*(1/5*tan(d*x+c)^5+2/3*tan(d*x+c)^3+tan(d*x+c))
```

maxima [B] time = 0.46, size = 102, normalized size = 2.04

$$a^3 x + \frac{\left(3 \tan (dx + c)^5 - 5 \tan (dx + c)^3 - 15 dx - 15 c + 15 \tan (dx + c)\right) a^3}{15 d} + \frac{\left(\tan (dx + c)^3 + 3 dx + 3 c - 3 \tan\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(
d*x + c))*a^3/d + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^3/d - 3
*(d*x + c - tan(d*x + c))*a^3/d
```

mupad [B] time = 11.47, size = 36, normalized size = 0.72

$$\frac{a^3 \tan (c + d\,x) \left(3 \tan (c + d\,x)^4 + 10 \tan (c + d\,x)^2 + 15\right)}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)^2)^3,x)
```

```
[Out] (a^3*tan(c + d*x)*(10*tan(c + d*x)^2 + 3*tan(c + d*x)^4 + 15))/(15*d)
```

sympy [A] time = 0.54, size = 54, normalized size = 1.08

$$\begin{cases} \frac{a^3 \tan^5{(c+dx)}}{5d} + \frac{2a^3 \tan^3{(c+dx)}}{3d} + \frac{a^3 \tan{(c+dx)}}{d} & \text{for } d \neq 0 \\ x \left(a \tan^2{(c)} + a\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*tan(c + d*x)**5/(5*d) + 2*a**3*tan(c + d*x)**3/(3*d) + a**3
*tan(c + d*x)/d, Ne(d, 0)), (x*(a*tan(c)**2 + a)**3, True))
```

3.181 $\int \left(a+a\tan^2(c+dx)\right)^2 dx$

Optimal. Leaf size=32

$$\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}$$

```
[Out] a^2*tan(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3657, 12, 3767}

$$\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+a\tan^2(c+dx)\right)^2 dx &= \int a^2\sec^4(c+dx)\,dx\\
&= a^2\int \sec^4(c+dx)\,dx\\
&= -\frac{a^2\,\text{Subst}\left(\int \left(1+x^2\right) dx, x, -\tan(c+dx)\right)}{d}\\
&= \frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 26, normalized size = 0.81

$$\frac{a^2\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 0.40, size = 29, normalized size = 0.91

$$\frac{a^2\tan(dx+c)^3+3\,a^2\tan(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*tan(d*x + c)^3 + 3*a^2*tan(d*x + c))/d
```

giac [B] time = 1.38, size = 133, normalized size = 4.16

$$-\frac{3\,a^2\tan(dx)^3\tan(c)^2+3\,a^2\tan(dx)^2\tan(c)^3+a^2\tan(dx)^3-3\,a^2\tan(dx)^2\tan(c)-3\,a^2\tan(dx)\tan(c)^2+a^2\,\mathrm{t}}{3\left(d\tan(dx)^3\tan(c)^3-3\,d\tan(dx)^2\tan(c)^2+3\,d\tan(dx)\tan(c)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*a^2*tan(d*x)^3*tan(c)^2 + 3*a^2*tan(d*x)^2*tan(c)^3 + a^2*tan(d*x)^
3 - 3*a^2*tan(d*x)^2*tan(c) - 3*a^2*tan(d*x)*tan(c)^2 + a^2*tan(c)^3 + 3*a^
2*tan(d*x) + 3*a^2*tan(c))/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan(c)^2
 + 3*d*tan(d*x)*tan(c) - d)
```

maple [A] time = 0.02, size = 25, normalized size = 0.78

$$\frac{a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*a^2*(1/3*tan(d*x+c)^3+tan(d*x+c))
```

maxima [A] time = 0.53, size = 59, normalized size = 1.84

$$a^2x+\frac{\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^2}{3\,d}-\frac{2\,(dx+c-\tan(dx+c))a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2/d - 2*(d*x
+ c - tan(d*x + c))*a^2/d
```

mupad [B] time = 11.57, size = 24, normalized size = 0.75

$$\frac{a^2\tan(c+d\,x)\,\left(\tan(c+d\,x)^2+3\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)^2)^2,x)
```

```
[Out] (a^2*tan(c + d*x)*(tan(c + d*x)^2 + 3))/(3*d)
```

sympy [A] time = 0.29, size = 37, normalized size = 1.16

$$\begin{cases} \frac{a^2\tan^3(c+dx)}{3d} + \frac{a^2\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a\tan^2(c) + a\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*tan(c + d*x)**3/(3*d) + a**2*tan(c + d*x)/d, Ne(d, 0)), (x*
(a*tan(c)**2 + a)**2, True))
```

3.182 $\int \frac{1}{a+a\tan^2(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

```
[Out] 1/2*x/a+1/2*cos(d*x+c)*sin(d*x+c)/a/d
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3657, 12, 2635, 8}

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-1),x]
```

```
[Out] x/(2*a) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+a\tan^2(c+dx)}\,dx &= \int \frac{\cos^2(c+dx)}{a}\,dx\\ &= \frac{\int \cos^2(c+dx)\,dx}{a}\\ &= \frac{\cos(c+dx)\sin(c+dx)}{2ad}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.84

$$\frac{2(c+dx)+\sin(2(c+dx))}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-1),x]
[Out] (2*(c + d*x) + Sin[2*(c + d*x)])/(4*a*d)
```

fricas [A] time = 0.42, size = 40, normalized size = 1.29

$$\frac{dx \tan{(dx+c)}^2 + dx + \tan{(dx+c)}}{2\left(ad\tan{(dx+c)}^2 + ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*(d*x*tan(d*x + c)^2 + d*x + tan(d*x + c))/(a*d*tan(d*x + c)^2 + a*d)
```

giac [A] time = 1.08, size = 37, normalized size = 1.19

$$\frac{\frac{dx+c}{a} + \frac{\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*((d*x + c)/a + tan(d*x + c)/((tan(d*x + c)^2 + 1)*a))/d
```

maple [A] time = 0.17, size = 43, normalized size = 1.39

$$\frac{\tan{(dx+c)}}{2ad\left(1+\tan^2{(dx+c)}\right)} + \frac{\arctan{(\tan{(dx+c)})}}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2),x)
[Out] 1/2/a/d*tan(d*x+c)/(1+tan(d*x+c)^2)+1/2/a/d*arctan(tan(d*x+c))
```

maxima [A] time = 0.48, size = 36, normalized size = 1.16

$$\frac{\frac{dx+c}{a} + \frac{\tan(dx+c)}{a\tan(dx+c)^2+a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2),x, algorithm="maxima")
[Out] 1/2*((d*x + c)/a + tan(d*x + c)/(a*tan(d*x + c)^2 + a))/d
```

mupad [B] time = 11.83, size = 26, normalized size = 0.84

$$\frac{\frac{\sin(2\,c+2\,d\,x)}{4\,a} + \frac{d\,x}{2\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)^2),x)
[Out] (sin(2*c + 2*d*x)/(4*a) + (d*x)/(2*a))/d
```

sympy [A] time = 0.54, size = 87, normalized size = 2.81

$$\begin{cases} \frac{dx\tan^2(c+dx)}{2ad\tan^2(c+dx)+2ad} + \frac{dx}{2ad\tan^2(c+dx)+2ad} + \frac{\tan(c+dx)}{2ad\tan^2(c+dx)+2ad} & \text{for } d \neq 0 \\ \frac{x}{a\tan^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((d*x*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**2 + 2*a*d) + d*x/(2*a*d
*tan(c + d*x)**2 + 2*a*d) + tan(c + d*x)/(2*a*d*tan(c + d*x)**2 + 2*a*d), N
e(d, 0)), (x/(a*tan(c)**2 + a), True))
```

3.183 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=55

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{3x}{8a^2}$$

```
[Out] 3/8*x/a^2+3/8*cos(d*x+c)*sin(d*x+c)/a^2/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a^2/d
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3657, 12, 2635, 8}

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{3x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-2),x]
```

```
[Out] (3*x)/(8*a^2) + (3*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) + (Cos[c + d*x]^3*S
in[c + d*x])/(4*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^2}\,dx &= \int \frac{\cos^4(c+dx)}{a^2}\,dx\\ &= \frac{\int \cos^4(c+dx)\,dx}{a^2}\\ &= \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}+\frac{3\int \cos^2(c+dx)\,dx}{4a^2}\\ &= \frac{3\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}+\frac{3\int 1\,dx}{8a^2}\\ &= \frac{3x}{8a^2}+\frac{3\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.65

$$\frac{12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx))}{32a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-2),x]
```

```
[Out] (12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])/(32*a^2*d)
```

fricas [A] time = 0.40, size = 84, normalized size = 1.53

$$\frac{3\,dx\tan\left(dx+c\right)^4+6\,dx\tan\left(dx+c\right)^2+3\,\tan\left(dx+c\right)^3+3\,dx+5\,\tan\left(dx+c\right)}{8\left(a^2d\tan\left(dx+c\right)^4+2\,a^2d\tan\left(dx+c\right)^2+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*d*x*tan(d*x + c)^4 + 6*d*x*tan(d*x + c)^2 + 3*tan(d*x + c)^3 + 3*d*x
 + 5*tan(d*x + c))/(a^2*d*tan(d*x + c)^4 + 2*a^2*d*tan(d*x + c)^2 + a^2*d)
```

giac [A] time = 1.37, size = 51, normalized size = 0.93

$$\frac{\frac{3\,(dx+c)}{a^2}+\frac{3\,\tan(dx+c)^3+5\,\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)^2a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(d*x + c)/a^2 + (3*tan(d*x + c)^3 + 5*tan(d*x + c))/((tan(d*x + c)^2
 + 1)^2*a^2))/d
```

maple [A] time = 0.35, size = 69, normalized size = 1.25

$$\frac{\tan\left(dx+c\right)}{4d\,a^2\left(1+\tan^2\left(dx+c\right)\right)^2}+\frac{3\tan\left(dx+c\right)}{8d\,a^2\left(1+\tan^2\left(dx+c\right)\right)}+\frac{3\arctan\left(\tan\left(dx+c\right)\right)}{8d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/4/d/a^2*tan(d*x+c)/(1+tan(d*x+c)^2)^2+3/8/d/a^2*tan(d*x+c)/(1+tan(d*x+c)^
2)+3/8/d/a^2*arctan(tan(d*x+c))
```

maxima [A] time = 0.78, size = 67, normalized size = 1.22

$$\frac{\frac{3\,\tan(dx+c)^3+5\,\tan(dx+c)}{a^2\tan(dx+c)^4+2\,a^2\tan(dx+c)^2+a^2}+\frac{3\,(dx+c)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*tan(d*x + c)^3 + 5*tan(d*x + c))/(a^2*tan(d*x + c)^4 + 2*a^2*tan(d*
x + c)^2 + a^2) + 3*(d*x + c)/a^2)/d
```

mupad [B] time = 11.90, size = 35, normalized size = 0.64

$$\frac{2\,\sin\left(2\,c+2\,d\,x\right)+\frac{\sin(4\,c+4\,d\,x)}{4}+3\,d\,x}{8\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)^2)^2,x)
```

```
[Out] (2*sin(2*c + 2*d*x) + sin(4*c + 4*d*x)/4 + 3*d*x)/(8*a^2*d)
```

sympy [A] time = 0.90, size = 248, normalized size = 4.51

$$\begin{cases} \frac{3dx\tan^4(c+dx)}{8a^2d\tan^4(c+dx)+16a^2d\tan^2(c+dx)+8a^2d} + \frac{6dx\tan^2(c+dx)}{8a^2d\tan^4(c+dx)+16a^2d\tan^2(c+dx)+8a^2d} + \frac{3dx}{8a^2d\tan^4(c+dx)+16a^2d\tan^2(c+dx)+8a^2d} + \frac{}{8a^2} \\ \frac{x}{\left(a\tan^2(c)+a\right)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((3*d*x*tan(c + d*x)**4/(8*a**2*d*tan(c + d*x)**4 + 16*a**2*d*tan(
c + d*x)**2 + 8*a**2*d) + 6*d*x*tan(c + d*x)**2/(8*a**2*d*tan(c + d*x)**4 +
 16*a**2*d*tan(c + d*x)**2 + 8*a**2*d) + 3*d*x/(8*a**2*d*tan(c + d*x)**4 +
16*a**2*d*tan(c + d*x)**2 + 8*a**2*d) + 3*tan(c + d*x)**3/(8*a**2*d*tan(c +
 d*x)**4 + 16*a**2*d*tan(c + d*x)**2 + 8*a**2*d) + 5*tan(c + d*x)/(8*a**2*d
*tan(c + d*x)**4 + 16*a**2*d*tan(c + d*x)**2 + 8*a**2*d), Ne(d, 0)), (x/(a*
tan(c)**2 + a)**2, True))
```

3.184 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=79

$$\frac{\sin(c+dx)\cos^5(c+dx)}{6a^3d}+\frac{5\sin(c+dx)\cos^3(c+dx)}{24a^3d}+\frac{5\sin(c+dx)\cos(c+dx)}{16a^3d}+\frac{5x}{16a^3}$$

```
[Out] 5/16*x/a^3+5/16*cos(d*x+c)*sin(d*x+c)/a^3/d+5/24*cos(d*x+c)^3*sin(d*x+c)/a^
3/d+1/6*cos(d*x+c)^5*sin(d*x+c)/a^3/d
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3657, 12, 2635, 8}

$$\frac{\sin(c+dx)\cos^5(c+dx)}{6a^3d}+\frac{5\sin(c+dx)\cos^3(c+dx)}{24a^3d}+\frac{5\sin(c+dx)\cos(c+dx)}{16a^3d}+\frac{5x}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-3),x]
```

```
[Out] (5*x)/(16*a^3) + (5*Cos[c + d*x]*Sin[c + d*x])/(16*a^3*d) + (5*Cos[c + d*x]
^3*Sin[c + d*x])/(24*a^3*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(6*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^3}\,dx = \int \frac{\cos^6(c+dx)}{a^3}\,dx$$
$$= \frac{\int \cos^6(c+dx)\,dx}{a^3}$$
$$= \frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d} + \frac{5\int \cos^4(c+dx)\,dx}{6a^3}$$
$$= \frac{5\cos^3(c+dx)\sin(c+dx)}{24a^3d} + \frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d} + \frac{5\int \cos^2(c+dx)\,dx}{8a^3}$$
$$= \frac{5\cos(c+dx)\sin(c+dx)}{16a^3d} + \frac{5\cos^3(c+dx)\sin(c+dx)}{24a^3d} + \frac{\cos^5(c+dx)\sin(c+d}{6a^3d}$$
$$= \frac{5x}{16a^3} + \frac{5\cos(c+dx)\sin(c+dx)}{16a^3d} + \frac{5\cos^3(c+dx)\sin(c+dx)}{24a^3d} + \frac{\cos^5(c+dx)\,\text{si}}{6a^3d}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 0.58

$$\frac{45\sin(2(c+dx)) + 9\sin(4(c+dx)) + \sin(6(c+dx)) + 60c + 60dx}{192a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-3),x]
```

```
[Out] (60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)
])/(192*a^3*d)
```

fricas [A] time = 0.39, size = 120, normalized size = 1.52

$$\frac{15\,dx\tan(dx+c)^6 + 45\,dx\tan(dx+c)^4 + 15\,\tan(dx+c)^5 + 45\,dx\tan(dx+c)^2 + 40\,\tan(dx+c)^3 + 15\,dx +}{48\left(a^3d\tan(dx+c)^6 + 3\,a^3d\tan(dx+c)^4 + 3\,a^3d\tan(dx+c)^2 + a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(15*d*x*tan(d*x + c)^6 + 45*d*x*tan(d*x + c)^4 + 15*tan(d*x + c)^5 + 4
5*d*x*tan(d*x + c)^2 + 40*tan(d*x + c)^3 + 15*d*x + 33*tan(d*x + c))/(a^3*d
*tan(d*x + c)^6 + 3*a^3*d*tan(d*x + c)^4 + 3*a^3*d*tan(d*x + c)^2 + a^3*d)
```

giac [A] time = 1.63, size = 61, normalized size = 0.77

$$\frac{\frac{15\,(dx+c)}{a^3} + \frac{15\,\tan(dx+c)^5 + 40\,\tan(dx+c)^3 + 33\,\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)^3 a^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/48*(15*(d*x + c)/a^3 + (15*tan(d*x + c)^5 + 40*tan(d*x + c)^3 + 33*tan(d*
x + c))/((tan(d*x + c)^2 + 1)^3*a^3))/d
```

maple [A] time = 0.34, size = 95, normalized size = 1.20

$$\frac{\tan(dx+c)}{6d\,a^3\left(1+\tan^2(dx+c)\right)^3} + \frac{5\tan(dx+c)}{24d\,a^3\left(1+\tan^2(dx+c)\right)^2} + \frac{5\tan(dx+c)}{16d\,a^3\left(1+\tan^2(dx+c)\right)} + \frac{5\arctan(\tan(dx+c))}{16d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^3,x)
```

```
[Out] 1/6/d/a^3*tan(d*x+c)/(1+tan(d*x+c)^2)^3+5/24/d/a^3*tan(d*x+c)/(1+tan(d*x+c)
^2)^2+5/16/d/a^3*tan(d*x+c)/(1+tan(d*x+c)^2)+5/16/d/a^3*arctan(tan(d*x+c))
```

maxima [A] time = 0.94, size = 90, normalized size = 1.14

$$\frac{\frac{15\tan(dx+c)^5+40\tan(dx+c)^3+33\tan(dx+c)}{a^3\tan(dx+c)^6+3a^3\tan(dx+c)^4+3a^3\tan(dx+c)^2+a^3}+\frac{15(dx+c)}{a^3}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/48*((15*tan(d*x + c)^5 + 40*tan(d*x + c)^3 + 33*tan(d*x + c))/(a^3*tan(d*
x + c)^6 + 3*a^3*tan(d*x + c)^4 + 3*a^3*tan(d*x + c)^2 + a^3) + 15*(d*x + c
)/a^3)/d
```

mupad [B] time = 11.93, size = 51, normalized size = 0.65

$$\frac{5x}{16a^3}+\frac{\cos(c+dx)^6\left(\frac{5\tan(c+dx)^5}{16}+\frac{5\tan(c+dx)^3}{6}+\frac{11\tan(c+dx)}{16}\right)}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)^2)^3,x)
```

```
[Out] (5*x)/(16*a^3) + (cos(c + d*x)^6*((11*tan(c + d*x))/16 + (5*tan(c + d*x)^3)
/6 + (5*tan(c + d*x)^5)/16))/(a^3*d)
```

sympy [A] time = 1.40, size = 454, normalized size = 5.75

$$\begin{cases}\frac{15dx\tan^6(c+dx)}{48a^3d\tan^6(c+dx)+144a^3d\tan^4(c+dx)+144a^3d\tan^2(c+dx)+48a^3d}+\frac{45dx\tan^4(c+dx)}{48a^3d\tan^6(c+dx)+144a^3d\tan^4(c+dx)+144a^3d\tan^2(c+dx)+48a^3d}+\frac{}{48a^3d} \\ \frac{x}{\left(a\tan^2(c)+a\right)^3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((15*d*x*tan(c + d*x)**6/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*t
an(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*a**3*d) + 45*d*x*tan(c + d
*x)**4/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d
*tan(c + d*x)**2 + 48*a**3*d) + 45*d*x*tan(c + d*x)**2/(48*a**3*d*tan(c + d
*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*a**3*
d) + 15*d*x/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a
**3*d*tan(c + d*x)**2 + 48*a**3*d) + 15*tan(c + d*x)**5/(48*a**3*d*tan(c +
d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*a**3
*d) + 40*tan(c + d*x)**3/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*
x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*a**3*d) + 33*tan(c + d*x)/(48*a**3*
d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2
 + 48*a**3*d), Ne(d, 0)), (x/(a*tan(c)**2 + a)**3, True))
```

3.185 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=74

$$\frac{(a-b)\tan^4(e+fx)}{4f}-\frac{(a-b)\tan^2(e+fx)}{2f}-\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^6(e+fx)}{6f}$$

```
[Out] -(a-b)*ln(cos(f*x+e))/f-1/2*(a-b)*tan(f*x+e)^2/f+1/4*(a-b)*tan(f*x+e)^4/f+1
/6*b*tan(f*x+e)^6/f
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3631, 3473, 3475}

$$\frac{(a-b)\tan^4(e+fx)}{4f}-\frac{(a-b)\tan^2(e+fx)}{2f}-\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^6(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)*Log[Cos[e + f*x]])/f) - ((a - b)*Tan[e + f*x]^2)/(2*f) + ((a - b
)*Tan[e + f*x]^4)/(4*f) + (b*Tan[e + f*x]^6)/(6*f)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{b\tan^6(e+fx)}{6f}+(a-b)\int \tan^5(e+fx)\,dx\\ &= \frac{(a-b)\tan^4(e+fx)}{4f}+\frac{b\tan^6(e+fx)}{6f}+(-a+b)\int \tan^3(e+fx)\,dx\\ &= -\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{(a-b)\tan^4(e+fx)}{4f}+\frac{b\tan^6(e+fx)}{6f}+(a\\ &= -\frac{(a-b)\log(\cos(e+fx))}{f}-\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{(a-b)\tan^4(e+\end{aligned}$$

Mathematica [A] time = 0.26, size = 63, normalized size = 0.85

$$\frac{3(a-b)\tan^4(e+fx)-6(a-b)\tan^2(e+fx)+12(b-a)\log(\cos(e+fx))+2b\tan^6(e+fx)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (12*(-a + b)*Log[Cos[e + f*x]] - 6*(a - b)*Tan[e + f*x]^2 + 3*(a - b)*Tan[e
 + f*x]^4 + 2*b*Tan[e + f*x]^6)/(12*f)
```

fricas [A] time = 0.45, size = 67, normalized size = 0.91

$$\frac{2\,b\tan\left(fx+e\right)^{6}+3\,(a-b)\tan\left(fx+e\right)^{4}-6\,(a-b)\tan\left(fx+e\right)^{2}-6\,(a-b)\log\left(\frac{1}{\tan\left(fx+e\right)^{2}+1}\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b*tan(f*x + e)^6 + 3*(a - b)*tan(f*x + e)^4 - 6*(a - b)*tan(f*x + e
)^2 - 6*(a - b)*log(1/(tan(f*x + e)^2 + 1)))/f
```

giac [B] time = 49.43, size = 1719, normalized size = 23.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/12*(6*a*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*ta
n(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan
(e)^6 - 6*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*ta
n(e)^6 + 9*a*tan(f*x)^6*tan(e)^6 - 11*b*tan(f*x)^6*tan(e)^6 - 36*a*log(4*(t
an(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 36*b*log(4*
(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)
^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 6*a*tan(f
*x)^6*tan(e)^4 - 6*b*tan(f*x)^6*tan(e)^4 - 42*a*tan(f*x)^5*tan(e)^5 + 54*b*
tan(f*x)^5*tan(e)^5 + 6*a*tan(f*x)^4*tan(e)^6 - 6*b*tan(f*x)^4*tan(e)^6 + 9
0*a*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
+ tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 -
 90*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^
2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4
 - 3*a*tan(f*x)^6*tan(e)^2 + 3*b*tan(f*x)^6*tan(e)^2 - 36*a*tan(f*x)^5*tan(
e)^3 + 36*b*tan(f*x)^5*tan(e)^3 + 69*a*tan(f*x)^4*tan(e)^4 - 99*b*tan(f*x)^
4*tan(e)^4 - 36*a*tan(f*x)^3*tan(e)^5 + 36*b*tan(f*x)^3*tan(e)^5 - 3*a*tan(
f*x)^2*tan(e)^6 + 3*b*tan(f*x)^2*tan(e)^6 - 120*a*log(4*(tan(f*x)^4*tan(e)^
2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan
(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 120*b*log(4*(tan(f*x)^4*tan(
e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*
tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 2*b*tan(f*x)^6 + 6*a*tan(
f*x)^5*tan(e) - 18*b*tan(f*x)^5*tan(e) + 60*a*tan(f*x)^4*tan(e)^2 - 90*b*ta
n(f*x)^4*tan(e)^2 - 72*a*tan(f*x)^3*tan(e)^3 + 72*b*tan(f*x)^3*tan(e)^3 + 6
0*a*tan(f*x)^2*tan(e)^4 - 90*b*tan(f*x)^2*tan(e)^4 + 6*a*tan(f*x)*tan(e)^5
- 18*b*tan(f*x)*tan(e)^5 - 2*b*tan(e)^6 + 90*a*log(4*(tan(f*x)^4*tan(e)^2 -
 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
 + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 90*b*log(4*(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(
e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 3*a*tan(f*x)^4 + 3*b*tan(f*x)
^4 - 36*a*tan(f*x)^3*tan(e) + 36*b*tan(f*x)^3*tan(e) + 69*a*tan(f*x)^2*tan(
e)^2 - 99*b*tan(f*x)^2*tan(e)^2 - 36*a*tan(f*x)*tan(e)^3 + 36*b*tan(f*x)*ta
n(e)^3 - 3*a*tan(e)^4 + 3*b*tan(e)^4 - 36*a*log(4*(tan(f*x)^4*tan(e)^2 - 2*
```

```
tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 36*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*ta
n(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)
/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 6*a*tan(f*x)^2 - 6*b*tan(f*x)^2 - 42*a*t
an(f*x)*tan(e) + 54*b*tan(f*x)*tan(e) + 6*a*tan(e)^2 - 6*b*tan(e)^2 + 6*a*l
og(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan
(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) - 6*b*log(4*(tan(f*x)^4*ta
n(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x
)*tan(e) + 1)/(tan(e)^2 + 1)) + 9*a - 11*b)/(f*tan(f*x)^6*tan(e)^6 - 6*f*ta
n(f*x)^5*tan(e)^5 + 15*f*tan(f*x)^4*tan(e)^4 - 20*f*tan(f*x)^3*tan(e)^3 + 1
5*f*tan(f*x)^2*tan(e)^2 - 6*f*tan(f*x)*tan(e) + f)
```

maple [A] time = 0.04, size = 106, normalized size = 1.43

$$\frac{b\left(\tan^{6}\left(fx+e\right)\right)}{6f}+\frac{\left(\tan^{4}\left(fx+e\right)\right)a}{4f}-\frac{b\left(\tan^{4}\left(fx+e\right)\right)}{4f}-\frac{a\left(\tan^{2}\left(fx+e\right)\right)}{2f}+\frac{b\left(\tan^{2}\left(fx+e\right)\right)}{2f}+\frac{\ln\left(1+\tan^{2}\left(f\right.\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/6*b*tan(f*x+e)^6/f+1/4/f*tan(f*x+e)^4*a-1/4*b*tan(f*x+e)^4/f-1/2/f*a*tan(
f*x+e)^2+1/2*b*tan(f*x+e)^2/f+1/2/f*ln(1+tan(f*x+e)^2)*a-1/2/f*ln(1+tan(f*x
+e)^2)*b
```

maxima [A] time = 0.44, size = 99, normalized size = 1.34

$$-\frac{6\,(a-b)\log\left(\sin\left(fx+e\right)^{2}-1\right)-\frac{6\,(2\,a-3\,b)\sin\left(fx+e\right)^{4}-3\,(7\,a-9\,b)\sin\left(fx+e\right)^{2}+9\,a-11\,b}{\sin\left(fx+e\right)^{6}-3\,\sin\left(fx+e\right)^{4}+3\,\sin\left(fx+e\right)^{2}-1}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*(a - b)*log(sin(f*x + e)^2 - 1) - (6*(2*a - 3*b)*sin(f*x + e)^4 -
3*(7*a - 9*b)*sin(f*x + e)^2 + 9*a - 11*b)/(sin(f*x + e)^6 - 3*sin(f*x + e)
^4 + 3*sin(f*x + e)^2 - 1))/f
```

mupad [B] time = 11.71, size = 68, normalized size = 0.92

$$\frac{\tan\left(e+f\,x\right)^{4}\,\left(\frac{a}{4}-\frac{b}{4}\right)-\tan\left(e+f\,x\right)^{2}\,\left(\frac{a}{2}-\frac{b}{2}\right)+\frac{b\tan\left(e+f\,x\right)^{6}}{6}+\ln\left(\tan\left(e+f\,x\right)^{2}+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (tan(e + f*x)^4*(a/4 - b/4) - tan(e + f*x)^2*(a/2 - b/2) + (b*tan(e + f*x)^
6)/6 + log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f
```

sympy [A] time = 0.73, size = 116, normalized size = 1.57

$$\begin{cases}\frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^{4}\left(e+fx\right)}{4f}-\frac{a\tan^{2}\left(e+fx\right)}{2f}-\frac{b\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{b\tan^{6}\left(e+fx\right)}{6f}-\frac{b\tan^{4}\left(e+fx\right)}{4f}+\frac{b\tan^{2}\left(e+fx\right)}{2f} & \text{fo} \\ x\left(a+b\tan^{2}\left(e\right)\right)\tan^{5}\left(e\right) & \text{ot}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*t
an(e + f*x)**2/(2*f) - b*log(tan(e + f*x)**2 + 1)/(2*f) + b*tan(e + f*x)**6
/(6*f) - b*tan(e + f*x)**4/(4*f) + b*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(
a + b*tan(e)**2)*tan(e)**5, True))
```

3.186 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=53

$$\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^4(e+fx)}{4f}$$

```
[Out] (a-b)*ln(cos(f*x+e))/f+1/2*(a-b)*tan(f*x+e)^2/f+1/4*b*tan(f*x+e)^4/f
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3631, 3473, 3475}

$$\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a - b)*Log[Cos[e + f*x]])/f + ((a - b)*Tan[e + f*x]^2)/(2*f) + (b*Tan[e +
 f*x]^4)/(4*f)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{b\tan^4(e+fx)}{4f}+(a-b)\int \tan^3(e+fx)\,dx\\ &= \frac{(a-b)\tan^2(e+fx)}{2f}+\frac{b\tan^4(e+fx)}{4f}+(-a+b)\int \tan(e+fx)\,dx\\ &= \frac{(a-b)\log(\cos(e+fx))}{f}+\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{b\tan^4(e+fx)}{4f}\end{aligned}$$

Mathematica [A] time = 0.17, size = 65, normalized size = 1.23

$$\frac{a\left(\tan^2(e+fx)+2\log(\cos(e+fx))\right)}{2f}-\frac{b\left(-\tan^4(e+fx)+2\tan^2(e+fx)+4\log(\cos(e+fx))\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (a*(2*Log[Cos[e + f*x]] + Tan[e + f*x]^2))/(2*f) - (b*(4*Log[Cos[e + f*x]]
+ 2*Tan[e + f*x]^2 - Tan[e + f*x]^4))/(4*f)
```

fricas [A] time = 0.42, size = 51, normalized size = 0.96

$$\frac{b\tan\left(fx+e\right)^{4}+2\,(a-b)\tan\left(fx+e\right)^{2}+2\,(a-b)\log\left(\frac{1}{\tan\left(fx+e\right)^{2}+1}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(b*tan(f*x + e)^4 + 2*(a - b)*tan(f*x + e)^2 + 2*(a - b)*log(1/(tan(f*x
 + e)^2 + 1)))/f
```

giac [B] time = 9.92, size = 1071, normalized size = 20.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(
e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e
)^4 - 2*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan
(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(
e)^4 + 2*a*tan(f*x)^4*tan(e)^4 - 3*b*tan(f*x)^4*tan(e)^4 - 8*a*log(4*(tan(f
*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2
*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 8*b*log(4*(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 2*a*tan(f*x)^4
*tan(e)^2 - 2*b*tan(f*x)^4*tan(e)^2 - 4*a*tan(f*x)^3*tan(e)^3 + 8*b*tan(f*x
)^3*tan(e)^3 + 2*a*tan(f*x)^2*tan(e)^4 - 2*b*tan(f*x)^2*tan(e)^4 + 12*a*log
(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f
*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 12*b*l
og(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan
(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + b*ta
n(f*x)^4 - 4*a*tan(f*x)^3*tan(e) + 8*b*tan(f*x)^3*tan(e) + 4*a*tan(f*x)^2*t
an(e)^2 - 4*b*tan(f*x)^2*tan(e)^2 - 4*a*tan(f*x)*tan(e)^3 + 8*b*tan(f*x)*ta
n(e)^3 + b*tan(e)^4 - 8*a*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
*tan(f*x)*tan(e) + 8*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*ta
n(f*x)*tan(e) + 2*a*tan(f*x)^2 - 2*b*tan(f*x)^2 - 4*a*tan(f*x)*tan(e) + 8*b
*tan(f*x)*tan(e) + 2*a*tan(e)^2 - 2*b*tan(e)^2 + 2*a*log(4*(tan(f*x)^4*tan(
e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*
tan(e) + 1)/(tan(e)^2 + 1)) - 2*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)
^2 + 1)) + 2*a - 3*b)/(f*tan(f*x)^4*tan(e)^4 - 4*f*tan(f*x)^3*tan(e)^3 + 6*
f*tan(f*x)^2*tan(e)^2 - 4*f*tan(f*x)*tan(e) + f)
```

maple [A] time = 0.03, size = 78, normalized size = 1.47

$$\frac{b\left(\tan^{4}\left(fx+e\right)\right)}{4f}+\frac{a\left(\tan^{2}\left(fx+e\right)\right)}{2f}-\frac{b\left(\tan^{2}\left(fx+e\right)\right)}{2f}-\frac{\ln\left(1+\tan^{2}\left(fx+e\right)\right)a}{2f}+\frac{\ln\left(1+\tan^{2}\left(fx+e\right)\right)b}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/4*b*tan(f*x+e)^4/f+1/2/f*a*tan(f*x+e)^2-1/2*b*tan(f*x+e)^2/f-1/2/f*ln(1+t
an(f*x+e)^2)*a+1/2/f*ln(1+tan(f*x+e)^2)*b
```

maxima [A] time = 0.54, size = 70, normalized size = 1.32

$$\frac{2(a-b)\log\left(\sin\left(fx+e\right)^{2}-1\right)-\frac{2(a-2b)\sin\left(fx+e\right)^{2}-2a+3b}{\sin\left(fx+e\right)^{4}-2\sin\left(fx+e\right)^{2}+1}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a - b)*log(sin(f*x + e)^2 - 1) - (2*(a - 2*b)*sin(f*x + e)^2 - 2*a
+ 3*b)/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

mupad [B] time = 11.70, size = 57, normalized size = 1.08

$$\frac{b\tan\left(e+fx\right)^{4}}{4f}-\frac{\ln\left(\tan\left(e+fx\right)^{2}+1\right)\left(\frac{a}{2}-\frac{b}{2}\right)}{f}+\frac{\tan\left(e+fx\right)^{2}\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (b*tan(e + f*x)^4)/(4*f) - (log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f + (tan(e
 + f*x)^2*(a/2 - b/2))/f
```

sympy [A] time = 0.39, size = 88, normalized size = 1.66

$$\begin{cases} -\frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^{2}\left(e+fx\right)}{2f}+\frac{b\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{b\tan^{4}\left(e+fx\right)}{4f}-\frac{b\tan^{2}\left(e+fx\right)}{2f} & \text{for } f\neq 0 \\ x\left(a+b\tan^{2}\left(e\right)\right)\tan^{3}\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b*
log(tan(e + f*x)**2 + 1)/(2*f) + b*tan(e + f*x)**4/(4*f) - b*tan(e + f*x)**
2/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)*tan(e)**3, True))
```

3.187 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=34

$$\frac{b\tan^2(e+fx)}{2f}-\frac{(a-b)\log(\cos(e+fx))}{f}$$

```
[Out] -(a-b)*ln(cos(f*x+e))/f+1/2*b*tan(f*x+e)^2/f
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3631, 3475}

$$\frac{b\tan^2(e+fx)}{2f}-\frac{(a-b)\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)*Log[Cos[e + f*x]])/f) + (b*Tan[e + f*x]^2)/(2*f)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)dx&=\frac{b\tan^2(e+fx)}{2f}+(a-b)\int \tan(e+fx)\,dx\\&=-\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^2(e+fx)}{2f}\end{aligned}$$

Mathematica [A] time = 0.07, size = 40, normalized size = 1.18

$$\frac{b\left(\tan^2(e+fx)+2\log(\cos(e+fx))\right)}{2f}-\frac{a\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*(2*Log[Cos[e + f*x]] + Tan[e + f*x]^2))/(2*
f)
```

fricas [A] time = 0.42, size = 36, normalized size = 1.06

$$\frac{b\tan\left(fx+e\right)^2-(a-b)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")

[Out] 1/2*(b*tan(f*x + e)^2 - (a - b)*log(1/(tan(f*x + e)^2 + 1)))/f

giac [B] time = 2.59, size = 500, normalized size = 14.71

$$-\frac{a\log\left(\frac{4\left(\tan\left(fx\right)^4\tan(e)^2-2\tan\left(fx\right)^3\tan(e)+\tan\left(fx\right)^2\tan(e)^2+\tan\left(fx\right)^2-2\tan\left(fx\right)\tan(e)+1\right)}{\tan(e)^2+1}\right)\tan\left(fx\right)^2\tan(e)^2-b\log\left(\frac{4\left(\tan(f\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")

[Out] -1/2*(a*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e
)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)
^2 - b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)
^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^
2 - b*tan(f*x)^2*tan(e)^2 - 2*a*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*t
an(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
 + 1))*tan(f*x)*tan(e) + 2*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(
e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 +
1))*tan(f*x)*tan(e) - b*tan(f*x)^2 - b*tan(e)^2 + a*log(4*(tan(f*x)^4*tan(e
)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*t
an(e) + 1)/(tan(e)^2 + 1)) - b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*ta
n(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
+ 1)) - b)/(f*tan(f*x)^2*tan(e)^2 - 2*f*tan(f*x)*tan(e) + f)

maple [A] time = 0.03, size = 50, normalized size = 1.47

$$\frac{b\left(\tan^2\left(fx+e\right)\right)}{2f}+\frac{\ln\left(1+\tan^2\left(fx+e\right)\right)a}{2f}-\frac{\ln\left(1+\tan^2\left(fx+e\right)\right)b}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)*(a+b*tan(f*x+e)^2),x)

[Out] 1/2*b*tan(f*x+e)^2/f+1/2/f*ln(1+tan(f*x+e)^2)*a-1/2/f*ln(1+tan(f*x+e)^2)*b

maxima [A] time = 0.59, size = 37, normalized size = 1.09

$$-\frac{(a-b)\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{b}{\sin\left(fx+e\right)^2-1}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")

[Out] -1/2*((a - b)*log(sin(f*x + e)^2 - 1) + b/(sin(f*x + e)^2 - 1))/f

mupad [B] time = 11.75, size = 37, normalized size = 1.09

$$\frac{b\tan\left(e+fx\right)^2}{2f}+\frac{\ln\left(\tan\left(e+fx\right)^2+1\right)\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (b*tan(e + f*x)^2)/(2*f) + (log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f
```

sympy [**A**] time = 0.20, size = 60, normalized size = 1.76

$$\begin{cases} \frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} - \frac{b\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{b\tan^2\left(e+fx\right)}{2f} & \text{for } f \neq 0 \\ x\left(a + b\tan^2\left(e\right)\right)\tan\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) - b*log(tan(e + f*x)**2 + 1)/(2
*f) + b*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)*tan(e), True
))
```

3.188 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=26

$$\frac{a\log(\sin(e+fx))}{f}-\frac{b\log(\cos(e+fx))}{f}$$

```
[Out] -b*ln(cos(f*x+e))/f+a*ln(sin(f*x+e))/f
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3625, 3475}

$$\frac{a\log(\sin(e+fx))}{f}-\frac{b\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2),x]
[Out] -((b*Log[Cos[e + f*x]])/f) + (a*Log[Sin[e + f*x]])/f
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3625

```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/tan[(e_.) + (f_.)*(x_)], x_Sym
bol] :> Dist[A, Int[1/Tan[e + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x],
x] /; FreeQ[{e, f, A, C}, x] && NeQ[A, C]
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= a\int \cot(e+fx)\,dx+b\int \tan(e+fx)\,dx\\ &= -\frac{b\log(\cos(e+fx))}{f}+\frac{a\log(\sin(e+fx))}{f}\end{aligned}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 1.31

$$\frac{a(\log(\tan(e+fx))+\log(\cos(e+fx)))}{f}-\frac{b\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2),x]
[Out] -((b*Log[Cos[e + f*x]])/f) + (a*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]]))/f
```

fricas [A] time = 0.46, size = 46, normalized size = 1.77

$$\frac{a\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)-b\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) - b*log(1/(tan(f*x + e)^2 +
 1)))/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)1/f*(a/2*ln(sin(f*x+exp(1))^2)-b/2*ln(abs(sin(f*x+exp(1))^2-1)))
```

maple [A] time = 0.57, size = 27, normalized size = 1.04

$$-\frac{b\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2),x)
```

```
[Out] -b*ln(cos(f*x+e))/f+a*ln(sin(f*x+e))/f
```

maxima [A] time = 0.63, size = 31, normalized size = 1.19

$$-\frac{b\log\left(\sin\left(fx+e\right)^{2}-1\right)-a\log\left(\sin\left(fx+e\right)^{2}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*log(sin(f*x + e)^2 - 1) - a*log(sin(f*x + e)^2))/f
```

mupad [B] time = 11.66, size = 36, normalized size = 1.38

$$\frac{a\ \ln\left(\tan\left(e+f\,x\right)\right)}{f}-\frac{\ln\left(\tan\left(e+f\,x\right)^{2}+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (a*log(tan(e + f*x)))/f - (log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f
```

sympy [A] time = 0.41, size = 58, normalized size = 2.23

$$\begin{cases} -\frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{a\log\left(\tan\left(e+fx\right)\right)}{f}+\frac{b\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f} & \text{for}\ f\neq 0\\ x\left(a+b\tan^{2}\left(e\right)\right)\cot\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*log(tan(e + f*x))/f + b*lo
g(tan(e + f*x)**2 + 1)/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)*cot(e), True)
)
```

3.189 $\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=34

$$-\frac{(a-b)\log(\sin(e+fx))}{f}-\frac{a\cot^2(e+fx)}{2f}$$

```
[Out] -1/2*a*cot(f*x+e)^2/f-(a-b)*ln(sin(f*x+e))/f
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3629, 12, 3475}

$$-\frac{(a-b)\log(\sin(e+fx))}{f}-\frac{a\cot^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(a*Cot[e + f*x]^2)/(2*f) - ((a - b)*Log[Sin[e + f*x]])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= -\frac{a\cot^2(e+fx)}{2f}-\int (a-b)\cot(e+fx)\,dx\\
&= -\frac{a\cot^2(e+fx)}{2f}-(a-b)\int \cot(e+fx)\,dx\\
&= -\frac{a\cot^2(e+fx)}{2f}-\frac{(a-b)\log(\sin(e+fx))}{f}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 56, normalized size = 1.65

$$\frac{b(\log(\tan(e+fx))+\log(\cos(e+fx)))}{f}-\frac{a\left(\cot^2(e+fx)+2\log(\tan(e+fx))+2\log(\cos(e+fx))\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (b*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]]))/f - (a*(Cot[e + f*x]^2 + 2*Log[
Cos[e + f*x]] + 2*Log[Tan[e + f*x]]))/(2*f)
```

fricas [A] time = 0.42, size = 61, normalized size = 1.79

$$-\frac{(a-b)\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2+a\tan\left(fx+e\right)^2+a}{2\,f\tan\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*((a - b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + a*t
an(f*x + e)^2 + a)/(f*tan(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(-(1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1)))*a/16+(4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a-4*
(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-a)*1/16/(1-cos(f*x+exp(1)))*(1+co
s(f*x+exp(1)))+(a-b)/2*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+(
-a+b)/4*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [A] time = 0.66, size = 41, normalized size = 1.21

$$\frac{b\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{a\left(\cot^2\left(fx+e\right)\right)}{2f}-\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*b*ln(sin(f*x+e))-1/2*a*cot(f*x+e)^2/f-a*ln(sin(f*x+e))/f
```

maxima [A] time = 0.31, size = 31, normalized size = 0.91

$$-\frac{(a-b)\log\left(\sin\left(fx+e\right)^2\right)+\frac{a}{\sin\left(fx+e\right)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*((a - b)*log(sin(f*x + e)^2) + a/sin(f*x + e)^2)/f
```

mupad [B] time = 11.62, size = 54, normalized size = 1.59

$$\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}-\frac{\ln\left(\tan\left(e+f\,x\right)\right)\,\left(a-b\right)}{f}-\frac{a\cot\left(e+f\,x\right)^2}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f - (log(tan(e + f*x))*(a - b))/f - (
a*cot(e + f*x)^2)/(2*f)
```

sympy [A] time = 1.29, size = 97, normalized size = 2.85

$$\begin{cases} \tilde{\infty}ax & \text{for } e = 0 \wedge f = 0 \\ x\left(a + b\tan^2(e)\right)\cot^3(e) & \text{for } f = 0 \\ \tilde{\infty}ax & \text{for } e = -fx \\ \frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} - \frac{a\log\left(\tan\left(e+fx\right)\right)}{f} - \frac{a}{2f\tan^2\left(e+fx\right)} - \frac{b\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{b\log\left(\tan\left(e+fx\right)\right)}{f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*a*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)*cot(e)**3, E
q(f, 0)), (zoo*a*x, Eq(e, -f*x)), (a*log(tan(e + f*x)**2 + 1)/(2*f) - a*log
(tan(e + f*x))/f - a/(2*f*tan(e + f*x)**2) - b*log(tan(e + f*x)**2 + 1)/(2*
f) + b*log(tan(e + f*x))/f, True))
```

3.190 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)dx$

Optimal. Leaf size=53

$$\frac{(a-b)\cot^2(e+fx)}{2f}+\frac{(a-b)\log(\sin(e+fx))}{f}-\frac{a\cot^4(e+fx)}{4f}$$

```
[Out] 1/2*(a-b)*cot(f*x+e)^2/f-1/4*a*cot(f*x+e)^4/f+(a-b)*ln(sin(f*x+e))/f
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3629, 12, 3473, 3475}

$$\frac{(a-b)\cot^2(e+fx)}{2f}+\frac{(a-b)\log(\sin(e+fx))}{f}-\frac{a\cot^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a - b)*Cot[e + f*x]^2)/(2*f) - (a*Cot[e + f*x]^4)/(4*f) + ((a - b)*Log[Si
n[e + f*x]])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)dx &= -\frac{a\cot^4(e+fx)}{4f}-\int (a-b)\cot^3(e+fx)\,dx\\ &= -\frac{a\cot^4(e+fx)}{4f}-(a-b)\int \cot^3(e+fx)\,dx\\ &= \frac{(a-b)\cot^2(e+fx)}{2f}-\frac{a\cot^4(e+fx)}{4f}-(-a+b)\int \cot(e+fx)\,dx\\ &= \frac{(a-b)\cot^2(e+fx)}{2f}-\frac{a\cot^4(e+fx)}{4f}+\frac{(a-b)\log(\sin(e+fx))}{f}\end{aligned}$$

Mathematica [**A**] time = 0.23, size = 56, normalized size = 1.06

$$\frac{2(a-b)\cot^2(e+fx)+4(a-b)(\log(\tan(e+fx))+\log(\cos(e+fx)))-a\cot^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (2*(a - b)*Cot[e + f*x]^2 - a*Cot[e + f*x]^4 + 4*(a - b)*(Log[Cos[e + f*x]]
 + Log[Tan[e + f*x]]))/(4*f)
```

fricas [**A**] time = 0.41, size = 85, normalized size = 1.60

$$\frac{2\,(a-b)\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^4+(3\,a-2\,b)\tan\left(fx+e\right)^4+2\,(a-b)\tan\left(fx+e\right)^2-a}{4\,f\tan\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a - b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + (3
*a - 2*b)*tan(f*x + e)^4 + 2*(a - b)*tan(f*x + e)^2 - a)/(f*tan(f*x + e)^4)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*((-32*((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1))))^2*a+384*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
)*a-256*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b)/4096+(-48*((1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1))))^2*a+48*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
)^2*b+12*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a-8*(1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1)))*b-a)*1/128/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2+(-
a+b)/2*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+(a-b)/4*ln(abs(1-
cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [**A**] time = 0.53, size = 69, normalized size = 1.30

$$-\frac{b\left(\cot^2\left(fx+e\right)\right)}{2f}-\frac{b\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{a\left(\cot^4\left(fx+e\right)\right)}{4f}+\frac{a\left(\cot^2\left(fx+e\right)\right)}{2f}+\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/2/f*b*cot(f*x+e)^2-1/f*b*ln(sin(f*x+e))-1/4*a*cot(f*x+e)^4/f+1/2*a*cot(f
*x+e)^2/f+a*ln(sin(f*x+e))/f
```

maxima [**A**] time = 0.50, size = 52, normalized size = 0.98

$$\frac{2\,(a-b)\log\left(\sin\left(fx+e\right)^2\right)+\frac{2\,(2\,a-b)\sin\left(fx+e\right)^2-a}{\sin\left(fx+e\right)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a - b)*log(sin(f*x + e)^2) + (2*(2*a - b)*sin(f*x + e)^2 - a)/sin(f
*x + e)^4)/f
```

mupad [B] time = 11.65, size = 74, normalized size = 1.40

$$\frac{\ln\left(\tan\left(e+f\,x\right)\right)\,\left(a-b\right)}{f}-\frac{\frac{a}{4}-\tan\left(e+f\,x\right)^2\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f\tan\left(e+f\,x\right)^4}-\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (log(tan(e + f*x))*(a - b))/f - (a/4 - tan(e + f*x)^2*(a/2 - b/2))/(f*tan(e
 + f*x)^4) - (log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f
```

sympy [A] time = 2.93, size = 124, normalized size = 2.34

$$\begin{cases} \tilde{\infty}ax \\ x\left(a+b\tan^2\left(e\right)\right)\cot^5\left(e\right) \\ \tilde{\infty}ax \\ -\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\log\left(\tan\left(e+fx\right)\right)}{f}+\frac{a}{2f\tan^2\left(e+fx\right)}-\frac{a}{4f\tan^4\left(e+fx\right)}+\frac{b\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-\frac{b\log\left(\tan\left(e+fx\right)\right)}{f}-\frac{b}{2f\tan^2\left(e\right.} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*a*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)*cot(e)**5, E
q(f, 0)), (zoo*a*x, Eq(e, -f*x)), (-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*lo
g(tan(e + f*x))/f + a/(2*f*tan(e + f*x)**2) - a/(4*f*tan(e + f*x)**4) + b*l
og(tan(e + f*x)**2 + 1)/(2*f) - b*log(tan(e + f*x))/f - b/(2*f*tan(e + f*x)
**2), True))
```

3.191 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=80

$$\frac{(a-b)\tan^5(e+fx)}{5f}-\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan(e+fx)}{f}-x(a-b)+\frac{b\tan^7(e+fx)}{7f}$$

```
[Out] -(a-b)*x+(a-b)*tan(f*x+e)/f-1/3*(a-b)*tan(f*x+e)^3/f+1/5*(a-b)*tan(f*x+e)^5
/f+1/7*b*tan(f*x+e)^7/f
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3631, 3473, 8}

$$\frac{(a-b)\tan^5(e+fx)}{5f}-\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan(e+fx)}{f}-x(a-b)+\frac{b\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a - b)*x) + ((a - b)*Tan[e + f*x])/f - ((a - b)*Tan[e + f*x]^3)/(3*f) +
((a - b)*Tan[e + f*x]^5)/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{b\tan^7(e+fx)}{7f}+(a-b)\int \tan^6(e+fx)\,dx\\ &= \frac{(a-b)\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}+(-a+b)\int \tan^4(e+fx)\,dx\\ &= -\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}+(a\\ &= \frac{(a-b)\tan(e+fx)}{f}-\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan^5(e+fx)}{5f}+\\ &= -(a-b)x+\frac{(a-b)\tan(e+fx)}{f}-\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan}{5}\end{aligned}$$

Mathematica [A] time = 0.05, size = 129, normalized size = 1.61

$$-\frac{a\tan^{-1}(\tan(e+fx))}{f}+\frac{a\tan^5(e+fx)}{5f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}+\frac{b\tan^{-1}(\tan(e+fx))}{f}+\frac{b\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*ArcTan[Tan[e + f*x]])/f) + (b*ArcTan[Tan[e + f*x]])/f + (a*Tan[e + f*x
])/f - (b*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3*f) + (b*Tan[e + f*x]^3)/(
3*f) + (a*Tan[e + f*x]^5)/(5*f) - (b*Tan[e + f*x]^5)/(5*f) + (b*Tan[e + f*x
]^7)/(7*f)
```

fricas [A] time = 0.41, size = 69, normalized size = 0.86

$$\frac{15\,b\tan\left(fx+e\right)^{7}+21\,(a-b)\tan\left(fx+e\right)^{5}-35\,(a-b)\tan\left(fx+e\right)^{3}-105\,(a-b)fx+105\,(a-b)\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*(a - b)*tan(f*x + e)^5 - 35*(a - b)*tan(f*x
 + e)^3 - 105*(a - b)*f*x + 105*(a - b)*tan(f*x + e))/f
```

giac [B] time = 86.83, size = 1087, normalized size = 13.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/105*(105*a*f*x*tan(f*x)^7*tan(e)^7 - 105*b*f*x*tan(f*x)^7*tan(e)^7 - 735
*a*f*x*tan(f*x)^6*tan(e)^6 + 735*b*f*x*tan(f*x)^6*tan(e)^6 + 105*a*tan(f*x)
^7*tan(e)^6 - 105*b*tan(f*x)^7*tan(e)^6 + 105*a*tan(f*x)^6*tan(e)^7 - 105*b
*tan(f*x)^6*tan(e)^7 + 2205*a*f*x*tan(f*x)^5*tan(e)^5 - 2205*b*f*x*tan(f*x)
^5*tan(e)^5 - 35*a*tan(f*x)^7*tan(e)^4 + 35*b*tan(f*x)^7*tan(e)^4 - 735*a*t
an(f*x)^6*tan(e)^5 + 735*b*tan(f*x)^6*tan(e)^5 - 735*a*tan(f*x)^5*tan(e)^6
+ 735*b*tan(f*x)^5*tan(e)^6 - 35*a*tan(f*x)^4*tan(e)^7 + 35*b*tan(f*x)^4*ta
n(e)^7 - 3675*a*f*x*tan(f*x)^4*tan(e)^4 + 3675*b*f*x*tan(f*x)^4*tan(e)^4 +
21*a*tan(f*x)^7*tan(e)^2 - 21*b*tan(f*x)^7*tan(e)^2 + 245*a*tan(f*x)^6*tan(
e)^3 - 245*b*tan(f*x)^6*tan(e)^3 + 2205*a*tan(f*x)^5*tan(e)^4 - 2205*b*tan(
f*x)^5*tan(e)^4 + 2205*a*tan(f*x)^4*tan(e)^5 - 2205*b*tan(f*x)^4*tan(e)^5 +
 245*a*tan(f*x)^3*tan(e)^6 - 245*b*tan(f*x)^3*tan(e)^6 + 21*a*tan(f*x)^2*ta
n(e)^7 - 21*b*tan(f*x)^2*tan(e)^7 + 3675*a*f*x*tan(f*x)^3*tan(e)^3 - 3675*b
*f*x*tan(f*x)^3*tan(e)^3 + 15*b*tan(f*x)^7 - 42*a*tan(f*x)^6*tan(e) + 147*b
*tan(f*x)^6*tan(e) - 420*a*tan(f*x)^5*tan(e)^2 + 735*b*tan(f*x)^5*tan(e)^2
- 3150*a*tan(f*x)^4*tan(e)^3 + 3675*b*tan(f*x)^4*tan(e)^3 - 3150*a*tan(f*x)
^3*tan(e)^4 + 3675*b*tan(f*x)^3*tan(e)^4 - 420*a*tan(f*x)^2*tan(e)^5 + 735*
b*tan(f*x)^2*tan(e)^5 - 42*a*tan(f*x)*tan(e)^6 + 147*b*tan(f*x)*tan(e)^6 +
15*b*tan(e)^7 - 2205*a*f*x*tan(f*x)^2*tan(e)^2 + 2205*b*f*x*tan(f*x)^2*tan(
e)^2 + 21*a*tan(f*x)^5 - 21*b*tan(f*x)^5 + 245*a*tan(f*x)^4*tan(e) - 245*b*
tan(f*x)^4*tan(e) + 2205*a*tan(f*x)^3*tan(e)^2 - 2205*b*tan(f*x)^3*tan(e)^2
 + 2205*a*tan(f*x)^2*tan(e)^3 - 2205*b*tan(f*x)^2*tan(e)^3 + 245*a*tan(f*x)
*tan(e)^4 - 245*b*tan(f*x)*tan(e)^4 + 21*a*tan(e)^5 - 21*b*tan(e)^5 + 735*a
*f*x*tan(f*x)*tan(e) - 735*b*f*x*tan(f*x)*tan(e) - 35*a*tan(f*x)^3 + 35*b*t
an(f*x)^3 - 735*a*tan(f*x)^2*tan(e) + 735*b*tan(f*x)^2*tan(e) - 735*a*tan(f
*x)*tan(e)^2 + 735*b*tan(f*x)*tan(e)^2 - 35*a*tan(e)^3 + 35*b*tan(e)^3 - 10
5*a*f*x + 105*b*f*x + 105*a*tan(f*x) - 105*b*tan(f*x) + 105*a*tan(e) - 105*
b*tan(e))/(f*tan(f*x)^7*tan(e)^7 - 7*f*tan(f*x)^6*tan(e)^6 + 21*f*tan(f*x)^
5*tan(e)^5 - 35*f*tan(f*x)^4*tan(e)^4 + 35*f*tan(f*x)^3*tan(e)^3 - 21*f*tan
(f*x)^2*tan(e)^2 + 7*f*tan(f*x)*tan(e) - f)
```

maple [A] time = 0.03, size = 120, normalized size = 1.50

$$\frac{b\left(\tan^7\left(fx+e\right)\right)}{7f}+\frac{a\left(\tan^5\left(fx+e\right)\right)}{5f}-\frac{b\left(\tan^5\left(fx+e\right)\right)}{5f}-\frac{a\left(\tan^3\left(fx+e\right)\right)}{3f}+\frac{b\left(\tan^3\left(fx+e\right)\right)}{3f}+\frac{a\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/7*b*tan(f*x+e)^7/f+1/5/f*a*tan(f*x+e)^5-1/5*b*tan(f*x+e)^5/f-1/3/f*a*tan(
f*x+e)^3+1/3*b*tan(f*x+e)^3/f+1/f*a*tan(f*x+e)-b*tan(f*x+e)/f-1/f*arctan(ta
n(f*x+e))*a+1/f*arctan(tan(f*x+e))*b
```

maxima [A] time = 0.92, size = 72, normalized size = 0.90

$$\frac{15\,b\tan\left(fx+e\right)^7+21\,(a-b)\tan\left(fx+e\right)^5-35\,(a-b)\tan\left(fx+e\right)^3-105\left(fx+e\right)(a-b)+105\,(a-b)\tan\left(\right.}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*(a - b)*tan(f*x + e)^5 - 35*(a - b)*tan(f*x
 + e)^3 - 105*(f*x + e)*(a - b) + 105*(a - b)*tan(f*x + e))/f
```

mupad [B] time = 11.57, size = 70, normalized size = 0.88

$$\frac{\frac{b\tan\left(e+f\,x\right)^7}{7}+\left(\frac{a}{5}-\frac{b}{5}\right)\,\tan\left(e+f\,x\right)^5+\left(\frac{b}{3}-\frac{a}{3}\right)\,\tan\left(e+f\,x\right)^3+(a-b)\,\tan\left(e+f\,x\right)-f\,x\,(a-b)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (tan(e + f*x)^5*(a/5 - b/5) - tan(e + f*x)^3*(a/3 - b/3) + tan(e + f*x)*(a
- b) + (b*tan(e + f*x)^7)/7 - f*x*(a - b))/f
```

sympy [A] time = 0.96, size = 109, normalized size = 1.36

$$\begin{cases} -ax+\frac{a\tan^5\left(e+fx\right)}{5f}-\frac{a\tan^3\left(e+fx\right)}{3f}+\frac{a\tan\left(e+fx\right)}{f}+bx+\frac{b\tan^7\left(e+fx\right)}{7f}-\frac{b\tan^5\left(e+fx\right)}{5f}+\frac{b\tan^3\left(e+fx\right)}{3f}-\frac{b\tan\left(e+fx\right)}{f} & \text{for } f \\ x\left(a+b\tan^2\left(e\right)\right)\tan^6\left(e\right) & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((-a*x + a*tan(e + f*x)**5/(5*f) - a*tan(e + f*x)**3/(3*f) + a*tan
(e + f*x)/f + b*x + b*tan(e + f*x)**7/(7*f) - b*tan(e + f*x)**5/(5*f) + b*t
an(e + f*x)**3/(3*f) - b*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)*ta
n(e)**6, True))
```

3.192 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=60

$$\frac{(a-b)\tan^3(e+fx)}{3f}-\frac{(a-b)\tan(e+fx)}{f}+x(a-b)+\frac{b\tan^5(e+fx)}{5f}$$

```
[Out] (a-b)*x-(a-b)*tan(f*x+e)/f+1/3*(a-b)*tan(f*x+e)^3/f+1/5*b*tan(f*x+e)^5/f
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3631, 3473, 8}

$$\frac{(a-b)\tan^3(e+fx)}{3f}-\frac{(a-b)\tan(e+fx)}{f}+x(a-b)+\frac{b\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (a - b)*x - ((a - b)*Tan[e + f*x])/f + ((a - b)*Tan[e + f*x]^3)/(3*f) + (b*
Tan[e + f*x]^5)/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)dx &= \frac{b\tan^5(e+fx)}{5f}+(a-b)\int \tan^4(e+fx)\,dx\\ &= \frac{(a-b)\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}+(-a+b)\int \tan^2(e+fx)\,dx\\ &= -\frac{(a-b)\tan(e+fx)}{f}+\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}+(a-b)\\ &= (a-b)x-\frac{(a-b)\tan(e+fx)}{f}+\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}\end{aligned}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 1.62

$$\frac{a\tan^{-1}(\tan(e+fx))}{f}+\frac{a\tan^3(e+fx)}{3f}-\frac{a\tan(e+fx)}{f}-\frac{b\tan^{-1}(\tan(e+fx))}{f}+\frac{b\tan^5(e+fx)}{5f}-\frac{b\tan^3(e+fx)}{3f}+\frac{b\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (a*ArcTan[Tan[e + f*x]])/f - (b*ArcTan[Tan[e + f*x]])/f - (a*Tan[e + f*x])/
f + (b*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f) - (b*Tan[e + f*x]^3)/(3*f
) + (b*Tan[e + f*x]^5)/(5*f)
```

fricas [A] time = 0.42, size = 54, normalized size = 0.90

$$\frac{3\,b\tan\left(fx+e\right)^{5}+5\,(a-b)\tan\left(fx+e\right)^{3}+15\,(a-b)fx-15\,(a-b)\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*(a - b)*tan(f*x + e)^3 + 15*(a - b)*f*x - 15*(
a - b)*tan(f*x + e))/f
```

giac [B] time = 20.50, size = 633, normalized size = 10.55

$$\frac{15\,afx\tan\left(fx\right)^{5}\tan(e)^{5}-15\,bfx\tan\left(fx\right)^{5}\tan(e)^{5}-75\,afx\tan\left(fx\right)^{4}\tan(e)^{4}+75\,bfx\tan\left(fx\right)^{4}\tan(e)^{4}+15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*a*f*x*tan(f*x)^5*tan(e)^5 - 15*b*f*x*tan(f*x)^5*tan(e)^5 - 75*a*f*
x*tan(f*x)^4*tan(e)^4 + 75*b*f*x*tan(f*x)^4*tan(e)^4 + 15*a*tan(f*x)^5*tan(
e)^4 - 15*b*tan(f*x)^5*tan(e)^4 + 15*a*tan(f*x)^4*tan(e)^5 - 15*b*tan(f*x)^
4*tan(e)^5 + 150*a*f*x*tan(f*x)^3*tan(e)^3 - 150*b*f*x*tan(f*x)^3*tan(e)^3
- 5*a*tan(f*x)^5*tan(e)^2 + 5*b*tan(f*x)^5*tan(e)^2 - 75*a*tan(f*x)^4*tan(e
)^3 + 75*b*tan(f*x)^4*tan(e)^3 - 75*a*tan(f*x)^3*tan(e)^4 + 75*b*tan(f*x)^3
*tan(e)^4 - 5*a*tan(f*x)^2*tan(e)^5 + 5*b*tan(f*x)^2*tan(e)^5 - 150*a*f*x*t
an(f*x)^2*tan(e)^2 + 150*b*f*x*tan(f*x)^2*tan(e)^2 - 3*b*tan(f*x)^5 + 10*a*
tan(f*x)^4*tan(e) - 25*b*tan(f*x)^4*tan(e) + 120*a*tan(f*x)^3*tan(e)^2 - 15
0*b*tan(f*x)^3*tan(e)^2 + 120*a*tan(f*x)^2*tan(e)^3 - 150*b*tan(f*x)^2*tan(
e)^3 + 10*a*tan(f*x)*tan(e)^4 - 25*b*tan(f*x)*tan(e)^4 - 3*b*tan(e)^5 + 75*
a*f*x*tan(f*x)*tan(e) - 75*b*f*x*tan(f*x)*tan(e) - 5*a*tan(f*x)^3 + 5*b*tan
(f*x)^3 - 75*a*tan(f*x)^2*tan(e) + 75*b*tan(f*x)^2*tan(e) - 75*a*tan(f*x)*t
an(e)^2 + 75*b*tan(f*x)*tan(e)^2 - 5*a*tan(e)^3 + 5*b*tan(e)^3 - 15*a*f*x +
 15*b*f*x + 15*a*tan(f*x) - 15*b*tan(f*x) + 15*a*tan(e) - 15*b*tan(e))/(f*t
an(f*x)^5*tan(e)^5 - 5*f*tan(f*x)^4*tan(e)^4 + 10*f*tan(f*x)^3*tan(e)^3 - 1
0*f*tan(f*x)^2*tan(e)^2 + 5*f*tan(f*x)*tan(e) - f)
```

maple [A] time = 0.04, size = 92, normalized size = 1.53

$$\frac{b\left(\tan^{5}\left(fx+e\right)\right)}{5f}+\frac{a\left(\tan^{3}\left(fx+e\right)\right)}{3f}-\frac{b\left(\tan^{3}\left(fx+e\right)\right)}{3f}-\frac{a\tan\left(fx+e\right)}{f}+\frac{b\tan\left(fx+e\right)}{f}+\frac{\arctan\left(\tan\left(fx+e\right)\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/5*b*tan(f*x+e)^5/f+1/3/f*a*tan(f*x+e)^3-1/3*b*tan(f*x+e)^3/f-1/f*a*tan(f*
x+e)+b*tan(f*x+e)/f+1/f*arctan(tan(f*x+e))*a-1/f*arctan(tan(f*x+e))*b
```

maxima [A] time = 1.76, size = 57, normalized size = 0.95

$$\frac{3\,b\tan\left(fx+e\right)^{5}+5\,(a-b)\tan\left(fx+e\right)^{3}+15\left(fx+e\right)(a-b)-15\,(a-b)\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*(a - b)*tan(f*x + e)^3 + 15*(f*x + e)*(a - b)
- 15*(a - b)*tan(f*x + e))/f
```

mupad [B] time = 11.63, size = 53, normalized size = 0.88

$$\frac{\frac{b\tan\left(e+f\,x\right)^{5}}{5}+\left(\frac{a}{3}-\frac{b}{3}\right)\,\tan\left(e+f\,x\right)^{3}+(b-a)\,\tan\left(e+f\,x\right)+f\,x\,(a-b)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (tan(e + f*x)^3*(a/3 - b/3) - tan(e + f*x)*(a - b) + (b*tan(e + f*x)^5)/5 +
 f*x*(a - b))/f
```

sympy [A] time = 0.53, size = 82, normalized size = 1.37

$$\begin{cases} ax+\frac{a\tan^{3}\left(e+fx\right)}{3f}-\frac{a\tan\left(e+fx\right)}{f}-bx+\frac{b\tan^{5}\left(e+fx\right)}{5f}-\frac{b\tan^{3}\left(e+fx\right)}{3f}+\frac{b\tan\left(e+fx\right)}{f} & \text{for}\ f\neq 0 \\ x\left(a+b\tan^{2}\left(e\right)\right)\tan^{4}\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((a*x + a*tan(e + f*x)**3/(3*f) - a*tan(e + f*x)/f - b*x + b*tan(e
 + f*x)**5/(5*f) - b*tan(e + f*x)**3/(3*f) + b*tan(e + f*x)/f, Ne(f, 0)), (
x*(a + b*tan(e)**2)*tan(e)**4, True))
```

3.193 $\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=40

$$\frac{(a-b)\tan(e+fx)}{f}-x(a-b)+\frac{b\tan^3(e+fx)}{3f}$$

```
[Out] -(a-b)*x+(a-b)*tan(f*x+e)/f+1/3*b*tan(f*x+e)^3/f
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3631, 3473, 8}

$$\frac{(a-b)\tan(e+fx)}{f}-x(a-b)+\frac{b\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a - b)*x) + ((a - b)*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3631

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(
m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{b\tan^3(e+fx)}{3f}+(a-b)\int \tan^2(e+fx)\,dx\\ &= \frac{(a-b)\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}+(-a+b)\int 1\,dx\\ &= -(a-b)x+\frac{(a-b)\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.62

$$-\frac{a\tan^{-1}(\tan(e+fx))}{f}+\frac{a\tan(e+fx)}{f}+\frac{b\tan^{-1}(\tan(e+fx))}{f}+\frac{b\tan^3(e+fx)}{3f}-\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*ArcTan[Tan[e + f*x]])/f) + (b*ArcTan[Tan[e + f*x]])/f + (a*Tan[e + f*x
])/f - (b*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*f)
```

fricas [A] time = 0.43, size = 38, normalized size = 0.95

$$\frac{b\tan\left(fx+e\right)^{3}-3\,(a-b)fx+3\,(a-b)\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*tan(f*x + e)^3 - 3*(a - b)*f*x + 3*(a - b)*tan(f*x + e))/f
```

giac [B] time = 2.53, size = 289, normalized size = 7.22

$$-\frac{3\,afx\tan\left(fx\right)^{3}\tan(e)^{3}-3\,bfx\tan\left(fx\right)^{3}\tan(e)^{3}-9\,afx\tan\left(fx\right)^{2}\tan(e)^{2}+9\,bfx\tan\left(fx\right)^{2}\tan(e)^{2}+3\,a\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*f*x*tan(f*x)^3*tan(e)^3 - 3*b*f*x*tan(f*x)^3*tan(e)^3 - 9*a*f*x*t
an(f*x)^2*tan(e)^2 + 9*b*f*x*tan(f*x)^2*tan(e)^2 + 3*a*tan(f*x)^3*tan(e)^2
- 3*b*tan(f*x)^3*tan(e)^2 + 3*a*tan(f*x)^2*tan(e)^3 - 3*b*tan(f*x)^2*tan(e)
^3 + 9*a*f*x*tan(f*x)*tan(e) - 9*b*f*x*tan(f*x)*tan(e) + b*tan(f*x)^3 - 6*a
*tan(f*x)^2*tan(e) + 9*b*tan(f*x)^2*tan(e) - 6*a*tan(f*x)*tan(e)^2 + 9*b*ta
n(f*x)*tan(e)^2 + b*tan(e)^3 - 3*a*f*x + 3*b*f*x + 3*a*tan(f*x) - 3*b*tan(f
*x) + 3*a*tan(e) - 3*b*tan(e))/(f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan(
e)^2 + 3*f*tan(f*x)*tan(e) - f)
```

maple [A] time = 0.04, size = 64, normalized size = 1.60

$$\frac{b\left(\tan^{3}\left(fx+e\right)\right)}{3f}+\frac{a\tan\left(fx+e\right)}{f}-\frac{b\tan\left(fx+e\right)}{f}-\frac{\arctan\left(\tan\left(fx+e\right)\right)a}{f}+\frac{\arctan\left(\tan\left(fx+e\right)\right)b}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/3*b*tan(f*x+e)^3/f+1/f*a*tan(f*x+e)-b*tan(f*x+e)/f-1/f*arctan(tan(f*x+e))
*a+1/f*arctan(tan(f*x+e))*b
```

maxima [A] time = 0.78, size = 41, normalized size = 1.02

$$\frac{b\tan\left(fx+e\right)^{3}-3\left(fx+e\right)(a-b)+3\,(a-b)\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*(a - b) + 3*(a - b)*tan(f*x + e))/f
```

mupad [B] time = 11.54, size = 37, normalized size = 0.92

$$\frac{\frac{b\tan\left(e+f\,x\right)^{3}}{3}+\left(a-b\right)\,\tan\left(e+f\,x\right)-f\,x\,\left(a-b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (tan(e + f*x)*(a - b) + (b*tan(e + f*x)^3)/3 - f*x*(a - b))/f
```

sympy [A] time = 0.29, size = 54, normalized size = 1.35

$$\begin{cases} -ax + \frac{a\tan\left(e+fx\right)}{f} + bx + \frac{b\tan^3\left(e+fx\right)}{3f} - \frac{b\tan\left(e+fx\right)}{f} & \text{for } f \neq 0 \\ x\left(a + b\tan^2\left(e\right)\right)\tan^2\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((-a*x + a*tan(e + f*x)/f + b*x + b*tan(e + f*x)**3/(3*f) - b*tan(
e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)*tan(e)**2, True))
```

3.194 $\int \left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=19

$$ax+\frac{b\tan(e+fx)}{f}-bx$$

```
[Out] a*x-b*x+b*tan(f*x+e)/f
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3473, 8}

$$ax+\frac{b\tan(e+fx)}{f}-bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[e + f*x]^2,x]
```

```
[Out] a*x - b*x + (b*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^2(e+fx)\right)\,dx &= ax+b\int \tan^2(e+fx)\,dx\\ &= ax+\frac{b\tan(e+fx)}{f}-b\int 1\,dx\\ &= ax-bx+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.47

$$ax-\frac{b\tan^{-1}(\tan(e+fx))}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[e + f*x]^2,x]
```

```
[Out] a*x - (b*ArcTan[Tan[e + f*x]])/f + (b*Tan[e + f*x])/f
```

fricas [A] time = 0.40, size = 21, normalized size = 1.11

$$\frac{(a-b)fx+b\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] ((a - b)*f*x + b*tan(f*x + e))/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)b*(-4*f*x*tan(exp(1))*tan(f*x)+4*f*x-pi*sign(2*tan(exp
(1))^2*tan(f*x)+2*tan(exp(1))*tan(f*x)^2-2*tan(exp(1))-2*tan(f*x))*tan(exp(
1))*tan(f*x)+pi*sign(2*tan(exp(1))^2*tan(f*x)+2*tan(exp(1))*tan(f*x)^2-2*ta
n(exp(1))-2*tan(f*x))-pi*tan(exp(1))*tan(f*x)+pi+2*atan((tan(exp(1))*tan(f*
x)-1)/(tan(exp(1))+tan(f*x)))*tan(exp(1))*tan(f*x)-2*atan((tan(exp(1))*tan(
f*x)-1)/(tan(exp(1))+tan(f*x)))+2*atan((tan(exp(1))+tan(f*x))/(tan(exp(1))*
tan(f*x)-1))*tan(exp(1))*tan(f*x)-2*atan((tan(exp(1))+tan(f*x))/(tan(exp(1)
)*tan(f*x)-1))-4*tan(exp(1))-4*tan(f*x))/(4*f*tan(exp(1))*tan(f*x)-4*f)+a*x
```

maple [A] time = 0.03, size = 29, normalized size = 1.53

$$ax + \frac{b\tan\left(fx+e\right)}{f} - \frac{\arctan\left(\tan\left(fx+e\right)\right)b}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(f*x+e)^2,x)
```

```
[Out] a*x+b*tan(f*x+e)/f-1/f*arctan(tan(f*x+e))*b
```

maxima [A] time = 0.79, size = 23, normalized size = 1.21

$$ax - \frac{\left(fx + e - \tan\left(fx+e\right)\right)b}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x - (f*x + e - tan(f*x + e))*b/f
```

mupad [B] time = 11.44, size = 21, normalized size = 1.11

$$\frac{b\tan\left(e+f\,x\right)+f\,x\,\left(a-b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*tan(e + f*x)^2,x)
```

```
[Out] (b*tan(e + f*x) + f*x*(a - b))/f
```

sympy [A] time = 0.14, size = 20, normalized size = 1.05

$$ax + b\left(\begin{cases} -x + \frac{\tan\left(e+fx\right)}{f} & \text{for}\ f \neq 0 \\ x\tan^{2}\left(e\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)**2,x)
```

```
[Out] a*x + b*Piecewise((-x + tan(e + f*x)/f, Ne(f, 0)), (x*tan(e)**2, True))
```

3.195 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=21

$$-(x(a-b))-\frac{a\cot(e+fx)}{f}$$

```
[Out] -(a-b)*x-a*cot(f*x+e)/f
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3629, 8}

$$x(-(a-b))-\frac{a\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a - b)*x) - (a*Cot[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= -\frac{a\cot(e+fx)}{f}+\int(-a+b)\,dx\\ &= -(a-b)x-\frac{a\cot(e+fx)}{f}\end{aligned}$$

Mathematica [C] time = 0.02, size = 34, normalized size = 1.62

$$bx-\frac{a\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] b*x - (a*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^2])/f
```

fricas [A] time = 0.41, size = 29, normalized size = 1.38

$$-\frac{(a-b)fx\tan\left(fx+e\right)+a}{f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -((a - b)*f*x*tan(f*x + e) + a)/(f*tan(f*x + e))
```

giac [B] time = 2.32, size = 46, normalized size = 2.19

$$-\frac{2\left(fx+e\right)(a-b)-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+\frac{a}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(f*x + e)*(a - b) - a*tan(1/2*f*x + 1/2*e) + a/tan(1/2*f*x + 1/2*e)
)/f
```

maple [A] time = 0.51, size = 31, normalized size = 1.48

$$\frac{\left(fx+e\right)b+a\left(-\cot\left(fx+e\right)-fx-e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*((f*x+e)*b+a*(-cot(f*x+e)-f*x-e))
```

maxima [A] time = 0.67, size = 27, normalized size = 1.29

$$-\frac{\left(fx+e\right)(a-b)+\frac{a}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -((f*x + e)*(a - b) + a/tan(f*x + e))/f
```

mupad [B] time = 11.47, size = 21, normalized size = 1.00

$$-x\,\left(a-b\right)-\frac{a\cot\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2),x)
```

```
[Out] - x*(a - b) - (a*cot(e + f*x))/f
```

sympy [A] time = 0.78, size = 46, normalized size = 2.19

$$\begin{cases} \tilde{\infty}ax & \text{for}\ \left(e=0\vee e=-fx\right)\wedge\left(e=-fx\vee f=0\right) \\ x\left(a+b\tan^{2}\left(e\right)\right)\cot^{2}\left(e\right) & \text{for}\ f=0 \\ -ax-\frac{a}{f\tan\left(e+fx\right)}+bx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x))), (
x*(a + b*tan(e)**2)*cot(e)**2, Eq(f, 0)), (-a*x - a/(f*tan(e + f*x)) + b*x,
 True))
```

3.196 $\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)dx$

Optimal. Leaf size=39

$$\frac{(a-b)\cot(e+fx)}{f}+x(a-b)-\frac{a\cot^3(e+fx)}{3f}$$

[Out] (a-b)*x+(a-b)*cot(f*x+e)/f-1/3*a*cot(f*x+e)^3/f

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3629, 12, 3473, 8}

$$\frac{(a-b)\cot(e+fx)}{f}+x(a-b)-\frac{a\cot^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]

[Out] (a - b)*x + ((a - b)*Cot[e + f*x])/f - (a*Cot[e + f*x]^3)/(3*f)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)dx &= -\frac{a\cot^3(e+fx)}{3f}-\int (a-b)\cot^2(e+fx)\,dx\\
&= -\frac{a\cot^3(e+fx)}{3f}-(a-b)\int \cot^2(e+fx)\,dx\\
&= \frac{(a-b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}-(-a+b)\int 1\,dx\\
&= (a-b)x+\frac{(a-b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 65, normalized size = 1.67

$$-\frac{a\cot^3(e+fx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(e+fx)\right)}{3f}-\frac{b\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/3*(a*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2])/f
 - (b*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^2])/f
```

fricas [A] time = 0.42, size = 49, normalized size = 1.26

$$\frac{3\,(a-b)fx\tan\left(fx+e\right)^3+3\,(a-b)\tan\left(fx+e\right)^2-a}{3\,f\tan\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a - b)*f*x*tan(f*x + e)^3 + 3*(a - b)*tan(f*x + e)^2 - a)/(f*tan(f*
x + e)^3)
```

giac [B] time = 2.72, size = 106, normalized size = 2.72

$$\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+24\left(fx+e\right)(a-b)-15\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+12\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+\frac{15\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-12\,b\,\text{ta}}{\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*f*x + 1/2*e)^3 + 24*(f*x + e)*(a - b) - 15*a*tan(1/2*f*x +
1/2*e) + 12*b*tan(1/2*f*x + 1/2*e) + (15*a*tan(1/2*f*x + 1/2*e)^2 - 12*b*ta
n(1/2*f*x + 1/2*e)^2 - a)/tan(1/2*f*x + 1/2*e)^3)/f
```

maple [A] time = 0.59, size = 47, normalized size = 1.21

$$\frac{b\left(-\cot\left(fx+e\right)-fx-e\right)+a\left(-\frac{\left(\cot^3\left(fx+e\right)\right)}{3}+\cot\left(fx+e\right)+fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(b*(-cot(f*x+e)-f*x-e)+a*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e))
```

maxima [A] time = 0.77, size = 46, normalized size = 1.18

$$\frac{3\left(fx+e\right)(a-b)+\frac{3\,(a-b)\tan\left(fx+e\right)^2-a}{\tan\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

[Out] 1/3*(3*(f*x + e)*(a - b) + (3*(a - b)*tan(f*x + e)^2 - a)/tan(f*x + e)^3)/f

mupad [B] time = 11.74, size = 40, normalized size = 1.03

$$x\,(a-b)-\frac{\frac{a}{3}-\tan\left(e+f\,x\right)^2\,(a-b)}{f\,\tan\left(e+f\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2),x)

[Out] x*(a - b) - (a/3 - tan(e + f*x)^2*(a - b))/(f*tan(e + f*x)^3)

sympy [A] time = 1.71, size = 70, normalized size = 1.79

$$\begin{cases} \tilde{\infty}ax & \text{for}\ \left(e=0\vee e=-fx\right)\wedge\left(e=-fx\vee f=0\right) \\ x\left(a+b\tan^2\left(e\right)\right)\cot^4\left(e\right) & \text{for}\ f=0 \\ ax+\frac{a}{f\tan\left(e+fx\right)}-\frac{a}{3f\tan^3\left(e+fx\right)}-bx-\frac{b}{f\tan\left(e+fx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2),x)

[Out] Piecewise((zoo*a*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x))), (x*(a + b*tan(e)**2)*cot(e)**4, Eq(f, 0)), (a*x + a/(f*tan(e + f*x)) - a/(3*f*tan(e + f*x)**3) - b*x - b/(f*tan(e + f*x)), True))

3.197 $\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{(a-b)\cot^3(e+fx)}{3f}-\frac{(a-b)\cot(e+fx)}{f}-x(a-b)-\frac{a\cot^5(e+fx)}{5f}$$

```
[Out] -(a-b)*x-(a-b)*cot(f*x+e)/f+1/3*(a-b)*cot(f*x+e)^3/f-1/5*a*cot(f*x+e)^5/f
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3629, 12, 3473, 8}

$$\frac{(a-b)\cot^3(e+fx)}{3f}-\frac{(a-b)\cot(e+fx)}{f}-x(a-b)-\frac{a\cot^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a - b)*x) - ((a - b)*Cot[e + f*x])/f + ((a - b)*Cot[e + f*x]^3)/(3*f) - (a*Cot[e + f*x]^5)/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3629

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m
 + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)dx = -\frac{a\cot^5(e+fx)}{5f} - \int (a-b)\cot^4(e+fx)\,dx$$
$$= -\frac{a\cot^5(e+fx)}{5f} - (a-b)\int \cot^4(e+fx)\,dx$$
$$= \frac{(a-b)\cot^3(e+fx)}{3f} - \frac{a\cot^5(e+fx)}{5f} - (-a+b)\int \cot^2(e+fx)\,dx$$
$$= -\frac{(a-b)\cot(e+fx)}{f} + \frac{(a-b)\cot^3(e+fx)}{3f} - \frac{a\cot^5(e+fx)}{5f} - (a-b)$$
$$= -(a-b)x - \frac{(a-b)\cot(e+fx)}{f} + \frac{(a-b)\cot^3(e+fx)}{3f} - \frac{a\cot^5(e+fx)}{5f}$$

Mathematica [C] time = 0.05, size = 69, normalized size = 1.13

$$-\frac{a\cot^5(e+fx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(e+fx)\right)}{5f} - \frac{b\cot^3(e+fx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/5*(a*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2])/f
 - (b*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2])/(3*
f)
```

fricas [A] time = 0.40, size = 64, normalized size = 1.05

$$-\frac{15\,(a-b)fx\tan\left(fx+e\right)^5 + 15\,(a-b)\tan\left(fx+e\right)^4 - 5\,(a-b)\tan\left(fx+e\right)^2 + 3\,a}{15\,f\tan\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(15*(a - b)*f*x*tan(f*x + e)^5 + 15*(a - b)*tan(f*x + e)^4 - 5*(a - b
)*tan(f*x + e)^2 + 3*a)/(f*tan(f*x + e)^5)
```

giac [B] time = 3.26, size = 168, normalized size = 2.75

$$\frac{3\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5 - 35\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + 20\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 480\left(fx+e\right)(a-b) + 330\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/480*(3*a*tan(1/2*f*x + 1/2*e)^5 - 35*a*tan(1/2*f*x + 1/2*e)^3 + 20*b*tan(
1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*(a - b) + 330*a*tan(1/2*f*x + 1/2*e) - 3
00*b*tan(1/2*f*x + 1/2*e) - (330*a*tan(1/2*f*x + 1/2*e)^4 - 300*b*tan(1/2*f
*x + 1/2*e)^4 - 35*a*tan(1/2*f*x + 1/2*e)^2 + 20*b*tan(1/2*f*x + 1/2*e)^2 +
 3*a)/tan(1/2*f*x + 1/2*e)^5)/f
```

maple [A] time = 0.44, size = 67, normalized size = 1.10

$$\frac{b\left(-\frac{\left(\cot^3\left(fx+e\right)\right)}{3}+\cot\left(fx+e\right)+fx+e\right)+a\left(-\frac{\left(\cot^5\left(fx+e\right)\right)}{5}+\frac{\left(\cot^3\left(fx+e\right)\right)}{3}-\cot\left(fx+e\right)-fx-e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*(b*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)+a*(-1/5*cot(f*x+e)^5+1/3*cot(f*
x+e)^3-cot(f*x+e)-f*x-e))
```

maxima [A] time = 0.79, size = 61, normalized size = 1.00

$$-\frac{15\left(fx+e\right)(a-b)+\frac{15\,(a-b)\tan\left(fx+e\right)^4-5\,(a-b)\tan\left(fx+e\right)^2+3\,a}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(f*x + e)*(a - b) + (15*(a - b)*tan(f*x + e)^4 - 5*(a - b)*tan(f*
x + e)^2 + 3*a)/tan(f*x + e)^5)/f
```

mupad [B] time = 11.94, size = 57, normalized size = 0.93

$$-x\,\left(a-b\right)-\frac{\left(a-b\right)\,\tan\left(e+f\,x\right)^4+\left(\frac{b}{3}-\frac{a}{3}\right)\,\tan\left(e+f\,x\right)^2+\frac{a}{5}}{f\,\tan\left(e+f\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2),x)
```

```
[Out] - x*(a - b) - (a/5 - tan(e + f*x)^2*(a/3 - b/3) + tan(e + f*x)^4*(a - b))/(
f*tan(e + f*x)^5)
```

sympy [A] time = 4.42, size = 97, normalized size = 1.59

$$\begin{cases} \tilde{\infty}ax & \text{for}\ \left(e=0\vee e=-fx\right)\wedge\left(e\right. \\ x\left(a+b\tan^2\left(e\right)\right)\cot^6\left(e\right) & \text{for}\ f=0 \\ -ax-\frac{a}{f\tan\left(e+fx\right)}+\frac{a}{3f\tan^3\left(e+fx\right)}-\frac{a}{5f\tan^5\left(e+fx\right)}+bx+\frac{b}{f\tan\left(e+fx\right)}-\frac{b}{3f\tan^3\left(e+fx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x))), (
x*(a + b*tan(e)**2)*cot(e)**6, Eq(f, 0)), (-a*x - a/(f*tan(e + f*x)) + a/(3
*f*tan(e + f*x)**3) - a/(5*f*tan(e + f*x)**5) + b*x + b/(f*tan(e + f*x)) -
b/(3*f*tan(e + f*x)**3), True))
```

3.198 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=105

$$\frac{b(2a-b)\tan^6(e+fx)}{6f}+\frac{(a-b)^2\tan^4(e+fx)}{4f}-\frac{(a-b)^2\tan^2(e+fx)}{2f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^8(e+fx)}{8f}$$

```
[Out] -(a-b)^2*ln(cos(f*x+e))/f-1/2*(a-b)^2*tan(f*x+e)^2/f+1/4*(a-b)^2*tan(f*x+e)
^4/f+1/6*(2*a-b)*b*tan(f*x+e)^6/f+1/8*b^2*tan(f*x+e)^8/f
```

Rubi [A] time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 446, 88}

$$\frac{b(2a-b)\tan^6(e+fx)}{6f}+\frac{(a-b)^2\tan^4(e+fx)}{4f}-\frac{(a-b)^2\tan^2(e+fx)}{2f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^8(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a - b)^2*Log[Cos[e + f*x]])/f) - ((a - b)^2*Tan[e + f*x]^2)/(2*f) + ((a
 - b)^2*Tan[e + f*x]^4)/(4*f) + ((2*a - b)*b*Tan[e + f*x]^6)/(6*f) + (b^2*T
an[e + f*x]^8)/(8*f)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^5\left(a+bx^2\right)^2}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^2}{1+x} dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(-(a-b)^2+(a-b)^2x+(2a-b)bx^2+b^2x^3+\frac{(a-b)^2}{1+x}\right) dx,$$
$$= -\frac{(a-b)^2\log(\cos(e+fx))}{f} - \frac{(a-b)^2\tan^2(e+fx)}{2f} + \frac{(a-b)^2\tan^4}{4f}$$

Mathematica [A] time = 0.35, size = 89, normalized size = 0.85

$$\frac{4b(2a-b)\tan^6(e+fx)+6(a-b)^2\tan^4(e+fx)-12(a-b)^2\tan^2(e+fx)-24(a-b)^2\log(\cos(e+fx))+3b^2}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-24*(a - b)^2*Log[Cos[e + f*x]] - 12*(a - b)^2*Tan[e + f*x]^2 + 6*(a - b)^
2*Tan[e + f*x]^4 + 4*(2*a - b)*b*Tan[e + f*x]^6 + 3*b^2*Tan[e + f*x]^8)/(24
*f)
```

fricas [A] time = 0.46, size = 107, normalized size = 1.02

$$\frac{3\,b^2\tan\left(fx+e\right)^8+4\left(2\,ab-b^2\right)\tan\left(fx+e\right)^6+6\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^4-12\left(a^2-2\,ab+b^2\right)\tan\left(fx\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^2*tan(f*x + e)^8 + 4*(2*a*b - b^2)*tan(f*x + e)^6 + 6*(a^2 - 2*a*
b + b^2)*tan(f*x + e)^4 - 12*(a^2 - 2*a*b + b^2)*tan(f*x + e)^2 - 12*(a^2 -
 2*a*b + b^2)*log(1/(tan(f*x + e)^2 + 1)))/f
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 198, normalized size = 1.89

$$\frac{b^2\left(\tan^8\left(fx+e\right)\right)}{8f}+\frac{ab\left(\tan^6\left(fx+e\right)\right)}{3f}-\frac{b^2\left(\tan^6\left(fx+e\right)\right)}{6f}+\frac{\left(\tan^4\left(fx+e\right)\right)a^2}{4f}-\frac{\left(\tan^4\left(fx+e\right)\right)ab}{2f}+\frac{b^2\left(\tan^4\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/8*b^2*tan(f*x+e)^8/f+1/3/f*a*b*tan(f*x+e)^6-1/6*b^2*tan(f*x+e)^6/f+1/4/f*
tan(f*x+e)^4*a^2-1/2/f*tan(f*x+e)^4*a*b+1/4/f*b^2*tan(f*x+e)^4-1/2/f*tan(f*
x+e)^2*a^2+1/f*tan(f*x+e)^2*a*b-1/2*b^2*tan(f*x+e)^2/f+1/2/f*ln(1+tan(f*x+e
)^2)*a^2-1/f*ln(1+tan(f*x+e)^2)*a*b+1/2/f*ln(1+tan(f*x+e)^2)*b^2
```

maxima [A] time = 0.56, size = 162, normalized size = 1.54

$$-\frac{12\left(a^2-2\,ab+b^2\right)\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{24\left(a^2-3\,ab+2\,b^2\right)\sin\left(fx+e\right)^6-6\left(11\,a^2-30\,ab+18\,b^2\right)\sin\left(fx+e\right)^4+4\left(15\,a^2-38\,ab+22\,b^2\right)}{\sin\left(fx+e\right)^8-4\,\sin\left(fx+e\right)^6+6\,\sin\left(fx+e\right)^4-4\,\sin\left(fx+e\right)^2+}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(12*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2 - 1) - (24*(a^2 - 3*a*b +
2*b^2)*sin(f*x + e)^6 - 6*(11*a^2 - 30*a*b + 18*b^2)*sin(f*x + e)^4 + 4*(15
*a^2 - 38*a*b + 22*b^2)*sin(f*x + e)^2 - 18*a^2 + 44*a*b - 25*b^2)/(sin(f*x
 + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e)^2 + 1))/f
```

mupad [B] time = 11.59, size = 113, normalized size = 1.08

$$\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)+\tan\left(e+f\,x\right)^6\,\left(\frac{a\,b}{3}-\frac{b^2}{6}\right)+\frac{b^2\,\tan\left(e+f\,x\right)^8}{8}-\tan\left(e+f\,x\right)^2\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)+}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/2) + tan(e + f*x)^6*((a*b)/3 -
b^2/6) + (b^2*tan(e + f*x)^8)/8 - tan(e + f*x)^2*(a^2/2 - a*b + b^2/2) + ta
n(e + f*x)^4*(a^2/4 - (a*b)/2 + b^2/4))/f
```

sympy [A] time = 1.34, size = 206, normalized size = 1.96

$$\begin{cases}\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\tan^4\left(e+fx\right)}{4f}-\frac{a^2\tan^2\left(e+fx\right)}{2f}-\frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f}+\frac{ab\tan^6\left(e+fx\right)}{3f}-\frac{ab\tan^4\left(e+fx\right)}{2f}+\frac{ab\tan^2\left(e+fx\right)}{f}+ \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan^5\left(e\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**4/(4*f)
 - a**2*tan(e + f*x)**2/(2*f) - a*b*log(tan(e + f*x)**2 + 1)/f + a*b*tan(e
+ f*x)**6/(3*f) - a*b*tan(e + f*x)**4/(2*f) + a*b*tan(e + f*x)**2/f + b**2*
log(tan(e + f*x)**2 + 1)/(2*f) + b**2*tan(e + f*x)**8/(8*f) - b**2*tan(e +
f*x)**6/(6*f) + b**2*tan(e + f*x)**4/(4*f) - b**2*tan(e + f*x)**2/(2*f), Ne
(f, 0)), (x*(a + b*tan(e)**2)**2*tan(e)**5, True))
```

3.199 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx$

Optimal. Leaf size=82

$$\frac{b(2a-b)\tan^4(e+fx)}{4f}+\frac{(a-b)^2\tan^2(e+fx)}{2f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^6(e+fx)}{6f}$$

```
[Out] (a-b)^2*ln(cos(f*x+e))/f+1/2*(a-b)^2*tan(f*x+e)^2/f+1/4*(2*a-b)*b*tan(f*x+e
)^4/f+1/6*b^2*tan(f*x+e)^6/f
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 446, 77}

$$\frac{b(2a-b)\tan^4(e+fx)}{4f}+\frac{(a-b)^2\tan^2(e+fx)}{2f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^6(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((a - b)^2*Log[Cos[e + f*x]])/f + ((a - b)^2*Tan[e + f*x]^2)/(2*f) + ((2*a
- b)*b*Tan[e + f*x]^4)/(4*f) + (b^2*Tan[e + f*x]^6)/(6*f)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^3\left(a+bx^2\right)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x(a+bx)^2}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left((a-b)^2+(2a-b)bx+b^2x^2-\frac{(a-b)^2}{1+x}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{(a-b)^2\tan^2(e+fx)}{2f}+\frac{(2a-b)b\tan^4(e-}{4f}$$

Mathematica [A] time = 0.28, size = 72, normalized size = 0.88

$$\frac{3b(2a-b)\tan^4(e+fx)+6(a-b)^2\tan^2(e+fx)+12(a-b)^2\log(\cos(e+fx))+2b^2\tan^6(e+fx)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (12*(a - b)^2*Log[Cos[e + f*x]] + 6*(a - b)^2*Tan[e + f*x]^2 + 3*(2*a - b)*
b*Tan[e + f*x]^4 + 2*b^2*Tan[e + f*x]^6)/(12*f)
```

fricas [A] time = 0.47, size = 86, normalized size = 1.05

$$\frac{2\,b^2\tan\left(fx+e\right)^6+3\left(2\,ab-b^2\right)\tan\left(fx+e\right)^4+6\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^2+6\left(a^2-2\,ab+b^2\right)\log\left(\frac{1}{\tan\left(fx+\right.}\right.}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^2*tan(f*x + e)^6 + 3*(2*a*b - b^2)*tan(f*x + e)^4 + 6*(a^2 - 2*a*
b + b^2)*tan(f*x + e)^2 + 6*(a^2 - 2*a*b + b^2)*log(1/(tan(f*x + e)^2 + 1))
)/f
```

giac [B] time = 33.84, size = 2600, normalized size = 31.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/12*(6*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*ta
n(e)^6 - 12*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^
6*tan(e)^6 + 6*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*
x)^6*tan(e)^6 + 6*a^2*tan(f*x)^6*tan(e)^6 - 18*a*b*tan(f*x)^6*tan(e)^6 + 11
*b^2*tan(f*x)^6*tan(e)^6 - 36*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)
^2 + 1))*tan(f*x)^5*tan(e)^5 + 72*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*
x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(ta
```

n(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 36*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*ta
n(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)
/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 6*a^2*tan(f*x)^6*tan(e)^4 - 12*a*b*t
an(f*x)^6*tan(e)^4 + 6*b^2*tan(f*x)^6*tan(e)^4 - 24*a^2*tan(f*x)^5*tan(e)^5
 + 84*a*b*tan(f*x)^5*tan(e)^5 - 54*b^2*tan(f*x)^5*tan(e)^5 + 6*a^2*tan(f*x)
^4*tan(e)^6 - 12*a*b*tan(f*x)^4*tan(e)^6 + 6*b^2*tan(f*x)^4*tan(e)^6 + 90*a
^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +
 tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 -
180*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e
)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)
^4 + 90*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*ta
n(e)^4 + 6*a*b*tan(f*x)^6*tan(e)^2 - 3*b^2*tan(f*x)^6*tan(e)^2 - 24*a^2*tan
(f*x)^5*tan(e)^3 + 72*a*b*tan(f*x)^5*tan(e)^3 - 36*b^2*tan(f*x)^5*tan(e)^3
+ 42*a^2*tan(f*x)^4*tan(e)^4 - 138*a*b*tan(f*x)^4*tan(e)^4 + 99*b^2*tan(f*x
)^4*tan(e)^4 - 24*a^2*tan(f*x)^3*tan(e)^5 + 72*a*b*tan(f*x)^3*tan(e)^5 - 36
*b^2*tan(f*x)^3*tan(e)^5 + 6*a*b*tan(f*x)^2*tan(e)^6 - 3*b^2*tan(f*x)^2*tan
(e)^6 - 120*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^
3*tan(e)^3 + 240*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan
(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(
f*x)^3*tan(e)^3 - 120*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
*tan(f*x)^3*tan(e)^3 + 2*b^2*tan(f*x)^6 - 12*a*b*tan(f*x)^5*tan(e) + 18*b^2
*tan(f*x)^5*tan(e) + 36*a^2*tan(f*x)^4*tan(e)^2 - 120*a*b*tan(f*x)^4*tan(e)
^2 + 90*b^2*tan(f*x)^4*tan(e)^2 - 48*a^2*tan(f*x)^3*tan(e)^3 + 144*a*b*tan(
f*x)^3*tan(e)^3 - 72*b^2*tan(f*x)^3*tan(e)^3 + 36*a^2*tan(f*x)^2*tan(e)^4 -
 120*a*b*tan(f*x)^2*tan(e)^4 + 90*b^2*tan(f*x)^2*tan(e)^4 - 12*a*b*tan(f*x)
*tan(e)^5 + 18*b^2*tan(f*x)*tan(e)^5 + 2*b^2*tan(e)^6 + 90*a^2*log(4*(tan(f
*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2
*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 180*a*b*log(4*(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 90*b^2*log
(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f
*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 6*a*b*
tan(f*x)^4 - 3*b^2*tan(f*x)^4 - 24*a^2*tan(f*x)^3*tan(e) + 72*a*b*tan(f*x)^
3*tan(e) - 36*b^2*tan(f*x)^3*tan(e) + 42*a^2*tan(f*x)^2*tan(e)^2 - 138*a*b*
tan(f*x)^2*tan(e)^2 + 99*b^2*tan(f*x)^2*tan(e)^2 - 24*a^2*tan(f*x)*tan(e)^3
 + 72*a*b*tan(f*x)*tan(e)^3 - 36*b^2*tan(f*x)*tan(e)^3 + 6*a*b*tan(e)^4 - 3
*b^2*tan(e)^4 - 36*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*ta
n(f*x)*tan(e) + 72*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*ta
n(f*x)*tan(e) - 36*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*ta
n(f*x)*tan(e) + 6*a^2*tan(f*x)^2 - 12*a*b*tan(f*x)^2 + 6*b^2*tan(f*x)^2 - 2
4*a^2*tan(f*x)*tan(e) + 84*a*b*tan(f*x)*tan(e) - 54*b^2*tan(f*x)*tan(e) + 6
*a^2*tan(e)^2 - 12*a*b*tan(e)^2 + 6*b^2*tan(e)^2 + 6*a^2*log(4*(tan(f*x)^4*
tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f
*x)*tan(e) + 1)/(tan(e)^2 + 1)) - 12*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan
(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/
(tan(e)^2 + 1)) + 6*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) +
 6*a^2 - 18*a*b + 11*b^2)/(f*tan(f*x)^6*tan(e)^6 - 6*f*tan(f*x)^5*tan(e)^5
+ 15*f*tan(f*x)^4*tan(e)^4 - 20*f*tan(f*x)^3*tan(e)^3 + 15*f*tan(f*x)^2*tan
(e)^2 - 6*f*tan(f*x)*tan(e) + f)

maple [A] time = 0.04, size = 151, normalized size = 1.84

$$\frac{b^2\left(\tan^6\left(fx+e\right)\right)}{6f}+\frac{\left(\tan^4\left(fx+e\right)\right)ab}{2f}-\frac{b^2\left(\tan^4\left(fx+e\right)\right)}{4f}+\frac{\left(\tan^2\left(fx+e\right)\right)a^2}{2f}-\frac{\left(\tan^2\left(fx+e\right)\right)ab}{f}+\frac{b^2\left(\tan^2\left(fx\right.\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/6*b^2*tan(f*x+e)^6/f+1/2/f*tan(f*x+e)^4*a*b-1/4/f*b^2*tan(f*x+e)^4+1/2/f*
tan(f*x+e)^2*a^2-1/f*tan(f*x+e)^2*a*b+1/2*b^2*tan(f*x+e)^2/f-1/2/f*ln(1+tan
(f*x+e)^2)*a^2+1/f*ln(1+tan(f*x+e)^2)*a*b-1/2/f*ln(1+tan(f*x+e)^2)*b^2
```

maxima [A] time = 0.78, size = 127, normalized size = 1.55

$$\frac{6\left(a^2-2\,ab+b^2\right)\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{6\left(a^2-4\,ab+3\,b^2\right)\sin\left(fx+e\right)^4-3\left(4\,a^2-14\,ab+9\,b^2\right)\sin\left(fx+e\right)^2+6\,a^2-18\,ab+11\,b^2}{\sin\left(fx+e\right)^6-3\,\sin\left(fx+e\right)^4+3\,\sin\left(fx+e\right)^2-1}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2 - 1) - (6*(a^2 - 4*a*b + 3*b
^2)*sin(f*x + e)^4 - 3*(4*a^2 - 14*a*b + 9*b^2)*sin(f*x + e)^2 + 6*a^2 - 18
*a*b + 11*b^2)/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1))/
f
```

mupad [B] time = 11.52, size = 97, normalized size = 1.18

$$\frac{\tan\left(e+f\,x\right)^4\,\left(\frac{a\,b}{2}-\frac{b^2}{4}\right)}{f}-\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f}+\frac{b^2\tan\left(e+f\,x\right)^6}{6\,f}+\frac{\tan\left(e+f\,x\right)^2\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)^4*((a*b)/2 - b^2/4))/f - (log(tan(e + f*x)^2 + 1)*(a^2/2 - a*
b + b^2/2))/f + (b^2*tan(e + f*x)^6)/(6*f) + (tan(e + f*x)^2*(a^2/2 - a*b +
 b^2/2))/f
```

sympy [A] time = 0.79, size = 160, normalized size = 1.95

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\tan^2\left(e+fx\right)}{2f}+\frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f}+\frac{ab\tan^4\left(e+fx\right)}{2f}-\frac{ab\tan^2\left(e+fx\right)}{f}-\frac{b^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{b^2\tan^6}{6} \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan^3\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((-a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**2/(2*f
) + a*b*log(tan(e + f*x)**2 + 1)/f + a*b*tan(e + f*x)**4/(2*f) - a*b*tan(e
+ f*x)**2/f - b**2*log(tan(e + f*x)**2 + 1)/(2*f) + b**2*tan(e + f*x)**6/(6
*f) - b**2*tan(e + f*x)**4/(4*f) + b**2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (
x*(a + b*tan(e)**2)**2*tan(e)**3, True))
```

3.200 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=62

$$\frac{b(a-b)\tan^2(e+fx)}{2f}+\frac{\left(a+b\tan^2(e+fx)\right)^2}{4f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

```
[Out] -(a-b)^2*ln(cos(f*x+e))/f+1/2*(a-b)*b*tan(f*x+e)^2/f+1/4*(a+b*tan(f*x+e)^2)
^2/f
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3670, 444, 43}

$$\frac{b(a-b)\tan^2(e+fx)}{2f}+\frac{\left(a+b\tan^2(e+fx)\right)^2}{4f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a - b)^2*Log[Cos[e + f*x]])/f) + ((a - b)*b*Tan[e + f*x]^2)/(2*f) + (a
+ b*Tan[e + f*x]^2)^2/(4*f)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x(a+bx^2)^2}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{1+x}dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left((a-b)b+\frac{(a-b)^2}{1+x}+b(a+bx)\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{(a-b)b\tan^2(e+fx)}{2f}+\frac{\left(a+b\tan^2(e+f\right.}{4f}$$

Mathematica [A] time = 0.22, size = 54, normalized size = 0.87

$$\frac{2b(2a-b)\tan^2(e+fx)-4(a-b)^2\log(\cos(e+fx))+b^2\tan^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-4*(a - b)^2*Log[Cos[e + f*x]] + 2*(2*a - b)*b*Tan[e + f*x]^2 + b^2*Tan[e
+ f*x]^4)/(4*f)
```

fricas [A] time = 0.42, size = 64, normalized size = 1.03

$$\frac{b^2\tan\left(fx+e\right)^4+2\left(2ab-b^2\right)\tan\left(fx+e\right)^2-2\left(a^2-2ab+b^2\right)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*tan(f*x + e)^4 + 2*(2*a*b - b^2)*tan(f*x + e)^2 - 2*(a^2 - 2*a*b +
 b^2)*log(1/(tan(f*x + e)^2 + 1)))/f
```

giac [B] time = 9.91, size = 1510, normalized size = 24.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*ta
n(e)^4 - 4*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^
2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4
*tan(e)^4 + 2*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*
x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x
)^4*tan(e)^4 - 4*a*b*tan(f*x)^4*tan(e)^4 + 3*b^2*tan(f*x)^4*tan(e)^4 - 8*a^
2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +
tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 1
6*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^
2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3
 - 8*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(
```

```
e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e
)^3 - 4*a*b*tan(f*x)^4*tan(e)^2 + 2*b^2*tan(f*x)^4*tan(e)^2 + 8*a*b*tan(f*x
)^3*tan(e)^3 - 8*b^2*tan(f*x)^3*tan(e)^3 - 4*a*b*tan(f*x)^2*tan(e)^4 + 2*b^
2*tan(f*x)^2*tan(e)^4 + 12*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*ta
n(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
+ 1))*tan(f*x)^2*tan(e)^2 - 24*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^
3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e
)^2 + 1))*tan(f*x)^2*tan(e)^2 + 12*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f
*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(t
an(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - b^2*tan(f*x)^4 + 8*a*b*tan(f*x)^3*tan(e
) - 8*b^2*tan(f*x)^3*tan(e) - 8*a*b*tan(f*x)^2*tan(e)^2 + 4*b^2*tan(f*x)^2*
tan(e)^2 + 8*a*b*tan(f*x)*tan(e)^3 - 8*b^2*tan(f*x)*tan(e)^3 - b^2*tan(e)^4
 - 8*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(
e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e)
+ 16*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(
e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e)
- 8*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e
)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) -
 4*a*b*tan(f*x)^2 + 2*b^2*tan(f*x)^2 + 8*a*b*tan(f*x)*tan(e) - 8*b^2*tan(f*
x)*tan(e) - 4*a*b*tan(e)^2 + 2*b^2*tan(e)^2 + 2*a^2*log(4*(tan(f*x)^4*tan(e
)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*t
an(e) + 1)/(tan(e)^2 + 1)) - 4*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^
3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e
)^2 + 1)) + 2*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*
x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) - 4*a*b
 + 3*b^2)/(f*tan(f*x)^4*tan(e)^4 - 4*f*tan(f*x)^3*tan(e)^3 + 6*f*tan(f*x)^2
*tan(e)^2 - 4*f*tan(f*x)*tan(e) + f)
```

maple [A] time = 0.03, size = 104, normalized size = 1.68

$$\frac{b^2\left(\tan^4\left(fx+e\right)\right)}{4f}+\frac{\left(\tan^2\left(fx+e\right)\right)ab}{f}-\frac{b^2\left(\tan^2\left(fx+e\right)\right)}{2f}+\frac{\ln\left(1+\tan^2\left(fx+e\right)\right)a^2}{2f}-\frac{\ln\left(1+\tan^2\left(fx+e\right)\right)a}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/4/f*b^2*tan(f*x+e)^4+1/f*tan(f*x+e)^2*a*b-1/2*b^2*tan(f*x+e)^2/f+1/2/f*ln
(1+tan(f*x+e)^2)*a^2-1/f*ln(1+tan(f*x+e)^2)*a*b+1/2/f*ln(1+tan(f*x+e)^2)*b^
2
```

maxima [A] time = 0.56, size = 82, normalized size = 1.32

$$-\frac{2\left(a^2-2\,ab+b^2\right)\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{4\left(ab-b^2\right)\sin\left(fx+e\right)^2-4\,ab+3\,b^2}{\sin\left(fx+e\right)^4-2\,\sin\left(fx+e\right)^2+1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2 - 1) + (4*(a*b - b^2)*sin(f*
x + e)^2 - 4*a*b + 3*b^2)/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

mupad [B] time = 11.89, size = 68, normalized size = 1.10

$$\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f}+\frac{\tan\left(e+f\,x\right)^2\,\left(a\,b-\frac{b^2}{2}\right)}{f}+\frac{b^2\,\tan\left(e+f\,x\right)^4}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/2))/f + (tan(e + f*x)^2*(a*b -
b^2/2))/f + (b^2*tan(e + f*x)^4)/(4*f)
```

sympy [**A**] time = 0.42, size = 112, normalized size = 1.81

$$\begin{cases} \frac{a^2 \log\left(\tan^2\left(e+fx\right)+1\right)}{2f} - \frac{ab \log\left(\tan^2\left(e+fx\right)+1\right)}{f} + \frac{ab \tan^2\left(e+fx\right)}{f} + \frac{b^2 \log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{b^2 \tan^4\left(e+fx\right)}{4f} - \frac{b^2 \tan^2\left(e+fx\right)}{2f} & \text{for } f \neq \\ x\left(a+b\tan^2\left(e\right)\right)^2 \tan\left(e\right) & \text{otherv} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) - a*b*log(tan(e + f*x)**2 +
1)/f + a*b*tan(e + f*x)**2/f + b**2*log(tan(e + f*x)**2 + 1)/(2*f) + b**2*t
an(e + f*x)**4/(4*f) - b**2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan
(e)**2)**2*tan(e), True))
```

3.201 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=51

$$\frac{a^2\log(\tan(e+fx))}{f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^2(e+fx)}{2f}$$

```
[Out] (a-b)^2*ln(cos(f*x+e))/f+a^2*ln(tan(f*x+e))/f+1/2*b^2*tan(f*x+e)^2/f
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 446, 72}

$$\frac{a^2\log(\tan(e+fx))}{f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((a - b)^2*Log[Cos[e + f*x]])/f + (a^2*Log[Tan[e + f*x]])/f + (b^2*Tan[e +
f*x]^2)/(2*f)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x(1+x)}\,dx,x,\tan^2(e+fx)\right)}{2f}\\ &= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2}{x}-\frac{(a-b)^2}{1+x}\right)dx,x,\tan^2(e+fx)\right)}{2f}\\ &= \frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{a^2\log(\tan(e+fx))}{f}+\frac{b^2\tan^2(e+fx)}{2f}\end{aligned}$$

Mathematica [A] time = 0.13, size = 65, normalized size = 1.27

$$\frac{a^2(\log(\tan(e+fx))+\log(\cos(e+fx)))}{f}-\frac{2ab\log(\cos(e+fx))}{f}+\frac{b^2\left(\tan^2(e+fx)+2\log(\cos(e+fx))\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-2*a*b*Log[Cos[e + f*x]])/f + (a^2*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]])
)/f + (b^2*(2*Log[Cos[e + f*x]] + Tan[e + f*x]^2))/(2*f)
```

fricas [A] time = 0.42, size = 69, normalized size = 1.35

$$\frac{b^2\tan\left(fx+e\right)^2+a^2\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)-\left(2\,ab-b^2\right)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*tan(f*x + e)^2 + a^2*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) - (2
*a*b - b^2)*log(1/(tan(f*x + e)^2 + 1)))/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)1/f*(a^2/2*ln(sin(f*x+exp(1))^2)+(b^2-2*b*a)/2*ln(abs(sin(f*x+exp(1
))^2-1))+(-sin(f*x+exp(1))^2*b^2+2*sin(f*x+exp(1))^2*b*a-2*b*a)/2/(sin(f*x+
exp(1))^2-1))
```

maple [A] time = 0.64, size = 60, normalized size = 1.18

$$\frac{b^2\left(\tan^2\left(fx+e\right)\right)}{2f}+\frac{b^2\ln\left(\cos\left(fx+e\right)\right)}{f}-\frac{2ab\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{a^2\ln\left(\sin\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/2*b^2*tan(f*x+e)^2/f+1/f*b^2*ln(cos(f*x+e))-2/f*a*b*ln(cos(f*x+e))+1/f*a^
2*ln(sin(f*x+e))
```

maxima [A] time = 0.60, size = 59, normalized size = 1.16

$$\frac{a^2\log\left(\sin\left(fx+e\right)^2\right)-\left(2\,ab-b^2\right)\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{b^2}{\sin\left(fx+e\right)^2-1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*log(sin(f*x + e)^2) - (2*a*b - b^2)*log(sin(f*x + e)^2 - 1) - b^2/
(sin(f*x + e)^2 - 1))/f
```

mupad [B] time = 11.95, size = 62, normalized size = 1.22

$$\frac{b^2\tan\left(e+f\,x\right)^2}{2\,f}-\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f}+\frac{a^2\,\ln\left(\tan\left(e+f\,x\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (b^2*tan(e + f*x)^2)/(2*f) - (log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/2)
)/f + (a^2*log(tan(e + f*x)))/f
```

sympy [A] time = 1.14, size = 97, normalized size = 1.90

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\log\left(\tan\left(e+fx\right)\right)}{f}+\frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f}-\frac{b^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{b^2\tan^2\left(e+fx\right)}{2f} & \text{for}\ f\neq 0 \\ x\left(a+b\tan^2\left(e\right)\right)^2\cot\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((-a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*log(tan(e + f*x))/f
+ a*b*log(tan(e + f*x)**2 + 1)/f - b**2*log(tan(e + f*x)**2 + 1)/(2*f) + b*
*2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)**2*cot(e), True))
```

3.202 $\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=56

$$-\frac{a^2\cot^2(e+fx)}{2f}-\frac{a(a-2b)\log(\tan(e+fx))}{f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

```
[Out] -1/2*a^2*cot(f*x+e)^2/f-(a-b)^2*ln(cos(f*x+e))/f-a*(a-2*b)*ln(tan(f*x+e))/f
```

Rubi [A] time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 446, 88}

$$-\frac{a^2\cot^2(e+fx)}{2f}-\frac{a(a-2b)\log(\tan(e+fx))}{f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(a^2*Cot[e + f*x]^2)/(2*f) - ((a - b)^2*Log[Cos[e + f*x]])/f - (a*(a - 2*b
)*Log[Tan[e + f*x]])/f
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^3\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(1+x)}\,dx,x,\tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^2}-\frac{a(a-2b)}{x}+\frac{(a-b)^2}{1+x}\right)dx,x,\tan^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\cot^2(e+fx)}{2f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}-\frac{a(a-2b)\log(\tan(e}{f}$$

Mathematica [A] time = 0.25, size = 51, normalized size = 0.91

$$-\frac{a^2\cot^2(e+fx)+2a(a-2b)\log(\tan(e+fx))+2(a-b)^2\log(\cos(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(a^2*Cot[e + f*x]^2 + 2*(a - b)^2*Log[Cos[e + f*x]] + 2*a*(a - 2*b)*Lo
g[Tan[e + f*x]])/f
```

fricas [A] time = 0.44, size = 93, normalized size = 1.66

$$-\frac{b^2\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2+a^2\tan\left(fx+e\right)^2+\left(a^2-2\,ab\right)\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2+a^2}{2\,f\tan\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*log(1/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + a^2*tan(f*x + e)^2 +
 (a^2 - 2*a*b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + a^
2)/(f*tan(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-b^2/4*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos
(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))-(-a^2+2*a*b-b^2)/4*ln(abs((1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))+2))-
((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+e
xp(1))))*a^2/16)
```

maple [A] time = 0.74, size = 62, normalized size = 1.11

$$-\frac{b^2\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{2ab\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{a^2\left(\cot^2\left(fx+e\right)\right)}{2f}-\frac{a^2\ln\left(\sin\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/f*b^2*ln(cos(f*x+e))+2/f*a*b*ln(sin(f*x+e))-1/2*a^2*cot(f*x+e)^2/f-1/f*a
^2*ln(sin(f*x+e))
```

maxima [A] time = 0.69, size = 51, normalized size = 0.91

$$-\frac{b^2\log\left(\sin\left(fx+e\right)^2-1\right)+\left(a^2-2\,ab\right)\log\left(\sin\left(fx+e\right)^2\right)+\frac{a^2}{\sin\left(fx+e\right)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*log(sin(f*x + e)^2 - 1) + (a^2 - 2*a*b)*log(sin(f*x + e)^2) + a^2
/sin(f*x + e)^2)/f
```

mupad [B] time = 12.09, size = 68, normalized size = 1.21

$$\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f}+\frac{\ln\left(\tan\left(e+f\,x\right)\right)\left(2\,a\,b-a^2\right)}{f}-\frac{a^2\cot\left(e+f\,x\right)^2}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/2))/f + (log(tan(e + f*x))*(2*a
*b - a^2))/f - (a^2*cot(e + f*x)^2)/(2*f)
```

sympy [A] time = 2.80, size = 129, normalized size = 2.30

$$\begin{cases} \tilde{\infty}a^2x \\ x\left(a+b\tan^2\left(e\right)\right)^2\cot^3\left(e\right) \\ \tilde{\infty}a^2x \\ \frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-\frac{a^2\log\left(\tan\left(e+fx\right)\right)}{f}-\frac{a^2}{2f\tan^2\left(e+fx\right)}-\frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f}+\frac{2ab\log\left(\tan\left(e+fx\right)\right)}{f}+\frac{b^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)**2*cot(e)
**3, Eq(f, 0)), (zoo*a**2*x, Eq(e, -f*x)), (a**2*log(tan(e + f*x)**2 + 1)/(
2*f) - a**2*log(tan(e + f*x))/f - a**2/(2*f*tan(e + f*x)**2) - a*b*log(tan(
e + f*x)**2 + 1)/f + 2*a*b*log(tan(e + f*x))/f + b**2*log(tan(e + f*x)**2 +
 1)/(2*f), True))
```

3.203 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=76

$$-\frac{a^2\cot^4(e+fx)}{4f}+\frac{a(a-2b)\cot^2(e+fx)}{2f}+\frac{(a-b)^2\log(\tan(e+fx))}{f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

```
[Out] 1/2*a*(a-2*b)*cot(f*x+e)^2/f-1/4*a^2*cot(f*x+e)^4/f+(a-b)^2*ln(cos(f*x+e))/
f+(a-b)^2*ln(tan(f*x+e))/f
```

Rubi [A] time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 446, 88}

$$-\frac{a^2\cot^4(e+fx)}{4f}+\frac{a(a-2b)\cot^2(e+fx)}{2f}+\frac{(a-b)^2\log(\tan(e+fx))}{f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a*(a - 2*b)*Cot[e + f*x]^2)/(2*f) - (a^2*Cot[e + f*x]^4)/(4*f) + ((a - b)^
2*Log[Cos[e + f*x]])/f + ((a - b)^2*Log[Tan[e + f*x]])/f
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+bx^2\right)^2}{x^5\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int\frac{(a+bx)^2}{x^3(1+x)}\,dx,x,\tan^2(e+fx)\right)}{2f}$$
$$=\frac{\text{Subst}\left(\int\left(\frac{a^2}{x^3}-\frac{a(a-2b)}{x^2}+\frac{(a-b)^2}{x}-\frac{(a-b)^2}{1+x}\right)dx,x,\tan^2(e+fx)\right)}{2f}$$
$$=\frac{a(a-2b)\cot^2(e+fx)}{2f}-\frac{a^2\cot^4(e+fx)}{4f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

Mathematica [A] time = 0.30, size = 61, normalized size = 0.80

$$\frac{-a^2\cot^4(e+fx)+2a(a-2b)\cot^2(e+fx)+4(a-b)^2(\log(\tan(e+fx))+\log(\cos(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (2*a*(a - 2*b)*Cot[e + f*x]^2 - a^2*Cot[e + f*x]^4 + 4*(a - b)^2*(Log[Cos[e
 + f*x]] + Log[Tan[e + f*x]]))/(4*f)
```

fricas [A] time = 0.44, size = 99, normalized size = 1.30

$$\frac{2\left(a^2-2ab+b^2\right)\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^4+\left(3a^2-4ab\right)\tan\left(fx+e\right)^4+2\left(a^2-2ab\right)\tan\left(fx+e\right)^2-a^2}{4f\tan\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^2 - 2*a*b + b^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x
 + e)^4 + (3*a^2 - 4*a*b)*tan(f*x + e)^4 + 2*(a^2 - 2*a*b)*tan(f*x + e)^2 -
 a^2)/(f*tan(f*x + e)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*((-32*((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1))))^2*a^2+384*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1
)))*a^2-512*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a*b)/4096+(-48*((1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2+96*((1-cos(f*x+exp(1)))/(1+cos(f*x+
exp(1))))^2*a*b-48*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2+12*(1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2-16*(1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1)))*a*b-a^2)*1/128/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2+(-a^2+2*a
*b-b^2)/2*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+(a^2-2*a*b+b^2
)/4*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [A] time = 0.54, size = 91, normalized size = 1.20

$$\frac{b^2\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{ab\left(\cot^2\left(fx+e\right)\right)}{f}-\frac{2ab\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{a^2\left(\cot^4\left(fx+e\right)\right)}{4f}+\frac{a^2\left(\cot^2\left(fx+e\right)\right)}{2f}+\frac{a^2\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*b^2*ln(sin(f*x+e))-1/f*a*b*cot(f*x+e)^2-2/f*a*b*ln(sin(f*x+e))-1/4*a^2*
cot(f*x+e)^4/f+1/2*a^2*cot(f*x+e)^2/f+1/f*a^2*ln(sin(f*x+e))
```

maxima [A] time = 0.32, size = 61, normalized size = 0.80

$$\frac{2\left(a^2-2\,ab+b^2\right)\log\left(\sin\left(fx+e\right)^2\right)+\frac{4\left(a^2-ab\right)\sin\left(fx+e\right)^2-a^2}{\sin\left(fx+e\right)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2) + (4*(a^2 - a*b)*sin(f*x + e
)^2 - a^2)/sin(f*x + e)^4)/f
```

mupad [B] time = 12.18, size = 91, normalized size = 1.20

$$\frac{\ln\left(\tan\left(e+f\,x\right)\right)\,\left(a^2-2\,a\,b+b^2\right)}{f}-\frac{\frac{a^2}{4}+\tan\left(e+f\,x\right)^2\,\left(a\,b-\frac{a^2}{2}\right)}{f\,\tan\left(e+f\,x\right)^4}-\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (log(tan(e + f*x))*(a^2 - 2*a*b + b^2))/f - (a^2/4 + tan(e + f*x)^2*(a*b -
a^2/2))/(f*tan(e + f*x)^4) - (log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/2)
)/f
```

sympy [A] time = 8.54, size = 172, normalized size = 2.26

$$\begin{cases} \tilde{\infty}a^2x \\ x\left(a+b\tan^2\left(e\right)\right)^2\cot^5\left(e\right) \\ \tilde{\infty}a^2x \\ -\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\log\left(\tan\left(e+fx\right)\right)}{f}+\frac{a^2}{2f\tan^2\left(e+fx\right)}-\frac{a^2}{4f\tan^4\left(e+fx\right)}+\frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f}-\frac{2ab\log\left(\tan\left(e+fx\right)\right)}{f}- \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)**2*cot(e)
**5, Eq(f, 0)), (zoo*a**2*x, Eq(e, -f*x)), (-a**2*log(tan(e + f*x)**2 + 1)/
(2*f) + a**2*log(tan(e + f*x))/f + a**2/(2*f*tan(e + f*x)**2) - a**2/(4*f*t
an(e + f*x)**4) + a*b*log(tan(e + f*x)**2 + 1)/f - 2*a*b*log(tan(e + f*x))/
f - a*b/(f*tan(e + f*x)**2) - b**2*log(tan(e + f*x)**2 + 1)/(2*f) + b**2*lo
g(tan(e + f*x))/f, True))
```

3.204 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=113

$$\frac{b(2a-b)\tan^7(e+fx)}{7f}+\frac{(a-b)^2\tan^5(e+fx)}{5f}-\frac{(a-b)^2\tan^3(e+fx)}{3f}+\frac{(a-b)^2\tan(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan^9(e+}{9f}$$

```
[Out] -(a-b)^2*x+(a-b)^2*tan(f*x+e)/f-1/3*(a-b)^2*tan(f*x+e)^3/f+1/5*(a-b)^2*tan(
f*x+e)^5/f+1/7*(2*a-b)*b*tan(f*x+e)^7/f+1/9*b^2*tan(f*x+e)^9/f
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 461, 203}

$$\frac{b(2a-b)\tan^7(e+fx)}{7f}+\frac{(a-b)^2\tan^5(e+fx)}{5f}-\frac{(a-b)^2\tan^3(e+fx)}{3f}+\frac{(a-b)^2\tan(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan^9(e+}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a - b)^2*x) + ((a - b)^2*Tan[e + f*x])/f - ((a - b)^2*Tan[e + f*x]^3)/(3
*f) + ((a - b)^2*Tan[e + f*x]^5)/(5*f) + ((2*a - b)*b*Tan[e + f*x]^7)/(7*f)
 + (b^2*Tan[e + f*x]^9)/(9*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+bx^2\right)^2}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left((a-b)^2-(a-b)^2x^2+(a-b)^2x^4+(2a-b)bx^6+b^2x^8 -\right.\right.}{f}$$
$$= \frac{(a-b)^2\tan(e+fx)}{f} - \frac{(a-b)^2\tan^3(e+fx)}{3f} + \frac{(a-b)^2\tan^5(e+fx}{5f}$$
$$= -(a-b)^2x + \frac{(a-b)^2\tan(e+fx)}{f} - \frac{(a-b)^2\tan^3(e+fx)}{3f} + \frac{(a-b)}{}$$

Mathematica [B] time = 0.08, size = 243, normalized size = 2.15

$$-\frac{a^2\tan^{-1}(\tan(e+fx))}{f}+\frac{a^2\tan^5(e+fx)}{5f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan(e+fx)}{f}+\frac{2ab\tan^{-1}(\tan(e+fx))}{f}+\frac{2ab\tan^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTan[Tan[e + f*x]])/f) + (2*a*b*ArcTan[Tan[e + f*x]])/f - (b^2*Arc
Tan[Tan[e + f*x]])/f + (a^2*Tan[e + f*x])/f - (2*a*b*Tan[e + f*x])/f + (b^2
*Tan[e + f*x])/f - (a^2*Tan[e + f*x]^3)/(3*f) + (2*a*b*Tan[e + f*x]^3)/(3*f
) - (b^2*Tan[e + f*x]^3)/(3*f) + (a^2*Tan[e + f*x]^5)/(5*f) - (2*a*b*Tan[e
+ f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^5)/(5*f) + (2*a*b*Tan[e + f*x]^7)/(7*f)
 - (b^2*Tan[e + f*x]^7)/(7*f) + (b^2*Tan[e + f*x]^9)/(9*f)
```

fricas [A] time = 0.44, size = 115, normalized size = 1.02

$$\frac{35\,b^2\tan\left(fx+e\right)^9+45\left(2\,ab-b^2\right)\tan\left(fx+e\right)^7+63\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^5-105\left(a^2-2\,ab+b^2\right)\tan}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 45*(2*a*b - b^2)*tan(f*x + e)^7 + 63*(a^2 -
2*a*b + b^2)*tan(f*x + e)^5 - 105*(a^2 - 2*a*b + b^2)*tan(f*x + e)^3 - 315*
(a^2 - 2*a*b + b^2)*f*x + 315*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 226, normalized size = 2.00

$$\frac{b^2\left(\tan^9\left(fx+e\right)\right)}{9f}+\frac{2\left(\tan^7\left(fx+e\right)\right)ab}{7f}-\frac{b^2\left(\tan^7\left(fx+e\right)\right)}{7f}+\frac{\left(\tan^5\left(fx+e\right)\right)a^2}{5f}-\frac{2\left(\tan^5\left(fx+e\right)\right)ab}{5f}+\frac{b^2\left(\text{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/9*b^2*tan(f*x+e)^9/f+2/7/f*tan(f*x+e)^7*a*b-1/7*b^2*tan(f*x+e)^7/f+1/5/f*
tan(f*x+e)^5*a^2-2/5/f*tan(f*x+e)^5*a*b+1/5*b^2*tan(f*x+e)^5/f-1/3/f*tan(f*
x+e)^3*a^2+2/3/f*tan(f*x+e)^3*a*b-1/3*b^2*tan(f*x+e)^3/f+1/f*a^2*tan(f*x+e)
-2*a*b*tan(f*x+e)/f+b^2*tan(f*x+e)/f-1/f*arctan(tan(f*x+e))*a^2+2/f*arctan(
tan(f*x+e))*a*b-1/f*arctan(tan(f*x+e))*b^2
```

maxima [A] time = 0.98, size = 118, normalized size = 1.04

$$\frac{35\,b^2\tan\left(fx+e\right)^9+45\left(2\,ab-b^2\right)\tan\left(fx+e\right)^7+63\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^5-105\left(a^2-2\,ab+b^2\right)\tan\left(\right.}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 45*(2*a*b - b^2)*tan(f*x + e)^7 + 63*(a^2 -
2*a*b + b^2)*tan(f*x + e)^5 - 105*(a^2 - 2*a*b + b^2)*tan(f*x + e)^3 - 315*
(a^2 - 2*a*b + b^2)*(f*x + e) + 315*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f
```

mupad [B] time = 12.09, size = 155, normalized size = 1.37

$$\frac{\tan\left(e+f\,x\right)^7\,\left(\frac{2\,a\,b}{7}-\frac{b^2}{7}\right)}{f}-\frac{\mathrm{atan}\left(\frac{\tan\left(e+f\,x\right)\,{\left(a-b\right)}^2}{a^2-2\,a\,b+b^2}\right)\,{\left(a-b\right)}^2}{f}+\frac{\tan\left(e+f\,x\right)\,\left(a^2-2\,a\,b+b^2\right)}{f}+\frac{b^2\,\tan\left(e+f\,x\right)^9}{9\,f}-\frac{\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)^7*((2*a*b)/7 - b^2/7))/f - (atan((tan(e + f*x)*(a - b)^2)/(a^
2 - 2*a*b + b^2))*(a - b)^2)/f + (tan(e + f*x)*(a^2 - 2*a*b + b^2))/f + (b^
2*tan(e + f*x)^9)/(9*f) - (tan(e + f*x)^3*(a^2/3 - (2*a*b)/3 + b^2/3))/f +
(tan(e + f*x)^5*(a^2/5 - (2*a*b)/5 + b^2/5))/f
```

sympy [A] time = 1.87, size = 212, normalized size = 1.88

$$\begin{cases} -a^2x+\frac{a^2\tan^5\left(e+fx\right)}{5f}-\frac{a^2\tan^3\left(e+fx\right)}{3f}+\frac{a^2\tan\left(e+fx\right)}{f}+2abx+\frac{2ab\tan^7\left(e+fx\right)}{7f}-\frac{2ab\tan^5\left(e+fx\right)}{5f}+\frac{2ab\tan^3\left(e+fx\right)}{3f}-\frac{2ab\tan\left(e+\right.}{f} \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan^6\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((-a**2*x + a**2*tan(e + f*x)**5/(5*f) - a**2*tan(e + f*x)**3/(3*f
) + a**2*tan(e + f*x)/f + 2*a*b*x + 2*a*b*tan(e + f*x)**7/(7*f) - 2*a*b*tan
(e + f*x)**5/(5*f) + 2*a*b*tan(e + f*x)**3/(3*f) - 2*a*b*tan(e + f*x)/f - b
**2*x + b**2*tan(e + f*x)**9/(9*f) - b**2*tan(e + f*x)**7/(7*f) + b**2*tan(
e + f*x)**5/(5*f) - b**2*tan(e + f*x)**3/(3*f) + b**2*tan(e + f*x)/f, Ne(f,
 0)), (x*(a + b*tan(e)**2)**2*tan(e)**6, True))
```

3.205 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{b(2a-b)\tan^5(e+fx)}{5f}+\frac{(a-b)^2\tan^3(e+fx)}{3f}-\frac{(a-b)^2\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^7(e+fx)}{7f}$$

```
[Out] (a-b)^2*x-(a-b)^2*tan(f*x+e)/f+1/3*(a-b)^2*tan(f*x+e)^3/f+1/5*(2*a-b)*b*tan
(f*x+e)^5/f+1/7*b^2*tan(f*x+e)^7/f
```

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 461, 203}

$$\frac{b(2a-b)\tan^5(e+fx)}{5f}+\frac{(a-b)^2\tan^3(e+fx)}{3f}-\frac{(a-b)^2\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a - b)^2*x - ((a - b)^2*Tan[e + f*x])/f + ((a - b)^2*Tan[e + f*x]^3)/(3*f)
 + ((2*a - b)*b*Tan[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(-(a-b)^2+(a-b)^2x^2+(2a-b)bx^4+b^2x^6+\frac{a^2-2ab+b^2}{1+x^2}\right) dx\right.}{f}$$
$$= -\frac{(a-b)^2\tan(e+fx)}{f}+\frac{(a-b)^2\tan^3(e+fx)}{3f}+\frac{(2a-b)b\tan^5(e+fx}{5f}$$
$$= (a-b)^2x-\frac{(a-b)^2\tan(e+fx)}{f}+\frac{(a-b)^2\tan^3(e+fx)}{3f}+\frac{(2a-b)b\tan}{5}$$

Mathematica [B] time = 0.08, size = 190, normalized size = 2.09

$$\frac{a^2\tan^{-1}(\tan(e+fx))}{f}+\frac{a^2\tan^3(e+fx)}{3f}-\frac{a^2\tan(e+fx)}{f}-\frac{2ab\tan^{-1}(\tan(e+fx))}{f}+\frac{2ab\tan^5(e+fx)}{5f}-\frac{2ab\tan^3(e+}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a^2*ArcTan[Tan[e + f*x]])/f - (2*a*b*ArcTan[Tan[e + f*x]])/f + (b^2*ArcTan
[Tan[e + f*x]])/f - (a^2*Tan[e + f*x])/f + (2*a*b*Tan[e + f*x])/f - (b^2*Ta
n[e + f*x])/f + (a^2*Tan[e + f*x]^3)/(3*f) - (2*a*b*Tan[e + f*x]^3)/(3*f) +
 (b^2*Tan[e + f*x]^3)/(3*f) + (2*a*b*Tan[e + f*x]^5)/(5*f) - (b^2*Tan[e + f
*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

fricas [A] time = 0.41, size = 94, normalized size = 1.03

$$\frac{15\,b^2\tan\left(fx+e\right)^7+21\left(2\,ab-b^2\right)\tan\left(fx+e\right)^5+35\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^3+105\left(a^2-2\,ab+b^2\right)fx-1}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b - b^2)*tan(f*x + e)^5 + 35*(a^2 -
2*a*b + b^2)*tan(f*x + e)^3 + 105*(a^2 - 2*a*b + b^2)*f*x - 105*(a^2 - 2*a*
b + b^2)*tan(f*x + e))/f
```

giac [B] time = 35.41, size = 1684, normalized size = 18.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/105*(105*a^2*f*x*tan(f*x)^7*tan(e)^7 - 210*a*b*f*x*tan(f*x)^7*tan(e)^7 +
105*b^2*f*x*tan(f*x)^7*tan(e)^7 - 735*a^2*f*x*tan(f*x)^6*tan(e)^6 + 1470*a*
b*f*x*tan(f*x)^6*tan(e)^6 - 735*b^2*f*x*tan(f*x)^6*tan(e)^6 + 105*a^2*tan(f
*x)^7*tan(e)^6 - 210*a*b*tan(f*x)^7*tan(e)^6 + 105*b^2*tan(f*x)^7*tan(e)^6
+ 105*a^2*tan(f*x)^6*tan(e)^7 - 210*a*b*tan(f*x)^6*tan(e)^7 + 105*b^2*tan(f
*x)^6*tan(e)^7 + 2205*a^2*f*x*tan(f*x)^5*tan(e)^5 - 4410*a*b*f*x*tan(f*x)^5
*tan(e)^5 + 2205*b^2*f*x*tan(f*x)^5*tan(e)^5 - 35*a^2*tan(f*x)^7*tan(e)^4 +
 70*a*b*tan(f*x)^7*tan(e)^4 - 35*b^2*tan(f*x)^7*tan(e)^4 - 735*a^2*tan(f*x)
^6*tan(e)^5 + 1470*a*b*tan(f*x)^6*tan(e)^5 - 735*b^2*tan(f*x)^6*tan(e)^5 -
```

```
735*a^2*tan(f*x)^5*tan(e)^6 + 1470*a*b*tan(f*x)^5*tan(e)^6 - 735*b^2*tan(f*
x)^5*tan(e)^6 - 35*a^2*tan(f*x)^4*tan(e)^7 + 70*a*b*tan(f*x)^4*tan(e)^7 - 3
5*b^2*tan(f*x)^4*tan(e)^7 - 3675*a^2*f*x*tan(f*x)^4*tan(e)^4 + 7350*a*b*f*x
*tan(f*x)^4*tan(e)^4 - 3675*b^2*f*x*tan(f*x)^4*tan(e)^4 - 42*a*b*tan(f*x)^7
*tan(e)^2 + 21*b^2*tan(f*x)^7*tan(e)^2 + 140*a^2*tan(f*x)^6*tan(e)^3 - 490*
a*b*tan(f*x)^6*tan(e)^3 + 245*b^2*tan(f*x)^6*tan(e)^3 + 1995*a^2*tan(f*x)^5
*tan(e)^4 - 4410*a*b*tan(f*x)^5*tan(e)^4 + 2205*b^2*tan(f*x)^5*tan(e)^4 + 1
995*a^2*tan(f*x)^4*tan(e)^5 - 4410*a*b*tan(f*x)^4*tan(e)^5 + 2205*b^2*tan(f
*x)^4*tan(e)^5 + 140*a^2*tan(f*x)^3*tan(e)^6 - 490*a*b*tan(f*x)^3*tan(e)^6
+ 245*b^2*tan(f*x)^3*tan(e)^6 - 42*a*b*tan(f*x)^2*tan(e)^7 + 21*b^2*tan(f*x
)^2*tan(e)^7 + 3675*a^2*f*x*tan(f*x)^3*tan(e)^3 - 7350*a*b*f*x*tan(f*x)^3*t
an(e)^3 + 3675*b^2*f*x*tan(f*x)^3*tan(e)^3 - 15*b^2*tan(f*x)^7 + 84*a*b*tan
(f*x)^6*tan(e) - 147*b^2*tan(f*x)^6*tan(e) - 210*a^2*tan(f*x)^5*tan(e)^2 +
840*a*b*tan(f*x)^5*tan(e)^2 - 735*b^2*tan(f*x)^5*tan(e)^2 - 2730*a^2*tan(f*
x)^4*tan(e)^3 + 6300*a*b*tan(f*x)^4*tan(e)^3 - 3675*b^2*tan(f*x)^4*tan(e)^3
 - 2730*a^2*tan(f*x)^3*tan(e)^4 + 6300*a*b*tan(f*x)^3*tan(e)^4 - 3675*b^2*t
an(f*x)^3*tan(e)^4 - 210*a^2*tan(f*x)^2*tan(e)^5 + 840*a*b*tan(f*x)^2*tan(e
)^5 - 735*b^2*tan(f*x)^2*tan(e)^5 + 84*a*b*tan(f*x)*tan(e)^6 - 147*b^2*tan(
f*x)*tan(e)^6 - 15*b^2*tan(e)^7 - 2205*a^2*f*x*tan(f*x)^2*tan(e)^2 + 4410*a
*b*f*x*tan(f*x)^2*tan(e)^2 - 2205*b^2*f*x*tan(f*x)^2*tan(e)^2 - 42*a*b*tan(
f*x)^5 + 21*b^2*tan(f*x)^5 + 140*a^2*tan(f*x)^4*tan(e) - 490*a*b*tan(f*x)^4
*tan(e) + 245*b^2*tan(f*x)^4*tan(e) + 1995*a^2*tan(f*x)^3*tan(e)^2 - 4410*a
*b*tan(f*x)^3*tan(e)^2 + 2205*b^2*tan(f*x)^3*tan(e)^2 + 1995*a^2*tan(f*x)^2
*tan(e)^3 - 4410*a*b*tan(f*x)^2*tan(e)^3 + 2205*b^2*tan(f*x)^2*tan(e)^3 + 1
40*a^2*tan(f*x)*tan(e)^4 - 490*a*b*tan(f*x)*tan(e)^4 + 245*b^2*tan(f*x)*tan
(e)^4 - 42*a*b*tan(e)^5 + 21*b^2*tan(e)^5 + 735*a^2*f*x*tan(f*x)*tan(e) - 1
470*a*b*f*x*tan(f*x)*tan(e) + 735*b^2*f*x*tan(f*x)*tan(e) - 35*a^2*tan(f*x)
^3 + 70*a*b*tan(f*x)^3 - 35*b^2*tan(f*x)^3 - 735*a^2*tan(f*x)^2*tan(e) + 14
70*a*b*tan(f*x)^2*tan(e) - 735*b^2*tan(f*x)^2*tan(e) - 735*a^2*tan(f*x)*tan
(e)^2 + 1470*a*b*tan(f*x)*tan(e)^2 - 735*b^2*tan(f*x)*tan(e)^2 - 35*a^2*tan
(e)^3 + 70*a*b*tan(e)^3 - 35*b^2*tan(e)^3 - 105*a^2*f*x + 210*a*b*f*x - 105
*b^2*f*x + 105*a^2*tan(f*x) - 210*a*b*tan(f*x) + 105*b^2*tan(f*x) + 105*a^2
*tan(e) - 210*a*b*tan(e) + 105*b^2*tan(e))/(f*tan(f*x)^7*tan(e)^7 - 7*f*tan
(f*x)^6*tan(e)^6 + 21*f*tan(f*x)^5*tan(e)^5 - 35*f*tan(f*x)^4*tan(e)^4 + 35
*f*tan(f*x)^3*tan(e)^3 - 21*f*tan(f*x)^2*tan(e)^2 + 7*f*tan(f*x)*tan(e) - f
)
```

maple [B] time = 0.03, size = 179, normalized size = 1.97

$$\frac{b^2\left(\tan^7\left(fx+e\right)\right)}{7f}+\frac{2\left(\tan^5\left(fx+e\right)\right)ab}{5f}-\frac{b^2\left(\tan^5\left(fx+e\right)\right)}{5f}+\frac{\left(\tan^3\left(fx+e\right)\right)a^2}{3f}-\frac{2\left(\tan^3\left(fx+e\right)\right)ab}{3f}+\frac{b^2\left(\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/7*b^2*tan(f*x+e)^7/f+2/5/f*tan(f*x+e)^5*a*b-1/5*b^2*tan(f*x+e)^5/f+1/3/f*
tan(f*x+e)^3*a^2-2/3/f*tan(f*x+e)^3*a*b+1/3*b^2*tan(f*x+e)^3/f-1/f*a^2*tan(
f*x+e)+2*a*b*tan(f*x+e)/f-b^2*tan(f*x+e)/f+1/f*arctan(tan(f*x+e))*a^2-2/f*a
rctan(tan(f*x+e))*a*b+1/f*arctan(tan(f*x+e))*b^2
```

maxima [A] time = 0.65, size = 97, normalized size = 1.07

$$\frac{15\,b^2\tan\left(fx+e\right)^7+21\left(2\,ab-b^2\right)\tan\left(fx+e\right)^5+35\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^3+105\left(a^2-2\,ab+b^2\right)\left(f\right.}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b - b^2)*tan(f*x + e)^5 + 35*(a^2 -
2*a*b + b^2)*tan(f*x + e)^3 + 105*(a^2 - 2*a*b + b^2)*(f*x + e) - 105*(a^2
- 2*a*b + b^2)*tan(f*x + e))/f
```

mupad [B] time = 12.13, size = 127, normalized size = 1.40

$$\frac{\operatorname{atan}\left(\frac{\tan\left(e+f\,x\right)\,\left(a-b\right)^2}{a^2-2\,a\,b+b^2}\right)\,\left(a-b\right)^2}{f}+\frac{\tan\left(e+f\,x\right)^5\,\left(\frac{2\,a\,b}{5}-\frac{b^2}{5}\right)}{f}-\frac{\tan\left(e+f\,x\right)\,\left(a^2-2\,a\,b+b^2\right)}{f}+\frac{b^2\,\tan\left(e+f\,x\right)^7}{7\,f}+\frac{\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (atan((tan(e + f*x)*(a - b)^2)/(a^2 - 2*a*b + b^2))*(a - b)^2)/f + (tan(e +
 f*x)^5*((2*a*b)/5 - b^2/5))/f - (tan(e + f*x)*(a^2 - 2*a*b + b^2))/f + (b^
2*tan(e + f*x)^7)/(7*f) + (tan(e + f*x)^3*(a^2/3 - (2*a*b)/3 + b^2/3))/f
```

sympy [A] time = 1.08, size = 165, normalized size = 1.81

$$\begin{cases} a^2x + \frac{a^2\tan^3\left(e+fx\right)}{3f} - \frac{a^2\tan\left(e+fx\right)}{f} - 2abx + \frac{2ab\tan^5\left(e+fx\right)}{5f} - \frac{2ab\tan^3\left(e+fx\right)}{3f} + \frac{2ab\tan\left(e+fx\right)}{f} + b^2x + \frac{b^2\tan^7\left(e+fx\right)}{7f} - \frac{b^2\tan^5}{5} \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan^4\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*x + a**2*tan(e + f*x)**3/(3*f) - a**2*tan(e + f*x)/f - 2*a*
b*x + 2*a*b*tan(e + f*x)**5/(5*f) - 2*a*b*tan(e + f*x)**3/(3*f) + 2*a*b*tan
(e + f*x)/f + b**2*x + b**2*tan(e + f*x)**7/(7*f) - b**2*tan(e + f*x)**5/(5
*f) + b**2*tan(e + f*x)**3/(3*f) - b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a +
b*tan(e)**2)**2*tan(e)**4, True))
```

3.206 $\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=69

$$\frac{b(2a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)^2\tan(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan^5(e+fx)}{5f}$$

```
[Out] -(a-b)^2*x+(a-b)^2*tan(f*x+e)/f+1/3*(2*a-b)*b*tan(f*x+e)^3/f+1/5*b^2*tan(f*
x+e)^5/f
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 461, 203}

$$\frac{b(2a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)^2\tan(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a - b)^2*x) + ((a - b)^2*Tan[e + f*x])/f + ((2*a - b)*b*Tan[e + f*x]^3)/
(3*f) + (b^2*Tan[e + f*x]^5)/(5*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left((a-b)^2+(2a-b)bx^2+b^2x^4+\frac{-a^2+2ab-b^2}{1+x^2}\right)dx, x, \tan(e+f\right.}{f}$$

$$= \frac{(a-b)^2\tan(e+fx)}{f}+\frac{(2a-b)b\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}-\frac{(a}{}$$

$$= -(a-b)^2x+\frac{(a-b)^2\tan(e+fx)}{f}+\frac{(2a-b)b\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(}{5}$$

Mathematica [A] time = 0.05, size = 137, normalized size = 1.99

$$-\frac{a^2\tan^{-1}(\tan(e+fx))}{f}+\frac{a^2\tan(e+fx)}{f}+\frac{2ab\tan^{-1}(\tan(e+fx))}{f}+\frac{2ab\tan^3(e+fx)}{3f}-\frac{2ab\tan(e+fx)}{f}-\frac{b^2\tan^{-1}(t}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTan[Tan[e + f*x]])/f) + (2*a*b*ArcTan[Tan[e + f*x]])/f - (b^2*Arc
Tan[Tan[e + f*x]])/f + (a^2*Tan[e + f*x])/f - (2*a*b*Tan[e + f*x])/f + (b^2
*Tan[e + f*x])/f + (2*a*b*Tan[e + f*x]^3)/(3*f) - (b^2*Tan[e + f*x]^3)/(3*f
) + (b^2*Tan[e + f*x]^5)/(5*f)
```

fricas [A] time = 0.41, size = 73, normalized size = 1.06

$$\frac{3\,b^2\tan\left(fx+e\right)^5+5\left(2\,ab-b^2\right)\tan\left(fx+e\right)^3-15\left(a^2-2\,ab+b^2\right)fx+15\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 5*(2*a*b - b^2)*tan(f*x + e)^3 - 15*(a^2 - 2*a
*b + b^2)*f*x + 15*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f
```

giac [B] time = 8.87, size = 937, normalized size = 13.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/15*(15*a^2*f*x*tan(f*x)^5*tan(e)^5 - 30*a*b*f*x*tan(f*x)^5*tan(e)^5 + 15
*b^2*f*x*tan(f*x)^5*tan(e)^5 - 75*a^2*f*x*tan(f*x)^4*tan(e)^4 + 150*a*b*f*x
*tan(f*x)^4*tan(e)^4 - 75*b^2*f*x*tan(f*x)^4*tan(e)^4 + 15*a^2*tan(f*x)^5*t
an(e)^4 - 30*a*b*tan(f*x)^5*tan(e)^4 + 15*b^2*tan(f*x)^5*tan(e)^4 + 15*a^2*
tan(f*x)^4*tan(e)^5 - 30*a*b*tan(f*x)^4*tan(e)^5 + 15*b^2*tan(f*x)^4*tan(e)
^5 + 150*a^2*f*x*tan(f*x)^3*tan(e)^3 - 300*a*b*f*x*tan(f*x)^3*tan(e)^3 + 15
0*b^2*f*x*tan(f*x)^3*tan(e)^3 + 10*a*b*tan(f*x)^5*tan(e)^2 - 5*b^2*tan(f*x)
^5*tan(e)^2 - 60*a^2*tan(f*x)^4*tan(e)^3 + 150*a*b*tan(f*x)^4*tan(e)^3 - 75
*b^2*tan(f*x)^4*tan(e)^3 - 60*a^2*tan(f*x)^3*tan(e)^4 + 150*a*b*tan(f*x)^3*
tan(e)^4 - 75*b^2*tan(f*x)^3*tan(e)^4 + 10*a*b*tan(f*x)^2*tan(e)^5 - 5*b^2*
tan(f*x)^2*tan(e)^5 - 150*a^2*f*x*tan(f*x)^2*tan(e)^2 + 300*a*b*f*x*tan(f*x
```

```
)^2*tan(e)^2 - 150*b^2*f*x*tan(f*x)^2*tan(e)^2 + 3*b^2*tan(f*x)^5 - 20*a*b*
tan(f*x)^4*tan(e) + 25*b^2*tan(f*x)^4*tan(e) + 90*a^2*tan(f*x)^3*tan(e)^2 -
 240*a*b*tan(f*x)^3*tan(e)^2 + 150*b^2*tan(f*x)^3*tan(e)^2 + 90*a^2*tan(f*x
)^2*tan(e)^3 - 240*a*b*tan(f*x)^2*tan(e)^3 + 150*b^2*tan(f*x)^2*tan(e)^3 -
20*a*b*tan(f*x)*tan(e)^4 + 25*b^2*tan(f*x)*tan(e)^4 + 3*b^2*tan(e)^5 + 75*a
^2*f*x*tan(f*x)*tan(e) - 150*a*b*f*x*tan(f*x)*tan(e) + 75*b^2*f*x*tan(f*x)*
tan(e) + 10*a*b*tan(f*x)^3 - 5*b^2*tan(f*x)^3 - 60*a^2*tan(f*x)^2*tan(e) +
150*a*b*tan(f*x)^2*tan(e) - 75*b^2*tan(f*x)^2*tan(e) - 60*a^2*tan(f*x)*tan(
e)^2 + 150*a*b*tan(f*x)*tan(e)^2 - 75*b^2*tan(f*x)*tan(e)^2 + 10*a*b*tan(e)
^3 - 5*b^2*tan(e)^3 - 15*a^2*f*x + 30*a*b*f*x - 15*b^2*f*x + 15*a^2*tan(f*x
) - 30*a*b*tan(f*x) + 15*b^2*tan(f*x) + 15*a^2*tan(e) - 30*a*b*tan(e) + 15*
b^2*tan(e))/(f*tan(f*x)^5*tan(e)^5 - 5*f*tan(f*x)^4*tan(e)^4 + 10*f*tan(f*x
)^3*tan(e)^3 - 10*f*tan(f*x)^2*tan(e)^2 + 5*f*tan(f*x)*tan(e) - f)
```

maple [B] time = 0.03, size = 132, normalized size = 1.91

$$\frac{b^2 \left(\tan^5 \left(fx+e\right)\right)}{5f}+\frac{2 \left(\tan^3 \left(fx+e\right)\right) ab}{3f}-\frac{b^2 \left(\tan^3 \left(fx+e\right)\right)}{3f}+\frac{a^2 \tan \left(fx+e\right)}{f}-\frac{2ab \tan \left(fx+e\right)}{f}+\frac{b^2 \tan \left(fx+\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/5*b^2*tan(f*x+e)^5/f+2/3/f*tan(f*x+e)^3*a*b-1/3*b^2*tan(f*x+e)^3/f+1/f*a^
2*tan(f*x+e)-2*a*b*tan(f*x+e)/f+b^2*tan(f*x+e)/f-1/f*arctan(tan(f*x+e))*a^2
+2/f*arctan(tan(f*x+e))*a*b-1/f*arctan(tan(f*x+e))*b^2
```

maxima [A] time = 1.04, size = 76, normalized size = 1.10

$$\frac{3\,b^2 \tan \left(fx+e\right)^5+5\left(2\,ab-b^2\right)\tan \left(fx+e\right)^3-15\left(a^2-2\,ab+b^2\right)\left(fx+e\right)+15\left(a^2-2\,ab+b^2\right)\tan \left(fx+e\right.}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 5*(2*a*b - b^2)*tan(f*x + e)^3 - 15*(a^2 - 2*a
*b + b^2)*(f*x + e) + 15*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f
```

mupad [B] time = 11.67, size = 100, normalized size = 1.45

$$\frac{\tan \left(e+f\,x\right)^3\,\left(\frac{2\,a\,b}{3}-\frac{b^2}{3}\right)}{f}-\frac{\mathrm{atan}\left(\frac{\tan\left(e+f\,x\right)\,{\left(a-b\right)}^2}{a^2-2\,a\,b+b^2}\right)\,{\left(a-b\right)}^2}{f}+\frac{\tan \left(e+f\,x\right)\,\left(a^2-2\,a\,b+b^2\right)}{f}+\frac{b^2\,\tan \left(e+f\,x\right)^5}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)^3*((2*a*b)/3 - b^2/3))/f - (atan((tan(e + f*x)*(a - b)^2)/(a^
2 - 2*a*b + b^2))*(a - b)^2)/f + (tan(e + f*x)*(a^2 - 2*a*b + b^2))/f + (b^
2*tan(e + f*x)^5)/(5*f)
```

sympy [A] time = 0.63, size = 117, normalized size = 1.70

$$\begin{cases} -a^2x+\frac{a^2\tan\left(e+fx\right)}{f}+2abx+\frac{2ab\tan^3\left(e+fx\right)}{3f}-\frac{2ab\tan\left(e+fx\right)}{f}-b^2x+\frac{b^2\tan^5\left(e+fx\right)}{5f}-\frac{b^2\tan^3\left(e+fx\right)}{3f}+\frac{b^2\tan\left(e+fx\right)}{f} & \text{fo} \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan^2\left(e\right) & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((-a**2*x + a**2*tan(e + f*x)/f + 2*a*b*x + 2*a*b*tan(e + f*x)**3/
(3*f) - 2*a*b*tan(e + f*x)/f - b**2*x + b**2*tan(e + f*x)**5/(5*f) - b**2*t
an(e + f*x)**3/(3*f) + b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)
**2*tan(e)**2, True))
```

3.207 $\int \left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=46

$$\frac{b(2a-b)\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] (a-b)^2*x+(2*a-b)*b*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 390, 203}

$$\frac{b(2a-b)\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a - b)^2*x + ((2*a - b)*b*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{(a-b)^2}{1+x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+\right.}{f}\\
&= (a-b)^2x+\frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] time = 0.59, size = 73, normalized size = 1.59

$$\frac{\tan(e+fx)\left(b\left(6a-b\left(3-\tan^2(e+fx)\right)\right)+\frac{3(a-b)^2\tanh^{-1}\left(\sqrt{-\tan^2(e+fx)}\right)}{\sqrt{-\tan^2(e+fx)}}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (Tan[e + f*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Tan[e + f*x]^2]])/Sqrt[-Tan[e + f
*x]^2] + b*(6*a - b*(3 - Tan[e + f*x]^2))))/(3*f)
```

fricas [A] time = 0.42, size = 51, normalized size = 1.11

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(a^2-2\,ab+b^2\right)fx+3\left(2\,ab-b^2\right)\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*x + 3*(2*a*b - b^2)*tan(f
*x + e))/f
```

giac [B] time = 1.84, size = 382, normalized size = 8.30

$$\frac{3\,a^2fx\tan\left(fx\right)^3\tan(e)^3-6\,abfx\tan\left(fx\right)^3\tan(e)^3+3\,b^2fx\tan\left(fx\right)^3\tan(e)^3-9\,a^2fx\tan\left(fx\right)^2\tan(e)^2+18\,ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*f*x*tan(f*x)^3*tan(e)^3 - 6*a*b*f*x*tan(f*x)^3*tan(e)^3 + 3*b^2*
f*x*tan(f*x)^3*tan(e)^3 - 9*a^2*f*x*tan(f*x)^2*tan(e)^2 + 18*a*b*f*x*tan(f*
x)^2*tan(e)^2 - 9*b^2*f*x*tan(f*x)^2*tan(e)^2 - 6*a*b*tan(f*x)^3*tan(e)^2 +
 3*b^2*tan(f*x)^3*tan(e)^2 - 6*a*b*tan(f*x)^2*tan(e)^3 + 3*b^2*tan(f*x)^2*t
an(e)^3 + 9*a^2*f*x*tan(f*x)*tan(e) - 18*a*b*f*x*tan(f*x)*tan(e) + 9*b^2*f*
x*tan(f*x)*tan(e) - b^2*tan(f*x)^3 + 12*a*b*tan(f*x)^2*tan(e) - 9*b^2*tan(f
*x)^2*tan(e) + 12*a*b*tan(f*x)*tan(e)^2 - 9*b^2*tan(f*x)*tan(e)^2 - b^2*tan
(e)^3 - 3*a^2*f*x + 6*a*b*f*x - 3*b^2*f*x - 6*a*b*tan(f*x) + 3*b^2*tan(f*x)
 - 6*a*b*tan(e) + 3*b^2*tan(e))/(f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan
(e)^2 + 3*f*tan(f*x)*tan(e) - f)
```

maple [A] time = 0.03, size = 87, normalized size = 1.89

$$\frac{b^2\left(\tan^3\left(fx+e\right)\right)}{3f}+\frac{2ab\tan\left(fx+e\right)}{f}-\frac{b^2\tan\left(fx+e\right)}{f}+\frac{\arctan\left(\tan\left(fx+e\right)\right)a^2}{f}-\frac{2\arctan\left(\tan\left(fx+e\right)\right)ab}{f}+\frac{\text{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/3*b^2*tan(f*x+e)^3/f+2*a*b*tan(f*x+e)/f-b^2*tan(f*x+e)/f+1/f*arctan(tan(f
*x+e))*a^2-2/f*arctan(tan(f*x+e))*a*b+1/f*arctan(tan(f*x+e))*b^2
```

maxima [A] time = 0.85, size = 58, normalized size = 1.26

$$a^2x-\frac{2\left(fx+e-\tan\left(fx+e\right)\right)ab}{f}+\frac{\left(\tan\left(fx+e\right)^3+3\,fx+3\,e-3\,\tan\left(fx+e\right)\right)b^2}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 2*(f*x + e - tan(f*x + e))*a*b/f + 1/3*(tan(f*x + e)^3 + 3*f*x + 3*
e - 3*tan(f*x + e))*b^2/f
```

mupad [B] time = 12.02, size = 76, normalized size = 1.65

$$\frac{\tan\left(e+f\,x\right)\,\left(2\,a\,b-b^2\right)}{f}+\frac{\mathrm{atan}\left(\frac{\tan\left(e+f\,x\right)\,{\left(a-b\right)}^2}{a^2-2\,a\,b+b^2}\right)\,{\left(a-b\right)}^2}{f}+\frac{b^2\,{\tan\left(e+f\,x\right)}^3}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)*(2*a*b - b^2))/f + (atan((tan(e + f*x)*(a - b)^2)/(a^2 - 2*a*
b + b^2))*(a - b)^2)/f + (b^2*tan(e + f*x)^3)/(3*f)
```

sympy [A] time = 0.32, size = 68, normalized size = 1.48

$$\begin{cases} a^2x - 2abx + \frac{2ab\tan\left(e+fx\right)}{f} + b^2x + \frac{b^2\tan^3\left(e+fx\right)}{3f} - \frac{b^2\tan\left(e+fx\right)}{f} & \text{for}\ f \neq 0 \\ x\left(a + b\tan^2\left(e\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*x + 2*a*b*tan(e + f*x)/f + b**2*x + b**2*tan(e +
f*x)**3/(3*f) - b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)**2, Tr
ue))
```

3.208 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=38

$$-\frac{a^2\cot(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan(e+fx)}{f}$$

```
[Out] -(a-b)^2*x-a^2*cot(f*x+e)/f+b^2*tan(f*x+e)/f
```

Rubi [A] time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 461, 203}

$$-\frac{a^2\cot(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a - b)^2*x) - (a^2*Cot[e + f*x])/f + (b^2*Tan[e + f*x])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^2\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2}{x^2}-\frac{(a-b)^2}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{a^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}-\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e\right.}{f}\\
&= -(a-b)^2x-\frac{a^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [C] time = 0.11, size = 66, normalized size = 1.74

$$-\frac{a^2\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(e+fx)\right)}{f}+2abx-\frac{b^2\tan^{-1}(\tan(e+fx))}{f}+\frac{b^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] 2*a*b*x - (b^2*ArcTan[Tan[e + f*x]])/f - (a^2*Cot[e + f*x]*Hypergeometric2F
1[-1/2, 1, 1/2, -Tan[e + f*x]^2])/f + (b^2*Tan[e + f*x])/f
```

fricas [A] time = 0.41, size = 50, normalized size = 1.32

$$-\frac{\left(a^2-2\,ab+b^2\right)fx\tan\left(fx+e\right)-b^2\tan\left(fx+e\right)^2+a^2}{f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 - 2*a*b + b^2)*f*x*tan(f*x + e) - b^2*tan(f*x + e)^2 + a^2)/(f*tan(f
*x + e))
```

giac [A] time = 3.07, size = 49, normalized size = 1.29

$$\frac{b^2\tan\left(fx+e\right)-\left(a^2-2\,ab+b^2\right)\left(fx+e\right)-\frac{a^2}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] (b^2*tan(f*x + e) - (a^2 - 2*a*b + b^2)*(f*x + e) - a^2/tan(f*x + e))/f
```

maple [A] time = 0.48, size = 53, normalized size = 1.39

$$\frac{b^2\left(\tan\left(fx+e\right)-fx-e\right)+2ab\left(fx+e\right)+a^2\left(-\cot\left(fx+e\right)-fx-e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(b^2*(tan(f*x+e)-f*x-e)+2*a*b*(f*x+e)+a^2*(-cot(f*x+e)-f*x-e))
```

maxima [A] time = 0.56, size = 46, normalized size = 1.21

$$\frac{b^2\tan\left(fx+e\right)-\left(a^2-2\,ab+b^2\right)\left(fx+e\right)-\frac{a^2}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] (b^2*tan(f*x + e) - (a^2 - 2*a*b + b^2)*(f*x + e) - a^2/tan(f*x + e))/f
```

mupad [B] time = 12.02, size = 70, normalized size = 1.84

$$\frac{b^2\tan\left(e+f\,x\right)}{f}-\frac{\mathrm{atan}\left(\frac{\tan\left(e+f\,x\right)\,{\left(a-b\right)}^2}{a^2-2\,a\,b+b^2}\right)\,{\left(a-b\right)}^2}{f}-\frac{a^2}{f\tan\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (b^2*tan(e + f*x))/f - (atan((tan(e + f*x)*(a - b)^2)/(a^2 - 2*a*b + b^2))*
(a - b)^2)/f - a^2/(f*tan(e + f*x))
```

sympy [A] time = 1.70, size = 73, normalized size = 1.92

$$\begin{cases} \tilde{\infty} a^2 x & \text{for } \left(e = 0 \vee e = -fx\right) \wedge \left(e = -fx \vee f = 0\right) \\ x \left(a + b \tan^2(e)\right)^2 \cot^2(e) & \text{for } f = 0 \\ -a^2 x - \frac{a^2}{f \tan(e+fx)} + 2abx - b^2 x + \frac{b^2 \tan(e+fx)}{f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x)))
, (x*(a + b*tan(e)**2)**2*cot(e)**2, Eq(f, 0)), (-a**2*x - a**2/(f*tan(e +
f*x)) + 2*a*b*x - b**2*x + b**2*tan(e + f*x)/f, True))
```

3.209 $\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=44

$$-\frac{a^2\cot^3(e+fx)}{3f}+\frac{a(a-2b)\cot(e+fx)}{f}+x(a-b)^2$$

```
[Out] (a-b)^2*x+a*(a-2*b)*cot(f*x+e)/f-1/3*a^2*cot(f*x+e)^3/f
```

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 461, 203}

$$-\frac{a^2\cot^3(e+fx)}{3f}+\frac{a(a-2b)\cot(e+fx)}{f}+x(a-b)^2$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a - b)^2*x + (a*(a - 2*b)*Cot[e + f*x])/f - (a^2*Cot[e + f*x]^3)/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^4\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^4}-\frac{a(a-2b)}{x^2}+\frac{(a-b)^2}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{a(a-2b)\cot(e+fx)}{f}-\frac{a^2\cot^3(e+fx)}{3f}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,d\right.}{f}\\
&= (a-b)^2x+\frac{a(a-2b)\cot(e+fx)}{f}-\frac{a^2\cot^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] time = 1.31, size = 71, normalized size = 1.61

$$-\frac{\cot(e+fx)\left(a\left(a\cot^2(e+fx)-3a+6b\right)+3(a-b)^2\sqrt{-\tan^2(e+fx)}\,\tanh^{-1}\left(\sqrt{-\tan^2(e+fx)}\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/3*(Cot[e + f*x]*(a*(-3*a + 6*b + a*Cot[e + f*x]^2) + 3*(a - b)^2*ArcTanh
[Sqrt[-Tan[e + f*x]^2]]*Sqrt[-Tan[e + f*x]^2]))/f
```

fricas [A] time = 0.42, size = 60, normalized size = 1.36

$$\frac{3\left(a^2-2\,ab+b^2\right)fx\tan\left(fx+e\right)^3+3\left(a^2-2\,ab\right)\tan\left(fx+e\right)^2-a^2}{3\,f\tan\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^2 - 2*a*b + b^2)*f*x*tan(f*x + e)^3 + 3*(a^2 - 2*a*b)*tan(f*x + e
)^2 - a^2)/(f*tan(f*x + e)^3)
```

giac [B] time = 4.73, size = 122, normalized size = 2.77

$$\frac{a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+24\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+24\left(a^2-2\,ab+b^2\right)\left(fx+e\right)+\frac{15\,a^2\tan\left(\frac{1}{2}\,fx\right.}{}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*f*x + 1/2*e)^3 - 15*a^2*tan(1/2*f*x + 1/2*e) + 24*a*b*tan
(1/2*f*x + 1/2*e) + 24*(a^2 - 2*a*b + b^2)*(f*x + e) + (15*a^2*tan(1/2*f*x
+ 1/2*e)^2 - 24*a*b*tan(1/2*f*x + 1/2*e)^2 - a^2)/tan(1/2*f*x + 1/2*e)^3)/f
```

maple [A] time = 0.53, size = 60, normalized size = 1.36

$$\frac{b^2\left(fx+e\right)+2ab\left(-\cot\left(fx+e\right)-fx-e\right)+a^2\left(-\frac{\left(\cot^3\left(fx+e\right)\right)}{3}+\cot\left(fx+e\right)+fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(b^2*(f*x+e)+2*a*b*(-cot(f*x+e)-f*x-e)+a^2*(-1/3*cot(f*x+e)^3+cot(f*x+e
)+f*x+e))
```

maxima [A] time = 0.78, size = 57, normalized size = 1.30

$$\frac{3\left(a^2-2\,ab+b^2\right)\left(fx+e\right)+\frac{3\left(a^2-2\,ab\right)\tan\left(fx+e\right)^2-a^2}{\tan\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^2 - 2*a*b + b^2)*(f*x + e) + (3*(a^2 - 2*a*b)*tan(f*x + e)^2 - a^
2)/tan(f*x + e)^3)/f
```

mupad [B] time = 11.61, size = 58, normalized size = 1.32

$$a^2\,x+b^2\,x+\frac{a^2\,\cot\left(e+f\,x\right)}{f}-2\,a\,b\,x-\frac{a^2\,{\cot\left(e+f\,x\right)}^3}{3\,f}-\frac{2\,a\,b\,\cot\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] a^2*x + b^2*x + (a^2*cot(e + f*x))/f - 2*a*b*x - (a^2*cot(e + f*x)^3)/(3*f)
 - (2*a*b*cot(e + f*x))/f
```

sympy [A] time = 4.12, size = 90, normalized size = 2.05

$$\begin{cases} \tilde{\infty} a^{2} x & \text{for}\: \left(e = 0 \vee e = - f x\right) \wedge \left(e = - f x \vee f = 0\right) \\ x \left(a + b \tan^{2}{\left(e \right)}\right)^{2} \cot^{4}{\left(e \right)} & \text{for}\: f = 0 \\ a^{2} x + \frac{a^{2}}{f \tan{\left(e + f x \right)}} - \frac{a^{2}}{3 f \tan^{3}{\left(e + f x \right)}} - 2 a b x - \frac{2 a b}{f \tan{\left(e + f x \right)}} + b^{2} x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x)))
, (x*(a + b*tan(e)**2)**2*cot(e)**4, Eq(f, 0)), (a**2*x + a**2/(f*tan(e + f
*x)) - a**2/(3*f*tan(e + f*x)**3) - 2*a*b*x - 2*a*b/(f*tan(e + f*x)) + b**2
*x, True))
```

3.210 $\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=68

$$-\frac{a^2\cot^5(e+fx)}{5f}+\frac{a(a-2b)\cot^3(e+fx)}{3f}-\frac{(a-b)^2\cot(e+fx)}{f}-x(a-b)^2$$

```
[Out] -(a-b)^2*x-(a-b)^2*cot(f*x+e)/f+1/3*a*(a-2*b)*cot(f*x+e)^3/f-1/5*a^2*cot(f*
x+e)^5/f
```

Rubi [A] time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 461, 203}

$$-\frac{a^2\cot^5(e+fx)}{5f}+\frac{a(a-2b)\cot^3(e+fx)}{3f}-\frac{(a-b)^2\cot(e+fx)}{f}-x(a-b)^2$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a - b)^2*x) - ((a - b)^2*Cot[e + f*x])/f + (a*(a - 2*b)*Cot[e + f*x]^3)/
(3*f) - (a^2*Cot[e + f*x]^5)/(5*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^6\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^6}-\frac{a(a-2b)}{x^4}+\frac{(a-b)^2}{x^2}-\frac{(a-b)^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{(a-b)^2\cot(e+fx)}{f}+\frac{a(a-2b)\cot^3(e+fx)}{3f}-\frac{a^2\cot^5(e+fx)}{5f}-$$
$$= -(a-b)^2x-\frac{(a-b)^2\cot(e+fx)}{f}+\frac{a(a-2b)\cot^3(e+fx)}{3f}-\frac{a^2\cot}{}$$

Mathematica [C] time = 0.10, size = 104, normalized size = 1.53

$$-\frac{a^2\cot^5(e+fx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(e+fx)\right)}{5f}-\frac{2ab\cot^3(e+fx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(e+fx)\right)}{3f}-\frac{b^2\cot(e+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/5*(a^2*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2])
/f - (2*a*b*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2
])/(3*f) - (b^2*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^
2])/f
```

fricas [A] time = 0.41, size = 81, normalized size = 1.19

$$-\frac{15\left(a^2-2ab+b^2\right)fx\tan\left(fx+e\right)^5+15\left(a^2-2ab+b^2\right)\tan\left(fx+e\right)^4-5\left(a^2-2ab\right)\tan\left(fx+e\right)^2+3a^2}{15f\tan\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(15*(a^2 - 2*a*b + b^2)*f*x*tan(f*x + e)^5 + 15*(a^2 - 2*a*b + b^2)*t
an(f*x + e)^4 - 5*(a^2 - 2*a*b)*tan(f*x + e)^2 + 3*a^2)/(f*tan(f*x + e)^5)
```

giac [B] time = 7.94, size = 222, normalized size = 3.26

$$3a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-35a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+40ab\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+330a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-600ab\,\mathrm{ta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*f*x + 1/2*e)^5 - 35*a^2*tan(1/2*f*x + 1/2*e)^3 + 40*a*
b*tan(1/2*f*x + 1/2*e)^3 + 330*a^2*tan(1/2*f*x + 1/2*e) - 600*a*b*tan(1/2*f
*x + 1/2*e) + 240*b^2*tan(1/2*f*x + 1/2*e) - 480*(a^2 - 2*a*b + b^2)*(f*x +
 e) - (330*a^2*tan(1/2*f*x + 1/2*e)^4 - 600*a*b*tan(1/2*f*x + 1/2*e)^4 + 24
0*b^2*tan(1/2*f*x + 1/2*e)^4 - 35*a^2*tan(1/2*f*x + 1/2*e)^2 + 40*a*b*tan(1
/2*f*x + 1/2*e)^2 + 3*a^2)/tan(1/2*f*x + 1/2*e)^5)/f
```

maple [A] time = 0.69, size = 91, normalized size = 1.34

$$\frac{b^2\left(-\cot\left(fx+e\right)-fx-e\right)+2ab\left(-\frac{\left(\cot^3\left(fx+e\right)\right)}{3}+\cot\left(fx+e\right)+fx+e\right)+a^2\left(-\frac{\left(\cot^5\left(fx+e\right)\right)}{5}+\frac{\left(\cot^3\left(fx+e\right)\right)}{3}-\cot\left(f\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(b^2*(-cot(f*x+e)-f*x-e)+2*a*b*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)+a^2
*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e))
```

maxima [A] time = 0.80, size = 78, normalized size = 1.15

$$-\frac{15\left(a^2-2\,ab+b^2\right)\left(fx+e\right)+\frac{15\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^4-5\left(a^2-2\,ab\right)\tan\left(fx+e\right)^2+3\,a^2}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/15*(15*(a^2 - 2*a*b + b^2)*(f*x + e) + (15*(a^2 - 2*a*b + b^2)*tan(f*x +
 e)^4 - 5*(a^2 - 2*a*b)*tan(f*x + e)^2 + 3*a^2)/tan(f*x + e)^5)/f
```

mupad [B] time = 11.52, size = 76, normalized size = 1.12

$$2\,a\,b\,x-b^2\,x-\frac{\cot\left(e+f\,x\right)^5\,\left(\tan\left(e+f\,x\right)^4\,\left(a^2-2\,a\,b+b^2\right)+\frac{a^2}{5}+\tan\left(e+f\,x\right)^2\,\left(\frac{2\,a\,b}{3}-\frac{a^2}{3}\right)\right)}{f}-a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] 2*a*b*x - b^2*x - (cot(e + f*x)^5*(tan(e + f*x)^4*(a^2 - 2*a*b + b^2) + a^2
/5 + tan(e + f*x)^2*((2*a*b)/3 - a^2/3)))/f - a^2*x
```

sympy [A] time = 8.46, size = 134, normalized size = 1.97

$$\begin{cases} \tilde{\infty} a^2 x & \text{for } (e = \\ x\left(a+b\tan^2(e)\right)^2\cot^6(e) & \text{for } f = 0 \\ -a^2x-\frac{a^2}{f\tan(e+fx)}+\frac{a^2}{3f\tan^3(e+fx)}-\frac{a^2}{5f\tan^5(e+fx)}+2abx+\frac{2ab}{f\tan(e+fx)}-\frac{2ab}{3f\tan^3(e+fx)}-b^2x-\frac{b^2}{f\tan(e+fx)} & \text{otherwis} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x)))
, (x*(a + b*tan(e)**2)**2*cot(e)**6, Eq(f, 0)), (-a**2*x - a**2/(f*tan(e +
f*x)) + a**2/(3*f*tan(e + f*x)**3) - a**2/(5*f*tan(e + f*x)**5) + 2*a*b*x +
 2*a*b/(f*tan(e + f*x)) - 2*a*b/(3*f*tan(e + f*x)**3) - b**2*x - b**2/(f*ta
n(e + f*x)), True))
```

3.211 $\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=71

$$-\frac{a^2\log\left(a+b\tan^2(e+fx)\right)}{2b^2f(a-b)}-\frac{\log(\cos(e+fx))}{f(a-b)}+\frac{\tan^2(e+fx)}{2bf}$$

```
[Out] -ln(cos(f*x+e))/(a-b)/f-1/2*a^2*ln(a+b*tan(f*x+e)^2)/(a-b)/b^2/f+1/2*tan(f*
x+e)^2/b/f
```

Rubi [A] time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 446, 72}

$$-\frac{a^2\log\left(a+b\tan^2(e+fx)\right)}{2b^2f(a-b)}-\frac{\log(\cos(e+fx))}{f(a-b)}+\frac{\tan^2(e+fx)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(Log[Cos[e + f*x]]/((a - b)*f)) - (a^2*Log[a + b*Tan[e + f*x]^2])/(2*(a -
b)*b^2*f) + Tan[e + f*x]^2/(2*b*f)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b}+\frac{1}{(a-b)(1+x)}-\frac{a^2}{(a-b)b(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{\log(\cos(e+fx))}{(a-b)f} - \frac{a^2\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)b^2f} + \frac{\tan^2(e+fx)}{2bf}$$

Mathematica [A] time = 0.17, size = 64, normalized size = 0.90

$$\frac{-\frac{a^2\log\left(a+b\tan^2(e+fx)\right)}{b^2(a-b)} - \frac{2\log(\cos(e+fx))}{a-b} + \frac{\tan^2(e+fx)}{b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((-2*Log[Cos[e + f*x]])/(a - b) - (a^2*Log[a + b*Tan[e + f*x]^2])/((a - b)*
b^2) + Tan[e + f*x]^2/b)/(2*f)
```

fricas [A] time = 0.48, size = 92, normalized size = 1.30

$$-\frac{a^2\log\left(\frac{b\tan\left(fx+e\right)^2+a}{\tan\left(fx+e\right)^2+1}\right) - \left(ab-b^2\right)\tan\left(fx+e\right)^2 - \left(a^2-b^2\right)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)}{2\left(ab^2-b^3\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)) - (a*b - b^2)*ta
n(f*x + e)^2 - (a^2 - b^2)*log(1/(tan(f*x + e)^2 + 1)))/((a*b^2 - b^3)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)2/f*(-1/(-4*a+4*b)*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f
*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))+2))-a^3/(4*a^2*b^2-4*
a*b^3)*ln(abs(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1))
)*(1+cos(f*x+exp(1))))*a-2*a+4*b))+(a+b)*1/4/b^2*ln(abs((1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))+(-((1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))
))*a-((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(
```

```
f*x+exp(1))))*b+2*a+6*b)*1/4/b^2/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1
/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))
```

maple [A] time = 0.18, size = 72, normalized size = 1.01

$$\frac{\tan^2\left(fx+e\right)}{2bf}-\frac{a^2\ln\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}{2\left(a-b\right)b^2f}+\frac{\ln\left(1+\tan^2\left(fx+e\right)\right)}{2f\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/2*tan(f*x+e)^2/b/f-1/2*a^2*ln(a+b*tan(f*x+e)^2)/(a-b)/b^2/f+1/2/f/(a-b)*l
n(1+tan(f*x+e)^2)
```

maxima [A] time = 0.76, size = 76, normalized size = 1.07

$$-\frac{\frac{a^2\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{ab^2-b^3}-\frac{(a+b)\log\left(\sin\left(fx+e\right)^2-1\right)}{b^2}+\frac{1}{b\sin\left(fx+e\right)^2-b}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*log(-(a - b)*sin(f*x + e)^2 + a)/(a*b^2 - b^3) - (a + b)*log(sin(
f*x + e)^2 - 1)/b^2 + 1/(b*sin(f*x + e)^2 - b))/f
```

mupad [B] time = 11.86, size = 74, normalized size = 1.04

$$\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f\,\left(a-b\right)}+\frac{\tan\left(e+f\,x\right)^2}{2\,b\,f}-\frac{a^2\,\ln\left(b\,\tan\left(e+f\,x\right)^2+a\right)}{2\,f\,\left(a\,b^2-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2),x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*f*(a - b)) + tan(e + f*x)^2/(2*b*f) - (a^2*log(a
 + b*tan(e + f*x)^2))/(2*f*(a*b^2 - b^3))
```

sympy [A] time = 16.36, size = 348, normalized size = 4.90

$$\begin{cases} \tilde{\infty}x\tan^3(e) & \text{for } a = \\ -\frac{2\log\left(\tan^2\left(e+fx\right)+1\right)\tan^2\left(e+fx\right)}{2bf\tan^2\left(e+fx\right)+2bf}-\frac{2\log\left(\tan^2\left(e+fx\right)+1\right)}{2bf\tan^2\left(e+fx\right)+2bf}+\frac{\tan^4\left(e+fx\right)}{2bf\tan^2\left(e+fx\right)+2bf}-\frac{2}{2bf\tan^2\left(e+fx\right)+2bf} & \text{for } a = \\ \frac{\frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{\tan^4\left(e+fx\right)}{4f}-\frac{\tan^2\left(e+fx\right)}{2f}}{a} & \text{for } b = \\ \frac{x\tan^5(e)}{a+b\tan^2(e)} & \text{for } f \\ -\frac{a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan\left(e+fx\right)\right)}{2ab^2f-2b^3f}-\frac{a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tan\left(e+fx\right)\right)}{2ab^2f-2b^3f}+\frac{ab\tan^2\left(e+fx\right)}{2ab^2f-2b^3f}+\frac{b^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2ab^2f-2b^3f}-\frac{b^2\tan^2\left(e+fx\right)}{2ab^2f-2b^3f} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*tan(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-2*log(tan(e
+ f*x)**2 + 1)*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*b*f) - 2*log(tan(
e + f*x)**2 + 1)/(2*b*f*tan(e + f*x)**2 + 2*b*f) + tan(e + f*x)**4/(2*b*f*t
an(e + f*x)**2 + 2*b*f) - 2/(2*b*f*tan(e + f*x)**2 + 2*b*f), Eq(a, b)), ((l
og(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x)**4/(4*f) - tan(e + f*x)**2/(2*
f))/a, Eq(b, 0)), (x*tan(e)**5/(a + b*tan(e)**2), Eq(f, 0)), (-a**2*log(-I*
sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*a*b**2*f - 2*b**3*f) - a**2*log(I*sqrt
(a)*sqrt(1/b) + tan(e + f*x))/(2*a*b**2*f - 2*b**3*f) + a*b*tan(e + f*x)**2
/(2*a*b**2*f - 2*b**3*f) + b**2*log(tan(e + f*x)**2 + 1)/(2*a*b**2*f - 2*b*
*3*f) - b**2*tan(e + f*x)**2/(2*a*b**2*f - 2*b**3*f), True))
```

3.212 $\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{a\log\left(a+b\tan^2(e+fx)\right)}{2bf(a-b)}+\frac{\log(\cos(e+fx))}{f(a-b)}$$

```
[Out] ln(cos(f*x+e))/(a-b)/f+1/2*a*ln(a+b*tan(f*x+e)^2)/(a-b)/b/f
```

Rubi [A] time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 446, 72}

$$\frac{a\log\left(a+b\tan^2(e+fx)\right)}{2bf(a-b)}+\frac{\log(\cos(e+fx))}{f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] Log[Cos[e + f*x]]/((a - b)*f) + (a*Log[a + b*Tan[e + f*x]^2])/(2*(a - b)*b*
f)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a-b)(1+x)} + \frac{a}{(a-b)(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\log(\cos(e+fx))}{(a-b)f} + \frac{a\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)bf}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.82

$$\frac{a\log\left(a+b\tan^2(e+fx)\right)+2b\log(\cos(e+fx))}{2abf-2b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
[Out] (2*b*Log[Cos[e + f*x]] + a*Log[a + b*Tan[e + f*x]^2])/(2*a*b*f - 2*b^2*f)
```

fricas [A] time = 0.45, size = 65, normalized size = 1.30

$$\frac{a\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)-(a-b)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\left(ab-b^2\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
[Out] 1/2*(a*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)) - (a - b)*log(1/(ta
n(f*x + e)^2 + 1)))/((a*b - b^2)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)2/f*(-1/4/b*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))-1/(4*a-4*b)*ln(abs((1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))
)+2))-a^2/(-4*a^2*b+4*a*b^2)*ln(abs(((1-cos(f*x+exp(1))))/(1+cos(f*x+exp(1))
)+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*a-2*a+4*b)))
```

maple [A] time = 0.14, size = 54, normalized size = 1.08

$$\frac{a\ln\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}{2\left(a-b\right)bf}-\frac{\ln\left(1+\tan^2\left(fx+e\right)\right)}{2f\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/2*a*ln(a+b*tan(f*x+e)^2)/(a-b)/b/f-1/2/f/(a-b)*ln(1+tan(f*x+e)^2)
```

maxima [A] time = 0.31, size = 53, normalized size = 1.06

$$\frac{\frac{a\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{ab-b^2}-\frac{\log\left(\sin\left(fx+e\right)^2-1\right)}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*log(-(a - b)*sin(f*x + e)^2 + a)/(a*b - b^2) - log(sin(f*x + e)^2 -
1)/b)/f
```

mupad [B] time = 11.78, size = 54, normalized size = 1.08

$$\frac{a\,\ln\left(b\tan\left(e+f\,x\right)^2+a\right)}{2\,f\,\left(a\,b-b^2\right)}-\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f\,\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (a*log(a + b*tan(e + f*x)^2))/(2*f*(a*b - b^2)) - log(tan(e + f*x)^2 + 1)/(
2*f*(a - b))
```

sympy [A] time = 3.67, size = 240, normalized size = 4.80

$$\begin{cases} \tilde{\infty}x\tan\left(e\right) & \text{for}\: a=0\wedge b=0\wedge f=0 \\ \frac{-\frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{\tan^2\left(e+fx\right)}{2f}}{a} & \text{for}\: b=0 \\ \frac{\log\left(\tan^2\left(e+fx\right)+1\right)\tan^2\left(e+fx\right)}{2bf\tan^2\left(e+fx\right)+2bf}+\frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2bf\tan^2\left(e+fx\right)+2bf}+\frac{1}{2bf\tan^2\left(e+fx\right)+2bf} & \text{for}\: a=b \\ \frac{x\tan^3\left(e\right)}{a+b\tan^2\left(e\right)} & \text{for}\: f=0 \\ \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan\left(e+fx\right)\right)}{2abf-2b^2f}+\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tan\left(e+fx\right)\right)}{2abf-2b^2f}-\frac{b\log\left(\tan^2\left(e+fx\right)+1\right)}{2abf-2b^2f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-log(tan(e + f*
x)**2 + 1)/(2*f) + tan(e + f*x)**2/(2*f))/a, Eq(b, 0)), (log(tan(e + f*x)**
2 + 1)*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*b*f) + log(tan(e + f*x)**
2 + 1)/(2*b*f*tan(e + f*x)**2 + 2*b*f) + 1/(2*b*f*tan(e + f*x)**2 + 2*b*f),
 Eq(a, b)), (x*tan(e)**3/(a + b*tan(e)**2), Eq(f, 0)), (a*log(-I*sqrt(a)*sq
rt(1/b) + tan(e + f*x))/(2*a*b*f - 2*b**2*f) + a*log(I*sqrt(a)*sqrt(1/b) +
tan(e + f*x))/(2*a*b*f - 2*b**2*f) - b*log(tan(e + f*x)**2 + 1)/(2*a*b*f -
2*b**2*f), True))
```

3.213 $\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=36

$$-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)}$$

```
[Out] -1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)/f
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3670, 444, 36, 31}

$$-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2(a-b)f} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \tan^2(e+fx)\right)}{2(a-b)f}$$
$$= -\frac{\log(\cos(e+fx))}{(a-b)f} - \frac{\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)f}$$

Mathematica [A] time = 0.04, size = 37, normalized size = 1.03

$$-\frac{\log\left(a+b\tan^2(e+fx)\right)+2\log(\cos(e+fx))}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2),x]
[Out] -1/2*(2*Log[Cos[e + f*x]] + Log[a + b*Tan[e + f*x]^2])/((a - b)*f)
```

fricas [A] time = 0.43, size = 38, normalized size = 1.06

$$-\frac{\log\left(\frac{b\tan\left(fx+e\right)^2+a}{\tan\left(fx+e\right)^2+1}\right)}{2\,(a-b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
[Out] -1/2*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1))/((a - b)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(1/(2*a-2*b)*ln(abs((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1)))+1))-1/(4*a-4*b)*ln(((1-cos(f*x+exp(1)))/(1+cos(
f*x+exp(1))))^2*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1)))*b+a))
```

maple [A] time = 0.19, size = 50, normalized size = 1.39

$$-\frac{\ln\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}{2f\,(a-b)}+\frac{\ln\left(1+\tan^2\left(fx+e\right)\right)}{2f\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/2/f/(a-b)*ln(a+b*tan(f*x+e)^2)+1/2/f/(a-b)*ln(1+tan(f*x+e)^2)
```

maxima [A] time = 0.31, size = 30, normalized size = 0.83

$$-\frac{\log\left(-(a-b)\sin\left(fx+e\right)^{2}+a\right)}{2\,(a-b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log(-(a - b)*sin(f*x + e)^2 + a)/((a - b)*f)
```

mupad [B] time = 11.85, size = 66, normalized size = 1.83

$$-\frac{\operatorname{atan}\left(\frac{a\tan\left(e+f\,x\right)^{2}\,1\mathrm{i}-b\tan\left(e+f\,x\right)^{2}\,1\mathrm{i}}{2\,a+a\tan\left(e+f\,x\right)^{2}+b\tan\left(e+f\,x\right)^{2}}\right)\,1\mathrm{i}}{f\,\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*tan(e + f*x)^2),x)
```

```
[Out] -(atan((a*tan(e + f*x)^2*1i - b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*x)^2
+ b*tan(e + f*x)^2))*1i)/(f*(a - b))
```

sympy [A] time = 2.11, size = 143, normalized size = 3.97

$$\begin{cases} \frac{\tilde{\infty}x}{\tan{(e)}} & \text{for}\: a = 0 \wedge b = 0 \wedge f = 0 \\ \frac{\log{\left(\tan^{2}{\left(e+fx\right)}+1\right)}}{2af} & \text{for}\: b = 0 \\ -\frac{1}{2bf\tan^{2}{\left(e+fx\right)}+2bf} & \text{for}\: a = b \\ \frac{x\tan{(e)}}{a+b\tan^{2}{(e)}} & \text{for}\: f = 0 \\ -\frac{\log{\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan{\left(e+fx\right)}\right)}}{2af-2bf} - \frac{\log{\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tan{\left(e+fx\right)}\right)}}{2af-2bf} + \frac{\log{\left(\tan^{2}{\left(e+fx\right)}+1\right)}}{2af-2bf} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(tan(e + f*x)
**2 + 1)/(2*a*f), Eq(b, 0)), (-1/(2*b*f*tan(e + f*x)**2 + 2*b*f), Eq(a, b))
, (x*tan(e)/(a + b*tan(e)**2), Eq(f, 0)), (-log(-I*sqrt(a)*sqrt(1/b) + tan(
e + f*x))/(2*a*f - 2*b*f) - log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*a*f
- 2*b*f) + log(tan(e + f*x)**2 + 1)/(2*a*f - 2*b*f), True))
```

3.214 $\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)} dx$

Optimal. Leaf size=64

$$\frac{b\log\left(a+b\tan^2(e+fx)\right)}{2af(a-b)}+\frac{\log(\cos(e+fx))}{f(a-b)}+\frac{\log(\tan(e+fx))}{af}$$

```
[Out] ln(cos(f*x+e))/(a-b)/f+ln(tan(f*x+e))/a/f+1/2*b*ln(a+b*tan(f*x+e)^2)/a/(a-b
)/f
```

Rubi [A] time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 446, 72}

$$\frac{b\log\left(a+b\tan^2(e+fx)\right)}{2af(a-b)}+\frac{\log(\cos(e+fx))}{f(a-b)}+\frac{\log(\tan(e+fx))}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] Log[Cos[e + f*x]]/((a - b)*f) + Log[Tan[e + f*x]]/(a*f) + (b*Log[a + b*Tan[
e + f*x]^2])/(2*a*(a - b)*f)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax} - \frac{1}{(a-b)(1+x)} + \frac{b^2}{a(a-b)(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\log(\cos(e+fx))}{(a-b)f} + \frac{\log(\tan(e+fx))}{af} + \frac{b\log\left(a+b\tan^2(e+fx)\right)}{2a(a-b)f}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.89

$$\frac{b\log\left(a+b\tan^2(e+fx)\right)+2(a-b)\log(\tan(e+fx))+2a\log(\cos(e+fx))}{2af(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (2*a*Log[Cos[e + f*x]] + 2*(a - b)*Log[Tan[e + f*x]] + b*Log[a + b*Tan[e + 
f*x]^2])/(2*a*(a - b)*f)
```

fricas [A] time = 0.45, size = 72, normalized size = 1.12

$$\frac{(a-b)\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)+b\log\left(\frac{b\tan\left(fx+e\right)^2+a}{\tan\left(fx+e\right)^2+1}\right)}{2\left(a^2-ab\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a - b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + b*log((b*tan(f*x + 
e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^2 - a*b)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)1/f*(1/2/a*ln(sin(f*x+exp(1))^2)+b/(-2*b*a+2*a^2)*ln(abs(sin(f*x+ex
p(1))^2*b-sin(f*x+exp(1))^2*a+a)))
```

maple [A] time = 0.73, size = 76, normalized size = 1.19

$$\frac{b\ln\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)}{2fa\left(a-b\right)}+\frac{\ln\left(-1+\cos\left(fx+e\right)\right)}{2fa}+\frac{\ln\left(1+\cos\left(fx+e\right)\right)}{2fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/2/f/a*b/(a-b)*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+1/2/f/a*ln(-1+cos(f*x+e
))+1/2/f/a*ln(1+cos(f*x+e))
```

maxima [A] time = 0.83, size = 49, normalized size = 0.77

$$\frac{\frac{b\log\left(-(a-b)\sin\left(fx+e\right)^{2}+a\right)}{a^{2}-ab}+\frac{\log\left(\sin\left(fx+e\right)^{2}\right)}{a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*log(-(a - b)*sin(f*x + e)^2 + a)/(a^2 - a*b) + log(sin(f*x + e)^2)/a
)/f
```

mupad [B] time = 11.71, size = 68, normalized size = 1.06

$$\frac{\ln\left(\tan\left(e+f\,x\right)\right)}{a\,f}-\frac{\ln\left({\tan\left(e+f\,x\right)}^2+1\right)}{2\,f\,\left(a-b\right)}-\frac{b\,\ln\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}{2\,f\,\left(a\,b-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*tan(e + f*x)^2),x)
```

```
[Out] log(tan(e + f*x))/(a*f) - log(tan(e + f*x)^2 + 1)/(2*f*(a - b)) - (b*log(a
+ b*tan(e + f*x)^2))/(2*f*(a*b - a^2))
```

sympy [A] time = 8.38, size = 398, normalized size = 6.22

$$\begin{cases} \frac{\tilde{\infty}x\cot(e)}{\tan^{2}(e)} \\ \frac{\frac{\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}-\frac{\log\left(\tan\left(e+fx\right)\right)}{f}-\frac{1}{2f\tan^{2}\left(e+fx\right)}}{b} \\ -\frac{\log\left(\tan^{2}\left(e+fx\right)+1\right)\tan^{2}\left(e+fx\right)}{2af\tan^{2}\left(e+fx\right)+2af}-\frac{\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2af\tan^{2}\left(e+fx\right)+2af}+\frac{2\log\left(\tan\left(e+fx\right)\right)\tan^{2}\left(e+fx\right)}{2af\tan^{2}\left(e+fx\right)+2af}+\frac{2\log\left(\tan\left(e+fx\right)\right)}{2af\tan^{2}\left(e+fx\right)+2af}+\frac{1}{2af\tan^{2}\left(e+fx\right)+2} \\ \frac{x\cot(e)}{a+b\tan^{2}(e)} \\ \frac{-\frac{\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{\log\left(\tan\left(e+fx\right)\right)}{f}}{a} \\ -\frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2a^{2}f-2abf}+\frac{2a\log\left(\tan\left(e+fx\right)\right)}{2a^{2}f-2abf}+\frac{b\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan\left(e+fx\right)\right)}{2a^{2}f-2abf}+\frac{b\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tan\left(e+fx\right)\right)}{2a^{2}f-2abf}-\frac{2b\log\left(\tan\left(e+fx\right)\right)}{2a^{2}f-2abf} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*cot(e)/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((log(t
an(e + f*x)**2 + 1)/(2*f) - log(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2))/
b, Eq(a, 0)), (-log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a*f*tan(e + f*x
)**2 + 2*a*f) - log(tan(e + f*x)**2 + 1)/(2*a*f*tan(e + f*x)**2 + 2*a*f) +
2*log(tan(e + f*x))*tan(e + f*x)**2/(2*a*f*tan(e + f*x)**2 + 2*a*f) + 2*log
(tan(e + f*x))/(2*a*f*tan(e + f*x)**2 + 2*a*f) + 1/(2*a*f*tan(e + f*x)**2 +
 2*a*f), Eq(a, b)), (x*cot(e)/(a + b*tan(e)**2), Eq(f, 0)), ((-log(tan(e +
f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f)/a, Eq(b, 0)), (-a*log(tan(e + f*x
```

```
)**2 + 1)/(2*a**2*f - 2*a*b*f) + 2*a*log(tan(e + f*x))/(2*a**2*f - 2*a*b*f)
 + b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*a**2*f - 2*a*b*f) + b*log(
I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*a**2*f - 2*a*b*f) - 2*b*log(tan(e +
f*x))/(2*a**2*f - 2*a*b*f), True))
```

3.215 $\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=89

$$-\frac{b^2\log\left(a+b\tan^2(e+fx)\right)}{2a^2f(a-b)}-\frac{(a+b)\log(\tan(e+fx))}{a^2f}-\frac{\log(\cos(e+fx))}{f(a-b)}-\frac{\cot^2(e+fx)}{2af}$$

```
[Out] -1/2*cot(f*x+e)^2/a/f-ln(cos(f*x+e))/(a-b)/f-(a+b)*ln(tan(f*x+e))/a^2/f-1/2
*b^2*ln(a+b*tan(f*x+e)^2)/a^2/(a-b)/f
```

Rubi [A] time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 446, 72}

$$-\frac{b^2\log\left(a+b\tan^2(e+fx)\right)}{2a^2f(a-b)}-\frac{(a+b)\log(\tan(e+fx))}{a^2f}-\frac{\log(\cos(e+fx))}{f(a-b)}-\frac{\cot^2(e+fx)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -Cot[e + f*x]^2/(2*a*f) - Log[Cos[e + f*x]]/((a - b)*f) - ((a + b)*Log[Tan[
e + f*x]])/(a^2*f) - (b^2*Log[a + b*Tan[e + f*x]^2])/(2*a^2*(a - b)*f)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{-a-b}{a^2x}+\frac{1}{(a-b)(1+x)}-\frac{b^3}{a^2(a-b)(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{\cot^2(e+fx)}{2af}-\frac{\log(\cos(e+fx))}{(a-b)f}-\frac{(a+b)\log(\tan(e+fx))}{a^2f}-\frac{b^2\log\left(a+b\tan^2(e+\right.}{2a^2(a-b)f}$$

Mathematica [A] time = 0.26, size = 63, normalized size = 0.71

$$-\frac{\frac{b^2\log\left(a\cot^2(e+fx)+b\right)}{a^2(a-b)}+\frac{2\log(\sin(e+fx))}{a-b}+\frac{\cot^2(e+fx)}{a}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/2*(Cot[e + f*x]^2/a + (b^2*Log[b + a*Cot[e + f*x]^2])/(a^2*(a - b)) + (2
*Log[Sin[e + f*x]])/(a - b))/f
```

fricas [A] time = 0.46, size = 128, normalized size = 1.44

$$-\frac{b^2\log\left(\frac{b\tan\left(fx+e\right)^2+a}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2+\left(a^2-b^2\right)\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2+\left(a^2-ab\right)\tan\left(fx+e\right)^2+a^2-}{2\left(a^3-a^2b\right)f\tan\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 +
 (a^2 - b^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + (a^2
 - a*b)*tan(f*x + e)^2 + a^2 - a*b)/((a^3 - a^2*b)*f*tan(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(-(1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1)))*1/16/a+(4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+
4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-a)*1/16/a^2/(1-cos(f*x+exp(1)))
*(1+cos(f*x+exp(1)))+1/(2*a-2*b)*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1)))+1))-b^2/(4*a^3-4*a^2*b)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2
*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1)))*b+a)+(-a-b)*1/4/a^2*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+e
xp(1)))))
```

maple [A] time = 0.78, size = 150, normalized size = 1.69

$$-\frac{b^2 \ln\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)}{2f\,a^2\,(a-b)}+\frac{1}{4fa\left(-1+\cos\left(fx+e\right)\right)}-\frac{\ln\left(-1+\cos\left(fx+e\right)\right)}{2fa}-\frac{\ln\left(-1+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/2/f*b^2/a^2/(a-b)*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+1/4/f/a/(-1+cos(f*
x+e))-1/2/f/a*ln(-1+cos(f*x+e))-1/2/f/a^2*ln(-1+cos(f*x+e))*b-1/4/f/a/(1+co
s(f*x+e))-1/2/f/a*ln(1+cos(f*x+e))-1/2/f/a^2*ln(1+cos(f*x+e))*b
```

maxima [A] time = 0.54, size = 68, normalized size = 0.76

$$-\frac{\frac{b^2\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^3-a^2b}+\frac{(a+b)\log\left(\sin\left(fx+e\right)^2\right)}{a^2}+\frac{1}{a\sin\left(fx+e\right)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - a^2*b) + (a + b)*log(sin(
f*x + e)^2)/a^2 + 1/(a*sin(f*x + e)^2))/f
```

mupad [B] time = 11.60, size = 89, normalized size = 1.00

$$\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f\,\left(a-b\right)}-\frac{\cot\left(e+f\,x\right)^2}{2\,a\,f}-\frac{\ln\left(\tan\left(e+f\,x\right)\right)\,\left(a+b\right)}{a^2\,f}-\frac{b^2\,\ln\left(b\tan\left(e+f\,x\right)^2+a\right)}{2\,a^2\,f\,\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2),x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*f*(a - b)) - cot(e + f*x)^2/(2*a*f) - (log(tan(e
 + f*x))*(a + b))/(a^2*f) - (b^2*log(a + b*tan(e + f*x)^2))/(2*a^2*f*(a - b
))
```

sympy [A] time = 28.09, size = 743, normalized size = 8.35

$$\begin{cases} \tilde{\infty} x \\ \frac{-\frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{\log\left(\tan\left(e+fx\right)\right)}{f}+\frac{1}{2f\tan^2\left(e+fx\right)}-\frac{1}{4f\tan^4\left(e+fx\right)}}{b} \\ \frac{2\log\left(\tan^2\left(e+fx\right)+1\right)\tan^4\left(e+fx\right)}{2af\tan^4\left(e+fx\right)+2af\tan^2\left(e+fx\right)}+\frac{2\log\left(\tan^2\left(e+fx\right)+1\right)\tan^2\left(e+fx\right)}{2af\tan^4\left(e+fx\right)+2af\tan^2\left(e+fx\right)}-\frac{4\log\left(\tan\left(e+fx\right)\right)\tan^4\left(e+fx\right)}{2af\tan^4\left(e+fx\right)+2af\tan^2\left(e+fx\right)}-\frac{4\log\left(\tan\left(e+fx\right)\right)\tan^2\left(e+\cdots\right)}{2af\tan^4\left(e+fx\right)+2af\tan^2\left(\cdots\right)}\cdots \\ \frac{\tilde{\infty} x}{a} \\ \frac{x\cot^3\left(e\right)}{a+b\tan^2\left(e\right)} \\ \frac{\frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-\frac{\log\left(\tan\left(e+fx\right)\right)}{f}-\frac{1}{2f\tan^2\left(e+fx\right)}}{a} \\ \frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)\tan^2\left(e+fx\right)}{2a^3f\tan^2\left(e+fx\right)-2a^2bf\tan^2\left(e+fx\right)}-\frac{2a^2\log\left(\tan\left(e+fx\right)\right)\tan^2\left(e+fx\right)}{2a^3f\tan^2\left(e+fx\right)-2a^2bf\tan^2\left(e+fx\right)}-\frac{a^2}{2a^3f\tan^2\left(e+fx\right)-2a^2bf\tan^2\left(e+fx\right)}+\frac{ab}{2a^3f\tan^2\left(e+fx\right)-2\cdots}\cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((-log(tan(e
+ f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f + 1/(2*f*tan(e + f*x)**2) - 1/(4
*f*tan(e + f*x)**4))/b, Eq(a, 0)), (2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)
**4/(2*a*f*tan(e + f*x)**4 + 2*a*f*tan(e + f*x)**2) + 2*log(tan(e + f*x)**2
 + 1)*tan(e + f*x)**2/(2*a*f*tan(e + f*x)**4 + 2*a*f*tan(e + f*x)**2) - 4*l
og(tan(e + f*x))*tan(e + f*x)**4/(2*a*f*tan(e + f*x)**4 + 2*a*f*tan(e + f*x
)**2) - 4*log(tan(e + f*x))*tan(e + f*x)**2/(2*a*f*tan(e + f*x)**4 + 2*a*f*
tan(e + f*x)**2) - 2*tan(e + f*x)**2/(2*a*f*tan(e + f*x)**4 + 2*a*f*tan(e +
 f*x)**2) - 1/(2*a*f*tan(e + f*x)**4 + 2*a*f*tan(e + f*x)**2), Eq(a, b)), (
zoo*x/a, Eq(e, -f*x)), (x*cot(e)**3/(a + b*tan(e)**2), Eq(f, 0)), ((log(tan
(e + f*x)**2 + 1)/(2*f) - log(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2))/a,
 Eq(b, 0)), (a**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a**3*f*tan(e
+ f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) - 2*a**2*log(tan(e + f*x))*tan(e +
f*x)**2/(2*a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) - a**2/(2*a
**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) + a*b/(2*a**3*f*tan(e +
 f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) - b**2*log(-I*sqrt(a)*sqrt(1/b) + ta
n(e + f*x))*tan(e + f*x)**2/(2*a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e +
f*x)**2) - b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(2*
a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) + 2*b**2*log(tan(e + f
*x))*tan(e + f*x)**2/(2*a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2
), True))
```

3.216 $\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=115

$$\frac{b^3\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)}+\frac{(a+b)\cot^2(e+fx)}{2a^2f}+\frac{\left(a^2+ab+b^2\right)\log(\tan(e+fx))}{a^3f}+\frac{\log(\cos(e+fx))}{f(a-b)}-\frac{\cot^4(e}{4a}$$

```
[Out] 1/2*(a+b)*cot(f*x+e)^2/a^2/f-1/4*cot(f*x+e)^4/a/f+ln(cos(f*x+e))/(a-b)/f+(a
^2+a*b+b^2)*ln(tan(f*x+e))/a^3/f+1/2*b^3*ln(a+b*tan(f*x+e)^2)/a^3/(a-b)/f
```

Rubi [A] time = 0.14, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 446, 72}

$$\frac{b^3\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)}+\frac{\left(a^2+ab+b^2\right)\log(\tan(e+fx))}{a^3f}+\frac{(a+b)\cot^2(e+fx)}{2a^2f}+\frac{\log(\cos(e+fx))}{f(a-b)}-\frac{\cot^4(e}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a + b)*Cot[e + f*x]^2)/(2*a^2*f) - Cot[e + f*x]^4/(4*a*f) + Log[Cos[e + f
*x]]/((a - b)*f) + ((a^2 + a*b + b^2)*Log[Tan[e + f*x]])/(a^3*f) + (b^3*Log
[a + b*Tan[e + f*x]^2])/(2*a^3*(a - b)*f)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^3} + \frac{-a-b}{a^2x^2} + \frac{a^2+ab+b^2}{a^3x} - \frac{1}{(a-b)(1+x)} + \frac{b^4}{a^3(a-b)(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{(a+b)\cot^2(e+fx)}{2a^2f} - \frac{\cot^4(e+fx)}{4af} + \frac{\log(\cos(e+fx))}{(a-b)f} + \frac{(a^2+ab+b^2)\log(\tan(e+$$

Mathematica [A] time = 0.37, size = 83, normalized size = 0.72

$$-\frac{-\frac{b^3\log\left(a\cot^2(e+fx)+b\right)}{a^3(a-b)} - \frac{(a+b)\cot^2(e+fx)}{a^2} - \frac{2\log(\sin(e+fx))}{a-b} + \frac{\cot^4(e+fx)}{2a}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/2*(-(((a + b)*Cot[e + f*x]^2)/a^2) + Cot[e + f*x]^4/(2*a) - (b^3*Log[b +
 a*Cot[e + f*x]^2])/(a^3*(a - b)) - (2*Log[Sin[e + f*x]])/(a - b))/f
```

fricas [A] time = 0.45, size = 163, normalized size = 1.42

$$\frac{2\,b^3\log\left(\frac{b\tan\left(fx+e\right)^2+a}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^4 + 2\left(a^3-b^3\right)\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^4 + \left(3\,a^3-a^2b-2\,ab^2\right)\tan\left(fx\right.}{4\left(a^4-a^3b\right)f\tan\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4
+ 2*(a^3 - b^3)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + (
3*a^3 - a^2*b - 2*a*b^2)*tan(f*x + e)^4 - a^3 + a^2*b + 2*(a^3 - a*b^2)*tan
(f*x + e)^2)/((a^4 - a^3*b)*f*tan(f*x + e)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*((-32*((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1))))^2*a+384*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
)*a+256*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b)*1/4096/a^2+(-48*((1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2-48*((1-cos(f*x+exp(1)))/(1+cos(f*x+
exp(1))))^2*a*b-48*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2+12*(1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2+8*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp
```

```
(1)))*a*b-a^2)*1/128/a^3/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2-1/(2*a
-2*b)*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+b^3/(4*a^4-4*a^3*b
)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-2*(1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a)+(a^2+a*b
+b^2)*1/4/a^3*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1)))))
```

maple [B] time = 0.92, size = 264, normalized size = 2.30

$$\frac{b^3 \ln\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)}{2f\,a^3\,(a-b)}-\frac{1}{16fa\left(-1+\cos\left(fx+e\right)\right)^2}-\frac{7}{16fa\left(-1+\cos\left(fx+e\right)\right)}-\frac{}{4f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/2/f*b^3/a^3/(a-b)*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-1/16/f/a/(-1+cos(f*
x+e))^2-7/16/f/a/(-1+cos(f*x+e))-1/4/f/a^2/(-1+cos(f*x+e))*b+1/2/f/a*ln(-1+
cos(f*x+e))+1/2/f/a^2*ln(-1+cos(f*x+e))*b+1/2/f/a^3*ln(-1+cos(f*x+e))*b^2-1
/16/f/a/(1+cos(f*x+e))^2+7/16/f/a/(1+cos(f*x+e))+1/4/f/a^2/(1+cos(f*x+e))*b
+1/2/f/a*ln(1+cos(f*x+e))+1/2/f/a^2*ln(1+cos(f*x+e))*b+1/2/f/a^3*ln(1+cos(f
*x+e))*b^2
```

maxima [A] time = 0.46, size = 96, normalized size = 0.83

$$\frac{\frac{2\,b^3 \log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^4-a^3 b}+\frac{2\left(a^2+ab+b^2\right)\log\left(\sin\left(fx+e\right)^2\right)}{a^3}+\frac{2\,(2\,a+b)\sin\left(fx+e\right)^2-a}{a^2\sin\left(fx+e\right)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^3*log(-(a - b)*sin(f*x + e)^2 + a)/(a^4 - a^3*b) + 2*(a^2 + a*b +
b^2)*log(sin(f*x + e)^2)/a^3 + (2*(2*a + b)*sin(f*x + e)^2 - a)/(a^2*sin(f*
x + e)^4))/f
```

mupad [B] time = 11.78, size = 118, normalized size = 1.03

$$\frac{\ln\left(\tan\left(e+f\,x\right)\right)\,\left(a^2+a\,b+b^2\right)}{a^3\,f}-\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f\,\left(a-b\right)}-\frac{b^3\,\ln\left(b\tan\left(e+f\,x\right)^2+a\right)}{f\,\left(2\,a^3\,b-2\,a^4\right)}-\frac{\cot\left(e+f\,x\right)^4\,\left(\frac{1}{4\,a}-\frac{t}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (log(tan(e + f*x))*(a*b + a^2 + b^2))/(a^3*f) - log(tan(e + f*x)^2 + 1)/(2*
f*(a - b)) - (b^3*log(a + b*tan(e + f*x)^2))/(f*(2*a^3*b - 2*a^4)) - (cot(e
 + f*x)^4*(1/(4*a) - (tan(e + f*x)^2*(a + b))/(2*a^2)))/f
```

sympy [A] time = 94.47, size = 908, normalized size = 7.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((log(tan(e +
 f*x)**2 + 1)/(2*f) - log(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2) + 1/(4*
```

```
f*tan(e + f*x)**4) - 1/(6*f*tan(e + f*x)**6))/b, Eq(a, 0)), (-6*log(tan(e +
 f*x)**2 + 1)*tan(e + f*x)**6/(4*a*f*tan(e + f*x)**6 + 4*a*f*tan(e + f*x)**
4) - 6*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**4/(4*a*f*tan(e + f*x)**6 + 4*
a*f*tan(e + f*x)**4) + 12*log(tan(e + f*x))*tan(e + f*x)**6/(4*a*f*tan(e +
f*x)**6 + 4*a*f*tan(e + f*x)**4) + 12*log(tan(e + f*x))*tan(e + f*x)**4/(4*
a*f*tan(e + f*x)**6 + 4*a*f*tan(e + f*x)**4) + 6*tan(e + f*x)**4/(4*a*f*tan
(e + f*x)**6 + 4*a*f*tan(e + f*x)**4) + 3*tan(e + f*x)**2/(4*a*f*tan(e + f*
x)**6 + 4*a*f*tan(e + f*x)**4) - 1/(4*a*f*tan(e + f*x)**6 + 4*a*f*tan(e + f
*x)**4), Eq(a, b)), (zoo*x/a, Eq(e, -f*x)), (x*cot(e)**5/(a + b*tan(e)**2),
 Eq(f, 0)), ((-log(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f + 1/(2*
f*tan(e + f*x)**2) - 1/(4*f*tan(e + f*x)**4))/a, Eq(b, 0)), (-2*a**3*log(ta
n(e + f*x)**2 + 1)*tan(e + f*x)**4/(4*a**4*f*tan(e + f*x)**4 - 4*a**3*b*f*t
an(e + f*x)**4) + 4*a**3*log(tan(e + f*x))*tan(e + f*x)**4/(4*a**4*f*tan(e
+ f*x)**4 - 4*a**3*b*f*tan(e + f*x)**4) + 2*a**3*tan(e + f*x)**2/(4*a**4*f*
tan(e + f*x)**4 - 4*a**3*b*f*tan(e + f*x)**4) - a**3/(4*a**4*f*tan(e + f*x)
**4 - 4*a**3*b*f*tan(e + f*x)**4) + a**2*b/(4*a**4*f*tan(e + f*x)**4 - 4*a*
*3*b*f*tan(e + f*x)**4) - 2*a*b**2*tan(e + f*x)**2/(4*a**4*f*tan(e + f*x)**
4 - 4*a**3*b*f*tan(e + f*x)**4) + 2*b**3*log(-I*sqrt(a)*sqrt(1/b) + tan(e +
 f*x))*tan(e + f*x)**4/(4*a**4*f*tan(e + f*x)**4 - 4*a**3*b*f*tan(e + f*x)*
*4) + 2*b**3*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**
4*f*tan(e + f*x)**4 - 4*a**3*b*f*tan(e + f*x)**4) - 4*b**3*log(tan(e + f*x)
)*tan(e + f*x)**4/(4*a**4*f*tan(e + f*x)**4 - 4*a**3*b*f*tan(e + f*x)**4),
True))
```

3.217 $\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=85

$$\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{5/2}f(a-b)}-\frac{(a+b)\tan(e+fx)}{b^2f}-\frac{x}{a-b}+\frac{\tan^3(e+fx)}{3bf}$$

```
[Out] -x/(a-b)+a^(5/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)/b^(5/2)/f-(a+b)*t
an(f*x+e)/b^2/f+1/3*tan(f*x+e)^3/b/f
```

Rubi [A] time = 0.19, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 479, 582, 522, 203, 205}

$$\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{5/2}f(a-b)}-\frac{(a+b)\tan(e+fx)}{b^2f}-\frac{x}{a-b}+\frac{\tan^3(e+fx)}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(x/(a - b)) + (a^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*b^
(5/2)*f) - ((a + b)*Tan[e + f*x])/(b^2*f) + Tan[e + f*x]^3/(3*b*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
```

```
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\tan^3(e+fx)}{3bf} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a+3(a+b)x^2\right)}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{3bf} \\
&= -\frac{(a+b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf} + \frac{\text{Subst}\left(\int \frac{3a(a+b)+3\left(a^2+ab+b^2\right)x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx\right.}{3b^2f} \\
&= -\frac{(a+b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f} + \frac{a^3\,\text{Subst}\left(\right.}{} \\
&= -\frac{x}{a-b} + \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)b^{5/2}f} - \frac{(a+b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf}
\end{aligned}
$$

Mathematica [A] time = 0.84, size = 92, normalized size = 1.08

$$
\frac{\sqrt{b}\left((a-b)\tan(e+fx)\left(3a-b\sec^2(e+fx)+4b\right)+3b^2(e+fx)\right)-3a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{3b^{5/2}f(b-a)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-3*a^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[b]*(3*b^2*(e + f*
x) + (a - b)*(3*a + 4*b - b*Sec[e + f*x]^2)*Tan[e + f*x]))/(3*b^(5/2)*(-a +
 b)*f)
```

fricas [A] time = 0.49, size = 278, normalized size = 3.27

$$
\left[-\frac{12\,b^2fx - 4\left(ab-b^2\right)\tan\left(fx+e\right)^3 + 3\,a^2\sqrt{-\frac{a}{b}}\,\log\left(\frac{b^2\tan\left(fx+e\right)^4 - 6\,ab\tan\left(fx+e\right)^2 + a^2 - 4\left(b^2\tan\left(fx+e\right)^3 - ab\tan\left(fx+e\right)\right)\sqrt{-\frac{a}{b}}}{b^2\tan\left(fx+e\right)^4 + 2\,ab\tan\left(fx+e\right)^2 + a^2}\right.}{12\left(ab^2-b^3\right)f}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(12*b^2*f*x - 4*(a*b - b^2)*tan(f*x + e)^3 + 3*a^2*sqrt(-a/b)*log((b
^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(b^2*tan(f*x + e)^3 - a*
b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^
2)) + 12*(a^2 - b^2)*tan(f*x + e))/((a*b^2 - b^3)*f), -1/6*(6*b^2*f*x - 2*(
a*b - b^2)*tan(f*x + e)^3 - 3*a^2*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)^2 -
a)*sqrt(a/b)/(a*tan(f*x + e))) + 6*(a^2 - b^2)*tan(f*x + e))/((a*b^2 - b^3)
*f)]
```

giac [A] time = 21.52, size = 118, normalized size = 1.39

$$\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)a^3}{(ab^2-b^3)\sqrt{ab}}-\frac{3(fx+e)}{a-b}+\frac{b^2\tan(fx+e)^3-3\,ab\tan(fx+e)-3\,b^2\tan(fx+e)}{b^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b)))*a^3/((a*b^2 - b^3)*sqrt(a*b)) - 3*(f*x + e)/(a - b) + (b^2*tan(f*x + e
)^3 - 3*a*b*tan(f*x + e) - 3*b^2*tan(f*x + e))/b^3)/f
```

maple [A] time = 0.18, size = 102, normalized size = 1.20

$$\frac{\tan^3(fx+e)}{3bf}-\frac{a\tan(fx+e)}{f\,b^2}-\frac{\tan(fx+e)}{bf}+\frac{a^3\arctan\left(\frac{\tan(fx+e)b}{\sqrt{ab}}\right)}{f\,b^2\,(a-b)\sqrt{ab}}-\frac{\arctan(\tan(fx+e))}{f\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/3*tan(f*x+e)^3/b/f-1/f/b^2*a*tan(f*x+e)-tan(f*x+e)/b/f+1/f/b^2*a^3/(a-b)/
(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/f/(a-b)*arctan(tan(f*x+e))
```

maxima [A] time = 0.90, size = 83, normalized size = 0.98

$$\frac{\frac{3\,a^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(ab^2-b^3)\sqrt{ab}}-\frac{3(fx+e)}{a-b}+\frac{b\tan(fx+e)^3-3\,(a+b)\tan(fx+e)}{b^2}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^3*arctan(b*tan(f*x + e)/sqrt(a*b))/((a*b^2 - b^3)*sqrt(a*b)) - 3*(
f*x + e)/(a - b) + (b*tan(f*x + e)^3 - 3*(a + b)*tan(f*x + e))/b^2)/f
```

mupad [B] time = 12.00, size = 1310, normalized size = 15.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2),x)
```

```
[Out] tan(e + f*x)^3/(3*b*f) + (2*atan((((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4*a
^4*b^3)/b^3 - (tan(e + f*x)*(4*a*b^7 - 4*b^8 + 4*a^2*b^6 - 4*a^3*b^5)*2i)/(
b^3*(2*a - 2*b)))*1i)/(2*a - 2*b) + (2*tan(e + f*x)*(a^6 + b^6))/b^3)/(2*a
- 2*b) - ((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e + f*
x)*(4*a*b^7 - 4*b^8 + 4*a^2*b^6 - 4*a^3*b^5)*2i)/(b^3*(2*a - 2*b)))*1i)/(2*
```

```
a - 2*b) - (2*tan(e + f*x)*(a^6 + b^6))/b^3)/(2*a - 2*b))/((((((4*a*b^6 - 4
*a^2*b^5 - 4*a^3*b^4 + 4*a^4*b^3)/b^3 - (tan(e + f*x)*(4*a*b^7 - 4*b^8 + 4*
a^2*b^6 - 4*a^3*b^5)*2i)/(b^3*(2*a - 2*b)))*1i)/(2*a - 2*b) + (2*tan(e + f*
x)*(a^6 + b^6))/b^3)*1i)/(2*a - 2*b) - (2*(a^4*b + a^5 + a^3*b^2))/b^3 + ((
(((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e + f*x)*(4*a*b^
7 - 4*b^8 + 4*a^2*b^6 - 4*a^3*b^5)*2i)/(b^3*(2*a - 2*b)))*1i)/(2*a - 2*b) -
 (2*tan(e + f*x)*(a^6 + b^6))/b^3)*1i)/(2*a - 2*b))))/(f*(2*a - 2*b)) - (ta
n(e + f*x)*(a + b))/(b^2*f) - (atan((((((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 +
 4*a^4*b^3)/b^3 + (tan(e + f*x)*(-a^5*b^5)^(1/2)*(4*a*b^7 - 4*b^8 + 4*a^2*b
^6 - 4*a^3*b^5))/(b^3*(a*b^5 - b^6)))*(-a^5*b^5)^(1/2))/(2*(a*b^5 - b^6)) -
 (2*tan(e + f*x)*(a^6 + b^6))/b^3)*(-a^5*b^5)^(1/2)*1i)/(2*(a*b^5 - b^6)) -
 (((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4*a^4*b^3)/b^3 - (tan(e + f*x)*(-a^
5*b^5)^(1/2)*(4*a*b^7 - 4*b^8 + 4*a^2*b^6 - 4*a^3*b^5))/(b^3*(a*b^5 - b^6))
)*(-a^5*b^5)^(1/2))/(2*(a*b^5 - b^6)) + (2*tan(e + f*x)*(a^6 + b^6))/b^3)*(
-a^5*b^5)^(1/2)*1i)/(2*(a*b^5 - b^6)))/((((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^
4 + 4*a^4*b^3)/b^3 + (tan(e + f*x)*(-a^5*b^5)^(1/2)*(4*a*b^7 - 4*b^8 + 4*a^
2*b^6 - 4*a^3*b^5))/(b^3*(a*b^5 - b^6)))*(-a^5*b^5)^(1/2))/(2*(a*b^5 - b^6)
) - (2*tan(e + f*x)*(a^6 + b^6))/b^3)*(-a^5*b^5)^(1/2))/(2*(a*b^5 - b^6)) -
 (2*(a^4*b + a^5 + a^3*b^2))/b^3 + (((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4
*a^4*b^3)/b^3 - (tan(e + f*x)*(-a^5*b^5)^(1/2)*(4*a*b^7 - 4*b^8 + 4*a^2*b^6
 - 4*a^3*b^5))/(b^3*(a*b^5 - b^6)))*(-a^5*b^5)^(1/2))/(2*(a*b^5 - b^6)) + (
2*tan(e + f*x)*(a^6 + b^6))/b^3)*(-a^5*b^5)^(1/2))/(2*(a*b^5 - b^6))))*(-a^
5*b^5)^(1/2)*1i)/(f*(a*b^5 - b^6))
```

sympy [A] time = 35.00, size = 685, normalized size = 8.06

$$\begin{cases} \tilde{\infty} x \tan^4(e) \\ \frac{-x+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan(e+fx)}{f}}{a} \\ \frac{x+\frac{\tan^3(e+fx)}{3f}-\frac{\tan(e+fx)}{f}}{b} \\ \frac{15fx\tan^2(e+fx)}{6bf\tan^2(e+fx)+6bf}+\frac{15fx}{6bf\tan^2(e+fx)+6bf}+\frac{2\tan^5(e+fx)}{6bf\tan^2(e+fx)+6bf}-\frac{10\tan^3(e+fx)}{6bf\tan^2(e+fx)+6bf}-\frac{15\tan(e+fx)}{6bf\tan^2(e+fx)+6bf} \\ \frac{x\tan^6(e)}{a+b\tan^2(e)} \\ -\frac{6ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}\tan(e+fx)}{6ia^{\frac{3}{2}}b^3f\sqrt{\frac{1}{b}}-6i\sqrt{a}b^4f\sqrt{\frac{1}{b}}}+\frac{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}\tan^3(e+fx)}{6ia^{\frac{3}{2}}b^3f\sqrt{\frac{1}{b}}-6i\sqrt{a}b^4f\sqrt{\frac{1}{b}}}-\frac{6i\sqrt{a}b^3fx\sqrt{\frac{1}{b}}}{6ia^{\frac{3}{2}}b^3f\sqrt{\frac{1}{b}}-6i\sqrt{a}b^4f\sqrt{\frac{1}{b}}}-\frac{2i\sqrt{a}b^3\sqrt{\frac{1}{b}}\tan^3(e+fx)}{6ia^{\frac{3}{2}}b^3f\sqrt{\frac{1}{b}}-6i\sqrt{a}b^4f\sqrt{\frac{1}{b}}}+\frac{6i\sqrt{a}b^3\sqrt{\frac{1}{b}}\tan(}{6ia^{\frac{3}{2}}b^3f\sqrt{\frac{1}{b}}-6i\sqrt{a}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e +
 f*x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e + f*x)/f)/a, Eq(b, 0)), ((x
+ tan(e + f*x)**3/(3*f) - tan(e + f*x)/f)/b, Eq(a, 0)), (15*f*x*tan(e + f*x
)**2/(6*b*f*tan(e + f*x)**2 + 6*b*f) + 15*f*x/(6*b*f*tan(e + f*x)**2 + 6*b*
f) + 2*tan(e + f*x)**5/(6*b*f*tan(e + f*x)**2 + 6*b*f) - 10*tan(e + f*x)**3
/(6*b*f*tan(e + f*x)**2 + 6*b*f) - 15*tan(e + f*x)/(6*b*f*tan(e + f*x)**2 +
 6*b*f), Eq(a, b)), (x*tan(e)**6/(a + b*tan(e)**2), Eq(f, 0)), (-6*I*a**(5/
2)*b*sqrt(1/b)*tan(e + f*x)/(6*I*a**(3/2)*b**3*f*sqrt(1/b) - 6*I*sqrt(a)*b*
*4*f*sqrt(1/b)) + 2*I*a**(3/2)*b**2*sqrt(1/b)*tan(e + f*x)**3/(6*I*a**(3/2)
*b**3*f*sqrt(1/b) - 6*I*sqrt(a)*b**4*f*sqrt(1/b)) - 6*I*sqrt(a)*b**3*f*x*sq
rt(1/b)/(6*I*a**(3/2)*b**3*f*sqrt(1/b) - 6*I*sqrt(a)*b**4*f*sqrt(1/b)) - 2*
I*sqrt(a)*b**3*sqrt(1/b)*tan(e + f*x)**3/(6*I*a**(3/2)*b**3*f*sqrt(1/b) - 6
*I*sqrt(a)*b**4*f*sqrt(1/b)) + 6*I*sqrt(a)*b**3*sqrt(1/b)*tan(e + f*x)/(6*I
```

```
*a**(3/2)*b**3*f*sqrt(1/b) - 6*I*sqrt(a)*b**4*f*sqrt(1/b)) + 3*a**3*log(-I*
sqrt(a)*sqrt(1/b) + tan(e + f*x))/(6*I*a**(3/2)*b**3*f*sqrt(1/b) - 6*I*sqrt
(a)*b**4*f*sqrt(1/b)) - 3*a**3*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(6*I
*a**(3/2)*b**3*f*sqrt(1/b) - 6*I*sqrt(a)*b**4*f*sqrt(1/b)), True))
```

3.218 $\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=63

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{3/2}f(a-b)}+\frac{x}{a-b}+\frac{\tan(e+fx)}{bf}$$

```
[Out] x/(a-b)-a^(3/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)/b^(3/2)/f+tan(f*x+
e)/b/f
```

Rubi [A] time = 0.11, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 479, 522, 203, 205}

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{3/2}f(a-b)}+\frac{x}{a-b}+\frac{\tan(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] x/(a - b) - (a^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*b^(3/
2)*f) + Tan[e + f*x]/(b*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\tan(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{bf}$$
$$= \frac{\tan(e+fx)}{bf} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+\right.}{(a-b)bf}$$
$$= \frac{x}{a-b} - \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)b^{3/2}f} + \frac{\tan(e+fx)}{bf}$$

Mathematica [A] time = 0.29, size = 70, normalized size = 1.11

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{b^{3/2}f(a-b)} + \frac{e+fx}{f(a-b)} + \frac{\tan(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (e + f*x)/((a - b)*f) - (a^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((
a - b)*b^(3/2)*f) + Tan[e + f*x]/(b*f)
```

fricas [A] time = 0.45, size = 220, normalized size = 3.49

$$\left[\frac{4\,bfx - a\sqrt{-\frac{a}{b}}\,\log\left(\frac{b^2\tan(fx+e)^4 - 6\,ab\tan(fx+e)^2 + a^2 + 4\left(b^2\tan(fx+e)^3 - ab\tan(fx+e)\right)\sqrt{-\frac{a}{b}}}{b^2\tan(fx+e)^4 + 2\,ab\tan(fx+e)^2 + a^2}\right) + 4\,(a-b)\tan(fx+e)}{4\left(ab-b^2\right)f}, \frac{2\,bf\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*f*x - a*sqrt(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2
 + a^2 + 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x
 + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) + 4*(a - b)*tan(f*x + e))/((a*b - b^
2)*f), 1/2*(2*b*f*x - a*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/
b)/(a*tan(f*x + e))) + 2*(a - b)*tan(f*x + e))/((a*b - b^2)*f)]
```

giac [A] time = 2.98, size = 86, normalized size = 1.37

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)a^2}{(ab-b^2)\sqrt{ab}} - \frac{fx+e}{a-b} - \frac{\tan(fx+e)}{b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*
a^2/((a*b - b^2)*sqrt(a*b)) - (f*x + e)/(a - b) - tan(f*x + e)/b)/f
```

maple [A] time = 0.13, size = 70, normalized size = 1.11

$$\frac{\tan\left(fx+e\right)}{bf}-\frac{a^{2}\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{fb\left(a-b\right)\sqrt{ab}}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{f\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x)
```

```
[Out] tan(f*x+e)/b/f-1/f/b*a^2/(a-b)/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))
+1/f/(a-b)*arctan(tan(f*x+e))
```

maxima [A] time = 1.01, size = 65, normalized size = 1.03

$$-\frac{\frac{a^{2}\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(ab-b^{2}\right)\sqrt{ab}}-\frac{fx+e}{a-b}-\frac{\tan\left(fx+e\right)}{b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(a^2*arctan(b*tan(f*x + e)/sqrt(a*b))/((a*b - b^2)*sqrt(a*b)) - (f*x + e)/
(a - b) - tan(f*x + e)/b)/f
```

mupad [B] time = 12.18, size = 1212, normalized size = 19.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2),x)
```

```
[Out] tan(e + f*x)/(b*f) - (2*atan((((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b - (tan
(e + f*x)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3)*2i)/(b*(2*a - 2*b)))*1i
)/(2*a - 2*b) + (2*tan(e + f*x)*(a^4 + b^4))/b)/(2*a - 2*b) - ((((4*a*b^4 -
 8*a^2*b^3 + 4*a^3*b^2)/b + (tan(e + f*x)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*
a^3*b^3)*2i)/(b*(2*a - 2*b)))*1i)/(2*a - 2*b) - (2*tan(e + f*x)*(a^4 + b^4)
)/b)/(2*a - 2*b))/((((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b - (tan(e + f*x)*
(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3)*2i)/(b*(2*a - 2*b)))*1i)/(2*a - 2
*b) + (2*tan(e + f*x)*(a^4 + b^4))/b)*1i)/(2*a - 2*b) - (2*(a^2*b + a^3))/b
 + (((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b + (tan(e + f*x)*(4*a*b^5 - 4*b^6
 + 4*a^2*b^4 - 4*a^3*b^3)*2i)/(b*(2*a - 2*b)))*1i)/(2*a - 2*b) - (2*tan(e +
 f*x)*(a^4 + b^4))/b)*1i)/(2*a - 2*b))))/(f*(2*a - 2*b)) + (atan((((-a^3*b^
3)^(1/2)*((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b + (tan(e + f*x)*(-a^3*b^3)^
(1/2)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3))/(b*(a*b^3 - b^4)))*(-a^3*b
^3)^(1/2))/(2*(a*b^3 - b^4)) - (2*tan(e + f*x)*(a^4 + b^4))/b)*1i)/(2*(a*b^
3 - b^4)) - ((-a^3*b^3)^(1/2)*((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b - (tan
(e + f*x)*(-a^3*b^3)^(1/2)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3))/(b*(a
*b^3 - b^4)))*(-a^3*b^3)^(1/2))/(2*(a*b^3 - b^4)) + (2*tan(e + f*x)*(a^4 +
b^4))/b)*1i)/(2*(a*b^3 - b^4)))/(((-a^3*b^3)^(1/2)*((((4*a*b^4 - 8*a^2*b^3
+ 4*a^3*b^2)/b + (tan(e + f*x)*(-a^3*b^3)^(1/2)*(4*a*b^5 - 4*b^6 + 4*a^2*b^
4 - 4*a^3*b^3))/(b*(a*b^3 - b^4)))*(-a^3*b^3)^(1/2))/(2*(a*b^3 - b^4)) - (2
*tan(e + f*x)*(a^4 + b^4))/b))/(2*(a*b^3 - b^4)) - (2*(a^2*b + a^3))/b + ((
-a^3*b^3)^(1/2)*((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b - (tan(e + f*x)*(-a^
```

```
3*b^3)^(1/2)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3))/(b*(a*b^3 - b^4)))*
(-a^3*b^3)^(1/2))/(2*(a*b^3 - b^4)) + (2*tan(e + f*x)*(a^4 + b^4))/b))/(2*(
a*b^3 - b^4))))*(-a^3*b^3)^(1/2)*1i)/(f*(a*b^3 - b^4))
```

sympy [A] time = 6.76, size = 493, normalized size = 7.83

$$\begin{cases} \tilde{\infty} x \tan^2(e) \\ \frac{x+\frac{\tan^3(e+fx)}{3f}-\frac{\tan(e+fx)}{f}}{a} \\ \frac{-x+\frac{\tan(e+fx)}{f}}{b} \\ -\frac{3fx\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf}-\frac{3fx}{2bf\tan^2(e+fx)+2bf}+\frac{2\tan^3(e+fx)}{2bf\tan^2(e+fx)+2bf}+\frac{3\tan(e+fx)}{2bf\tan^2(e+fx)+2bf} \\ \frac{x\tan^4(e)}{a+b\tan^2(e)} \\ \frac{2ia^{\frac{3}{2}}b\sqrt{\frac{1}{b}}\tan(e+fx)}{2ia^{\frac{3}{2}}b^2f\sqrt{\frac{1}{b}}-2i\sqrt{a}b^3f\sqrt{\frac{1}{b}}}+\frac{2i\sqrt{a}b^2fx\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}b^2f\sqrt{\frac{1}{b}}-2i\sqrt{a}b^3f\sqrt{\frac{1}{b}}}-\frac{2i\sqrt{a}b^2\sqrt{\frac{1}{b}}\tan(e+fx)}{2ia^{\frac{3}{2}}b^2f\sqrt{\frac{1}{b}}-2i\sqrt{a}b^3f\sqrt{\frac{1}{b}}}-\frac{a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan(e+fx)\right)}{2ia^{\frac{3}{2}}b^2f\sqrt{\frac{1}{b}}-2i\sqrt{a}b^3f\sqrt{\frac{1}{b}}}+\frac{a^2\log\left(i\sqrt{a}\ldots\right.}{2ia^{\frac{3}{2}}b^2f\sqrt{\frac{1}{b}}\ldots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((x + tan(e +
f*x)**3/(3*f) - tan(e + f*x)/f)/a, Eq(b, 0)), ((-x + tan(e + f*x)/f)/b, Eq(
a, 0)), (-3*f*x*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*b*f) - 3*f*x/(2*
b*f*tan(e + f*x)**2 + 2*b*f) + 2*tan(e + f*x)**3/(2*b*f*tan(e + f*x)**2 + 2
*b*f) + 3*tan(e + f*x)/(2*b*f*tan(e + f*x)**2 + 2*b*f), Eq(a, b)), (x*tan(e
)**4/(a + b*tan(e)**2), Eq(f, 0)), (2*I*a**(3/2)*b*sqrt(1/b)*tan(e + f*x)/(
2*I*a**(3/2)*b**2*f*sqrt(1/b) - 2*I*sqrt(a)*b**3*f*sqrt(1/b)) + 2*I*sqrt(a)
*b**2*f*x*sqrt(1/b)/(2*I*a**(3/2)*b**2*f*sqrt(1/b) - 2*I*sqrt(a)*b**3*f*sqr
t(1/b)) - 2*I*sqrt(a)*b**2*sqrt(1/b)*tan(e + f*x)/(2*I*a**(3/2)*b**2*f*sqrt
(1/b) - 2*I*sqrt(a)*b**3*f*sqrt(1/b)) - a**2*log(-I*sqrt(a)*sqrt(1/b) + tan
(e + f*x))/(2*I*a**(3/2)*b**2*f*sqrt(1/b) - 2*I*sqrt(a)*b**3*f*sqrt(1/b)) +
 a**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*I*a**(3/2)*b**2*f*sqrt(1/b
) - 2*I*sqrt(a)*b**3*f*sqrt(1/b)), True))
```

3.219 $\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}f(a-b)}-\frac{x}{a-b}$$

```
[Out] -x/(a-b)+arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*a^(1/2)/(a-b)/f/b^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3670, 481, 203, 205}

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}f(a-b)}-\frac{x}{a-b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(x/(a - b)) + (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*Sq
rt[b]*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f} + \frac{a\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f}$$
$$= -\frac{x}{a-b} + \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)\sqrt{b}f}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.98

$$\frac{\tan^{-1}(\tan(e+fx)) - \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}}}{bf-af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (ArcTan[Tan[e + f*x]] - (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/Sq
rt[b])/(-(a*f) + b*f)
```

fricas [A] time = 0.45, size = 181, normalized size = 3.62

$$\left[-\frac{4fx + \sqrt{-\frac{a}{b}}\log\left(\frac{b^2\tan(fx+e)^4 - 6ab\tan(fx+e)^2 + a^2 - 4\left(b^2\tan(fx+e)^3 - ab\tan(fx+e)\right)\sqrt{-\frac{a}{b}}}{b^2\tan(fx+e)^4 + 2ab\tan(fx+e)^2 + a^2}\right)}{4(a-b)f}, -\frac{2fx - \sqrt{\frac{a}{b}}\arctan\left(\frac{(b\tan(fx+ \cdots}{2a\tan \cdots}\right.}{2(a-b)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*f*x + sqrt(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
a^2 - 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x +
e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a - b)*f), -1/2*(2*f*x - sqrt(a/b)*a
rctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/b)/(a*tan(f*x + e))))/((a - b)*f)]
```

giac [A] time = 2.08, size = 67, normalized size = 1.34

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(b) + \arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)a}{\sqrt{ab}(a-b)} - \frac{fx+e}{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*a
/(sqrt(a*b)*(a - b)) - (f*x + e)/(a - b))/f
```

maple [A] time = 0.16, size = 52, normalized size = 1.04

$$\frac{a\arctan\left(\frac{\tan(fx+e)b}{\sqrt{ab}}\right)}{f(a-b)\sqrt{ab}} - \frac{\arctan\left(\tan\left(fx+e\right)\right)}{f(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f*a/(a-b)/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/f/(a-b)*arctan(t
an(f*x+e))
```

maxima [A] time = 0.74, size = 47, normalized size = 0.94

$$\frac{\frac{a \arctan\left(\frac{b \tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a-b)} - \frac{fx+e}{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] (a*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (f*x + e)/(a - b)
)/f
```

mupad [B] time = 11.57, size = 135, normalized size = 2.70

$$-\frac{2\,\mathrm{atan}\left(\frac{\tan\left(e+f\,x\right)\,\left(2\,a^2\,b+2\,b^3\right)+\frac{\tan\left(e+f\,x\right)\,\left(-8\,a^3\,b^2+8\,a^2\,b^3+8\,a\,b^4-8\,b^5\right)}{{\left(2\,a-2\,b\right)}^2}}{a\,b\,\left(2\,a-2\,b\right)}\right)}{f\,\left(2\,a-2\,b\right)}-\frac{\mathrm{atanh}\left(\frac{\tan\left(e+f\,x\right)\,\sqrt{-a\,b}}{a}\right)\,\sqrt{-a\,b}}{f\,\left(a\,b-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2),x)
```

```
[Out] - (2*atan((tan(e + f*x)*(2*a^2*b + 2*b^3) + (tan(e + f*x)*(8*a*b^4 - 8*b^5
+ 8*a^2*b^3 - 8*a^3*b^2))/(2*a - 2*b)^2)/(a*b*(2*a - 2*b))))/(f*(2*a - 2*b)
) - (atanh((tan(e + f*x)*(-a*b)^(1/2))/a)*(-a*b)^(1/2))/(f*(a*b - b^2))
```

sympy [A] time = 2.29, size = 292, normalized size = 5.84

$$\begin{cases} \tilde{\infty} x & \text{for}\: a = 0 \wedge b = 0 \wedge f = 0 \\ \frac{x}{b} & \text{for}\: a = 0 \\ \frac{f x \tan^{2}{\left(e + f x \right)}}{2 b f \tan^{2}{\left(e + f x \right)} + 2 b f} + \frac{f x}{2 b f \tan^{2}{\left(e + f x \right)} + 2 b f} - \frac{\tan{\left(e + f x \right)}}{2 b f \tan^{2}{\left(e + f x \right)} + 2 b f} & \text{for}\: a = b \\ \frac{x \tan^{2}{\left(e \right)}}{a + b \tan^{2}{\left(e \right)}} & \text{for}\: f = 0 \\ \frac{- x + \frac{\tan{\left(e + f x \right)}}{f}}{a} & \text{for}\: b = 0 \\ - \frac{2 i \sqrt{a} b f x \sqrt{\frac{1}{b}}}{2 i a^{\frac{3}{2}} b f \sqrt{\frac{1}{b}} - 2 i \sqrt{a} b^{2} f \sqrt{\frac{1}{b}}} + \frac{a \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \tan{\left(e + f x \right)} \right)}}{2 i a^{\frac{3}{2}} b f \sqrt{\frac{1}{b}} - 2 i \sqrt{a} b^{2} f \sqrt{\frac{1}{b}}} - \frac{a \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \tan{\left(e + f x \right)} \right)}}{2 i a^{\frac{3}{2}} b f \sqrt{\frac{1}{b}} - 2 i \sqrt{a} b^{2} f \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/b, Eq(a, 0)), (f*x*ta
n(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*b*f) + f*x/(2*b*f*tan(e + f*x)**2
+ 2*b*f) - tan(e + f*x)/(2*b*f*tan(e + f*x)**2 + 2*b*f), Eq(a, b)), (x*tan(
e)**2/(a + b*tan(e)**2), Eq(f, 0)), ((-x + tan(e + f*x)/f)/a, Eq(b, 0)), (-
```

```
2*I*sqrt(a)*b*f*x*sqrt(1/b)/(2*I*a**(3/2)*b*f*sqrt(1/b) - 2*I*sqrt(a)*b**2*
f*sqrt(1/b)) + a*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*I*a**(3/2)*b*f
*sqrt(1/b) - 2*I*sqrt(a)*b**2*f*sqrt(1/b)) - a*log(I*sqrt(a)*sqrt(1/b) + ta
n(e + f*x))/(2*I*a**(3/2)*b*f*sqrt(1/b) - 2*I*sqrt(a)*b**2*f*sqrt(1/b)), Tr
ue))
```

3.220 $\int \frac{1}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\,f(a-b)}$$

```
[Out] x/(a-b)-arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/(a-b)/f/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3660, 3675, 205}

$$\frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\,f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-1),x]
```

```
[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a -
b)*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3660

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\tan^2(e+fx)}\,dx &= \frac{x}{a-b}-\frac{b\int \frac{\sec^2(e+fx)}{a+b\tan^2(e+fx)}\,dx}{a-b}\\ &= \frac{x}{a-b}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\tan(e+fx)\right)}{(a-b)f}\\ &= \frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)f}\end{aligned}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.98

$$\frac{\tan^{-1}(\tan(e+fx))-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}}}{af-bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-1),x]
```

```
[Out] (ArcTan[Tan[e + f*x]] - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/Sq
rt[a])/(a*f - b*f)
```

fricas [A] time = 0.46, size = 182, normalized size = 3.64

$$\left[\frac{4\,fx-\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan\left(fx+e\right)^4-6\,ab\tan\left(fx+e\right)^2+a^2+4\left(ab\tan\left(fx+e\right)^3-a^2\tan\left(fx+e\right)\right)\sqrt{-\frac{b}{a}}}{b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2}\right)}{4\,(a-b)f},\frac{2\,fx-\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan\left(fx+e\right)^2-\right.}{2\,b\tan\left(fx\right.}\right.}{2\,(a-b)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x - sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a
^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e
)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a - b)*f), 1/2*(2*f*x - sqrt(b/a)*arc
tan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a - b)*f)]
```

giac [A] time = 1.55, size = 68, normalized size = 1.36

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,(a-b)}-\frac{fx+e}{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*
b/(sqrt(a*b)*(a - b)) - (f*x + e)/(a - b))/f
```

maple [A] time = 0.26, size = 52, normalized size = 1.04

$$-\frac{b\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{f\,(a-b)\sqrt{ab}}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{f\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f*b/(a-b)/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+1/f/(a-b)*arctan(
tan(f*x+e))
```

maxima [A] time = 0.76, size = 48, normalized size = 0.96

$$-\frac{\frac{b\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a-b)}-\frac{fx+e}{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (f*x + e)/(a - b
))/f
```

mupad [B] time = 12.30, size = 948, normalized size = 18.96

$$-\frac{\operatorname{atan}\left(\frac{\frac{-4b^3\tan(e+fx)+\frac{\left(4b^4-8ab^3+4a^2b^2+\frac{\tan(e+fx)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1i}{2a-2b}\right)1i}{2a-2b}}{2a-2b}+\frac{-4b^3\tan(e+fx)+\frac{\left(8ab^3-4b^4-4a^2b^2+\frac{\tan(e+fx)\left(-8a^3b^2+8a^2b^3\cdots\right.}{2a-2b}\right.}{2a-2b}}{2a-2b}}{\frac{\left(-4b^3\tan(e+fx)+\frac{\left(4b^4-8ab^3+4a^2b^2+\frac{\tan(e+fx)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1i}{2a-2b}\right)1i}{2a-2b}\right)1i}{2a-2b}-\frac{\left(-4b^3\tan(e+fx)+\frac{\left(8ab^3-4b^4-4a^2b^2+\frac{\tan(e+fx)\left(-8a^3b^2+8a^2b^3\cdots\right.}{2a-2b}\right.}{2a-2b}\right.}{2a-2b}}\right.}{f\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (atan((((-a*b)^(1/2)*(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 +
 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^
3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))*1i)/(a*b - a^2) + ((-a*b)^(1/2)*
(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(4*a*b^3 - 2*b^4 - 2*a^2*b^2 + (tan(e +
 f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2
))))/(2*(a*b - a^2)))*1i)/(a*b - a^2))/(((-a*b)^(1/2)*(2*b^3*tan(e + f*x) -
 ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 + 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8
*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))
)/(a*b - a^2) - ((-a*b)^(1/2)*(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(4*a*b^3
- 2*b^4 - 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b
^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2))))/(a*b - a^2)))*(-a*b)^(
1/2)*1i)/(a*f*(a - b)) - atan(((((4*b^4 - 8*a*b^3 + 4*a^2*b^2 + (tan(e + f*
x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*
b) - 4*b^3*tan(e + f*x))/(2*a - 2*b) + (((8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (ta
n(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(
2*a - 2*b) - 4*b^3*tan(e + f*x))/(2*a - 2*b))/(((((4*b^4 - 8*a*b^3 + 4*a^2*
b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*
b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))*1i)/(2*a - 2*b) - ((((8*a*b^3 - 4
*b^4 - 4*a^2*b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*
1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))*1i)/(2*a - 2*b)))/(f
*(a - b))
```

sympy [A] time = 2.31, size = 280, normalized size = 5.60

$$\begin{cases} \frac{\tilde{\infty} x}{\tan^{2}{(e)}} & \text{for}\: a = 0 \wedge b = 0 \wedge f = 0 \\ \frac{- x - \frac{1}{f \tan{\left(e + f x \right)}}}{b} & \text{for}\: a = 0 \\ \frac{f x \tan^{2}{\left(e + f x \right)}}{2 b f \tan^{2}{\left(e + f x \right)} + 2 b f} + \frac{f x}{2 b f \tan^{2}{\left(e + f x \right)} + 2 b f} + \frac{\tan{\left(e + f x \right)}}{2 b f \tan^{2}{\left(e + f x \right)} + 2 b f} & \text{for}\: a = b \\ \frac{x}{a + b \tan^{2}{(e)}} & \text{for}\: f = 0 \\ \frac{x}{a} & \text{for}\: b = 0 \\ \frac{2 i \sqrt{a} f x \sqrt{\frac{1}{b}}}{2 i a^{\frac{3}{2}} f \sqrt{\frac{1}{b}} - 2 i \sqrt{a} b f \sqrt{\frac{1}{b}}} - \frac{\log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \tan{\left(e + f x \right)} \right)}}{2 i a^{\frac{3}{2}} f \sqrt{\frac{1}{b}} - 2 i \sqrt{a} b f \sqrt{\frac{1}{b}}} + \frac{\log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \tan{\left(e + f x \right)} \right)}}{2 i a^{\frac{3}{2}} f \sqrt{\frac{1}{b}} - 2 i \sqrt{a} b f \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x - 1/(f*ta
n(e + f*x)))/b, Eq(a, 0)), (f*x*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*
b*f) + f*x/(2*b*f*tan(e + f*x)**2 + 2*b*f) + tan(e + f*x)/(2*b*f*tan(e + f*
x)**2 + 2*b*f), Eq(a, b)), (x/(a + b*tan(e)**2), Eq(f, 0)), (x/a, Eq(b, 0))
, (2*I*sqrt(a)*f*x*sqrt(1/b)/(2*I*a**(3/2)*f*sqrt(1/b) - 2*I*sqrt(a)*b*f*sq
rt(1/b)) - log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*I*a**(3/2)*f*sqrt(1/
b) - 2*I*sqrt(a)*b*f*sqrt(1/b)) + log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(
2*I*a**(3/2)*f*sqrt(1/b) - 2*I*sqrt(a)*b*f*sqrt(1/b)), True))
```

3.221 $\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f(a-b)}-\frac{x}{a-b}-\frac{\cot(e+fx)}{af}$$

```
[Out] -x/(a-b)+b^(3/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(3/2)/(a-b)/f-cot(f*x
+e)/a/f
```

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 480, 522, 203, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f(a-b)}-\frac{x}{a-b}-\frac{\cot(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(x/(a - b)) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(3/2)*(a
 - b)*f) - Cot[e + f*x]/(a*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)}{af} + \frac{\text{Subst}\left(\int \frac{-a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= -\frac{\cot(e+fx)}{af} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e\right.}{a(a-b)f}$$

$$= -\frac{x}{a-b} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)f} - \frac{\cot(e+fx)}{af}$$

Mathematica [A] time = 0.26, size = 68, normalized size = 1.06

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) - \sqrt{a}\,((a-b)\cot(e+fx) + a(e+fx))}{a^{3/2}f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - Sqrt[a]*(a*(e + f*x) + (a
 - b)*Cot[e + f*x]))/(a^(3/2)*(a - b)*f)
```

fricas [A] time = 0.47, size = 243, normalized size = 3.80

$$\left[-\frac{4\,afx\tan\left(fx+e\right) + b\sqrt{-\frac{b}{a}}\,\log\left(\frac{b^2\tan\left(fx+e\right)^4 - 6\,ab\tan\left(fx+e\right)^2 + a^2 - 4\left(ab\tan\left(fx+e\right)^3 - a^2\tan\left(fx+e\right)\right)\sqrt{-\frac{b}{a}}}{b^2\tan\left(fx+e\right)^4 + 2\,ab\tan\left(fx+e\right)^2 + a^2}\right)\tan\left(fx+e\right) + 4}{4\left(a^2-ab\right)f\tan\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*f*x*tan(f*x + e) + b*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*
tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a)
)/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2))*tan(f*x + e) + 4*a - 4
*b)/((a^2 - a*b)*f*tan(f*x + e)), -1/2*(2*a*f*x*tan(f*x + e) - b*sqrt(b/a)*
arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e)))*tan(f*x + e)
+ 2*a - 2*b)/((a^2 - a*b)*f*tan(f*x + e))]
```

giac [A] time = 2.48, size = 86, normalized size = 1.34

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(b) + \arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)b^2}{\left(a^2-ab\right)\sqrt{ab}} - \frac{fx+e}{a-b} - \frac{1}{a\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*b
^2/((a^2 - a*b)*sqrt(a*b)) - (f*x + e)/(a - b) - 1/(a*tan(f*x + e)))/f
```

maple [A] time = 0.61, size = 73, normalized size = 1.14

$$\frac{b^{2}\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{fa\left(a-b\right)\sqrt{ab}}-\frac{1}{fa\tan\left(fx+e\right)}-\frac{\arctan\left(\tan\left(fx+e\right)\right)}{f\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f/a*b^2/(a-b)/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/f/a/tan(f*x+
e)-1/f/(a-b)*arctan(tan(f*x+e))
```

maxima [A] time = 1.07, size = 65, normalized size = 1.02

$$\frac{\frac{b^{2}\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^{2}-ab\right)\sqrt{ab}}-\frac{fx+e}{a-b}-\frac{1}{a\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] (b^2*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2 - a*b)*sqrt(a*b)) - (f*x + e)/(
a - b) - 1/(a*tan(f*x + e)))/f
```

mupad [B] time = 11.77, size = 438, normalized size = 6.84

$$\frac{a^{2}b-a^{3}}{f\left(a^{4}\tan\left(e+fx\right)-a^{3}b\tan\left(e+fx\right)\right)}+\frac{\mathrm{atan}\left(\frac{a^{6}b\tan\left(e+fx\right)\sqrt{-a^{3}b^{3}}\,1\mathrm{i}-a^{3}b^{4}\tan\left(e+fx\right)\sqrt{-a^{3}b^{3}}\,1\mathrm{i}}{a^{5}b^{5}-a^{8}b^{2}}\right)\sqrt{-a^{3}b^{3}}\,1\mathrm{i}-a^{3}\mathrm{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (a^2*b - a^3)/(f*(a^4*tan(e + f*x) - a^3*b*tan(e + f*x))) + (atan((a^6*b*ta
n(e + f*x)*(-a^3*b^3)^(1/2)*1i - a^3*b^4*tan(e + f*x)*(-a^3*b^3)^(1/2)*1i)/
(a^5*b^5 - a^8*b^2))*(-a^3*b^3)^(1/2)*1i - a^3*atan(((((4*a^5*b^4 - 4*a^4*b
^5 + 4*a^6*b^3 - 4*a^7*b^2 + (tan(e + f*x)*(8*a^5*b^5 - 8*a^6*b^4 - 8*a^7*b
^3 + 8*a^8*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) + tan(e + f*x)*(2*a^3*b^5
+ 2*a^5*b^3))/(2*a - 2*b) + (((4*a^4*b^5 - 4*a^5*b^4 - 4*a^6*b^3 + 4*a^7*b^
2 + (tan(e + f*x)*(8*a^5*b^5 - 8*a^6*b^4 - 8*a^7*b^3 + 8*a^8*b^2)*1i)/(2*a
- 2*b))*1i)/(2*a - 2*b) + tan(e + f*x)*(2*a^3*b^5 + 2*a^5*b^3))/(2*a - 2*b)
)/(2*a^3*b^4 + 2*a^4*b^3 + 2*a^5*b^2)))/(f*(a^3*b - a^4))
```

sympy [A] time = 14.92, size = 570, normalized size = 8.91

$$\begin{cases} \tilde{\infty} x \\ \frac{x+\frac{1}{f\tan\left(e+fx\right)}-\frac{1}{3f\tan^{3}\left(e+fx\right)}}{b} \\ -\frac{3fx\tan^{3}\left(e+fx\right)}{2bf\tan^{3}\left(e+fx\right)+2bf\tan\left(e+fx\right)}-\frac{3fx\tan\left(e+fx\right)}{2bf\tan^{3}\left(e+fx\right)+2bf\tan\left(e+fx\right)}-\frac{3\tan^{2}\left(e+fx\right)}{2bf\tan^{3}\left(e+fx\right)+2bf\tan\left(e+fx\right)}-\frac{2}{2bf\tan^{3}\left(e+fx\right)+2bf\tan\left(e+fx\right.} \\ \frac{\tilde{\infty} x}{a} \\ \frac{x\cot^{2}\left(e\right)}{a+b\tan^{2}\left(e\right)} \\ \frac{-x-\frac{\cot\left(e+fx\right)}{f}}{a} \\ -\frac{2ia^{\frac{3}{2}}fx\sqrt{\frac{1}{b}}\tan\left(e+fx\right)}{2ia^{\frac{5}{2}}f\sqrt{\frac{1}{b}}\tan\left(e+fx\right)-2ia^{\frac{3}{2}}bf\sqrt{\frac{1}{b}}\tan\left(e+fx\right)}-\frac{2ia^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{2ia^{\frac{5}{2}}f\sqrt{\frac{1}{b}}\tan\left(e+fx\right)-2ia^{\frac{3}{2}}bf\sqrt{\frac{1}{b}}\tan\left(e+fx\right)}+\frac{2i\sqrt{a}b\sqrt{\frac{1}{b}}}{2ia^{\frac{5}{2}}f\sqrt{\frac{1}{b}}\tan\left(e+fx\right)-2ia^{\frac{3}{2}}bf\sqrt{\frac{1}{b}}\tan\left(e\right.} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((x + 1/(f*ta
n(e + f*x)) - 1/(3*f*tan(e + f*x)**3))/b, Eq(a, 0)), (-3*f*x*tan(e + f*x)**
3/(2*b*f*tan(e + f*x)**3 + 2*b*f*tan(e + f*x)) - 3*f*x*tan(e + f*x)/(2*b*f*
tan(e + f*x)**3 + 2*b*f*tan(e + f*x)) - 3*tan(e + f*x)**2/(2*b*f*tan(e + f*
x)**3 + 2*b*f*tan(e + f*x)) - 2/(2*b*f*tan(e + f*x)**3 + 2*b*f*tan(e + f*x)
), Eq(a, b)), (zoo*x/a, Eq(e, -f*x)), (x*cot(e)**2/(a + b*tan(e)**2), Eq(f,
 0)), ((-x - cot(e + f*x)/f)/a, Eq(b, 0)), (-2*I*a**(3/2)*f*x*sqrt(1/b)*tan
(e + f*x)/(2*I*a**(5/2)*f*sqrt(1/b)*tan(e + f*x) - 2*I*a**(3/2)*b*f*sqrt(1/
b)*tan(e + f*x)) - 2*I*a**(3/2)*sqrt(1/b)/(2*I*a**(5/2)*f*sqrt(1/b)*tan(e +
 f*x) - 2*I*a**(3/2)*b*f*sqrt(1/b)*tan(e + f*x)) + 2*I*sqrt(a)*b*sqrt(1/b)/
(2*I*a**(5/2)*f*sqrt(1/b)*tan(e + f*x) - 2*I*a**(3/2)*b*f*sqrt(1/b)*tan(e +
 f*x)) + b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)/(2*I*a**(5
/2)*f*sqrt(1/b)*tan(e + f*x) - 2*I*a**(3/2)*b*f*sqrt(1/b)*tan(e + f*x)) - b
*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)/(2*I*a**(5/2)*f*sqrt(
1/b)*tan(e + f*x) - 2*I*a**(3/2)*b*f*sqrt(1/b)*tan(e + f*x)), True))
```

3.222 $\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f(a-b)}+\frac{(a+b)\cot(e+fx)}{a^2f}+\frac{x}{a-b}-\frac{\cot^3(e+fx)}{3af}$$

```
[Out] x/(a-b)-b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(5/2)/(a-b)/f+(a+b)*co
t(f*x+e)/a^2/f-1/3*cot(f*x+e)^3/a/f
```

Rubi [A] time = 0.17, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 480, 583, 522, 203, 205}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f(a-b)}+\frac{(a+b)\cot(e+fx)}{a^2f}+\frac{x}{a-b}-\frac{\cot^3(e+fx)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] x/(a - b) - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(5/2)*(a -
b)*f) + ((a + b)*Cot[e + f*x])/(a^2*f) - Cot[e + f*x]^3/(3*a*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
```

```
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3af} + \frac{\text{Subst}\left(\int \frac{-3(a+b)-3bx^2}{x^2(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{3af}$$

$$= \frac{(a+b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af} - \frac{\text{Subst}\left(\int \frac{-3(a^2+ab+b^2)-3b(a+b)x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+
}{3a^2f}$$

$$= \frac{(a+b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f} - \frac{b^3\,\text{Subst}}{}$$

$$= \frac{x}{a-b} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)f} + \frac{(a+b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af}$$

Mathematica [A] time = 0.70, size = 92, normalized size = 1.10

$$\frac{\sqrt{a}\left(3a^2(e+fx)-(a-b)\cot(e+fx)\left(a\csc^2(e+fx)-4a-3b\right)\right)-3b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{3a^{5/2}f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-3*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(3*a^2*(e + f*
x) - (a - b)*Cot[e + f*x]*(-4*a - 3*b + a*Csc[e + f*x]^2)))/(3*a^(5/2)*(a -
 b)*f)
```

fricas [A] time = 0.51, size = 308, normalized size = 3.67

$$\left[\frac{12\,a^2fx\tan\left(fx+e\right)^3 - 3\,b^2\sqrt{-\frac{b}{a}}\,\log\left(\frac{b^2\tan\left(fx+e\right)^4-6\,ab\tan\left(fx+e\right)^2+a^2+4\left(ab\tan\left(fx+e\right)^3-a^2\tan\left(fx+e\right)\right)\sqrt{-\frac{b}{a}}}{b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2}\right)\tan\left(fx+e\right.}{12\left(a^3-a^2b\right)f\tan\left(fx+e\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(12*a^2*f*x*tan(f*x + e)^3 - 3*b^2*sqrt(-b/a)*log((b^2*tan(f*x + e)^4
 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*s
qrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2))*tan(f*x + e)^
3 + 12*(a^2 - b^2)*tan(f*x + e)^2 - 4*a^2 + 4*a*b)/((a^3 - a^2*b)*f*tan(f*x
 + e)^3), 1/6*(6*a^2*f*x*tan(f*x + e)^3 - 3*b^2*sqrt(b/a)*arctan(1/2*(b*tan
(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e)))*tan(f*x + e)^3 + 6*(a^2 - b^2)
*tan(f*x + e)^2 - 2*a^2 + 2*a*b)/((a^3 - a^2*b)*f*tan(f*x + e)^3)]
```

giac [A] time = 3.25, size = 118, normalized size = 1.40

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)b^3}{\left(a^3-a^2b\right)\sqrt{ab}}-\frac{3\left(fx+e\right)}{a-b}-\frac{3a\tan\left(fx+e\right)^2+3b\tan\left(fx+e\right)^2-a}{a^2\tan\left(fx+e\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*b^3/((a^3 - a^2*b)*sqrt(a*b)) - 3*(f*x + e)/(a - b) - (3*a*tan(f*x +
e)^2 + 3*b*tan(f*x + e)^2 - a)/(a^2*tan(f*x + e)^3))/f
```

maple [A] time = 0.84, size = 104, normalized size = 1.24

$$-\frac{b^3\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{f\,a^2\left(a-b\right)\sqrt{ab}}-\frac{1}{3fa\tan\left(fx+e\right)^3}+\frac{1}{fa\tan\left(fx+e\right)}+\frac{b}{f\,a^2\tan\left(fx+e\right)}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{f\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x)
```

```
[Out] -1/f/a^2*b^3/(a-b)/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/3/f/a/tan
(f*x+e)^3+1/f/a/tan(f*x+e)+1/f/a^2/tan(f*x+e)*b+1/f/(a-b)*arctan(tan(f*x+e)
)
```

maxima [A] time = 0.96, size = 86, normalized size = 1.02

$$-\frac{\frac{3b^3\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^3-a^2b\right)\sqrt{ab}}-\frac{3\left(fx+e\right)}{a-b}-\frac{3\left(a+b\right)\tan\left(fx+e\right)^2-a}{a^2\tan\left(fx+e\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^3*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - a^2*b)*sqrt(a*b)) - 3*
(f*x + e)/(a - b) - (3*(a + b)*tan(f*x + e)^2 - a)/(a^2*tan(f*x + e)^3))/f
```

mupad [B] time = 12.20, size = 484, normalized size = 5.76

$$\frac{a^4b+\tan\left(e+fx\right)^2\left(3a^5-3a^3b^2\right)-a^5}{f\left(3a^6\tan\left(e+fx\right)^3-3a^5b\tan\left(e+fx\right)^3\right)}-\frac{\mathrm{atan}\left(\frac{a^{10}b\tan\left(e+fx\right)\sqrt{-a^5b^5}\,1\mathrm{i}-a^5b^6\tan\left(e+fx\right)\sqrt{-a^5b^5}\,1\mathrm{i}}{a^8b^8-a^{13}b^3}\right)\sqrt{-a^5b^5}\,3\mathrm{i}-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (a^4*b + tan(e + f*x)^2*(3*a^5 - 3*a^3*b^2) - a^5)/(f*(3*a^6*tan(e + f*x)^3
 - 3*a^5*b*tan(e + f*x)^3)) - (atan((a^10*b*tan(e + f*x)*(-a^5*b^5)^(1/2)*1
i - a^5*b^6*tan(e + f*x)*(-a^5*b^5)^(1/2)*1i)/(a^8*b^8 - a^13*b^3))*(-a^5*b
^5)^(1/2)*3i - 3*a^5*atan(((((4*a^9*b^5 - 4*a^8*b^6 + 4*a^11*b^3 - 4*a^12*b
^2 + (tan(e + f*x)*(8*a^10*b^5 - 8*a^11*b^4 - 8*a^12*b^3 + 8*a^13*b^2)*1i)/
(2*a - 2*b))*1i)/(2*a - 2*b) + tan(e + f*x)*(2*a^6*b^7 + 2*a^10*b^3))/(2*a
- 2*b) + (((4*a^8*b^6 - 4*a^9*b^5 - 4*a^11*b^3 + 4*a^12*b^2 + (tan(e + f*x)
*(8*a^10*b^5 - 8*a^11*b^4 - 8*a^12*b^3 + 8*a^13*b^2)*1i)/(2*a - 2*b))*1i)/(
2*a - 2*b) + tan(e + f*x)*(2*a^6*b^7 + 2*a^10*b^3))/(2*a - 2*b))/(2*a^6*b^6
 + 2*a^7*b^5 + 2*a^8*b^4 + 2*a^9*b^3 + 2*a^10*b^2)))/(f*(3*a^5*b - 3*a^6))
```

sympy [A] time = 57.33, size = 823, normalized size = 9.80

$$\begin{cases} \tilde{\infty} x \\ \frac{-x - \frac{1}{f\tan\left(e+fx\right)} + \frac{1}{3f\tan^{3}\left(e+fx\right)} - \frac{1}{5f\tan^{5}\left(e+fx\right)}}{b} \\ \frac{15fx\tan^{5}\left(e+fx\right)}{6bf\tan^{5}\left(e+fx\right)+6bf\tan^{3}\left(e+fx\right)} + \frac{15fx\tan^{3}\left(e+fx\right)}{6bf\tan^{5}\left(e+fx\right)+6bf\tan^{3}\left(e+fx\right)} + \frac{15\tan^{4}\left(e+fx\right)}{6bf\tan^{5}\left(e+fx\right)+6bf\tan^{3}\left(e+fx\right)} + \frac{10\tan^{2}\left(e+fx\right)}{6bf\tan^{5}\left(e+fx\right)+6bf\tan^{3}\left(e+ \right.} \\ \frac{\tilde{\infty} x}{a} \\ \frac{x\cot^{4}\left(e\right)}{a+b\tan^{2}\left(e\right)} \\ \frac{x - \frac{\cot^{3}\left(e+fx\right)}{3f} + \frac{\cot\left(e+fx\right)}{f}}{a} \\ \frac{6ia^{\frac{5}{2}}fx\sqrt{\frac{1}{b}}\tan^{3}\left(e+fx\right)}{6ia^{\frac{7}{2}}f\sqrt{\frac{1}{b}}\tan^{3}\left(e+fx\right)-6ia^{\frac{5}{2}}bf\sqrt{\frac{1}{b}}\tan^{3}\left(e+fx\right)} + \frac{6ia^{\frac{5}{2}}\sqrt{\frac{1}{b}}\tan^{2}\left(e+fx\right)}{6ia^{\frac{7}{2}}f\sqrt{\frac{1}{b}}\tan^{3}\left(e+fx\right)-6ia^{\frac{5}{2}}bf\sqrt{\frac{1}{b}}\tan^{3}\left(e+fx\right)} - \frac{2ia^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{6ia^{\frac{7}{2}}f\sqrt{\frac{1}{b}}\tan^{3}\left(e+fx\right)-6ia^{\frac{5}{2}}bf\sqrt{\frac{1}{b}}\tan} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((-x - 1/(f*t
an(e + f*x)) + 1/(3*f*tan(e + f*x)**3) - 1/(5*f*tan(e + f*x)**5))/b, Eq(a,
0)), (15*f*x*tan(e + f*x)**5/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3
) + 15*f*x*tan(e + f*x)**3/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3)
+ 15*tan(e + f*x)**4/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3) + 10*t
an(e + f*x)**2/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3) - 2/(6*b*f*t
an(e + f*x)**5 + 6*b*f*tan(e + f*x)**3), Eq(a, b)), (zoo*x/a, Eq(e, -f*x)),
 (x*cot(e)**4/(a + b*tan(e)**2), Eq(f, 0)), ((x - cot(e + f*x)**3/(3*f) + c
ot(e + f*x)/f)/a, Eq(b, 0)), (6*I*a**(5/2)*f*x*sqrt(1/b)*tan(e + f*x)**3/(6
*I*a**(7/2)*f*sqrt(1/b)*tan(e + f*x)**3 - 6*I*a**(5/2)*b*f*sqrt(1/b)*tan(e
+ f*x)**3) + 6*I*a**(5/2)*sqrt(1/b)*tan(e + f*x)**2/(6*I*a**(7/2)*f*sqrt(1/
b)*tan(e + f*x)**3 - 6*I*a**(5/2)*b*f*sqrt(1/b)*tan(e + f*x)**3) - 2*I*a**(
5/2)*sqrt(1/b)/(6*I*a**(7/2)*f*sqrt(1/b)*tan(e + f*x)**3 - 6*I*a**(5/2)*b*f
*sqrt(1/b)*tan(e + f*x)**3) + 2*I*a**(3/2)*b*sqrt(1/b)/(6*I*a**(7/2)*f*sqrt
(1/b)*tan(e + f*x)**3 - 6*I*a**(5/2)*b*f*sqrt(1/b)*tan(e + f*x)**3) - 6*I*s
qrt(a)*b**2*sqrt(1/b)*tan(e + f*x)**2/(6*I*a**(7/2)*f*sqrt(1/b)*tan(e + f*x
)**3 - 6*I*a**(5/2)*b*f*sqrt(1/b)*tan(e + f*x)**3) - 3*b**2*log(-I*sqrt(a)*
sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**3/(6*I*a**(7/2)*f*sqrt(1/b)*tan(e +
 f*x)**3 - 6*I*a**(5/2)*b*f*sqrt(1/b)*tan(e + f*x)**3) + 3*b**2*log(I*sqrt(
a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**3/(6*I*a**(7/2)*f*sqrt(1/b)*tan(
e + f*x)**3 - 6*I*a**(5/2)*b*f*sqrt(1/b)*tan(e + f*x)**3), True))
```

3.223 $\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=113

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f(a-b)}+\frac{(a+b)\cot^3(e+fx)}{3a^2f}-\frac{\left(a^2+ab+b^2\right)\cot(e+fx)}{a^3f}-\frac{x}{a-b}-\frac{\cot^5(e+fx)}{5af}$$

```
[Out] -x/(a-b)+b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(7/2)/(a-b)/f-(a^2+a*
b+b^2)*cot(f*x+e)/a^3/f+1/3*(a+b)*cot(f*x+e)^3/a^2/f-1/5*cot(f*x+e)^5/a/f
```

Rubi [A] time = 0.24, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 480, 583, 522, 203, 205}

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f(a-b)}-\frac{\left(a^2+ab+b^2\right)\cot(e+fx)}{a^3f}+\frac{(a+b)\cot^3(e+fx)}{3a^2f}-\frac{x}{a-b}-\frac{\cot^5(e+fx)}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(x/(a - b)) + (b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*(a
 - b)*f) - ((a^2 + a*b + b^2)*Cot[e + f*x])/(a^3*f) + ((a + b)*Cot[e + f*x]
^3)/(3*a^2*f) - Cot[e + f*x]^5/(5*a*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
```

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af} + \frac{\text{Subst}\left(\int \frac{-5(a+b)-5bx^2}{x^4(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= \frac{(a+b)\cot^3(e+fx)}{3a^2f} - \frac{\cot^5(e+fx)}{5af} - \frac{\text{Subst}\left(\int \frac{-15(a^2+ab+b^2)-15b(a+b)x^2}{x^2(1+x^2)(a+bx^2)}\,dx, x, \tan(e\right.}{15a^2f}$$

$$= -\frac{(a^2+ab+b^2)\cot(e+fx)}{a^3f} + \frac{(a+b)\cot^3(e+fx)}{3a^2f} - \frac{\cot^5(e+fx)}{5af} + \frac{\text{Subst}\left(\int \frac{-15}{}\right.}{}$$

$$= -\frac{(a^2+ab+b^2)\cot(e+fx)}{a^3f} + \frac{(a+b)\cot^3(e+fx)}{3a^2f} - \frac{\cot^5(e+fx)}{5af} - \frac{\text{Subst}\left(\int \frac{1}{1+x}\right.}{(}$$

$$= -\frac{x}{a-b} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}(a-b)f} - \frac{(a^2+ab+b^2)\cot(e+fx)}{a^3f} + \frac{(a+b)\cot^3(e+}{3a^2f}$$

Mathematica [A] time = 1.92, size = 121, normalized size = 1.07

$$\frac{\sqrt{a}\left(-15a^3(e+fx)-(a-b)\cot(e+fx)\left(3a^2\csc^4(e+fx)+23a^2-a(11a+5b)\csc^2(e+fx)+20ab+15b^2\right)\right)}{15a^{7/2}f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (15*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(-15*a^3*(e +
f*x) - (a - b)*Cot[e + f*x]*(23*a^2 + 20*a*b + 15*b^2 - a*(11*a + 5*b)*Csc[
e + f*x]^2 + 3*a^2*Csc[e + f*x]^4)))/(15*a^(7/2)*(a - b)*f)
```

fricas [A] time = 0.46, size = 352, normalized size = 3.12

$$\left[-\frac{60\,a^3fx\tan\left(fx+e\right)^5+15\,b^3\sqrt{-\frac{b}{a}}\,\log\left(\frac{b^2\tan\left(fx+e\right)^4-6\,ab\tan\left(fx+e\right)^2+a^2-4\left(ab\tan\left(fx+e\right)^3-a^2\tan\left(fx+e\right)\right)\sqrt{-\frac{b}{a}}}{b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2}\right)\tan\left(fx\right.}{60\left(a^4-a^3b\right)f\tan\left(fx+e\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(60*a^3*f*x*tan(f*x + e)^5 + 15*b^3*sqrt(-b/a)*log((b^2*tan(f*x + e)
^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))
*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2))*tan(f*x + e
)^5 + 60*(a^3 - b^3)*tan(f*x + e)^4 + 12*a^3 - 12*a^2*b - 20*(a^3 - a*b^2)*
tan(f*x + e)^2)/((a^4 - a^3*b)*f*tan(f*x + e)^5), -1/30*(30*a^3*f*x*tan(f*x
 + e)^5 - 15*b^3*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*t
an(f*x + e)))*tan(f*x + e)^5 + 30*(a^3 - b^3)*tan(f*x + e)^4 + 6*a^3 - 6*a^
2*b - 10*(a^3 - a*b^2)*tan(f*x + e)^2)/((a^4 - a^3*b)*f*tan(f*x + e)^5)]
```

giac [A] time = 3.85, size = 164, normalized size = 1.45

$$\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)b^4}{\left(a^4-a^3b\right)\sqrt{ab}}-\frac{15\left(fx+e\right)}{a-b}-\frac{15\,a^2\tan\left(fx+e\right)^4+15\,ab\tan\left(fx+e\right)^4+15\,b^2\tan\left(fx+e\right)^4-5\,a^2\tan\left(fx+e\right)^2-5\,ab\tan\left(fx\right.}{a^3\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b)))*b^4/((a^4 - a^3*b)*sqrt(a*b)) - 15*(f*x + e)/(a - b) - (15*a^2*tan(f
*x + e)^4 + 15*a*b*tan(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 - 5*a^2*tan(f*x +
 e)^2 - 5*a*b*tan(f*x + e)^2 + 3*a^2)/(a^3*tan(f*x + e)^5))/f
```

maple [A] time = 0.89, size = 158, normalized size = 1.40

$$\frac{b^4\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{f\,a^3\left(a-b\right)\sqrt{ab}}-\frac{1}{5fa\tan\left(fx+e\right)^5}+\frac{1}{3fa\tan\left(fx+e\right)^3}+\frac{b}{3f\,a^2\tan\left(fx+e\right)^3}-\frac{1}{fa\tan\left(fx+e\right)}-\frac{b}{f\,a^2\tan\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x)
```

```
[Out] 1/f/a^3*b^4/(a-b)/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/5/f/a/tan(
f*x+e)^5+1/3/f/a/tan(f*x+e)^3+1/3/f/a^2/tan(f*x+e)^3*b-1/f/a/tan(f*x+e)-1/f
/a^2/tan(f*x+e)*b-1/f/a^3/tan(f*x+e)*b^2-1/f/(a-b)*arctan(tan(f*x+e))
```

maxima [A] time = 0.71, size = 112, normalized size = 0.99

$$\frac{\frac{15\,b^4\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^4-a^3b\right)\sqrt{ab}}-\frac{15\left(fx+e\right)}{a-b}-\frac{15\left(a^2+ab+b^2\right)\tan\left(fx+e\right)^4-5\left(a^2+ab\right)\tan\left(fx+e\right)^2+3\,a^2}{a^3\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*b^4*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - a^3*b)*sqrt(a*b)) - 1
5*(f*x + e)/(a - b) - (15*(a^2 + a*b + b^2)*tan(f*x + e)^4 - 5*(a^2 + a*b)*
tan(f*x + e)^2 + 3*a^2)/(a^3*tan(f*x + e)^5))/f
```

mupad [B] time = 13.74, size = 524, normalized size = 4.64

$$\frac{\operatorname{atan}\left(\frac{a^{14}\,b\tan\left(e+f\,x\right)\,\sqrt{-a^7\,b^7}\,1\mathrm{i}-a^7\,b^8\tan\left(e+f\,x\right)\,\sqrt{-a^7\,b^7}\,1\mathrm{i}}{a^{11}\,b^{11}-a^{18}\,b^4}\right)\,\sqrt{-a^7\,b^7}\,15\mathrm{i}-15\,a^7\operatorname{atan}\left(\frac{\tan\left(e+f\,x\right)\left(2\,a^{15}\,b^3+2\,a^9\,b^9\right)+\frac{\left(4\,a^{13}\,b^6-4\,a^{12}\,b^7 \cdots\right.}{\cdots}}{\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (atan((a^14*b*tan(e + f*x)*(-a^7*b^7)^(1/2)*1i - a^7*b^8*tan(e + f*x)*(-a^7
*b^7)^(1/2)*1i)/(a^11*b^11 - a^18*b^4))*(-a^7*b^7)^(1/2)*15i - 15*a^7*atan(
((((4*a^13*b^6 - 4*a^12*b^7 + 4*a^16*b^3 - 4*a^17*b^2 + (tan(e + f*x)*(8*a^
15*b^5 - 8*a^16*b^4 - 8*a^17*b^3 + 8*a^18*b^2)*1i)/(2*a - 2*b))*1i)/(2*a -
2*b) + tan(e + f*x)*(2*a^9*b^9 + 2*a^15*b^3))/(2*a - 2*b) + (((4*a^12*b^7 -
 4*a^13*b^6 - 4*a^16*b^3 + 4*a^17*b^2 + (tan(e + f*x)*(8*a^15*b^5 - 8*a^16*
b^4 - 8*a^17*b^3 + 8*a^18*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) + tan(e + f
*x)*(2*a^9*b^9 + 2*a^15*b^3))/(2*a - 2*b))/(2*a^9*b^8 + 2*a^10*b^7 + 2*a^11
*b^6 + 2*a^12*b^5 + 2*a^13*b^4 + 2*a^14*b^3 + 2*a^15*b^2)))/(f*(15*a^7*b -
15*a^8)) + (3*a^6*b + tan(e + f*x)^2*(5*a^7 - 5*a^5*b^2) - tan(e + f*x)^4*(
15*a^7 - 15*a^4*b^3) - 3*a^7)/(f*(15*a^8*tan(e + f*x)^5 - 15*a^7*b*tan(e +
f*x)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.224 $\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{a^2}{2b^2f(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{a(a-2b)\log\left(a+b\tan^2(e+fx)\right)}{2b^2f(a-b)^2}-\frac{\log(\cos(e+fx))}{f(a-b)^2}$$

```
[Out] -ln(cos(f*x+e))/(a-b)^2/f+1/2*a*(a-2*b)*ln(a+b*tan(f*x+e)^2)/(a-b)^2/b^2/f+
1/2*a^2/(a-b)/b^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 446, 88}

$$\frac{a^2}{2b^2f(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{a(a-2b)\log\left(a+b\tan^2(e+fx)\right)}{2b^2f(a-b)^2}-\frac{\log(\cos(e+fx))}{f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(Log[Cos[e + f*x]]/((a - b)^2*f)) + (a*(a - 2*b)*Log[a + b*Tan[e + f*x]^2]
)/(2*(a - b)^2*b^2*f) + a^2/(2*(a - b)*b^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^2(1+x)} - \frac{a^2}{(a-b)b(a+bx)^2} + \frac{a(a-2b)}{(a-b)^2b(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{\log(\cos(e+fx))}{(a-b)^2f} + \frac{a(a-2b)\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)^2b^2f} + \frac{a^2}{2(a-b)b^2f\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 0.72, size = 73, normalized size = 0.81

$$\frac{\frac{a^2(a-b)}{b^2\left(a+b\tan^2(e+fx)\right)} + \frac{a(a-2b)\log\left(a+b\tan^2(e+fx)\right)}{b^2} - 2\log(\cos(e+fx))}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-2*Log[Cos[e + f*x]] + (a*(a - 2*b)*Log[a + b*Tan[e + f*x]^2])/b^2 + (a^2*
(a - b))/(b^2*(a + b*Tan[e + f*x]^2)))/(2*(a - b)^2*f)
```

fricas [B] time = 0.45, size = 186, normalized size = 2.07

$$-\frac{a^2b\tan\left(fx+e\right)^2 + a^2b - \left(a^3 - 2a^2b + \left(a^2b - 2ab^2\right)\tan\left(fx+e\right)^2\right)\log\left(\frac{b\tan\left(fx+e\right)^2+a}{\tan\left(fx+e\right)^2+1}\right) + \left(a^3 - 2a^2b + ab^2 + \ldots\right.}{2\left(\left(a^2b^3 - 2ab^4 + b^5\right)f\tan\left(fx+e\right)^2 + \left(a^3b^2 - 2a^2b^3 + ab^4\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b*tan(f*x + e)^2 + a^2*b - (a^3 - 2*a^2*b + (a^2*b - 2*a*b^2)*tan
(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)) + (a^3 - 2*a^
2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*log(1/(tan(f*x + e)^2
 + 1)))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3
+ a*b^4)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)2/f*(-1/4/b^2*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))-2))-1/(-4*a^2+8*a*b-4*b^2)
*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+co
```

```
s(f*x+exp(1)))+2))+(a^3-2*a^2*b)/(4*a^3*b^2-8*a^2*b^3+4*a*b^4)*ln(abs(((1-c
os(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)
)))*a-2*a+4*b))+(-((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(f*x+exp
(1)))*(1+cos(f*x+exp(1))))*a^3+2*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1
/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1))))*a^2*b+2*a^3-12*a^2*b+12*a*b^2)/(4
*a^2*b^2-8*a*b^3+4*b^4)/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1/(1-cos(
f*x+exp(1)))*(1+cos(f*x+exp(1))))*a-2*a+4*b))
```

maple [A] time = 0.18, size = 149, normalized size = 1.66

$$\frac{a^2 \ln\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}{2f\left(a-b\right)^2 b^2}-\frac{a \ln\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}{f\left(a-b\right)^2 b}+\frac{a^3}{2f\left(a-b\right)^2 b^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{}{2f\left(a-b\right)^2 b\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*a^2/(a-b)^2/b^2*ln(a+b*tan(f*x+e)^2)-1/f*a/(a-b)^2/b*ln(a+b*tan(f*x+e
)^2)+1/2/f*a^3/(a-b)^2/b^2/(a+b*tan(f*x+e)^2)-1/2/f*a^2/(a-b)^2/b/(a+b*tan(
f*x+e)^2)+1/2/f/(a-b)^2*ln(1+tan(f*x+e)^2)
```

maxima [A] time = 0.33, size = 128, normalized size = 1.42

$$-\frac{\frac{a^2}{a^3 b-2 a^2 b^2+a b^3-\left(a^3 b-3 a^2 b^2+3 a b^3-b^4\right)\sin\left(fx+e\right)^2}-\frac{\left(a^2-2 a b\right)\log\left(-\left(a-b\right)\sin\left(fx+e\right)^2+a\right)}{a^2 b^2-2 a b^3+b^4}+\frac{\log\left(\sin\left(fx+e\right)^2-1\right)}{b^2}}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2/(a^3*b - 2*a^2*b^2 + a*b^3 - (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*
sin(f*x + e)^2) - (a^2 - 2*a*b)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^2*b^2 -
 2*a*b^3 + b^4) + log(sin(f*x + e)^2 - 1)/b^2)/f
```

mupad [B] time = 11.59, size = 90, normalized size = 1.00

$$\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f\,\left(a-b\right)^2}+\frac{a^2}{2\,b^2\,f\,\left(b\,\tan\left(e+f\,x\right)^2+a\right)\,\left(a-b\right)}+\frac{a\,\ln\left(b\,\tan\left(e+f\,x\right)^2+a\right)\,\left(a-2\,b\right)}{2\,b^2\,f\,\left(a-b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^2) + a^2/(2*b^2*f*(a + b*tan(e + f*x)^
2)*(a - b)) + (a*log(a + b*tan(e + f*x)^2)*(a - 2*b))/(2*b^2*f*(a - b)^2)
```

sympy [A] time = 55.48, size = 1583, normalized size = 17.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x*tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (2*log(tan(e + f*
x)**2 + 1)*tan(e + f*x)**4/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x
)**2 + 4*b**2*f) + 4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*b**2*f*tan
(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) + 2*log(tan(e + f*x)**2
 + 1)/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) + 4*
```

```
tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b*
*2*f) + 3/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f),
 Eq(a, b)), ((log(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x)**4/(4*f) - tan(
e + f*x)**2/(2*f))/a**2, Eq(b, 0)), (x*tan(e)**5/(a + b*tan(e)**2)**2, Eq(f
, 0)), (a**3*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*a**3*b**2*f + 2*a*
*2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*
a*b**4*f + 2*b**5*f*tan(e + f*x)**2) + a**3*log(I*sqrt(a)*sqrt(1/b) + tan(e
 + f*x))/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4
*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) + a**3/(
2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*
tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) + a**2*b*log(-I*sq
rt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*b**2*f + 2*a**2*b**
3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4
*f + 2*b**5*f*tan(e + f*x)**2) - 2*a**2*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e
+ f*x))/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*
a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) + a**2*b*
log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*b**2*f + 2*
a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 +
2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) - 2*a**2*b*log(I*sqrt(a)*sqrt(1/b) +
 tan(e + f*x))/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3
*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) -
a**2*b/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a
*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) - 2*a*b**2
*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*b**2*f +
2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2
+ 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) - 2*a*b**2*log(I*sqrt(a)*sqrt(1/b)
 + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x
)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*t
an(e + f*x)**2) + a*b**2*log(tan(e + f*x)**2 + 1)/(2*a**3*b**2*f + 2*a**2*b
**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b*
*4*f + 2*b**5*f*tan(e + f*x)**2) + b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*
x)**2/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*
b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2), True))
```

3.225 $\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=69

$$\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^2}-\frac{a}{2bf(a-b)\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] 1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^2/f-1/2*a/(a-b)/b/f/(a+b*tan(f*
x+e)^2)
```

Rubi [A] time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 446, 77}

$$\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^2}-\frac{a}{2bf(a-b)\left(a+b\tan^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^2*f) - a/(2*(a - b)*b*f
*(a + b*Tan[e + f*x]^2))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a-b)^2(1+x)}+\frac{a}{(a-b)(a+bx)^2}+\frac{b}{(a-b)^2(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^2f} - \frac{a}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 0.58, size = 61, normalized size = 0.88

$$\frac{\frac{a(b-a)}{b\left(a+b\tan^2(e+fx)\right)}+\log\left(a+b\tan^2(e+fx)\right)+2\log(\cos(e+fx))}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
[Out] (2*Log[Cos[e + f*x]] + Log[a + b*Tan[e + f*x]^2] + (a*(-a + b))/(b*(a + b*T
an[e + f*x]^2)))/(2*(a - b)^2*f)
```

fricas [A] time = 0.43, size = 98, normalized size = 1.42

$$\frac{a\tan\left(fx+e\right)^2+\left(b\tan\left(fx+e\right)^2+a\right)\log\left(\frac{b\tan\left(fx+e\right)^2+a}{\tan\left(fx+e\right)^2+1}\right)+a}{2\left(\left(a^2b-2\,ab^2+b^3\right)f\tan\left(fx+e\right)^2+\left(a^3-2\,a^2b+ab^2\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
[Out] 1/2*(a*tan(f*x + e)^2 + (b*tan(f*x + e)^2 + a)*log((b*tan(f*x + e)^2 + a)/(
tan(f*x + e)^2 + 1)) + a)/((a^2*b - 2*a*b^2 + b^3)*f*tan(f*x + e)^2 + (a^3
- 2*a^2*b + a*b^2)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(-1/(2*a^2-4*a*b+2*b^2)*ln(abs((
1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+1/(4*a^2-8*a*b+4*b^2)*ln(((1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a)+(-((1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1))))^2*a+6*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a-8*(
1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-a)/(4*a^2-8*a*b+4*b^2)/(((1-cos(f*
x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)
))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a))
```

maple [A] time = 0.27, size = 109, normalized size = 1.58

$$\frac{\ln\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}{2f\left(a-b\right)^2}-\frac{a^2}{2f\left(a-b\right)^2b\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{a}{2f\left(a-b\right)^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{\ln\left(1+\tan^2\right.}{2f\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f/(a-b)^2*ln(a+b*tan(f*x+e)^2)-1/2/f*a^2/(a-b)^2/b/(a+b*tan(f*x+e)^2)+1
/2/f/(a-b)^2*a/(a+b*tan(f*x+e)^2)-1/2/f/(a-b)^2*ln(1+tan(f*x+e)^2)
```

maxima [A] time = 0.34, size = 88, normalized size = 1.28

$$\frac{\frac{a}{a^3-2a^2b+ab^2-\left(a^3-3a^2b+3ab^2-b^3\right)\sin\left(fx+e\right)^2}+\frac{\log\left(-\left(a-b\right)\sin\left(fx+e\right)^2+a\right)}{a^2-2ab+b^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a/(a^3 - 2*a^2*b + a*b^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sin(f*x + e
)^2) + log(-(a - b)*sin(f*x + e)^2 + a)/(a^2 - 2*a*b + b^2))/f
```

mupad [B] time = 11.60, size = 270, normalized size = 3.91

$$-\frac{\frac{a^2\cos\left(e+fx\right)^2}{2}+b^2\sin\left(e+fx\right)^2\operatorname{atan}\left(\frac{a\sin\left(e+fx\right)^2-b\sin\left(e+fx\right)^2}{a\cos\left(e+fx\right)^2 2\mathrm{i}+a\sin\left(e+fx\right)^2 1\mathrm{i}+b\sin\left(e+fx\right)^2 1\mathrm{i}}\right)1\mathrm{i}-\frac{ab\cos\left(e+fx\right)^2}{2}+ab\cos\left(e+fx\right.}{f\left(a^3b\cos\left(e+fx\right)^2-2a^2b^2\cos\left(e+fx\right)^2+a^2b^2\sin\left(e+fx\right)^2+ab^3\cos\left(e+fx\right)^2-2ab^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] -((a^2*cos(e + f*x)^2)/2 + b^2*sin(e + f*x)^2*atan((a*sin(e + f*x)^2 - b*si
n(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e + f*x)^2*1i + b*sin(e + f*x)^2
*1i))*1i - (a*b*cos(e + f*x)^2)/2 + a*b*cos(e + f*x)^2*atan((a*sin(e + f*x)
^2 - b*sin(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e + f*x)^2*1i + b*sin(e
 + f*x)^2*1i))*1i)/(f*(b^4*sin(e + f*x)^2 + a*b^3*cos(e + f*x)^2 + a^3*b*co
s(e + f*x)^2 - 2*a*b^3*sin(e + f*x)^2 - 2*a^2*b^2*cos(e + f*x)^2 + a^2*b^2*
sin(e + f*x)^2))
```

sympy [A] time = 26.51, size = 930, normalized size = 13.48

$$\begin{cases} \frac{\tilde{\infty}x}{\tan\left(e\right)} \\ \frac{-\frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{\tan^2\left(e+fx\right)}{2f}}{a^2} \\ -\frac{2\tan^2\left(e+fx\right)}{4b^2f\tan^4\left(e+fx\right)+8b^2f\tan^2\left(e+fx\right)+4b^2f}-\frac{1}{4b^2f\tan^4\left(e+fx\right)+8b^2f\tan^2\left(e+fx\right)+4b^2f} \\ \frac{x\tan^3\left(e\right)}{\left(a+b\tan^2\left(e\right)\right)^2} \\ -\frac{a^2}{2a^3bf+2a^2b^2f\tan^2\left(e+fx\right)-4a^2b^2f-4ab^3f\tan^2\left(e+fx\right)+2ab^3f+2b^4f\tan^2\left(e+fx\right)}+\frac{ab\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan\left(e+fx\right)\right)}{2a^3bf+2a^2b^2f\tan^2\left(e+fx\right)-4a^2b^2f-4ab^3f\tan^2\left(e+fx\right)+2a\ldots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-log(tan(e + f*
x)**2 + 1)/(2*f) + tan(e + f*x)**2/(2*f))/a**2, Eq(b, 0)), (-2*tan(e + f*x)
**2/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) - 1/(4
*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f), Eq(a, b)),
(x*tan(e)**3/(a + b*tan(e)**2)**2, Eq(f, 0)), (-a**2/(2*a**3*b*f + 2*a**2*b
**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b*
*3*f + 2*b**4*f*tan(e + f*x)**2) + a*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f
*x))/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3
*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)**2) + a*b*log(I*sqr
t(a)*sqrt(1/b) + tan(e + f*x))/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2
- 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e
+ f*x)**2) - a*b*log(tan(e + f*x)**2 + 1)/(2*a**3*b*f + 2*a**2*b**2*f*tan(e
 + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b*
*4*f*tan(e + f*x)**2) + a*b/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4
*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f
*x)**2) + b**2*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(2*
a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e
 + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)**2) + b**2*log(I*sqrt(a)*sq
rt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*b*f + 2*a**2*b**2*f*tan(e +
 f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4
*f*tan(e + f*x)**2) - b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a**3
*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f
*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)**2), True))
```

3.226 $\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=65

$$\frac{1}{2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^2}$$

```
[Out] -1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^2/f+1/2/(a-b)/f/(a+b*tan(f*x+e
)^2)
```

Rubi [A] time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 444, 44}

$$\frac{1}{2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^2*f) + 1/(2*(a - b)*f*
(a + b*Tan[e + f*x]^2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^2(1+x)} - \frac{b}{(a-b)(a+bx)^2} - \frac{b}{(a-b)^2(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^2f} + \frac{1}{2(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [**A**] time = 0.66, size = 57, normalized size = 0.88

$$-\frac{\frac{b-a}{a+b\tan^2(e+fx)} + \log\left(a+b\tan^2(e+fx)\right) + 2\log(\cos(e+fx))}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(2*Log[Cos[e + f*x]] + Log[a + b*Tan[e + f*x]^2] + (-a + b)/(a + b*Tan
[e + f*x]^2))/((a - b)^2*f)
```

fricas [**A**] time = 0.45, size = 98, normalized size = 1.51

$$-\frac{b\tan\left(fx+e\right)^2 + \left(b\tan\left(fx+e\right)^2 + a\right)\log\left(\frac{b\tan\left(fx+e\right)^2+a}{\tan\left(fx+e\right)^2+1}\right) + b}{2\left(\left(a^2b-2\,ab^2+b^3\right)f\tan\left(fx+e\right)^2 + \left(a^3-2\,a^2b+ab^2\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*tan(f*x + e)^2 + (b*tan(f*x + e)^2 + a)*log((b*tan(f*x + e)^2 + a)/
(tan(f*x + e)^2 + 1)) + b)/((a^2*b - 2*a*b^2 + b^3)*f*tan(f*x + e)^2 + (a^3
 - 2*a^2*b + a*b^2)*f)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(1/(2*a^2-4*a*b+2*b^2)*ln(abs((1
-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))-1/(4*a^2-8*a*b+4*b^2)*ln(((1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a)+(((1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1))))^2*a^2-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2+4
*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b^2+a^2)/(4*a^3-8*a^2*b+4*a*b^2)/(
((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-2*(1-cos(f*x+exp(1)))/(1+cos(
f*x+exp(1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a))
```

maple [A] time = 0.21, size = 104, normalized size = 1.60

$$-\frac{\ln\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}{2f\left(a-b\right)^2}+\frac{a}{2f\left(a-b\right)^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{b}{2f\left(a-b\right)^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{\ln\left(1+\tan^2\right.}{2f\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f/(a-b)^2*ln(a+b*tan(f*x+e)^2)+1/2/f/(a-b)^2*a/(a+b*tan(f*x+e)^2)-1/2/
f*b/(a-b)^2/(a+b*tan(f*x+e)^2)+1/2/f/(a-b)^2*ln(1+tan(f*x+e)^2)
```

maxima [A] time = 0.64, size = 88, normalized size = 1.35

$$-\frac{\frac{b}{a^3-2a^2b+ab^2-\left(a^3-3a^2b+3ab^2-b^3\right)\sin\left(fx+e\right)^2}+\frac{\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^2-2ab+b^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b/(a^3 - 2*a^2*b + a*b^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sin(f*x +
e)^2) + log(-(a - b)*sin(f*x + e)^2 + a)/(a^2 - 2*a*b + b^2))/f
```

mupad [B] time = 11.64, size = 195, normalized size = 3.00

$$-\frac{b\left(1+\tan\left(e+fx\right)^2\operatorname{atan}\left(\frac{a\tan\left(e+fx\right)^2 1\mathrm{i}-b\tan\left(e+fx\right)^2 1\mathrm{i}}{2a+a\tan\left(e+fx\right)^2+b\tan\left(e+fx\right)^2}\right)2\mathrm{i}\right)+a\left(-1+\operatorname{atan}\left(\frac{a\tan\left(e+fx\right)^2 1\mathrm{i}-b\tan\left(e+fx\right)^2 1\mathrm{i}}{2a+a\tan\left(e+fx\right)^2+b\tan\left(e+fx\right)^2}\right)2\mathrm{i}\right)}{f\left(2a^3+2a^2b\tan\left(e+fx\right)^2-4a^2b-4ab^2\tan\left(e+fx\right)^2+2ab^2+2b^3\tan\left(e+fx\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] -(b*(tan(e + f*x)^2*atan((a*tan(e + f*x)^2*1i - b*tan(e + f*x)^2*1i)/(2*a +
 a*tan(e + f*x)^2 + b*tan(e + f*x)^2))*2i + 1) + a*(atan((a*tan(e + f*x)^2*
1i - b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*x)^2 + b*tan(e + f*x)^2))*2i -
 1))/(f*(2*a*b^2 - 4*a^2*b + 2*a^3 + 2*b^3*tan(e + f*x)^2 - 4*a*b^2*tan(e +
 f*x)^2 + 2*a^2*b*tan(e + f*x)^2))
```

sympy [A] time = 26.39, size = 816, normalized size = 12.55

$$\begin{cases} \frac{\tilde{\infty} x}{\tan^3(e)} \\ -\frac{1}{4b^2f\tan^4\left(e+fx\right)+8b^2f\tan^2\left(e+fx\right)+4b^2f} \\ \frac{x\tan(e)}{\left(a+b\tan^2(e)\right)^2} \\ \frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2a^2f} \\ -\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tan\left(e+fx\right)\right)}{2a^3f+2a^2bf\tan^2\left(e+fx\right)-4a^2bf-4ab^2f\tan^2\left(e+fx\right)+2ab^2f+2b^3f\tan^2\left(e+fx\right)}-\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tan\left(e+fx\right)\right)}{2a^3f+2a^2bf\tan^2\left(e+fx\right)-4a^2bf-4ab^2f\tan^2\left(e+fx\right)+2ab^2f+2b\ldots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-1/(4*b**2*f*
tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f), Eq(a, b)), (x*tan(e
)/(a + b*tan(e)**2)**2, Eq(f, 0)), (log(tan(e + f*x)**2 + 1)/(2*a**2*f), Eq
(b, 0)), (-a*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*a**3*f + 2*a**2*b*
f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f +
2*b**3*f*tan(e + f*x)**2) - a*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(2*a*
*3*f + 2*a**2*b*f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2
 + 2*a*b**2*f + 2*b**3*f*tan(e + f*x)**2) + a*log(tan(e + f*x)**2 + 1)/(2*a
**3*f + 2*a**2*b*f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**
2 + 2*a*b**2*f + 2*b**3*f*tan(e + f*x)**2) + a/(2*a**3*f + 2*a**2*b*f*tan(e
 + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*b**3*
f*tan(e + f*x)**2) - b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x
)**2/(2*a**3*f + 2*a**2*b*f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e
 + f*x)**2 + 2*a*b**2*f + 2*b**3*f*tan(e + f*x)**2) - b*log(I*sqrt(a)*sqrt(
1/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*f + 2*a**2*b*f*tan(e + f*x)**2
 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*b**3*f*tan(e +
f*x)**2) + b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a**3*f + 2*a**2*b*
f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f +
2*b**3*f*tan(e + f*x)**2) - b/(2*a**3*f + 2*a**2*b*f*tan(e + f*x)**2 - 4*a*
*2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*b**3*f*tan(e + f*x)**2
), True))
```

3.227 $\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{b(2a-b)\log\left(a+b\tan^2(e+fx)\right)}{2a^2f(a-b)^2}+\frac{\log(\tan(e+fx))}{a^2f}-\frac{b}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{\log(\cos(e+fx))}{f(a-b)^2}$$

```
[Out] ln(cos(f*x+e))/(a-b)^2/f+ln(tan(f*x+e))/a^2/f+1/2*(2*a-b)*b*ln(a+b*tan(f*x+
e)^2)/a^2/(a-b)^2/f-1/2*b/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3670, 446, 72}

$$\frac{b(2a-b)\log\left(a+b\tan^2(e+fx)\right)}{2a^2f(a-b)^2}+\frac{\log(\tan(e+fx))}{a^2f}-\frac{b}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{\log(\cos(e+fx))}{f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] Log[Cos[e + f*x]]/((a - b)^2*f) + Log[Tan[e + f*x]]/(a^2*f) + ((2*a - b)*b*
Log[a + b*Tan[e + f*x]^2])/(2*a^2*(a - b)^2*f) - b/(2*a*(a - b)*f*(a + b*Ta
n[e + f*x]^2))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{1}{(a-b)^2(1+x)} + \frac{b^2}{a(a-b)(a+bx)^2} + \frac{(2a-b)b^2}{a^2(a-b)^2(a+bx)}\right) dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\log(\cos(e+fx))}{(a-b)^2f} + \frac{\log(\tan(e+fx))}{a^2f} + \frac{(2a-b)b\log\left(a+b\tan^2(e+fx)\right)}{2a^2(a-b)^2f} - \frac{}{2a}$$

Mathematica [A] time = 2.10, size = 90, normalized size = 0.87

$$\frac{\frac{\frac{b\left(\frac{a(b-a)}{a+b\tan^2(e+fx)}+(2a-b)\log\left(a+b\tan^2(e+fx)\right)\right)}{(a-b)^2}+2\log(\tan(e+fx))}{a^2}+\frac{2\log(\cos(e+fx))}{(a-b)^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((2*Log[Cos[e + f*x]])/(a - b)^2 + (2*Log[Tan[e + f*x]] + (b*((2*a - b)*Log
[a + b*Tan[e + f*x]^2] + (a*(-a + b))/(a + b*Tan[e + f*x]^2)))/(a - b)^2)/a
^2)/(2*f)
```

fricas [A] time = 0.48, size = 197, normalized size = 1.91

$$\frac{ab^2\tan\left(fx+e\right)^2 + ab^2 + \left(a^3 - 2a^2b + ab^2 + \left(a^2b - 2ab^2 + b^3\right)\tan\left(fx+e\right)^2\right)\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right) + \left(2a^2b - ab\right.}{2\left(\left(a^4b - 2a^3b^2 + a^2b^3\right)f\tan\left(fx+e\right)^2 + \left(a^5 - 2a^4b + a^3b^2\right)f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2*tan(f*x + e)^2 + a*b^2 + (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b
^2 + b^3)*tan(f*x + e)^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + (2*a^2
*b - a*b^2 + (2*a*b^2 - b^3)*tan(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(ta
n(f*x + e)^2 + 1)))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a^5
- 2*a^4*b + a^3*b^2)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)1/f*(1/2/a^2*ln(sin(f*x+exp(1))^2)+(-b^2+2*b*a)/(2*b^2*a^2-4*b*a^3+
2*a^4)*ln(abs(sin(f*x+exp(1))^2*b-sin(f*x+exp(1))^2*a+a))+(sin(f*x+exp(1))^
```

```
2*b^2-2*sin(f*x+exp(1))^2*b*a+2*b*a)/(2*b*a^2-2*a^3)/(sin(f*x+exp(1))^2*b-s
in(f*x+exp(1))^2*a+a))
```

maple [A] time = 0.98, size = 160, normalized size = 1.55

$$\frac{b^2}{2fa(a-b)^2\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)}+\frac{b\ln\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)}{fa(a-b)^2}-\frac{b^2\ln\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*b^2/a/(a-b)^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+1/f*b/a/(a-b)^2*ln(a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)-1/2/f*b^2/a^2/(a-b)^2*ln(a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)+1/2/f/a^2*ln(-1+cos(f*x+e))+1/2/f/a^2*ln(1+cos(f*x+e))
```

maxima [A] time = 0.54, size = 124, normalized size = 1.20

$$\frac{\frac{b^2}{a^4-2a^3b+a^2b^2-\left(a^4-3a^3b+3a^2b^2-ab^3\right)\sin\left(fx+e\right)^2}+\frac{\left(2ab-b^2\right)\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^4-2a^3b+a^2b^2}+\frac{\log\left(\sin\left(fx+e\right)^2\right)}{a^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2/(a^4 - 2*a^3*b + a^2*b^2 - (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sin
(f*x + e)^2) + (2*a*b - b^2)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^4 - 2*a^3*
b + a^2*b^2) + log(sin(f*x + e)^2)/a^2)/f
```

mupad [B] time = 11.73, size = 104, normalized size = 1.01

$$\frac{\ln\left(\tan\left(e+fx\right)\right)}{a^2f}-\frac{\ln\left(\tan\left(e+fx\right)^2+1\right)}{2f(a-b)^2}-\frac{b}{2af\left(b\tan\left(e+fx\right)^2+a\right)(a-b)}+\frac{b\,\ln\left(b\tan\left(e+fx\right)^2+a\right)\,(2a-}{2a^2f(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] log(tan(e + f*x))/(a^2*f) - log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^2) - b/(2*
a*f*(a + b*tan(e + f*x)^2)*(a - b)) + (b*log(a + b*tan(e + f*x)^2)*(2*a - b
))/(2*a^2*f*(a - b)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.228 $\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=132

$$-\frac{b^2(3a-2b)\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)^2}-\frac{(a+2b)\log(\tan(e+fx))}{a^3f}+\frac{b^2}{2a^2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{\cot^2(e+}{2a^2f}$$

```
[Out] -1/2*cot(f*x+e)^2/a^2/f-ln(cos(f*x+e))/(a-b)^2/f-(a+2*b)*ln(tan(f*x+e))/a^3
/f-1/2*(3*a-2*b)*b^2*ln(a+b*tan(f*x+e)^2)/a^3/(a-b)^2/f+1/2*b^2/a^2/(a-b)/f
/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 446, 88}

$$\frac{b^2}{2a^2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{b^2(3a-2b)\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)^2}-\frac{(a+2b)\log(\tan(e+fx))}{a^3f}-\frac{\cot^2(e+f}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -Cot[e + f*x]^2/(2*a^2*f) - Log[Cos[e + f*x]]/((a - b)^2*f) - ((a + 2*b)*Lo
g[Tan[e + f*x]])/(a^3*f) - ((3*a - 2*b)*b^2*Log[a + b*Tan[e + f*x]^2])/(2*a
^3*(a - b)^2*f) + b^2/(2*a^2*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^2} + \frac{-a-2b}{a^3x} + \frac{1}{(a-b)^2(1+x)} - \frac{b^3}{a^2(a-b)(a+bx)^2} - \frac{(3a-2b)b^3}{a^3(a-b)^2(a+bx)}\right) dx, x, \tan^2(e+ \right.}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2a^2f} - \frac{\log(\cos(e+fx))}{(a-b)^2f} - \frac{(a+2b)\log(\tan(e+fx))}{a^3f} - \frac{(3a-2b)b^2\log(}{2a^3(a}$$

Mathematica [A] time = 0.88, size = 98, normalized size = 0.74

$$-\frac{\frac{b^3}{a^3(a-b)\left(a\cot^2(e+fx)+b\right)} + \frac{b^2(3a-2b)\log\left(a\cot^2(e+fx)+b\right)}{a^3(a-b)^2} + \frac{\cot^2(e+fx)}{a^2} + \frac{2\log(\sin(e+fx))}{(a-b)^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(Cot[e + f*x]^2/a^2 + b^3/(a^3*(a - b)*(b + a*Cot[e + f*x]^2)) + ((3*a
 - 2*b)*b^2*Log[b + a*Cot[e + f*x]^2])/(a^3*(a - b)^2) + (2*Log[Sin[e + f*x
]])/(a - b)^2)/f
```

fricas [B] time = 0.49, size = 292, normalized size = 2.21

$$-\frac{\left(a^3b - 2a^2b^2 + 2ab^3\right)\tan\left(fx+e\right)^4 + a^4 - 2a^3b + a^2b^2 + \left(a^4 - a^3b - a^2b^2 + 2ab^3\right)\tan\left(fx+e\right)^2 + \left(\left(a^3b - 3ab\right.\right.}{2\left(\left(a^5b - 2a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^3*b - 2*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^4 + a^4 - 2*a^3*b + a^2*b^
2 + (a^4 - a^3*b - a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2 + ((a^3*b - 3*a*b^3 +
2*b^4)*tan(f*x + e)^4 + (a^4 - 3*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2)*log(tan
(f*x + e)^2/(tan(f*x + e)^2 + 1)) + ((3*a*b^3 - 2*b^4)*tan(f*x + e)^4 + (3*
a^2*b^2 - 2*a*b^3)*tan(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)
^2 + 1)))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a^6 - 2*a^5*b
+ a^4*b^2)*f*tan(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(-(1-cos(f*x+exp(1)
```

```
))/(1+cos(f*x+exp(1)))*1/16/a^2+(4*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
)^3*a^4+12*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^2*b^2-8*((1-cos(f*
x+exp(1)))/(1+cos(f*x+exp(1))))^3*a*b^3-11*((1-cos(f*x+exp(1)))/(1+cos(f*x+
exp(1))))^2*a^4+22*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^3*b-27*((1
-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2*b^2+16*((1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1))))^2*a*b^3+16*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b
^4+10*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^4-24*(1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1)))*a^3*b+42*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2*b^2-
20*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a*b^3-3*a^4+6*a^3*b-3*a^2*b^2)/(
48*a^5-96*a^4*b+48*a^3*b^2)/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a-
2*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a+4*((1-cos(f*x+exp(1)))/(1+c
os(f*x+exp(1))))^2*b+(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a)+1/(2*a^2-4*
a*b+2*b^2)*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+(-a-2*b)*1/4/
a^3*ln(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1))))+(-3*a*b^2+2*b^3)/(4*a
^5-8*a^4*b+4*a^3*b^2)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-2*(1
-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+ex
p(1)))*b+a))
```

maple [A] time = 1.00, size = 234, normalized size = 1.77

$$-\frac{b^3}{2f\,a^2\,(a-b)^2\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)}-\frac{3b^2\ln\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)}{2f\,a^2\,(a-b)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b^3/a^2/(a-b)^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-3/2/f*b^2/a^2/(a-b
)^2*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+1/f*b^3/a^3/(a-b)^2*ln(a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)+1/4/f/a^2/(-1+cos(f*x+e))-1/2/f/a^2*ln(-1+cos(f*x+e))-
1/f/a^3*ln(-1+cos(f*x+e))*b-1/4/f/a^2/(1+cos(f*x+e))-1/2/f/a^2*ln(1+cos(f*x
+e))-1/f/a^3*ln(1+cos(f*x+e))*b
```

maxima [A] time = 1.39, size = 187, normalized size = 1.42

$$-\frac{\frac{\left(3\,ab^2-2\,b^3\right)\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^5-2\,a^4b+a^3b^2}-\frac{a^3-2\,a^2b+ab^2-\left(a^3-3\,a^2b+3\,ab^2-2\,b^3\right)\sin\left(fx+e\right)^2}{\left(a^5-3\,a^4b+3\,a^3b^2-a^2b^3\right)\sin\left(fx+e\right)^4-\left(a^5-2\,a^4b+a^3b^2\right)\sin\left(fx+e\right)^2}+\frac{(a+2\,b)\log\left(\sin\left(fx+e\right)^2\right)}{a^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*a*b^2 - 2*b^3)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^5 - 2*a^4*b + a
^3*b^2) - (a^3 - 2*a^2*b + a*b^2 - (a^3 - 3*a^2*b + 3*a*b^2 - 2*b^3)*sin(f*
x + e)^2)/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sin(f*x + e)^4 - (a^5 - 2*
a^4*b + a^3*b^2)*sin(f*x + e)^2) + (a + 2*b)*log(sin(f*x + e)^2)/a^3)/f
```

mupad [B] time = 11.88, size = 144, normalized size = 1.09

$$\frac{\ln\left(b\tan\left(e+f\,x\right)^2+a\right)\left(\frac{b}{a^3}+\frac{1}{2\,a^2}-\frac{1}{2\,(a-b)^2}\right)}{f}-\frac{\frac{1}{2\,a}+\frac{\tan\left(e+f\,x\right)^2\,\left(a\,b-2\,b^2\right)}{2\,a^2\,(a-b)}}{f\,\left(b\tan\left(e+f\,x\right)^4+a\tan\left(e+f\,x\right)^2\right)}+\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f\,(a-b)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (log(a + b*tan(e + f*x)^2)*(b/a^3 + 1/(2*a^2) - 1/(2*(a - b)^2)))/f - (1/(2
*a) + (tan(e + f*x)^2*(a*b - 2*b^2))/(2*a^2*(a - b)))/(f*(a*tan(e + f*x)^2
```

```
+ b*tan(e + f*x)^4)) + log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^2) - (log(tan(e
 + f*x))*(a + 2*b))/(a^3*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.229 $\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=161

$$\frac{b^3(4a-3b)\log\left(a+b\tan^2(e+fx)\right)}{2a^4f(a-b)^2}-\frac{b^3}{2a^3f(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{(a+2b)\cot^2(e+fx)}{2a^3f}-\frac{\cot^4(e+fx)}{4a^2f}+$$

```
[Out] 1/2*(a+2*b)*cot(f*x+e)^2/a^3/f-1/4*cot(f*x+e)^4/a^2/f+ln(cos(f*x+e))/(a-b)^
2/f+(a^2+2*a*b+3*b^2)*ln(tan(f*x+e))/a^4/f+1/2*(4*a-3*b)*b^3*ln(a+b*tan(f*x
+e)^2)/a^4/(a-b)^2/f-1/2*b^3/a^3/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.18, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 446, 88}

$$-\frac{b^3}{2a^3f(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{b^3(4a-3b)\log\left(a+b\tan^2(e+fx)\right)}{2a^4f(a-b)^2}+\frac{\left(a^2+2ab+3b^2\right)\log(\tan(e+fx))}{a^4f}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b)*Cot[e + f*x]^2)/(2*a^3*f) - Cot[e + f*x]^4/(4*a^2*f) + Log[Cos[e
 + f*x]]/((a - b)^2*f) + ((a^2 + 2*a*b + 3*b^2)*Log[Tan[e + f*x]])/(a^4*f)
+ ((4*a - 3*b)*b^3*Log[a + b*Tan[e + f*x]^2])/(2*a^4*(a - b)^2*f) - b^3/(2*
a^3*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^3} + \frac{-a-2b}{a^3x^2} + \frac{a^2+2ab+3b^2}{a^4x} - \frac{1}{(a-b)^2(1+x)} + \frac{b^4}{a^3(a-b)(a+bx)^2} + \frac{(4a-3b)b^4}{a^4(a-b)^2(a+bx)}\right) dx,$$

$$= \frac{(a+2b)\cot^2(e+fx)}{2a^3f} - \frac{\cot^4(e+fx)}{4a^2f} + \frac{\log(\cos(e+fx))}{(a-b)^2f} + \frac{\left(a^2+2ab+3b^2\right)\log(t}{a^4f}$$

Mathematica [A] time = 1.09, size = 121, normalized size = 0.75

$$-\frac{-\frac{b^4}{a^4(a-b)\left(a\cot^2(e+fx)+b\right)} - \frac{b^3(4a-3b)\log\left(a\cot^2(e+fx)+b\right)}{a^4(a-b)^2} - \frac{(a+2b)\cot^2(e+fx)}{a^3} + \frac{\cot^4(e+fx)}{2a^2} - \frac{2\log(\sin(e+fx))}{(a-b)^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(-(((a + 2*b)*Cot[e + f*x]^2)/a^3) + Cot[e + f*x]^4/(2*a^2) - b^4/(a^4
*(a - b)*(b + a*Cot[e + f*x]^2)) - ((4*a - 3*b)*b^3*Log[b + a*Cot[e + f*x]^
2])/(a^4*(a - b)^2) - (2*Log[Sin[e + f*x]])/(a - b)^2)/f
```

fricas [B] time = 0.52, size = 347, normalized size = 2.16

$$\frac{\left(3\,a^4b - 2\,a^3b^2 - 5\,a^2b^3 + 6\,ab^4\right)\tan\left(fx+e\right)^6 - a^5 + 2\,a^4b - a^3b^2 + \left(3\,a^5 - 5\,a^3b^2 - 2\,a^2b^3 + 6\,ab^4\right)\tan\left(fx+e\right)^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((3*a^4*b - 2*a^3*b^2 - 5*a^2*b^3 + 6*a*b^4)*tan(f*x + e)^6 - a^5 + 2*a
^4*b - a^3*b^2 + (3*a^5 - 5*a^3*b^2 - 2*a^2*b^3 + 6*a*b^4)*tan(f*x + e)^4 +
 (2*a^5 - a^4*b - 4*a^3*b^2 + 3*a^2*b^3)*tan(f*x + e)^2 + 2*((a^4*b - 4*a*b
^4 + 3*b^5)*tan(f*x + e)^6 + (a^5 - 4*a^2*b^3 + 3*a*b^4)*tan(f*x + e)^4)*lo
g(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + 2*((4*a*b^4 - 3*b^5)*tan(f*x + e)^
6 + (4*a^2*b^3 - 3*a*b^4)*tan(f*x + e)^4)*log((b*tan(f*x + e)^2 + a)/(tan(f
*x + e)^2 + 1)))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^6 + (a^7 - 2
*a^6*b + a^5*b^2)*f*tan(f*x + e)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
```

```
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*((-4*((1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1))))^2*a^2*b^3+3*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1))))^2*a*b^4+8*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2*b^3-18*(1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1)))*a*b^4+8*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1)))*b^5-4*a^2*b^3+3*a*b^4)/(4*a^6-8*a^5*b+4*a^4*b^2)/(((1-cos(f*x+exp(1))
)/(1+cos(f*x+exp(1))))^2*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+4*(1
-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a)+(-48*((1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1))))^2*a^2-96*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a*b-144
*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^2+12*(1-cos(f*x+exp(1)))/(1+
cos(f*x+exp(1)))*a^2+16*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a*b-a^2)*1/
128/a^4/((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2+(-32*((1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1))))^2*a^2+384*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a
^2+512*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a*b)*1/4096/a^4-1/(2*a^2-4*a
*b+2*b^2)*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+(4*a*b^3-3*b^4
)/(4*a^6-8*a^5*b+4*a^4*b^2)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*
a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(
f*x+exp(1)))*b+a)+(a^2+2*a*b+3*b^2)*1/4/a^4*ln(abs(1-cos(f*x+exp(1)))/abs(1
+cos(f*x+exp(1)))))
```

maple [B] time = 0.99, size = 347, normalized size = 2.16

$$\frac{b^4}{2f\,a^3\,(a-b)^2\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)}+\frac{2b^3\ln\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)}{f\,a^3\,(a-b)^2}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*b^4/a^3/(a-b)^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+2/f*b^3/a^3/(a-b)^2
*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-3/2/f*b^4/a^4/(a-b)^2*ln(a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)-1/16/f/a^2/(-1+cos(f*x+e))^2-7/16/f/a^2/(-1+cos(f*x+e))
-1/2/f/a^3/(-1+cos(f*x+e))*b+1/2/f/a^2*ln(-1+cos(f*x+e))+1/f/a^3*ln(-1+cos(
f*x+e))*b+3/2/f/a^4*ln(-1+cos(f*x+e))*b^2-1/16/f/a^2/(1+cos(f*x+e))^2+7/16/
f/a^2/(1+cos(f*x+e))+1/2/f/a^3/(1+cos(f*x+e))*b+1/2/f/a^2*ln(1+cos(f*x+e))+
1/f/a^3*ln(1+cos(f*x+e))*b+3/2/f/a^4*ln(1+cos(f*x+e))*b^2
```

maxima [A] time = 1.19, size = 236, normalized size = 1.47

$$\frac{\frac{2\left(4\,ab^3-3\,b^4\right)\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^6-2\,a^5b+a^4b^2}+\frac{2\left(2\,a^4-4\,a^3b+4\,ab^3-3\,b^4\right)\sin\left(fx+e\right)^4+a^4-2\,a^3b+a^2b^2-\left(5\,a^4-7\,a^3b-a^2b^2+3\,ab^3\right)\sin\left(fx+e\right)^2}{\left(a^6-3\,a^5b+3\,a^4b^2-a^3b^3\right)\sin\left(fx+e\right)^6-\left(a^6-2\,a^5b+a^4b^2\right)\sin\left(fx+e\right)^4}+\frac{2\,(a\cdots}{\cdots}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(4*a*b^3 - 3*b^4)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^6 - 2*a^5*b +
a^4*b^2) + (2*(2*a^4 - 4*a^3*b + 4*a*b^3 - 3*b^4)*sin(f*x + e)^4 + a^4 - 2*
a^3*b + a^2*b^2 - (5*a^4 - 7*a^3*b - a^2*b^2 + 3*a*b^3)*sin(f*x + e)^2)/((a
^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*sin(f*x + e)^6 - (a^6 - 2*a^5*b + a^4*b
^2)*sin(f*x + e)^4) + 2*(a^2 + 2*a*b + 3*b^2)*log(sin(f*x + e)^2)/a^4)/f
```

mupad [B] time = 12.37, size = 191, normalized size = 1.19

$$\frac{\frac{\tan\left(e+f\,x\right)^2\,\left(2\,a+3\,b\right)}{4\,a^2}-\frac{1}{4\,a}+\frac{\tan\left(e+f\,x\right)^4\,\left(a^2\,b+a\,b^2-3\,b^3\right)}{2\,a^3\,\left(a-b\right)}}{f\,\left(b\,\tan\left(e+f\,x\right)^6+a\,\tan\left(e+f\,x\right)^4\right)}-\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f\,\left(a-b\right)^2}+\frac{\ln\left(\tan\left(e+f\,x\right)\right)\,\left(a^2+2\,a\,b+3\,b^2\right)}{a^4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] ((tan(e + f*x)^2*(2*a + 3*b))/(4*a^2) - 1/(4*a) + (tan(e + f*x)^4*(a*b^2 +
a^2*b - 3*b^3))/(2*a^3*(a - b)))/(f*(a*tan(e + f*x)^4 + b*tan(e + f*x)^6))
- log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^2) + (log(tan(e + f*x))*(2*a*b + a^2
 + 3*b^2))/(a^4*f) + (log(a + b*tan(e + f*x)^2)*(4*a*b^3 - 3*b^4))/(f*(2*a^
6 - 4*a^5*b + 2*a^4*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.230 $\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=130

$$-\frac{a^{3/2}(3a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2b^{5/2}f(a-b)^2}+\frac{(3a-2b)\tan(e+fx)}{2b^2f(a-b)}-\frac{a\tan^3(e+fx)}{2bf(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{x}{(a-b)^2}$$

```
[Out] -x/(a-b)^2-1/2*a^(3/2)*(3*a-5*b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)^2
/b^(5/2)/f+1/2*(3*a-2*b)*tan(f*x+e)/(a-b)/b^2/f-1/2*a*tan(f*x+e)^3/(a-b)/b/
f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.20, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 470, 582, 522, 203, 205}

$$-\frac{a^{3/2}(3a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2b^{5/2}f(a-b)^2}+\frac{(3a-2b)\tan(e+fx)}{2b^2f(a-b)}-\frac{a\tan^3(e+fx)}{2bf(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(x/(a - b)^2) - (a^(3/2)*(3*a - 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(2*(a - b)^2*b^(5/2)*f) + ((3*a - 2*b)*Tan[e + f*x])/(2*(a - b)*b^2*f) -
 (a*Tan[e + f*x]^3)/(2*(a - b)*b*f*(a + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan^3(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{x^2(3a+(3a-2b)x^2)}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2(a-b)bf}$$

$$= \frac{(3a-2b)\tan(e+fx)}{2(a-b)b^2f} - \frac{a\tan^3(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{a(3a-2b)+(3a^2-2a}{(1+x^2)(a+b}\right.}{2(a-}$$

$$= \frac{(3a-2b)\tan(e+fx)}{2(a-b)b^2f} - \frac{a\tan^3(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan\right.}{(a-b)^2f}$$

$$= -\frac{x}{(a-b)^2} - \frac{a^{3/2}(3a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^2b^{5/2}f} + \frac{(3a-2b)\tan(e+fx)}{2(a-b)b^2f} - \frac{a}{2(a-b)b}$$

Mathematica [A] time = 1.32, size = 118, normalized size = 0.91

$$\frac{-\frac{a^{3/2}(3a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{5/2}(a-b)^2} + \frac{a^2\sin(2(e+fx))}{b^2(a-b)((a-b)\cos(2(e+fx))+a+b)} - \frac{2(e+fx)}{(a-b)^2} + \frac{2\tan(e+fx)}{b^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((-2*(e + f*x))/(a - b)^2 - (a^(3/2)*(3*a - 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/((a - b)^2*b^(5/2)) + (a^2*Sin[2*(e + f*x)])/((a - b)*b^2*(a
+ b + (a - b)*Cos[2*(e + f*x)])) + (2*Tan[e + f*x])/b^2)/(2*f)
```

fricas [A] time = 0.47, size = 474, normalized size = 3.65

$$\left[-\frac{8\,b^3 f x \tan\left(fx+e\right)^2 + 8\,ab^2 fx - 8\left(a^2b - 2\,ab^2 + b^3\right)\tan\left(fx+e\right)^3 + \left(3\,a^3 - 5\,a^2b + \left(3\,a^2b - 5\,ab^2\right)\tan\left(fx\right.\right.}{8\left(\left(a^2b^3 - 2\,ab^4 + b^5\right)f\tan\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*b^3*f*x*tan(f*x + e)^2 + 8*a*b^2*f*x - 8*(a^2*b - 2*a*b^2 + b^3)*t
an(f*x + e)^3 + (3*a^3 - 5*a^2*b + (3*a^2*b - 5*a*b^2)*tan(f*x + e)^2)*sqrt
(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(b^2*tan(f*
x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*
x + e)^2 + a^2)) - 4*(3*a^3 - 5*a^2*b + 2*a*b^2)*tan(f*x + e))/((a^2*b^3 -
2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f), -1/4*(4
*b^3*f*x*tan(f*x + e)^2 + 4*a*b^2*f*x - 4*(a^2*b - 2*a*b^2 + b^3)*tan(f*x +
 e)^3 + (3*a^3 - 5*a^2*b + (3*a^2*b - 5*a*b^2)*tan(f*x + e)^2)*sqrt(a/b)*ar
ctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/b)/(a*tan(f*x + e))) - 2*(3*a^3 - 5*
a^2*b + 2*a*b^2)*tan(f*x + e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2
+ (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f)]
```

giac [A] time = 34.53, size = 149, normalized size = 1.15

$$\frac{\frac{a^2\tan\left(fx+e\right)}{\left(ab^2-b^3\right)\left(b\tan\left(fx+e\right)^2+a\right)} - \frac{\left(3\,a^3-5\,a^2b\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^2b^2-2\,ab^3+b^4\right)\sqrt{ab}} - \frac{2\left(fx+e\right)}{a^2-2\,ab+b^2} + \frac{2\tan\left(fx+e\right)}{b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*tan(f*x + e)/((a*b^2 - b^3)*(b*tan(f*x + e)^2 + a)) - (3*a^3 - 5*a
^2*b)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
)))/((a^2*b^2 - 2*a*b^3 + b^4)*sqrt(a*b)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2)
 + 2*tan(f*x + e)/b^2)/f
```

maple [A] time = 0.20, size = 184, normalized size = 1.42

$$\frac{\tan\left(fx+e\right)}{f\,b^2} + \frac{a^3\tan\left(fx+e\right)}{2f\,b^2\left(a-b\right)^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)} - \frac{a^2\tan\left(fx+e\right)}{2fb\left(a-b\right)^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)} - \frac{3a^3\arctan\left(\frac{\tan\left(fx+\right.}{\sqrt{ab}}\right.}{2f\,b^2\left(a-b\right)^2\sqrt{a\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/f/b^2*tan(f*x+e)+1/2/f*a^3/b^2/(a-b)^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)-1/2/
f*a^2/b/(a-b)^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)-3/2/f*a^3/b^2/(a-b)^2/(a*b)^(
1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+5/2/f*a^2/b/(a-b)^2/(a*b)^(1/2)*arcta
n(tan(f*x+e)*b/(a*b)^(1/2))-1/f/(a-b)^2*arctan(tan(f*x+e))
```

maxima [A] time = 1.64, size = 135, normalized size = 1.04

$$\frac{\frac{a^2\tan\left(fx+e\right)}{a^2b^2-ab^3+\left(ab^3-b^4\right)\tan\left(fx+e\right)^2} - \frac{\left(3\,a^3-5\,a^2b\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^2b^2-2\,ab^3+b^4\right)\sqrt{ab}} - \frac{2\left(fx+e\right)}{a^2-2\,ab+b^2} + \frac{2\tan\left(fx+e\right)}{b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*tan(f*x + e)/(a^2*b^2 - a*b^3 + (a*b^3 - b^4)*tan(f*x + e)^2) - (3
*a^3 - 5*a^2*b)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2*b^2 - 2*a*b^3 + b^4)
*sqrt(a*b)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2) + 2*tan(f*x + e)/b^2)/f
```

mupad [B] time = 13.01, size = 2581, normalized size = 19.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (2*atan((((((4*a*b^8 - 22*a^2*b^7 + 48*a^3*b^6 - 52*a^4*b^5 + 28*a^5*b^4 -
6*a^6*b^3)*1i)/(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3) - (tan(e + f*x)*(16*b^
10 - 48*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7 - 48*a^4*b^6 + 16*a^5*b^5))/(2*(b^5
 - 2*a*b^4 + a^2*b^3)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) + (
tan(e + f*x)*(9*a^6 - 30*a^5*b + 4*b^6 + 25*a^4*b^2))/(2*(b^5 - 2*a*b^4 + a
^2*b^3)))/(2*a^2 - 4*a*b + 2*b^2) - ((((4*a*b^8 - 22*a^2*b^7 + 48*a^3*b^6 -
 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^3)*1i)/(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*
b^3) + (tan(e + f*x)*(16*b^10 - 48*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7 - 48*a^4
*b^6 + 16*a^5*b^5))/(2*(b^5 - 2*a*b^4 + a^2*b^3)*(2*a^2 - 4*a*b + 2*b^2)))/
(2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(9*a^6 - 30*a^5*b + 4*b^6 + 25*a^4*
b^2))/(2*(b^5 - 2*a*b^4 + a^2*b^3)))/(2*a^2 - 4*a*b + 2*b^2))/((((((4*a*b^8
 - 22*a^2*b^7 + 48*a^3*b^6 - 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^3)*1i)/(3*a*
b^5 - b^6 - 3*a^2*b^4 + a^3*b^3) - (tan(e + f*x)*(16*b^10 - 48*a*b^9 + 32*a
^2*b^8 + 32*a^3*b^7 - 48*a^4*b^6 + 16*a^5*b^5))/(2*(b^5 - 2*a*b^4 + a^2*b^3
)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(9*
a^6 - 30*a^5*b + 4*b^6 + 25*a^4*b^2)*1i)/(2*(b^5 - 2*a*b^4 + a^2*b^3)))/(2*
a^2 - 4*a*b + 2*b^2) - ((9*a^5)/2 - (21*a^4*b)/2 + 5*a^2*b^3 + 2*a^3*b^2)/(
3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3) + (((((4*a*b^8 - 22*a^2*b^7 + 48*a^3*b
^6 - 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^3)*1i)/(3*a*b^5 - b^6 - 3*a^2*b^4 +
a^3*b^3) + (tan(e + f*x)*(16*b^10 - 48*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7 - 48
*a^4*b^6 + 16*a^5*b^5))/(2*(b^5 - 2*a*b^4 + a^2*b^3)*(2*a^2 - 4*a*b + 2*b^2
)))*1i)/(2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(9*a^6 - 30*a^5*b + 4*b^6 +
 25*a^4*b^2)*1i)/(2*(b^5 - 2*a*b^4 + a^2*b^3)))/(2*a^2 - 4*a*b + 2*b^2))))/
(f*(2*a^2 - 4*a*b + 2*b^2)) + tan(e + f*x)/(b^2*f) + (a^2*tan(e + f*x))/(2*
f*(a - b)*(a*b^2 + b^3*tan(e + f*x)^2)) - (atan((((3*a - 5*b)*(-a^3*b^5)^(1
/2)*((tan(e + f*x)*(9*a^6 - 30*a^5*b + 4*b^6 + 25*a^4*b^2))/(2*(b^5 - 2*a*b
^4 + a^2*b^3)) - (((4*a*b^8 - 22*a^2*b^7 + 48*a^3*b^6 - 52*a^4*b^5 + 28*a^5
*b^4 - 6*a^6*b^3)/(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3) - (tan(e + f*x)*(3*
a - 5*b)*(-a^3*b^5)^(1/2)*(16*b^10 - 48*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7 - 4
8*a^4*b^6 + 16*a^5*b^5))/(8*(b^5 - 2*a*b^4 + a^2*b^3)*(b^7 - 2*a*b^6 + a^2*
b^5)))*(3*a - 5*b)*(-a^3*b^5)^(1/2))/(4*(b^7 - 2*a*b^6 + a^2*b^5)))*1i)/(4*
(b^7 - 2*a*b^6 + a^2*b^5)) + ((3*a - 5*b)*(-a^3*b^5)^(1/2)*((tan(e + f*x)*(
9*a^6 - 30*a^5*b + 4*b^6 + 25*a^4*b^2))/(2*(b^5 - 2*a*b^4 + a^2*b^3)) + (((
4*a*b^8 - 22*a^2*b^7 + 48*a^3*b^6 - 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^3)/(3
*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3) + (tan(e + f*x)*(3*a - 5*b)*(-a^3*b^5)^
(1/2)*(16*b^10 - 48*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7 - 48*a^4*b^6 + 16*a^5*b
^5))/(8*(b^5 - 2*a*b^4 + a^2*b^3)*(b^7 - 2*a*b^6 + a^2*b^5)))*(3*a - 5*b)*(
-a^3*b^5)^(1/2))/(4*(b^7 - 2*a*b^6 + a^2*b^5)))*1i)/(4*(b^7 - 2*a*b^6 + a^2
*b^5)))/(((9*a^5)/2 - (21*a^4*b)/2 + 5*a^2*b^3 + 2*a^3*b^2)/(3*a*b^5 - b^6
- 3*a^2*b^4 + a^3*b^3) + ((3*a - 5*b)*(-a^3*b^5)^(1/2)*((tan(e + f*x)*(9*a^
6 - 30*a^5*b + 4*b^6 + 25*a^4*b^2))/(2*(b^5 - 2*a*b^4 + a^2*b^3)) - (((4*a*
b^8 - 22*a^2*b^7 + 48*a^3*b^6 - 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^3)/(3*a*b
^5 - b^6 - 3*a^2*b^4 + a^3*b^3) - (tan(e + f*x)*(3*a - 5*b)*(-a^3*b^5)^(1/2
)*(16*b^10 - 48*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7 - 48*a^4*b^6 + 16*a^5*b^5))
/(8*(b^5 - 2*a*b^4 + a^2*b^3)*(b^7 - 2*a*b^6 + a^2*b^5)))*(3*a - 5*b)*(-a^3
```

```
*b^5)^(1/2))/(4*(b^7 - 2*a*b^6 + a^2*b^5))))/(4*(b^7 - 2*a*b^6 + a^2*b^5))
- ((3*a - 5*b)*(-a^3*b^5)^(1/2)*((tan(e + f*x)*(9*a^6 - 30*a^5*b + 4*b^6 +
25*a^4*b^2))/(2*(b^5 - 2*a*b^4 + a^2*b^3)) + (((4*a*b^8 - 22*a^2*b^7 + 48*a
^3*b^6 - 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^3)/(3*a*b^5 - b^6 - 3*a^2*b^4 +
a^3*b^3) + (tan(e + f*x)*(3*a - 5*b)*(-a^3*b^5)^(1/2)*(16*b^10 - 48*a*b^9 +
 32*a^2*b^8 + 32*a^3*b^7 - 48*a^4*b^6 + 16*a^5*b^5))/(8*(b^5 - 2*a*b^4 + a^
2*b^3)*(b^7 - 2*a*b^6 + a^2*b^5)))*(3*a - 5*b)*(-a^3*b^5)^(1/2))/(4*(b^7 -
2*a*b^6 + a^2*b^5))))/(4*(b^7 - 2*a*b^6 + a^2*b^5))))*(3*a - 5*b)*(-a^3*b^5
)^(1/2)*1i)/(2*f*(b^7 - 2*a*b^6 + a^2*b^5))
```

sympy [A] time = 78.02, size = 3198, normalized size = 24.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x*tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e +
 f*x)/f)/b**2, Eq(a, 0)), (-15*f*x*tan(e + f*x)**4/(8*b**2*f*tan(e + f*x)**
4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) - 30*f*x*tan(e + f*x)**2/(8*b**2*
f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) - 15*f*x/(8*b**2*
f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 8*tan(e + f*x)*
*5/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 25*t
an(e + f*x)**3/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b*
*2*f) + 15*tan(e + f*x)/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)*
*2 + 8*b**2*f), Eq(a, b)), (x*tan(e)**6/(a + b*tan(e)**2)**2, Eq(f, 0)), ((
-x + tan(e + f*x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e + f*x)/f)/a**2,
Eq(b, 0)), (6*I*a**(7/2)*b*sqrt(1/b)*tan(e + f*x)/(4*I*a**(7/2)*b**3*f*sqrt
(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**4*f
*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b
**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) + 4*I*a**(5
/2)*b**2*sqrt(1/b)*tan(e + f*x)**3/(4*I*a**(7/2)*b**3*f*sqrt(1/b) + 4*I*a**
(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**4*f*sqrt(1/b) - 8*
I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**5*f*sqrt(1/b)
 + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) - 10*I*a**(5/2)*b**2*sqrt(
1/b)*tan(e + f*x)/(4*I*a**(7/2)*b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt
(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f
*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b*
*6*f*sqrt(1/b)*tan(e + f*x)**2) - 4*I*a**(3/2)*b**3*f*x*sqrt(1/b)/(4*I*a**(
7/2)*b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I
*a**(5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2
+ 4*I*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)
**2) - 8*I*a**(3/2)*b**3*sqrt(1/b)*tan(e + f*x)**3/(4*I*a**(7/2)*b**3*f*sqr
t(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**4*
f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*
b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) + 4*I*a**(
3/2)*b**3*sqrt(1/b)*tan(e + f*x)/(4*I*a**(7/2)*b**3*f*sqrt(1/b) + 4*I*a**(5
/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**4*f*sqrt(1/b) - 8*I*
a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**5*f*sqrt(1/b) +
 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) - 4*I*sqrt(a)*b**4*f*x*sqrt(
1/b)*tan(e + f*x)**2/(4*I*a**(7/2)*b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*s
qrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**
5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)
*b**6*f*sqrt(1/b)*tan(e + f*x)**2) + 4*I*sqrt(a)*b**4*sqrt(1/b)*tan(e + f*x
)**3/(4*I*a**(7/2)*b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e +
 f*x)**2 - 8*I*a**(5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*ta
n(e + f*x)**2 + 4*I*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b
)*tan(e + f*x)**2) - 3*a**4*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a
**(7/2)*b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 -
8*I*a**(5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)*
```

```
*2 + 4*I*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f
*x)**2) + 3*a**4*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b**3
*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)
*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**
(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) - 3*
a**3*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/
2)*b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a
**(5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 +
4*I*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**
2) + 5*a**3*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b**3*f
*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b
**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3
/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) + 3*a*
*3*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/2)*
b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(
5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I
*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2)
- 5*a**3*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b**3*f*sqr
t(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**4*
f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*
b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2) + 5*a**2*b
**2*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/2)*
b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(
5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I
*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e + f*x)**2)
- 5*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*
a**(7/2)*b**3*f*sqrt(1/b) + 4*I*a**(5/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 -
 8*I*a**(5/2)*b**4*f*sqrt(1/b) - 8*I*a**(3/2)*b**5*f*sqrt(1/b)*tan(e + f*x)
**2 + 4*I*a**(3/2)*b**5*f*sqrt(1/b) + 4*I*sqrt(a)*b**6*f*sqrt(1/b)*tan(e +
f*x)**2), True))
```

3.231 $\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=95

$$\frac{\sqrt{a}\,(a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2b^{3/2}f(a-b)^2}-\frac{a\tan(e+fx)}{2bf(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x}{(a-b)^2}$$

```
[Out] x/(a-b)^2+1/2*(a-3*b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*a^(1/2)/(a-b)^2/b^
(3/2)/f-1/2*a*tan(f*x+e)/(a-b)/b/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 470, 522, 203, 205}

$$\frac{\sqrt{a}\,(a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2b^{3/2}f(a-b)^2}-\frac{a\tan(e+fx)}{2bf(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] x/(a - b)^2 + (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2
*(a - b)^2*b^(3/2)*f) - (a*Tan[e + f*x])/(2*(a - b)*b*f*(a + b*Tan[e + f*x]
^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{a+(a-2b)x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2(a-b)bf}$$

$$= -\frac{a\tan(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)^2f} + \frac{(a(a-3b)}{}$$

$$= \frac{x}{(a-b)^2} + \frac{\sqrt{a}\,(a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^2b^{3/2}f} - \frac{a\tan(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 0.82, size = 94, normalized size = 0.99

$$\frac{\frac{\sqrt{a}\,(a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{3/2}} - \frac{a(a-b)\sin(2(e+fx))}{b((a-b)\cos(2(e+fx))+a+b)} + 2(e+fx)}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (2*(e + f*x) + (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/b
^(3/2) - (a*(a - b)*Sin[2*(e + f*x)])/(b*(a + b + (a - b)*Cos[2*(e + f*x)])
))/(2*(a - b)^2*f)
```

fricas [A] time = 0.49, size = 381, normalized size = 4.01

$$\left[\frac{8\,b^2fx\tan\left(fx+e\right)^2 + 8\,abfx - \left(\left(ab-3\,b^2\right)\tan\left(fx+e\right)^2 + a^2 - 3\,ab\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{b^2\tan\left(fx+e\right)^4 - 6\,ab\tan\left(fx+e\right)^2 + a^2 - 4\left(\right.}{b^2\tan\left(fx+e\right)^4 + 2\,ab}\right.}{8\left(\left(a^2b^2 - 2\,ab^3 + b^4\right)f\tan\left(fx+e\right)^2 + \left(a^3b - 2\,a^2b^2 + ab^3\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*b^2*f*x*tan(f*x + e)^2 + 8*a*b*f*x - ((a*b - 3*b^2)*tan(f*x + e)^2
+ a^2 - 3*a*b)*sqrt(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
a^2 - 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x +
e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a^2 - a*b)*tan(f*x + e))/((a^2*b^2
 - 2*a*b^3 + b^4)*f*tan(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f), 1/4*(4
*b^2*f*x*tan(f*x + e)^2 + 4*a*b*f*x + ((a*b - 3*b^2)*tan(f*x + e)^2 + a^2 -
```

```
3*a*b)*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/b)/(a*tan(f*x +
e))) - 2*(a^2 - a*b)*tan(f*x + e))/((a^2*b^2 - 2*a*b^3 + b^4)*f*tan(f*x + e
)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f)]
```

giac [A] time = 3.70, size = 127, normalized size = 1.34

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)\left(a^2-3\,ab\right)}{\left(a^2b-2\,ab^2+b^3\right)\sqrt{ab}}+\frac{2\left(fx+e\right)}{a^2-2\,ab+b^2}-\frac{a\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)\left(ab-b^2\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)
))*(a^2 - 3*a*b)/((a^2*b - 2*a*b^2 + b^3)*sqrt(a*b)) + 2*(f*x + e)/(a^2 - 2
*a*b + b^2) - a*tan(f*x + e)/((b*tan(f*x + e)^2 + a)*(a*b - b^2)))/f
```

maple [A] time = 0.20, size = 160, normalized size = 1.68

$$-\frac{a^2\tan\left(fx+e\right)}{2fb\left(a-b\right)^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{a\tan\left(fx+e\right)}{2f\left(a-b\right)^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{a^2\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{2fb\left(a-b\right)^2\sqrt{ab}}-\frac{3a\arctan\left(\right.}{2f\left(a-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*a^2/b/(a-b)^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2/f*a/(a-b)^2*tan(f*x+
e)/(a+b*tan(f*x+e)^2)+1/2/f*a^2/b/(a-b)^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(
a*b)^(1/2))-3/2/f*a/(a-b)^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+1/
f/(a-b)^2*arctan(tan(f*x+e))
```

maxima [A] time = 0.75, size = 114, normalized size = 1.20

$$-\frac{\frac{a\tan\left(fx+e\right)}{a^2b-ab^2+\left(ab^2-b^3\right)\tan\left(fx+e\right)^2}-\frac{\left(a^2-3\,ab\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^2b-2\,ab^2+b^3\right)\sqrt{ab}}-\frac{2\left(fx+e\right)}{a^2-2\,ab+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*tan(f*x + e)/(a^2*b - a*b^2 + (a*b^2 - b^3)*tan(f*x + e)^2) - (a^2
- 3*a*b)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2*b - 2*a*b^2 + b^3)*sqrt(a*b
)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2))/f
```

mupad [B] time = 13.52, size = 2358, normalized size = 24.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (2*atan((((((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2*a^5*b^2)*1i)/
(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) - (tan(e + f*x)*(16*b^8 - 48*a*b^7 + 32
*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(2*(a^2*b - 2*a*b^2 + b^3
)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(a^4 -
6*a^3*b + 4*b^4 + 9*a^2*b^2))/(2*(a^2*b - 2*a*b^2 + b^3)))/(2*a^2 - 4*a*b +
```

```
2*b^2) - ((((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2*a^5*b^2)*1i)
/(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) + (tan(e + f*x)*(16*b^8 - 48*a*b^7 + 3
2*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(2*(a^2*b - 2*a*b^2 + b^
3)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(a^4 -
 6*a^3*b + 4*b^4 + 9*a^2*b^2))/(2*(a^2*b - 2*a*b^2 + b^3)))/(2*a^2 - 4*a*b
+ 2*b^2))/((((((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2*a^5*b^2)*1
i)/(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) - (tan(e + f*x)*(16*b^8 - 48*a*b^7 +
 32*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(2*(a^2*b - 2*a*b^2 +
b^3)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*
(a^4 - 6*a^3*b + 4*b^4 + 9*a^2*b^2)*1i)/(2*(a^2*b - 2*a*b^2 + b^3)))/(2*a^2
 - 4*a*b + 2*b^2) + (((((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2*a
^5*b^2)*1i)/(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) + (tan(e + f*x)*(16*b^8 - 4
8*a*b^7 + 32*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(2*(a^2*b - 2
*a*b^2 + b^3)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) - (tan(
e + f*x)*(a^4 - 6*a^3*b + 4*b^4 + 9*a^2*b^2)*1i)/(2*(a^2*b - 2*a*b^2 + b^3)
))/(2*a^2 - 4*a*b + 2*b^2) + (3*a*b^2 - (5*a^2*b)/2 + a^3/2)/(3*a*b^3 + a^3
*b - b^4 - 3*a^2*b^2))))/(f*(2*a^2 - 4*a*b + 2*b^2)) + (atan(((((tan(e + f*
x)*(a^4 - 6*a^3*b + 4*b^4 + 9*a^2*b^2))/(2*(a^2*b - 2*a*b^2 + b^3)) - (((2*
a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2*a^5*b^2)/(3*a*b^3 + a^3*b -
b^4 - 3*a^2*b^2) - (tan(e + f*x)*(a - 3*b)*(-a*b^3)^(1/2)*(16*b^8 - 48*a*b^
7 + 32*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(8*(a^2*b - 2*a*b^2
 + b^3)*(b^5 - 2*a*b^4 + a^2*b^3)))*(a - 3*b)*(-a*b^3)^(1/2))/(4*(b^5 - 2*a
*b^4 + a^2*b^3)))*(a - 3*b)*(-a*b^3)^(1/2)*1i)/(4*(b^5 - 2*a*b^4 + a^2*b^3)
) + (((tan(e + f*x)*(a^4 - 6*a^3*b + 4*b^4 + 9*a^2*b^2))/(2*(a^2*b - 2*a*b^
2 + b^3)) + (((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2*a^5*b^2)/(3
*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) + (tan(e + f*x)*(a - 3*b)*(-a*b^3)^(1/2)*
(16*b^8 - 48*a*b^7 + 32*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(8
*(a^2*b - 2*a*b^2 + b^3)*(b^5 - 2*a*b^4 + a^2*b^3)))*(a - 3*b)*(-a*b^3)^(1/
2))/(4*(b^5 - 2*a*b^4 + a^2*b^3)))*(a - 3*b)*(-a*b^3)^(1/2)*1i)/(4*(b^5 - 2
*a*b^4 + a^2*b^3)))/((3*a*b^2 - (5*a^2*b)/2 + a^3/2)/(3*a*b^3 + a^3*b - b^4
 - 3*a^2*b^2) - (((tan(e + f*x)*(a^4 - 6*a^3*b + 4*b^4 + 9*a^2*b^2))/(2*(a^
2*b - 2*a*b^2 + b^3)) - (((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2
*a^5*b^2)/(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) - (tan(e + f*x)*(a - 3*b)*(-a
*b^3)^(1/2)*(16*b^8 - 48*a*b^7 + 32*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*
a^5*b^3))/(8*(a^2*b - 2*a*b^2 + b^3)*(b^5 - 2*a*b^4 + a^2*b^3)))*(a - 3*b)*
(-a*b^3)^(1/2))/(4*(b^5 - 2*a*b^4 + a^2*b^3)))*(a - 3*b)*(-a*b^3)^(1/2))/(4
*(b^5 - 2*a*b^4 + a^2*b^3)) + (((tan(e + f*x)*(a^4 - 6*a^3*b + 4*b^4 + 9*a^
2*b^2))/(2*(a^2*b - 2*a*b^2 + b^3)) + (((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 -
 8*a^4*b^3 + 2*a^5*b^2)/(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) + (tan(e + f*x)
*(a - 3*b)*(-a*b^3)^(1/2)*(16*b^8 - 48*a*b^7 + 32*a^2*b^6 + 32*a^3*b^5 - 48
*a^4*b^4 + 16*a^5*b^3))/(8*(a^2*b - 2*a*b^2 + b^3)*(b^5 - 2*a*b^4 + a^2*b^3
)))*(a - 3*b)*(-a*b^3)^(1/2))/(4*(b^5 - 2*a*b^4 + a^2*b^3)))*(a - 3*b)*(-a*
b^3)^(1/2))/(4*(b^5 - 2*a*b^4 + a^2*b^3))))*(a - 3*b)*(-a*b^3)^(1/2)*1i)/(2
*f*(b^5 - 2*a*b^4 + a^2*b^3)) - (a*tan(e + f*x))/(2*b*f*(a + b*tan(e + f*x)
^2)*(a - b))
```

sympy [A] time = 28.76, size = 2416, normalized size = 25.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/b**2, Eq(a, 0)), (3*f
*x*tan(e + f*x)**4/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 +
8*b**2*f) + 6*f*x*tan(e + f*x)**2/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan
(e + f*x)**2 + 8*b**2*f) + 3*f*x/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(
e + f*x)**2 + 8*b**2*f) - 5*tan(e + f*x)**3/(8*b**2*f*tan(e + f*x)**4 + 16*
b**2*f*tan(e + f*x)**2 + 8*b**2*f) - 3*tan(e + f*x)/(8*b**2*f*tan(e + f*x)*
*4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f), Eq(a, b)), (x*tan(e)**4/(a + b*
```

```
tan(e)**2)**2, Eq(f, 0)), ((x + tan(e + f*x)**3/(3*f) - tan(e + f*x)/f)/a**
2, Eq(b, 0)), (-2*I*a**(5/2)*b*sqrt(1/b)*tan(e + f*x)/(4*I*a**(7/2)*b**2*f*
sqrt(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b*
*3*f*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/
2)*b**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2) + 4*I*a
**(3/2)*b**2*f*x*sqrt(1/b)/(4*I*a**(7/2)*b**2*f*sqrt(1/b) + 4*I*a**(5/2)*b*
*3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**3*f*sqrt(1/b) - 8*I*a**(3/
2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**4*f*sqrt(1/b) + 4*I*s
qrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2) + 2*I*a**(3/2)*b**2*sqrt(1/b)*tan(
e + f*x)/(4*I*a**(7/2)*b**2*f*sqrt(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan
(e + f*x)**2 - 8*I*a**(5/2)*b**3*f*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b
)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt
(1/b)*tan(e + f*x)**2) + 4*I*sqrt(a)*b**3*f*x*sqrt(1/b)*tan(e + f*x)**2/(4*
I*a**(7/2)*b**2*f*sqrt(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2
 - 8*I*a**(5/2)*b**3*f*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*
x)**2 + 4*I*a**(3/2)*b**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e
+ f*x)**2) + a**3*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b*
*2*f*sqrt(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/
2)*b**3*f*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a
**(3/2)*b**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2) -
a**3*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b**2*f*sqrt(1/b)
 + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**3*f*sqrt
(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**4*f
*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2) + a**2*b*log(-I*
sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/2)*b**2*f*sqrt
(1/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**3*f
*sqrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b
**4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2) - 3*a**2*b*
log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b**2*f*sqrt(1/b) + 4
*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**3*f*sqrt(1/b
) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**4*f*sqr
t(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2) - a**2*b*log(I*sqrt(
a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/2)*b**2*f*sqrt(1/b)
 + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**3*f*sqrt
(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**4*f
*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2) + 3*a**2*b*log(I
*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b**2*f*sqrt(1/b) + 4*I*a**
(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**3*f*sqrt(1/b) - 8*
I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**4*f*sqrt(1/b)
 + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2) - 3*a*b**2*log(-I*sqrt(a)*
sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/2)*b**2*f*sqrt(1/b) +
4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**3*f*sqrt(1/
b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**4*f*sq
rt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2) + 3*a*b**2*log(I*sq
rt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/2)*b**2*f*sqrt(1
/b) + 4*I*a**(5/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**3*f*s
qrt(1/b) - 8*I*a**(3/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**
4*f*sqrt(1/b) + 4*I*sqrt(a)*b**5*f*sqrt(1/b)*tan(e + f*x)**2), True))
```

3.232 $\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}f(a-b)^2}+\frac{\tan(e+fx)}{2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{x}{(a-b)^2}$$

```
[Out] -x/(a-b)^2+1/2*(a+b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)^2/f/a^(1/2)/b
^(1/2)+1/2*tan(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3670, 471, 522, 203, 205}

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}f(a-b)^2}+\frac{\tan(e+fx)}{2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(x/(a - b)^2) + ((a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*Sqrt[a
]*(a - b)^2*Sqrt[b]*f) + Tan[e + f*x]/(2*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
```

```
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1-x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= \frac{\tan(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)^2 f} + \frac{(a+b)\,\text{Su}}{}$$

$$= -\frac{x}{(a-b)^2} + \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}\,(a-b)^2\sqrt{b}\,f} + \frac{\tan(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 0.59, size = 87, normalized size = 0.97

$$\frac{\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{(a-b)\sin(2(e+fx))}{(a-b)\cos(2(e+fx))+a+b} - 2(e+fx)}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-2*(e + f*x) + ((a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*S
qrt[b]) + ((a - b)*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)]))/(2
*(a - b)^2*f)
```

fricas [B] time = 0.47, size = 393, normalized size = 4.37

$$\left[-\frac{8\,ab^2fx\tan\left(fx+e\right)^2 + 8\,a^2bfx + \left(\left(ab+b^2\right)\tan\left(fx+e\right)^2 + a^2 + ab\right)\sqrt{-ab}\,\log\left(\frac{b^2\tan\left(fx+e\right)^4 - 6\,ab\tan\left(fx+e\right)^2 + }{b^2\tan\left(fx+e\right)^4}\right.}{8\left(\left(a^3b^2 - 2\,a^2b^3 + ab^4\right)f\tan\left(fx+e\right)^2 + \left(a^4b - 2\,a^3b^2 + \right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b^2*f*x*tan(f*x + e)^2 + 8*a^2*b*f*x + ((a*b + b^2)*tan(f*x + e)
^2 + a^2 + a*b)*sqrt(-a*b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
 a^2 - 4*(b*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(-a*b))/(b^2*tan(f*x + e)^
4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a^2*b - a*b^2)*tan(f*x + e))/((a^3*b^
2 - 2*a^2*b^3 + a*b^4)*f*tan(f*x + e)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*f),
 -1/4*(4*a*b^2*f*x*tan(f*x + e)^2 + 4*a^2*b*f*x - ((a*b + b^2)*tan(f*x + e)
^2 + a^2 + a*b)*sqrt(a*b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a*b)/(a*b*
tan(f*x + e))) - 2*(a^2*b - a*b^2)*tan(f*x + e))/((a^3*b^2 - 2*a^2*b^3 + a*
b^4)*f*tan(f*x + e)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*f)]
```

giac [A] time = 2.04, size = 112, normalized size = 1.24

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(a+b)}{(a^2-2\,ab+b^2)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}+\frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)(a-b)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)
))*(a + b)/((a^2 - 2*a*b + b^2)*sqrt(a*b)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2
) + tan(f*x + e)/((b*tan(f*x + e)^2 + a)*(a - b)))/f
```

maple [A] time = 0.22, size = 151, normalized size = 1.68

$$\frac{a\tan(fx+e)}{2f(a-b)^2\left(a+b\left(\tan^2(fx+e)\right)\right)}-\frac{b\tan(fx+e)}{2(a-b)^2 f\left(a+b\left(\tan^2(fx+e)\right)\right)}+\frac{a\arctan\left(\frac{\tan(fx+e)b}{\sqrt{ab}}\right)}{2f(a-b)^2\sqrt{ab}}+\frac{\arctan\left(\frac{\tan(fx+e)b}{\sqrt{ab}}\right)}{2f(a-b)^2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*a/(a-b)^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)-1/2*b*tan(f*x+e)/(a-b)^2/f/(a
+b*tan(f*x+e)^2)+1/2/f*a/(a-b)^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2
))+1/2/f/(a-b)^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))*b-1/f/(a-b)^2
*arctan(tan(f*x+e))
```

maxima [A] time = 0.77, size = 97, normalized size = 1.08

$$\frac{\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^2-2\,ab+b^2)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}+\frac{\tan(fx+e)}{(ab-b^2)\tan(fx+e)^2+a^2-ab}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a + b)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2 - 2*a*b + b^2)*sqrt(a*b
)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2) + tan(f*x + e)/((a*b - b^2)*tan(f*x +
e)^2 + a^2 - a*b))/f
```

mupad [B] time = 13.04, size = 2136, normalized size = 23.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] tan(e + f*x)/(2*f*(a + b*tan(e + f*x)^2)*(a - b)) - (2*atan((((((2*b^6 - 8*
a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)*1i)/(3*a*b^2 - 3*a^2*b + a^3 -
b^3) - (tan(e + f*x)*(16*b^7 - 48*a*b^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*
b^3 + 16*a^5*b^2))/(2*(a^2 - 2*a*b + b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2
- 4*a*b + 2*b^2) + (tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3))/(2*(a^2 - 2*a*b
 + b^2)))/(2*a^2 - 4*a*b + 2*b^2) - ((((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^
3*b^3 + 2*a^4*b^2)*1i)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (tan(e + f*x)*(16*
b^7 - 48*a*b^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(2*(a^
2 - 2*a*b + b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - (tan(e
```

```
+ f*x)*(2*a*b^2 + a^2*b + 5*b^3))/(2*(a^2 - 2*a*b + b^2)))/(2*a^2 - 4*a*b
+ 2*b^2))/((((((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)*1i)/(
3*a*b^2 - 3*a^2*b + a^3 - b^3) - (tan(e + f*x)*(16*b^7 - 48*a*b^6 + 32*a^2*
b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(2*(a^2 - 2*a*b + b^2)*(2*a^2
- 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(2*a*b^2 + a
^2*b + 5*b^3)*1i)/(2*(a^2 - 2*a*b + b^2)))/(2*a^2 - 4*a*b + 2*b^2) + (((((2
*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)*1i)/(3*a*b^2 - 3*a^2*b
+ a^3 - b^3) + (tan(e + f*x)*(16*b^7 - 48*a*b^6 + 32*a^2*b^5 + 32*a^3*b^4
- 48*a^4*b^3 + 16*a^5*b^2))/(2*(a^2 - 2*a*b + b^2)*(2*a^2 - 4*a*b + 2*b^2))
)*1i)/(2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3)*1i)
/(2*(a^2 - 2*a*b + b^2)))/(2*a^2 - 4*a*b + 2*b^2) + ((a*b)/2 + b^2/2)/(3*a*
b^2 - 3*a^2*b + a^3 - b^3))))/(f*(2*a^2 - 4*a*b + 2*b^2)) - (atan((((-a*b)^
(1/2)*(a + b)*((tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3))/(2*(a^2 - 2*a*b + b
^2)) - (((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)/(3*a*b^2 -
3*a^2*b + a^3 - b^3) - (tan(e + f*x)*(-a*b)^(1/2)*(a + b)*(16*b^7 - 48*a*b^
6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(8*(a^2 - 2*a*b + b
^2)*(a*b^3 + a^3*b - 2*a^2*b^2)))*(-a*b)^(1/2)*(a + b))/(4*(a*b^3 + a^3*b -
 2*a^2*b^2)))*1i)/(4*(a*b^3 + a^3*b - 2*a^2*b^2)) + ((-a*b)^(1/2)*(a + b)*(
(tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3))/(2*(a^2 - 2*a*b + b^2)) + (((2*b^6
 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)/(3*a*b^2 - 3*a^2*b + a^3 -
 b^3) + (tan(e + f*x)*(-a*b)^(1/2)*(a + b)*(16*b^7 - 48*a*b^6 + 32*a^2*b^5
+ 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(8*(a^2 - 2*a*b + b^2)*(a*b^3 + a^
3*b - 2*a^2*b^2)))*(-a*b)^(1/2)*(a + b))/(4*(a*b^3 + a^3*b - 2*a^2*b^2)))*1
i)/(4*(a*b^3 + a^3*b - 2*a^2*b^2)))/(((a*b)/2 + b^2/2)/(3*a*b^2 - 3*a^2*b +
 a^3 - b^3) - ((-a*b)^(1/2)*(a + b)*((tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3
))/(2*(a^2 - 2*a*b + b^2)) - (((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 +
2*a^4*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) - (tan(e + f*x)*(-a*b)^(1/2)*(a
+ b)*(16*b^7 - 48*a*b^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2
))/(8*(a^2 - 2*a*b + b^2)*(a*b^3 + a^3*b - 2*a^2*b^2)))*(-a*b)^(1/2)*(a + b
))/(4*(a*b^3 + a^3*b - 2*a^2*b^2))))/(4*(a*b^3 + a^3*b - 2*a^2*b^2)) + ((-a
*b)^(1/2)*(a + b)*((tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3))/(2*(a^2 - 2*a*b
 + b^2)) + (((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)/(3*a*b^
2 - 3*a^2*b + a^3 - b^3) + (tan(e + f*x)*(-a*b)^(1/2)*(a + b)*(16*b^7 - 48*
a*b^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(8*(a^2 - 2*a*b
 + b^2)*(a*b^3 + a^3*b - 2*a^2*b^2)))*(-a*b)^(1/2)*(a + b))/(4*(a*b^3 + a^3
*b - 2*a^2*b^2))))/(4*(a*b^3 + a^3*b - 2*a^2*b^2))))*(-a*b)^(1/2)*(a + b)*1
i)/(2*f*(a*b^3 + a^3*b - 2*a^2*b^2))
```

sympy [A] time = 28.29, size = 2375, normalized size = 26.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x - 1/(f*ta
n(e + f*x)))/b**2, Eq(a, 0)), (f*x*tan(e + f*x)**4/(8*b**2*f*tan(e + f*x)**
4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 2*f*x*tan(e + f*x)**2/(8*b**2*f
*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + f*x/(8*b**2*f*ta
n(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + tan(e + f*x)**3/(8*
b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) - tan(e + f*
x)/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f), Eq(a,
 b)), (x*tan(e)**2/(a + b*tan(e)**2)**2, Eq(f, 0)), ((-x + tan(e + f*x)/f)/
a**2, Eq(b, 0)), (-4*I*a**(3/2)*b*f*x*sqrt(1/b)/(4*I*a**(7/2)*b*f*sqrt(1/b)
 + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt
(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**3*f
*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(1/b)*tan(e + f*x)**2) + 2*I*a**(3/2)*b
*sqrt(1/b)*tan(e + f*x)/(4*I*a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2*f*s
qrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)*b**
3*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqrt(a)
```

```
*b**4*f*sqrt(1/b)*tan(e + f*x)**2) - 4*I*sqrt(a)*b**2*f*x*sqrt(1/b)*tan(e +
 f*x)**2/(4*I*a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e
+ f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)*t
an(e + f*x)**2 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(1/
b)*tan(e + f*x)**2) - 2*I*sqrt(a)*b**2*sqrt(1/b)*tan(e + f*x)/(4*I*a**(7/2)
*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/
2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a
**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(1/b)*tan(e + f*x)**2) +
a**2*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b*f*sqrt(1/b) +
 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt(1
/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**3*f*s
qrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(1/b)*tan(e + f*x)**2) - a**2*log(I*sqrt(
a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**
2*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2
)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sq
rt(a)*b**4*f*sqrt(1/b)*tan(e + f*x)**2) + a*b*log(-I*sqrt(a)*sqrt(1/b) + ta
n(e + f*x))*tan(e + f*x)**2/(4*I*a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2
*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)
*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqr
t(a)*b**4*f*sqrt(1/b)*tan(e + f*x)**2) + a*b*log(-I*sqrt(a)*sqrt(1/b) + tan
(e + f*x))/(4*I*a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(
e + f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)
*tan(e + f*x)**2 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(
1/b)*tan(e + f*x)**2) - a*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e +
 f*x)**2/(4*I*a**(7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e
+ f*x)**2 - 8*I*a**(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)*t
an(e + f*x)**2 + 4*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(1/
b)*tan(e + f*x)**2) - a*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(
7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a*
*(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 4
*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(1/b)*tan(e + f*x)**2
) + b**2*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(
7/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a*
*(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 4
*I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(1/b)*tan(e + f*x)**2
) - b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(7
/2)*b*f*sqrt(1/b) + 4*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**
(5/2)*b**2*f*sqrt(1/b) - 8*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 4*
I*a**(3/2)*b**3*f*sqrt(1/b) + 4*I*sqrt(a)*b**4*f*sqrt(1/b)*tan(e + f*x)**2)
, True))
```

3.233 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{b}\,(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}f(a-b)^2}-\frac{b\tan(e+fx)}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x}{(a-b)^2}$$

```
[Out] x/(a-b)^2-1/2*(3*a-b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(3/2)/(a
-b)^2/f-1/2*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = $\{3661, 414, 522, 203, 205\}$

$$-\frac{\sqrt{b}\,(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}f(a-b)^2}-\frac{b\tan(e+fx)}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-2),x]
```

```
[Out] x/(a - b)^2 - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2
*a^(3/2)*(a - b)^2*f) - (b*Tan[e + f*x])/(2*a*(a - b)*f*(a + b*Tan[e + f*x]
^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
```

```
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx=\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)}+\frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx,x,\tan(e+fx)\right)}{2a(a-b)f}$$

$$=-\frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)}+\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{(a-b)^2f}-\frac{((3a-b)b}{\ }$$

$$=\frac{x}{(a-b)^2}-\frac{(3a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2f}-\frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 1.04, size = 88, normalized size = 0.91

$$\frac{\frac{\sqrt{b}(b-3a)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}}+\frac{b(b-a)\tan(e+fx)}{a\left(a+b\tan^2(e+fx)\right)}+2\tan^{-1}(\tan(e+fx))}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-2),x]
```

```
[Out] (2*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a]])/a^(3/2) + (b*(-a + b)*Tan[e + f*x])/(a*(a + b*Tan[e + f*x]^2)))/
(2*(a - b)^2*f)
```

fricas [A] time = 0.46, size = 390, normalized size = 4.02

$$\left[\frac{8\,abfx\tan\left(fx+e\right)^2+8\,a^2fx-\left(\left(3\,ab-b^2\right)\tan\left(fx+e\right)^2+3\,a^2-ab\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan\left(fx+e\right)^4-6\,ab\tan\left(fx+e\right)^2+a^2+4\left(\cdots\right.}{b^2\tan\left(fx+e\right)^4+2\,ab\cdots}\right.}{8\left(\left(a^3b-2\,a^2b^2+ab^3\right)f\tan\left(fx+e\right)^2+\left(a^4-2\,a^3b+a^2b^2\right)\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*b*f*x*tan(f*x + e)^2 + 8*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2
+ 3*a^2 - a*b)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x +
e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a*b - b^2)*tan(f*x + e))/((a^3*b -
 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f), 1/4*(4
*a*b*f*x*tan(f*x + e)^2 + 4*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2 + 3*a^2
 - a*b)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x +
e))) - 2*(a*b - b^2)*tan(f*x + e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x +
 e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f)]
```

giac [A] time = 2.28, size = 127, normalized size = 1.31

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(3\,ab-b^2)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}+\frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)(a^2-ab)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
)))*(3*a*b - b^2)/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b)) - 2*(f*x + e)/(a^2 -
2*a*b + b^2) + b*tan(f*x + e)/((b*tan(f*x + e)^2 + a)*(a^2 - a*b)))/f
```

maple [A] time = 0.29, size = 160, normalized size = 1.65

$$-\frac{b\tan\left(fx+e\right)}{2\left(a-b\right)^2 f\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{b^2\tan\left(fx+e\right)}{2f\left(a-b\right)^2 a\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{3\arctan\left(\frac{\tan(fx+e)b}{\sqrt{ab}}\right)b}{2f\left(a-b\right)^2\sqrt{ab}}+\frac{b^2\arctan}{2f\left(a-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2*b*tan(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)+1/2/f*b^2/(a-b)^2/a*tan(f*x+
e)/(a+b*tan(f*x+e)^2)-3/2/f/(a-b)^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(
1/2))*b+1/2/f*b^2/(a-b)^2/a/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+1/
f/(a-b)^2*arctan(tan(f*x+e))
```

maxima [A] time = 0.76, size = 114, normalized size = 1.18

$$-\frac{\frac{b\tan(fx+e)}{a^3-a^2b+(a^2b-ab^2)\tan(fx+e)^2}+\frac{(3\,ab-b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*tan(f*x + e)/(a^3 - a^2*b + (a^2*b - a*b^2)*tan(f*x + e)^2) + (3*a*
b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b
)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2))/f
```

mupad [B] time = 13.52, size = 2489, normalized size = 25.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (2*atan((((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 -
4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(16*a^
2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2
*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2
) + (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2
)))/(2*a^2 - 4*a*b + 2*b^2) - ((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^
4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) +
 (tan(e + f*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*
b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(
```

```
2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^
4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^2))/((((((2*a*b^7 - 12*a^2*b^
6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 +
a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5
+ 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^
2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(b^5 - 6*a
*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^
2) + (((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a
^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(e + f*x)*(16*a^2*b
^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a
^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^
2) - (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2
*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - ((3*a*b^3)/2 - b^4/2)/(3*a^4*b - a^5 + a^
2*b^3 - 3*a^3*b^2))))/(f*(2*a^2 - 4*a*b + 2*b^2)) - (atan((((-a^3*b)^(1/2)*
((tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2))
- (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^
2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(-a^3*b)^(1/2)*(3*
a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16
*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b
)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a - b)*1i)/(4*(a^5 - 2
*a^4*b + a^3*b^2)) + ((-a^3*b)^(1/2)*((tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2
*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)) + (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5
 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^
2) + (tan(e + f*x)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a
^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2
)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b +
 a^3*b^2)))*(3*a - b)*1i)/(4*(a^5 - 2*a^4*b + a^3*b^2)))/(((3*a*b^3)/2 - b^
4/2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + ((-a^3*b)^(1/2)*((tan(e + f*x)
*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)) - (((2*a*b^7 -
 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b -
a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2
*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*
(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b)^(1/2)*(3*a -
 b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)
) - ((-a^3*b)^(1/2)*((tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 -
2*a^3*b + a^2*b^2)) + (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 1
8*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(e + f*x
)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b
^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b
+ a^3*b^2)))*(-a^3*b)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a
- b))/(4*(a^5 - 2*a^4*b + a^3*b^2))))*(-a^3*b)^(1/2)*(3*a - b)*1i)/(2*f*(a^
5 - 2*a^4*b + a^3*b^2)) - (b*tan(e + f*x))/(2*a*f*(a + b*tan(e + f*x)^2)*(a
 - b))
```

sympy [A] time = 27.87, size = 2322, normalized size = 23.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a**2, Eq(b,
 0)), ((x + 1/(f*tan(e + f*x)) - 1/(3*f*tan(e + f*x)**3))/b**2, Eq(a, 0)),
(3*f*x*tan(e + f*x)**4/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**
2 + 8*b**2*f) + 6*f*x*tan(e + f*x)**2/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f
*tan(e + f*x)**2 + 8*b**2*f) + 3*f*x/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*
tan(e + f*x)**2 + 8*b**2*f) + 3*tan(e + f*x)**3/(8*b**2*f*tan(e + f*x)**4 +
 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 5*tan(e + f*x)/(8*b**2*f*tan(e + f
*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f), Eq(a, b)), (x/(a + b*tan(e)
**2)**2, Eq(f, 0)), (4*I*a**(5/2)*f*x*sqrt(1/b)/(4*I*a**(9/2)*f*sqrt(1/b) +
```

```
4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) -
8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/
b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) + 4*I*a**(3/2)*b*f*x*sq
rt(1/b)*tan(e + f*x)**2/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1
/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt
(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f
*sqrt(1/b)*tan(e + f*x)**2) - 2*I*a**(3/2)*b*sqrt(1/b)*tan(e + f*x)/(4*I*a*
*(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7
/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**
(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) + 2
*I*sqrt(a)*b**2*sqrt(1/b)*tan(e + f*x)/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(
7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5
/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*
a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) - 3*a**2*log(-I*sqrt(a)*sqrt(1/b
) + tan(e + f*x))/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*ta
n(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*
tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(
1/b)*tan(e + f*x)**2) + 3*a**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I
*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a*
*(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*
a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2)
- 3*a*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(9
/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)
*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/
2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) + a*b*
log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a*
*(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**
(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*
I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) + 3*a*b*log(I*sqrt(a)*sqrt(1/b
) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*
b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b
**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3
/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2) - a*b*log(I*sqrt(a)*sqrt(1/b) + tan(e
 + f*x))/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x
)**2 - 8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f
*x)**2 + 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(
e + f*x)**2) + b**2*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**
2/(4*I*a**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 -
8*I*a**(7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2
+ 4*I*a**(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x
)**2) - b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*I*a
**(9/2)*f*sqrt(1/b) + 4*I*a**(7/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 8*I*a**(
7/2)*b*f*sqrt(1/b) - 8*I*a**(5/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 4*I*a*
*(5/2)*b**2*f*sqrt(1/b) + 4*I*a**(3/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2), T
rue))
```

3.234 $\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=128

$$\frac{b^{3/2}(5a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f(a-b)^2}-\frac{(2a-3b)\cot(e+fx)}{2a^2f(a-b)}-\frac{b\cot(e+fx)}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{x}{(a-b)^2}$$

```
[Out] -x/(a-b)^2+1/2*(5*a-3*b)*b^(3/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(5/2)
/(a-b)^2/f-1/2*(2*a-3*b)*cot(f*x+e)/a^2/(a-b)/f-1/2*b*cot(f*x+e)/a/(a-b)/f/
(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.19, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 472, 583, 522, 203, 205}

$$\frac{b^{3/2}(5a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f(a-b)^2}-\frac{(2a-3b)\cot(e+fx)}{2a^2f(a-b)}-\frac{b\cot(e+fx)}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(x/(a - b)^2) + ((5*a - 3*b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(2*a^(5/2)*(a - b)^2*f) - ((2*a - 3*b)*Cot[e + f*x])/(2*a^2*(a - b)*f) -
 (b*Cot[e + f*x])/(2*a*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-3b-3bx^2}{x^2(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a-b)f}$$

$$= -\frac{(2a-3b)\cot(e+fx)}{2a^2(a-b)f} - \frac{b\cot(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{2a^2+2ab-3b}{(1+x^2)}\right.}{2}$$

$$= -\frac{(2a-3b)\cot(e+fx)}{2a^2(a-b)f} - \frac{b\cot(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x\right.}{(a-b)^2}$$

$$= -\frac{x}{(a-b)^2} + \frac{(5a-3b)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}(a-b)^2f} - \frac{(2a-3b)\cot(e+fx)}{2a^2(a-b)f} - \frac{}{2a(a-}$$

Mathematica [A] time = 3.22, size = 117, normalized size = 0.91

$$\frac{\frac{b^{3/2}(5a-3b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)^2} + \frac{\frac{b^2(a-b)\sin(2(e+fx))}{a^2((a-b)\cos(2(e+fx))+a+b)} - 2(e+fx)}{(a-b)^2} - \frac{2\cot(e+fx)}{a^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (((5*a - 3*b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(5/2)*(a -
 b)^2) - (2*Cot[e + f*x])/a^2 + (-2*(e + f*x) + ((a - b)*b^2*Sin[2*(e + f*x
)])/(a^2*(a + b + (a - b)*Cos[2*(e + f*x)])))/(a - b)^2)/(2*f)
```

fricas [A] time = 0.48, size = 503, normalized size = 3.93

$$\left[-\frac{8\,a^2bfx\tan\left(fx+e\right)^3+8\,a^3fx\tan\left(fx+e\right)+8\,a^3-16\,a^2b+8\,ab^2+4\left(2\,a^2b-5\,ab^2+3\,b^3\right)\tan\left(fx+e\right)^2+\left(\right.}{8\left(\left(a^4b-2\,a^3b^2+a^2b^3\right)f\tan\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a^2*b*f*x*tan(f*x + e)^3 + 8*a^3*f*x*tan(f*x + e) + 8*a^3 - 16*a^2
*b + 8*a*b^2 + 4*(2*a^2*b - 5*a*b^2 + 3*b^3)*tan(f*x + e)^2 + ((5*a*b^2 - 3
*b^3)*tan(f*x + e)^3 + (5*a^2*b - 3*a*b^2)*tan(f*x + e))*sqrt(-b/a)*log((b^
2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2
*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2
)))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*
b^2)*f*tan(f*x + e)), -1/4*(4*a^2*b*f*x*tan(f*x + e)^3 + 4*a^3*f*x*tan(f*x
+ e) + 4*a^3 - 8*a^2*b + 4*a*b^2 + 2*(2*a^2*b - 5*a*b^2 + 3*b^3)*tan(f*x +
e)^2 - ((5*a*b^2 - 3*b^3)*tan(f*x + e)^3 + (5*a^2*b - 3*a*b^2)*tan(f*x + e)
)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/
((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*b^2)
*f*tan(f*x + e))]
```

giac [A] time = 2.60, size = 171, normalized size = 1.34

$$\frac{\frac{\left(5\,ab^2-3\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^4-2\,a^3b+a^2b^2\right)\sqrt{ab}}-\frac{2\left(fx+e\right)}{a^2-2\,ab+b^2}-\frac{2\,ab\tan\left(fx+e\right)^2-3\,b^2\tan\left(fx+e\right)^2+2\,a^2-2\,ab}{\left(b\tan\left(fx+e\right)^3+a\tan\left(fx+e\right)\right)\left(a^3-a^2b\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*a*b^2 - 3*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b)))/((a^4 - 2*a^3*b + a^2*b^2)*sqrt(a*b)) - 2*(f*x + e)/(a
^2 - 2*a*b + b^2) - (2*a*b*tan(f*x + e)^2 - 3*b^2*tan(f*x + e)^2 + 2*a^2 -
2*a*b)/((b*tan(f*x + e)^3 + a*tan(f*x + e))*(a^3 - a^2*b)))/f
```

maple [A] time = 0.83, size = 187, normalized size = 1.46

$$\frac{b^2\tan\left(fx+e\right)}{2f\left(a-b\right)^2a\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{b^3\tan\left(fx+e\right)}{2f\,a^2\left(a-b\right)^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{5b^2\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{2f\left(a-b\right)^2a\sqrt{ab}}-\frac{3b^3\arctan\left(\right.}{2f\,a^2\left(a-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*b^2/(a-b)^2/a*tan(f*x+e)/(a+b*tan(f*x+e)^2)-1/2/f*b^3/a^2/(a-b)^2*tan
(f*x+e)/(a+b*tan(f*x+e)^2)+5/2/f*b^2/(a-b)^2/a/(a*b)^(1/2)*arctan(tan(f*x+e
)*b/(a*b)^(1/2))-3/2/f*b^3/a^2/(a-b)^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b
)^(1/2))-1/f/a^2/tan(f*x+e)-1/f/(a-b)^2*arctan(tan(f*x+e))
```

maxima [A] time = 0.71, size = 151, normalized size = 1.18

$$\frac{\frac{\left(5\,ab^2-3\,b^3\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^4-2\,a^3b+a^2b^2\right)\sqrt{ab}}-\frac{\left(2\,ab-3\,b^2\right)\tan\left(fx+e\right)^2+2\,a^2-2\,ab}{\left(a^3b-a^2b^2\right)\tan\left(fx+e\right)^3+\left(a^4-a^3b\right)\tan\left(fx+e\right)}-\frac{2\left(fx+e\right)}{a^2-2\,ab+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((5*a*b^2 - 3*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - 2*a^3*b + a
^2*b^2)*sqrt(a*b)) - ((2*a*b - 3*b^2)*tan(f*x + e)^2 + 2*a^2 - 2*a*b)/((a^3
*b - a^2*b^2)*tan(f*x + e)^3 + (a^4 - a^3*b)*tan(f*x + e)) - 2*(f*x + e)/(a
^2 - 2*a*b + b^2))/f
```

mupad [B] time = 14.23, size = 2674, normalized size = 20.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] - (1/a + (tan(e + f*x)^2*(2*a*b - 3*b^2))/(2*a^2*(a - b)))/(f*(a*tan(e + f*
x) + b*tan(e + f*x)^3)) - (2*atan(((((1280*a^9*b^9 - 192*a^8*b^10 - 3520*a^
10*b^8 + 4992*a^11*b^7 - 3520*a^12*b^6 + 512*a^13*b^5 + 960*a^14*b^4 - 640*
a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(256*a^10*b^10 - 1536*a^11*b^9 + 35
84*a^12*b^8 - 3584*a^13*b^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3
 - 256*a^18*b^2)*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*b^2) +
 tan(e + f*x)*(144*a^6*b^10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b^7 + 1
744*a^10*b^6 - 592*a^11*b^5 + 192*a^12*b^4 - 64*a^13*b^3))/(2*a^2 - 4*a*b +
 2*b^2) + (((192*a^8*b^10 - 1280*a^9*b^9 + 3520*a^10*b^8 - 4992*a^11*b^7 +
3520*a^12*b^6 - 512*a^13*b^5 - 960*a^14*b^4 + 640*a^15*b^3 - 128*a^16*b^2 +
 (tan(e + f*x)*(256*a^10*b^10 - 1536*a^11*b^9 + 3584*a^12*b^8 - 3584*a^13*b
^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3 - 256*a^18*b^2)*1i)/(2*a
^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*b^2) + tan(e + f*x)*(144*a^6*b^
10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b^7 + 1744*a^10*b^6 - 592*a^11*b
^5 + 192*a^12*b^4 - 64*a^13*b^3))/(2*a^2 - 4*a*b + 2*b^2))/(((((192*a^8*b^1
0 - 1280*a^9*b^9 + 3520*a^10*b^8 - 4992*a^11*b^7 + 3520*a^12*b^6 - 512*a^13
*b^5 - 960*a^14*b^4 + 640*a^15*b^3 - 128*a^16*b^2 + (tan(e + f*x)*(256*a^10
*b^10 - 1536*a^11*b^9 + 3584*a^12*b^8 - 3584*a^13*b^7 + 3584*a^15*b^5 - 358
4*a^16*b^4 + 1536*a^17*b^3 - 256*a^18*b^2)*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)
/(2*a^2 - 4*a*b + 2*b^2) + tan(e + f*x)*(144*a^6*b^10 - 912*a^7*b^9 + 2272*
a^8*b^8 - 2784*a^9*b^7 + 1744*a^10*b^6 - 592*a^11*b^5 + 192*a^12*b^4 - 64*a
^13*b^3))*1i)/(2*a^2 - 4*a*b + 2*b^2) - ((((1280*a^9*b^9 - 192*a^8*b^10 - 3
520*a^10*b^8 + 4992*a^11*b^7 - 3520*a^12*b^6 + 512*a^13*b^5 + 960*a^14*b^4
- 640*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(256*a^10*b^10 - 1536*a^11*b^
9 + 3584*a^12*b^8 - 3584*a^13*b^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^
17*b^3 - 256*a^18*b^2)*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*
b^2) + tan(e + f*x)*(144*a^6*b^10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b
^7 + 1744*a^10*b^6 - 592*a^11*b^5 + 192*a^12*b^4 - 64*a^13*b^3))*1i)/(2*a^2
 - 4*a*b + 2*b^2) + 144*a^6*b^8 - 624*a^7*b^7 + 976*a^8*b^6 - 656*a^9*b^5 +
 160*a^10*b^4)))/(f*(2*a^2 - 4*a*b + 2*b^2)) - (atan((((tan(e + f*x)*(144*a
^6*b^10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b^7 + 1744*a^10*b^6 - 592*a
^11*b^5 + 192*a^12*b^4 - 64*a^13*b^3) + ((5*a - 3*b)*(-a^5*b^3)^(1/2)*(1280
*a^9*b^9 - 192*a^8*b^10 - 3520*a^10*b^8 + 4992*a^11*b^7 - 3520*a^12*b^6 + 5
12*a^13*b^5 + 960*a^14*b^4 - 640*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(5
*a - 3*b)*(-a^5*b^3)^(1/2)*(256*a^10*b^10 - 1536*a^11*b^9 + 3584*a^12*b^8 -
 3584*a^13*b^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3 - 256*a^18*b
^2))/(4*(a^7 - 2*a^6*b + a^5*b^2))))/(4*(a^7 - 2*a^6*b + a^5*b^2)))*(5*a -
3*b)*(-a^5*b^3)^(1/2)*1i)/(4*(a^7 - 2*a^6*b + a^5*b^2)) + ((tan(e + f*x)*(1
44*a^6*b^10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b^7 + 1744*a^10*b^6 - 5
92*a^11*b^5 + 192*a^12*b^4 - 64*a^13*b^3) + ((5*a - 3*b)*(-a^5*b^3)^(1/2)*(
192*a^8*b^10 - 1280*a^9*b^9 + 3520*a^10*b^8 - 4992*a^11*b^7 + 3520*a^12*b^6
 - 512*a^13*b^5 - 960*a^14*b^4 + 640*a^15*b^3 - 128*a^16*b^2 + (tan(e + f*x
)*(5*a - 3*b)*(-a^5*b^3)^(1/2)*(256*a^10*b^10 - 1536*a^11*b^9 + 3584*a^12*b
^8 - 3584*a^13*b^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3 - 256*a^
```

```
18*b^2))/(4*(a^7 - 2*a^6*b + a^5*b^2))))/(4*(a^7 - 2*a^6*b + a^5*b^2)))*(5*
a - 3*b)*(-a^5*b^3)^(1/2)*1i)/(4*(a^7 - 2*a^6*b + a^5*b^2)))/(144*a^6*b^8 -
 624*a^7*b^7 + 976*a^8*b^6 - 656*a^9*b^5 + 160*a^10*b^4 - ((tan(e + f*x)*(1
44*a^6*b^10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b^7 + 1744*a^10*b^6 - 5
92*a^11*b^5 + 192*a^12*b^4 - 64*a^13*b^3) + ((5*a - 3*b)*(-a^5*b^3)^(1/2)*(
1280*a^9*b^9 - 192*a^8*b^10 - 3520*a^10*b^8 + 4992*a^11*b^7 - 3520*a^12*b^6
 + 512*a^13*b^5 + 960*a^14*b^4 - 640*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x
)*(5*a - 3*b)*(-a^5*b^3)^(1/2)*(256*a^10*b^10 - 1536*a^11*b^9 + 3584*a^12*b
^8 - 3584*a^13*b^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3 - 256*a^
18*b^2))/(4*(a^7 - 2*a^6*b + a^5*b^2))))/(4*(a^7 - 2*a^6*b + a^5*b^2)))*(5*
a - 3*b)*(-a^5*b^3)^(1/2))/(4*(a^7 - 2*a^6*b + a^5*b^2)) + ((tan(e + f*x)*(
144*a^6*b^10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b^7 + 1744*a^10*b^6 -
592*a^11*b^5 + 192*a^12*b^4 - 64*a^13*b^3) + ((5*a - 3*b)*(-a^5*b^3)^(1/2)*
(192*a^8*b^10 - 1280*a^9*b^9 + 3520*a^10*b^8 - 4992*a^11*b^7 + 3520*a^12*b^
6 - 512*a^13*b^5 - 960*a^14*b^4 + 640*a^15*b^3 - 128*a^16*b^2 + (tan(e + f*
x)*(5*a - 3*b)*(-a^5*b^3)^(1/2)*(256*a^10*b^10 - 1536*a^11*b^9 + 3584*a^12*
b^8 - 3584*a^13*b^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3 - 256*a
^18*b^2))/(4*(a^7 - 2*a^6*b + a^5*b^2))))/(4*(a^7 - 2*a^6*b + a^5*b^2)))*(5
*a - 3*b)*(-a^5*b^3)^(1/2))/(4*(a^7 - 2*a^6*b + a^5*b^2))))*(5*a - 3*b)*(-a
^5*b^3)^(1/2)*1i)/(2*f*(a^7 - 2*a^6*b + a^5*b^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.235 $\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal. Leaf size=169

$$-\frac{b^{5/2}(7a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f(a-b)^2}-\frac{(2a-5b)\cot^3(e+fx)}{6a^2f(a-b)}+\frac{(2a^2+2ab-5b^2)\cot(e+fx)}{2a^3f(a-b)}-\frac{b\cot^3(e+}{2af(a-b)\left(a+b\mathrm{t}\right.}$$

```
[Out] x/(a-b)^2-1/2*(7*a-5*b)*b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(7/2)/
(a-b)^2/f+1/2*(2*a^2+2*a*b-5*b^2)*cot(f*x+e)/a^3/(a-b)/f-1/6*(2*a-5*b)*cot(
f*x+e)^3/a^2/(a-b)/f-1/2*b*cot(f*x+e)^3/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.29, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 472, 583, 522, 203, 205}

$$-\frac{b^{5/2}(7a-5b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f(a-b)^2}+\frac{(2a^2+2ab-5b^2)\cot(e+fx)}{2a^3f(a-b)}-\frac{(2a-5b)\cot^3(e+fx)}{6a^2f(a-b)}-\frac{b\cot^3(e+}{2af(a-b)\left(a+b\mathrm{t}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] x/(a - b)^2 - ((7*a - 5*b)*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/
(2*a^(7/2)*(a - b)^2*f) + ((2*a^2 + 2*a*b - 5*b^2)*Cot[e + f*x])/(2*a^3*(a
- b)*f) - ((2*a - 5*b)*Cot[e + f*x]^3)/(6*a^2*(a - b)*f) - (b*Cot[e + f*x]^
3)/(2*a*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-5b-5bx^2}{x^4(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a-b)f}$$

$$= -\frac{(2a-5b)\cot^3(e+fx)}{6a^2(a-b)f} - \frac{b\cot^3(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{3(2a^2+2ab-5b^2}{x^2(1+x^2}\right.}{6}$$

$$= \frac{\left(2a^2+2ab-5b^2\right)\cot(e+fx)}{2a^3(a-b)f} - \frac{(2a-5b)\cot^3(e+fx)}{6a^2(a-b)f} - \frac{b\cot^3(e+fx)}{2a(a-b)f\left(a+b\tan^2(e\right.}$$

$$= \frac{\left(2a^2+2ab-5b^2\right)\cot(e+fx)}{2a^3(a-b)f} - \frac{(2a-5b)\cot^3(e+fx)}{6a^2(a-b)f} - \frac{b\cot^3(e+fx)}{2a(a-b)f\left(a+b\tan^2(e\right.}$$

$$= \frac{x}{(a-b)^2} - \frac{(7a-5b)b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}(a-b)^2f} + \frac{\left(2a^2+2ab-5b^2\right)\cot(e+fx)}{2a^3(a-b)f} - \frac{(2a}{}$$

Mathematica [A] time = 3.60, size = 137, normalized size = 0.81

$$\frac{\frac{3b^{5/2}(5b-7a)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}(a-b)^2} + \frac{3\left(2(e+fx)-\frac{b^3(a-b)\sin(2(e+fx))}{a^3((a-b)\cos(2(e+fx))+a+b)}\right)}{(a-b)^2} - \frac{2\cot(e+fx)\left(a\csc^2(e+fx)-4a-6b\right)}{a^3}}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((3*b^(5/2)*(-7*a + 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*(
a - b)^2) - (2*Cot[e + f*x]*(-4*a - 6*b + a*Csc[e + f*x]^2))/a^3 + (3*(2*(e
 + f*x) - ((a - b)*b^3*Sin[2*(e + f*x)])/(a^3*(a + b + (a - b)*Cos[2*(e + f
*x)]))))/(a - b)^2)/(6*f)
```

fricas [A] time = 0.52, size = 596, normalized size = 3.53

$$\left[\frac{24\,a^3bfx\tan\left(fx+e\right)^5+24\,a^4fx\tan\left(fx+e\right)^3+12\left(2\,a^3b-7\,ab^3+5\,b^4\right)\tan\left(fx+e\right)^4-8\,a^4+16\,a^3b-8\,a}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(24*a^3*b*f*x*tan(f*x + e)^5 + 24*a^4*f*x*tan(f*x + e)^3 + 12*(2*a^3*
b - 7*a*b^3 + 5*b^4)*tan(f*x + e)^4 - 8*a^4 + 16*a^3*b - 8*a^2*b^2 + 8*(3*a
^4 - a^3*b - 7*a^2*b^2 + 5*a*b^3)*tan(f*x + e)^2 - 3*((7*a*b^3 - 5*b^4)*tan
(f*x + e)^5 + (7*a^2*b^2 - 5*a*b^3)*tan(f*x + e)^3)*sqrt(-b/a)*log((b^2*tan
(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(
f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/(
(a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^4*b^2)*
f*tan(f*x + e)^3), 1/12*(12*a^3*b*f*x*tan(f*x + e)^5 + 12*a^4*f*x*tan(f*x +
 e)^3 + 6*(2*a^3*b - 7*a*b^3 + 5*b^4)*tan(f*x + e)^4 - 4*a^4 + 8*a^3*b - 4*
a^2*b^2 + 4*(3*a^4 - a^3*b - 7*a^2*b^2 + 5*a*b^3)*tan(f*x + e)^2 - 3*((7*a*
b^3 - 5*b^4)*tan(f*x + e)^5 + (7*a^2*b^2 - 5*a*b^3)*tan(f*x + e)^3)*sqrt(b/
a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a^5*b -
 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^4*b^2)*f*tan(f*
x + e)^3)]
```

giac [A] time = 6.54, size = 179, normalized size = 1.06

$$-\frac{\frac{3\,b^3\tan\left(fx+e\right)}{\left(a^4-a^3b\right)\left(b\tan\left(fx+e\right)^2+a\right)}+\frac{3\left(7\,ab^3-5\,b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^5-2\,a^4b+a^3b^2\right)\sqrt{ab}}-\frac{6\left(fx+e\right)}{a^2-2\,ab+b^2}-\frac{2\left(3\,a\tan\left(fx+e\right)^2+6\,b\tan\left(fx+e\right)^2-a\right)}{a^3\tan\left(fx+e\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^3*tan(f*x + e)/((a^4 - a^3*b)*(b*tan(f*x + e)^2 + a)) + 3*(7*a*b^
3 - 5*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqr
t(a*b)))/((a^5 - 2*a^4*b + a^3*b^2)*sqrt(a*b)) - 6*(f*x + e)/(a^2 - 2*a*b +
 b^2) - 2*(3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 - a)/(a^3*tan(f*x + e)^3
))/f
```

maple [A] time = 0.94, size = 218, normalized size = 1.29

$$-\frac{b^3\tan\left(fx+e\right)}{2f\,a^2\left(a-b\right)^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{b^4\tan\left(fx+e\right)}{2f\,a^3\left(a-b\right)^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{7b^3\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{2f\,a^2\left(a-b\right)^2\sqrt{ab}}+\frac{5b^4\,\mathrm{ar}}{2f\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b^3/a^2/(a-b)^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2/f*b^4/a^3/(a-b)^2*
tan(f*x+e)/(a+b*tan(f*x+e)^2)-7/2/f*b^3/a^2/(a-b)^2/(a*b)^(1/2)*arctan(tan(
f*x+e)*b/(a*b)^(1/2))+5/2/f*b^4/a^3/(a-b)^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b
/(a*b)^(1/2))-1/3/f/a^2/tan(f*x+e)^3+1/f/a^2/tan(f*x+e)+2/f/a^3/tan(f*x+e)*
b+1/f/(a-b)^2*arctan(tan(f*x+e))
```

maxima [A] time = 0.77, size = 193, normalized size = 1.14

$$-\frac{\frac{3\left(7\,ab^3-5\,b^4\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^5-2\,a^4b+a^3b^2\right)\sqrt{ab}}-\frac{3\left(2\,a^2b+2\,ab^2-5\,b^3\right)\tan\left(fx+e\right)^4-2\,a^3+2\,a^2b+2\left(3\,a^3+2\,a^2b-5\,ab^2\right)\tan\left(fx+e\right)^2}{\left(a^4b-a^3b^2\right)\tan\left(fx+e\right)^5+\left(a^5-a^4b\right)\tan\left(fx+e\right)^3}-\frac{6\left(fx+e\right)}{a^2-2\,ab+b^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(7*a*b^3 - 5*b^4)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5 - 2*a^4*b
+ a^3*b^2)*sqrt(a*b)) - (3*(2*a^2*b + 2*a*b^2 - 5*b^3)*tan(f*x + e)^4 - 2*a
^3 + 2*a^2*b + 2*(3*a^3 + 2*a^2*b - 5*a*b^2)*tan(f*x + e)^2)/((a^4*b - a^3*
b^2)*tan(f*x + e)^5 + (a^5 - a^4*b)*tan(f*x + e)^3) - 6*(f*x + e)/(a^2 - 2*
a*b + b^2))/f
```

mupad [B] time = 15.75, size = 2000, normalized size = 11.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (2*atan((2*tan(e + f*x)*(2320*a^10*b^11 - 400*a^9*b^12 - 5344*a^11*b^10 + 6
112*a^12*b^9 - 3472*a^13*b^8 + 784*a^14*b^7 - 64*a^15*b^6 + 192*a^16*b^5 -
192*a^17*b^4 + 64*a^18*b^3 + (256*a^15*b^10 - 1536*a^16*b^9 + 3584*a^17*b^8
 - 3584*a^18*b^7 + 3584*a^20*b^5 - 3584*a^21*b^4 + 1536*a^22*b^3 - 256*a^23
*b^2)/(2*a^2 - 4*a*b + 2*b^2)^2))/((2*a^2 - 4*a*b + 2*b^2)*((2*(320*a^12*b^
11 - 2048*a^13*b^10 + 5440*a^14*b^9 - 7680*a^15*b^8 + 6208*a^16*b^7 - 3200*
a^17*b^6 + 1728*a^18*b^5 - 1280*a^19*b^4 + 640*a^20*b^3 - 128*a^21*b^2))/(2
*a^2 - 4*a*b + 2*b^2)^2 - 400*a^9*b^10 + 1520*a^10*b^9 - 1904*a^11*b^8 + 62
4*a^12*b^7 + 384*a^13*b^6 - 224*a^14*b^5))))/(f*(2*a^2 - 4*a*b + 2*b^2)) +
((tan(e + f*x)^2*(3*a + 5*b))/(3*a^2) - 1/(3*a) + (tan(e + f*x)^4*(2*a*b^2
+ 2*a^2*b - 5*b^3))/(2*a^3*(a - b)))/(f*(a*tan(e + f*x)^3 + b*tan(e + f*x)^
5)) + (atan((((tan(e + f*x)*(400*a^9*b^12 - 2320*a^10*b^11 + 5344*a^11*b^10
 - 6112*a^12*b^9 + 3472*a^13*b^8 - 784*a^14*b^7 + 64*a^15*b^6 - 192*a^16*b^
5 + 192*a^17*b^4 - 64*a^18*b^3) + ((7*a - 5*b)*(-a^7*b^5)^(1/2)*(2048*a^13*
b^10 - 320*a^12*b^11 - 5440*a^14*b^9 + 7680*a^15*b^8 - 6208*a^16*b^7 + 3200
*a^17*b^6 - 1728*a^18*b^5 + 1280*a^19*b^4 - 640*a^20*b^3 + 128*a^21*b^2 + (
tan(e + f*x)*(7*a - 5*b)*(-a^7*b^5)^(1/2)*(256*a^15*b^10 - 1536*a^16*b^9 +
3584*a^17*b^8 - 3584*a^18*b^7 + 3584*a^20*b^5 - 3584*a^21*b^4 + 1536*a^22*b
^3 - 256*a^23*b^2))/(4*(a^9 - 2*a^8*b + a^7*b^2))))/(4*(a^9 - 2*a^8*b + a^7
*b^2)))*(7*a - 5*b)*(-a^7*b^5)^(1/2)*1i)/(4*(a^9 - 2*a^8*b + a^7*b^2)) + ((
tan(e + f*x)*(400*a^9*b^12 - 2320*a^10*b^11 + 5344*a^11*b^10 - 6112*a^12*b^
9 + 3472*a^13*b^8 - 784*a^14*b^7 + 64*a^15*b^6 - 192*a^16*b^5 + 192*a^17*b^
4 - 64*a^18*b^3) + ((7*a - 5*b)*(-a^7*b^5)^(1/2)*(320*a^12*b^11 - 2048*a^13
*b^10 + 5440*a^14*b^9 - 7680*a^15*b^8 + 6208*a^16*b^7 - 3200*a^17*b^6 + 172
8*a^18*b^5 - 1280*a^19*b^4 + 640*a^20*b^3 - 128*a^21*b^2 + (tan(e + f*x)*(7
*a - 5*b)*(-a^7*b^5)^(1/2)*(256*a^15*b^10 - 1536*a^16*b^9 + 3584*a^17*b^8 -
 3584*a^18*b^7 + 3584*a^20*b^5 - 3584*a^21*b^4 + 1536*a^22*b^3 - 256*a^23*b
^2))/(4*(a^9 - 2*a^8*b + a^7*b^2))))/(4*(a^9 - 2*a^8*b + a^7*b^2)))*(7*a -
5*b)*(-a^7*b^5)^(1/2)*1i)/(4*(a^9 - 2*a^8*b + a^7*b^2)))/(400*a^9*b^10 - 15
20*a^10*b^9 + 1904*a^11*b^8 - 624*a^12*b^7 - 384*a^13*b^6 + 224*a^14*b^5 -
((tan(e + f*x)*(400*a^9*b^12 - 2320*a^10*b^11 + 5344*a^11*b^10 - 6112*a^12*
b^9 + 3472*a^13*b^8 - 784*a^14*b^7 + 64*a^15*b^6 - 192*a^16*b^5 + 192*a^17*
b^4 - 64*a^18*b^3) + ((7*a - 5*b)*(-a^7*b^5)^(1/2)*(2048*a^13*b^10 - 320*a^
12*b^11 - 5440*a^14*b^9 + 7680*a^15*b^8 - 6208*a^16*b^7 + 3200*a^17*b^6 - 1
728*a^18*b^5 + 1280*a^19*b^4 - 640*a^20*b^3 + 128*a^21*b^2 + (tan(e + f*x)*
(7*a - 5*b)*(-a^7*b^5)^(1/2)*(256*a^15*b^10 - 1536*a^16*b^9 + 3584*a^17*b^8
```

```
 - 3584*a^18*b^7 + 3584*a^20*b^5 - 3584*a^21*b^4 + 1536*a^22*b^3 - 256*a^23
*b^2))/(4*(a^9 - 2*a^8*b + a^7*b^2))))/(4*(a^9 - 2*a^8*b + a^7*b^2)))*(7*a
- 5*b)*(-a^7*b^5)^(1/2))/(4*(a^9 - 2*a^8*b + a^7*b^2)) + ((tan(e + f*x)*(40
0*a^9*b^12 - 2320*a^10*b^11 + 5344*a^11*b^10 - 6112*a^12*b^9 + 3472*a^13*b^
8 - 784*a^14*b^7 + 64*a^15*b^6 - 192*a^16*b^5 + 192*a^17*b^4 - 64*a^18*b^3)
 + ((7*a - 5*b)*(-a^7*b^5)^(1/2)*(320*a^12*b^11 - 2048*a^13*b^10 + 5440*a^1
4*b^9 - 7680*a^15*b^8 + 6208*a^16*b^7 - 3200*a^17*b^6 + 1728*a^18*b^5 - 128
0*a^19*b^4 + 640*a^20*b^3 - 128*a^21*b^2 + (tan(e + f*x)*(7*a - 5*b)*(-a^7*
b^5)^(1/2)*(256*a^15*b^10 - 1536*a^16*b^9 + 3584*a^17*b^8 - 3584*a^18*b^7 +
 3584*a^20*b^5 - 3584*a^21*b^4 + 1536*a^22*b^3 - 256*a^23*b^2))/(4*(a^9 - 2
*a^8*b + a^7*b^2))))/(4*(a^9 - 2*a^8*b + a^7*b^2)))*(7*a - 5*b)*(-a^7*b^5)^
(1/2))/(4*(a^9 - 2*a^8*b + a^7*b^2))))*(7*a - 5*b)*(-a^7*b^5)^(1/2)*1i)/(2*
f*(a^9 - 2*a^8*b + a^7*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)

[Out] Timed out
```

3.236 $\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=218

$$\frac{b^{7/2}(9a-7b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f(a-b)^2}-\frac{(2a-7b)\cot^5(e+fx)}{10a^2f(a-b)}+\frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{6a^3f(a-b)}-\frac{\left(2a^3+2a^2b+2ab^2-\right.}{2a^4f(a-}$$

```
[Out] -x/(a-b)^2+1/2*(9*a-7*b)*b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(9/2)
/(a-b)^2/f-1/2*(2*a^3+2*a^2*b+2*a*b^2-7*b^3)*cot(f*x+e)/a^4/(a-b)/f+1/6*(2*
a^2+2*a*b-7*b^2)*cot(f*x+e)^3/a^3/(a-b)/f-1/10*(2*a-7*b)*cot(f*x+e)^5/a^2/(
a-b)/f-1/2*b*cot(f*x+e)^5/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.34, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3670, 472, 583, 522, 203, 205}

$$\frac{b^{7/2}(9a-7b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f(a-b)^2}+\frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{6a^3f(a-b)}-\frac{\left(2a^2b+2a^3+2ab^2-7b^3\right)\cot(e+fx)}{2a^4f(a-b)}-\frac{(2a-}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(x/(a - b)^2) + ((9*a - 7*b)*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(2*a^(9/2)*(a - b)^2*f) - ((2*a^3 + 2*a^2*b + 2*a*b^2 - 7*b^3)*Cot[e + f
*x])/(2*a^4*(a - b)*f) + ((2*a^2 + 2*a*b - 7*b^2)*Cot[e + f*x]^3)/(6*a^3*(a
 - b)*f) - ((2*a - 7*b)*Cot[e + f*x]^5)/(10*a^2*(a - b)*f) - (b*Cot[e + f*x
]^5)/(2*a*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6\left(1+x^2\right)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-7b-7bx^2}{x^6\left(1+x^2\right)\left(a+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2a(a-b)f}$$

$$= -\frac{(2a-7b)\cot^5(e+fx)}{10a^2(a-b)f} - \frac{b\cot^5(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{5\left(2a^2+2ab\right.}{x^4\left(1\right.}\right.}{}$$

$$= \frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{6a^3(a-b)f} - \frac{(2a-7b)\cot^5(e+fx)}{10a^2(a-b)f} - \frac{b\cot^5(e+f}{2a(a-b)f\left(a+b\tan\right.}$$

$$= -\frac{\left(2a^3+2a^2b+2ab^2-7b^3\right)\cot(e+fx)}{2a^4(a-b)f} + \frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{6a^3(a-b)f} - \frac{(2a}{}$$

$$= -\frac{\left(2a^3+2a^2b+2ab^2-7b^3\right)\cot(e+fx)}{2a^4(a-b)f} + \frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{6a^3(a-b)f} - \frac{(2a}{}$$

$$= -\frac{x}{(a-b)^2} + \frac{(9a-7b)b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}(a-b)^2f} - \frac{\left(2a^3+2a^2b+2ab^2-7b^3\right)\cot(}{2a^4(a-b)f}$$

Mathematica [A] time = 6.28, size = 231, normalized size = 1.06

$$\frac{b^{7/2}(9a-7b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f(a-b)^2} + \frac{b^4\sin(2(e+fx))}{2a^4f(a-b)(a\cos(2(e+fx))+a-b\cos(2(e+fx))+b)} + \frac{\csc^3(e+fx)(11a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((e + f*x)/((a - b)^2*f)) + ((9*a - 7*b)*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f
*x])/Sqrt[a]])/(2*a^(9/2)*(a - b)^2*f) + ((-23*a^2*Cos[e + f*x] - 40*a*b*Co
s[e + f*x] - 45*b^2*Cos[e + f*x])*Csc[e + f*x])/(15*a^4*f) + ((11*a*Cos[e +
 f*x] + 10*b*Cos[e + f*x])*Csc[e + f*x]^3)/(15*a^3*f) - (Cot[e + f*x]*Csc[e
 + f*x]^4)/(5*a^2*f) + (b^4*Sin[2*(e + f*x)])/(2*a^4*(a - b)*f*(a + b + a*C
os[2*(e + f*x)] - b*Cos[2*(e + f*x)]))
```

fricas [A] time = 0.54, size = 672, normalized size = 3.08

$$\left[-\frac{120\,a^4bfx\tan\left(fx+e\right)^7+120\,a^5fx\tan\left(fx+e\right)^5+60\left(2\,a^4b-9\,ab^4+7\,b^5\right)\tan\left(fx+e\right)^6+24\,a^5-48\,a^4b+2\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/120*(120*a^4*b*f*x*tan(f*x + e)^7 + 120*a^5*f*x*tan(f*x + e)^5 + 60*(2*
a^4*b - 9*a*b^4 + 7*b^5)*tan(f*x + e)^6 + 24*a^5 - 48*a^4*b + 24*a^3*b^2 +
40*(3*a^5 - a^4*b - 9*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^4 - 8*(5*a^5 - 3*a^4*
b - 9*a^3*b^2 + 7*a^2*b^3)*tan(f*x + e)^2 + 15*((9*a*b^4 - 7*b^5)*tan(f*x +
 e)^7 + (9*a^2*b^3 - 7*a*b^4)*tan(f*x + e)^5)*sqrt(-b/a)*log((b^2*tan(f*x +
 e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x +
e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a^6*b
 - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^7 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(
f*x + e)^5), -1/60*(60*a^4*b*f*x*tan(f*x + e)^7 + 60*a^5*f*x*tan(f*x + e)^5
 + 30*(2*a^4*b - 9*a*b^4 + 7*b^5)*tan(f*x + e)^6 + 12*a^5 - 24*a^4*b + 12*a
^3*b^2 + 20*(3*a^5 - a^4*b - 9*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^4 - 4*(5*a^5
 - 3*a^4*b - 9*a^3*b^2 + 7*a^2*b^3)*tan(f*x + e)^2 - 15*((9*a*b^4 - 7*b^5)*
tan(f*x + e)^7 + (9*a^2*b^3 - 7*a*b^4)*tan(f*x + e)^5)*sqrt(b/a)*arctan(1/2
*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a^6*b - 2*a^5*b^2 +
a^4*b^3)*f*tan(f*x + e)^7 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^5)]
```

giac [A] time = 7.35, size = 225, normalized size = 1.03

$$\frac{\frac{15\,b^4\tan\left(fx+e\right)}{\left(a^5-a^4b\right)\left(b\tan\left(fx+e\right)^2+a\right)}+\frac{15\left(9\,ab^4-7\,b^5\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^6-2\,a^5b+a^4b^2\right)\sqrt{ab}}-\frac{30\left(fx+e\right)}{a^2-2\,ab+b^2}-\frac{2\left(15\,a^2\tan\left(fx+e\right)^4+30\,ab\tan\left(fx+e\right)^4+45\cdots\right)}{\cdots}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/30*(15*b^4*tan(f*x + e)/((a^5 - a^4*b)*(b*tan(f*x + e)^2 + a)) + 15*(9*a*
b^4 - 7*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/s
qrt(a*b)))/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(a*b)) - 30*(f*x + e)/(a^2 - 2*a*
b + b^2) - 2*(15*a^2*tan(f*x + e)^4 + 30*a*b*tan(f*x + e)^4 + 45*b^2*tan(f*
x + e)^4 - 5*a^2*tan(f*x + e)^2 - 10*a*b*tan(f*x + e)^2 + 3*a^2)/(a^4*tan(f
*x + e)^5))/f
```

maple [A] time = 0.94, size = 272, normalized size = 1.25

$$\frac{b^4\tan\left(fx+e\right)}{2f\,a^3\left(a-b\right)^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{b^5\tan\left(fx+e\right)}{2f\,a^4\left(a-b\right)^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{9b^4\arctan\left(\frac{\tan\left(fx+e\right)b}{\sqrt{ab}}\right)}{2f\,a^3\left(a-b\right)^2\sqrt{ab}}-\frac{7b^5\arctan\cdots}{2f\,a^4\left(a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*b^4/a^3/(a-b)^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)-1/2/f*b^5/a^4/(a-b)^2*t
an(f*x+e)/(a+b*tan(f*x+e)^2)+9/2/f*b^4/a^3/(a-b)^2/(a*b)^(1/2)*arctan(tan(f
*x+e)*b/(a*b)^(1/2))-7/2/f*b^5/a^4/(a-b)^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/
(a*b)^(1/2))-1/5/f/a^2/tan(f*x+e)^5+1/3/f/a^2/tan(f*x+e)^3+2/3/f/a^3/tan(f*
x+e)^3*b-1/f/a^2/tan(f*x+e)-2/f/a^3/tan(f*x+e)*b-3/f/a^4/tan(f*x+e)*b^2-1/f
/(a-b)^2*arctan(tan(f*x+e))
```

maxima [A] time = 0.75, size = 239, normalized size = 1.10

$$\frac{\frac{15\left(9\,ab^4-7\,b^5\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^6-2\,a^5b+a^4b^2\right)\sqrt{ab}}-\frac{15\left(2\,a^3b+2\,a^2b^2+2\,ab^3-7\,b^4\right)\tan\left(fx+e\right)^6+10\left(3\,a^4+2\,a^3b+2\,a^2b^2-7\,ab^3\right)\tan\left(fx+e\right)^4+6\,a^4-6\,a^3b-2\left(5\,a^4+2\right.}{\left(a^5b-a^4b^2\right)\tan\left(fx+e\right)^7+\left(a^6-a^5b\right)\tan\left(fx+e\right)^5}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*(9*a*b^4 - 7*b^5)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^6 - 2*a^5*b
 + a^4*b^2)*sqrt(a*b)) - (15*(2*a^3*b + 2*a^2*b^2 + 2*a*b^3 - 7*b^4)*tan(f*
x + e)^6 + 10*(3*a^4 + 2*a^3*b + 2*a^2*b^2 - 7*a*b^3)*tan(f*x + e)^4 + 6*a^
4 - 6*a^3*b - 2*(5*a^4 + 2*a^3*b - 7*a^2*b^2)*tan(f*x + e)^2)/((a^5*b - a^4
*b^2)*tan(f*x + e)^7 + (a^6 - a^5*b)*tan(f*x + e)^5) - 30*(f*x + e)/(a^2 -
2*a*b + b^2))/f
```

mupad [B] time = 16.00, size = 3030, normalized size = 13.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] - (1/(5*a) + (tan(e + f*x)^4*(5*a*b + 3*a^2 + 7*b^2))/(3*a^3) - (tan(e + f*
x)^2*(5*a + 7*b))/(15*a^2) + (tan(e + f*x)^6*(2*a*b^3 + 2*a^3*b - 7*b^4 + 2
*a^2*b^2))/(2*a^4*(a - b)))/(f*(a*tan(e + f*x)^5 + b*tan(e + f*x)^7)) - (2*
atan(((tan(e + f*x)*(784*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 - 1072
0*a^15*b^11 + 5904*a^16*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 192*a^21*b^5 +
 192*a^22*b^4 - 64*a^23*b^3) + ((2816*a^17*b^11 - 448*a^16*b^12 - 7360*a^18
*b^10 + 10240*a^19*b^9 - 8000*a^20*b^8 + 3200*a^21*b^7 + 64*a^22*b^6 - 1280
*a^23*b^5 + 1280*a^24*b^4 - 640*a^25*b^3 + 128*a^26*b^2 + (tan(e + f*x)*(25
6*a^20*b^10 - 1536*a^21*b^9 + 3584*a^22*b^8 - 3584*a^23*b^7 + 3584*a^25*b^5
 - 3584*a^26*b^4 + 1536*a^27*b^3 - 256*a^28*b^2)*1i)/(2*a^2 - 4*a*b + 2*b^2
))*1i)/(2*a^2 - 4*a*b + 2*b^2))/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(78
4*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 - 10720*a^15*b^11 + 5904*a^16
*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 192*a^21*b^5 + 192*a^22*b^4 - 64*a^23
*b^3) + ((448*a^16*b^12 - 2816*a^17*b^11 + 7360*a^18*b^10 - 10240*a^19*b^9
+ 8000*a^20*b^8 - 3200*a^21*b^7 - 64*a^22*b^6 + 1280*a^23*b^5 - 1280*a^24*b
^4 + 640*a^25*b^3 - 128*a^26*b^2 + (tan(e + f*x)*(256*a^20*b^10 - 1536*a^21
*b^9 + 3584*a^22*b^8 - 3584*a^23*b^7 + 3584*a^25*b^5 - 3584*a^26*b^4 + 1536
*a^27*b^3 - 256*a^28*b^2)*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b +
 2*b^2))/(2*a^2 - 4*a*b + 2*b^2))/(((tan(e + f*x)*(784*a^12*b^14 - 4368*a^1
3*b^13 + 9696*a^14*b^12 - 10720*a^15*b^11 + 5904*a^16*b^10 - 1296*a^17*b^9
+ 64*a^20*b^6 - 192*a^21*b^5 + 192*a^22*b^4 - 64*a^23*b^3) + ((448*a^16*b^1
2 - 2816*a^17*b^11 + 7360*a^18*b^10 - 10240*a^19*b^9 + 8000*a^20*b^8 - 3200
*a^21*b^7 - 64*a^22*b^6 + 1280*a^23*b^5 - 1280*a^24*b^4 + 640*a^25*b^3 - 12
8*a^26*b^2 + (tan(e + f*x)*(256*a^20*b^10 - 1536*a^21*b^9 + 3584*a^22*b^8 -
 3584*a^23*b^7 + 3584*a^25*b^5 - 3584*a^26*b^4 + 1536*a^27*b^3 - 256*a^28*b
^2)*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 -
4*a*b + 2*b^2) - ((tan(e + f*x)*(784*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14
```

```
*b^12 - 10720*a^15*b^11 + 5904*a^16*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 19
2*a^21*b^5 + 192*a^22*b^4 - 64*a^23*b^3) + ((2816*a^17*b^11 - 448*a^16*b^12
 - 7360*a^18*b^10 + 10240*a^19*b^9 - 8000*a^20*b^8 + 3200*a^21*b^7 + 64*a^2
2*b^6 - 1280*a^23*b^5 + 1280*a^24*b^4 - 640*a^25*b^3 + 128*a^26*b^2 + (tan(
e + f*x)*(256*a^20*b^10 - 1536*a^21*b^9 + 3584*a^22*b^8 - 3584*a^23*b^7 + 3
584*a^25*b^5 - 3584*a^26*b^4 + 1536*a^27*b^3 - 256*a^28*b^2)*1i)/(2*a^2 - 4
*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*b^2) + 7
84*a^12*b^12 - 2800*a^13*b^11 + 3312*a^14*b^10 - 1296*a^15*b^9 + 224*a^16*b
^8 - 512*a^17*b^7 + 288*a^18*b^6)))/(f*(2*a^2 - 4*a*b + 2*b^2)) - (atan((((
tan(e + f*x)*(784*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 - 10720*a^15*
b^11 + 5904*a^16*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 192*a^21*b^5 + 192*a^
22*b^4 - 64*a^23*b^3) + ((9*a - 7*b)*(-a^9*b^7)^(1/2)*(2816*a^17*b^11 - 448
*a^16*b^12 - 7360*a^18*b^10 + 10240*a^19*b^9 - 8000*a^20*b^8 + 3200*a^21*b^
7 + 64*a^22*b^6 - 1280*a^23*b^5 + 1280*a^24*b^4 - 640*a^25*b^3 + 128*a^26*b
^2 + (tan(e + f*x)*(9*a - 7*b)*(-a^9*b^7)^(1/2)*(256*a^20*b^10 - 1536*a^21*
b^9 + 3584*a^22*b^8 - 3584*a^23*b^7 + 3584*a^25*b^5 - 3584*a^26*b^4 + 1536*
a^27*b^3 - 256*a^28*b^2))/(4*(a^11 - 2*a^10*b + a^9*b^2))))/(4*(a^11 - 2*a^
10*b + a^9*b^2)))*(9*a - 7*b)*(-a^9*b^7)^(1/2)*1i)/(4*(a^11 - 2*a^10*b + a^
9*b^2)) + ((tan(e + f*x)*(784*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 -
 10720*a^15*b^11 + 5904*a^16*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 192*a^21*
b^5 + 192*a^22*b^4 - 64*a^23*b^3) + ((9*a - 7*b)*(-a^9*b^7)^(1/2)*(448*a^16
*b^12 - 2816*a^17*b^11 + 7360*a^18*b^10 - 10240*a^19*b^9 + 8000*a^20*b^8 -
3200*a^21*b^7 - 64*a^22*b^6 + 1280*a^23*b^5 - 1280*a^24*b^4 + 640*a^25*b^3
- 128*a^26*b^2 + (tan(e + f*x)*(9*a - 7*b)*(-a^9*b^7)^(1/2)*(256*a^20*b^10
- 1536*a^21*b^9 + 3584*a^22*b^8 - 3584*a^23*b^7 + 3584*a^25*b^5 - 3584*a^26
*b^4 + 1536*a^27*b^3 - 256*a^28*b^2))/(4*(a^11 - 2*a^10*b + a^9*b^2))))/(4*
(a^11 - 2*a^10*b + a^9*b^2)))*(9*a - 7*b)*(-a^9*b^7)^(1/2)*1i)/(4*(a^11 - 2
*a^10*b + a^9*b^2)))/(784*a^12*b^12 - 2800*a^13*b^11 + 3312*a^14*b^10 - 129
6*a^15*b^9 + 224*a^16*b^8 - 512*a^17*b^7 + 288*a^18*b^6 - ((tan(e + f*x)*(7
84*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 - 10720*a^15*b^11 + 5904*a^1
6*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 192*a^21*b^5 + 192*a^22*b^4 - 64*a^2
3*b^3) + ((9*a - 7*b)*(-a^9*b^7)^(1/2)*(2816*a^17*b^11 - 448*a^16*b^12 - 73
60*a^18*b^10 + 10240*a^19*b^9 - 8000*a^20*b^8 + 3200*a^21*b^7 + 64*a^22*b^6
 - 1280*a^23*b^5 + 1280*a^24*b^4 - 640*a^25*b^3 + 128*a^26*b^2 + (tan(e + f
*x)*(9*a - 7*b)*(-a^9*b^7)^(1/2)*(256*a^20*b^10 - 1536*a^21*b^9 + 3584*a^22
*b^8 - 3584*a^23*b^7 + 3584*a^25*b^5 - 3584*a^26*b^4 + 1536*a^27*b^3 - 256*
a^28*b^2))/(4*(a^11 - 2*a^10*b + a^9*b^2))))/(4*(a^11 - 2*a^10*b + a^9*b^2)
))*(9*a - 7*b)*(-a^9*b^7)^(1/2))/(4*(a^11 - 2*a^10*b + a^9*b^2)) + ((tan(e
+ f*x)*(784*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 - 10720*a^15*b^11 +
 5904*a^16*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 192*a^21*b^5 + 192*a^22*b^4
 - 64*a^23*b^3) + ((9*a - 7*b)*(-a^9*b^7)^(1/2)*(448*a^16*b^12 - 2816*a^17*
b^11 + 7360*a^18*b^10 - 10240*a^19*b^9 + 8000*a^20*b^8 - 3200*a^21*b^7 - 64
*a^22*b^6 + 1280*a^23*b^5 - 1280*a^24*b^4 + 640*a^25*b^3 - 128*a^26*b^2 + (
tan(e + f*x)*(9*a - 7*b)*(-a^9*b^7)^(1/2)*(256*a^20*b^10 - 1536*a^21*b^9 +
3584*a^22*b^8 - 3584*a^23*b^7 + 3584*a^25*b^5 - 3584*a^26*b^4 + 1536*a^27*b
^3 - 256*a^28*b^2))/(4*(a^11 - 2*a^10*b + a^9*b^2))))/(4*(a^11 - 2*a^10*b +
 a^9*b^2)))*(9*a - 7*b)*(-a^9*b^7)^(1/2))/(4*(a^11 - 2*a^10*b + a^9*b^2))))
*(9*a - 7*b)*(-a^9*b^7)^(1/2)*1i)/(2*f*(a^11 - 2*a^10*b + a^9*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.237 $\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=108

$$\frac{a^2}{4b^2f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{a(a-2b)}{2b^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

```
[Out] -1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^3/f+1/4*a^2/(a-b)/b^2/f/(a+b*t
an(f*x+e)^2)^2-1/2*a*(a-2*b)/(a-b)^2/b^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 446, 88}

$$\frac{a^2}{4b^2f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{a(a-2b)}{2b^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^3*f) + a^2/(4*(a - b)*
b^2*f*(a + b*Tan[e + f*x]^2)^2) - (a*(a - 2*b))/(2*(a - b)^2*b^2*f*(a + b*T
an[e + f*x]^2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^3(1+x)} - \frac{a^2}{(a-b)b(a+bx)^3} + \frac{a(a-2b)}{(a-b)^2b(a+bx)^2} + \frac{b}{(-a+b)^3(a+bx)}\right)dx, x, \tan^2(e+f$$
$$= -\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^3f} + \frac{a^2}{4(a-b)b^2f\left(a+b\tan^2(e+fx)\right)^2} - \frac{}{2(a-}$$

Mathematica [A] time = 1.16, size = 97, normalized size = 0.90

$$\frac{\frac{a^2(a-b)^2}{b^2\left(a+b\tan^2(e+fx)\right)^2} - \frac{2a(a-2b)(a-b)}{b^2\left(a+b\tan^2(e+fx)\right)} - 2\log\left(a+b\tan^2(e+fx)\right) - 4\log(\cos(e+fx))}{4f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-4*Log[Cos[e + f*x]] - 2*Log[a + b*Tan[e + f*x]^2] + (a^2*(a - b)^2)/(b^2*
(a + b*Tan[e + f*x]^2)^2) - (2*a*(a - 2*b)*(a - b))/(b^2*(a + b*Tan[e + f*x
]^2)))/(4*(a - b)^3*f)
```

fricas [B] time = 0.45, size = 206, normalized size = 1.91

$$\frac{\left(a^2-4ab\right)\tan\left(fx+e\right)^4 - 2\left(a^2+2ab\right)\tan\left(fx+e\right)^2 - 3a^2 - 2\left(b^2\tan\left(fx+e\right)^4 + 2ab\tan\left(fx+e\right)^2 + a^2\right)\log}{4\left(\left(a^3b^2-3a^2b^3+3ab^4-b^5\right)f\tan\left(fx+e\right)^4 + 2\left(a^4b-3a^3b^2+3a^2b^3-ab^4\right)f\tan\left(fx+e\right)^2 + \left(a^5-3a^4b+3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 - 4*a*b)*tan(f*x + e)^4 - 2*(a^2 + 2*a*b)*tan(f*x + e)^2 - 3*a^2
- 2*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*log((b*tan(f*x + e)^2
 + a)/(tan(f*x + e)^2 + 1)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f
*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 + (a
^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(1/(2*a^3-6*a^2*b+6*a*b^2-2*b^3)
*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))-1/(4*a^3-12*a^2*b+12*a*
b^2-4*b^3)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-2*(1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a)
```

```
+(3*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a^2-20*((1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1))))^3*a^2+32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*
a*b+50*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2-128*((1-cos(f*x+exp(
1)))/(1+cos(f*x+exp(1))))^2*a*b+96*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))
)^2*b^2-20*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2+32*(1-cos(f*x+exp(1)
))/(1+cos(f*x+exp(1)))*a*b+3*a^2)/(8*a^3-24*a^2*b+24*a*b^2-8*b^3)/(((1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a)^2)
```

maple [B] time = 0.20, size = 234, normalized size = 2.17

$$-\frac{\ln\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}{2f\left(a-b\right)^3}-\frac{a^3}{2f\left(a-b\right)^3b^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{3a^2}{2f\left(a-b\right)^3b\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{}{f\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/2/f/(a-b)^3*ln(a+b*tan(f*x+e)^2)-1/2/f/(a-b)^3*a^3/b^2/(a+b*tan(f*x+e)^2
)+3/2/f/(a-b)^3*a^2/b/(a+b*tan(f*x+e)^2)-1/f/(a-b)^3*a/(a+b*tan(f*x+e)^2)+1
/4/f/(a-b)^3*a^4/b^2/(a+b*tan(f*x+e)^2)^2-1/2/f/(a-b)^3*a^3/b/(a+b*tan(f*x+
e)^2)^2+1/4/f/(a-b)^3*a^2/(a+b*tan(f*x+e)^2)^2+1/2/f/(a-b)^3*ln(1+tan(f*x+e
)^2)
```

maxima [A] time = 0.54, size = 189, normalized size = 1.75

$$\frac{\frac{4\left(a^2-ab\right)\sin\left(fx+e\right)^2-3a^2}{a^5-3a^4b+3a^3b^2-a^2b^3+\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5\right)\sin\left(fx+e\right)^4-2\left(a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4\right)\sin\left(fx+e\right)^2}-\frac{2\log\left(-(a-b)\sin\left(fx+\right.\right.}{a^3-3a^2b+3ab^2-}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*(a^2 - a*b)*sin(f*x + e)^2 - 3*a^2)/(a^5 - 3*a^4*b + 3*a^3*b^2 - a^
2*b^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sin(f*x +
 e)^4 - 2*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sin(f*x + e)^2) -
 2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

mupad [B] time = 12.52, size = 577, normalized size = 5.34

$$-\frac{a^3b\cos\left(e+fx\right)^4-\frac{a^4\cos\left(e+fx\right)^4}{4}-\frac{3a^2b^2\cos\left(e+fx\right)^4}{4}+b^4\sin\left(e+fx\right)^4\mathrm{atan}\left(\frac{a\sin\left(e+fx\right)^2-b\sin\left(e+fx\right)^2}{a\cos\left(e+fx\right)^2 2\mathrm{i}+a\sin\left(e+fx\right)^2 1\mathrm{i}+b\sin\left(e+f\right.}\right.}{f\left(-a^5b^2\cos\left(e+fx\right)^4+3a^4b^3\cos\left(e+fx\right)^4-2a^4b^3\cos\left(e+fx\right)^2\sin\left(e+fx\right)^2-3a^3b^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] -(a^3*b*cos(e + f*x)^4 - (a^4*cos(e + f*x)^4)/4 - (3*a^2*b^2*cos(e + f*x)^4
)/4 + b^4*sin(e + f*x)^4*atan((a*sin(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(
e + f*x)^2*2i + a*sin(e + f*x)^2*1i + b*sin(e + f*x)^2*1i))*1i - a*b^3*cos(
e + f*x)^2*sin(e + f*x)^2 - (a^3*b*cos(e + f*x)^2*sin(e + f*x)^2)/2 + a^2*b
^2*cos(e + f*x)^4*atan((a*sin(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(e + f*x
)^2*2i + a*sin(e + f*x)^2*1i + b*sin(e + f*x)^2*1i))*1i + (3*a^2*b^2*cos(e
+ f*x)^2*sin(e + f*x)^2)/2 + a*b^3*cos(e + f*x)^2*sin(e + f*x)^2*atan((a*si
n(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e + f*x)^2*1i
 + b*sin(e + f*x)^2*1i))*2i)/(f*(b^7*sin(e + f*x)^4 - 3*a*b^6*sin(e + f*x)^
4 + a^2*b^5*cos(e + f*x)^4 - 3*a^3*b^4*cos(e + f*x)^4 + 3*a^4*b^3*cos(e + f
```

```
*x)^4 - a^5*b^2*cos(e + f*x)^4 + 3*a^2*b^5*sin(e + f*x)^4 - a^3*b^4*sin(e +
 f*x)^4 + 2*a*b^6*cos(e + f*x)^2*sin(e + f*x)^2 - 6*a^2*b^5*cos(e + f*x)^2*
sin(e + f*x)^2 + 6*a^3*b^4*cos(e + f*x)^2*sin(e + f*x)^2 - 2*a^4*b^3*cos(e
+ f*x)^2*sin(e + f*x)^2))
```

sympy [A] time = 133.67, size = 3346, normalized size = 30.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-3*tan(e + f*x)*
*4/(6*b**3*f*tan(e + f*x)**6 + 18*b**3*f*tan(e + f*x)**4 + 18*b**3*f*tan(e
+ f*x)**2 + 6*b**3*f) - 3*tan(e + f*x)**2/(6*b**3*f*tan(e + f*x)**6 + 18*b*
*3*f*tan(e + f*x)**4 + 18*b**3*f*tan(e + f*x)**2 + 6*b**3*f) - 1/(6*b**3*f*
tan(e + f*x)**6 + 18*b**3*f*tan(e + f*x)**4 + 18*b**3*f*tan(e + f*x)**2 + 6
*b**3*f), Eq(a, b)), ((log(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x)**4/(4*
f) - tan(e + f*x)**2/(2*f))/a**3, Eq(b, 0)), (x*tan(e)**5/(a + b*tan(e)**2)
**3, Eq(f, 0)), (-a**4/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*
a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**
2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e
+ f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e
+ f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 2*a**3*b*tan(e + f*x)**2/(4*a**5*b*
*2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e
 + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5
*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*
b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)
**4) + 4*a**3*b/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b*
*3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*
a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)*
*2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)*
*2 - 4*b**7*f*tan(e + f*x)**4) - 2*a**2*b**2*log(-I*sqrt(a)*sqrt(1/b) + tan
(e + f*x))/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f
+ 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*
b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 -
4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 -
4*b**7*f*tan(e + f*x)**4) - 2*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f
*x))/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a*
*3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f
 - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2
*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7
*f*tan(e + f*x)**4) + 2*a**2*b**2*log(tan(e + f*x)**2 + 1)/(4*a**5*b**2*f +
 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x
)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan
(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f
*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) +
 6*a**2*b**2*tan(e + f*x)**2/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2
 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e +
f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*
tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*
tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 3*a**2*b**2/(4*a**5*b**2*f +
8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)
**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(
e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*
tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) -
4*a*b**3*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**5*b
**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(
e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**
5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a
```

```
*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x
)**4) - 4*a*b**3*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4
*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4
*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a
**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f
 + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(
e + f*x)**4) + 4*a*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*a**5*b*
*2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e
 + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5
*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*
b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)
**4) - 4*a*b**3*tan(e + f*x)**2/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)
**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e
 + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5
*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6
*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 2*b**4*log(-I*sqrt(a)*sqrt
(1/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e
+ f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f
*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**
2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*
a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 2*b**4*log(I*sqrt(a)
*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**5*b**2*f + 8*a**4*b**3*f*t
an(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b
**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 2
4*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4
 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) + 2*b**4*log(tan(
e + f*x)**2 + 1)*tan(e + f*x)**4/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x
)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(
e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**
5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**
6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4), True))
```

3.238 $\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=97

$$-\frac{a}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{1}{2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

```
[Out] 1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^3/f-1/4*a/(a-b)/b/f/(a+b*tan(f*
x+e)^2)^2-1/2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 446, 77}

$$-\frac{a}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{1}{2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^3*f) - a/(4*(a - b)*b*f
*(a + b*Tan[e + f*x]^2)^2) - 1/(2*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a-b)^3(1+x)} + \frac{a}{(a-b)(a+bx)^3} + \frac{b}{(a-b)^2(a+bx)^2} + \frac{b}{(a-b)^3(a+bx)}\right) dx, x, \tan^2(e+\right.}{2f}$$

$$= \frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^3 f} - \frac{a}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} - \frac{}{2(a}$$

Mathematica [A] time = 0.81, size = 87, normalized size = 0.90

$$\frac{-\frac{a(a-b)^2}{b\left(a+b\tan^2(e+fx)\right)^2} - \frac{2(a-b)}{a+b\tan^2(e+fx)} + 2\log\left(a+b\tan^2(e+fx)\right) + 4\log(\cos(e+fx))}{4f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (4*Log[Cos[e + f*x]] + 2*Log[a + b*Tan[e + f*x]^2] - (a*(a - b)^2)/(b*(a +
b*Tan[e + f*x]^2)^2) - (2*(a - b))/(a + b*Tan[e + f*x]^2))/(4*(a - b)^3*f)
```

fricas [B] time = 0.45, size = 212, normalized size = 2.19

$$\frac{(ab+2b^2)\tan\left(fx+e\right)^4 + 2\left(a^2+ab+b^2\right)\tan\left(fx+e\right)^2 + 2a^2 + ab + 2\left(b^2\tan\left(fx+e\right)^4 + 2ab\tan\left(fx+e\right)\right.}{4\left(\left(a^3b^2-3a^2b^3+3ab^4-b^5\right)f\tan\left(fx+e\right)^4 + 2\left(a^4b-3a^3b^2+3a^2b^3-ab^4\right)f\tan\left(fx+e\right)^2 + \left(a^5-3a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a*b + 2*b^2)*tan(f*x + e)^4 + 2*(a^2 + a*b + b^2)*tan(f*x + e)^2 + 2*
a^2 + a*b + 2*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*log((b*tan(
f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^
5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x +
 e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(-1/(2*a^3-6*a^2*b+6*a*b^2-2*b^3
)*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+1/(4*a^3-12*a^2*b+12*a
*b^2-4*b^3)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-2*(1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a
)+(-3*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a^3+20*((1-cos(f*x+exp(1)
```

```
))/(1+cos(f*x+exp(1))))^3*a^3-32*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^
3*a^2*b-34*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^3+80*((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1))))^2*a^2*b-48*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1))))^2*a*b^2-16*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^3+20*(1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^3-32*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1)))*a^2*b-3*a^3)/(8*a^4-24*a^3*b+24*a^2*b^2-8*a*b^3)/(((1-cos(f*x+exp(1))
)/(1+cos(f*x+exp(1))))^2*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+4*(1
-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a)^2)
```

maple [B] time = 0.20, size = 193, normalized size = 1.99

$$\frac{\ln\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}{2f(a-b)^3}-\frac{a}{2f(a-b)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{b}{2f(a-b)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{}{4f(a-b)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] 1/2/f/(a-b)^3*ln(a+b*tan(f*x+e)^2)-1/2/f/(a-b)^3*a/(a+b*tan(f*x+e)^2)+1/2/f
/(a-b)^3/(a+b*tan(f*x+e)^2)*b-1/4/f/(a-b)^3*a^3/b/(a+b*tan(f*x+e)^2)^2+1/2/
f/(a-b)^3*a^2/(a+b*tan(f*x+e)^2)^2-1/4/f/(a-b)^3*a*b/(a+b*tan(f*x+e)^2)^2-1
/2/f/(a-b)^3*ln(1+tan(f*x+e)^2)
```

maxima [B] time = 0.43, size = 194, normalized size = 2.00

$$-\frac{\frac{2\left(a^2-b^2\right)\sin\left(fx+e\right)^2-2a^2-ab}{a^5-3a^4b+3a^3b^2-a^2b^3+\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5\right)\sin\left(fx+e\right)^4-2\left(a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4\right)\sin\left(fx+e\right)^2}-\frac{2\log\left(-(a-b)\sin\left(fx+e\right)\right.}{a^3-3a^2b+3ab^2-b^3}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((2*(a^2 - b^2)*sin(f*x + e)^2 - 2*a^2 - a*b)/(a^5 - 3*a^4*b + 3*a^3*b
^2 - a^2*b^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*si
n(f*x + e)^4 - 2*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sin(f*x +
e)^2) - 2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))
/f
```

mupad [B] time = 12.52, size = 532, normalized size = 5.48

$$\frac{\frac{a^3\cos\left(e+fx\right)^4}{4}-\frac{ab^2\cos\left(e+fx\right)^4}{4}+b^3\sin\left(e+fx\right)^4\operatorname{atan}\left(\frac{a\sin\left(e+fx\right)^2-b\sin\left(e+fx\right)^2}{a\cos\left(e+fx\right)^2 2i+a\sin\left(e+fx\right)^2 1i+b\sin\left(e+fx\right)^2 1i}\right)1i-\cdots}{f\left(-a^5b\cos\left(e+fx\right)^4+3a^4b^2\cos\left(e+fx\right)^4-2a^4b^2\cos\left(e+fx\right)^2\sin\left(e+fx\right)^2-3a^3b^3\cos\left(e+fx\right)^4+6a^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] ((a^3*cos(e + f*x)^4)/4 - (a*b^2*cos(e + f*x)^4)/4 + b^3*sin(e + f*x)^4*ata
n((a*sin(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e + f*
x)^2*1i + b*sin(e + f*x)^2*1i))*1i - (b^3*cos(e + f*x)^2*sin(e + f*x)^2)/2
+ a^2*b*cos(e + f*x)^4*atan((a*sin(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(e
+ f*x)^2*2i + a*sin(e + f*x)^2*1i + b*sin(e + f*x)^2*1i))*1i + (a*b^2*cos(e
 + f*x)^2*sin(e + f*x)^2)/2 + a*b^2*cos(e + f*x)^2*sin(e + f*x)^2*atan((a*s
in(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e + f*x)^2*1
i + b*sin(e + f*x)^2*1i))*2i)/(f*(b^6*sin(e + f*x)^4 - a^5*b*cos(e + f*x)^4
 - 3*a*b^5*sin(e + f*x)^4 + a^2*b^4*cos(e + f*x)^4 - 3*a^3*b^3*cos(e + f*x)
^4 + 3*a^4*b^2*cos(e + f*x)^4 + 3*a^2*b^4*sin(e + f*x)^4 - a^3*b^3*sin(e +
```

```
f*x)^4 + 2*a*b^5*cos(e + f*x)^2*sin(e + f*x)^2 - 6*a^2*b^4*cos(e + f*x)^2*s
in(e + f*x)^2 + 6*a^3*b^3*cos(e + f*x)^2*sin(e + f*x)^2 - 2*a^4*b^2*cos(e +
 f*x)^2*sin(e + f*x)^2))
```

sympy [A] time = 132.46, size = 2849, normalized size = 29.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-log(tan(e +
 f*x)**2 + 1)/(2*f) + tan(e + f*x)**2/(2*f))/a**3, Eq(b, 0)), (-3*tan(e + f
*x)**2/(12*b**3*f*tan(e + f*x)**6 + 36*b**3*f*tan(e + f*x)**4 + 36*b**3*f*t
an(e + f*x)**2 + 12*b**3*f) - 1/(12*b**3*f*tan(e + f*x)**6 + 36*b**3*f*tan(
e + f*x)**4 + 36*b**3*f*tan(e + f*x)**2 + 12*b**3*f), Eq(a, b)), (x*tan(e)*
*3/(a + b*tan(e)**2)**3, Eq(f, 0)), (-a**3/(4*a**5*b*f + 8*a**4*b**2*f*tan(
e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3
*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a
**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 -
8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 2*a**2*b*log(-I*sq
rt(a)*sqrt(1/b) + tan(e + f*x))/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2
 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e +
f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*
tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*
tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 2*a**2*b*log(I*sqrt(a)*sqrt(1
/b) + tan(e + f*x))/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b
**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12
*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)
**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)
**2 - 4*b**6*f*tan(e + f*x)**4) - 2*a**2*b*log(tan(e + f*x)**2 + 1)/(4*a**5
*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e
 + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4
*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*
b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)
**4) + 4*a*b**2*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(4
*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*
tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2
*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f +
12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e +
 f*x)**4) + 4*a*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**
2/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**
3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*
a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*
f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan
(e + f*x)**4) - 4*a*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*a**5*b
*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e +
 f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f
*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b*
*5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**
4) - 2*a*b**2*tan(e + f*x)**2/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 -
 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*
x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*ta
n(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*ta
n(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + a*b**2/(4*a**5*b*f + 8*a**4*b**
2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a
**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**
4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*
x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 2*b**3*log
(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**5*b*f + 8*a**4*
```

```
b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 2
4*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x
)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e +
 f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 2*b**3*
log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**5*b*f + 8*a**
4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 -
 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f
*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e
 + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) - 2*b**
3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**4/(4*a**5*b*f + 8*a**4*b**2*f*tan(
e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3
*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a
**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 -
8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 2*b**3*tan(e + f*x
)**2/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*
b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f -
12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b*
*4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*
tan(e + f*x)**4), True))
```

3.239 $\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=93

$$\frac{1}{2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{1}{4f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

```
[Out] -1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^3/f+1/4/(a-b)/f/(a+b*tan(f*x+e
)^2)^2+1/2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3670, 444, 44}

$$\frac{1}{2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{1}{4f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^3*f) + 1/(4*(a - b)*f*
(a + b*Tan[e + f*x]^2)^2) + 1/(2*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^3(1+x)} - \frac{b}{(a-b)(a+bx)^3} - \frac{b}{(a-b)^2(a+bx)^2} + \frac{b}{(-a+b)^3(a+bx)}\right) dx, x, \tan^2(e+fx)\right.}{2f}$$

$$= -\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^3 f} + \frac{1}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{}{2(a-b}$$

Mathematica [A] time = 0.75, size = 82, normalized size = 0.88

$$\frac{\frac{(a-b)^2}{\left(a+b\tan^2(e+fx)\right)^2} + \frac{2(a-b)}{a+b\tan^2(e+fx)} - 2\log\left(a+b\tan^2(e+fx)\right) - 4\log(\cos(e+fx))}{4f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-4*Log[Cos[e + f*x]] - 2*Log[a + b*Tan[e + f*x]^2] + (a - b)^2/(a + b*Tan[
e + f*x]^2)^2 + (2*(a - b))/(a + b*Tan[e + f*x]^2))/(4*(a - b)^3*f)
```

fricas [B] time = 0.45, size = 206, normalized size = 2.22

$$-\frac{3b^2\tan\left(fx+e\right)^4 + 2\left(2ab+b^2\right)\tan\left(fx+e\right)^2 + 4ab - b^2 + 2\left(b^2\tan\left(fx+e\right)^4 + 2ab\tan\left(fx+e\right)^2 + a^2\right)\log}{4\left(\left(a^3b^2 - 3a^2b^3 + 3ab^4 - b^5\right)f\tan\left(fx+e\right)^4 + 2\left(a^4b - 3a^3b^2 + 3a^2b^3 - ab^4\right)f\tan\left(fx+e\right)^2 + \left(a^5 - 3a^4b + 3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*b^2*tan(f*x + e)^4 + 2*(2*a*b + b^2)*tan(f*x + e)^2 + 4*a*b - b^2 +
 2*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*log((b*tan(f*x + e)^2
+ a)/(tan(f*x + e)^2 + 1)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f*
x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 + (a^
5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(1/(2*a^3-6*a^2*b+6*a*b^2-2*b^3)
*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))-1/(4*a^3-12*a^2*b+12*a*
b^2-4*b^3)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-2*(1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a)
+(3*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a^4-12*((1-cos(f*x+exp(1)))
```

```
/(1+cos(f*x+exp(1))))^3*a^4+8*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a
^3*b+24*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^2*b^2-8*((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1))))^3*a*b^3+18*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp
(1))))^2*a^4-16*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^3*b-48*((1-co
s(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2*b^2+80*((1-cos(f*x+exp(1)))/(1+co
s(f*x+exp(1))))^2*a*b^3-16*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b^4-
12*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^4+8*(1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1)))*a^3*b+24*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2*b^2-8*(1
-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a*b^3+3*a^4)/(8*a^5-24*a^4*b+24*a^3*b
^2-8*a^2*b^3)/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-2*(1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a
)^2)
```

maple [B] time = 0.24, size = 190, normalized size = 2.04

$$-\frac{\ln\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}{2f\left(a-b\right)^3}+\frac{a}{2f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}-\frac{b}{2f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)}+\frac{}{4f\left(a-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -1/2/f/(a-b)^3*ln(a+b*tan(f*x+e)^2)+1/2/f/(a-b)^3*a/(a+b*tan(f*x+e)^2)-1/2/
f/(a-b)^3/(a+b*tan(f*x+e)^2)*b+1/4/f/(a-b)^3*a^2/(a+b*tan(f*x+e)^2)^2-1/2/f
/(a-b)^3*a*b/(a+b*tan(f*x+e)^2)^2+1/4/f*b^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2+1/
2/f/(a-b)^3*ln(1+tan(f*x+e)^2)
```

maxima [B] time = 0.53, size = 192, normalized size = 2.06

$$\frac{\frac{4\left(ab-b^2\right)\sin\left(fx+e\right)^2-4ab+b^2}{a^5-3a^4b+3a^3b^2-a^2b^3+\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5\right)\sin\left(fx+e\right)^4-2\left(a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4\right)\sin\left(fx+e\right)^2}-\frac{2\log\left(-(a-b)\sin\left(fx+\right.\right.}{a^3-3a^2b+3ab^2-}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*(a*b - b^2)*sin(f*x + e)^2 - 4*a*b + b^2)/(a^5 - 3*a^4*b + 3*a^3*b^
2 - a^2*b^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sin
(f*x + e)^4 - 2*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sin(f*x + e
)^2) - 2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/
f
```

mupad [B] time = 12.46, size = 375, normalized size = 4.03

$$\frac{a^2\left(-3+\mathrm{atan}\left(\frac{a\tan\left(e+fx\right)^2 1\mathrm{i}-b\tan\left(e+fx\right)^2 1\mathrm{i}}{2a+a\tan\left(e+fx\right)^2+b\tan\left(e+fx\right)^2}\right)4\mathrm{i}\right)+b^2\left(2\tan\left(e+fx\right)^2-1+\tan\left(e+fx\right)^4\mathrm{atan}\left(\frac{a\tan\left(e\right.}{2a+a\mathrm{t}}\right.\right.}{f\left(-4a^5-8a^4b\tan\left(e+fx\right)^2+12a^4b-4a^3b^2\tan\left(e+fx\right)^4+24a^3b^2\tan\left(e+fx\right)^2-12a^3b^2+12a^2b^3\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] (a^2*(atan((a*tan(e + f*x)^2*1i - b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*x
)^2 + b*tan(e + f*x)^2))*4i - 3) + b^2*(tan(e + f*x)^4*atan((a*tan(e + f*x)
^2*1i - b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*x)^2 + b*tan(e + f*x)^2))*4
i + 2*tan(e + f*x)^2 - 1) + a*b*(tan(e + f*x)^2*atan((a*tan(e + f*x)^2*1i -
 b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*x)^2 + b*tan(e + f*x)^2))*8i - 2*t
an(e + f*x)^2 + 4))/(f*(12*a^4*b - 4*a^5 + 4*a^2*b^3 - 12*a^3*b^2 + 4*b^5*t
```

```
an(e + f*x)^4 + 8*a*b^4*tan(e + f*x)^2 - 8*a^4*b*tan(e + f*x)^2 - 12*a*b^4*
tan(e + f*x)^4 - 24*a^2*b^3*tan(e + f*x)^2 + 24*a^3*b^2*tan(e + f*x)^2 + 12
*a^2*b^3*tan(e + f*x)^4 - 4*a^3*b^2*tan(e + f*x)^4))
```

sympy [A] time = 133.35, size = 2876, normalized size = 30.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**5, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(tan(e + f
*x)**2 + 1)/(2*a**3*f), Eq(b, 0)), (-1/(6*b**3*f*tan(e + f*x)**6 + 18*b**3*
f*tan(e + f*x)**4 + 18*b**3*f*tan(e + f*x)**2 + 6*b**3*f), Eq(a, b)), (x*ta
n(e)/(a + b*tan(e)**2)**3, Eq(f, 0)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + t
an(e + f*x))/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*
b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f -
12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b*
*3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*
tan(e + f*x)**4) - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(4*a**5*f
 + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4
 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e +
 f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan
(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) + 2*a
**2*log(tan(e + f*x)**2 + 1)/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a*
*4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 1
2*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x
)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x
)**2 - 4*b**5*f*tan(e + f*x)**4) + 3*a**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*
x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e
+ f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*
f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*
f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 4*a*b*log(-I*sqrt(a)*sqrt(1
/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2
- 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)
**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(
e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(
e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 4*a*b*log(I*sqrt(a)*sqrt(1/b) + t
an(e + f*x))*tan(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a*
*4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 1
2*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x
)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x
)**2 - 4*b**5*f*tan(e + f*x)**4) + 4*a*b*log(tan(e + f*x)**2 + 1)*tan(e + f
*x)**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f
*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**
2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f +
 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e
+ f*x)**4) + 2*a*b*tan(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 -
 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)*
*2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e
 + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e
 + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 4*a*b/(4*a**5*f + 8*a**4*b*f*tan(e
 + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*t
an(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*
b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*
b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 2*b**2*log(-I*sqrt(a)*
sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**5*f + 8*a**4*b*f*tan(e + f*
x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e
+ f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*
f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*
```

```
f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 2*b**2*log(I*sqrt(a)*sqrt(1
/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2
- 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)
**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(
e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(
e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) + 2*b**2*log(tan(e + f*x)**2 + 1)*t
an(e + f*x)**4/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**
3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f
- 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*
b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*
f*tan(e + f*x)**4) - 2*b**2*tan(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*tan(e +
f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(
e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**
3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**
4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) + b**2/(4*a**5*f + 8*a**4*b
*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*
b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 +
24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**
4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4), True))
```

3.240 $\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=148

$$\frac{\log(\tan(e+fx))}{a^3f}-\frac{b(2a-b)}{2a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{b\left(3a^2-3ab+b^2\right)\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)^3}-\frac{}{4af(a-b)\left(a\right.}$$

```
[Out] ln(cos(f*x+e))/(a-b)^3/f+ln(tan(f*x+e))/a^3/f+1/2*b*(3*a^2-3*a*b+b^2)*ln(a+
b*tan(f*x+e)^2)/a^3/(a-b)^3/f-1/4*b/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/2*(2*a
-b)*b/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.17, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3670, 446, 72}

$$\frac{b\left(3a^2-3ab+b^2\right)\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)^3}-\frac{b(2a-b)}{2a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{\log(\tan(e+fx))}{a^3f}-\frac{}{4af(a-b)\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] Log[Cos[e + f*x]]/((a - b)^3*f) + Log[Tan[e + f*x]]/(a^3*f) + (b*(3*a^2 - 3
*a*b + b^2)*Log[a + b*Tan[e + f*x]^2])/(2*a^3*(a - b)^3*f) - b/(4*a*(a - b)
*f*(a + b*Tan[e + f*x]^2)^2) - ((2*a - b)*b)/(2*a^2*(a - b)^2*f*(a + b*Tan[
e + f*x]^2))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{1}{(a-b)^3(1+x)} + \frac{b^2}{a(a-b)(a+bx)^3} + \frac{(2a-b)b^2}{a^2(a-b)^2(a+bx)^2} + \frac{b^2(3a^2-3ab+b^2)}{a^3(a-b)^3(a+bx)}\right) dx, x,\right.}{2f}$$
$$= \frac{\log(\cos(e+fx))}{(a-b)^3 f} + \frac{\log(\tan(e+fx))}{a^3 f} + \frac{b\left(3a^2-3ab+b^2\right)\log\left(a+b\tan^2(e+f\right.}{2a^3(a-b)^3 f}$$

Mathematica [A] time = 1.82, size = 126, normalized size = 0.85

$$\frac{\frac{\frac{b\left(2\left(3a^2-3ab+b^2\right)\log\left(a+b\tan^2(e+fx)\right)-\frac{a(a-b)\left(2b(2a-b)\tan^2(e+fx)+a(5a-3b)\right)}{\left(a+b\tan^2(e+fx)\right)^2}\right)}{(a-b)^3}+4\log(\tan(e+fx))}{a^3}+\frac{4\log(\cos(e+fx))}{(a-b)^3}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((4*Log[Cos[e + f*x]])/(a - b)^3 + (4*Log[Tan[e + f*x]] + (b*(2*(3*a^2 - 3*
a*b + b^2)*Log[a + b*Tan[e + f*x]^2] - (a*(a - b)*(a*(5*a - 3*b) + 2*(2*a -
 b)*b*Tan[e + f*x]^2))/(a + b*Tan[e + f*x]^2)^2))/(a - b)^3)/a^3)/(4*f)
```

fricas [B] time = 0.50, size = 422, normalized size = 2.85

$$\frac{6\,a^3b^2 - 3\,a^2b^3 + \left(5\,a^2b^3 - 2\,ab^4\right)\tan\left(fx+e\right)^4 + 2\left(3\,a^3b^2 + a^2b^3 - ab^4\right)\tan\left(fx+e\right)^2 + 2\left(a^5 - 3\,a^4b + 3\,a^3b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*a^3*b^2 - 3*a^2*b^3 + (5*a^2*b^3 - 2*a*b^4)*tan(f*x + e)^4 + 2*(3*a^
3*b^2 + a^2*b^3 - a*b^4)*tan(f*x + e)^2 + 2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^
2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3
*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*log(tan(f*x + e)^2/(tan(f*x +
 e)^2 + 1)) + 2*(3*a^4*b - 3*a^3*b^2 + a^2*b^3 + (3*a^2*b^3 - 3*a*b^4 + b^5
)*tan(f*x + e)^4 + 2*(3*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*log((b
*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b
^4 - a^3*b^5)*f*tan(f*x + e)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4
)*f*tan(f*x + e)^2 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)1/f*(1/2/a^3*ln(sin(f*x+exp(1))^2)+(-b^3+3*b^2*a-3*b*a^2)/(2*b^3*a^
3-6*b^2*a^4+6*b*a^5-2*a^6)*ln(abs(sin(f*x+exp(1))^2*b-sin(f*x+exp(1))^2*a+a
))+(3*sin(f*x+exp(1))^4*b^4-12*sin(f*x+exp(1))^4*b^3*a+18*sin(f*x+exp(1))^4
*b^2*a^2-9*sin(f*x+exp(1))^4*b*a^3+8*sin(f*x+exp(1))^2*b^3*a-24*sin(f*x+exp
(1))^2*b^2*a^2+18*sin(f*x+exp(1))^2*b*a^3+6*b^2*a^2-9*b*a^3)/(4*b^2*a^3-8*b
*a^4+4*a^5)/(sin(f*x+exp(1))^2*b-sin(f*x+exp(1))^2*a+a)^2)
```

maple [B] time = 0.86, size = 289, normalized size = 1.95

$$\frac{3b^2}{2fa(a-b)^3\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)}-\frac{b^3}{2fa^2(a-b)^3\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] 3/2/f*b^2/a/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-1/2/f*b^3/a^2/(a-b)^3
/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+3/2/f*b/a/(a-b)^3*ln(a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)-3/2/f*b^2/a^2/(a-b)^3*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+1/2
/f*b^3/a^3/(a-b)^3*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-1/4/f*b^3/a/(a-b)^3/
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2+1/2/f/a^3*ln(-1+cos(f*x+e))+1/2/f/a^3*l
n(1+cos(f*x+e))
```

maxima [A] time = 0.41, size = 250, normalized size = 1.69

$$\frac{\frac{2\left(3a^2b-3ab^2+b^3\right)\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^6-3a^5b+3a^4b^2-a^3b^3}+\frac{6a^2b^2-3ab^3-2\left(3a^2b^2-4ab^3+b^4\right)\sin\left(fx+e\right)^2}{a^7-3a^6b+3a^5b^2-a^4b^3+\left(a^7-5a^6b+10a^5b^2-10a^4b^3+5a^3b^4-a^2b^5\right)\sin\left(fx+e\right)^4-2\left(a^7-4a^6b+6a^5b^2-\right.}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*a^2*b - 3*a*b^2 + b^3)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^6 - 3*
a^5*b + 3*a^4*b^2 - a^3*b^3) + (6*a^2*b^2 - 3*a*b^3 - 2*(3*a^2*b^2 - 4*a*b^
3 + b^4)*sin(f*x + e)^2)/(a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3 + (a^7 - 5*a^
6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*sin(f*x + e)^4 - 2*(a^
7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*sin(f*x + e)^2) + 2*log(sin(
f*x + e)^2)/a^3)/f
```

mupad [B] time = 12.59, size = 181, normalized size = 1.22

$$\frac{\ln\left(\tan\left(e+fx\right)\right)}{a^3f}-\frac{\ln\left(\tan\left(e+fx\right)^2+1\right)}{2f(a-b)^3}-\frac{\frac{5ab-3b^2}{4a\left(a^2-2ab+b^2\right)}+\frac{b\tan\left(e+fx\right)^2\left(2ab-b^2\right)}{2a^2\left(a^2-2ab+b^2\right)}}{f\left(a^2+2ab\tan\left(e+fx\right)^2+b^2\tan\left(e+fx\right)^4\right)}+\frac{b\ln\left(b\tan\left(e+fx\right)\right.}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] log(tan(e + f*x))/(a^3*f) - log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^3) - ((5*a
*b - 3*b^2)/(4*a*(a^2 - 2*a*b + b^2)) + (b*tan(e + f*x)^2*(2*a*b - b^2))/(2
*a^2*(a^2 - 2*a*b + b^2)))/(f*(a^2 + b^2*tan(e + f*x)^4 + 2*a*b*tan(e + f*x
)^2)) + (b*log(a + b*tan(e + f*x)^2)*(3*a^2 - 3*a*b + b^2))/(2*a^3*f*(a - b
)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.241 $\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=181

$$-\frac{(a+3b)\log(\tan(e+fx))}{a^4 f}+\frac{b^2(3a-2b)}{2a^3 f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{\cot^2(e+fx)}{2a^3 f}+\frac{b^2}{4a^2 f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-$$

```
[Out] -1/2*cot(f*x+e)^2/a^3/f-ln(cos(f*x+e))/(a-b)^3/f-(a+3*b)*ln(tan(f*x+e))/a^4
/f-1/2*b^2*(6*a^2-8*a*b+3*b^2)*ln(a+b*tan(f*x+e)^2)/a^4/(a-b)^3/f+1/4*b^2/a
^2/(a-b)/f/(a+b*tan(f*x+e)^2)^2+1/2*(3*a-2*b)*b^2/a^3/(a-b)^2/f/(a+b*tan(f*
x+e)^2)
```

Rubi [A] time = 0.21, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 88}

$$\frac{b^2(3a-2b)}{2a^3 f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{b^2}{4a^2 f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{b^2\left(6a^2-8ab+3b^2\right)\log\left(a+b\tan^2(e+f\right.}{2a^4 f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -Cot[e + f*x]^2/(2*a^3*f) - Log[Cos[e + f*x]]/((a - b)^3*f) - ((a + 3*b)*Lo
g[Tan[e + f*x]])/(a^4*f) - (b^2*(6*a^2 - 8*a*b + 3*b^2)*Log[a + b*Tan[e + f
*x]^2])/(2*a^4*(a - b)^3*f) + b^2/(4*a^2*(a - b)*f*(a + b*Tan[e + f*x]^2)^2
) + ((3*a - 2*b)*b^2)/(2*a^3*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x^2} + \frac{-a-3b}{a^4x} + \frac{1}{(a-b)^3(1+x)} - \frac{b^3}{a^2(a-b)(a+bx)^3} - \frac{(3a-2b)b^3}{a^3(a-b)^2(a+bx)^2} - \frac{b^3\left(6a^2-8ab+3\right.}{a^4(a-b)^3(a+b}\right.\right.}{2f}$$
$$= -\frac{\cot^2(e+fx)}{2a^3f} - \frac{\log(\cos(e+fx))}{(a-b)^3f} - \frac{(a+3b)\log(\tan(e+fx))}{a^4f} - \frac{b^2\left(6a^2-8ab\right.}{}$$

Mathematica [A] time = 2.06, size = 144, normalized size = 0.80

$$-\frac{-\frac{b^4}{2a^4(a-b)\left(a\cot^2(e+fx)+b\right)^2} + \frac{b^3(4a-3b)}{a^4(a-b)^2\left(a\cot^2(e+fx)+b\right)} + \frac{\cot^2(e+fx)}{a^3} + \frac{b^2\left(6a^2-8ab+3b^2\right)\log\left(a\cot^2(e+fx)+b\right)}{a^4(a-b)^3} + \frac{2\log(\sin(e+fx))}{(a-b)^3}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -1/2*(Cot[e + f*x]^2/a^3 - b^4/(2*a^4*(a - b)*(b + a*Cot[e + f*x]^2)^2) + (
(4*a - 3*b)*b^3)/(a^4*(a - b)^2*(b + a*Cot[e + f*x]^2)) + (b^2*(6*a^2 - 8*a
*b + 3*b^2)*Log[b + a*Cot[e + f*x]^2])/(a^4*(a - b)^3) + (2*Log[Sin[e + f*x
]])/(a - b)^3)/f
```

fricas [B] time = 0.58, size = 545, normalized size = 3.01

$$-\frac{\left(2a^4b^2-6a^3b^3+13a^2b^4-6ab^5\right)\tan\left(fx+e\right)^6+2a^6-6a^5b+6a^4b^2-2a^3b^3+2\left(2a^5b-5a^4b^2+7a^3b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*a^4*b^2 - 6*a^3*b^3 + 13*a^2*b^4 - 6*a*b^5)*tan(f*x + e)^6 + 2*a^6
 - 6*a^5*b + 6*a^4*b^2 - 2*a^3*b^3 + 2*(2*a^5*b - 5*a^4*b^2 + 7*a^3*b^3 + 2
*a^2*b^4 - 3*a*b^5)*tan(f*x + e)^4 + (2*a^6 - 2*a^5*b - 6*a^4*b^2 + 18*a^3*
b^3 - 9*a^2*b^4)*tan(f*x + e)^2 + 2*((a^4*b^2 - 6*a^2*b^4 + 8*a*b^5 - 3*b^6
)*tan(f*x + e)^6 + 2*(a^5*b - 6*a^3*b^3 + 8*a^2*b^4 - 3*a*b^5)*tan(f*x + e)
^4 + (a^6 - 6*a^4*b^2 + 8*a^3*b^3 - 3*a^2*b^4)*tan(f*x + e)^2)*log(tan(f*x
+ e)^2/(tan(f*x + e)^2 + 1)) + 2*((6*a^2*b^4 - 8*a*b^5 + 3*b^6)*tan(f*x + e
)^6 + 2*(6*a^3*b^3 - 8*a^2*b^4 + 3*a*b^5)*tan(f*x + e)^4 + (6*a^4*b^2 - 8*a
^3*b^3 + 3*a^2*b^4)*tan(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e
)^2 + 1)))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^6 +
2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^4 + (a^9 - 3*a^8
*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*((4*(1-cos(f*x+exp(
1)))/(1+cos(f*x+exp(1)))*a+12*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b-a)*
1/16/a^4/(1-cos(f*x+exp(1)))*(1+cos(f*x+exp(1)))+(18*((1-cos(f*x+exp(1)))/(
1+cos(f*x+exp(1))))^4*a^4*b^2-24*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^
4*a^3*b^3+9*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a^2*b^4-72*((1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^4*b^2+208*((1-cos(f*x+exp(1)))/(1+cos
(f*x+exp(1))))^3*a^3*b^3-172*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^
2*b^4+48*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a*b^5+108*((1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1))))^2*a^4*b^2-368*((1-cos(f*x+exp(1)))/(1+cos(f*x
+exp(1))))^2*a^3*b^3+502*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^2*b^
4-288*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a*b^5+64*((1-cos(f*x+exp(
1)))/(1+cos(f*x+exp(1))))^2*b^6-72*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*
a^4*b^2+208*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^3*b^3-172*(1-cos(f*x+
exp(1)))/(1+cos(f*x+exp(1)))*a^2*b^4+48*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1)))*a*b^5+18*a^4*b^2-24*a^3*b^3+9*a^2*b^4)/(8*a^7-24*a^6*b+24*a^5*b^2-8*a^
4*b^3)/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a-2*(1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*b+a)^2-(1-
cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*1/16/a^3+1/(2*a^3-6*a^2*b+6*a*b^2-2*b^
3)*ln(abs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+(-a-3*b)*1/4/a^4*ln(a
bs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1))))+(-6*a^2*b^2+8*a*b^3-3*b^4)/(4
*a^7-12*a^6*b+12*a^5*b^2-4*a^4*b^3)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(
1))))^2*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1)))*b+a))
```

maple [B] time = 1.00, size = 362, normalized size = 2.00

$$-\frac{2b^3}{f\,a^2\,(a-b)^3\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)}+\frac{b^4}{f\,a^3\,(a-b)^3\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -2/f*b^3/a^2/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+1/f*b^4/a^3/(a-b)^3/
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-3/f*b^2/a^2/(a-b)^3*ln(a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)+4/f*b^3/a^3/(a-b)^3*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-3/2/
f*b^4/a^4/(a-b)^3*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+1/4/f*b^4/a^2/(a-b)^3
/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2+1/4/f/a^3/(-1+cos(f*x+e))-1/2/f/a^3*ln
(-1+cos(f*x+e))-3/2/f/a^4*ln(-1+cos(f*x+e))*b-1/4/f/a^3/(1+cos(f*x+e))-1/2/
f/a^3*ln(1+cos(f*x+e))-3/2/f/a^4*ln(1+cos(f*x+e))*b
```

maxima [A] time = 0.97, size = 345, normalized size = 1.91

$$-\frac{\frac{2\left(6\,a^2b^2-8\,ab^3+3\,b^4\right)\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^7-3\,a^6b+3\,a^5b^2-a^4b^3}+\frac{2\,a^5-6\,a^4b+6\,a^3b^2-2\,a^2b^3+2\left(a^5-5\,a^4b+10\,a^3b^2-14\,a^2b^3+11\,ab^4-3\,b^5\right)\sin\left(fx+e\right)^4-\left(4\,a^5-16\right.}{\left(a^8-5\,a^7b+10\,a^6b^2-10\,a^5b^3+5\,a^4b^4-a^3b^5\right)\sin\left(fx+e\right)^6-2\left(a^8-4\,a^7b+6\,a^6b^2-4\,a^5b^3+a^4b^4\right)\sin\left(f\right.}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(6*a^2*b^2 - 8*a*b^3 + 3*b^4)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^7
 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3) + (2*a^5 - 6*a^4*b + 6*a^3*b^2 - 2*a^2*b^
3 + 2*(a^5 - 5*a^4*b + 10*a^3*b^2 - 14*a^2*b^3 + 11*a*b^4 - 3*b^5)*sin(f*x
```

```
+ e)^4 - (4*a^5 - 16*a^4*b + 24*a^3*b^2 - 24*a^2*b^3 + 9*a*b^4)*sin(f*x + e
)^2)/((a^8 - 5*a^7*b + 10*a^6*b^2 - 10*a^5*b^3 + 5*a^4*b^4 - a^3*b^5)*sin(f
*x + e)^6 - 2*(a^8 - 4*a^7*b + 6*a^6*b^2 - 4*a^5*b^3 + a^4*b^4)*sin(f*x + e
)^4 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*sin(f*x + e)^2) + 2*(a + 3*b)*l
og(sin(f*x + e)^2)/a^4)/f
```

mupad [B] time = 13.46, size = 229, normalized size = 1.27

$$\frac{\ln\left(\tan\left(e+f\,x\right)^2+1\right)}{2\,f\,(a-b)^3}-\frac{\frac{1}{2\,a}+\frac{\tan\left(e+f\,x\right)^4\,\left(a^2\,b^2-5\,a\,b^3+3\,b^4\right)}{2\,a^3\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\tan\left(e+f\,x\right)^2\,\left(4\,a^2\,b-15\,a\,b^2+9\,b^3\right)}{4\,a^2\,\left(a^2-2\,a\,b+b^2\right)}}{f\,\left(a^2\tan\left(e+f\,x\right)^2+2\,a\,b\tan\left(e+f\,x\right)^4+b^2\tan\left(e+f\,x\right)^6\right)}-\frac{\ln\left(\tan\left(e+f\,x\right)\right)\,(a+}{a^4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^3) - (1/(2*a) + (tan(e + f*x)^4*(3*b^4
 - 5*a*b^3 + a^2*b^2))/(2*a^3*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)^2*(4*a^2
*b - 15*a*b^2 + 9*b^3))/(4*a^2*(a^2 - 2*a*b + b^2)))/(f*(a^2*tan(e + f*x)^2
 + b^2*tan(e + f*x)^6 + 2*a*b*tan(e + f*x)^4)) - (log(tan(e + f*x))*(a + 3*
b))/(a^4*f) - (b^2*log(a + b*tan(e + f*x)^2)*(6*a^2 - 8*a*b + 3*b^2))/(2*a^
4*f*(a - b)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.242 $\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=210

$$-\frac{b^3(4a-3b)}{2a^4f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{(a+3b)\cot^2(e+fx)}{2a^4f}-\frac{b^3}{4a^3f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{\cot^4(e+fx)}{4a^3f}+\frac{(a^2$$

```
[Out] 1/2*(a+3*b)*cot(f*x+e)^2/a^4/f-1/4*cot(f*x+e)^4/a^3/f+ln(cos(f*x+e))/(a-b)^
3/f+(a^2+3*a*b+6*b^2)*ln(tan(f*x+e))/a^5/f+1/2*b^3*(10*a^2-15*a*b+6*b^2)*ln
(a+b*tan(f*x+e)^2)/a^5/(a-b)^3/f-1/4*b^3/a^3/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1
/2*(4*a-3*b)*b^3/a^4/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.24, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 88}

$$-\frac{b^3(4a-3b)}{2a^4f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{b^3}{4a^3f(a-b)\left(a+b\tan^2(e+fx)\right)^2}+\frac{b^3\left(10a^2-15ab+6b^2\right)\log\left(a+b\tan^2(e\right.}{2a^5f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((a + 3*b)*Cot[e + f*x]^2)/(2*a^4*f) - Cot[e + f*x]^4/(4*a^3*f) + Log[Cos[e
 + f*x]]/((a - b)^3*f) + ((a^2 + 3*a*b + 6*b^2)*Log[Tan[e + f*x]])/(a^5*f)
+ (b^3*(10*a^2 - 15*a*b + 6*b^2)*Log[a + b*Tan[e + f*x]^2])/(2*a^5*(a - b)^
3*f) - b^3/(4*a^3*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - ((4*a - 3*b)*b^3)/(
2*a^4*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x^3} + \frac{-a-3b}{a^4x^2} + \frac{a^2+3ab+6b^2}{a^5x} - \frac{1}{(a-b)^3(1+x)} + \frac{b^4}{a^3(a-b)(a+bx)^3} + \frac{(4a-3b)b^4}{a^4(a-b)^2(a+bx)^2}\right.\right.}{2f}$$

$$= \frac{(a+3b)\cot^2(e+fx)}{2a^4f} - \frac{\cot^4(e+fx)}{4a^3f} + \frac{\log(\cos(e+fx))}{(a-b)^3f} + \frac{\left(a^2+3ab+6b^2\right)\text{lo}}{a^5f}$$

Mathematica [A] time = 2.65, size = 178, normalized size = 0.85

$$\frac{\frac{(a+3b)\cot^2(e+fx)}{a^4} - \frac{\cot^4(e+fx)}{2a^3} + \frac{4\left(a^2+3ab+6b^2\right)\log(\tan(e+fx)) + \frac{b^3\left(2\left(10a^2-15ab+6b^2\right)\log\left(a+b\tan^2(e+fx)\right) - \frac{a(a-b)\left(2b(4a-3b)\tan^2(e+fx)+a(9a-7b)\right)}{\left(a+b\tan^2(e+fx)\right)^2}\right)}{(a-b)^3}}{2a^5}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (((a + 3*b)*Cot[e + f*x]^2)/a^4 - Cot[e + f*x]^4/(2*a^3) + (2*Log[Cos[e + f
*x]])/(a - b)^3 + (4*(a^2 + 3*a*b + 6*b^2)*Log[Tan[e + f*x]] + (b^3*(2*(10*
a^2 - 15*a*b + 6*b^2)*Log[a + b*Tan[e + f*x]^2] - (a*(a - b)*(a*(9*a - 7*b)
 + 2*(4*a - 3*b)*b*Tan[e + f*x]^2))/(a + b*Tan[e + f*x]^2)^2))/(a - b)^3)/(
2*a^5))/(2*f)
```

fricas [B] time = 0.62, size = 611, normalized size = 2.91

$$3\left(a^5b^2 - a^4b^3 - 3a^3b^4 + 8a^2b^5 - 4ab^6\right)\tan\left(fx+e\right)^8 - a^7 + 3a^6b - 3a^5b^2 + a^4b^3 + 2\left(3a^6b - 2a^5b^2 - 9a^4b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(a^5*b^2 - a^4*b^3 - 3*a^3*b^4 + 8*a^2*b^5 - 4*a*b^6)*tan(f*x + e)^8
 - a^7 + 3*a^6*b - 3*a^5*b^2 + a^4*b^3 + 2*(3*a^6*b - 2*a^5*b^2 - 9*a^4*b^3
 + 14*a^3*b^4 + 3*a^2*b^5 - 6*a*b^6)*tan(f*x + e)^6 + (3*a^7 + a^6*b - 10*a
^5*b^2 - 6*a^4*b^3 + 33*a^3*b^4 - 18*a^2*b^5)*tan(f*x + e)^4 + 2*(a^7 - a^6
*b - 3*a^5*b^2 + 5*a^4*b^3 - 2*a^3*b^4)*tan(f*x + e)^2 + 2*((a^5*b^2 - 10*a
^2*b^5 + 15*a*b^6 - 6*b^7)*tan(f*x + e)^8 + 2*(a^6*b - 10*a^3*b^4 + 15*a^2*
b^5 - 6*a*b^6)*tan(f*x + e)^6 + (a^7 - 10*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5)
*tan(f*x + e)^4)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + 2*((10*a^2*b^5
- 15*a*b^6 + 6*b^7)*tan(f*x + e)^8 + 2*(10*a^3*b^4 - 15*a^2*b^5 + 6*a*b^6)*
tan(f*x + e)^6 + (10*a^4*b^3 - 15*a^3*b^4 + 6*a^2*b^5)*tan(f*x + e)^4)*log(
(b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6
*b^4 - a^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b
^4)*f*tan(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e
)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*((-32*((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1))))^2*a^3+384*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1
)))*a^3+768*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^2*b)*1/4096/a^6+(-16*
((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^6*a^7-160*((1-cos(f*x+exp(1)))/(1
+cos(f*x+exp(1))))^6*a^4*b^3+240*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^
6*a^3*b^4-96*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^6*a^2*b^5+76*((1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*a^7-140*((1-cos(f*x+exp(1)))/(1+cos(f*
x+exp(1))))^5*a^6*b-36*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*a^5*b^2+
700*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*a^4*b^3-1624*((1-cos(f*x+ex
p(1)))/(1+cos(f*x+exp(1))))^5*a^3*b^4+1152*((1-cos(f*x+exp(1)))/(1+cos(f*x+
exp(1))))^5*a^2*b^5-256*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^5*a*b^6-1
45*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a^7+403*((1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1))))^4*a^6*b-211*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))
^4*a^5*b^2-1487*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a^4*b^3+3296*((
1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*a^3*b^4-2560*((1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1))))^4*a^2*b^5+256*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))
))^4*a*b^6+256*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^4*b^7+140*((1-cos(
f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^7-412*((1-cos(f*x+exp(1)))/(1+cos(f*x
+exp(1))))^3*a^6*b+204*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^5*b^2+
1356*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a^4*b^3-3272*((1-cos(f*x+e
xp(1)))/(1+cos(f*x+exp(1))))^3*a^3*b^4+2496*((1-cos(f*x+exp(1)))/(1+cos(f*x
+exp(1))))^3*a^2*b^5-640*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a*b^6-
70*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^7+178*((1-cos(f*x+exp(1)))
/(1+cos(f*x+exp(1))))^2*a^6*b-34*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^
2*a^5*b^2-586*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^4*b^3+752*((1-c
os(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*a^3*b^4-272*((1-cos(f*x+exp(1)))/(1+
cos(f*x+exp(1))))^2*a^2*b^5+16*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^7-
32*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^6*b+32*(1-cos(f*x+exp(1)))/(1+
cos(f*x+exp(1)))*a^4*b^3-16*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a^3*b^4
-a^7+3*a^6*b-3*a^5*b^2+a^4*b^3)/(128*a^8-384*a^7*b+384*a^6*b^2-128*a^5*b^3)
/(((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^3*a-2*((1-cos(f*x+exp(1)))/(1+c
os(f*x+exp(1))))^2*a+4*((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1))))^2*b+(1-cos
(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a)^2-1/(2*a^3-6*a^2*b+6*a*b^2-2*b^3)*ln(a
bs((1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))+1))+(a^2+3*a*b+6*b^2)*1/4/a^5*ln
(abs(1-cos(f*x+exp(1)))/abs(1+cos(f*x+exp(1))))+(10*a^2*b^3-15*a*b^4+6*b^5)
/(4*a^8-12*a^7*b+12*a^6*b^2-4*a^5*b^3)*ln(((1-cos(f*x+exp(1)))/(1+cos(f*x+e
xp(1))))^2*a-2*(1-cos(f*x+exp(1)))/(1+cos(f*x+exp(1)))*a+4*(1-cos(f*x+exp(1
)))/(1+cos(f*x+exp(1)))*b+a))
```

maple [B] time = 0.92, size = 477, normalized size = 2.27

$$\frac{5b^4}{2f\,a^3\,(a-b)^3\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)}-\frac{3b^5}{2f\,a^4\,(a-b)^3\left(a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] 5/2/f*b^4/a^3/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-3/2/f*b^5/a^4/(a-b)
^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+5/f*b^3/a^3/(a-b)^3*ln(a*cos(f*x+e)^2-
```

```
cos(f*x+e)^2*b+b)-15/2/f*b^4/a^4/(a-b)^3*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)+3/f*b^5/a^5/(a-b)^3*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-1/4/f*b^5/a^3/(a-
b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2-1/16/f/a^3/(-1+cos(f*x+e))^2-7/16/
f/a^3/(-1+cos(f*x+e))-3/4/f/a^4/(-1+cos(f*x+e))*b+1/2/f/a^3*ln(-1+cos(f*x+e
))+3/2/f/a^4*ln(-1+cos(f*x+e))*b+3/f/a^5*ln(-1+cos(f*x+e))*b^2-1/16/f/a^3/(
1+cos(f*x+e))^2+7/16/f/a^3/(1+cos(f*x+e))+3/4/f/a^4/(1+cos(f*x+e))*b+1/2/f/
a^3*ln(1+cos(f*x+e))+3/2/f/a^4*ln(1+cos(f*x+e))*b+3/f/a^5*ln(1+cos(f*x+e))*
b^2
```

maxima [B] time = 0.38, size = 416, normalized size = 1.98

$$\frac{\frac{2\left(10\,a^2b^3-15\,ab^4+6\,b^5\right)\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{a^8-3\,a^7b+3\,a^6b^2-a^5b^3}+\frac{2\left(2\,a^6-7\,a^5b+5\,a^4b^2+10\,a^3b^3-25\,a^2b^4+21\,ab^5-6\,b^6\right)\sin\left(fx+e\right)^6-a^6+3\,a^5b-3\,a^4b^2+a^3b^3-\cdots}{\left(a^9-5\,a^8b+10\,a^7b^2-10\,a^6b^3+5\,a^5b^4-a^4b^5\right)\sin\left(fx+e\right)^8-2\left(a^{\cdots}\right.}}{4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(10*a^2*b^3 - 15*a*b^4 + 6*b^5)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^
8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3) + (2*(2*a^6 - 7*a^5*b + 5*a^4*b^2 + 10*a
^3*b^3 - 25*a^2*b^4 + 21*a*b^5 - 6*b^6)*sin(f*x + e)^6 - a^6 + 3*a^5*b - 3*
a^4*b^2 + a^3*b^3 - (9*a^6 - 25*a^5*b + 10*a^4*b^2 + 30*a^3*b^3 - 45*a^2*b^
4 + 18*a*b^5)*sin(f*x + e)^4 + 2*(3*a^6 - 7*a^5*b + 3*a^4*b^2 + 3*a^3*b^3 -
 2*a^2*b^4)*sin(f*x + e)^2)/((a^9 - 5*a^8*b + 10*a^7*b^2 - 10*a^6*b^3 + 5*a
^5*b^4 - a^4*b^5)*sin(f*x + e)^8 - 2*(a^9 - 4*a^8*b + 6*a^7*b^2 - 4*a^6*b^3
 + a^5*b^4)*sin(f*x + e)^6 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*sin(f*x
+ e)^4) + 2*(a^2 + 3*a*b + 6*b^2)*log(sin(f*x + e)^2)/a^5)/f
```

mupad [B] time = 13.51, size = 269, normalized size = 1.28

$$\frac{\frac{\tan\left(e+f\,x\right)^2\,(a+2\,b)}{2\,a^2}-\frac{1}{4\,a}+\frac{\tan\left(e+f\,x\right)^6\,\left(a^3\,b^2+a^2\,b^3-9\,a\,b^4+6\,b^5\right)}{2\,a^4\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\tan\left(e+f\,x\right)^4\,\left(4\,a^3\,b+3\,a^2\,b^2-27\,a\,b^3+18\,b^4\right)}{4\,a^3\,\left(a^2-2\,a\,b+b^2\right)}}{f\,\left(a^2\,\tan\left(e+f\,x\right)^4+2\,a\,b\,\tan\left(e+f\,x\right)^6+b^2\,\tan\left(e+f\,x\right)^8\right)}-\frac{\ln\left(b\,\tan\left(e+f\,x\right)^2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^2*(a + 2*b))/(2*a^2) - 1/(4*a) + (tan(e + f*x)^6*(6*b^5 - 9*
a*b^4 + a^2*b^3 + a^3*b^2))/(2*a^4*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)^4*(
4*a^3*b - 27*a*b^3 + 18*b^4 + 3*a^2*b^2))/(4*a^3*(a^2 - 2*a*b + b^2)))/(f*(
a^2*tan(e + f*x)^4 + b^2*tan(e + f*x)^8 + 2*a*b*tan(e + f*x)^6)) - (log(a +
 b*tan(e + f*x)^2)*((3*b)/(2*a^4) + 1/(2*a^3) - 1/(2*(a - b)^3) + (3*b^2)/a
^5))/f - log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^3) + (log(tan(e + f*x))*(3*a*
b + a^2 + 6*b^2))/(a^5*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.243 $\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{a}\left(3a^2-10ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8b^{5/2}f(a-b)^3}-\frac{a(3a-7b)\tan(e+fx)}{8b^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{a\tan^3(e+fx)}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}$$

```
[Out] -x/(a-b)^3+1/8*(3*a^2-10*a*b+15*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*a^(
1/2)/(a-b)^3/b^(5/2)/f-1/4*a*tan(f*x+e)^3/(a-b)/b/f/(a+b*tan(f*x+e)^2)^2-1/
8*a*(3*a-7*b)*tan(f*x+e)/(a-b)^2/b^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.23, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3670, 470, 578, 522, 203, 205}

$$\frac{\sqrt{a}\left(3a^2-10ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8b^{5/2}f(a-b)^3}-\frac{a(3a-7b)\tan(e+fx)}{8b^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{a\tan^3(e+fx)}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(x/(a - b)^3) + (Sqrt[a]*(3*a^2 - 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a]])/(8*(a - b)^3*b^(5/2)*f) - (a*Tan[e + f*x]^3)/(4*(a - b)*b*
f*(a + b*Tan[e + f*x]^2)^2) - (a*(3*a - 7*b)*Tan[e + f*x])/(8*(a - b)^2*b^2
*f*(a + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan^3(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3a+(3a-4b)x^2\right)}{(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4(a-b)bf}$$

$$= -\frac{a\tan^3(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} - \frac{a(3a-7b)\tan(e+fx)}{8(a-b)^2b^2f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}}{}$$

$$= -\frac{a\tan^3(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} - \frac{a(3a-7b)\tan(e+fx)}{8(a-b)^2b^2f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}}{}$$

$$= -\frac{x}{(a-b)^3} + \frac{\sqrt{a}\left(3a^2-10ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8(a-b)^3b^{5/2}f} - \frac{a\tan^3(e+}{4(a-b)bf\left(a+b\tan\right.}$$

Mathematica [A] time = 2.37, size = 142, normalized size = 0.93

$$\frac{-\frac{a(a-b)\sin(2(e+fx))\left(3\left(a^2-4ab+3b^2\right)\cos(2(e+fx))+3a^2-2ab-9b^2\right)}{b^2((a-b)\cos(2(e+fx))+a+b)^2} + \frac{\sqrt{a}\left(3a^2-10ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{5/2}} - 8(e+fx)}{8f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-8*(e + f*x) + (Sqrt[a]*(3*a^2 - 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a]])/b^(5/2) - (a*(a - b)*(3*a^2 - 2*a*b - 9*b^2 + 3*(a^2 - 4*a*
b + 3*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(b^2*(a + b + (a - b)*Cos[2*
(e + f*x)])^2))/(8*(a - b)^3*f)
```

fricas [B] time = 0.48, size = 743, normalized size = 4.86

$$\left[-\frac{32\,b^4 f x \tan\left(f x + e\right)^4 + 64\,a b^3 f x \tan\left(f x + e\right)^2 + 32\,a^2 b^2 f x + 4\left(5\,a^3 b - 14\,a^2 b^2 + 9\,a b^3\right)\tan\left(f x + e\right)^3 + \left(\left(3\,a\right.\right.}{32\left(\left(a^3 b^4 - 3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*b^4*f*x*tan(f*x + e)^4 + 64*a*b^3*f*x*tan(f*x + e)^2 + 32*a^2*b^
2*f*x + 4*(5*a^3*b - 14*a^2*b^2 + 9*a*b^3)*tan(f*x + e)^3 + ((3*a^2*b^2 - 1
0*a*b^3 + 15*b^4)*tan(f*x + e)^4 + 3*a^4 - 10*a^3*b + 15*a^2*b^2 + 2*(3*a^3
*b - 10*a^2*b^2 + 15*a*b^3)*tan(f*x + e)^2)*sqrt(-a/b)*log((b^2*tan(f*x + e
)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e)
)*sqrt(-a/b))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) + 4*(3*a^4
 - 10*a^3*b + 7*a^2*b^2)*tan(f*x + e))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^
7)*f*tan(f*x + e)^4 + 2*(a^4*b^3 - 3*a^3*b^4 + 3*a^2*b^5 - a*b^6)*f*tan(f*x
 + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f), -1/16*(16*b^4*f*x
*tan(f*x + e)^4 + 32*a*b^3*f*x*tan(f*x + e)^2 + 16*a^2*b^2*f*x + 2*(5*a^3*b
 - 14*a^2*b^2 + 9*a*b^3)*tan(f*x + e)^3 - ((3*a^2*b^2 - 10*a*b^3 + 15*b^4)*
tan(f*x + e)^4 + 3*a^4 - 10*a^3*b + 15*a^2*b^2 + 2*(3*a^3*b - 10*a^2*b^2 +
15*a*b^3)*tan(f*x + e)^2)*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(
a/b)/(a*tan(f*x + e))) + 2*(3*a^4 - 10*a^3*b + 7*a^2*b^2)*tan(f*x + e))/((a
^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*f*tan(f*x + e)^4 + 2*(a^4*b^3 - 3*a^3*b
^4 + 3*a^2*b^5 - a*b^6)*f*tan(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
 - a^2*b^5)*f)]
```

giac [A] time = 33.17, size = 215, normalized size = 1.41

$$\frac{\frac{\left(3\,a^3-10\,a^2 b+15\,a b^2\right)\left(\pi\left\lfloor\frac{f x+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(f x+e\right)}{\sqrt{a b}}\right)\right)}{\left(a^3 b^2-3\,a^2 b^3+3\,a b^4-b^5\right)\sqrt{a b}}-\frac{8\left(f x+e\right)}{a^3-3\,a^2 b+3\,a b^2-b^3}-\frac{5\,a^2 b\tan\left(f x+e\right)^3-9\,a b^2\tan\left(f x+e\right)^3+3\,a^3\tan\left(f x+e\right)-7\,a^2 b\,t}{\left(a^2 b^2-2\,a b^3+b^4\right)\left(b\tan\left(f x+e\right)^2+a\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^3 - 10*a^2*b + 15*a*b^2)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + a
rctan(b*tan(f*x + e)/sqrt(a*b)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sqr
t(a*b)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (5*a^2*b*tan(f*x +
e)^3 - 9*a*b^2*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) - 7*a^2*b*tan(f*x + e))/
((a^2*b^2 - 2*a*b^3 + b^4)*(b*tan(f*x + e)^2 + a)^2))/f
```

maple [B] time = 0.24, size = 351, normalized size = 2.29

$$-\frac{5a^3\left(\tan^3\left(f x+e\right)\right)}{8f\left(a-b\right)^3\left(a+b\left(\tan^2\left(f x+e\right)\right)\right)^2 b}+\frac{7a^2\left(\tan^3\left(f x+e\right)\right)}{4f\left(a-b\right)^3\left(a+b\left(\tan^2\left(f x+e\right)\right)\right)^2}-\frac{9ab\left(\tan^3\left(f x+e\right)\right)}{8f\left(a-b\right)^3\left(a+b\left(\tan^2\left(f x+e\right)\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -5/8/f*a^3/(a-b)^3/(a+b*tan(f*x+e)^2)^2/b*tan(f*x+e)^3+7/4/f*a^2/(a-b)^3/(a
+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-9/8/f*a/(a-b)^3/(a+b*tan(f*x+e)^2)^2*b*tan(
f*x+e)^3-3/8/f*a^4/(a-b)^3/(a+b*tan(f*x+e)^2)^2/b^2*tan(f*x+e)+5/4/f*a^3/(a
```

```
-b)^3/(a+b*tan(f*x+e)^2)^2/b*tan(f*x+e)-7/8/f*a^2/(a-b)^3/(a+b*tan(f*x+e)^2
)^2*tan(f*x+e)+3/8/f*a^3/(a-b)^3/b^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^
(1/2))-5/4/f*a^2/(a-b)^3/b/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+15/
8/f*a/(a-b)^3/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/f/(a-b)^3*arct
an(tan(f*x+e))
```

maxima [A] time = 0.84, size = 229, normalized size = 1.50

$$\frac{\frac{\left(3\,a^{3}-10\,a^{2}b+15\,ab^{2}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^{3}b^{2}-3\,a^{2}b^{3}+3\,ab^{4}-b^{5}\right)\sqrt{ab}}-\frac{\left(5\,a^{2}b-9\,ab^{2}\right)\tan\left(fx+e\right)^{3}+\left(3\,a^{3}-7\,a^{2}b\right)\tan\left(fx+e\right)}{a^{4}b^{2}-2\,a^{3}b^{3}+a^{2}b^{4}+\left(a^{2}b^{4}-2\,ab^{5}+b^{6}\right)\tan\left(fx+e\right)^{4}+2\left(a^{3}b^{3}-2\,a^{2}b^{4}+ab^{5}\right)\tan\left(fx+e\right)^{2}}-\frac{8\left(fx+e\right)}{a^{3}-3\,a^{2}b+3\,a}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^3 - 10*a^2*b + 15*a*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3*b
^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sqrt(a*b)) - ((5*a^2*b - 9*a*b^2)*tan(f*x +
 e)^3 + (3*a^3 - 7*a^2*b)*tan(f*x + e))/(a^4*b^2 - 2*a^3*b^3 + a^2*b^4 + (a
^2*b^4 - 2*a*b^5 + b^6)*tan(f*x + e)^4 + 2*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*ta
n(f*x + e)^2) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

mupad [B] time = 15.52, size = 3838, normalized size = 25.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^3*(9*a*b - 5*a^2))/(8*(a^2*b - 2*a*b^2 + b^3)) + (a*tan(e +
f*x)*(7*a*b - 3*a^2))/(8*b*(a^2*b - 2*a*b^2 + b^3)))/(f*(a^2 + b^2*tan(e +
f*x)^4 + 2*a*b*tan(e + f*x)^2)) - (2*atan((((((224*a*b^10 - 1440*a^2*b^9 +
3936*a^3*b^8 - 5920*a^4*b^7 + 5280*a^5*b^6 - 2784*a^6*b^5 + 800*a^7*b^4 - 9
6*a^8*b^3)/(64*(b^9 - 6*a*b^8 + 15*a^2*b^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^
5*b^4 + a^6*b^3)) - (tan(e + f*x)*(1280*a*b^11 - 256*b^12 - 2304*a^2*b^10 +
 1280*a^3*b^9 + 1280*a^4*b^8 - 2304*a^5*b^7 + 1280*a^6*b^6 - 256*a^7*b^5)*1
i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a
^3*b^4 + a^4*b^3)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (tan(e + f*x)
*(9*a^6 - 60*a^5*b + 64*b^6 + 225*a^2*b^4 - 300*a^3*b^3 + 190*a^4*b^2))/(32
*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))/(6*a*b^2 - 6*a^2*b + 2
*a^3 - 2*b^3) - ((((224*a*b^10 - 1440*a^2*b^9 + 3936*a^3*b^8 - 5920*a^4*b^7
 + 5280*a^5*b^6 - 2784*a^6*b^5 + 800*a^7*b^4 - 96*a^8*b^3)/(64*(b^9 - 6*a*b
^8 + 15*a^2*b^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^5*b^4 + a^6*b^3)) + (tan(e
+ f*x)*(1280*a*b^11 - 256*b^12 - 2304*a^2*b^10 + 1280*a^3*b^9 + 1280*a^4*b^
8 - 2304*a^5*b^7 + 1280*a^6*b^6 - 256*a^7*b^5)*1i)/(32*(6*a*b^2 - 6*a^2*b +
 2*a^3 - 2*b^3)*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))*1i)/(6*
a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(9*a^6 - 60*a^5*b + 64*b^6
 + 225*a^2*b^4 - 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7 - 4*a*b^6 + 6*a^2*b^5
 - 4*a^3*b^4 + a^4*b^3)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))/((120*a*b^4
- 51*a^4*b + 9*a^5 - 185*a^2*b^3 + 139*a^3*b^2)/(32*(b^9 - 6*a*b^8 + 15*a^2
*b^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^5*b^4 + a^6*b^3)) + (((((224*a*b^10 -
1440*a^2*b^9 + 3936*a^3*b^8 - 5920*a^4*b^7 + 5280*a^5*b^6 - 2784*a^6*b^5 +
800*a^7*b^4 - 96*a^8*b^3)/(64*(b^9 - 6*a*b^8 + 15*a^2*b^7 - 20*a^3*b^6 + 15
*a^4*b^5 - 6*a^5*b^4 + a^6*b^3)) - (tan(e + f*x)*(1280*a*b^11 - 256*b^12 -
2304*a^2*b^10 + 1280*a^3*b^9 + 1280*a^4*b^8 - 2304*a^5*b^7 + 1280*a^6*b^6 -
 256*a^7*b^5)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(b^7 - 4*a*b^6 +
6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)
+ (tan(e + f*x)*(9*a^6 - 60*a^5*b + 64*b^6 + 225*a^2*b^4 - 300*a^3*b^3 + 19
0*a^4*b^2))/(32*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))*1i)/(6*
```

```
a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (((((224*a*b^10 - 1440*a^2*b^9 + 3936*a^
3*b^8 - 5920*a^4*b^7 + 5280*a^5*b^6 - 2784*a^6*b^5 + 800*a^7*b^4 - 96*a^8*b
^3)/(64*(b^9 - 6*a*b^8 + 15*a^2*b^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^5*b^4 +
 a^6*b^3)) + (tan(e + f*x)*(1280*a*b^11 - 256*b^12 - 2304*a^2*b^10 + 1280*a
^3*b^9 + 1280*a^4*b^8 - 2304*a^5*b^7 + 1280*a^6*b^6 - 256*a^7*b^5)*1i)/(32*
(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4
+ a^4*b^3)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(9*a^6
 - 60*a^5*b + 64*b^6 + 225*a^2*b^4 - 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7 -
 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^
3 - 2*b^3))))/(f*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)) - (atan((((-a*b^5)^(1
/2)*((tan(e + f*x)*(9*a^6 - 60*a^5*b + 64*b^6 + 225*a^2*b^4 - 300*a^3*b^3 +
 190*a^4*b^2))/(32*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)) + ((-
a*b^5)^(1/2)*((224*a*b^10 - 1440*a^2*b^9 + 3936*a^3*b^8 - 5920*a^4*b^7 + 52
80*a^5*b^6 - 2784*a^6*b^5 + 800*a^7*b^4 - 96*a^8*b^3)/(64*(b^9 - 6*a*b^8 +
15*a^2*b^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^5*b^4 + a^6*b^3)) - (tan(e + f*x
)*(-a*b^5)^(1/2)*(3*a^2 - 10*a*b + 15*b^2)*(1280*a*b^11 - 256*b^12 - 2304*a
^2*b^10 + 1280*a^3*b^9 + 1280*a^4*b^8 - 2304*a^5*b^7 + 1280*a^6*b^6 - 256*a
^7*b^5))/(512*(3*a*b^7 - b^8 - 3*a^2*b^6 + a^3*b^5)*(b^7 - 4*a*b^6 + 6*a^2*
b^5 - 4*a^3*b^4 + a^4*b^3)))*(3*a^2 - 10*a*b + 15*b^2))/(16*(3*a*b^7 - b^8
- 3*a^2*b^6 + a^3*b^5)))*(3*a^2 - 10*a*b + 15*b^2)*1i)/(16*(3*a*b^7 - b^8 -
 3*a^2*b^6 + a^3*b^5)) + ((-a*b^5)^(1/2)*((tan(e + f*x)*(9*a^6 - 60*a^5*b +
 64*b^6 + 225*a^2*b^4 - 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7 - 4*a*b^6 + 6*
a^2*b^5 - 4*a^3*b^4 + a^4*b^3)) - ((-a*b^5)^(1/2)*((224*a*b^10 - 1440*a^2*b
^9 + 3936*a^3*b^8 - 5920*a^4*b^7 + 5280*a^5*b^6 - 2784*a^6*b^5 + 800*a^7*b^
4 - 96*a^8*b^3)/(64*(b^9 - 6*a*b^8 + 15*a^2*b^7 - 20*a^3*b^6 + 15*a^4*b^5 -
 6*a^5*b^4 + a^6*b^3)) + (tan(e + f*x)*(-a*b^5)^(1/2)*(3*a^2 - 10*a*b + 15*
b^2)*(1280*a*b^11 - 256*b^12 - 2304*a^2*b^10 + 1280*a^3*b^9 + 1280*a^4*b^8
- 2304*a^5*b^7 + 1280*a^6*b^6 - 256*a^7*b^5))/(512*(3*a*b^7 - b^8 - 3*a^2*b
^6 + a^3*b^5)*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))*(3*a^2 -
10*a*b + 15*b^2))/(16*(3*a*b^7 - b^8 - 3*a^2*b^6 + a^3*b^5)))*(3*a^2 - 10*a
*b + 15*b^2)*1i)/(16*(3*a*b^7 - b^8 - 3*a^2*b^6 + a^3*b^5)))/((120*a*b^4 -
51*a^4*b + 9*a^5 - 185*a^2*b^3 + 139*a^3*b^2)/(32*(b^9 - 6*a*b^8 + 15*a^2*b
^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^5*b^4 + a^6*b^3)) + ((-a*b^5)^(1/2)*((ta
n(e + f*x)*(9*a^6 - 60*a^5*b + 64*b^6 + 225*a^2*b^4 - 300*a^3*b^3 + 190*a^4
*b^2))/(32*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)) + ((-a*b^5)^(
1/2)*((224*a*b^10 - 1440*a^2*b^9 + 3936*a^3*b^8 - 5920*a^4*b^7 + 5280*a^5*b
^6 - 2784*a^6*b^5 + 800*a^7*b^4 - 96*a^8*b^3)/(64*(b^9 - 6*a*b^8 + 15*a^2*b
^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^5*b^4 + a^6*b^3)) - (tan(e + f*x)*(-a*b^
5)^(1/2)*(3*a^2 - 10*a*b + 15*b^2)*(1280*a*b^11 - 256*b^12 - 2304*a^2*b^10
+ 1280*a^3*b^9 + 1280*a^4*b^8 - 2304*a^5*b^7 + 1280*a^6*b^6 - 256*a^7*b^5))
/(512*(3*a*b^7 - b^8 - 3*a^2*b^6 + a^3*b^5)*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*
a^3*b^4 + a^4*b^3)))*(3*a^2 - 10*a*b + 15*b^2))/(16*(3*a*b^7 - b^8 - 3*a^2*
b^6 + a^3*b^5)))*(3*a^2 - 10*a*b + 15*b^2))/(16*(3*a*b^7 - b^8 - 3*a^2*b^6
+ a^3*b^5)) - ((-a*b^5)^(1/2)*((tan(e + f*x)*(9*a^6 - 60*a^5*b + 64*b^6 + 2
25*a^2*b^4 - 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4
*a^3*b^4 + a^4*b^3)) - ((-a*b^5)^(1/2)*((224*a*b^10 - 1440*a^2*b^9 + 3936*a
^3*b^8 - 5920*a^4*b^7 + 5280*a^5*b^6 - 2784*a^6*b^5 + 800*a^7*b^4 - 96*a^8*
b^3)/(64*(b^9 - 6*a*b^8 + 15*a^2*b^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^5*b^4
+ a^6*b^3)) + (tan(e + f*x)*(-a*b^5)^(1/2)*(3*a^2 - 10*a*b + 15*b^2)*(1280*
a*b^11 - 256*b^12 - 2304*a^2*b^10 + 1280*a^3*b^9 + 1280*a^4*b^8 - 2304*a^5*
b^7 + 1280*a^6*b^6 - 256*a^7*b^5))/(512*(3*a*b^7 - b^8 - 3*a^2*b^6 + a^3*b^
5)*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))*(3*a^2 - 10*a*b + 15
*b^2))/(16*(3*a*b^7 - b^8 - 3*a^2*b^6 + a^3*b^5)))*(3*a^2 - 10*a*b + 15*b^2
))/(16*(3*a*b^7 - b^8 - 3*a^2*b^6 + a^3*b^5))))*(-a*b^5)^(1/2)*(3*a^2 - 10*
a*b + 15*b^2)*1i)/(8*f*(3*a*b^7 - b^8 - 3*a^2*b^6 + a^3*b^5))
```

sympy [**A**] time = 141.17, size = 9823, normalized size = 64.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/b**3, Eq(a, 0)), (15*
f*x*tan(e + f*x)**6/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4
 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 45*f*x*tan(e + f*x)**4/(48*b**
3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)*
*2 + 48*b**3*f) + 45*f*x*tan(e + f*x)**2/(48*b**3*f*tan(e + f*x)**6 + 144*b
**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 15*f*x/(4
8*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e +
f*x)**2 + 48*b**3*f) - 33*tan(e + f*x)**5/(48*b**3*f*tan(e + f*x)**6 + 144*
b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) - 40*tan(e
 + f*x)**3/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b*
*3*f*tan(e + f*x)**2 + 48*b**3*f) - 15*tan(e + f*x)/(48*b**3*f*tan(e + f*x)
**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f),
 Eq(a, b)), (x*tan(e)**6/(a + b*tan(e)**2)**3, Eq(f, 0)), ((-x + tan(e + f*
x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e + f*x)/f)/a**3, Eq(b, 0)), (-6*
I*a**(9/2)*b*sqrt(1/b)*tan(e + f*x)/(16*I*a**(11/2)*b**3*f*sqrt(1/b) + 32*I
*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*sqrt(1/b)
 + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**5*f*sq
rt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**(5/2)*b*
*6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x
)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(
e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)
*b**8*f*sqrt(1/b)*tan(e + f*x)**4) - 10*I*a**(7/2)*b**2*sqrt(1/b)*tan(e + f
*x)**3/(16*I*a**(11/2)*b**3*f*sqrt(1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*ta
n(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(
1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*
I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**
4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**6*f*s
qrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b
**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x
)**4) + 20*I*a**(7/2)*b**2*sqrt(1/b)*tan(e + f*x)/(16*I*a**(11/2)*b**3*f*sq
rt(1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b*
*4*f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(
7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48
*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/
b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*
sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2
 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) - 16*I*a**(5/2)*b**3*f*x*s
qrt(1/b)/(16*I*a**(11/2)*b**3*f*sqrt(1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*
tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqr
t(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4
8*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)
**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**6*f
*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)
*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f
*x)**4) + 28*I*a**(5/2)*b**3*sqrt(1/b)*tan(e + f*x)**3/(16*I*a**(11/2)*b**3
*f*sqrt(1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/
2)*b**4*f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I
*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b)
 - 48*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sq
rt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b*
*7*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x
)**2 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) - 14*I*a**(5/2)*b**3*
sqrt(1/b)*tan(e + f*x)/(16*I*a**(11/2)*b**3*f*sqrt(1/b) + 32*I*a**(9/2)*b**
4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*sqrt(1/b) + 16*I*a**(7
/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e
 + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**(5/2)*b**6*f*sqrt(1/b
)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a
**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**4 -
```

```
 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**8*f*sqrt(
1/b)*tan(e + f*x)**4) - 32*I*a**(3/2)*b**4*f*x*sqrt(1/b)*tan(e + f*x)**2/(1
6*I*a**(11/2)*b**3*f*sqrt(1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x
)**2 - 48*I*a**(9/2)*b**4*f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(
e + f*x)**4 - 96*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2
)*b**5*f*sqrt(1/b) - 48*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*
a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b)
+ 48*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqr
t(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) - 1
8*I*a**(3/2)*b**4*sqrt(1/b)*tan(e + f*x)**3/(16*I*a**(11/2)*b**3*f*sqrt(1/b
) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*s
qrt(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b
**5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**
(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan
(e + f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1
/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I
*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) - 16*I*sqrt(a)*b**5*f*x*sqrt(1/b
)*tan(e + f*x)**4/(16*I*a**(11/2)*b**3*f*sqrt(1/b) + 32*I*a**(9/2)*b**4*f*s
qrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*sqrt(1/b) + 16*I*a**(7/2)*b
**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*
x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**(5/2)*b**6*f*sqrt(1/b)*tan
(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/
2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I
*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*
tan(e + f*x)**4) + 3*a**5*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a*
*(11/2)*b**3*f*sqrt(1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 -
 48*I*a**(9/2)*b**4*f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*
x)**4 - 96*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5
*f*sqrt(1/b) - 48*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/
2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I
*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)
*tan(e + f*x)**2 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) - 3*a**5*
log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(11/2)*b**3*f*sqrt(1/b) +
32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*sqrt(
1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**5*
f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**(5/2
)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e +
 f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1/b)*
tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqr
t(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) + 6*a**4*b*log(-I*sqrt(a)*sqrt(1/b)
+ tan(e + f*x))*tan(e + f*x)**2/(16*I*a**(11/2)*b**3*f*sqrt(1/b) + 32*I*a**
(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*sqrt(1/b) + 1
6*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**5*f*sqrt(1
/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**(5/2)*b**6*f
*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2
 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e +
f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**
8*f*sqrt(1/b)*tan(e + f*x)**4) - 10*a**4*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e
 + f*x))/(16*I*a**(11/2)*b**3*f*sqrt(1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*
tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqr
t(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 4
8*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)
**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**6*f
*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)
*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f
*x)**4) - 6*a**4*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/
(16*I*a**(11/2)*b**3*f*sqrt(1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f
*x)**2 - 48*I*a**(9/2)*b**4*f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*ta
n(e + f*x)**4 - 96*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7
```

```
/2)*b**5*f*sqrt(1/b) - 48*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*
I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b
) + 48*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*s
qrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) +
 10*a**4*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(11/2)*b**3*f*s
qrt(1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b
**4*f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**
(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 4
8*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1
/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f
*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2
 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) + 3*a**3*b**2*log(-I*sqrt
(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(11/2)*b**3*f*sqrt(1
/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f
*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)
*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a
**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*t
an(e + f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt
(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16
*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) - 20*a**3*b**2*log(-I*sqrt(a)*
sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16*I*a**(11/2)*b**3*f*sqrt(1/b)
+ 32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*sqr
t(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**
5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**(5
/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e
 + f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1/b
)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*s
qrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) + 15*a**3*b**2*log(-I*sqrt(a)*sqrt
(1/b) + tan(e + f*x))/(16*I*a**(11/2)*b**3*f*sqrt(1/b) + 32*I*a**(9/2)*b**4
*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*sqrt(1/b) + 16*I*a**(7/
2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e
+ f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**(5/2)*b**6*f*sqrt(1/b)
*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a*
*(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**4 -
32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**8*f*sqrt(1
/b)*tan(e + f*x)**4) - 3*a**3*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*
tan(e + f*x)**4/(16*I*a**(11/2)*b**3*f*sqrt(1/b) + 32*I*a**(9/2)*b**4*f*sqr
t(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*sqrt(1/b) + 16*I*a**(7/2)*b**
5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)
**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e
 + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)
*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a
**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*ta
n(e + f*x)**4) + 20*a**3*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e
 + f*x)**2/(16*I*a**(11/2)*b**3*f*sqrt(1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b
)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*s
qrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 +
 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*
x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**6
*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/
2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e +
 f*x)**4) - 15*a**3*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(
11/2)*b**3*f*sqrt(1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 4
8*I*a**(9/2)*b**4*f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)
**4 - 96*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f
*sqrt(1/b) - 48*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)
*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a
**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*t
an(e + f*x)**2 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) - 10*a**2*b
```

```
**3*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(11/2
)*b**3*f*sqrt(1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*
a**(9/2)*b**4*f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4
 - 96*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqr
t(1/b) - 48*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**
6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3
/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e
 + f*x)**2 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) + 30*a**2*b**3*
log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16*I*a**(11/2)*b*
*3*f*sqrt(1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(
9/2)*b**4*f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96
*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/
b) - 48*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*
sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*
b**7*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f
*x)**2 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) + 10*a**2*b**3*log(
I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(11/2)*b**3*f*
sqrt(1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*
b**4*f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a*
*(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) -
48*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(
1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*
f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**
2 - 16*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) - 30*a**2*b**3*log(I*sqr
t(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16*I*a**(11/2)*b**3*f*sqrt(
1/b) + 32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*
f*sqrt(1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2
)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*
a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*
tan(e + f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqr
t(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 1
6*I*sqrt(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) + 15*a*b**4*log(-I*sqrt(a)*sq
rt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(11/2)*b**3*f*sqrt(1/b) +
32*I*a**(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*sqrt(
1/b) + 16*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**5*
f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**(5/2
)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e +
 f*x)**2 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1/b)*
tan(e + f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqr
t(a)*b**8*f*sqrt(1/b)*tan(e + f*x)**4) - 15*a*b**4*log(I*sqrt(a)*sqrt(1/b)
+ tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(11/2)*b**3*f*sqrt(1/b) + 32*I*a**
(9/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**4*f*sqrt(1/b) + 1
6*I*a**(7/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**5*f*sqrt(1
/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**5*f*sqrt(1/b) - 48*I*a**(5/2)*b**6*f
*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2
 - 16*I*a**(5/2)*b**6*f*sqrt(1/b) + 48*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e +
f*x)**4 - 32*I*a**(3/2)*b**7*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**
8*f*sqrt(1/b)*tan(e + f*x)**4), True))
```

3.244 $\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=145

$$\frac{\left(a^2-6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{3/2}f(a-b)^3}+\frac{(a-5b)\tan(e+fx)}{8bf(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{a\tan(e+fx)}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}+\frac{}{(a-}$$

```
[Out] x/(a-b)^3+1/8*(a^2-6*a*b-3*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)^3/
b^(3/2)/f/a^(1/2)-1/4*a*tan(f*x+e)/(a-b)/b/f/(a+b*tan(f*x+e)^2)^2+1/8*(a-5*
b)*tan(f*x+e)/(a-b)^2/b/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.18, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 470, 527, 522, 203, 205}

$$\frac{\left(a^2-6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{3/2}f(a-b)^3}+\frac{(a-5b)\tan(e+fx)}{8bf(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{a\tan(e+fx)}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}+\frac{}{(a-}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] x/(a - b)^3 + ((a^2 - 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]]
)/(8*Sqrt[a]*(a - b)^3*b^(3/2)*f) - (a*Tan[e + f*x])/(4*(a - b)*b*f*(a + b*
Tan[e + f*x]^2)^2) + ((a - 5*b)*Tan[e + f*x])/(8*(a - b)^2*b*f*(a + b*Tan[e
 + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+(a-4b)x^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4(a-b)bf}$$

$$= -\frac{a\tan(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} + \frac{(a-5b)\tan(e+fx)}{8(a-b)^2bf\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a\tan(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} + \frac{(a-5b)\tan(e+fx)}{8(a-b)^2bf\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{x}{(a-b)^3} + \frac{\left(a^2-6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}(a-b)^3b^{3/2}f} - \frac{a\tan(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2}$$

Mathematica [A] time = 2.09, size = 136, normalized size = 0.94

$$\frac{-\frac{(a-b)\sin(2(e+fx))\left(\left(a^2+4ab-5b^2\right)\cos(2(e+fx))+a^2+2ab+5b^2\right)}{b((a-b)\cos(2(e+fx))+a+b)^2} + \frac{\left(a^2-6ab-3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}} + 8(e+fx)}{8f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (8*(e + f*x) + ((a^2 - 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(Sqrt[a]*b^(3/2)) - ((a - b)*(a^2 + 2*a*b + 5*b^2 + (a^2 + 4*a*b - 5*b^2
)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(b*(a + b + (a - b)*Cos[2*(e + f*x)])
^2))/(8*(a - b)^3*f)
```

fricas [B] time = 0.50, size = 749, normalized size = 5.17

$$\left[\frac{32\,ab^4fx\tan\left(fx+e\right)^4+64\,a^2b^3fx\tan\left(fx+e\right)^2+32\,a^3b^2fx+4\left(a^3b^2-6\,a^2b^3+5\,ab^4\right)\tan\left(fx+e\right)^3-\left(\left(a\right.\right.}{32\left(\left(a^4b^4-3\,a^3b^5+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*a*b^4*f*x*tan(f*x + e)^4 + 64*a^2*b^3*f*x*tan(f*x + e)^2 + 32*a^3
*b^2*f*x + 4*(a^3*b^2 - 6*a^2*b^3 + 5*a*b^4)*tan(f*x + e)^3 - ((a^2*b^2 - 6
*a*b^3 - 3*b^4)*tan(f*x + e)^4 + a^4 - 6*a^3*b - 3*a^2*b^2 + 2*(a^3*b - 6*a
^2*b^2 - 3*a*b^3)*tan(f*x + e)^2)*sqrt(-a*b)*log((b^2*tan(f*x + e)^4 - 6*a*
b*tan(f*x + e)^2 + a^2 - 4*(b*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(-a*b))/
(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a^4*b + 2*a^3*b^2 -
 3*a^2*b^3)*tan(f*x + e))/((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(
f*x + e)^4 + 2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f*tan(f*x + e)^2
 + (a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f), 1/16*(16*a*b^4*f*x*tan(f
*x + e)^4 + 32*a^2*b^3*f*x*tan(f*x + e)^2 + 16*a^3*b^2*f*x + 2*(a^3*b^2 - 6
*a^2*b^3 + 5*a*b^4)*tan(f*x + e)^3 + ((a^2*b^2 - 6*a*b^3 - 3*b^4)*tan(f*x +
 e)^4 + a^4 - 6*a^3*b - 3*a^2*b^2 + 2*(a^3*b - 6*a^2*b^2 - 3*a*b^3)*tan(f*x
 + e)^2)*sqrt(a*b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a*b)/(a*b*tan(f*x
 + e))) - 2*(a^4*b + 2*a^3*b^2 - 3*a^2*b^3)*tan(f*x + e))/((a^4*b^4 - 3*a^3
*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^4 + 2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3
*b^5 - a^2*b^6)*f*tan(f*x + e)^2 + (a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b
^5)*f)]
```

giac [A] time = 4.02, size = 199, normalized size = 1.37

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)\left(a^2-6\,ab-3\,b^2\right)}{\left(a^3b-3\,a^2b^2+3\,ab^3-b^4\right)\sqrt{ab}}+\frac{8\left(fx+e\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{ab\tan\left(fx+e\right)^3-5\,b^2\tan\left(fx+e\right)^3-a^2\tan\left(fx+e\right)-3\,ab\tan\left(fx+e\right.}{\left(a^2b-2\,ab^2+b^3\right)\left(b\tan\left(fx+e\right)^2+a\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)
))*(a^2 - 6*a*b - 3*b^2)/((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*sqrt(a*b)) +
8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (a*b*tan(f*x + e)^3 - 5*b^2*t
an(f*x + e)^3 - a^2*tan(f*x + e) - 3*a*b*tan(f*x + e))/((a^2*b - 2*a*b^2 +
b^3)*(b*tan(f*x + e)^2 + a)^2))/f
```

maple [B] time = 0.21, size = 338, normalized size = 2.33

$$\frac{a^2\left(\tan^3\left(fx+e\right)\right)}{8f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{3ab\left(\tan^3\left(fx+e\right)\right)}{4f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{5\left(\tan^3\left(fx+e\right)\right)b^2}{8f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] 1/8/f*a^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-3/4/f*a/(a-b)^3/(a+b*ta
n(f*x+e)^2)^2*b*tan(f*x+e)^3+5/8/f/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^
3*b^2-1/8/f*a^3/(a-b)^3/(a+b*tan(f*x+e)^2)^2/b*tan(f*x+e)-1/4/f*a^2/(a-b)^3
```

```
/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)+3/8/f/(a-b)^3/(a+b*tan(f*x+e)^2)^2*a*b*tan
(f*x+e)+1/8/f*a^2/(a-b)^3/b/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-3/
4/f*a/(a-b)^3/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-3/8/f/(a-b)^3*b/
(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+1/f/(a-b)^3*arctan(tan(f*x+e))
```

maxima [A] time = 0.88, size = 212, normalized size = 1.46

$$\frac{\frac{\left(a^{2}-6\,ab-3\,b^{2}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^{3}b-3\,a^{2}b^{2}+3\,ab^{3}-b^{4}\right)\sqrt{ab}}+\frac{\left(ab-5\,b^{2}\right)\tan\left(fx+e\right)^{3}-\left(a^{2}+3\,ab\right)\tan\left(fx+e\right)}{a^{4}b-2\,a^{3}b^{2}+a^{2}b^{3}+\left(a^{2}b^{3}-2\,ab^{4}+b^{5}\right)\tan\left(fx+e\right)^{4}+2\left(a^{3}b^{2}-2\,a^{2}b^{3}+ab^{4}\right)\tan\left(fx+e\right)^{2}}+\frac{8\left(fx+e\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((a^2 - 6*a*b - 3*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3*b - 3*a^2
*b^2 + 3*a*b^3 - b^4)*sqrt(a*b)) + ((a*b - 5*b^2)*tan(f*x + e)^3 - (a^2 + 3
*a*b)*tan(f*x + e))/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^2*b^3 - 2*a*b^4 + b^5
)*tan(f*x + e)^4 + 2*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*tan(f*x + e)^2) + 8*(f*x
 + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

mupad [B] time = 15.49, size = 3667, normalized size = 25.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^3*(a - 5*b))/(8*(a^2 - 2*a*b + b^2)) - (a*tan(e + f*x)*(a +
3*b))/(8*(a^2*b - 2*a*b^2 + b^3)))/(f*(a^2 + b^2*tan(e + f*x)^4 + 2*a*b*tan
(e + f*x)^2)) - (2*atan((((((544*a*b^8 - 96*b^9 - 1248*a^2*b^7 + 1440*a^3*b
^6 - 800*a^4*b^5 + 96*a^5*b^4 + 96*a^6*b^3 - 32*a^7*b^2)/(64*(a^6*b - 6*a*b
^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2)) - (tan(e + f*
x)*(1280*a*b^9 - 256*b^10 - 2304*a^2*b^8 + 1280*a^3*b^7 + 1280*a^4*b^6 - 23
04*a^5*b^5 + 1280*a^6*b^4 - 256*a^7*b^3)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3
 - 2*b^3)*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*1i)/(6*a*b^2 -
6*a^2*b + 2*a^3 - 2*b^3) + (tan(e + f*x)*(36*a*b^3 - 12*a^3*b + a^4 + 73*b^
4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))/(6*a
*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - ((((544*a*b^8 - 96*b^9 - 1248*a^2*b^7 + 1
440*a^3*b^6 - 800*a^4*b^5 + 96*a^5*b^4 + 96*a^6*b^3 - 32*a^7*b^2)/(64*(a^6*
b - 6*a*b^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2)) + (t
an(e + f*x)*(1280*a*b^9 - 256*b^10 - 2304*a^2*b^8 + 1280*a^3*b^7 + 1280*a^4
*b^6 - 2304*a^5*b^5 + 1280*a^6*b^4 - 256*a^7*b^3)*1i)/(32*(6*a*b^2 - 6*a^2*
b + 2*a^3 - 2*b^3)*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*1i)/(6
*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(36*a*b^3 - 12*a^3*b + a^
4 + 73*b^4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^
2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))/((((((544*a*b^8 - 96*b^9 - 1248*a
^2*b^7 + 1440*a^3*b^6 - 800*a^4*b^5 + 96*a^5*b^4 + 96*a^6*b^3 - 32*a^7*b^2)
/(64*(a^6*b - 6*a*b^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*
b^2)) - (tan(e + f*x)*(1280*a*b^9 - 256*b^10 - 2304*a^2*b^8 + 1280*a^3*b^7
+ 1280*a^4*b^6 - 2304*a^5*b^5 + 1280*a^6*b^4 - 256*a^7*b^3)*1i)/(32*(6*a*b^
2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2
)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (tan(e + f*x)*(36*a*b^3 - 12*
a^3*b + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3
- 4*a^3*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (27*a*b^2 - 11*a^2
*b + a^3 + 15*b^3)/(32*(a^6*b - 6*a*b^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 1
5*a^4*b^3 - 6*a^5*b^2)) + (((((544*a*b^8 - 96*b^9 - 1248*a^2*b^7 + 1440*a^3
*b^6 - 800*a^4*b^5 + 96*a^5*b^4 + 96*a^6*b^3 - 32*a^7*b^2)/(64*(a^6*b - 6*a
*b^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2)) + (tan(e +
```

```
f*x)*(1280*a*b^9 - 256*b^10 - 2304*a^2*b^8 + 1280*a^3*b^7 + 1280*a^4*b^6 -
2304*a^5*b^5 + 1280*a^6*b^4 - 256*a^7*b^3)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a
^3 - 2*b^3)*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*1i)/(6*a*b^2
- 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(36*a*b^3 - 12*a^3*b + a^4 + 73*
b^4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*1i
)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))))/(f*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*
b^3)) + (atan((((-a*b^3)^(1/2)*((tan(e + f*x)*(36*a*b^3 - 12*a^3*b + a^4 +
73*b^4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2))
+ (((544*a*b^8 - 96*b^9 - 1248*a^2*b^7 + 1440*a^3*b^6 - 800*a^4*b^5 + 96*a^
5*b^4 + 96*a^6*b^3 - 32*a^7*b^2)/(64*(a^6*b - 6*a*b^6 + b^7 + 15*a^2*b^5 -
20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2)) - (tan(e + f*x)*(-a*b^3)^(1/2)*(6*a*b
 - a^2 + 3*b^2)*(1280*a*b^9 - 256*b^10 - 2304*a^2*b^8 + 1280*a^3*b^7 + 1280
*a^4*b^6 - 2304*a^5*b^5 + 1280*a^6*b^4 - 256*a^7*b^3))/(512*(a*b^6 - 3*a^2*
b^5 + 3*a^3*b^4 - a^4*b^3)*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2))
)*(-a*b^3)^(1/2)*(6*a*b - a^2 + 3*b^2))/(16*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4
- a^4*b^3)))*(6*a*b - a^2 + 3*b^2)*1i)/(16*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4 -
 a^4*b^3)) + ((-a*b^3)^(1/2)*((tan(e + f*x)*(36*a*b^3 - 12*a^3*b + a^4 + 73
*b^4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)) -
(((544*a*b^8 - 96*b^9 - 1248*a^2*b^7 + 1440*a^3*b^6 - 800*a^4*b^5 + 96*a^5*
b^4 + 96*a^6*b^3 - 32*a^7*b^2)/(64*(a^6*b - 6*a*b^6 + b^7 + 15*a^2*b^5 - 20
*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2)) + (tan(e + f*x)*(-a*b^3)^(1/2)*(6*a*b -
 a^2 + 3*b^2)*(1280*a*b^9 - 256*b^10 - 2304*a^2*b^8 + 1280*a^3*b^7 + 1280*a
^4*b^6 - 2304*a^5*b^5 + 1280*a^6*b^4 - 256*a^7*b^3))/(512*(a*b^6 - 3*a^2*b^
5 + 3*a^3*b^4 - a^4*b^3)*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*
(-a*b^3)^(1/2)*(6*a*b - a^2 + 3*b^2))/(16*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4 -
a^4*b^3)))*(6*a*b - a^2 + 3*b^2)*1i)/(16*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4 - a
^4*b^3)))/((27*a*b^2 - 11*a^2*b + a^3 + 15*b^3)/(32*(a^6*b - 6*a*b^6 + b^7
+ 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2)) - ((-a*b^3)^(1/2)*((ta
n(e + f*x)*(36*a*b^3 - 12*a^3*b + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(a^4*b -
4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)) + (((544*a*b^8 - 96*b^9 - 1248*a^2*
b^7 + 1440*a^3*b^6 - 800*a^4*b^5 + 96*a^5*b^4 + 96*a^6*b^3 - 32*a^7*b^2)/(6
4*(a^6*b - 6*a*b^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2
)) - (tan(e + f*x)*(-a*b^3)^(1/2)*(6*a*b - a^2 + 3*b^2)*(1280*a*b^9 - 256*b
^10 - 2304*a^2*b^8 + 1280*a^3*b^7 + 1280*a^4*b^6 - 2304*a^5*b^5 + 1280*a^6*
b^4 - 256*a^7*b^3))/(512*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4 - a^4*b^3)*(a^4*b -
 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-a*b^3)^(1/2)*(6*a*b - a^2 + 3*b
^2))/(16*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4 - a^4*b^3)))*(6*a*b - a^2 + 3*b^2))
/(16*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4 - a^4*b^3)) + ((-a*b^3)^(1/2)*((tan(e +
 f*x)*(36*a*b^3 - 12*a^3*b + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b
^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)) - (((544*a*b^8 - 96*b^9 - 1248*a^2*b^7 +
 1440*a^3*b^6 - 800*a^4*b^5 + 96*a^5*b^4 + 96*a^6*b^3 - 32*a^7*b^2)/(64*(a^
6*b - 6*a*b^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2)) +
(tan(e + f*x)*(-a*b^3)^(1/2)*(6*a*b - a^2 + 3*b^2)*(1280*a*b^9 - 256*b^10 -
 2304*a^2*b^8 + 1280*a^3*b^7 + 1280*a^4*b^6 - 2304*a^5*b^5 + 1280*a^6*b^4 -
 256*a^7*b^3))/(512*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4 - a^4*b^3)*(a^4*b - 4*a*
b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-a*b^3)^(1/2)*(6*a*b - a^2 + 3*b^2))/
(16*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4 - a^4*b^3)))*(6*a*b - a^2 + 3*b^2))/(16*
(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4 - a^4*b^3))))*(-a*b^3)^(1/2)*(6*a*b - a^2 +
3*b^2)*1i)/(8*f*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4 - a^4*b^3))
```

sympy [A] time = 140.86, size = 9811, normalized size = 67.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x - 1/(f*ta
n(e + f*x)))/b**3, Eq(a, 0)), (3*f*x*tan(e + f*x)**6/(48*b**3*f*tan(e + f*x
)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f)
```

```
 + 9*f*x*tan(e + f*x)**4/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*
x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 9*f*x*tan(e + f*x)**2/(48
*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f
*x)**2 + 48*b**3*f) + 3*f*x/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e +
 f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 3*tan(e + f*x)**5/(48*
b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*
x)**2 + 48*b**3*f) - 8*tan(e + f*x)**3/(48*b**3*f*tan(e + f*x)**6 + 144*b**
3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) - 3*tan(e + f
*x)/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*ta
n(e + f*x)**2 + 48*b**3*f), Eq(a, b)), (x*tan(e)**4/(a + b*tan(e)**2)**3, E
q(f, 0)), ((x + tan(e + f*x)**3/(3*f) - tan(e + f*x)/f)/a**3, Eq(b, 0)), (-
2*I*a**(7/2)*b*sqrt(1/b)*tan(e + f*x)/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32
*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/
b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*
sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*
b**5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f
*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*ta
n(e + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(
a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4) + 16*I*a**(5/2)*b**2*f*x*sqrt(1/b)/(16
*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)
**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e
 + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)
*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a
**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) +
 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt
(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4) + 2*
I*a**(5/2)*b**2*sqrt(1/b)*tan(e + f*x)**3/(16*I*a**(11/2)*b**2*f*sqrt(1/b)
+ 32*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sqr
t(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**
4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5
/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e
 + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b
)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*s
qrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4) - 4*I*a**(5/2)*b**2*sqrt(1/b)*tan(
e + f*x)/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*
tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a**(7/2)*b**4*f*sqr
t(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 4
8*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)
**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f
*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)
*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f
*x)**4) + 32*I*a**(3/2)*b**3*f*x*sqrt(1/b)*tan(e + f*x)**2/(16*I*a**(11/2)*
b**2*f*sqrt(1/b) + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a*
*(9/2)*b**3*f*sqrt(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 -
96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*sqrt(
1/b) - 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**5*
f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**(3/2
)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e +
 f*x)**2 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4) - 12*I*a**(3/2)*b
**3*sqrt(1/b)*tan(e + f*x)**3/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9
/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*
I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b
)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*s
qrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 -
 16*I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*
x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*
f*sqrt(1/b)*tan(e + f*x)**4) + 6*I*a**(3/2)*b**3*sqrt(1/b)*tan(e + f*x)/(16
*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)
**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e
```

```
 + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)
*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a
**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) +
 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt
(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4) + 16
*I*sqrt(a)*b**4*f*x*sqrt(1/b)*tan(e + f*x)**4/(16*I*a**(11/2)*b**2*f*sqrt(1
/b) + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f
*sqrt(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)
*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a
**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*t
an(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt
(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16
*I*sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4) + 10*I*sqrt(a)*b**4*sqrt(1/b)*
tan(e + f*x)**3/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*b**3*f*sqr
t(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a**(7/2)*b**
4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)
**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e
 + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)
*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a
**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*ta
n(e + f*x)**4) + a**4*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(11
/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*
I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**
4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*s
qrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b
**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**
(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan
(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4) - a**4*log(I*
sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a
**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) +
 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt
(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5
*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)*
*2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e
+ f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b
**7*f*sqrt(1/b)*tan(e + f*x)**4) + 2*a**3*b*log(-I*sqrt(a)*sqrt(1/b) + tan(
e + f*x))*tan(e + f*x)**2/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*
b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a*
*(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*ta
n(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(
1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*
I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**
4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sq
rt(1/b)*tan(e + f*x)**4) - 6*a**3*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x)
)/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e +
 f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*
tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**
(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 + 9
6*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1
/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**6*f
*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4)
 - 2*a**3*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16*I*a
**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2
- 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f
*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**
4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5
/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) + 48*
I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b
)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4) + 6*a**3
```

```
*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(11/2)*b**2*f*sqrt(1/b)
 + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sq
rt(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b*
*4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(
5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(
e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/
b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*
sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4) + a**2*b**2*log(-I*sqrt(a)*sqrt(1
/b) + tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I
*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b)
 + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sq
rt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b*
*5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x
)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(
e + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)
*b**7*f*sqrt(1/b)*tan(e + f*x)**4) - 12*a**2*b**2*log(-I*sqrt(a)*sqrt(1/b)
+ tan(e + f*x))*tan(e + f*x)**2/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**
(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 1
6*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1
/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f
*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2
 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e +
f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**
7*f*sqrt(1/b)*tan(e + f*x)**4) - 3*a**2*b**2*log(-I*sqrt(a)*sqrt(1/b) + tan
(e + f*x))/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*b**3*f*sqrt(1/b
)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a**(7/2)*b**4*f*s
qrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 +
 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*
x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5
*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/
2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*tan(e +
 f*x)**4) - a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)*
*4/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e
+ f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)
*tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a*
*(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 +
96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(
1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**6*
f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4
) + 12*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(1
6*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x
)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(
e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2
)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*
a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)
+ 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqr
t(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4) + 3
*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(11/2)*b**2*f*s
qrt(1/b) + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b
**3*f*sqrt(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**
(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 4
8*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1
/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f
*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2
 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4) - 6*a*b**3*log(-I*sqrt(a)
*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(11/2)*b**2*f*sqrt(1/b)
 + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sq
rt(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b*
*4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(
```

```
5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(
e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/
b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*
sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4) - 6*a*b**3*log(-I*sqrt(a)*sqrt(1/
b) + tan(e + f*x))*tan(e + f*x)**2/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*
a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b)
+ 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqr
t(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**
5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)
**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e
 + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*
b**7*f*sqrt(1/b)*tan(e + f*x)**4) + 6*a*b**3*log(I*sqrt(a)*sqrt(1/b) + tan(
e + f*x))*tan(e + f*x)**4/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*
b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a*
*(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*ta
n(e + f*x)**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(
1/b)*tan(e + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*
I*a**(5/2)*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**
4 - 32*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sq
rt(1/b)*tan(e + f*x)**4) + 6*a*b**3*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))
*tan(e + f*x)**2/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*b**3*f*sq
rt(1/b)*tan(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a**(7/2)*b*
*4*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x
)**2 + 48*I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(
e + f*x)**4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2
)*b**5*f*sqrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*
a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*t
an(e + f*x)**4) - 3*b**4*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f
*x)**4/(16*I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*ta
n(e + f*x)**2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(
1/b)*tan(e + f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*
I*a**(7/2)*b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**
4 + 96*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*s
qrt(1/b) + 48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b
**6*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x
)**4) + 3*b**4*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(16*
I*a**(11/2)*b**2*f*sqrt(1/b) + 32*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)*
*2 - 48*I*a**(9/2)*b**3*f*sqrt(1/b) + 16*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e
+ f*x)**4 - 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(7/2)*
b**4*f*sqrt(1/b) - 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a*
*(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b) +
48*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(3/2)*b**6*f*sqrt(
1/b)*tan(e + f*x)**2 - 16*I*sqrt(a)*b**7*f*sqrt(1/b)*tan(e + f*x)**4), True
))
```

3.245 $\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{\left(3a^2+6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}f(a-b)^3}+\frac{(3a+b)\tan(e+fx)}{8af(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{\tan(e+fx)}{4f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{x}{(a-b)^3}$$

```
[Out] -x/(a-b)^3+1/8*(3*a^2+6*a*b-b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(3/2)
/(a-b)^3/f/b^(1/2)+1/4*tan(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^2+1/8*(3*a+b)*
tan(f*x+e)/a/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.15, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 471, 527, 522, 203, 205}

$$\frac{\left(3a^2+6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}f(a-b)^3}+\frac{(3a+b)\tan(e+fx)}{8af(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{\tan(e+fx)}{4f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{x}{(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(x/(a - b)^3) + ((3*a^2 + 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a]])/(8*a^(3/2)*(a - b)^3*Sqrt[b]*f) + Tan[e + f*x]/(4*(a - b)*f*(a + b*Tan
[e + f*x]^2)^2) + ((3*a + b)*Tan[e + f*x])/(8*a*(a - b)^2*f*(a + b*Tan[e +
f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)\left(a+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{1-3x^2}{\left(1+x^2\right)\left(a+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= \frac{\tan(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{(3a+b)\tan(e+fx)}{8a(a-b)^2f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tan(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{(3a+b)\tan(e+fx)}{8a(a-b)^2f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{x}{(a-b)^3} + \frac{\left(3a^2+6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^3\sqrt{b}f} + \frac{\tan(e+fx)}{4(a-b)f\left(a+b\tan^2(e+f\right.}$$

Mathematica [A] time = 2.11, size = 139, normalized size = 0.97

$$\frac{\frac{(a-b)\sin(2(e+fx))\left(\left(5a^2-4ab-b^2\right)\cos(2(e+fx))+5a^2+2ab+b^2\right)}{a((a-b)\cos(2(e+fx))+a+b)^2} + \frac{\left(3a^2+6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} - 8(e+fx)}{8f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-8*(e + f*x) + ((3*a^2 + 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
]])/(a^(3/2)*Sqrt[b]) + ((a - b)*(5*a^2 + 2*a*b + b^2 + (5*a^2 - 4*a*b - b^
2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a*(a + b + (a - b)*Cos[2*(e + f*x)]
)^2))/(8*(a - b)^3*f)
```

fricas [B] time = 0.76, size = 759, normalized size = 5.27

$$\left[-\frac{32\,a^2b^3fx\tan\left(fx+e\right)^4+64\,a^3b^2fx\tan\left(fx+e\right)^2+32\,a^4bfx-4\left(3\,a^3b^2-2\,a^2b^3-ab^4\right)\tan\left(fx+e\right)^3+\left(\left(3\,a\right.\right.}{32\left(\left(a^5b^3-3\,a^4b^4+3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*a^2*b^3*f*x*tan(f*x + e)^4 + 64*a^3*b^2*f*x*tan(f*x + e)^2 + 32*
a^4*b*f*x - 4*(3*a^3*b^2 - 2*a^2*b^3 - a*b^4)*tan(f*x + e)^3 + ((3*a^2*b^2
+ 6*a*b^3 - b^4)*tan(f*x + e)^4 + 3*a^4 + 6*a^3*b - a^2*b^2 + 2*(3*a^3*b +
6*a^2*b^2 - a*b^3)*tan(f*x + e)^2)*sqrt(-a*b)*log((b^2*tan(f*x + e)^4 - 6*a
*b*tan(f*x + e)^2 + a^2 - 4*(b*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(-a*b))
/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(5*a^4*b - 6*a^3*b^
2 + a^2*b^3)*tan(f*x + e))/((a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f*t
an(f*x + e)^4 + 2*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e
)^2 + (a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f), -1/16*(16*a^2*b^3*f*x*t
an(f*x + e)^4 + 32*a^3*b^2*f*x*tan(f*x + e)^2 + 16*a^4*b*f*x - 2*(3*a^3*b^2
 - 2*a^2*b^3 - a*b^4)*tan(f*x + e)^3 - ((3*a^2*b^2 + 6*a*b^3 - b^4)*tan(f*x
 + e)^4 + 3*a^4 + 6*a^3*b - a^2*b^2 + 2*(3*a^3*b + 6*a^2*b^2 - a*b^3)*tan(f
*x + e)^2)*sqrt(a*b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a*b)/(a*b*tan(f
*x + e))) - 2*(5*a^4*b - 6*a^3*b^2 + a^2*b^3)*tan(f*x + e))/((a^5*b^3 - 3*a
^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f*tan(f*x + e)^4 + 2*(a^6*b^2 - 3*a^5*b^3 + 3
*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^2 + (a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4
*b^4)*f)]
```

giac [A] time = 3.58, size = 200, normalized size = 1.39

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)\left(3\,a^2+6\,ab-b^2\right)}{\left(a^4-3\,a^3b+3\,a^2b^2-ab^3\right)\sqrt{ab}}-\frac{8\left(fx+e\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{3\,ab\tan\left(fx+e\right)^3+b^2\tan\left(fx+e\right)^3+5\,a^2\tan\left(fx+e\right)-ab\tan\left(fx+e\right)}{\left(a^3-2\,a^2b+ab^2\right)\left(b\tan\left(fx+e\right)^2+a\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)
))*(3*a^2 + 6*a*b - b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt(a*b)) -
8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (3*a*b*tan(f*x + e)^3 + b^2*t
an(f*x + e)^3 + 5*a^2*tan(f*x + e) - a*b*tan(f*x + e))/((a^3 - 2*a^2*b + a*
b^2)*(b*tan(f*x + e)^2 + a)^2))/f
```

maple [B] time = 0.24, size = 339, normalized size = 2.35

$$\frac{3ab\left(\tan^3\left(fx+e\right)\right)}{8f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{\left(\tan^3\left(fx+e\right)\right)b^2}{4f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{b^3\left(\tan^3\left(fx+e\right)\right)}{8f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2a}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] 3/8/f*a/(a-b)^3/(a+b*tan(f*x+e)^2)^2*b*tan(f*x+e)^3-1/4/f/(a-b)^3/(a+b*tan(
f*x+e)^2)^2*tan(f*x+e)^3*b^2-1/8/f/(a-b)^3/(a+b*tan(f*x+e)^2)^2*b^3/a*tan(f
*x+e)^3+5/8/f*a^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)-3/4/f/(a-b)^3/(a+
```

```
b*tan(f*x+e)^2)^2*a*b*tan(f*x+e)+1/8/f/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x
+e)*b^2+3/8/f*a/(a-b)^3/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+3/4/f/
(a-b)^3*b/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/8/f/(a-b)^3/a/(a*b
)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))*b^2-1/f/(a-b)^3*arctan(tan(f*x+e))
```

maxima [A] time = 0.88, size = 213, normalized size = 1.48

$$\frac{\frac{\left(3\,a^{2}+6\,ab-b^{2}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^{4}-3\,a^{3}b+3\,a^{2}b^{2}-ab^{3}\right)\sqrt{ab}}+\frac{\left(3\,ab+b^{2}\right)\tan\left(fx+e\right)^{3}+\left(5\,a^{2}-ab\right)\tan\left(fx+e\right)}{a^{5}-2\,a^{4}b+a^{3}b^{2}+\left(a^{3}b^{2}-2\,a^{2}b^{3}+ab^{4}\right)\tan\left(fx+e\right)^{4}+2\left(a^{4}b-2\,a^{3}b^{2}+a^{2}b^{3}\right)\tan\left(fx+e\right)^{2}}-\frac{8\left(fx+e\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + 6*a*b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - 3*a^3*b
 + 3*a^2*b^2 - a*b^3)*sqrt(a*b)) + ((3*a*b + b^2)*tan(f*x + e)^3 + (5*a^2 -
 a*b)*tan(f*x + e))/(a^5 - 2*a^4*b + a^3*b^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4
)*tan(f*x + e)^4 + 2*(a^4*b - 2*a^3*b^2 + a^2*b^3)*tan(f*x + e)^2) - 8*(f*x
 + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

mupad [B] time = 15.49, size = 3817, normalized size = 26.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)*(5*a - b))/(8*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)^3*(3*a*b
+ b^2))/(8*a*(a^2 - 2*a*b + b^2)))/(f*(a^2 + b^2*tan(e + f*x)^4 + 2*a*b*tan
(e + f*x)^2)) - (2*atan((((((32*a*b^9 - 352*a^2*b^8 + 1440*a^3*b^7 - 3040*a
^4*b^6 + 3680*a^5*b^5 - 2592*a^6*b^4 + 992*a^7*b^3 - 160*a^8*b^2)/(64*(a^8
- 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) -
(tan(e + f*x)*(256*a^2*b^9 - 1280*a^3*b^8 + 2304*a^4*b^7 - 1280*a^5*b^6 - 1
280*a^6*b^5 + 2304*a^7*b^4 - 1280*a^8*b^3 + 256*a^9*b^2)*1i)/(32*(6*a*b^2 -
 6*a^2*b + 2*a^3 - 2*b^3)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)
))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(9*a^4*b - 12*a*
b^4 + b^5 + 94*a^2*b^3 + 36*a^3*b^2))/(32*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*
b^3 + 6*a^4*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - ((((32*a*b^9 - 352
*a^2*b^8 + 1440*a^3*b^7 - 3040*a^4*b^6 + 3680*a^5*b^5 - 2592*a^6*b^4 + 992*
a^7*b^3 - 160*a^8*b^2)/(64*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^
4 - 20*a^5*b^3 + 15*a^6*b^2)) + (tan(e + f*x)*(256*a^2*b^9 - 1280*a^3*b^8 +
 2304*a^4*b^7 - 1280*a^5*b^6 - 1280*a^6*b^5 + 2304*a^7*b^4 - 1280*a^8*b^3 +
 256*a^9*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^6 - 4*a^5*b +
a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)
+ (tan(e + f*x)*(9*a^4*b - 12*a*b^4 + b^5 + 94*a^2*b^3 + 36*a^3*b^2))/(32*(
a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a
^3 - 2*b^3))/((((((32*a*b^9 - 352*a^2*b^8 + 1440*a^3*b^7 - 3040*a^4*b^6 + 3
680*a^5*b^5 - 2592*a^6*b^4 + 992*a^7*b^3 - 160*a^8*b^2)/(64*(a^8 - 6*a^7*b
+ a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) - (tan(e + f
*x)*(256*a^2*b^9 - 1280*a^3*b^8 + 2304*a^4*b^7 - 1280*a^5*b^6 - 1280*a^6*b^
5 + 2304*a^7*b^4 - 1280*a^8*b^3 + 256*a^9*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b +
 2*a^3 - 2*b^3)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*1i)/(6*
a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(9*a^4*b - 12*a*b^4 + b^5
+ 94*a^2*b^3 + 36*a^3*b^2))/(32*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^
4*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (3*a*b^3 + 9*a^3*b - b^4
 + 21*a^2*b^2)/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a
^5*b^3 + 15*a^6*b^2)) + (((((32*a*b^9 - 352*a^2*b^8 + 1440*a^3*b^7 - 3040*a
^4*b^6 + 3680*a^5*b^5 - 2592*a^6*b^4 + 992*a^7*b^3 - 160*a^8*b^2)/(64*(a^8
```

```
- 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) +
(tan(e + f*x)*(256*a^2*b^9 - 1280*a^3*b^8 + 2304*a^4*b^7 - 1280*a^5*b^6 - 1
280*a^6*b^5 + 2304*a^7*b^4 - 1280*a^8*b^3 + 256*a^9*b^2)*1i)/(32*(6*a*b^2 -
 6*a^2*b + 2*a^3 - 2*b^3)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)
))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (tan(e + f*x)*(9*a^4*b - 12*a*
b^4 + b^5 + 94*a^2*b^3 + 36*a^3*b^2))/(32*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*
b^3 + 6*a^4*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))))/(f*(6*a*b^2 -
 6*a^2*b + 2*a^3 - 2*b^3)) - (atan(((((tan(e + f*x)*(9*a^4*b - 12*a*b^4 + b
^5 + 94*a^2*b^3 + 36*a^3*b^2))/(32*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6
*a^4*b^2)) - ((-a^3*b)^(1/2)*((32*a*b^9 - 352*a^2*b^8 + 1440*a^3*b^7 - 3040
*a^4*b^6 + 3680*a^5*b^5 - 2592*a^6*b^4 + 992*a^7*b^3 - 160*a^8*b^2)/(64*(a^
8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2))
- (tan(e + f*x)*(-a^3*b)^(1/2)*(6*a*b + 3*a^2 - b^2)*(256*a^2*b^9 - 1280*a^
3*b^8 + 2304*a^4*b^7 - 1280*a^5*b^6 - 1280*a^6*b^5 + 2304*a^7*b^4 - 1280*a^
8*b^3 + 256*a^9*b^2))/(512*(a^6*b - a^3*b^4 + 3*a^4*b^3 - 3*a^5*b^2)*(a^6 -
 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(6*a*b + 3*a^2 - b^2))/(16*(a
^6*b - a^3*b^4 + 3*a^4*b^3 - 3*a^5*b^2)))*(-a^3*b)^(1/2)*(6*a*b + 3*a^2 - b
^2)*1i)/(16*(a^6*b - a^3*b^4 + 3*a^4*b^3 - 3*a^5*b^2)) + (((tan(e + f*x)*(9
*a^4*b - 12*a*b^4 + b^5 + 94*a^2*b^3 + 36*a^3*b^2))/(32*(a^6 - 4*a^5*b + a^
2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)) + ((-a^3*b)^(1/2)*((32*a*b^9 - 352*a^2*b^8
+ 1440*a^3*b^7 - 3040*a^4*b^6 + 3680*a^5*b^5 - 2592*a^6*b^4 + 992*a^7*b^3 -
 160*a^8*b^2)/(64*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^
5*b^3 + 15*a^6*b^2)) + (tan(e + f*x)*(-a^3*b)^(1/2)*(6*a*b + 3*a^2 - b^2)*(
256*a^2*b^9 - 1280*a^3*b^8 + 2304*a^4*b^7 - 1280*a^5*b^6 - 1280*a^6*b^5 + 2
304*a^7*b^4 - 1280*a^8*b^3 + 256*a^9*b^2))/(512*(a^6*b - a^3*b^4 + 3*a^4*b^
3 - 3*a^5*b^2)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(6*a*b +
 3*a^2 - b^2))/(16*(a^6*b - a^3*b^4 + 3*a^4*b^3 - 3*a^5*b^2)))*(-a^3*b)^(1/
2)*(6*a*b + 3*a^2 - b^2)*1i)/(16*(a^6*b - a^3*b^4 + 3*a^4*b^3 - 3*a^5*b^2))
)/((3*a*b^3 + 9*a^3*b - b^4 + 21*a^2*b^2)/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*
a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) + (((tan(e + f*x)*(9*a^4*b
 - 12*a*b^4 + b^5 + 94*a^2*b^3 + 36*a^3*b^2))/(32*(a^6 - 4*a^5*b + a^2*b^4
- 4*a^3*b^3 + 6*a^4*b^2)) - ((-a^3*b)^(1/2)*((32*a*b^9 - 352*a^2*b^8 + 1440
*a^3*b^7 - 3040*a^4*b^6 + 3680*a^5*b^5 - 2592*a^6*b^4 + 992*a^7*b^3 - 160*a
^8*b^2)/(64*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3
+ 15*a^6*b^2)) - (tan(e + f*x)*(-a^3*b)^(1/2)*(6*a*b + 3*a^2 - b^2)*(256*a^
2*b^9 - 1280*a^3*b^8 + 2304*a^4*b^7 - 1280*a^5*b^6 - 1280*a^6*b^5 + 2304*a^
7*b^4 - 1280*a^8*b^3 + 256*a^9*b^2))/(512*(a^6*b - a^3*b^4 + 3*a^4*b^3 - 3*
a^5*b^2)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(6*a*b + 3*a^2
 - b^2))/(16*(a^6*b - a^3*b^4 + 3*a^4*b^3 - 3*a^5*b^2)))*(-a^3*b)^(1/2)*(6*
a*b + 3*a^2 - b^2))/(16*(a^6*b - a^3*b^4 + 3*a^4*b^3 - 3*a^5*b^2)) - (((tan
(e + f*x)*(9*a^4*b - 12*a*b^4 + b^5 + 94*a^2*b^3 + 36*a^3*b^2))/(32*(a^6 -
4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)) + ((-a^3*b)^(1/2)*((32*a*b^9 -
352*a^2*b^8 + 1440*a^3*b^7 - 3040*a^4*b^6 + 3680*a^5*b^5 - 2592*a^6*b^4 + 9
92*a^7*b^3 - 160*a^8*b^2)/(64*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4
*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) + (tan(e + f*x)*(-a^3*b)^(1/2)*(6*a*b + 3*
a^2 - b^2)*(256*a^2*b^9 - 1280*a^3*b^8 + 2304*a^4*b^7 - 1280*a^5*b^6 - 1280
*a^6*b^5 + 2304*a^7*b^4 - 1280*a^8*b^3 + 256*a^9*b^2))/(512*(a^6*b - a^3*b^
4 + 3*a^4*b^3 - 3*a^5*b^2)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2
)))*(6*a*b + 3*a^2 - b^2))/(16*(a^6*b - a^3*b^4 + 3*a^4*b^3 - 3*a^5*b^2)))*
(-a^3*b)^(1/2)*(6*a*b + 3*a^2 - b^2))/(16*(a^6*b - a^3*b^4 + 3*a^4*b^3 - 3*
a^5*b^2))))*(-a^3*b)^(1/2)*(6*a*b + 3*a^2 - b^2)*1i)/(8*f*(a^6*b - a^3*b^4
+ 3*a^4*b^3 - 3*a^5*b^2))
```

sympy [A] time = 140.01, size = 9763, normalized size = 67.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e +
 f*x)/f)/a**3, Eq(b, 0)), ((x + 1/(f*tan(e + f*x)) - 1/(3*f*tan(e + f*x)**3
))/b**3, Eq(a, 0)), (3*f*x*tan(e + f*x)**6/(48*b**3*f*tan(e + f*x)**6 + 144
*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 9*f*x*t
an(e + f*x)**4/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 14
4*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 9*f*x*tan(e + f*x)**2/(48*b**3*f*ta
n(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 4
8*b**3*f) + 3*f*x/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 +
 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 3*tan(e + f*x)**5/(48*b**3*f*tan
(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48
*b**3*f) + 8*tan(e + f*x)**3/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e
+ f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) - 3*tan(e + f*x)/(48*b*
*3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)
**2 + 48*b**3*f), Eq(a, b)), (x*tan(e)**2/(a + b*tan(e)**2)**3, Eq(f, 0)),
(-16*I*a**(7/2)*b*f*x*sqrt(1/b)/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11
/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16
*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/
b)*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*
sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2
- 16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f
*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**
6*f*sqrt(1/b)*tan(e + f*x)**4) + 10*I*a**(7/2)*b*sqrt(1/b)*tan(e + f*x)/(16
*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**
2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e
+ f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*
b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a*
*(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) +
48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(
1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4) - 32
*I*a**(5/2)*b**2*f*x*sqrt(1/b)*tan(e + f*x)**2/(16*I*a**(13/2)*b*f*sqrt(1/b
) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f
*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)
*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a
**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*t
an(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt
(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16
*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4) + 6*I*a**(5/2)*b**2*sqrt(1/b)
*tan(e + f*x)**3/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2)*b**2*f*sqrt
(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)*b**
3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)
**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e
 + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(7/2)
*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a
**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/b)*t
an(e + f*x)**4) - 12*I*a**(5/2)*b**2*sqrt(1/b)*tan(e + f*x)/(16*I*a**(13/2)
*b*f*sqrt(1/b) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**
(11/2)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 -
96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(
1/b) - 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*b**4*
f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2
)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e +
 f*x)**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4) - 16*I*a**(3/2)*
b**3*f*x*sqrt(1/b)*tan(e + f*x)**4/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**
(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) +
 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt
(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4
*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)*
*2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e
+ f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*
```

```
b**6*f*sqrt(1/b)*tan(e + f*x)**4) - 4*I*a**(3/2)*b**3*sqrt(1/b)*tan(e + f*x
)**3/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e
+ f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt(1/b
)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a
**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 +
 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*sqrt
(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*b**5
*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)*
*4) + 2*I*a**(3/2)*b**3*sqrt(1/b)*tan(e + f*x)/(16*I*a**(13/2)*b*f*sqrt(1/b
) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f
*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)
*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a
**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*t
an(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt
(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16
*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4) - 2*I*sqrt(a)*b**4*sqrt(1/b)*
tan(e + f*x)**3/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2)*b**2*f*sqrt(
1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)*b**3
*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)*
*2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e
+ f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(7/2)*
b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a*
*(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/b)*ta
n(e + f*x)**4) + 3*a**4*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(
13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*
I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)*
*4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*
sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*
b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a*
*(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*ta
n(e + f*x)**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4) - 3*a**4*lo
g(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*
a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b
) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*s
qrt(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b
**4*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*
x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan
(e + f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/
2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4) + 6*a**3*b*log(-I*sqrt(a)*sqrt(1/b) +
tan(e + f*x))*tan(e + f*x)**2/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2
)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I
*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)
*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sq
rt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 -
16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x
)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**6*
f*sqrt(1/b)*tan(e + f*x)**4) + 6*a**3*b*log(-I*sqrt(a)*sqrt(1/b) + tan(e +
f*x))/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e
 + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt(1/
b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*
a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4
+ 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*sqr
t(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*b**
5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)
**4) - 6*a**3*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16
*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**
2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e
+ f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*
b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a*
```

```
*(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) +
48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(
1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4) - 6*
a**3*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(13/2)*b*f*sqrt(1/b
) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f
*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)
*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a
**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*t
an(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt
(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16
*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4) + 3*a**2*b**2*log(-I*sqrt(a)*
sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(13/2)*b*f*sqrt(1/b) + 3
2*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt
(1/b) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3
*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/
2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e
+ f*x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)
*tan(e + f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a*
*(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4) + 12*a**2*b**2*log(-I*sqrt(a)*sqrt
(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*
a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b
) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*s
qrt(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b
**4*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*
x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan
(e + f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/
2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4) - a**2*b**2*log(-I*sqrt(a)*sqrt(1/b) +
 tan(e + f*x))/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2)*b**2*f*sqrt(1
/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)*b**3*
f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**
2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e +
 f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(7/2)*b
**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**
(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/b)*tan
(e + f*x)**4) - 3*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e +
 f*x)**4/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*ta
n(e + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt
(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48
*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)*
*4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*
sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*
b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f
*x)**4) - 12*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)
**2/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e +
 f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)
*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a*
*(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 +
96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(
1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*b**5*
f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**
4) + a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(13/2)*b*f*
sqrt(1/b) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2
)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*
a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b)
- 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqr
t(1/b)*tan(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**
5*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)
**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4) + 6*a*b**3*log(-I*sqr
t(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(13/2)*b*f*sqrt(1/b
```

```
) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f
*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)
*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a
**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*t
an(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt
(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16
*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4) - 2*a*b**3*log(-I*sqrt(a)*sqr
t(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I
*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/
b) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*
sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*
b**4*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f
*x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*ta
n(e + f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3
/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4) - 6*a*b**3*log(I*sqrt(a)*sqrt(1/b) +
tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2
)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I
*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)
*tan(e + f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sq
rt(1/b)*tan(e + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 -
16*I*a**(7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x
)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**6*
f*sqrt(1/b)*tan(e + f*x)**4) + 2*a*b**3*log(I*sqrt(a)*sqrt(1/b) + tan(e + f
*x))*tan(e + f*x)**2/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2)*b**2*f*
sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)
*b**3*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e +
f*x)**2 + 48*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sqrt(1/b)*t
an(e + f*x)**4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(
7/2)*b**4*f*sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 32
*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/
b)*tan(e + f*x)**4) - b**4*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e +
 f*x)**4/(16*I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*ta
n(e + f*x)**2 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt
(1/b)*tan(e + f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48
*I*a**(9/2)*b**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)*
*4 + 96*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*
sqrt(1/b) + 48*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*
b**5*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f
*x)**4) + b**4*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(16*
I*a**(13/2)*b*f*sqrt(1/b) + 32*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2
 - 48*I*a**(11/2)*b**2*f*sqrt(1/b) + 16*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e +
 f*x)**4 - 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(9/2)*b
**3*f*sqrt(1/b) - 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**
(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(7/2)*b**4*f*sqrt(1/b) + 4
8*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(5/2)*b**5*f*sqrt(1
/b)*tan(e + f*x)**2 - 16*I*a**(3/2)*b**6*f*sqrt(1/b)*tan(e + f*x)**4), True
))
```

3.246 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3} dx$

Optimal. Leaf size=150

$$-\frac{b(7a-3b)\tan(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}f(a-b)^3}-\frac{b\tan(e+fx)}{4af(a-b)\left(a+b\tan^2(e+f\right.}$$

```
[Out] x/(a-b)^3-1/8*(15*a^2-10*a*b+3*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1
/2)/a^(5/2)/(a-b)^3/f-1/4*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8*(
7*a-3*b)*b*tan(f*x+e)/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.15, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3661, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}f(a-b)^3}-\frac{b(7a-3b)\tan(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{b\tan(e+fx)}{4af(a-b)\left(a+b\tan^2(e+f\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-3),x]
```

```
[Out] x/(a - b)^3 - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*f) - (b*Tan[e + f*x])/(4*a*(a - b)*f*(a
+ b*Tan[e + f*x]^2)^2) - ((7*a - 3*b)*b*Tan[e + f*x])/(8*a^2*(a - b)^2*f*(a
 + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-3b-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-3b)b\tan(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-3b)b\tan(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{x}{(a-b)^3} - \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3f} - \frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e\right.}$$

Mathematica [A] time = 1.94, size = 138, normalized size = 0.92

$$-\frac{\frac{b(7a-3b)(a-b)\tan(e+fx)}{a^2\left(a+b\tan^2(e+fx)\right)} + \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2b(a-b)^2\tan(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^2} - 8\tan^{-1}(\tan(e+fx))}{8f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-3),x]
```

```
[Out] -1/8*(-8*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(
Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Tan[e + f*x])/(a*(
a + b*Tan[e + f*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Tan[e + f*x])/(a^2*(a + b
*Tan[e + f*x]^2)))/((a - b)^3*f)
```

fricas [B] time = 0.53, size = 742, normalized size = 4.95

$$\left[\frac{32\,a^2b^2fx\tan\left(fx+e\right)^4+64\,a^3bfx\tan\left(fx+e\right)^2+32\,a^4fx-4\left(7\,a^2b^2-10\,ab^3+3\,b^4\right)\tan\left(fx+e\right)^3-\left(\left(15\right.\right.}{32\left(\left(a^5b^2-3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*a^2*b^2*f*x*tan(f*x + e)^4 + 64*a^3*b*f*x*tan(f*x + e)^2 + 32*a^4
*f*x - 4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 10*
a*b^3 + 3*b^4)*tan(f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b
 - 10*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4
 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*s
qrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(9*a^3*b
- 14*a^2*b^2 + 5*a*b^3)*tan(f*x + e))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*ta
n(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/16*(16*a^2*b^2*f
*x*tan(f*x + e)^4 + 32*a^3*b*f*x*tan(f*x + e)^2 + 16*a^4*f*x - 2*(7*a^2*b^2
 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(
f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b - 10*a^2*b^2 + 3*a
*b^3)*tan(f*x + e)^2)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)
/(b*tan(f*x + e))) - 2*(9*a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(f*x + e))/((a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 -
 a^4*b^3)*f)]
```

giac [A] time = 1.75, size = 213, normalized size = 1.42

$$-\frac{\frac{\left(15\,a^2b-10\,ab^2+3\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^5-3\,a^4b+3\,a^3b^2-a^2b^3\right)\sqrt{ab}}-\frac{8\left(fx+e\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{7\,ab^2\tan\left(fx+e\right)^3-3\,b^3\tan\left(fx+e\right)^3+9\,a^2b\tan\left(fx+e\right)-}{\left(a^4-2\,a^3b+a^2b^2\right)\left(b\tan\left(fx+e\right)^2+a\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sq
rt(a*b)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (7*a*b^2*tan(f*x +
 e)^3 - 3*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) - 5*a*b^2*tan(f*x + e))
/((a^4 - 2*a^3*b + a^2*b^2)*(b*tan(f*x + e)^2 + a)^2))/f
```

maple [B] time = 0.35, size = 350, normalized size = 2.33

$$-\frac{7\left(\tan^3\left(fx+e\right)\right)b^2}{8f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{5b^3\left(\tan^3\left(fx+e\right)\right)}{4f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2a}-\frac{3b^4\left(\tan^3\left(fx+e\right)\right)}{8f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3*b^2+5/4/f/(a-b)^3/(a+b*tan
(f*x+e)^2)^2*b^3/a*tan(f*x+e)^3-3/8/f*b^4/(a-b)^3/(a+b*tan(f*x+e)^2)^2/a^2*
tan(f*x+e)^3-9/8/f/(a-b)^3/(a+b*tan(f*x+e)^2)^2*a*b*tan(f*x+e)+7/4/f/(a-b)^
```

```
3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)*b^2-5/8/f*b^3/(a-b)^3/(a+b*tan(f*x+e)^2)^
2/a*tan(f*x+e)-15/8/f/(a-b)^3*b/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2)
)+5/4/f/(a-b)^3/a/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))*b^2-3/8/f*b^
3/(a-b)^3/a^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+1/f/(a-b)^3*arct
an(tan(f*x+e))
```

maxima [A] time = 1.05, size = 227, normalized size = 1.51

$$-\frac{\frac{\left(15\,a^{2}b-10\,ab^{2}+3\,b^{3}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^{5}-3\,a^{4}b+3\,a^{3}b^{2}-a^{2}b^{3}\right)\sqrt{ab}}+\frac{\left(7\,ab^{2}-3\,b^{3}\right)\tan\left(fx+e\right)^{3}+\left(9\,a^{2}b-5\,ab^{2}\right)\tan\left(fx+e\right)}{a^{6}-2\,a^{5}b+a^{4}b^{2}+\left(a^{4}b^{2}-2\,a^{3}b^{3}+a^{2}b^{4}\right)\tan\left(fx+e\right)^{4}+2\left(a^{5}b-2\,a^{4}b^{2}+a^{3}b^{3}\right)\tan\left(fx+e\right)^{2}}-\frac{8\left(fx+e\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5
- 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(a*b)) + ((7*a*b^2 - 3*b^3)*tan(f*x +
e)^3 + (9*a^2*b - 5*a*b^2)*tan(f*x + e))/(a^6 - 2*a^5*b + a^4*b^2 + (a^4*b^
2 - 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 + 2*(a^5*b - 2*a^4*b^2 + a^3*b^3)*t
an(f*x + e)^2) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

mupad [B] time = 15.35, size = 3901, normalized size = 26.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] (atan((((-a^5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300
*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b
^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^
6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^1
0 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2))
- (tan(e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 128
0*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 128
0*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8
 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10
*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b
 + 3*b^2)*1i)/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^5*b)^(1/2)*
((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3)
)/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((96*a^2*b^10 -
 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 +
5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 -
 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(e + f*x)*(-a^5*b
)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^
7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b
^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 -
4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a
^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2)*1i)/(16*(3*a^
7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))/((51*a*b^5 - 9*b^6 - 115*a^2*b^4 + 105*a
^3*b^3)/(32*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3
 + 15*a^8*b^2)) - ((-a^5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^
2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^
3 + 6*a^6*b^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^
7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^
2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15
*a^8*b^2)) - (tan(e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^
4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^
```

9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2))))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*1i)/(8*f*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) - ((tan(e + f*x)^3*(7*a*b^2 - 3*b^3))/(8*a^2*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)*(9*a*b - 5*b^2))/(8*a*(a^2 - 2*a*b + b^2)))/(f*(a^2 + b^2*tan(e + f*x)^4 + 2*a*b*tan(e + f*x)^2)) - (2*atan((((((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) - (tan(e + f*x)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - ((((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(e + f*x)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))/((51*a*b^5 - 9*b^6 - 115*a^2*b^4 + 105*a^3*b^3)/(32*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (((((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) - (tan(e + f*x)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (((((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(e + f*x)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))))/(f*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))

sympy [A] time = 138.07, size = 9629, normalized size = 64.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**6, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a**3, Eq(b,
 0)), ((-x - 1/(f*tan(e + f*x)) + 1/(3*f*tan(e + f*x)**3) - 1/(5*f*tan(e +
f*x)**5))/b**3, Eq(a, 0)), (15*f*x*tan(e + f*x)**6/(48*b**3*f*tan(e + f*x)*
*6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) +
 45*f*x*tan(e + f*x)**4/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x
)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 45*f*x*tan(e + f*x)**2/(48
*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f
*x)**2 + 48*b**3*f) + 15*f*x/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e
+ f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 15*tan(e + f*x)**5/(4
8*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e +
f*x)**2 + 48*b**3*f) + 40*tan(e + f*x)**3/(48*b**3*f*tan(e + f*x)**6 + 144*
b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 33*tan(e
 + f*x)/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*
f*tan(e + f*x)**2 + 48*b**3*f), Eq(a, b)), (x/(a + b*tan(e)**2)**3, Eq(f, 0
)), (16*I*a**(9/2)*f*x*sqrt(1/b)/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/
2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(
11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*ta
n(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt
(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16
*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)*
*4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*
sqrt(1/b)*tan(e + f*x)**4) + 32*I*a**(7/2)*b*f*x*sqrt(1/b)*tan(e + f*x)**2/
(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2
- 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*
x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b*
*2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(
9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48
*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/
b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) - 18*I
*a**(7/2)*b*sqrt(1/b)*tan(e + f*x)/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(1
3/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a*
*(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*
tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sq
rt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 -
16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x
)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*
f*sqrt(1/b)*tan(e + f*x)**4) + 16*I*a**(5/2)*b**2*f*x*sqrt(1/b)*tan(e + f*x
)**4/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x
)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e
 + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/
2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I
*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b)
 + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sq
rt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) -
 14*I*a**(5/2)*b**2*sqrt(1/b)*tan(e + f*x)**3/(16*I*a**(15/2)*f*sqrt(1/b) +
 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b
) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f
*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2
)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e +
 f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*
tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**
(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) + 28*I*a**(5/2)*b**2*sqrt(1/b)*tan(
```

```
e + f*x)/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e +
 f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*t
an(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**
(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 +
96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(
1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*
f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**
4) + 20*I*a**(3/2)*b**3*sqrt(1/b)*tan(e + f*x)**3/(16*I*a**(15/2)*f*sqrt(1/
b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt
(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b*
*2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**
(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan
(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1
/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I
*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) - 10*I*a**(3/2)*b**3*sqrt(1/b)*
tan(e + f*x)/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan
(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/
b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I
*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**
4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*s
qrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b
**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*
x)**4) - 6*I*sqrt(a)*b**4*sqrt(1/b)*tan(e + f*x)**3/(16*I*a**(15/2)*f*sqrt(
1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sq
rt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*
b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a
**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*t
an(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt
(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16
*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) - 15*a**4*log(-I*sqrt(a)*sqrt
(1/b) + tan(e + f*x))/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt
(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*
f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)*
*2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e
 + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)
*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a
**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*t
an(e + f*x)**4) + 15*a**4*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**
(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a*
*(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 9
6*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt
(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3
*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/
2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e
+ f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) - 30*a**3*b*log
(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16*I*a**(15/2)*f*sqr
t(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*
sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2
)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I
*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)
*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sq
rt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 -
16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) + 10*a**3*b*log(-I*sqrt(a)*
sqrt(1/b) + tan(e + f*x))/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*
sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b
**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f
*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*t
an(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(
9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32
```

```
*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/
b)*tan(e + f*x)**4) + 30*a**3*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan
(e + f*x)**2/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan
(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/
b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I
*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**
4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*s
qrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b
**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*
x)**4) - 10*a**3*b*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(15/2)*
f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)
*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**
(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) -
 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt
(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4
*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)*
*2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) - 15*a**2*b**2*log(-I*
sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(15/2)*f*sqrt(1/
b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt
(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b*
*2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**
(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan
(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1
/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I
*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) + 20*a**2*b**2*log(-I*sqrt(a)*s
qrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I
*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 1
6*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt
(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**
3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)
**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e
 + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)
*b**5*f*sqrt(1/b)*tan(e + f*x)**4) - 3*a**2*b**2*log(-I*sqrt(a)*sqrt(1/b) +
 tan(e + f*x))/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*t
an(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(
1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48
*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)
**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f
*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)
*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e +
f*x)**4) + 15*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x
)**4/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x
)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e
 + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/
2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I
*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b)
 + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sq
rt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) -
 20*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16*I
*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*
I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4
 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*
sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*
b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a*
*(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*ta
n(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) + 3*a**2*b*
*2*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))/(16*I*a**(15/2)*f*sqrt(1/b) + 32
*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) +
 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sq
```

```
rt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b
**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*
x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan
(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/
2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) + 10*a*b**3*log(-I*sqrt(a)*sqrt(1/b) +
 tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)
*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11
/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(
e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1
/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I
*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4
 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sq
rt(1/b)*tan(e + f*x)**4) - 6*a*b**3*log(-I*sqrt(a)*sqrt(1/b) + tan(e + f*x)
)*tan(e + f*x)**2/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b
)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sq
rt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 +
 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f
*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**
3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7
/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e
 + f*x)**4) - 10*a*b**3*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x
)**4/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x
)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e
 + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/
2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I
*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b)
 + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sq
rt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) +
 6*a*b**3*log(I*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**2/(16*I*a**
(15/2)*f*sqrt(1/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a*
*(13/2)*b*f*sqrt(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 9
6*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt
(1/b) - 48*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3
*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/
2)*b**4*f*sqrt(1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e
+ f*x)**2 - 16*I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) - 3*b**4*log(-I
*sqrt(a)*sqrt(1/b) + tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(15/2)*f*sqrt(1
/b) + 32*I*a**(13/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqr
t(1/b) + 16*I*a**(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b
**2*f*sqrt(1/b)*tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a*
*(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*ta
n(e + f*x)**2 - 16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(
1/b)*tan(e + f*x)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*
I*a**(5/2)*b**5*f*sqrt(1/b)*tan(e + f*x)**4) + 3*b**4*log(I*sqrt(a)*sqrt(1/
b) + tan(e + f*x))*tan(e + f*x)**4/(16*I*a**(15/2)*f*sqrt(1/b) + 32*I*a**(1
3/2)*b*f*sqrt(1/b)*tan(e + f*x)**2 - 48*I*a**(13/2)*b*f*sqrt(1/b) + 16*I*a*
*(11/2)*b**2*f*sqrt(1/b)*tan(e + f*x)**4 - 96*I*a**(11/2)*b**2*f*sqrt(1/b)*
tan(e + f*x)**2 + 48*I*a**(11/2)*b**2*f*sqrt(1/b) - 48*I*a**(9/2)*b**3*f*sq
rt(1/b)*tan(e + f*x)**4 + 96*I*a**(9/2)*b**3*f*sqrt(1/b)*tan(e + f*x)**2 -
16*I*a**(9/2)*b**3*f*sqrt(1/b) + 48*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x
)**4 - 32*I*a**(7/2)*b**4*f*sqrt(1/b)*tan(e + f*x)**2 - 16*I*a**(5/2)*b**5*
f*sqrt(1/b)*tan(e + f*x)**4), True))
```

3.247 $\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=189

$$-\frac{b(9a-5b)\cot(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{b^{3/2}\left(35a^2-42ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f(a-b)^3}-\frac{\left(8a^2-27ab+15b^2\right)\cot(e+}{8a^3f(a-b)^2}$$

```
[Out] -x/(a-b)^3+1/8*b^(3/2)*(35*a^2-42*a*b+15*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(
1/2))/a^(7/2)/(a-b)^3/f-1/8*(8*a^2-27*a*b+15*b^2)*cot(f*x+e)/a^3/(a-b)^2/f-
1/4*b*cot(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8*(9*a-5*b)*b*cot(f*x+e)/
a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.29, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3670, 472, 579, 583, 522, 203, 205}

$$\frac{b^{3/2}\left(35a^2-42ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f(a-b)^3}-\frac{\left(8a^2-27ab+15b^2\right)\cot(e+fx)}{8a^3f(a-b)^2}-\frac{b(9a-5b)\cot(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(x/(a - b)^3) + (b^(3/2)*(35*a^2 - 42*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tan[e
+ f*x])/Sqrt[a]])/(8*a^(7/2)*(a - b)^3*f) - ((8*a^2 - 27*a*b + 15*b^2)*Cot[
e + f*x])/(8*a^3*(a - b)^2*f) - (b*Cot[e + f*x])/(4*a*(a - b)*f*(a + b*Tan[
e + f*x]^2)^2) - ((9*a - 5*b)*b*Cot[e + f*x])/(8*a^2*(a - b)^2*f*(a + b*Tan
[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-5b-5bx^2}{x^2(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a-b)f}$$

$$= -\frac{b\cot(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(9a-5b)b\cot(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}}{}$$

$$= -\frac{\left(8a^2-27ab+15b^2\right)\cot(e+fx)}{8a^3(a-b)^2f} - \frac{b\cot(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(9}{8a^2(a-}$$

$$= -\frac{\left(8a^2-27ab+15b^2\right)\cot(e+fx)}{8a^3(a-b)^2f} - \frac{b\cot(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(9}{8a^2(a-}$$

$$= -\frac{x}{(a-b)^3} + \frac{b^{3/2}\left(35a^2-42ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}(a-b)^3f} - \frac{\left(8a^2-27ab+15b\right.}{8a^3(a-b}$$

Mathematica [A] time = 2.40, size = 174, normalized size = 0.92

$$\frac{\frac{b^2(13a-7b)\sin(2(e+fx))}{a^3(a-b)^2((a-b)\cos(2(e+fx))+a+b)} - \frac{8\cot(e+fx)}{a^3} - \frac{4b^3\sin(2(e+fx))}{a^2(a-b)^2((a-b)\cos(2(e+fx))+a+b)^2} + \frac{b^{3/2}(35a^2-42ab+15b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}(a-b)^3} + \frac{8(e+fx)}{(b-a)^3}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((8*(e + f*x))/(-a + b)^3 + (b^(3/2)*(35*a^2 - 42*a*b + 15*b^2)*ArcTan[(Sqr
t[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*(a - b)^3) - (8*Cot[e + f*x])/a^3 - (
4*b^3*Sin[2*(e + f*x)])/(a^2*(a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)])^2
) + ((13*a - 7*b)*b^2*Sin[2*(e + f*x)])/(a^3*(a - b)^2*(a + b + (a - b)*Cos
[2*(e + f*x)])))/(8*f)
```

fricas [B] time = 0.51, size = 881, normalized size = 4.66

$$\left[-\frac{32\,a^3b^2fx\tan\left(fx+e\right)^5 + 64\,a^4bfx\tan\left(fx+e\right)^3 + 32\,a^5fx\tan\left(fx+e\right) + 32\,a^5 - 96\,a^4b + 96\,a^3b^2 - 32\,a^2b^3 \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*a^3*b^2*f*x*tan(f*x + e)^5 + 64*a^4*b*f*x*tan(f*x + e)^3 + 32*a^
5*f*x*tan(f*x + e) + 32*a^5 - 96*a^4*b + 96*a^3*b^2 - 32*a^2*b^3 + 4*(8*a^3
*b^2 - 35*a^2*b^3 + 42*a*b^4 - 15*b^5)*tan(f*x + e)^4 + 4*(16*a^4*b - 61*a^
3*b^2 + 70*a^2*b^3 - 25*a*b^4)*tan(f*x + e)^2 + ((35*a^2*b^3 - 42*a*b^4 + 1
5*b^5)*tan(f*x + e)^5 + 2*(35*a^3*b^2 - 42*a^2*b^3 + 15*a*b^4)*tan(f*x + e)
^3 + (35*a^4*b - 42*a^3*b^2 + 15*a^2*b^3)*tan(f*x + e))*sqrt(-b/a)*log((b^2
*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*
tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)
))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^5 + 2*(a^7*b
 - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^3 + (a^8 - 3*a^7*b + 3*a
^6*b^2 - a^5*b^3)*f*tan(f*x + e)), -1/16*(16*a^3*b^2*f*x*tan(f*x + e)^5 + 3
2*a^4*b*f*x*tan(f*x + e)^3 + 16*a^5*f*x*tan(f*x + e) + 16*a^5 - 48*a^4*b +
48*a^3*b^2 - 16*a^2*b^3 + 2*(8*a^3*b^2 - 35*a^2*b^3 + 42*a*b^4 - 15*b^5)*ta
n(f*x + e)^4 + 2*(16*a^4*b - 61*a^3*b^2 + 70*a^2*b^3 - 25*a*b^4)*tan(f*x +
e)^2 - ((35*a^2*b^3 - 42*a*b^4 + 15*b^5)*tan(f*x + e)^5 + 2*(35*a^3*b^2 - 4
2*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^3 + (35*a^4*b - 42*a^3*b^2 + 15*a^2*b^3)
*tan(f*x + e))*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan
(f*x + e))))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^5
+ 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^3 + (a^8 - 3*a
^7*b + 3*a^6*b^2 - a^5*b^3)*f*tan(f*x + e))]
```

giac [A] time = 5.48, size = 231, normalized size = 1.22

$$\frac{\frac{(35\,a^2b^2-42\,ab^3+15\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^6-3\,a^5b+3\,a^4b^2-a^3b^3)\sqrt{ab}} - \frac{8\,(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3} + \frac{11\,ab^3\tan(fx+e)^3-7\,b^4\tan(fx+e)^3+13\,a^2b^2\tan(fx+e)-\cdots}{(a^5-2\,a^4b+a^3b^2)\left(b\tan(fx+e)^2+a\right)^2}\cdots}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

[Out] 1/8*((35*a^2*b^2 - 42*a*b^3 + 15*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*sqrt(a*b)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (11*a*b^3*tan(f*x + e)^3 - 7*b^4*tan(f*x + e)^3 + 13*a^2*b^2*tan(f*x + e) - 9*a*b^3*tan(f*x + e))/((a^5 - 2*a^4*b + a^3*b^2)*(b*tan(f*x + e)^2 + a)^2) - 8/(a^3*tan(f*x + e)))/f

maple [B] time = 0.98, size = 379, normalized size = 2.01

$$\frac{11b^3\left(\tan^3\left(fx+e\right)\right)}{8f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2 a}-\frac{9b^4\left(\tan^3\left(fx+e\right)\right)}{4f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2 a^2}+\frac{7b^5\left(\tan^3\left(fx+e\right)\right)}{8f\,a^3\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x)

[Out] 11/8/f/(a-b)^3/(a+b*tan(f*x+e)^2)^2*b^3/a*tan(f*x+e)^3-9/4/f*b^4/(a-b)^3/(a+b*tan(f*x+e)^2)^2/a^2*tan(f*x+e)^3+7/8/f*b^5/a^3/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+13/8/f/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)*b^2-11/4/f*b^3/(a-b)^3/(a+b*tan(f*x+e)^2)^2/a*tan(f*x+e)+9/8/f*b^4/a^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)+35/8/f/(a-b)^3/a/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))*b^2-21/4/f*b^3/(a-b)^3/a^2/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))+15/8/f*b^4/a^3/(a-b)^3/(a*b)^(1/2)*arctan(tan(f*x+e)*b/(a*b)^(1/2))-1/f/a^3/tan(f*x+e)-1/f/(a-b)^3*arctan(tan(f*x+e))

maxima [A] time = 0.82, size = 275, normalized size = 1.46

$$\frac{\frac{\left(35\,a^2b^2-42\,ab^3+15\,b^4\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^6-3\,a^5b+3\,a^4b^2-a^3b^3\right)\sqrt{ab}}-\frac{\left(8\,a^2b^2-27\,ab^3+15\,b^4\right)\tan\left(fx+e\right)^4+8\,a^4-16\,a^3b+8\,a^2b^2+\left(16\,a^3b-45\,a^2b^2+25\,ab^3\right)\tan\left(fx+e\right)^2}{\left(a^5b^2-2\,a^4b^3+a^3b^4\right)\tan\left(fx+e\right)^5+2\left(a^6b-2\,a^5b^2+a^4b^3\right)\tan\left(fx+e\right)^3+\left(a^7-2\,a^6b+a^5b^2\right)\tan\left(fx+e\right)}-}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")

[Out] 1/8*((35*a^2*b^2 - 42*a*b^3 + 15*b^4)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*sqrt(a*b)) - ((8*a^2*b^2 - 27*a*b^3 + 15*b^4)*tan(f*x + e)^4 + 8*a^4 - 16*a^3*b + 8*a^2*b^2 + (16*a^3*b - 45*a^2*b^2 + 25*a*b^3)*tan(f*x + e)^2)/((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*tan(f*x + e)^5 + 2*(a^6*b - 2*a^5*b^2 + a^4*b^3)*tan(f*x + e)^3 + (a^7 - 2*a^6*b + a^5*b^2)*tan(f*x + e)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f

mupad [B] time = 15.05, size = 915, normalized size = 4.84

$$-\frac{2\,\mathrm{atan}\left(\frac{2\tan\left(e+f\,x\right)\left(\frac{262144\,a^{28}\,b^2-2883584\,a^{27}\,b^3+14155776\,a^{26}\,b^4-40370176\,a^{25}\,b^5+72089600\,a^{24}\,b^6-77856768\,a^{23}\,b^7+34603008\,a^{22}\,b^8+34603008\,a^{21}\,b^9-77\ldots}{\left(2\,a^3-6\,a^2\,b+6\,a\,b^2-2\,b^3\right)^2}\right.}{\left(2\,a^3-6\,a^2\,b+6\,a\,b^2-2\,b^3\right)\left(\frac{2\left(131072\,a^{25}\,b^2-1179648\,a^{24}\,b^3+4145152\,a^{23}\,b^4-5160960\,a^{22}\,b^5-10567680\,a^{21}\,b^6+58638\ldots\right.}{}\right.}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^3,x)

[Out] (atan((b^5*tan(e + f*x)*(-a^7*b^3)^(3/2)*225i - a*b^4*tan(e + f*x)*(-a^7*b^3)^(3/2)*1260i + a^4*b*tan(e + f*x)*(-a^7*b^3)^(3/2)*1225i + a^14*b*tan(e + f*x)*(-a^7*b^3)^(1/2)*64i + a^2*b^3*tan(e + f*x)*(-a^7*b^3)^(3/2)*2814i - a^3*b^2*tan(e + f*x)*(-a^7*b^3)^(3/2)*2940i)/(225*a^11*b^9 - 1260*a^12*b^8

```
+ 2814*a^13*b^7 - 2940*a^14*b^6 + 1225*a^15*b^5 - 64*a^18*b^2))*(-a^7*b^3)^
(1/2)*(35*a^2 - 42*a*b + 15*b^2)*1i)/(8*f*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8
*b^2)) - (1/a + (tan(e + f*x)^4*(15*b^4 - 27*a*b^3 + 8*a^2*b^2))/(8*a^3*(a^
2 - 2*a*b + b^2)) + (tan(e + f*x)^2*(16*a^2*b - 45*a*b^2 + 25*b^3))/(8*a^2*
(a^2 - 2*a*b + b^2)))/(f*(a^2*tan(e + f*x) + b^2*tan(e + f*x)^5 + 2*a*b*tan
(e + f*x)^3)) - (2*atan((2*tan(e + f*x)*((262144*a^15*b^15 - 2883584*a^16*b
^14 + 14155776*a^17*b^13 - 40370176*a^18*b^12 + 72089600*a^19*b^11 - 778567
68*a^20*b^10 + 34603008*a^21*b^9 + 34603008*a^22*b^8 - 77856768*a^23*b^7 +
72089600*a^24*b^6 - 40370176*a^25*b^5 + 14155776*a^26*b^4 - 2883584*a^27*b^
3 + 262144*a^28*b^2)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)^2 - 230400*a^9*b^1
5 + 2672640*a^10*b^14 - 14078976*a^11*b^13 + 44261376*a^12*b^12 - 91801600*
a^13*b^11 + 131051520*a^14*b^10 - 130287616*a^15*b^9 + 89219072*a^16*b^8 -
40743936*a^17*b^7 + 11847680*a^18*b^6 - 2237440*a^19*b^5 + 393216*a^20*b^4
- 65536*a^21*b^3))/((6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*((2*(245760*a^12*b^
15 - 2899968*a^13*b^14 + 15613952*a^14*b^13 - 50577408*a^15*b^12 + 10928128
0*a^16*b^11 - 164659200*a^17*b^10 + 174882816*a^18*b^9 - 127893504*a^19*b^8
 + 58638336*a^20*b^7 - 10567680*a^21*b^6 - 5160960*a^22*b^5 + 4145152*a^23*
b^4 - 1179648*a^24*b^3 + 131072*a^25*b^2))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b
^3)^2 + 230400*a^9*b^12 - 1981440*a^10*b^11 + 7443456*a^11*b^10 - 15879168*
a^12*b^9 + 20933632*a^13*b^8 - 17363968*a^14*b^7 + 8788992*a^15*b^6 - 24586
24*a^16*b^5 + 286720*a^17*b^4))))/(f*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.248 $\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3} dx$

Optimal. Leaf size=240

$$-\frac{b(11a-7b)\cot^3(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{b^{5/2}\left(63a^2-90ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f(a-b)^3}-\frac{\left(8a^2-55ab+35b^2\right)\cot^3(}{24a^3f(a-b)^2}$$

```
[Out] x/(a-b)^3-1/8*b^(5/2)*(63*a^2-90*a*b+35*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1
/2))/a^(9/2)/(a-b)^3/f+1/8*(8*a^3+8*a^2*b-55*a*b^2+35*b^3)*cot(f*x+e)/a^4/(
a-b)^2/f-1/24*(8*a^2-55*a*b+35*b^2)*cot(f*x+e)^3/a^3/(a-b)^2/f-1/4*b*cot(f*
x+e)^3/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8*(11*a-7*b)*b*cot(f*x+e)^3/a^2/(a-
b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.36, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3670, 472, 579, 583, 522, 203, 205}

$$-\frac{b^{5/2}\left(63a^2-90ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f(a-b)^3}-\frac{\left(8a^2-55ab+35b^2\right)\cot^3(e+fx)}{24a^3f(a-b)^2}+\frac{\left(8a^2b+8a^3-55ab^2+35\right.}{8a^4f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] x/(a - b)^3 - (b^(5/2)*(63*a^2 - 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tan[e + f
*x])/Sqrt[a]])/(8*a^(9/2)*(a - b)^3*f) + ((8*a^3 + 8*a^2*b - 55*a*b^2 + 35*
b^3)*Cot[e + f*x])/(8*a^4*(a - b)^2*f) - ((8*a^2 - 55*a*b + 35*b^2)*Cot[e +
 f*x]^3)/(24*a^3*(a - b)^2*f) - (b*Cot[e + f*x]^3)/(4*a*(a - b)*f*(a + b*Ta
n[e + f*x]^2)^2) - ((11*a - 7*b)*b*Cot[e + f*x]^3)/(8*a^2*(a - b)^2*f*(a +
b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-7b-7bx^2}{x^4(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(11a-7b)b\cot^3(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \text{Subst}$$

$$= -\frac{\left(8a^2-55ab+35b^2\right)\cot^3(e+fx)}{24a^3(a-b)^2f} - \frac{b\cot^3(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(1}{8a^2(a}$$

$$= \frac{\left(8a^3+8a^2b-55ab^2+35b^3\right)\cot(e+fx)}{8a^4(a-b)^2f} - \frac{\left(8a^2-55ab+35b^2\right)\cot^3(e+fx)}{24a^3(a-b)^2f} -$$

$$= \frac{\left(8a^3+8a^2b-55ab^2+35b^3\right)\cot(e+fx)}{8a^4(a-b)^2f} - \frac{\left(8a^2-55ab+35b^2\right)\cot^3(e+fx)}{24a^3(a-b)^2f} -$$

$$= \frac{x}{(a-b)^3} - \frac{b^{5/2}\left(63a^2-90ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}(a-b)^3f} + \frac{\left(8a^3+8a^2b-55ab^2\right.}{8a^4(a}$$

Mathematica [A] time = 4.98, size = 184, normalized size = 0.77

$$\frac{-\frac{8\cot(e+fx)\left(a\csc^2(e+fx)-4a-9b\right)}{a^4} - \frac{3b^{5/2}\left(63a^2-90ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{9/2}(a-b)^3} + \frac{3\left(8(e+fx)-\frac{b^3(a-b)\sin(2(e+fx))\left(\left(17a^2-28ab+11b^2\right)\cos(2(e+fx)\right.}{a^4((a-b)\cos(2(e+fx))+a+b)^2}\right.}{(a-b)^3}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((-3*b^(5/2)*(63*a^2 - 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a]])/(a^(9/2)*(a - b)^3) - (8*Cot[e + f*x]*(-4*a - 9*b + a*Csc[e + f*x]^2))
/a^4 + (3*(8*(e + f*x) - ((a - b)*b^3*(17*a^2 + 2*a*b - 11*b^2 + (17*a^2 -
28*a*b + 11*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a^4*(a + b + (a - b)*
Cos[2*(e + f*x)])^2)))/(a - b)^3)/(24*f)
```

fricas [B] time = 0.55, size = 1006, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(96*a^4*b^2*f*x*tan(f*x + e)^7 + 192*a^5*b*f*x*tan(f*x + e)^5 + 96*a^
6*f*x*tan(f*x + e)^3 + 12*(8*a^4*b^2 - 63*a^2*b^4 + 90*a*b^5 - 35*b^6)*tan(
f*x + e)^6 - 32*a^6 + 96*a^5*b - 96*a^4*b^2 + 32*a^3*b^3 + 4*(48*a^5*b - 8*
a^4*b^2 - 315*a^3*b^3 + 450*a^2*b^4 - 175*a*b^5)*tan(f*x + e)^4 + 32*(3*a^6
```

```
 - 2*a^5*b - 12*a^4*b^2 + 18*a^3*b^3 - 7*a^2*b^4)*tan(f*x + e)^2 - 3*((63*a
^2*b^4 - 90*a*b^5 + 35*b^6)*tan(f*x + e)^7 + 2*(63*a^3*b^3 - 90*a^2*b^4 + 3
5*a*b^5)*tan(f*x + e)^5 + (63*a^4*b^2 - 90*a^3*b^3 + 35*a^2*b^4)*tan(f*x +
e)^3)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(
a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*
a*b*tan(f*x + e)^2 + a^2)))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*
tan(f*x + e)^7 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)
^5 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^3), 1/48*(48*a^4*
b^2*f*x*tan(f*x + e)^7 + 96*a^5*b*f*x*tan(f*x + e)^5 + 48*a^6*f*x*tan(f*x +
 e)^3 + 6*(8*a^4*b^2 - 63*a^2*b^4 + 90*a*b^5 - 35*b^6)*tan(f*x + e)^6 - 16*
a^6 + 48*a^5*b - 48*a^4*b^2 + 16*a^3*b^3 + 2*(48*a^5*b - 8*a^4*b^2 - 315*a^
3*b^3 + 450*a^2*b^4 - 175*a*b^5)*tan(f*x + e)^4 + 16*(3*a^6 - 2*a^5*b - 12*
a^4*b^2 + 18*a^3*b^3 - 7*a^2*b^4)*tan(f*x + e)^2 - 3*((63*a^2*b^4 - 90*a*b^
5 + 35*b^6)*tan(f*x + e)^7 + 2*(63*a^3*b^3 - 90*a^2*b^4 + 35*a*b^5)*tan(f*x
 + e)^5 + (63*a^4*b^2 - 90*a^3*b^3 + 35*a^2*b^4)*tan(f*x + e)^3)*sqrt(b/a)*
arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a^7*b^2 -
3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^7 + 2*(a^8*b - 3*a^7*b^2 +
3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^5 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^
3)*f*tan(f*x + e)^3)]
```

giac [A] time = 7.73, size = 261, normalized size = 1.09

$$-\frac{\frac{3\left(63\,a^2b^3-90\,ab^4+35\,b^5\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^7-3\,a^6b+3\,a^5b^2-a^4b^3\right)\sqrt{ab}}-\frac{24\left(fx+e\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{3\left(15\,ab^4\tan\left(fx+e\right)^3-11\,b^5\tan\left(fx+e\right)^3+17\,a^2b^3\tan\left(\ldots\right.}{\left(a^6-2\,a^5b+a^4b^2\right)\left(b\tan\left(fx+e\right)^2+\ldots\right.}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(63*a^2*b^3 - 90*a*b^4 + 35*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b
^3)*sqrt(a*b)) - 24*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 3*(15*a*b^4
*tan(f*x + e)^3 - 11*b^5*tan(f*x + e)^3 + 17*a^2*b^3*tan(f*x + e) - 13*a*b^
4*tan(f*x + e))/((a^6 - 2*a^5*b + a^4*b^2)*(b*tan(f*x + e)^2 + a)^2) - 8*(3
*a*tan(f*x + e)^2 + 9*b*tan(f*x + e)^2 - a)/(a^4*tan(f*x + e)^3))/f
```

maple [A] time = 0.81, size = 413, normalized size = 1.72

$$-\frac{15b^4\left(\tan^3\left(fx+e\right)\right)}{8f\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2a^2}+\frac{13b^5\left(\tan^3\left(fx+e\right)\right)}{4f\,a^3\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{11b^6\left(\tan^3\left(fx+e\right)\right)}{8f\,a^4\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+\ldots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] -15/8/f*b^4/(a-b)^3/(a+b*tan(f*x+e)^2)^2/a^2*tan(f*x+e)^3+13/4/f*b^5/a^3/(a
-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-11/8/f*b^6/a^4/(a-b)^3/(a+b*tan(f*x
+e)^2)^2*tan(f*x+e)^3-17/8/f*b^3/(a-b)^3/(a+b*tan(f*x+e)^2)^2/a*tan(f*x+e)+
15/4/f*b^4/a^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)-13/8/f*b^5/a^3/(a-b)
^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)-63/8/f*b^3/(a-b)^3/a^2/(a*b)^(1/2)*arcta
n(tan(f*x+e)*b/(a*b)^(1/2))+45/4/f*b^4/a^3/(a-b)^3/(a*b)^(1/2)*arctan(tan(f
*x+e)*b/(a*b)^(1/2))-35/8/f*b^5/a^4/(a-b)^3/(a*b)^(1/2)*arctan(tan(f*x+e)*b
/(a*b)^(1/2))-1/3/f/a^3/tan(f*x+e)^3+1/f/a^3/tan(f*x+e)+3/f/a^4/tan(f*x+e)*
b+1/f/(a-b)^3*arctan(tan(f*x+e))
```

maxima [A] time = 0.58, size = 332, normalized size = 1.38

$$-\frac{\frac{3\left(63\,a^{2}b^{3}-90\,ab^{4}+35\,b^{5}\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^{7}-3\,a^{6}b+3\,a^{5}b^{2}-a^{4}b^{3}\right)\sqrt{ab}}-\frac{3\left(8\,a^{3}b^{2}+8\,a^{2}b^{3}-55\,ab^{4}+35\,b^{5}\right)\tan\left(fx+e\right)^{6}-8\,a^{5}+16\,a^{4}b-8\,a^{3}b^{2}+\left(48\,a^{4}b+40\,a^{3}b^{2}-275\,a^{2}b^{3}+1\cdots\right.}{\left(a^{6}b^{2}-2\,a^{5}b^{3}+a^{4}b^{4}\right)\tan\left(fx+e\right)^{7}+2\left(a^{7}b-2\,a^{6}b^{2}+a^{5}b^{3}\right)\tan\left(fx+e\cdots\right.}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(3*(63*a^2*b^3 - 90*a*b^4 + 35*b^5)*arctan(b*tan(f*x + e)/sqrt(a*b))/
((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*sqrt(a*b)) - (3*(8*a^3*b^2 + 8*a^2*b
^3 - 55*a*b^4 + 35*b^5)*tan(f*x + e)^6 - 8*a^5 + 16*a^4*b - 8*a^3*b^2 + (48
*a^4*b + 40*a^3*b^2 - 275*a^2*b^3 + 175*a*b^4)*tan(f*x + e)^4 + 8*(3*a^5 +
a^4*b - 11*a^3*b^2 + 7*a^2*b^3)*tan(f*x + e)^2)/((a^6*b^2 - 2*a^5*b^3 + a^4
*b^4)*tan(f*x + e)^7 + 2*(a^7*b - 2*a^6*b^2 + a^5*b^3)*tan(f*x + e)^5 + (a^
8 - 2*a^7*b + a^6*b^2)*tan(f*x + e)^3) - 24*(f*x + e)/(a^3 - 3*a^2*b + 3*a*
b^2 - b^3))/f
```

mupad [B] time = 15.36, size = 986, normalized size = 4.11

$$2\,\mathrm{atan}\left(\frac{2\tan\left(e+f\,x\right)\left(\frac{262144\,a^{33}\,b^{2}-2883584\,a^{32}\,b^{3}+14155776\,a^{31}\,b^{4}-40370176\,a^{30}\,b^{5}+72089600\,a^{29}\,b^{6}-77856768\,a^{28}\,b^{7}+34603008\,a^{27}\,b^{8}+34603008\,a^{26}\,b^{9}-7785\cdots}{\left(2\,a^{3}-6\,a^{2}\,b+6\,a\,b^{2}-2\,b^{3}\right)^{2}}\cdots\right.}{\left(2\,a^{3}-6\,a^{2}\,b+6\,a\,b^{2}-2\,b^{3}\right)\left(\frac{2\left(131072\,a^{30}\,b^{2}-1179648\,a^{29}\,b^{3}+4718592\,a^{28}\,b^{4}-12042240\,a^{27}\,b^{5}+27279360\,a^{26}\,b^{6}-6751846\cdots\right.}{\cdots}\right.}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] (2*atan((2*tan(e + f*x)*((262144*a^20*b^15 - 2883584*a^21*b^14 + 14155776*a
^22*b^13 - 40370176*a^23*b^12 + 72089600*a^24*b^11 - 77856768*a^25*b^10 + 3
4603008*a^26*b^9 + 34603008*a^27*b^8 - 77856768*a^28*b^7 + 72089600*a^29*b^
6 - 40370176*a^30*b^5 + 14155776*a^31*b^4 - 2883584*a^32*b^3 + 262144*a^33*
b^2)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)^2 - 1254400*a^12*b^17 + 13977600*a
^13*b^16 - 70333440*a^14*b^15 + 210329600*a^15*b^14 - 413730816*a^16*b^13 +
 559067136*a^17*b^12 - 525322240*a^18*b^11 + 338780160*a^19*b^10 - 14351257
6*a^20*b^9 + 36390912*a^21*b^8 - 5047296*a^22*b^7 + 1310720*a^23*b^6 - 9830
40*a^24*b^5 + 393216*a^25*b^4 - 65536*a^26*b^3))/((6*a*b^2 - 6*a^2*b + 2*a^
3 - 2*b^3)*((2*(573440*a^16*b^16 - 6635520*a^17*b^15 + 34947072*a^18*b^14 -
 110542848*a^19*b^13 + 233275392*a^20*b^12 - 344883200*a^21*b^11 + 36519936
0*a^22*b^10 - 279281664*a^23*b^9 + 155959296*a^24*b^8 - 67518464*a^25*b^7 +
 27279360*a^26*b^6 - 12042240*a^27*b^5 + 4718592*a^28*b^4 - 1179648*a^29*b^
3 + 131072*a^30*b^2))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)^2 + 1254400*a^12*
b^14 - 10214400*a^13*b^13 + 35927040*a^14*b^12 - 70650880*a^15*b^11 + 83495
936*a^16*b^10 - 58242048*a^17*b^9 + 20216832*a^18*b^8 - 17408*a^19*b^7 - 22
85568*a^20*b^6 + 516096*a^21*b^5))))/(f*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)
) + ((tan(e + f*x)^2*(3*a + 7*b))/(3*a^2) - 1/(3*a) + (tan(e + f*x)^6*(35*b
^5 - 55*a*b^4 + 8*a^2*b^3 + 8*a^3*b^2))/(8*a^4*(a^2 - 2*a*b + b^2)) + (tan(
e + f*x)^4*(48*a^3*b - 275*a*b^3 + 175*b^4 + 40*a^2*b^2))/(24*a^3*(a^2 - 2*
a*b + b^2)))/(f*(a^2*tan(e + f*x)^3 + b^2*tan(e + f*x)^7 + 2*a*b*tan(e + f*
x)^5)) - (atan((b^5*tan(e + f*x)*(-a^9*b^5)^(3/2)*1225i - a*b^4*tan(e + f*x
)*(-a^9*b^5)^(3/2)*6300i + a^4*b*tan(e + f*x)*(-a^9*b^5)^(3/2)*3969i + a^18
*b*tan(e + f*x)*(-a^9*b^5)^(1/2)*64i + a^2*b^3*tan(e + f*x)*(-a^9*b^5)^(3/2
)*12510i - a^3*b^2*tan(e + f*x)*(-a^9*b^5)^(3/2)*11340i)/(1225*a^14*b^12 -
6300*a^15*b^11 + 12510*a^16*b^10 - 11340*a^17*b^9 + 3969*a^18*b^8 - 64*a^23
*b^3))*(-a^9*b^5)^(1/2)*(63*a^2 - 90*a*b + 35*b^2)*1i)/(8*f*(3*a^11*b - a^1
2 + a^9*b^3 - 3*a^10*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.249 $\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=297

$$-\frac{b(13a-9b)\cot^5(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{b^{7/2}\left(99a^2-154ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f(a-b)^3}-\frac{\left(8a^2-91ab+63b^2\right)\cot^5}{40a^3f(a-b)^2}$$

```
[Out] -x/(a-b)^3+1/8*b^(7/2)*(99*a^2-154*a*b+63*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^
(1/2))/a^(11/2)/(a-b)^3/f-1/8*(8*a^4+8*a^3*b+8*a^2*b^2-91*a*b^3+63*b^4)*cot
(f*x+e)/a^5/(a-b)^2/f+1/24*(8*a^3+8*a^2*b-91*a*b^2+63*b^3)*cot(f*x+e)^3/a^4
/(a-b)^2/f-1/40*(8*a^2-91*a*b+63*b^2)*cot(f*x+e)^5/a^3/(a-b)^2/f-1/4*b*cot(
f*x+e)^5/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8*(13*a-9*b)*b*cot(f*x+e)^5/a^2/(
a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A] time = 0.47, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3670, 472, 579, 583, 522, 203, 205}

$$\frac{b^{7/2}\left(99a^2-154ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f(a-b)^3}-\frac{\left(8a^2-91ab+63b^2\right)\cot^5(e+fx)}{40a^3f(a-b)^2}+\frac{\left(8a^2b+8a^3-91ab^2+63b\right.}{24a^4f(a-b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(x/(a - b)^3) + (b^(7/2)*(99*a^2 - 154*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e
 + f*x])/Sqrt[a]])/(8*a^(11/2)*(a - b)^3*f) - ((8*a^4 + 8*a^3*b + 8*a^2*b^2
 - 91*a*b^3 + 63*b^4)*Cot[e + f*x])/(8*a^5*(a - b)^2*f) + ((8*a^3 + 8*a^2*b
 - 91*a*b^2 + 63*b^3)*Cot[e + f*x]^3)/(24*a^4*(a - b)^2*f) - ((8*a^2 - 91*a
*b + 63*b^2)*Cot[e + f*x]^5)/(40*a^3*(a - b)^2*f) - (b*Cot[e + f*x]^5)/(4*a
*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - ((13*a - 9*b)*b*Cot[e + f*x]^5)/(8*a
^2*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-9b-9bx^2}{x^6(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(13a-9b)b\cot^5(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \text{Subst}$$

$$= -\frac{(8a^2-91ab+63b^2)\cot^5(e+fx)}{40a^3(a-b)^2f} - \frac{b\cot^5(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(1}{8a^2(a}$$

$$= \frac{(8a^3+8a^2b-91ab^2+63b^3)\cot^3(e+fx)}{24a^4(a-b)^2f} - \frac{(8a^2-91ab+63b^2)\cot^5(e+fx)}{40a^3(a-b)^2f}$$

$$= -\frac{(8a^4+8a^3b+8a^2b^2-91ab^3+63b^4)\cot(e+fx)}{8a^5(a-b)^2f} + \frac{(8a^3+8a^2b-91ab^2+63}{24a^4(a-b)}$$

$$= -\frac{(8a^4+8a^3b+8a^2b^2-91ab^3+63b^4)\cot(e+fx)}{8a^5(a-b)^2f} + \frac{(8a^3+8a^2b-91ab^2+63}{24a^4(a-b)}$$

$$= -\frac{x}{(a-b)^3} + \frac{b^{7/2}(99a^2-154ab+63b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}(a-b)^3f} - \frac{(8a^4+8a^3b+8a}{}$$

Mathematica [B] time = 6.37, size = 949, normalized size = 3.20

$$\frac{\left(-3184\cos(e+fx)a^7 - 1536\cos(3(e+fx))a^7 - 704\cos(5(e+fx))a^7 - 536\cos(7(e+fx))a^7 - 184\cos(9(e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (b^(7/2)*(99*a^2 - 154*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]]
)/(8*a^(11/2)*(a - b)^3*f) + (Csc[e + f*x]^5*(-3184*a^7*Cos[e + f*x] + 7440
*a^6*b*Cos[e + f*x] - 12000*a^5*b^2*Cos[e + f*x] + 10240*a^4*b^3*Cos[e + f*
x] + 6450*a^3*b^4*Cos[e + f*x] + 714*a^2*b^5*Cos[e + f*x] - 22890*a*b^6*Cos
[e + f*x] + 13230*b^7*Cos[e + f*x] - 1536*a^7*Cos[3*(e + f*x)] + 7648*a^6*b
*Cos[3*(e + f*x)] - 2912*a^5*b^2*Cos[3*(e + f*x)] - 1152*a^4*b^3*Cos[3*(e +
 f*x)] - 14872*a^3*b^4*Cos[3*(e + f*x)] - 12796*a^2*b^5*Cos[3*(e + f*x)] +
52080*a*b^6*Cos[3*(e + f*x)] - 26460*b^7*Cos[3*(e + f*x)] - 704*a^7*Cos[5*(
e + f*x)] + 2656*a^6*b*Cos[5*(e + f*x)] - 4128*a^5*b^2*Cos[5*(e + f*x)] - 3
712*a^4*b^3*Cos[5*(e + f*x)] + 5504*a^3*b^4*Cos[5*(e + f*x)] + 27684*a^2*b^
5*Cos[5*(e + f*x)] - 46200*a*b^6*Cos[5*(e + f*x)] + 18900*b^7*Cos[5*(e + f*
x)] - 536*a^7*Cos[7*(e + f*x)] + 248*a^6*b*Cos[7*(e + f*x)] + 768*a^5*b^2*C
os[7*(e + f*x)] + 128*a^4*b^3*Cos[7*(e + f*x)] + 6553*a^3*b^4*Cos[7*(e + f*
x)] - 21441*a^2*b^5*Cos[7*(e + f*x)] + 20895*a*b^6*Cos[7*(e + f*x)] - 6615*
```

```
b^7*Cos[7*(e + f*x)] - 184*a^7*Cos[9*(e + f*x)] + 440*a^6*b*Cos[9*(e + f*x)
] - 160*a^5*b^2*Cos[9*(e + f*x)] + 640*a^4*b^3*Cos[9*(e + f*x)] - 3635*a^3*
b^4*Cos[9*(e + f*x)] + 5839*a^2*b^5*Cos[9*(e + f*x)] - 3885*a*b^6*Cos[9*(e
+ f*x)] + 945*b^7*Cos[9*(e + f*x)] - 720*a^7*(e + f*x)*Sin[e + f*x] - 3360*
a^6*b*(e + f*x)*Sin[e + f*x] - 15120*a^5*b^2*(e + f*x)*Sin[e + f*x] - 480*a
^7*(e + f*x)*Sin[3*(e + f*x)] + 10080*a^5*b^2*(e + f*x)*Sin[3*(e + f*x)] +
480*a^7*(e + f*x)*Sin[5*(e + f*x)] + 1920*a^6*b*(e + f*x)*Sin[5*(e + f*x)]
- 4320*a^5*b^2*(e + f*x)*Sin[5*(e + f*x)] + 120*a^7*(e + f*x)*Sin[7*(e + f*
x)] - 1200*a^6*b*(e + f*x)*Sin[7*(e + f*x)] + 1080*a^5*b^2*(e + f*x)*Sin[7*
(e + f*x)] - 120*a^7*(e + f*x)*Sin[9*(e + f*x)] + 240*a^6*b*(e + f*x)*Sin[9
*(e + f*x)] - 120*a^5*b^2*(e + f*x)*Sin[9*(e + f*x)]))/(7680*a^5*(a - b)^3*
f*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])^2)
```

fricas [A] time = 0.61, size = 1114, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/480*(480*a^5*b^2*f*x*tan(f*x + e)^9 + 960*a^6*b*f*x*tan(f*x + e)^7 + 48
0*a^7*f*x*tan(f*x + e)^5 + 60*(8*a^5*b^2 - 99*a^2*b^5 + 154*a*b^6 - 63*b^7)
*tan(f*x + e)^8 + 96*a^7 - 288*a^6*b + 288*a^5*b^2 - 96*a^4*b^3 + 20*(48*a^
6*b - 8*a^5*b^2 - 495*a^3*b^4 + 770*a^2*b^5 - 315*a*b^6)*tan(f*x + e)^6 + 3
2*(15*a^7 - 10*a^6*b + 3*a^5*b^2 - 99*a^4*b^3 + 154*a^3*b^4 - 63*a^2*b^5)*t
an(f*x + e)^4 - 32*(5*a^7 - 6*a^6*b - 12*a^5*b^2 + 22*a^4*b^3 - 9*a^3*b^4)*
tan(f*x + e)^2 + 15*((99*a^2*b^5 - 154*a*b^6 + 63*b^7)*tan(f*x + e)^9 + 2*(
99*a^3*b^4 - 154*a^2*b^5 + 63*a*b^6)*tan(f*x + e)^7 + (99*a^4*b^3 - 154*a^3
*b^4 + 63*a^2*b^5)*tan(f*x + e)^5)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a
*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b
/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a^8*b^2 - 3*a^7*
b^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^9 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*
b^3 - a^6*b^4)*f*tan(f*x + e)^7 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*
tan(f*x + e)^5), -1/240*(240*a^5*b^2*f*x*tan(f*x + e)^9 + 480*a^6*b*f*x*tan
(f*x + e)^7 + 240*a^7*f*x*tan(f*x + e)^5 + 30*(8*a^5*b^2 - 99*a^2*b^5 + 154
*a*b^6 - 63*b^7)*tan(f*x + e)^8 + 48*a^7 - 144*a^6*b + 144*a^5*b^2 - 48*a^4
*b^3 + 10*(48*a^6*b - 8*a^5*b^2 - 495*a^3*b^4 + 770*a^2*b^5 - 315*a*b^6)*ta
n(f*x + e)^6 + 16*(15*a^7 - 10*a^6*b + 3*a^5*b^2 - 99*a^4*b^3 + 154*a^3*b^4
 - 63*a^2*b^5)*tan(f*x + e)^4 - 16*(5*a^7 - 6*a^6*b - 12*a^5*b^2 + 22*a^4*b
^3 - 9*a^3*b^4)*tan(f*x + e)^2 - 15*((99*a^2*b^5 - 154*a*b^6 + 63*b^7)*tan(
f*x + e)^9 + 2*(99*a^3*b^4 - 154*a^2*b^5 + 63*a*b^6)*tan(f*x + e)^7 + (99*a
^4*b^3 - 154*a^3*b^4 + 63*a^2*b^5)*tan(f*x + e)^5)*sqrt(b/a)*arctan(1/2*(b*
tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a^8*b^2 - 3*a^7*b^3 + 3*
a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^9 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^
6*b^4)*f*tan(f*x + e)^7 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x
+ e)^5)]
```

giac [A] time = 10.51, size = 307, normalized size = 1.03

$$\frac{\frac{15\left(99\,a^2b^4-154\,ab^5+63\,b^6\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{\left(a^8-3\,a^7b+3\,a^6b^2-a^5b^3\right)\sqrt{ab}}-\frac{120\left(fx+e\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{15\left(19\,ab^5\tan\left(fx+e\right)^3-15\,b^6\tan\left(fx+e\right)^3+21\,a^2b^4\tan\cdots\right.}{\left(a^7-2\,a^6b+a^5b^2\right)\left(b\tan\left(fx+e\right)^2\cdots\right.}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(99*a^2*b^4 - 154*a*b^5 + 63*b^6)*(pi*floor((f*x + e)/pi + 1/2)*s
gn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^8 - 3*a^7*b + 3*a^6*b^2 - a^5
```

```
*b^3)*sqrt(a*b)) - 120*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 15*(19*a
*b^5*tan(f*x + e)^3 - 15*b^6*tan(f*x + e)^3 + 21*a^2*b^4*tan(f*x + e) - 17*
a*b^5*tan(f*x + e))/((a^7 - 2*a^6*b + a^5*b^2)*(b*tan(f*x + e)^2 + a)^2) -
8*(15*a^2*tan(f*x + e)^4 + 45*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 -
5*a^2*tan(f*x + e)^2 - 15*a*b*tan(f*x + e)^2 + 3*a^2)/(a^5*tan(f*x + e)^5))
/f
```

maple [A] time = 1.08, size = 466, normalized size = 1.57

$$\frac{19b^5\left(\tan^3\left(fx+e\right)\right)}{8f\,a^3\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}-\frac{17b^6\left(\tan^3\left(fx+e\right)\right)}{4f\,a^4\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^2}+\frac{15b^7\left(\tan^3\left(fx+e\right)\right)}{8f\,a^5\left(a-b\right)^3\left(a+b\left(\tan^2\left(fx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x)
```

```
[Out] 19/8/f*b^5/a^3/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-17/4/f*b^6/a^4/(a-
b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+15/8/f*b^7/a^5/(a-b)^3/(a+b*tan(f*x+
e)^2)^2*tan(f*x+e)^3+21/8/f*b^4/a^2/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)
-19/4/f*b^5/a^3/(a-b)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)+17/8/f*b^6/a^4/(a-b
)^3/(a+b*tan(f*x+e)^2)^2*tan(f*x+e)+99/8/f*b^4/a^3/(a-b)^3/(a*b)^(1/2)*arct
an(tan(f*x+e)*b/(a*b)^(1/2))-77/4/f*b^5/a^4/(a-b)^3/(a*b)^(1/2)*arctan(tan(
f*x+e)*b/(a*b)^(1/2))+63/8/f*b^6/a^5/(a-b)^3/(a*b)^(1/2)*arctan(tan(f*x+e)*
b/(a*b)^(1/2))-1/5/f/a^3/tan(f*x+e)^5+1/3/f/a^3/tan(f*x+e)^3+1/f/a^4/tan(f*
x+e)^3*b-1/f/a^3/tan(f*x+e)-3/f/a^4/tan(f*x+e)*b-6/f/a^5/tan(f*x+e)*b^2-1/f
/(a-b)^3*arctan(tan(f*x+e))
```

maxima [A] time = 1.39, size = 396, normalized size = 1.33

$$\frac{\frac{15\left(99\,a^2b^4-154\,ab^5+63\,b^6\right)\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\left(a^8-3\,a^7b+3\,a^6b^2-a^5b^3\right)\sqrt{ab}}-\frac{15\left(8\,a^4b^2+8\,a^3b^3+8\,a^2b^4-91\,ab^5+63\,b^6\right)\tan\left(fx+e\right)^8+5\left(48\,a^5b+40\,a^4b^2+40\,a^3b^3-455\,a^2b^4+3\right.}{\left(a^7b^2-2\,a^6b^3+a^5b^4\right)\tan\left(f\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(15*(99*a^2*b^4 - 154*a*b^5 + 63*b^6)*arctan(b*tan(f*x + e)/sqrt(a*b)
)/((a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*sqrt(a*b)) - (15*(8*a^4*b^2 + 8*a^
3*b^3 + 8*a^2*b^4 - 91*a*b^5 + 63*b^6)*tan(f*x + e)^8 + 5*(48*a^5*b + 40*a^
4*b^2 + 40*a^3*b^3 - 455*a^2*b^4 + 315*a*b^5)*tan(f*x + e)^6 + 24*a^6 - 48*
a^5*b + 24*a^4*b^2 + 8*(15*a^6 + 5*a^5*b + 8*a^4*b^2 - 91*a^3*b^3 + 63*a^2*
b^4)*tan(f*x + e)^4 - 8*(5*a^6 - a^5*b - 13*a^4*b^2 + 9*a^3*b^3)*tan(f*x +
e)^2)/((a^7*b^2 - 2*a^6*b^3 + a^5*b^4)*tan(f*x + e)^9 + 2*(a^8*b - 2*a^7*b^
2 + a^6*b^3)*tan(f*x + e)^7 + (a^9 - 2*a^8*b + a^7*b^2)*tan(f*x + e)^5) - 1
20*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

mupad [B] time = 16.13, size = 2507, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] (atan((b^5*tan(e + f*x)*(-a^11*b^7)^(3/2)*3969i - a*b^4*tan(e + f*x)*(-a^11
*b^7)^(3/2)*19404i + a^4*b*tan(e + f*x)*(-a^11*b^7)^(3/2)*9801i + a^22*b*ta
n(e + f*x)*(-a^11*b^7)^(1/2)*64i + a^2*b^3*tan(e + f*x)*(-a^11*b^7)^(3/2)*3
6190i - a^3*b^2*tan(e + f*x)*(-a^11*b^7)^(3/2)*30492i)/(3969*a^17*b^15 - 19
```

404*a^18*b^14 + 36190*a^19*b^13 - 30492*a^20*b^12 + 9801*a^21*b^11 - 64*a^2
8*b^4))*(-a^11*b^7)^(1/2)*(99*a^2 - 154*a*b + 63*b^2)*1i)/(8*f*(3*a^13*b -
a^14 + a^11*b^3 - 3*a^12*b^2)) - (1/(5*a) + (tan(e + f*x)^4*(35*a*b + 15*a^
2 + 63*b^2))/(15*a^3) - (tan(e + f*x)^2*(5*a + 9*b))/(15*a^2) + (tan(e + f*
x)^6*(48*a^4*b - 455*a*b^4 + 315*b^5 + 40*a^2*b^3 + 40*a^3*b^2))/(24*a^4*(a
^2 - 2*a*b + b^2)) + (tan(e + f*x)^8*(63*b^6 - 91*a*b^5 + 8*a^2*b^4 + 8*a^3
*b^3 + 8*a^4*b^2))/(8*a^5*(a^2 - 2*a*b + b^2)))/(f*(a^2*tan(e + f*x)^5 + b^
2*tan(e + f*x)^9 + 2*a*b*tan(e + f*x)^7)) - (2*atan(((((1032192*a^20*b^17 -
11812864*a^21*b^16 + 61489152*a^22*b^15 - 192135168*a^23*b^14 + 400392192*
a^24*b^13 - 584220672*a^25*b^12 + 608862208*a^26*b^11 - 452296704*a^27*b^10
+ 231653376*a^28*b^9 - 71122944*a^29*b^8 + 606208*a^30*b^7 + 14893056*a^31
*b^6 - 11010048*a^32*b^5 + 4718592*a^33*b^4 - 1179648*a^34*b^3 + 131072*a^3
5*b^2 + (tan(e + f*x)*(262144*a^25*b^15 - 2883584*a^26*b^14 + 14155776*a^27
*b^13 - 40370176*a^28*b^12 + 72089600*a^29*b^11 - 77856768*a^30*b^10 + 3460
3008*a^31*b^9 + 34603008*a^32*b^8 - 77856768*a^33*b^7 + 72089600*a^34*b^6 -
40370176*a^35*b^5 + 14155776*a^36*b^4 - 2883584*a^37*b^3 + 262144*a^38*b^2
)*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 -
2*b^3) + tan(e + f*x)*(4064256*a^15*b^19 - 44255232*a^16*b^18 + 217240576*
a^17*b^17 - 632905728*a^18*b^16 + 1211615232*a^19*b^15 - 1592176640*a^20*b^
14 + 1454180352*a^21*b^13 - 911302656*a^22*b^12 + 374944768*a^23*b^11 - 914
41152*a^24*b^10 + 10101760*a^25*b^9 - 393216*a^26*b^8 + 983040*a^27*b^7 - 1
310720*a^28*b^6 + 983040*a^29*b^5 - 393216*a^30*b^4 + 65536*a^31*b^3))/(6*a
*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (((1032192*a^20*b^17 - 11812864*a^21*b^16
+ 61489152*a^22*b^15 - 192135168*a^23*b^14 + 400392192*a^24*b^13 - 5842206
72*a^25*b^12 + 608862208*a^26*b^11 - 452296704*a^27*b^10 + 231653376*a^28*b
^9 - 71122944*a^29*b^8 + 606208*a^30*b^7 + 14893056*a^31*b^6 - 11010048*a^3
2*b^5 + 4718592*a^33*b^4 - 1179648*a^34*b^3 + 131072*a^35*b^2 - (tan(e + f*
x)*(262144*a^25*b^15 - 2883584*a^26*b^14 + 14155776*a^27*b^13 - 40370176*a^
28*b^12 + 72089600*a^29*b^11 - 77856768*a^30*b^10 + 34603008*a^31*b^9 + 346
03008*a^32*b^8 - 77856768*a^33*b^7 + 72089600*a^34*b^6 - 40370176*a^35*b^5
+ 14155776*a^36*b^4 - 2883584*a^37*b^3 + 262144*a^38*b^2)*1i)/(6*a*b^2 - 6*
a^2*b + 2*a^3 - 2*b^3))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - tan(e + f
x)(4064256*a^15*b^19 - 44255232*a^16*b^18 + 217240576*a^17*b^17 - 6329057
28*a^18*b^16 + 1211615232*a^19*b^15 - 1592176640*a^20*b^14 + 1454180352*a^2
1*b^13 - 911302656*a^22*b^12 + 374944768*a^23*b^11 - 91441152*a^24*b^10 + 1
0101760*a^25*b^9 - 393216*a^26*b^8 + 983040*a^27*b^7 - 1310720*a^28*b^6 + 9
83040*a^29*b^5 - 393216*a^30*b^4 + 65536*a^31*b^3))/(6*a*b^2 - 6*a^2*b + 2*
a^3 - 2*b^3))/(((((1032192*a^20*b^17 - 11812864*a^21*b^16 + 61489152*a^22*b
^15 - 192135168*a^23*b^14 + 400392192*a^24*b^13 - 584220672*a^25*b^12 + 608
862208*a^26*b^11 - 452296704*a^27*b^10 + 231653376*a^28*b^9 - 71122944*a^29
*b^8 + 606208*a^30*b^7 + 14893056*a^31*b^6 - 11010048*a^32*b^5 + 4718592*a^
33*b^4 - 1179648*a^34*b^3 + 131072*a^35*b^2 + (tan(e + f*x)*(262144*a^25*b^
15 - 2883584*a^26*b^14 + 14155776*a^27*b^13 - 40370176*a^28*b^12 + 72089600
*a^29*b^11 - 77856768*a^30*b^10 + 34603008*a^31*b^9 + 34603008*a^32*b^8 - 7
7856768*a^33*b^7 + 72089600*a^34*b^6 - 40370176*a^35*b^5 + 14155776*a^36*b^
4 - 2883584*a^37*b^3 + 262144*a^38*b^2)*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*
b^3))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + tan(e + f*x)*(4064256*a^15*
b^19 - 44255232*a^16*b^18 + 217240576*a^17*b^17 - 632905728*a^18*b^16 + 121
1615232*a^19*b^15 - 1592176640*a^20*b^14 + 1454180352*a^21*b^13 - 911302656
*a^22*b^12 + 374944768*a^23*b^11 - 91441152*a^24*b^10 + 10101760*a^25*b^9 -
393216*a^26*b^8 + 983040*a^27*b^7 - 1310720*a^28*b^6 + 983040*a^29*b^5 - 3
93216*a^30*b^4 + 65536*a^31*b^3))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) +
((((1032192*a^20*b^17 - 11812864*a^21*b^16 + 61489152*a^22*b^15 - 19213516
8*a^23*b^14 + 400392192*a^24*b^13 - 584220672*a^25*b^12 + 608862208*a^26*b^
11 - 452296704*a^27*b^10 + 231653376*a^28*b^9 - 71122944*a^29*b^8 + 606208*
a^30*b^7 + 14893056*a^31*b^6 - 11010048*a^32*b^5 + 4718592*a^33*b^4 - 11796
48*a^34*b^3 + 131072*a^35*b^2 - (tan(e + f*x)*(262144*a^25*b^15 - 2883584*a
^26*b^14 + 14155776*a^27*b^13 - 40370176*a^28*b^12 + 72089600*a^29*b^11 - 7
7856768*a^30*b^10 + 34603008*a^31*b^9 + 34603008*a^32*b^8 - 77856768*a^33*b

```
^7 + 72089600*a^34*b^6 - 40370176*a^35*b^5 + 14155776*a^36*b^4 - 2883584*a^
37*b^3 + 262144*a^38*b^2)*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))*1i)/(6*a
*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - tan(e + f*x)*(4064256*a^15*b^19 - 4425523
2*a^16*b^18 + 217240576*a^17*b^17 - 632905728*a^18*b^16 + 1211615232*a^19*b
^15 - 1592176640*a^20*b^14 + 1454180352*a^21*b^13 - 911302656*a^22*b^12 + 3
74944768*a^23*b^11 - 91441152*a^24*b^10 + 10101760*a^25*b^9 - 393216*a^26*b
^8 + 983040*a^27*b^7 - 1310720*a^28*b^6 + 983040*a^29*b^5 - 393216*a^30*b^4
 + 65536*a^31*b^3))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - 4064256*a^15*
b^16 + 32062464*a^16*b^15 - 108860416*a^17*b^14 + 206072832*a^18*b^13 - 234
753024*a^19*b^12 + 161354752*a^20*b^11 - 64142336*a^21*b^10 + 16180224*a^22
*b^9 - 6733824*a^23*b^8 + 3694592*a^24*b^7 - 811008*a^25*b^6)))/(f*(6*a*b^2
 - 6*a^2*b + 2*a^3 - 2*b^3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

3.250 $\int \left(a+b\tan^2(c+dx)\right)^4\,dx$

Optimal. Leaf size=115

$$\frac{b^2\left(6a^2-4ab+b^2\right)\tan^3(c+dx)}{3d}+\frac{b(2a-b)\left(2a^2-2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^3(4a-b)\tan^5(c+dx)}{5d}+x(a-b)^4+\frac{b^4 \mathrm{ta}}{}$$

```
[Out] (a-b)^4*x+(2*a-b)*b*(2*a^2-2*a*b+b^2)*tan(d*x+c)/d+1/3*b^2*(6*a^2-4*a*b+b^2
)*tan(d*x+c)^3/d+1/5*(4*a-b)*b^3*tan(d*x+c)^5/d+1/7*b^4*tan(d*x+c)^7/d
```

Rubi [A] time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = $\{3661, 390, 203\}$

$$\frac{b^2\left(6a^2-4ab+b^2\right)\tan^3(c+dx)}{3d}+\frac{b(2a-b)\left(2a^2-2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^3(4a-b)\tan^5(c+dx)}{5d}+x(a-b)^4+\frac{b^4 \mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^4,x]
```

```
[Out] (a - b)^4*x + ((2*a - b)*b*(2*a^2 - 2*a*b + b^2)*Tan[c + d*x])/d + (b^2*(6*
a^2 - 4*a*b + b^2)*Tan[c + d*x]^3)/(3*d) + ((4*a - b)*b^3*Tan[c + d*x]^5)/(
5*d) + (b^4*Tan[c + d*x]^7)/(7*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^4}{1+x^2}dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left((2a-b)b\left(2a^2-2ab+b^2\right)+b^2\left(6a^2-4ab+b^2\right)x^2+(4a-b)b^3x^4+\right.\right.}{d}$$
$$= \frac{(2a-b)b\left(2a^2-2ab+b^2\right)\tan(c+dx)}{d} + \frac{b^2\left(6a^2-4ab+b^2\right)\tan^3(c+dx)}{3d} + \frac{(4a}{}$$
$$= (a-b)^4x + \frac{(2a-b)b\left(2a^2-2ab+b^2\right)\tan(c+dx)}{d} + \frac{b^2\left(6a^2-4ab+b^2\right)\tan^3(c}{3d}$$

Mathematica [A] time = 1.91, size = 137, normalized size = 1.19

$$\frac{\tan(c+dx)\left(b\left(35b\left(6a^2-4ab+b^2\right)\tan^2(c+dx)+105\left(4a^3-6a^2b+4ab^2-b^3\right)+21b^2(4a-b)\tan^4(c+dx)\right.\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^4,x]
```

```
[Out] (Tan[c + d*x]*((105*(a - b)^4*ArcTanh[Sqrt[-Tan[c + d*x]^2]])/Sqrt[-Tan[c +
 d*x]^2] + b*(105*(4*a^3 - 6*a^2*b + 4*a*b^2 - b^3) + 35*b*(6*a^2 - 4*a*b +
 b^2)*Tan[c + d*x]^2 + 21*(4*a - b)*b^2*Tan[c + d*x]^4 + 15*b^3*Tan[c + d*x
]^6)))/(105*d)
```

fricas [A] time = 0.41, size = 134, normalized size = 1.17

$$\frac{15\,b^4\tan\left(dx+c\right)^7+21\left(4\,ab^3-b^4\right)\tan\left(dx+c\right)^5+35\left(6\,a^2b^2-4\,ab^3+b^4\right)\tan\left(dx+c\right)^3+105\left(a^4-4\,a^3b+6\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^4*tan(d*x + c)^7 + 21*(4*a*b^3 - b^4)*tan(d*x + c)^5 + 35*(6*a^
2*b^2 - 4*a*b^3 + b^4)*tan(d*x + c)^3 + 105*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*d*x + 105*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*tan(d*x + c))/
d
```

giac [B] time = 35.01, size = 2209, normalized size = 19.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(105*a^4*d*x*tan(d*x)^7*tan(c)^7 - 420*a^3*b*d*x*tan(d*x)^7*tan(c)^7
+ 630*a^2*b^2*d*x*tan(d*x)^7*tan(c)^7 - 420*a*b^3*d*x*tan(d*x)^7*tan(c)^7 +
 105*b^4*d*x*tan(d*x)^7*tan(c)^7 - 735*a^4*d*x*tan(d*x)^6*tan(c)^6 + 2940*a
^3*b*d*x*tan(d*x)^6*tan(c)^6 - 4410*a^2*b^2*d*x*tan(d*x)^6*tan(c)^6 + 2940*
a*b^3*d*x*tan(d*x)^6*tan(c)^6 - 735*b^4*d*x*tan(d*x)^6*tan(c)^6 - 420*a^3*b
*tan(d*x)^7*tan(c)^6 + 630*a^2*b^2*tan(d*x)^7*tan(c)^6 - 420*a*b^3*tan(d*x)
^7*tan(c)^6 + 105*b^4*tan(d*x)^7*tan(c)^6 - 420*a^3*b*tan(d*x)^6*tan(c)^7 +
 630*a^2*b^2*tan(d*x)^6*tan(c)^7 - 420*a*b^3*tan(d*x)^6*tan(c)^7 + 105*b^4*
```

```
tan(d*x)^6*tan(c)^7 + 2205*a^4*d*x*tan(d*x)^5*tan(c)^5 - 8820*a^3*b*d*x*tan
(d*x)^5*tan(c)^5 + 13230*a^2*b^2*d*x*tan(d*x)^5*tan(c)^5 - 8820*a*b^3*d*x*t
an(d*x)^5*tan(c)^5 + 2205*b^4*d*x*tan(d*x)^5*tan(c)^5 - 210*a^2*b^2*tan(d*x
)^7*tan(c)^4 + 140*a*b^3*tan(d*x)^7*tan(c)^4 - 35*b^4*tan(d*x)^7*tan(c)^4 +
 2520*a^3*b*tan(d*x)^6*tan(c)^5 - 4410*a^2*b^2*tan(d*x)^6*tan(c)^5 + 2940*a
*b^3*tan(d*x)^6*tan(c)^5 - 735*b^4*tan(d*x)^6*tan(c)^5 + 2520*a^3*b*tan(d*x
)^5*tan(c)^6 - 4410*a^2*b^2*tan(d*x)^5*tan(c)^6 + 2940*a*b^3*tan(d*x)^5*tan
(c)^6 - 735*b^4*tan(d*x)^5*tan(c)^6 - 210*a^2*b^2*tan(d*x)^4*tan(c)^7 + 140
*a*b^3*tan(d*x)^4*tan(c)^7 - 35*b^4*tan(d*x)^4*tan(c)^7 - 3675*a^4*d*x*tan(
d*x)^4*tan(c)^4 + 14700*a^3*b*d*x*tan(d*x)^4*tan(c)^4 - 22050*a^2*b^2*d*x*t
an(d*x)^4*tan(c)^4 + 14700*a*b^3*d*x*tan(d*x)^4*tan(c)^4 - 3675*b^4*d*x*tan
(d*x)^4*tan(c)^4 - 84*a*b^3*tan(d*x)^7*tan(c)^2 + 21*b^4*tan(d*x)^7*tan(c)^
2 + 840*a^2*b^2*tan(d*x)^6*tan(c)^3 - 980*a*b^3*tan(d*x)^6*tan(c)^3 + 245*b
^4*tan(d*x)^6*tan(c)^3 - 6300*a^3*b*tan(d*x)^5*tan(c)^4 + 11970*a^2*b^2*tan
(d*x)^5*tan(c)^4 - 8820*a*b^3*tan(d*x)^5*tan(c)^4 + 2205*b^4*tan(d*x)^5*tan
(c)^4 - 6300*a^3*b*tan(d*x)^4*tan(c)^5 + 11970*a^2*b^2*tan(d*x)^4*tan(c)^5
- 8820*a*b^3*tan(d*x)^4*tan(c)^5 + 2205*b^4*tan(d*x)^4*tan(c)^5 + 840*a^2*b
^2*tan(d*x)^3*tan(c)^6 - 980*a*b^3*tan(d*x)^3*tan(c)^6 + 245*b^4*tan(d*x)^3
*tan(c)^6 - 84*a*b^3*tan(d*x)^2*tan(c)^7 + 21*b^4*tan(d*x)^2*tan(c)^7 + 367
5*a^4*d*x*tan(d*x)^3*tan(c)^3 - 14700*a^3*b*d*x*tan(d*x)^3*tan(c)^3 + 22050
*a^2*b^2*d*x*tan(d*x)^3*tan(c)^3 - 14700*a*b^3*d*x*tan(d*x)^3*tan(c)^3 + 36
75*b^4*d*x*tan(d*x)^3*tan(c)^3 - 15*b^4*tan(d*x)^7 + 168*a*b^3*tan(d*x)^6*t
an(c) - 147*b^4*tan(d*x)^6*tan(c) - 1260*a^2*b^2*tan(d*x)^5*tan(c)^2 + 1680
*a*b^3*tan(d*x)^5*tan(c)^2 - 735*b^4*tan(d*x)^5*tan(c)^2 + 8400*a^3*b*tan(d
*x)^4*tan(c)^3 - 16380*a^2*b^2*tan(d*x)^4*tan(c)^3 + 12600*a*b^3*tan(d*x)^4
*tan(c)^3 - 3675*b^4*tan(d*x)^4*tan(c)^3 + 8400*a^3*b*tan(d*x)^3*tan(c)^4 -
 16380*a^2*b^2*tan(d*x)^3*tan(c)^4 + 12600*a*b^3*tan(d*x)^3*tan(c)^4 - 3675
*b^4*tan(d*x)^3*tan(c)^4 - 1260*a^2*b^2*tan(d*x)^2*tan(c)^5 + 1680*a*b^3*ta
n(d*x)^2*tan(c)^5 - 735*b^4*tan(d*x)^2*tan(c)^5 + 168*a*b^3*tan(d*x)*tan(c)
^6 - 147*b^4*tan(d*x)*tan(c)^6 - 15*b^4*tan(c)^7 - 2205*a^4*d*x*tan(d*x)^2*
tan(c)^2 + 8820*a^3*b*d*x*tan(d*x)^2*tan(c)^2 - 13230*a^2*b^2*d*x*tan(d*x)^
2*tan(c)^2 + 8820*a*b^3*d*x*tan(d*x)^2*tan(c)^2 - 2205*b^4*d*x*tan(d*x)^2*t
an(c)^2 - 84*a*b^3*tan(d*x)^5 + 21*b^4*tan(d*x)^5 + 840*a^2*b^2*tan(d*x)^4*
tan(c) - 980*a*b^3*tan(d*x)^4*tan(c) + 245*b^4*tan(d*x)^4*tan(c) - 6300*a^3
*b*tan(d*x)^3*tan(c)^2 + 11970*a^2*b^2*tan(d*x)^3*tan(c)^2 - 8820*a*b^3*tan
(d*x)^3*tan(c)^2 + 2205*b^4*tan(d*x)^3*tan(c)^2 - 6300*a^3*b*tan(d*x)^2*tan
(c)^3 + 11970*a^2*b^2*tan(d*x)^2*tan(c)^3 - 8820*a*b^3*tan(d*x)^2*tan(c)^3
+ 2205*b^4*tan(d*x)^2*tan(c)^3 + 840*a^2*b^2*tan(d*x)*tan(c)^4 - 980*a*b^3*
tan(d*x)*tan(c)^4 + 245*b^4*tan(d*x)*tan(c)^4 - 84*a*b^3*tan(c)^5 + 21*b^4*
tan(c)^5 + 735*a^4*d*x*tan(d*x)*tan(c) - 2940*a^3*b*d*x*tan(d*x)*tan(c) + 4
410*a^2*b^2*d*x*tan(d*x)*tan(c) - 2940*a*b^3*d*x*tan(d*x)*tan(c) + 735*b^4*
d*x*tan(d*x)*tan(c) - 210*a^2*b^2*tan(d*x)^3 + 140*a*b^3*tan(d*x)^3 - 35*b^
4*tan(d*x)^3 + 2520*a^3*b*tan(d*x)^2*tan(c) - 4410*a^2*b^2*tan(d*x)^2*tan(c
) + 2940*a*b^3*tan(d*x)^2*tan(c) - 735*b^4*tan(d*x)^2*tan(c) + 2520*a^3*b*t
an(d*x)*tan(c)^2 - 4410*a^2*b^2*tan(d*x)*tan(c)^2 + 2940*a*b^3*tan(d*x)*tan
(c)^2 - 735*b^4*tan(d*x)*tan(c)^2 - 210*a^2*b^2*tan(c)^3 + 140*a*b^3*tan(c)
^3 - 35*b^4*tan(c)^3 - 105*a^4*d*x + 420*a^3*b*d*x - 630*a^2*b^2*d*x + 420*
a*b^3*d*x - 105*b^4*d*x - 420*a^3*b*tan(d*x) + 630*a^2*b^2*tan(d*x) - 420*a
*b^3*tan(d*x) + 105*b^4*tan(d*x) - 420*a^3*b*tan(c) + 630*a^2*b^2*tan(c) -
420*a*b^3*tan(c) + 105*b^4*tan(c))/(d*tan(d*x)^7*tan(c)^7 - 7*d*tan(d*x)^6*
tan(c)^6 + 21*d*tan(d*x)^5*tan(c)^5 - 35*d*tan(d*x)^4*tan(c)^4 + 35*d*tan(d
*x)^3*tan(c)^3 - 21*d*tan(d*x)^2*tan(c)^2 + 7*d*tan(d*x)*tan(c) - d)
```

maple [B] time = 0.04, size = 242, normalized size = 2.10

$$\frac{b^4\left(\tan^7(dx+c)\right)}{7d}+\frac{4\left(\tan^5(dx+c)\right)a\,b^3}{5d}-\frac{\left(\tan^5(dx+c)\right)b^4}{5d}+\frac{2\left(\tan^3(dx+c)\right)a^2b^2}{d}-\frac{4\left(\tan^3(dx+c)\right)a\,b^3}{3d}+\frac{b^4\left(\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^2)^4,x)
```

```
[Out] 1/7*b^4*tan(d*x+c)^7/d+4/5/d*tan(d*x+c)^5*a*b^3-1/5/d*tan(d*x+c)^5*b^4+2/d*
tan(d*x+c)^3*a^2*b^2-4/3/d*tan(d*x+c)^3*a*b^3+1/3*b^4*tan(d*x+c)^3/d+4/d*a^
3*b*tan(d*x+c)-6*a^2*b^2*tan(d*x+c)/d+4*a*b^3*tan(d*x+c)/d-1/d*b^4*tan(d*x+
c)+1/d*arctan(tan(d*x+c))*a^4-4/d*arctan(tan(d*x+c))*a^3*b+6/d*arctan(tan(d
*x+c))*a^2*b^2-4/d*arctan(tan(d*x+c))*a*b^3+1/d*arctan(tan(d*x+c))*b^4
```

maxima [A] time = 0.42, size = 162, normalized size = 1.41

$$a^4x-\frac{4\,(dx+c-\tan(dx+c))a^3b}{d}+\frac{2\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^2b^2}{d}+\frac{4\left(3\,\tan(dx+c)^5-5\,\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x - 4*(d*x + c - tan(d*x + c))*a^3*b/d + 2*(tan(d*x + c)^3 + 3*d*x + 3*
c - 3*tan(d*x + c))*a^2*b^2/d + 4/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 -
 15*d*x - 15*c + 15*tan(d*x + c))*a*b^3/d + 1/105*(15*tan(d*x + c)^7 - 21*t
an(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105*c - 105*tan(d*x + c))*b^4
/d
```

mupad [B] time = 11.38, size = 164, normalized size = 1.43

$$\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a-b)^4}{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}\right)\,(a-b)^4}{d}+\frac{b^4\tan(c+d\,x)^7}{7\,d}+\frac{\tan(c+d\,x)^3\,\left(2\,a^2\,b^2-\frac{4\,a\,b^3}{3}+\frac{b^4}{3}\right)}{d}+\frac{\tan(c+d\,x)^5\,\left(\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^4,x)
```

```
[Out] (atan((tan(c + d*x)*(a - b)^4)/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))
*(a - b)^4)/d + (b^4*tan(c + d*x)^7)/(7*d) + (tan(c + d*x)^3*(b^4/3 - (4*a*
b^3)/3 + 2*a^2*b^2))/d + (tan(c + d*x)^5*((4*a*b^3)/5 - b^4/5))/d + (tan(c
+ d*x)*(4*a*b^3 + 4*a^3*b - b^4 - 6*a^2*b^2))/d
```

sympy [A] time = 1.27, size = 209, normalized size = 1.82

$$\begin{cases} a^4x-4a^3bx+\frac{4a^3b\tan(c+dx)}{d}+6a^2b^2x+\frac{2a^2b^2\tan^3(c+dx)}{d}-\frac{6a^2b^2\tan(c+dx)}{d}-4ab^3x+\frac{4ab^3\tan^5(c+dx)}{5d}-\frac{4ab^3\tan^3(c+dx)}{3d} \\ x\left(a+b\tan^2(c)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**2)**4,x)
```

```
[Out] Piecewise((a**4*x - 4*a**3*b*x + 4*a**3*b*tan(c + d*x)/d + 6*a**2*b**2*x +
2*a**2*b**2*tan(c + d*x)**3/d - 6*a**2*b**2*tan(c + d*x)/d - 4*a*b**3*x + 4
*a*b**3*tan(c + d*x)**5/(5*d) - 4*a*b**3*tan(c + d*x)**3/(3*d) + 4*a*b**3*t
an(c + d*x)/d + b**4*x + b**4*tan(c + d*x)**7/(7*d) - b**4*tan(c + d*x)**5/
(5*d) + b**4*tan(c + d*x)**3/(3*d) - b**4*tan(c + d*x)/d, Ne(d, 0)), (x*(a
+ b*tan(c)**2)**4, True))
```

3.251 $\int \left(a+b\tan^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=77

$$\frac{b\left(3a^2-3ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2(3a-b)\tan^3(c+dx)}{3d}+x(a-b)^3+\frac{b^3\tan^5(c+dx)}{5d}$$

```
[Out] (a-b)^3*x+b*(3*a^2-3*a*b+b^2)*tan(d*x+c)/d+1/3*(3*a-b)*b^2*tan(d*x+c)^3/d+1
/5*b^3*tan(d*x+c)^5/d
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 390, 203}

$$\frac{b\left(3a^2-3ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2(3a-b)\tan^3(c+dx)}{3d}+x(a-b)^3+\frac{b^3\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^3,x]
```

```
[Out] (a - b)^3*x + (b*(3*a^2 - 3*a*b + b^2)*Tan[c + d*x])/d + ((3*a - b)*b^2*Tan
[c + d*x]^3)/(3*d) + (b^3*Tan[c + d*x]^5)/(5*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tan^2(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(b\left(3a^2-3ab+b^2\right)+(3a-b)b^2x^2+b^3x^4+\frac{(a-b)^3}{1+x^2}\right)dx,x,\tan(c+dx)\right)}{d}\\
&= \frac{b\left(3a^2-3ab+b^2\right)\tan(c+dx)}{d}+\frac{(3a-b)b^2\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}+\frac{(a-}{}\\
&= (a-b)^3x+\frac{b\left(3a^2-3ab+b^2\right)\tan(c+dx)}{d}+\frac{(3a-b)b^2\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+}{5d}
\end{aligned}$$

Mathematica [A] time = 0.93, size = 102, normalized size = 1.32

$$\frac{\tan(c+dx)\left(b\left(45a^2-15ab\left(3-\tan^2(c+dx)\right)+b^2\left(3\tan^4(c+dx)-5\tan^2(c+dx)+15\right)\right)+\frac{15(a-b)^3\tanh^{-1}\left(\sqrt{-\ldots}\right.}{\sqrt{-\tan^2(c+\ldots}}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^3,x]
```

```
[Out] (Tan[c + d*x]*((15*(a - b)^3*ArcTanh[Sqrt[-Tan[c + d*x]^2]])/Sqrt[-Tan[c +
d*x]^2] + b*(45*a^2 - 15*a*b*(3 - Tan[c + d*x]^2) + b^2*(15 - 5*Tan[c + d*x
]^2 + 3*Tan[c + d*x]^4))))/(15*d)
```

fricas [A] time = 0.42, size = 90, normalized size = 1.17

$$\frac{3\,b^3\tan\left(dx+c\right)^5+5\left(3\,ab^2-b^3\right)\tan\left(dx+c\right)^3+15\left(a^3-3\,a^2b+3\,ab^2-b^3\right)dx+15\left(3\,a^2b-3\,ab^2+b^3\right)\tan\left(\ldots\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^3*tan(d*x + c)^5 + 5*(3*a*b^2 - b^3)*tan(d*x + c)^3 + 15*(a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*d*x + 15*(3*a^2*b - 3*a*b^2 + b^3)*tan(d*x + c))/d
```

giac [B] time = 5.62, size = 1027, normalized size = 13.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*a^3*d*x*tan(d*x)^5*tan(c)^5 - 45*a^2*b*d*x*tan(d*x)^5*tan(c)^5 + 4
5*a*b^2*d*x*tan(d*x)^5*tan(c)^5 - 15*b^3*d*x*tan(d*x)^5*tan(c)^5 - 75*a^3*d
*x*tan(d*x)^4*tan(c)^4 + 225*a^2*b*d*x*tan(d*x)^4*tan(c)^4 - 225*a*b^2*d*x*
tan(d*x)^4*tan(c)^4 + 75*b^3*d*x*tan(d*x)^4*tan(c)^4 - 45*a^2*b*tan(d*x)^5*
tan(c)^4 + 45*a*b^2*tan(d*x)^5*tan(c)^4 - 15*b^3*tan(d*x)^5*tan(c)^4 - 45*a
^2*b*tan(d*x)^4*tan(c)^5 + 45*a*b^2*tan(d*x)^4*tan(c)^5 - 15*b^3*tan(d*x)^4
*tan(c)^5 + 150*a^3*d*x*tan(d*x)^3*tan(c)^3 - 450*a^2*b*d*x*tan(d*x)^3*tan(
c)^3 + 450*a*b^2*d*x*tan(d*x)^3*tan(c)^3 - 150*b^3*d*x*tan(d*x)^3*tan(c)^3
- 15*a*b^2*tan(d*x)^5*tan(c)^2 + 5*b^3*tan(d*x)^5*tan(c)^2 + 180*a^2*b*tan(
d*x)^4*tan(c)^3 - 225*a*b^2*tan(d*x)^4*tan(c)^3 + 75*b^3*tan(d*x)^4*tan(c)^
3 + 180*a^2*b*tan(d*x)^3*tan(c)^4 - 225*a*b^2*tan(d*x)^3*tan(c)^4 + 75*b^3*
tan(d*x)^3*tan(c)^4 - 15*a*b^2*tan(d*x)^2*tan(c)^5 + 5*b^3*tan(d*x)^2*tan(c
)^5 - 150*a^3*d*x*tan(d*x)^2*tan(c)^2 + 450*a^2*b*d*x*tan(d*x)^2*tan(c)^2 -
 450*a*b^2*d*x*tan(d*x)^2*tan(c)^2 + 150*b^3*d*x*tan(d*x)^2*tan(c)^2 - 3*b^
3*tan(d*x)^5 + 30*a*b^2*tan(d*x)^4*tan(c) - 25*b^3*tan(d*x)^4*tan(c) - 270*
a^2*b*tan(d*x)^3*tan(c)^2 + 360*a*b^2*tan(d*x)^3*tan(c)^2 - 150*b^3*tan(d*x
)^3*tan(c)^2 - 270*a^2*b*tan(d*x)^2*tan(c)^3 + 360*a*b^2*tan(d*x)^2*tan(c)^
3 - 150*b^3*tan(d*x)^2*tan(c)^3 + 30*a*b^2*tan(d*x)*tan(c)^4 - 25*b^3*tan(d
*x)*tan(c)^4 - 3*b^3*tan(c)^5 + 75*a^3*d*x*tan(d*x)*tan(c) - 225*a^2*b*d*x*
tan(d*x)*tan(c) + 225*a*b^2*d*x*tan(d*x)*tan(c) - 75*b^3*d*x*tan(d*x)*tan(c
) - 15*a*b^2*tan(d*x)^3 + 5*b^3*tan(d*x)^3 + 180*a^2*b*tan(d*x)^2*tan(c) -
225*a*b^2*tan(d*x)^2*tan(c) + 75*b^3*tan(d*x)^2*tan(c) + 180*a^2*b*tan(d*x)
*tan(c)^2 - 225*a*b^2*tan(d*x)*tan(c)^2 + 75*b^3*tan(d*x)*tan(c)^2 - 15*a*b
^2*tan(c)^3 + 5*b^3*tan(c)^3 - 15*a^3*d*x + 45*a^2*b*d*x - 45*a*b^2*d*x + 1
5*b^3*d*x - 45*a^2*b*tan(d*x) + 45*a*b^2*tan(d*x) - 15*b^3*tan(d*x) - 45*a^
2*b*tan(c) + 45*a*b^2*tan(c) - 15*b^3*tan(c))/(d*tan(d*x)^5*tan(c)^5 - 5*d*
```

```
tan(d*x)^4*tan(c)^4 + 10*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 +
 5*d*tan(d*x)*tan(c) - d)
```

maple [B] time = 0.03, size = 154, normalized size = 2.00

$$\frac{b^3\left(\tan^5(dx+c)\right)}{5d}+\frac{a\,b^2\left(\tan^3(dx+c)\right)}{d}-\frac{b^3\left(\tan^3(dx+c)\right)}{3d}+\frac{3a^2b\tan(dx+c)}{d}-\frac{3a\,b^2\tan(dx+c)}{d}+\frac{b^3\tan(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^2)^3,x)
```

```
[Out] 1/5*b^3*tan(d*x+c)^5/d+a*b^2*tan(d*x+c)^3/d-1/3/d*b^3*tan(d*x+c)^3+3/d*a^2*
b*tan(d*x+c)-3*a*b^2*tan(d*x+c)/d+1/d*b^3*tan(d*x+c)+1/d*arctan(tan(d*x+c))
*a^3-3/d*arctan(tan(d*x+c))*a^2*b+3/d*arctan(tan(d*x+c))*a*b^2-1/d*arctan(t
an(d*x+c))*b^3
```

maxima [A] time = 0.91, size = 104, normalized size = 1.35

$$a^3x-\frac{3(dx+c-\tan(dx+c))a^2b}{d}+\frac{\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)ab^2}{d}+\frac{\left(3\,\tan(dx+c)^5-5\,\tan(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 3*(d*x + c - tan(d*x + c))*a^2*b/d + (tan(d*x + c)^3 + 3*d*x + 3*c
- 3*tan(d*x + c))*a*b^2/d + 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*
d*x - 15*c + 15*tan(d*x + c))*b^3/d
```

mupad [B] time = 11.47, size = 115, normalized size = 1.49

$$\frac{b^3\tan(c+d\,x)^5}{5\,d}+\frac{\tan(c+d\,x)\,\left(3\,a^2\,b-3\,a\,b^2+b^3\right)}{d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a-b)^3}{a^3-3\,a^2\,b+3\,a\,b^2-b^3}\right)\,(a-b)^3}{d}+\frac{\tan(c+d\,x)^3\,\left(a\,b^2-\frac{b^3}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^3,x)
```

```
[Out] (b^3*tan(c + d*x)^5)/(5*d) + (tan(c + d*x)*(3*a^2*b - 3*a*b^2 + b^3))/d + (
atan((tan(c + d*x)*(a - b)^3)/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*(a - b)^3)/d
 + (tan(c + d*x)^3*(a*b^2 - b^3/3))/d
```

sympy [A] time = 0.67, size = 126, normalized size = 1.64

$$\begin{cases} a^3x-3a^2bx+\frac{3a^2b\tan(c+dx)}{d}+3ab^2x+\frac{ab^2\tan^3(c+dx)}{d}-\frac{3ab^2\tan(c+dx)}{d}-b^3x+\frac{b^3\tan^5(c+dx)}{5d}-\frac{b^3\tan^3(c+dx)}{3d}+\frac{b^3\tan(c+dx)}{d} \\ x\left(a+b\tan^2(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*x + 3*a**2*b*tan(c + d*x)/d + 3*a*b**2*x + a*b
**2*tan(c + d*x)**3/d - 3*a*b**2*tan(c + d*x)/d - b**3*x + b**3*tan(c + d*x
)**5/(5*d) - b**3*tan(c + d*x)**3/(3*d) + b**3*tan(c + d*x)/d, Ne(d, 0)), (
x*(a + b*tan(c)**2)**3, True))
```

3.252 $\int \left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=46

$$\frac{b(2a-b)\tan(c+dx)}{d}+x(a-b)^2+\frac{b^2\tan^3(c+dx)}{3d}$$

```
[Out] (a-b)^2*x+(2*a-b)*b*tan(d*x+c)/d+1/3*b^2*tan(d*x+c)^3/d
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 390, 203}

$$\frac{b(2a-b)\tan(c+dx)}{d}+x(a-b)^2+\frac{b^2\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a - b)^2*x + ((2*a - b)*b*Tan[c + d*x])/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tan^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{(a-b)^2}{1+x^2}\right)\,dx,x,\tan(c+dx)\right)}{d}\\
&= \frac{(2a-b)b\tan(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(c+
\end{aligned}$$

$$= (a-b)^2x+\frac{(2a-b)b\tan(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

Mathematica [A] time = 0.60, size = 73, normalized size = 1.59

$$\frac{\tan(c+dx)\left(b\left(6a-b\left(3-\tan^2(c+dx)\right)\right)+\frac{3(a-b)^2\tanh^{-1}\left(\sqrt{-\tan^2(c+dx)}\right)}{\sqrt{-\tan^2(c+dx)}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (Tan[c + d*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Tan[c + d*x]^2]])/Sqrt[-Tan[c + d
*x]^2] + b*(6*a - b*(3 - Tan[c + d*x]^2))))/(3*d)
```

fricas [A] time = 0.44, size = 51, normalized size = 1.11

$$\frac{b^2\tan(dx+c)^3+3\left(a^2-2ab+b^2\right)dx+3\left(2ab-b^2\right)\tan(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*d*x + 3*(2*a*b - b^2)*tan(d
*x + c))/d
```

giac [B] time = 1.48, size = 359, normalized size = 7.80

$$\frac{3a^2dx\tan(dx)^3\tan(c)^3-6abdx\tan(dx)^3\tan(c)^3+3b^2dx\tan(dx)^3\tan(c)^3-9a^2dx\tan(dx)^2\tan(c)^2+18abdx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*d*x*tan(d*x)^3*tan(c)^3 - 6*a*b*d*x*tan(d*x)^3*tan(c)^3 + 3*b^2*
d*x*tan(d*x)^3*tan(c)^3 - 9*a^2*d*x*tan(d*x)^2*tan(c)^2 + 18*a*b*d*x*tan(d*
x)^2*tan(c)^2 - 9*b^2*d*x*tan(d*x)^2*tan(c)^2 - 6*a*b*tan(d*x)^3*tan(c)^2 +
 3*b^2*tan(d*x)^3*tan(c)^2 - 6*a*b*tan(d*x)^2*tan(c)^3 + 3*b^2*tan(d*x)^2*t
an(c)^3 + 9*a^2*d*x*tan(d*x)*tan(c) - 18*a*b*d*x*tan(d*x)*tan(c) + 9*b^2*d*
x*tan(d*x)*tan(c) - b^2*tan(d*x)^3 + 12*a*b*tan(d*x)^2*tan(c) - 9*b^2*tan(d
*x)^2*tan(c) + 12*a*b*tan(d*x)*tan(c)^2 - 9*b^2*tan(d*x)*tan(c)^2 - b^2*tan
(c)^3 - 3*a^2*d*x + 6*a*b*d*x - 3*b^2*d*x - 6*a*b*tan(d*x) + 3*b^2*tan(d*x)
 - 6*a*b*tan(c) + 3*b^2*tan(c))/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan
(c)^2 + 3*d*tan(d*x)*tan(c) - d)
```

maple [A] time = 0.04, size = 87, normalized size = 1.89

$$\frac{b^2\left(\tan^3(dx+c)\right)}{3d}+\frac{2ab\tan(dx+c)}{d}-\frac{b^2\tan(dx+c)}{d}+\frac{\arctan(\tan(dx+c))\,a^2}{d}-\frac{2\arctan(\tan(dx+c))\,ab}{d}+\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/3*b^2*tan(d*x+c)^3/d+2*a*b*tan(d*x+c)/d-b^2*tan(d*x+c)/d+1/d*arctan(tan(d
*x+c))*a^2-2/d*arctan(tan(d*x+c))*a*b+1/d*arctan(tan(d*x+c))*b^2
```

maxima [A] time = 1.13, size = 58, normalized size = 1.26

$$a^2x-\frac{2(dx+c-\tan(dx+c))ab}{d}+\frac{\left(\tan(dx+c)^3+3dx+3c-3\tan(dx+c)\right)b^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 2*(d*x + c - tan(d*x + c))*a*b/d + 1/3*(tan(d*x + c)^3 + 3*d*x + 3*
c - 3*tan(d*x + c))*b^2/d
```

mupad [B] time = 11.42, size = 76, normalized size = 1.65

$$\frac{\tan(c+dx)\left(2ab-b^2\right)}{d}+\frac{\operatorname{atan}\left(\frac{\tan(c+dx)(a-b)^2}{a^2-2ab+b^2}\right)(a-b)^2}{d}+\frac{b^2\tan(c+dx)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] (tan(c + d*x)*(2*a*b - b^2))/d + (atan((tan(c + d*x)*(a - b)^2)/(a^2 - 2*a*
b + b^2))*(a - b)^2)/d + (b^2*tan(c + d*x)^3)/(3*d)
```

sympy [A] time = 0.31, size = 68, normalized size = 1.48

$$\begin{cases} a^2x-2abx+\frac{2ab\tan(c+dx)}{d}+b^2x+\frac{b^2\tan^3(c+dx)}{3d}-\frac{b^2\tan(c+dx)}{d} & \text{for } d\neq 0 \\ x\left(a+b\tan^2(c)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*x + 2*a*b*tan(c + d*x)/d + b**2*x + b**2*tan(c +
d*x)**3/(3*d) - b**2*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**2)**2, Tr
ue))
```

3.253 $\int \left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=19

$$ax+\frac{b\tan(c+dx)}{d}-bx$$

```
[Out] a*x-b*x+b*tan(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3473, 8}

$$ax+\frac{b\tan(c+dx)}{d}-bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[c + d*x]^2,x]
```

```
[Out] a*x - b*x + (b*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^2(c+dx)\right)\,dx &= ax+b\int \tan^2(c+dx)\,dx\\ &= ax+\frac{b\tan(c+dx)}{d}-b\int 1\,dx\\ &= ax-bx+\frac{b\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.47

$$ax-\frac{b\tan^{-1}(\tan(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[c + d*x]^2,x]
```

```
[Out] a*x - (b*ArcTan[Tan[c + d*x]])/d + (b*Tan[c + d*x])/d
```

fricas [A] time = 0.43, size = 21, normalized size = 1.11

$$\frac{(a-b)dx+b\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] ((a - b)*d*x + b*tan(d*x + c))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)b*(-4*d*x*tan(c)*tan(d*x)+4*d*x-pi*sign(2*tan(c)^2*tan
(d*x)+2*tan(c)*tan(d*x)^2-2*tan(c)-2*tan(d*x))*tan(c)*tan(d*x)+pi*sign(2*ta
n(c)^2*tan(d*x)+2*tan(c)*tan(d*x)^2-2*tan(c)-2*tan(d*x))-pi*tan(c)*tan(d*x)
+pi+2*atan((tan(c)*tan(d*x)-1)/(tan(c)+tan(d*x)))*tan(c)*tan(d*x)-2*atan((t
an(c)*tan(d*x)-1)/(tan(c)+tan(d*x)))+2*atan((tan(c)+tan(d*x))/(tan(c)*tan(d
*x)-1))*tan(c)*tan(d*x)-2*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))-4*tan
(c)-4*tan(d*x))/(4*d*tan(c)*tan(d*x)-4*d)+a*x
```

maple [A] time = 0.03, size = 29, normalized size = 1.53

$$ax+\frac{b\tan\left(dx+c\right)}{d}-\frac{b\arctan\left(\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(d*x+c)^2,x)
```

```
[Out] a*x+b*tan(d*x+c)/d-1/d*b*arctan(tan(d*x+c))
```

maxima [A] time = 0.82, size = 23, normalized size = 1.21

$$ax-\frac{(dx+c-\tan\left(dx+c\right))b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] a*x - (d*x + c - tan(d*x + c))*b/d
```

mupad [B] time = 11.44, size = 21, normalized size = 1.11

$$\frac{b\tan\left(c+d\,x\right)+d\,x\,\left(a-b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*tan(c + d*x)^2,x)
```

```
[Out] (b*tan(c + d*x) + d*x*(a - b))/d
```

sympy [A] time = 0.14, size = 20, normalized size = 1.05

$$ax+b\left(\begin{cases}-x+\frac{\tan\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\tan^{2}\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)**2,x)
```

```
[Out] a*x + b*Piecewise((-x + tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)**2, True))
```

3.254 $\int \frac{1}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)}$$

[Out] x/(a-b)-arctan(b^(1/2)*tan(d*x+c)/a^(1/2))*b^(1/2)/(a-b)/d/a^(1/2)

Rubi [A] time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3660, 3675, 205}

$$\frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tan[c + d*x]^2)^(-1),x]

[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)*d)

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3660

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b\tan^2(c+dx)}\,dx &= \frac{x}{a-b}-\frac{b\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx}{a-b}\\
&= \frac{x}{a-b}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\tan(c+dx)\right)}{(a-b)d}\\
&= \frac{x}{a-b}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)d}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 49, normalized size = 0.98

$$\frac{\tan^{-1}(\tan(c+dx)) - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}}{ad-bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^(-1),x]
```

```
[Out] (ArcTan[Tan[c + d*x]] - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/Sq
rt[a])/(a*d - b*d)
```

fricas [A] time = 0.48, size = 182, normalized size = 3.64

$$\left[\frac{4\,dx - \sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(dx+c)^4 - 6\,ab\tan(dx+c)^2 + a^2 + 4\left(ab\tan(dx+c)^3 - a^2\tan(dx+c)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(dx+c)^4 + 2\,ab\tan(dx+c)^2 + a^2}\right)}{4\,(a-b)d}, \frac{2\,dx - \sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(dx+c)^2 - a\right)}{2\,b\tan(dx+c)}\right.}{2\,(a-b)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*d*x - sqrt(-b/a)*log((b^2*tan(d*x + c)^4 - 6*a*b*tan(d*x + c)^2 + a
^2 + 4*(a*b*tan(d*x + c)^3 - a^2*tan(d*x + c))*sqrt(-b/a))/(b^2*tan(d*x + c
)^4 + 2*a*b*tan(d*x + c)^2 + a^2)))/((a - b)*d), 1/2*(2*d*x - sqrt(b/a)*arc
tan(1/2*(b*tan(d*x + c)^2 - a)*sqrt(b/a)/(b*tan(d*x + c))))/((a - b)*d)]
```

giac [A] time = 0.82, size = 65, normalized size = 1.30

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,(a-b)} - \frac{dx+c}{a-b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))*
b/(sqrt(a*b)*(a - b)) - (d*x + c)/(a - b))/d
```

maple [A] time = 0.20, size = 52, normalized size = 1.04

$$-\frac{b\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)}{d\,(a-b)\sqrt{ab}} + \frac{\arctan\left(\tan\left(dx+c\right)\right)}{d\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^2),x)
```

```
[Out] -1/d*b/(a-b)/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))+1/d/(a-b)*arctan(
tan(d*x+c))
```

maxima [A] time = 0.78, size = 48, normalized size = 0.96

$$-\frac{\frac{b\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a-b)} - \frac{dx+c}{a-b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (d*x + c)/(a - b
))/d
```

mupad [B] time = 11.90, size = 948, normalized size = 18.96

$$-\frac{\operatorname{atan}\left(\frac{\frac{-4\,b^3\,\tan(c+d\,x)+\frac{\left(4\,b^4-8\,a\,b^3+4\,a^2\,b^2+\frac{\tan(c+d\,x)\,\left(-8\,a^3\,b^2+8\,a^2\,b^3+8\,a\,b^4-8\,b^5\right)\,1i}{2\,a-2\,b}\right)\,1i}{2\,a-2\,b}}{2\,a-2\,b}+\frac{-4\,b^3\,\tan(c+d\,x)+\frac{\left(8\,a\,b^3-4\,b^4-4\,a^2\,b^2+\frac{\tan(c+d\,x)\,\left(-8\,a^3\,b^2+8\,a^2\,b^3+\cdots\right.}{2\,a-2\,b}\cdots\right.}{2\,a-2\,b}}{2\,a-2\,b}}{\frac{\left(-4\,b^3\,\tan(c+d\,x)+\frac{\left(4\,b^4-8\,a\,b^3+4\,a^2\,b^2+\frac{\tan(c+d\,x)\,\left(-8\,a^3\,b^2+8\,a^2\,b^3+8\,a\,b^4-8\,b^5\right)\,1i}{2\,a-2\,b}\right)\,1i}{2\,a-2\,b}\right)\,1i}{2\,a-2\,b}-\frac{\left(-4\,b^3\,\tan(c+d\,x)+\frac{\left(8\,a\,b^3-4\,b^4-4\,a^2\,b^2+\frac{\tan(c+d\,x)\,\left(-8\,a^3\,b^2+8\,a^2\,b^3+\cdots\right.}{2\,a-2\,b}\cdots\right.}{2\,a-2\,b}\cdots\right.}{2\,a-2\,b}}\right.\cdots}{d\,\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^2),x)
```

```
[Out] (atan((((-a*b)^(1/2)*(2*b^3*tan(c + d*x) - ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 +
 2*a^2*b^2 + (tan(c + d*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^
3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))*1i)/(a*b - a^2) + ((-a*b)^(1/2)*
(2*b^3*tan(c + d*x) - ((-a*b)^(1/2)*(4*a*b^3 - 2*b^4 - 2*a^2*b^2 + (tan(c +
 d*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2
))))/(2*(a*b - a^2)))*1i)/(a*b - a^2))/(((-a*b)^(1/2)*(2*b^3*tan(c + d*x) -
 ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 + 2*a^2*b^2 + (tan(c + d*x)*(-a*b)^(1/2)*(8
*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))
)/(a*b - a^2) - ((-a*b)^(1/2)*(2*b^3*tan(c + d*x) - ((-a*b)^(1/2)*(4*a*b^3
- 2*b^4 - 2*a^2*b^2 + (tan(c + d*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b
^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2))))/(a*b - a^2)))*(-a*b)^(
1/2)*1i)/(a*d*(a - b)) - atan(((((4*b^4 - 8*a*b^3 + 4*a^2*b^2 + (tan(c + d*
x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*
b) - 4*b^3*tan(c + d*x))/(2*a - 2*b) + (((8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (ta
n(c + d*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(
2*a - 2*b) - 4*b^3*tan(c + d*x))/(2*a - 2*b))/(((((4*b^4 - 8*a*b^3 + 4*a^2*
b^2 + (tan(c + d*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*
b))*1i)/(2*a - 2*b) - 4*b^3*tan(c + d*x))*1i)/(2*a - 2*b) - ((((8*a*b^3 - 4
*b^4 - 4*a^2*b^2 + (tan(c + d*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*
1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(c + d*x))*1i)/(2*a - 2*b)))/(d
*(a - b))
```

sympy [A] time = 2.40, size = 280, normalized size = 5.60

$$\begin{cases} \frac{\tilde{\infty} x}{\tan^{2}{\left(c \right)}} & \text{for}\: a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x}{a} & \text{for}\: b = 0 \\ \frac{- x - \frac{1}{d \tan{\left(c + d x \right)}}}{b} & \text{for}\: a = 0 \\ \frac{d x \tan^{2}{\left(c + d x \right)}}{2 b d \tan^{2}{\left(c + d x \right)} + 2 b d} + \frac{d x}{2 b d \tan^{2}{\left(c + d x \right)} + 2 b d} + \frac{\tan{\left(c + d x \right)}}{2 b d \tan^{2}{\left(c + d x \right)} + 2 b d} & \text{for}\: a = b \\ \frac{x}{a + b \tan^{2}{\left(c \right)}} & \text{for}\: d = 0 \\ \frac{2 i \sqrt{a} d x \sqrt{\frac{1}{b}}}{2 i a^{\frac{3}{2}} d \sqrt{\frac{1}{b}} - 2 i \sqrt{a} b d \sqrt{\frac{1}{b}}} - \frac{\log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \tan{\left(c + d x \right)} \right)}}{2 i a^{\frac{3}{2}} d \sqrt{\frac{1}{b}} - 2 i \sqrt{a} b d \sqrt{\frac{1}{b}}} + \frac{\log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \tan{\left(c + d x \right)} \right)}}{2 i a^{\frac{3}{2}} d \sqrt{\frac{1}{b}} - 2 i \sqrt{a} b d \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a, Eq(b, 0)
), ((-x - 1/(d*tan(c + d*x)))/b, Eq(a, 0)), (d*x*tan(c + d*x)**2/(2*b*d*tan
(c + d*x)**2 + 2*b*d) + d*x/(2*b*d*tan(c + d*x)**2 + 2*b*d) + tan(c + d*x)/
(2*b*d*tan(c + d*x)**2 + 2*b*d), Eq(a, b)), (x/(a + b*tan(c)**2), Eq(d, 0))
, (2*I*sqrt(a)*d*x*sqrt(1/b)/(2*I*a**(3/2)*d*sqrt(1/b) - 2*I*sqrt(a)*b*d*sq
rt(1/b)) - log(-I*sqrt(a)*sqrt(1/b) + tan(c + d*x))/(2*I*a**(3/2)*d*sqrt(1/
b) - 2*I*sqrt(a)*b*d*sqrt(1/b)) + log(I*sqrt(a)*sqrt(1/b) + tan(c + d*x))/(
2*I*a**(3/2)*d*sqrt(1/b) - 2*I*sqrt(a)*b*d*sqrt(1/b)), True))
```

3.255 $\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{b}\,(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2}-\frac{b\tan(c+dx)}{2ad(a-b)\left(a+b\tan^2(c+dx)\right)}+\frac{x}{(a-b)^2}$$

```
[Out] x/(a-b)^2-1/2*(3*a-b)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))*b^(1/2)/a^(3/2)/(a
-b)^2/d-1/2*b*tan(d*x+c)/a/(a-b)/d/(a+b*tan(d*x+c)^2)
```

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3661, 414, 522, 203, 205}

$$-\frac{\sqrt{b}\,(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2}-\frac{b\tan(c+dx)}{2ad(a-b)\left(a+b\tan^2(c+dx)\right)}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^(-2),x]
```

```
[Out] x/(a - b)^2 - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2
*a^(3/2)*(a - b)^2*d) - (b*Tan[c + d*x])/(2*a*(a - b)*d*(a + b*Tan[c + d*x]
^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
```

```
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)}{2a(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(c+dx)\right)}{2a(a-b)d}$$

$$= -\frac{b\tan(c+dx)}{2a(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a-b)^2d} - \frac{((3a-b}{}$$

$$= \frac{x}{(a-b)^2} - \frac{(3a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2d} - \frac{b\tan(c+dx)}{2a(a-b)d\left(a+b\tan^2(c+dx)\right)}$$

Mathematica [A] time = 1.05, size = 88, normalized size = 0.91

$$\frac{\frac{\sqrt{b}(b-3a)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b(b-a)\tan(c+dx)}{a\left(a+b\tan^2(c+dx)\right)} + 2\tan^{-1}(\tan(c+dx))}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^(-2),x]
```

```
[Out] (2*ArcTan[Tan[c + d*x]] + (Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])
/Sqrt[a]])/a^(3/2) + (b*(-a + b)*Tan[c + d*x])/(a*(a + b*Tan[c + d*x]^2)))/
(2*(a - b)^2*d)
```

fricas [A] time = 0.45, size = 390, normalized size = 4.02

$$\left[\frac{8\,abdx\tan(dx+c)^2 + 8\,a^2dx - \left(\left(3\,ab-b^2\right)\tan(dx+c)^2 + 3\,a^2 - ab\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(dx+c)^4 - 6\,ab\tan(dx+c)^2 + a^2 + 4}{b^2\tan(dx+c)^4 + 2\,a}\right.}{8\left(\left(a^3b - 2\,a^2b^2 + ab^3\right)d\tan(dx+c)^2 + \left(a^4 - 2\,a^3b + a^2b^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*b*d*x*tan(d*x + c)^2 + 8*a^2*d*x - ((3*a*b - b^2)*tan(d*x + c)^2
+ 3*a^2 - a*b)*sqrt(-b/a)*log((b^2*tan(d*x + c)^4 - 6*a*b*tan(d*x + c)^2 +
a^2 + 4*(a*b*tan(d*x + c)^3 - a^2*tan(d*x + c))*sqrt(-b/a))/(b^2*tan(d*x +
c)^4 + 2*a*b*tan(d*x + c)^2 + a^2)) - 4*(a*b - b^2)*tan(d*x + c))/((a^3*b -
 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^2 + (a^4 - 2*a^3*b + a^2*b^2)*d), 1/4*(4
*a*b*d*x*tan(d*x + c)^2 + 4*a^2*d*x - ((3*a*b - b^2)*tan(d*x + c)^2 + 3*a^2
 - a*b)*sqrt(b/a)*arctan(1/2*(b*tan(d*x + c)^2 - a)*sqrt(b/a)/(b*tan(d*x +
c))) - 2*(a*b - b^2)*tan(d*x + c))/((a^3*b - 2*a^2*b^2 + a*b^3)*d*tan(d*x +
 c)^2 + (a^4 - 2*a^3*b + a^2*b^2)*d)]
```

giac [A] time = 0.89, size = 122, normalized size = 1.26

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)(3\,ab-b^2)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(dx+c)}{a^2-2\,ab+b^2}+\frac{b\tan(dx+c)}{(b\tan(dx+c)^2+a)(a^2-ab)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b
)))*(3*a*b - b^2)/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b)) - 2*(d*x + c)/(a^2 -
2*a*b + b^2) + b*tan(d*x + c)/((b*tan(d*x + c)^2 + a)*(a^2 - a*b)))/d
```

maple [A] time = 0.31, size = 160, normalized size = 1.65

$$-\frac{b\tan(dx+c)}{2d(a-b)^2\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{b^2\tan(dx+c)}{2d(a-b)^2a\left(a+b\left(\tan^2(dx+c)\right)\right)}-\frac{3b\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)}{2d(a-b)^2\sqrt{ab}}+\frac{b^2\arctan\left(\frac{\tan(d}{\sqrt{}}\right.}{2d(a-b)^2a\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d*b/(a-b)^2*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2/d*b^2/(a-b)^2/a*tan(d*x+
c)/(a+b*tan(d*x+c)^2)-3/2/d*b/(a-b)^2/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)
^(1/2))+1/2/d*b^2/(a-b)^2/a/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))+1/
d/(a-b)^2*arctan(tan(d*x+c))
```

maxima [A] time = 0.86, size = 114, normalized size = 1.18

$$-\frac{\frac{b\tan(dx+c)}{a^3-a^2b+(a^2b-ab^2)\tan(dx+c)^2}+\frac{(3\,ab-b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(dx+c)}{a^2-2\,ab+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*tan(d*x + c)/(a^3 - a^2*b + (a^2*b - a*b^2)*tan(d*x + c)^2) + (3*a*
b - b^2)*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b
)) - 2*(d*x + c)/(a^2 - 2*a*b + b^2))/d
```

mupad [B] time = 13.15, size = 2489, normalized size = 25.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] (2*atan((((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 -
4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(c + d*x)*(16*a^
2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2
*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2
) + (tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2
)))/(2*a^2 - 4*a*b + 2*b^2) - ((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^
4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) +
 (tan(c + d*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*
b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(
2*a^2 - 4*a*b + 2*b^2) - (tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^
```

```
4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^2))/((((((2*a*b^7 - 12*a^2*b^
6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 +
a^2*b^3 - 3*a^3*b^2) - (tan(c + d*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5
+ 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^
2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(c + d*x)*(b^5 - 6*a
*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^
2) + (((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a
^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(c + d*x)*(16*a^2*b
^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a
^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^
2) - (tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2
*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - ((3*a*b^3)/2 - b^4/2)/(3*a^4*b - a^5 + a^
2*b^3 - 3*a^3*b^2))))/(d*(2*a^2 - 4*a*b + 2*b^2)) - (atan((((-a^3*b)^(1/2)*
((tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2))
- (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^
2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(c + d*x)*(-a^3*b)^(1/2)*(3*
a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16
*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b
)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a - b)*1i)/(4*(a^5 - 2
*a^4*b + a^3*b^2)) + ((-a^3*b)^(1/2)*((tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2
*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)) + (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5
 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^
2) + (tan(c + d*x)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a
^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2
)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b +
 a^3*b^2)))*(3*a - b)*1i)/(4*(a^5 - 2*a^4*b + a^3*b^2)))/(((3*a*b^3)/2 - b^
4/2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + ((-a^3*b)^(1/2)*((tan(c + d*x)
*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)) - (((2*a*b^7 -
 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b -
a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(c + d*x)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2
*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*
(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b)^(1/2)*(3*a -
 b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)
) - ((-a^3*b)^(1/2)*((tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 -
2*a^3*b + a^2*b^2)) + (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 1
8*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(c + d*x
)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b
^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b
+ a^3*b^2)))*(-a^3*b)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a
- b))/(4*(a^5 - 2*a^4*b + a^3*b^2))))*(-a^3*b)^(1/2)*(3*a - b)*1i)/(2*d*(a^
5 - 2*a^4*b + a^3*b^2)) - (b*tan(c + d*x))/(2*a*d*(a + b*tan(c + d*x)^2)*(a
 - b))
```

sympy [A] time = 20.59, size = 2322, normalized size = 23.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x + 1/(d*tan
(c + d*x)) - 1/(3*d*tan(c + d*x)**3))/b**2, Eq(a, 0)), (3*d*x*tan(c + d*x)*
*4/(8*b**2*d*tan(c + d*x)**4 + 16*b**2*d*tan(c + d*x)**2 + 8*b**2*d) + 6*d*
x*tan(c + d*x)**2/(8*b**2*d*tan(c + d*x)**4 + 16*b**2*d*tan(c + d*x)**2 + 8
*b**2*d) + 3*d*x/(8*b**2*d*tan(c + d*x)**4 + 16*b**2*d*tan(c + d*x)**2 + 8*
b**2*d) + 3*tan(c + d*x)**3/(8*b**2*d*tan(c + d*x)**4 + 16*b**2*d*tan(c + d
*x)**2 + 8*b**2*d) + 5*tan(c + d*x)/(8*b**2*d*tan(c + d*x)**4 + 16*b**2*d*t
an(c + d*x)**2 + 8*b**2*d), Eq(a, b)), (x/(a + b*tan(c)**2)**2, Eq(d, 0)),
(x/a**2, Eq(b, 0)), (4*I*a**(5/2)*d*x*sqrt(1/b)/(4*I*a**(9/2)*d*sqrt(1/b) +
 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) -
```

```
8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/
b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2) + 4*I*a**(3/2)*b*d*x*sq
rt(1/b)*tan(c + d*x)**2/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1
/b)*tan(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt
(1/b)*tan(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d
*sqrt(1/b)*tan(c + d*x)**2) - 2*I*a**(3/2)*b*sqrt(1/b)*tan(c + d*x)/(4*I*a*
*(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 8*I*a**(7
/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 4*I*a**
(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2) + 2
*I*sqrt(a)*b**2*sqrt(1/b)*tan(c + d*x)/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(
7/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5
/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*
a**(3/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2) - 3*a**2*log(-I*sqrt(a)*sqrt(1/b
) + tan(c + d*x))/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*ta
n(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*
tan(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(
1/b)*tan(c + d*x)**2) + 3*a**2*log(I*sqrt(a)*sqrt(1/b) + tan(c + d*x))/(4*I
*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 8*I*a*
*(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 4*I*
a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2)
- 3*a*b*log(-I*sqrt(a)*sqrt(1/b) + tan(c + d*x))*tan(c + d*x)**2/(4*I*a**(9
/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 8*I*a**(7/2)
*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 4*I*a**(5/
2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2) + a*b*
log(-I*sqrt(a)*sqrt(1/b) + tan(c + d*x))/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a*
*(7/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**
(5/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*
I*a**(3/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2) + 3*a*b*log(I*sqrt(a)*sqrt(1/b
) + tan(c + d*x))*tan(c + d*x)**2/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*
b*d*sqrt(1/b)*tan(c + d*x)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b
**2*d*sqrt(1/b)*tan(c + d*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3
/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2) - a*b*log(I*sqrt(a)*sqrt(1/b) + tan(c
 + d*x))/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c + d*x
)**2 - 8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c + d
*x)**2 + 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tan(
c + d*x)**2) + b**2*log(-I*sqrt(a)*sqrt(1/b) + tan(c + d*x))*tan(c + d*x)**
2/(4*I*a**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 -
8*I*a**(7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2
+ 4*I*a**(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tan(c + d*x
)**2) - b**2*log(I*sqrt(a)*sqrt(1/b) + tan(c + d*x))*tan(c + d*x)**2/(4*I*a
**(9/2)*d*sqrt(1/b) + 4*I*a**(7/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 8*I*a**(
7/2)*b*d*sqrt(1/b) - 8*I*a**(5/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 4*I*a*
*(5/2)*b**2*d*sqrt(1/b) + 4*I*a**(3/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2), T
rue))
```

3.256 $\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=150

$$-\frac{b(7a-3b)\tan(c+dx)}{8a^2d(a-b)^2\left(a+b\tan^2(c+dx)\right)}-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3}-\frac{b\tan(c+dx)}{4ad(a-b)\left(a+b\tan^2(c+d\right.}$$

```
[Out] x/(a-b)^3-1/8*(15*a^2-10*a*b+3*b^2)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))*b^(1
/2)/a^(5/2)/(a-b)^3/d-1/4*b*tan(d*x+c)/a/(a-b)/d/(a+b*tan(d*x+c)^2)^2-1/8*(
7*a-3*b)*b*tan(d*x+c)/a^2/(a-b)^2/d/(a+b*tan(d*x+c)^2)
```

Rubi [A] time = 0.14, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3661, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3}-\frac{b(7a-3b)\tan(c+dx)}{8a^2d(a-b)^2\left(a+b\tan^2(c+dx)\right)}-\frac{b\tan(c+dx)}{4ad(a-b)\left(a+b\tan^2(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^(-3),x]
```

```
[Out] x/(a - b)^3 - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tan[c + d*
x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*d) - (b*Tan[c + d*x])/(4*a*(a - b)*d*(a
+ b*Tan[c + d*x]^2)^2) - ((7*a - 3*b)*b*Tan[c + d*x])/(8*a^2*(a - b)^2*d*(a
 + b*Tan[c + d*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)}{4a(a-b)d\left(a+b\tan^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-3b-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(c+dx)\right)}{4a(a-b)d}$$

$$= -\frac{b\tan(c+dx)}{4a(a-b)d\left(a+b\tan^2(c+dx)\right)^2} - \frac{(7a-3b)b\tan(c+dx)}{8a^2(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{b\tan(c+dx)}{4a(a-b)d\left(a+b\tan^2(c+dx)\right)^2} - \frac{(7a-3b)b\tan(c+dx)}{8a^2(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{x}{(a-b)^3} - \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d} - \frac{b\tan(c+dx)}{4a(a-b)d\left(a+b\tan^2(c\right.}$$

Mathematica [A] time = 1.93, size = 138, normalized size = 0.92

$$-\frac{\frac{b(7a-3b)(a-b)\tan(c+dx)}{a^2\left(a+b\tan^2(c+dx)\right)} + \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2b(a-b)^2\tan(c+dx)}{a\left(a+b\tan^2(c+dx)\right)^2} - 8\tan^{-1}(\tan(c+dx))}{8d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^(-3),x]
```

```
[Out] -1/8*(-8*ArcTan[Tan[c + d*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(
Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Tan[c + d*x])/(a*(
a + b*Tan[c + d*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Tan[c + d*x])/(a^2*(a + b
*Tan[c + d*x]^2)))/((a - b)^3*d)
```

fricas [B] time = 0.49, size = 742, normalized size = 4.95

$$\left[\frac{32\,a^2b^2dx\tan\left(dx+c\right)^4+64\,a^3bdx\tan\left(dx+c\right)^2+32\,a^4dx-4\left(7\,a^2b^2-10\,ab^3+3\,b^4\right)\tan\left(dx+c\right)^3-\left(\left(15\,a^2\right.\right.}{32\left(\left(a^5b^2-3\,a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*a^2*b^2*d*x*tan(d*x + c)^4 + 64*a^3*b*d*x*tan(d*x + c)^2 + 32*a^4
*d*x - 4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(d*x + c)^3 - ((15*a^2*b^2 - 10*
a*b^3 + 3*b^4)*tan(d*x + c)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b
 - 10*a^2*b^2 + 3*a*b^3)*tan(d*x + c)^2)*sqrt(-b/a)*log((b^2*tan(d*x + c)^4
 - 6*a*b*tan(d*x + c)^2 + a^2 + 4*(a*b*tan(d*x + c)^3 - a^2*tan(d*x + c))*s
qrt(-b/a))/(b^2*tan(d*x + c)^4 + 2*a*b*tan(d*x + c)^2 + a^2)) - 4*(9*a^3*b
- 14*a^2*b^2 + 5*a*b^3)*tan(d*x + c))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*d*tan(d*x + c)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*ta
n(d*x + c)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d), 1/16*(16*a^2*b^2*d
*x*tan(d*x + c)^4 + 32*a^3*b*d*x*tan(d*x + c)^2 + 16*a^4*d*x - 2*(7*a^2*b^2
 - 10*a*b^3 + 3*b^4)*tan(d*x + c)^3 - ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(
d*x + c)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b - 10*a^2*b^2 + 3*a
*b^3)*tan(d*x + c)^2)*sqrt(b/a)*arctan(1/2*(b*tan(d*x + c)^2 - a)*sqrt(b/a)
/(b*tan(d*x + c))) - 2*(9*a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(d*x + c))/((a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*tan(d*x + c)^4 + 2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*d*tan(d*x + c)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 -
 a^4*b^3)*d)]
```

giac [A] time = 1.16, size = 205, normalized size = 1.37

$$-\frac{\frac{\left(15\,a^2b-10\,ab^2+3\,b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(dx+c\right)}{\sqrt{ab}}\right)\right)}{\left(a^5-3\,a^4b+3\,a^3b^2-a^2b^3\right)\sqrt{ab}}-\frac{8\left(dx+c\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{7\,ab^2\tan\left(dx+c\right)^3-3\,b^3\tan\left(dx+c\right)^3+9\,a^2b\tan\left(dx+c\right)-5\,a}{\left(a^4-2\,a^3b+a^2b^2\right)\left(b\tan\left(dx+c\right)^2+a\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(b) +
arctan(b*tan(d*x + c)/sqrt(a*b)))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sq
rt(a*b)) - 8*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (7*a*b^2*tan(d*x +
 c)^3 - 3*b^3*tan(d*x + c)^3 + 9*a^2*b*tan(d*x + c) - 5*a*b^2*tan(d*x + c))
/((a^4 - 2*a^3*b + a^2*b^2)*(b*tan(d*x + c)^2 + a)^2))/d
```

maple [B] time = 0.33, size = 350, normalized size = 2.33

$$-\frac{7b^2\left(\tan^3\left(dx+c\right)\right)}{8d\left(a-b\right)^3\left(a+b\left(\tan^2\left(dx+c\right)\right)\right)^2}+\frac{5b^3\left(\tan^3\left(dx+c\right)\right)}{4d\left(a-b\right)^3\left(a+b\left(\tan^2\left(dx+c\right)\right)\right)^2a}-\frac{3b^4\left(\tan^3\left(dx+c\right)\right)}{8d\left(a-b\right)^3\left(a+b\left(\tan^2\left(dx+c\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^2)^3,x)
```

```
[Out] -7/8/d*b^2/(a-b)^3/(a+b*tan(d*x+c)^2)^2*tan(d*x+c)^3+5/4/d*b^3/(a-b)^3/(a+b
*tan(d*x+c)^2)^2/a*tan(d*x+c)^3-3/8/d*b^4/(a-b)^3/(a+b*tan(d*x+c)^2)^2/a^2*
tan(d*x+c)^3-9/8/d*b/(a-b)^3/(a+b*tan(d*x+c)^2)^2*a*tan(d*x+c)+7/4/d*b^2/(a
-b)^3/(a+b*tan(d*x+c)^2)^2*tan(d*x+c)-5/8/d*b^3/(a-b)^3/(a+b*tan(d*x+c)^2)^
```

```
2/a*tan(d*x+c)-15/8/d*b/(a-b)^3/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2)
)+5/4/d*b^2/(a-b)^3/a/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))-3/8/d*b^
3/(a-b)^3/a^2/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))+1/d/(a-b)^3*arct
an(tan(d*x+c))
```

maxima [A] time = 1.01, size = 227, normalized size = 1.51

$$-\frac{\frac{\left(15\,a^2b-10\,ab^2+3\,b^3\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\left(a^5-3\,a^4b+3\,a^3b^2-a^2b^3\right)\sqrt{ab}}+\frac{\left(7\,ab^2-3\,b^3\right)\tan(dx+c)^3+\left(9\,a^2b-5\,ab^2\right)\tan(dx+c)}{a^6-2\,a^5b+a^4b^2+\left(a^4b^2-2\,a^3b^3+a^2b^4\right)\tan(dx+c)^4+2\left(a^5b-2\,a^4b^2+a^3b^3\right)\tan(dx+c)^2}-\frac{8\,(dx+c)}{a^3-3\,a^2b+3\,ab^2-}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^5
- 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(a*b)) + ((7*a*b^2 - 3*b^3)*tan(d*x +
c)^3 + (9*a^2*b - 5*a*b^2)*tan(d*x + c))/(a^6 - 2*a^5*b + a^4*b^2 + (a^4*b^
2 - 2*a^3*b^3 + a^2*b^4)*tan(d*x + c)^4 + 2*(a^5*b - 2*a^4*b^2 + a^3*b^3)*t
an(d*x + c)^2) - 8*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/d
```

mupad [B] time = 13.98, size = 3901, normalized size = 26.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^2)^3,x)
```

```
[Out] (atan((((-a^5*b)^(1/2)*((tan(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300
*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b
^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^
6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^1
0 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2))
- (tan(c + d*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 128
0*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 128
0*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8
 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10
*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b
 + 3*b^2)*1i)/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^5*b)^(1/2)*
((tan(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3)
)/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((96*a^2*b^10 -
 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 +
5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 -
 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(c + d*x)*(-a^5*b
)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^
7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b
^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 -
4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a
^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2)*1i)/(16*(3*a^
7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))/((51*a*b^5 - 9*b^6 - 115*a^2*b^4 + 105*a
^3*b^3)/(32*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3
 + 15*a^8*b^2)) - ((-a^5*b)^(1/2)*((tan(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^
2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^
3 + 6*a^6*b^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^
7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^
2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15
*a^8*b^2)) - (tan(c + d*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^
4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^
9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^
6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(
```

```
15*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a
^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^5*b
)^(1/2)*((tan(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*
a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((96*a^
2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^
7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a
^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(c + d*x)
*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*b^8 + 230
4*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 25
6*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^
4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/
(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2))/(16*
(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2))))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b +
3*b^2)*1i)/(8*d*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) - (2*atan((((((96*a^
2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^
7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a
^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) - (tan(c + d*x)
*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 +
 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b +
2*a^3 - 2*b^3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a
*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2
*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3
 + 6*a^6*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - ((((96*a^2*b^10 - 800
*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280
*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a
^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(c + d*x)*(256*a^4*b^
9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^
4 - 1280*a^10*b^3 + 256*a^11*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^
3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2
*b + 2*a^3 - 2*b^3) + (tan(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a
^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2
)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))/((51*a*b^5 - 9*b^6 - 115*a^2*b^4 +
 105*a^3*b^3)/(32*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a
^7*b^3 + 15*a^8*b^2)) + (((((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 681
6*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256
*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*
b^3 + 15*a^8*b^2)) - (tan(c + d*x)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b
^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*
b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^8 - 4*a^7*b + a^4*b^4 -
 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(c
+ d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a
^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b +
2*a^3 - 2*b^3) + (((((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b
^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b
^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 1
5*a^8*b^2)) + (tan(c + d*x)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 12
80*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2)*1i
)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*
b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (tan(c + d*x)*
(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*
a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 -
 2*b^3))))/(d*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)) - ((tan(c + d*x)^3*(7*a*
b^2 - 3*b^3))/(8*a^2*(a^2 - 2*a*b + b^2)) + (tan(c + d*x)*(9*a*b - 5*b^2))/
(8*a*(a^2 - 2*a*b + b^2)))/(d*(a^2 + b^2*tan(c + d*x)^4 + 2*a*b*tan(c + d*x
)^2))
```

sympy [A] time = 99.08, size = 9629, normalized size = 64.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(c)**6, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-x - 1/(d*ta
n(c + d*x)) + 1/(3*d*tan(c + d*x)**3) - 1/(5*d*tan(c + d*x)**5))/b**3, Eq(a
, 0)), (15*d*x*tan(c + d*x)**6/(48*b**3*d*tan(c + d*x)**6 + 144*b**3*d*tan(
c + d*x)**4 + 144*b**3*d*tan(c + d*x)**2 + 48*b**3*d) + 45*d*x*tan(c + d*x)
**4/(48*b**3*d*tan(c + d*x)**6 + 144*b**3*d*tan(c + d*x)**4 + 144*b**3*d*ta
n(c + d*x)**2 + 48*b**3*d) + 45*d*x*tan(c + d*x)**2/(48*b**3*d*tan(c + d*x)
**6 + 144*b**3*d*tan(c + d*x)**4 + 144*b**3*d*tan(c + d*x)**2 + 48*b**3*d)
+ 15*d*x/(48*b**3*d*tan(c + d*x)**6 + 144*b**3*d*tan(c + d*x)**4 + 144*b**3
*d*tan(c + d*x)**2 + 48*b**3*d) + 15*tan(c + d*x)**5/(48*b**3*d*tan(c + d*x
)**6 + 144*b**3*d*tan(c + d*x)**4 + 144*b**3*d*tan(c + d*x)**2 + 48*b**3*d)
 + 40*tan(c + d*x)**3/(48*b**3*d*tan(c + d*x)**6 + 144*b**3*d*tan(c + d*x)*
*4 + 144*b**3*d*tan(c + d*x)**2 + 48*b**3*d) + 33*tan(c + d*x)/(48*b**3*d*t
an(c + d*x)**6 + 144*b**3*d*tan(c + d*x)**4 + 144*b**3*d*tan(c + d*x)**2 +
48*b**3*d), Eq(a, b)), (x/(a + b*tan(c)**2)**3, Eq(d, 0)), (x/a**3, Eq(b, 0
)), (16*I*a**(9/2)*d*x*sqrt(1/b)/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/
2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(
11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*ta
n(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt
(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 - 16
*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)*
*4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*
sqrt(1/b)*tan(c + d*x)**4) + 32*I*a**(7/2)*b*d*x*sqrt(1/b)*tan(c + d*x)**2/
(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x)**2
- 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*
x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/2)*b*
*2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I*a**(
9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48
*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/
b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) - 18*I
*a**(7/2)*b*sqrt(1/b)*tan(c + d*x)/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(1
3/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a*
*(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*
tan(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sq
rt(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 -
16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x
)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*
d*sqrt(1/b)*tan(c + d*x)**4) + 16*I*a**(5/2)*b**2*d*x*sqrt(1/b)*tan(c + d*x
)**4/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x
)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c
 + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/
2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I
*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b)
 + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sq
rt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) -
 14*I*a**(5/2)*b**2*sqrt(1/b)*tan(c + d*x)**3/(16*I*a**(15/2)*d*sqrt(1/b) +
 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b
) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b**2*d
*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2
)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c +
 d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*
tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**
(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) + 28*I*a**(5/2)*b**2*sqrt(1/b)*tan(
c + d*x)/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c +
 d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*t
an(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**
(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 +
96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(
1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*
```

```
d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**
4) + 20*I*a**(3/2)*b**3*sqrt(1/b)*tan(c + d*x)**3/(16*I*a**(15/2)*d*sqrt(1/
b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt
(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b*
*2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**
(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan
(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1
/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I
*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) - 10*I*a**(3/2)*b**3*sqrt(1/b)*
tan(c + d*x)/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan
(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/
b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I
*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**
4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*s
qrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b
**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*
x)**4) - 6*I*sqrt(a)*b**4*sqrt(1/b)*tan(c + d*x)**3/(16*I*a**(15/2)*d*sqrt(
1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*sq
rt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*
b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a
**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*t
an(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt
(1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16
*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) - 15*a**4*log(-I*sqrt(a)*sqrt
(1/b) + tan(c + d*x))/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt
(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*
d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)*
*2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c
 + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(9/2)
*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4 - 32*I*a
**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*t
an(c + d*x)**4) + 15*a**4*log(I*sqrt(a)*sqrt(1/b) + tan(c + d*x))/(16*I*a**
(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a*
*(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 9
6*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt
(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3
*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/
2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c
+ d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) - 30*a**3*b*log
(-I*sqrt(a)*sqrt(1/b) + tan(c + d*x))*tan(c + d*x)**2/(16*I*a**(15/2)*d*sqr
t(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*
sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2
)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I
*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)
*tan(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sq
rt(1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 -
16*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) + 10*a**3*b*log(-I*sqrt(a)*
sqrt(1/b) + tan(c + d*x))/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*
sqrt(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b
**2*d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d
*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*t
an(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(
9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4 - 32
*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/
b)*tan(c + d*x)**4) + 30*a**3*b*log(I*sqrt(a)*sqrt(1/b) + tan(c + d*x))*tan
(c + d*x)**2/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan
(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/
b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I
*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**
```

```
4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*s
qrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b
**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*
x)**4) - 10*a**3*b*log(I*sqrt(a)*sqrt(1/b) + tan(c + d*x))/(16*I*a**(15/2)*
d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)
*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**
(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) -
 48*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt
(1/b)*tan(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4
*d*sqrt(1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)*
*2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) - 15*a**2*b**2*log(-I*
sqrt(a)*sqrt(1/b) + tan(c + d*x))*tan(c + d*x)**4/(16*I*a**(15/2)*d*sqrt(1/
b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt
(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b*
*2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**
(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan
(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1
/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I
*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) + 20*a**2*b**2*log(-I*sqrt(a)*s
qrt(1/b) + tan(c + d*x))*tan(c + d*x)**2/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I
*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 1
6*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt
(1/b)*tan(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**
3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)
**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c
 + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)
*b**5*d*sqrt(1/b)*tan(c + d*x)**4) - 3*a**2*b**2*log(-I*sqrt(a)*sqrt(1/b) +
 tan(c + d*x))/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*t
an(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(
1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48
*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)
**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(9/2)*b**3*d
*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)
*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c +
d*x)**4) + 15*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(c + d*x))*tan(c + d*x
)**4/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x
)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c
 + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/
2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I
*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b)
 + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sq
rt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) -
 20*a**2*b**2*log(I*sqrt(a)*sqrt(1/b) + tan(c + d*x))*tan(c + d*x)**2/(16*I
*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*
I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4
 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*
sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*
b**3*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a*
*(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*ta
n(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) + 3*a**2*b*
*2*log(I*sqrt(a)*sqrt(1/b) + tan(c + d*x))/(16*I*a**(15/2)*d*sqrt(1/b) + 32
*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) +
 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sq
rt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b
**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*
x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan
(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/
2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) + 10*a*b**3*log(-I*sqrt(a)*sqrt(1/b) +
 tan(c + d*x))*tan(c + d*x)**4/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)
```

```
*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11
/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(
c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1
/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I
*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4
 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sq
rt(1/b)*tan(c + d*x)**4) - 6*a*b**3*log(-I*sqrt(a)*sqrt(1/b) + tan(c + d*x)
)*tan(c + d*x)**2/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b
)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sq
rt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 +
 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d
*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(9/2)*b**
3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4 - 32*I*a**(7
/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c
 + d*x)**4) - 10*a*b**3*log(I*sqrt(a)*sqrt(1/b) + tan(c + d*x))*tan(c + d*x
)**4/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x
)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c
 + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/
2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I
*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b)
 + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sq
rt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) +
 6*a*b**3*log(I*sqrt(a)*sqrt(1/b) + tan(c + d*x))*tan(c + d*x)**2/(16*I*a**
(15/2)*d*sqrt(1/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a*
*(13/2)*b*d*sqrt(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 9
6*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt
(1/b) - 48*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3
*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/
2)*b**4*d*sqrt(1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c
+ d*x)**2 - 16*I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) - 3*b**4*log(-I
*sqrt(a)*sqrt(1/b) + tan(c + d*x))*tan(c + d*x)**4/(16*I*a**(15/2)*d*sqrt(1
/b) + 32*I*a**(13/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqr
t(1/b) + 16*I*a**(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b
**2*d*sqrt(1/b)*tan(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a*
*(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*ta
n(c + d*x)**2 - 16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(
1/b)*tan(c + d*x)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*
I*a**(5/2)*b**5*d*sqrt(1/b)*tan(c + d*x)**4) + 3*b**4*log(I*sqrt(a)*sqrt(1/
b) + tan(c + d*x))*tan(c + d*x)**4/(16*I*a**(15/2)*d*sqrt(1/b) + 32*I*a**(1
3/2)*b*d*sqrt(1/b)*tan(c + d*x)**2 - 48*I*a**(13/2)*b*d*sqrt(1/b) + 16*I*a*
*(11/2)*b**2*d*sqrt(1/b)*tan(c + d*x)**4 - 96*I*a**(11/2)*b**2*d*sqrt(1/b)*
tan(c + d*x)**2 + 48*I*a**(11/2)*b**2*d*sqrt(1/b) - 48*I*a**(9/2)*b**3*d*sq
rt(1/b)*tan(c + d*x)**4 + 96*I*a**(9/2)*b**3*d*sqrt(1/b)*tan(c + d*x)**2 -
16*I*a**(9/2)*b**3*d*sqrt(1/b) + 48*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x
)**4 - 32*I*a**(7/2)*b**4*d*sqrt(1/b)*tan(c + d*x)**2 - 16*I*a**(5/2)*b**5*
d*sqrt(1/b)*tan(c + d*x)**4), True))
```

3.257 $\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=54

$$\frac{1}{4}\tan^3(x)\sqrt{a\sec^2(x)}-\frac{3}{8}\tan(x)\sqrt{a\sec^2(x)}+\frac{3}{8}\cos(x)\sqrt{a\sec^2(x)}\,\tanh^{-1}(\sin(x))$$

```
[Out] 3/8*arctanh(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)-3/8*(a*sec(x)^2)^(1/2)*tan(x)
+1/4*(a*sec(x)^2)^(1/2)*tan(x)^3
```

Rubi [A] time = 0.11, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3657, 4125, 2611, 3770}

$$\frac{1}{4}\tan^3(x)\sqrt{a\sec^2(x)}-\frac{3}{8}\tan(x)\sqrt{a\sec^2(x)}+\frac{3}{8}\cos(x)\sqrt{a\sec^2(x)}\,\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^4*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (3*ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2])/8 - (3*Sqrt[a*Sec[x]^2]*Tan[x])
/8 + (Sqrt[a*Sec[x]^2]*Tan[x]^3)/4
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx = \int \sqrt{a\sec^2(x)}\,\tan^4(x)\,dx$$
$$= \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec(x)\tan^4(x)\,dx$$
$$= \frac{1}{4}\sqrt{a\sec^2(x)}\,\tan^3(x) - \frac{1}{4}\left(3\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec(x)\tan^2(x)\,dx$$
$$= -\frac{3}{8}\sqrt{a\sec^2(x)}\,\tan(x) + \frac{1}{4}\sqrt{a\sec^2(x)}\,\tan^3(x) + \frac{1}{8}\left(3\cos(x)\sqrt{a\sec^2(x)}\right)\int \mathrm{sec}$$
$$= \frac{3}{8}\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} - \frac{3}{8}\sqrt{a\sec^2(x)}\,\tan(x) + \frac{1}{4}\sqrt{a\sec^2(x)}\,\tan^3$$

Mathematica [A] time = 0.08, size = 32, normalized size = 0.59

$$\frac{1}{8}\sqrt{a\sec^2(x)}\,\left(2\tan^3(x) - 3\tan(x) + 3\cos(x)\tanh^{-1}(\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^4*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (Sqrt[a*Sec[x]^2]*(3*ArcTanh[Sin[x]]*Cos[x] - 3*Tan[x] + 2*Tan[x]^3))/8
```

fricas [A] time = 0.42, size = 56, normalized size = 1.04

$$\frac{1}{8}\sqrt{a\tan(x)^2+a}\left(2\tan(x)^3 - 3\tan(x)\right)+\frac{3}{16}\sqrt{a}\log\left(2a\tan(x)^2 + 2\sqrt{a\tan(x)^2+a}\sqrt{a}\tan(x) + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(a*tan(x)^2 + a)*(2*tan(x)^3 - 3*tan(x)) + 3/16*sqrt(a)*log(2*a*tan
(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a)
```

giac [A] time = 0.42, size = 48, normalized size = 0.89

$$\frac{1}{8}\sqrt{a\tan(x)^2+a}\left(2\tan(x)^2-3\right)\tan(x) - \frac{3}{8}\sqrt{a}\log\left(\left|-\sqrt{a}\tan(x) + \sqrt{a\tan(x)^2+a}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^4,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 - 3)*tan(x) - 3/8*sqrt(a)*log(abs(-sqr
t(a)*tan(x) + sqrt(a*tan(x)^2 + a)))
```

maple [A] time = 0.30, size = 56, normalized size = 1.04

$$\frac{\tan(x)\left(a+a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}{4a} - \frac{5\sqrt{a+a\left(\tan^2(x)\right)}\,\tan(x)}{8} + \frac{3\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(x) + \sqrt{a+a\left(\tan^2(x)\right)}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(x)^2)^(1/2)*tan(x)^4,x)
```

```
[Out] 1/4*tan(x)*(a+a*tan(x)^2)^(3/2)/a-5/8*(a+a*tan(x)^2)^(1/2)*tan(x)+3/8*a^(1/
2)*ln(a^(1/2)*tan(x)+(a+a*tan(x)^2)^(1/2))
```

maxima [B] time = 1.55, size = 860, normalized size = 15.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^4,x, algorithm="maxima")
```

```
[Out] -1/16*(4*(5*sin(7*x) - 3*sin(5*x) + 3*sin(3*x) - 5*sin(x))*cos(8*x) - 40*(2
*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*cos(7*x) - 16*(3*sin(5*x) - 3*sin(3*x)
 + 5*sin(x))*cos(6*x) + 24*(3*sin(4*x) + 2*sin(2*x))*cos(5*x) + 24*(3*sin(3
*x) - 5*sin(x))*cos(4*x) - 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*
cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6
*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2
*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x)
 + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2
*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) +
1) + 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x)^2
+ 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 12*(4*cos(2*x)
 + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x) + 3*sin(4*x)
 + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x) + 2*sin(2*x))*sin(6*x
) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) + 16*sin(2*x)^2 +
8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(5*cos(7*x) - 3
*cos(5*x) + 3*cos(3*x) - 5*cos(x))*sin(8*x) + 20*(4*cos(6*x) + 6*cos(4*x) +
 4*cos(2*x) + 1)*sin(7*x) + 16*(3*cos(5*x) - 3*cos(3*x) + 5*cos(x))*sin(6*x
) - 12*(6*cos(4*x) + 4*cos(2*x) + 1)*sin(5*x) - 24*(3*cos(3*x) - 5*cos(x))*
sin(4*x) + 12*(4*cos(2*x) + 1)*sin(3*x) - 48*cos(3*x)*sin(2*x) + 80*cos(x)*
sin(2*x) - 80*cos(2*x)*sin(x) - 20*sin(x))*sqrt(a)/(2*(4*cos(6*x) + 6*cos(4
*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) +
1)*cos(6*x) + 16*cos(6*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2
+ 16*cos(2*x)^2 + 4*(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8
*x)^2 + 16*(3*sin(4*x) + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)
^2 + 48*sin(4*x)*sin(2*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(x)^4 \sqrt{a \tan(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4*(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)^4*(a + a*tan(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \left(\tan^2(x) + 1\right)} \tan^4(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)**2)**(1/2)*tan(x)**4,x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*tan(x)**4, x)
```

3.258 $\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=30

$$\frac{\left(a\sec^2(x)\right)^{3/2}}{3a}-\sqrt{a\sec^2(x)}$$

[Out] 1/3*(a*sec(x)^2)^(3/2)/a-(a*sec(x)^2)^(1/2)

Rubi [A] time = 0.09, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3657, 4124, 43}

$$\frac{\left(a\sec^2(x)\right)^{3/2}}{3a}-\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Tan[x]^3*Sqrt[a + a*Tan[x]^2],x]

[Out] -Sqrt[a*Sec[x]^2] + (a*Sec[x]^2)^(3/2)/(3*a)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \sqrt{a\sec^2(x)}\,\tan^3(x)\,dx\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{-1+x}{\sqrt{ax}}\,dx,x,\sec^2(x)\right)\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \left(-\frac{1}{\sqrt{ax}}+\frac{\sqrt{ax}}{a}\right)dx,x,\sec^2(x)\right)\\
&= -\sqrt{a\sec^2(x)}+\frac{\left(a\sec^2(x)\right)^{3/2}}{3a}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 20, normalized size = 0.67

$$\frac{1}{3}\left(\sec^2(x)-3\right)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (Sqrt[a*Sec[x]^2]*(-3 + Sec[x]^2))/3
```

fricas [A] time = 0.38, size = 18, normalized size = 0.60

$$\frac{1}{3}\sqrt{a\tan(x)^2 + a}\left(\tan(x)^2 - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*tan(x)^2 + a)*(tan(x)^2 - 2)
```

giac [A] time = 0.30, size = 29, normalized size = 0.97

$$\frac{\left(a\tan(x)^2 + a\right)^{\frac{3}{2}} - 3\sqrt{a\tan(x)^2 + a}\,a}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/3*((a*tan(x)^2 + a)^(3/2) - 3*sqrt(a*tan(x)^2 + a)*a)/a
```

maple [A] time = 0.20, size = 29, normalized size = 0.97

$$\frac{\left(a + a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}{3a} - \sqrt{a + a\left(\tan^2(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(x)^2)^(1/2)*tan(x)^3,x)
```

```
[Out] 1/3/a*(a+a*tan(x)^2)^(3/2)-(a+a*tan(x)^2)^(1/2)
```

maxima [B] time = 0.79, size = 276, normalized size = 9.20

$$-\frac{2\left((3\cos(5x) + 2\cos(3x) + 3\cos(x))\cos(6x) + 3(3\cos(4x) + 3\cos(2x) + 1)\cos(5x) + 3(2\cos(3x) + 3\right.}{3\left(2(3\cos(4x) + 3\cos(2x) + 1)\cos(6x) + \cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -2/3*((3*cos(5*x) + 2*cos(3*x) + 3*cos(x))*cos(6*x) + 3*(3*cos(4*x) + 3*cos
(2*x) + 1)*cos(5*x) + 3*(2*cos(3*x) + 3*cos(x))*cos(4*x) + 2*(3*cos(2*x) +
1)*cos(3*x) + 9*cos(2*x)*cos(x) + (3*sin(5*x) + 2*sin(3*x) + 3*sin(x))*sin(
6*x) + 9*(sin(4*x) + sin(2*x))*sin(5*x) + 3*(2*sin(3*x) + 3*sin(x))*sin(4*x
) + 6*sin(3*x)*sin(2*x) + 9*sin(2*x)*sin(x) + 3*cos(x))*sqrt(a)/(2*(3*cos(4
*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(2*x) + 1)*cos(4*x) +
 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x))*sin(6*x) + sin(6*x)^
2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) + 1)
```

mupad [B] time = 11.64, size = 19, normalized size = 0.63

$$-\frac{\sqrt{2}\,\sqrt{a}\,\left(6\cos(x)^2 - 2\right)}{3\left(2\cos(x)^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3*(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] -(2^(1/2)*a^(1/2)*(6*cos(x)^2 - 2))/(3*(2*cos(x)^2)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\tan^{2}(x)+1\right)}\tan^{3}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)**2)**(1/2)*tan(x)**3,x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*tan(x)**3, x)
```

3.259 $\int \tan^2(x)\sqrt{a + a\tan^2(x)}\, dx$

Optimal. Leaf size=36

$$\frac{1}{2}\tan(x)\sqrt{a\sec^2(x)} - \frac{1}{2}\cos(x)\sqrt{a\sec^2(x)}\,\tanh^{-1}(\sin(x))$$

```
[Out] -1/2*arctanh(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)+1/2*(a*sec(x)^2)^(1/2)*tan(x
)
```

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3657, 4125, 2611, 3770}

$$\frac{1}{2}\tan(x)\sqrt{a\sec^2(x)} - \frac{1}{2}\cos(x)\sqrt{a\sec^2(x)}\,\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2])/2 + (Sqrt[a*Sec[x]^2]*Tan[x])/2
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \tan^2(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \sqrt{a\sec^2(x)}\,\tan^2(x)\,dx\\ &= \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec(x)\tan^2(x)\,dx\\ &= \frac{1}{2}\sqrt{a\sec^2(x)}\,\tan(x)-\frac{1}{2}\left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec(x)\,dx\\ &= -\frac{1}{2}\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)}+\frac{1}{2}\sqrt{a\sec^2(x)}\,\tan(x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 24, normalized size = 0.67

$$\frac{1}{2}\sqrt{a\sec^2(x)}\left(\tan(x)-\cos(x)\tanh^{-1}(\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2*Sqrt[a + a*Tan[x]^2],x]
[Out] (Sqrt[a*Sec[x]^2]*(-(ArcTanh[Sin[x]]*Cos[x]) + Tan[x]))/2
```

fricas [A] time = 0.42, size = 47, normalized size = 1.31

$$\frac{1}{4}\sqrt{a}\log\left(2\,a\tan(x)^2-2\sqrt{a\tan(x)^2+a}\,\sqrt{a}\,\tan(x)+a\right)+\frac{1}{2}\sqrt{a\tan(x)^2+a}\,\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^2,x, algorithm="fricas")
[Out] 1/4*sqrt(a)*log(2*a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a) +
 1/2*sqrt(a*tan(x)^2 + a)*tan(x)
```

giac [A] time = 0.31, size = 40, normalized size = 1.11

$$\frac{1}{2}\sqrt{a}\log\left(\left|-\sqrt{a}\,\tan(x)+\sqrt{a\tan(x)^2+a}\right|\right)+\frac{1}{2}\sqrt{a\tan(x)^2+a}\,\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^2,x, algorithm="giac")
[Out] 1/2*sqrt(a)*log(abs(-sqrt(a)*tan(x) + sqrt(a*tan(x)^2 + a))) + 1/2*sqrt(a*t
an(x)^2 + a)*tan(x)
```

maple [A] time = 0.16, size = 39, normalized size = 1.08

$$\frac{\sqrt{a+a\left(\tan^2(x)\right)}\,\tan(x)}{2}-\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(x)+\sqrt{a+a\left(\tan^2(x)\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(x)^2)^(1/2)*tan(x)^2,x)
[Out] 1/2*(a+a*tan(x)^2)^(1/2)*tan(x)-1/2*a^(1/2)*ln(a^(1/2)*tan(x)+(a+a*tan(x)^2
)^(1/2))
```

maxima [B] time = 0.88, size = 295, normalized size = 8.19

$$\left(4\left(\sin(3\,x)-\sin(x)\right)\cos(4\,x)-\left(2\left(2\cos(2\,x)+1\right)\cos(4\,x)+\cos(4\,x)^2+4\cos(2\,x)^2+\sin(4\,x)^2+4\sin(4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(3*x) - sin(x))*cos(4*x) - (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*
x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*c
os(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (2*(2*cos(2*x) + 1)*
cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4
*sin(2*x)^2 + 4*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(
cos(3*x) - cos(x))*sin(4*x) + 4*(2*cos(2*x) + 1)*sin(3*x) - 8*cos(3*x)*sin(
2*x) + 8*cos(x)*sin(2*x) - 8*cos(2*x)*sin(x) - 4*sin(x))*sqrt(a)/(2*(2*cos(
2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*si
n(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \tan(x)^2 \sqrt{a\tan(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2*(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)^2*(a + a*tan(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\tan^2(x) + 1\right)}\tan^2(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)**2)**(1/2)*tan(x)**2,x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*tan(x)**2, x)
```

3.260 $\int \tan(x)\sqrt{a + a\tan^2(x)}\, dx$

Optimal. Leaf size=10

$$\sqrt{a\sec^2(x)}$$

```
[Out] (a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3657, 4124, 32}

$$\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] Sqrt[a*Sec[x]^2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \tan(x)\sqrt{a + a\tan^2(x)}\, dx &= \int \sqrt{a\sec^2(x)}\tan(x)\, dx\\ &= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{\sqrt{ax}}\, dx, x, \sec^2(x)\right)\\ &= \sqrt{a\sec^2(x)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] Sqrt[a*Sec[x]^2]
```

fricas [A] time = 0.42, size = 10, normalized size = 1.00

$$\sqrt{a\tan(x)^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] sqrt(a*tan(x)^2 + a)
```

giac [A] time = 0.30, size = 10, normalized size = 1.00

$$\sqrt{a\tan(x)^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] sqrt(a*tan(x)^2 + a)
```

maple [A] time = 0.12, size = 11, normalized size = 1.10

$$\sqrt{a + a\left(\tan^2(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(x)^2)^(1/2)*tan(x),x)
```

```
[Out] (a+a*tan(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\tan(x)^2 + a}\,\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*tan(x)^2 + a)*tan(x), x)
```

mupad [B] time = 11.63, size = 10, normalized size = 1.00

$$\frac{\sqrt{a}}{\sqrt{\cos(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] a^(1/2)/(cos(x)^2)^(1/2)
```

sympy [A] time = 0.60, size = 10, normalized size = 1.00

$$\sqrt{a\tan^2{(x)} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)**2)**(1/2)*tan(x),x)
```

```
[Out] sqrt(a*tan(x)**2 + a)
```

3.261 $\int \cot(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=24

$$-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)$$

```
[Out] -arctanh((a*sec(x)^2)^(1/2)/a^(1/2))*a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3657, 4124, 63, 207}

$$-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a*Sec[x]^2/Sqrt[a]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \cot(x)\sqrt{a+a\tan^2(x)}\,dx = \int \cot(x)\sqrt{a\sec^2(x)}\,dx$$
$$= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx, x, \sec^2(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a\sec^2(x)}\right)$$
$$= -\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.25

$$\cos(x)\sqrt{a\sec^2(x)}\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + a*Tan[x]^2],x]
[Out] Cos[x]*(-Log[Cos[x/2]] + Log[Sin[x/2]])*Sqrt[a*Sec[x]^2]
```

fricas [A] time = 0.44, size = 63, normalized size = 2.62

$$\left[\frac{1}{2}\sqrt{a}\log\left(\frac{a\tan(x)^2-2\sqrt{a\tan(x)^2+a}\sqrt{a}+2a}{\tan(x)^2}\right), \sqrt{-a}\arctan\left(\frac{\sqrt{a\tan(x)^2+a}\sqrt{-a}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*sqrt(a)*log((a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a) + 2*a)/tan(x)
^2), sqrt(-a)*arctan(sqrt(a*tan(x)^2 + a)*sqrt(-a)/a)]
```

giac [A] time = 0.26, size = 24, normalized size = 1.00

$$\frac{a\arctan\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
[Out] a*arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.59, size = 23, normalized size = 0.96

$$\cos(x)\sqrt{\frac{a}{\cos(x)^2}}\,\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+a*tan(x)^2)^(1/2),x)
[Out] cos(x)*(a/cos(x)^2)^(1/2)*ln(-(-1+cos(x))/sin(x))
```

maxima [B] time = 1.04, size = 38, normalized size = 1.58

$$-\frac{1}{2}\sqrt{a}\left(\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)-\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*(log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - log(cos(x)^2 + sin(
x)^2 - 2*cos(x) + 1))
```

mupad [B] time = 0.18, size = 12, normalized size = 0.50

$$-\sqrt{a}\ \mathrm{atanh}\left(\sqrt{\frac{1}{\cos(x)^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] -a^(1/2)*atanh((1/cos(x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\tan^2(x)+1\right)}\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*cot(x), x)
```

3.262 $\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=14

$$-\cot(x)\sqrt{a\sec^2(x)}$$

```
[Out] -cot(x)*(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3657, 4125, 2606, 8}

$$-\cot(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(Cot[x]*Sqrt[a*Sec[x]^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \cot^2(x)\sqrt{a\sec^2(x)}\,dx\\
&= \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \cot(x)\csc(x)\,dx\\
&= -\left(\left(\cos(x)\sqrt{a\sec^2(x)}\right)\text{Subst}\left(\int 1\,dx, x, \csc(x)\right)\right)\\
&= -\cot(x)\sqrt{a\sec^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$-\cot(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2*Sqrt[a + a*Tan[x]^2],x]
[Out] -(Cot[x]*Sqrt[a*Sec[x]^2])
```

fricas [A] time = 0.39, size = 16, normalized size = 1.14

$$-\frac{\sqrt{a\tan(x)^2+a}}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(a*tan(x)^2 + a)/tan(x)
```

giac [B] time = 0.45, size = 32, normalized size = 2.29

$$\frac{2\,a^{\frac{3}{2}}}{\left(\sqrt{a}\,\tan(x)-\sqrt{a\tan(x)^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
[Out] 2*a^(3/2)/((sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2 - a)
```

maple [A] time = 0.53, size = 17, normalized size = 1.21

$$-\frac{\cos(x)\sqrt{\frac{a}{\cos(x)^2}}}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a+a*tan(x)^2)^(1/2),x)
[Out] -cos(x)*(a/cos(x)^2)^(1/2)/sin(x)
```

maxima [A] time = 0.85, size = 17, normalized size = 1.21

$$-\frac{\sqrt{\tan(x)^2+1}\,\sqrt{a}}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(tan(x)^2 + 1)*sqrt(a)/tan(x)
```

mupad [B] time = 11.98, size = 25, normalized size = 1.79

$$\frac{2\sqrt{a}\,\cos(x)\,\sin(x)}{\sqrt{\cos(x)^2}\,\left(2\cos(x)^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a + a*tan(x)^2)^(1/2),x)
[Out] (2*a^(1/2)*cos(x)*sin(x))/((cos(x)^2)^(1/2)*(2*cos(x)^2 - 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\tan^{2}(x)+1\right)}\,\cot^{2}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2*(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*cot(x)**2, x)
```

3.263 $\int \cot^3(x)\sqrt{a + a\tan^2(x)}\,dx$

Optimal. Leaf size=45

$$\frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)-\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)}$$

```
[Out] 1/2*arctanh((a*sec(x)^2)^(1/2)/a^(1/2))*a^(1/2)-1/2*cot(x)^2*(a*sec(x)^2)^(
1/2)
```

Rubi [A] time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3657, 4124, 51, 63, 207}

$$\frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)-\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]])/2 - (Cot[x]^2*Sqrt[a*Sec[x]^2])
/2
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \cot^3(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \cot^3(x)\sqrt{a\sec^2(x)}\,dx\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{(-1+x)^2\sqrt{ax}}\,dx,x,\sec^2(x)\right)\\
&= -\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)}-\frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx,x,\sec^2(x)\right)\\
&= -\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a\sec^2(x)}\right)\\
&= \frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)-\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 38, normalized size = 0.84

$$-\frac{1}{2}\cos(x)\sqrt{a\sec^2(x)}\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)+\cot(x)\csc(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -1/2*(Cos[x]*(Cot[x]*Csc[x] - Log[Cos[x/2]] + Log[Sin[x/2]])*Sqrt[a*Sec[x]^
2])
```

fricas [A] time = 0.43, size = 58, normalized size = 1.29

$$\frac{\sqrt{a}\log\left(\frac{a\tan(x)^2+2\sqrt{a\tan(x)^2+a}\sqrt{a}+2a}{\tan(x)^2}\right)\tan(x)^2-2\sqrt{a\tan(x)^2+a}}{4\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a)*log((a*tan(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(a) + 2*a)/tan(x)
^2)*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a))/tan(x)^2
```

giac [A] time = 0.27, size = 42, normalized size = 0.93

$$-\frac{a\arctan\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}-\frac{\sqrt{a\tan(x)^2+a}}{2\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/sqrt(-a) - 1/2*sqrt(a*tan(x)^2
 + a)/tan(x)^2
```

maple [A] time = 0.56, size = 51, normalized size = 1.13

$$\frac{\left(\left(\cos^2(x)\right)\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)-\cos(x)-\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)\right)\cos(x)\sqrt{\frac{a}{\cos(x)^2}}}{2\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*(a+a*tan(x)^2)^(1/2),x)
```

```
[Out] 1/2*(cos(x)^2*ln(-(-1+cos(x))/sin(x))-cos(x)-ln(-(-1+cos(x))/sin(x)))*cos(x
)*(a/cos(x)^2)^(1/2)/sin(x)^2
```

maxima [B] time = 0.86, size = 303, normalized size = 6.73

$$-\frac{\left(4\left(\cos\left(3x\right)+\cos(x)\right)\cos\left(4x\right)-4\left(2\cos\left(2x\right)-1\right)\cos\left(3x\right)-8\cos\left(2x\right)\cos(x)-\left(2\left(2\cos\left(2x\right)-1\right)\cos\left(4\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(cos(3*x) + cos(x))*cos(4*x) - 4*(2*cos(2*x) - 1)*cos(3*x) - 8*cos(
2*x)*cos(x) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - si
n(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*log(cos(x)^
2 + sin(x)^2 + 2*cos(x) + 1) + (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 -
4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x)
 - 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*(sin(3*x) + sin(x))*sin(4
*x) - 8*sin(3*x)*sin(2*x) - 8*sin(2*x)*sin(x) + 4*cos(x))*sqrt(a)/(2*(2*cos
(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*s
in(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)
```

mupad [B] time = 12.04, size = 37, normalized size = 0.82

$$\frac{\sqrt{a}\ \mathrm{atanh}\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{a}}\right)}{2}-\frac{\sqrt{a\tan(x)^2+a}}{2\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] (a^(1/2)*atanh((a + a*tan(x)^2)^(1/2)/a^(1/2)))/2 - (a + a*tan(x)^2)^(1/2)/
(2*tan(x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\tan^2\left(x\right)+1\right)}\cot^3\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3*(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*cot(x)**3, x)
```

3.264 $\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=34

$$\cot(x)\sqrt{a\sec^2(x)}-\frac{1}{3}\cot(x)\csc^2(x)\sqrt{a\sec^2(x)}$$

```
[Out] cot(x)*(a*sec(x)^2)^(1/2)-1/3*cot(x)*csc(x)^2*(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3657, 4125, 2606}

$$\cot(x)\sqrt{a\sec^2(x)}-\frac{1}{3}\cot(x)\csc^2(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] Cot[x]*Sqrt[a*Sec[x]^2] - (Cot[x]*Csc[x]^2*Sqrt[a*Sec[x]^2])/3
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx&=\int \cot^4(x)\sqrt{a\sec^2(x)}\,dx\\&=\left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \cot^3(x)\csc(x)\,dx\\&=-\left(\left(\cos(x)\sqrt{a\sec^2(x)}\right)\text{Subst}\left(\int\left(-1+x^2\right)\,dx,x,\csc(x)\right)\right)\\&=\cot(x)\sqrt{a\sec^2(x)}-\frac{1}{3}\cot(x)\csc^2(x)\sqrt{a\sec^2(x)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.65

$$-\frac{1}{3}\cot(x)\left(\csc^2(x)-3\right)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4*Sqrt[a + a*Tan[x]^2],x]
[Out] -1/3*(Cot[x]*(-3 + Csc[x]^2)*Sqrt[a*Sec[x]^2])
```

fricas [A] time = 0.42, size = 24, normalized size = 0.71

$$\frac{\sqrt{a\tan(x)^2+a}\left(2\tan(x)^2-1\right)}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 - 1)/tan(x)^3
```

giac [B] time = 0.27, size = 59, normalized size = 1.74

$$\frac{4\left(3\left(\sqrt{a}\tan(x)-\sqrt{a\tan(x)^2+a}\right)^2-a\right)a^{\frac{5}{2}}}{3\left(\left(\sqrt{a}\tan(x)-\sqrt{a\tan(x)^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
[Out] 4/3*(3*(sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2 - a)*a^(5/2)/((sqrt(a)*tan
(x) - sqrt(a*tan(x)^2 + a))^2 - a)^3
```

maple [A] time = 0.64, size = 25, normalized size = 0.74

$$-\frac{\left(3\left(\cos^2(x)\right)-2\right)\cos(x)\sqrt{\frac{a}{\cos(x)^2}}}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4*(a+a*tan(x)^2)^(1/2),x)
[Out] -1/3*(3*cos(x)^2-2)*cos(x)*(a/cos(x)^2)^(1/2)/sin(x)^3
```

maxima [A] time = 0.97, size = 29, normalized size = 0.85

$$\frac{\left(2\sqrt{a}\tan(x)^2-\sqrt{a}\right)\sqrt{\tan(x)^2+1}}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*(2*sqrt(a)*tan(x)^2 - sqrt(a))*sqrt(tan(x)^2 + 1)/tan(x)^3
```

mupad [B] time = 11.85, size = 40, normalized size = 1.18

$$\frac{\sqrt{2}\sqrt{a}\left(2\sin(2x)-6\sin(2x)\left(2\cos(x)^2-1\right)\right)}{24\sqrt{2\cos(x)^2}\left(\cos(x)^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4*(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] (2^(1/2)*a^(1/2)*(2*sin(2*x) - 6*sin(2*x)*(2*cos(x)^2 - 1)))/(24*(2*cos(x)^
2)^(1/2)*(cos(x)^2 - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\tan^2\left(x\right)+1\right)}\cot^4\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4*(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*cot(x)**4, x)
```

3.265 $\int \sqrt{a+a\tan^2(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{d}$$

[Out] arctanh(a^(1/2)*tan(d*x+c)/(a*sec(d*x+c)^2)^(1/2))*a^(1/2)/d

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{3657, 4122, 217, 206\}$

$$\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Tan[c + d*x]^2],x]

[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a*Sec[c + d*x]^2]])/d

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+a\tan^2(c+dx)}\,dx &= \int \sqrt{a\sec^2(c+dx)}\,dx\\
&= \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+ax^2}}\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{a\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{d}\\
&= \frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.86

$$\frac{\cos(c+dx)\sqrt{a\sec^2(c+dx)}\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Tan[c + d*x]^2],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Cos[c + d*x]*Sqrt[a*Sec[c + d*x]^2])/d
```

fricas [A] time = 0.43, size = 90, normalized size = 2.50

$$\left[\frac{\sqrt{a}\log\left(2\,a\tan\left(dx+c\right)^2+2\,\sqrt{a\tan\left(dx+c\right)^2+a}\,\sqrt{a}\,\tan\left(dx+c\right)+a\right)}{2\,d}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\tan(dx+c)^2+a}\,\sqrt{-a}}{a\tan(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(2*a*tan(d*x + c)^2 + 2*sqrt(a*tan(d*x + c)^2 + a)*sqrt(a)*
tan(d*x + c) + a)/d, -sqrt(-a)*arctan(sqrt(a*tan(d*x + c)^2 + a)*sqrt(-a)/(
a*tan(d*x + c)))/d]
```

giac [B] time = 1.33, size = 66, normalized size = 1.83

$$-\frac{\left(\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(log(abs(tan(1/2*d*x + 1/2*c) + 1))*sgn(tan(1/2*d*x + 1/2*c)^4 - 1) - log(
abs(tan(1/2*d*x + 1/2*c) - 1))*sgn(tan(1/2*d*x + 1/2*c)^4 - 1))*sqrt(a)/d
```

maple [A] time = 0.54, size = 34, normalized size = 0.94

$$\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan\left(dx+c\right)+\sqrt{a+a\left(\tan^2\left(dx+c\right)\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^(1/2),x)
```

```
[Out] 1/d*a^(1/2)*ln(a^(1/2)*tan(d*x+c)+(a+a*tan(d*x+c)^2)^(1/2))
```

maxima [B] time = 1.14, size = 65, normalized size = 1.81

$$\frac{\sqrt{a}\left(\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

mupad [B] time = 11.86, size = 41, normalized size = 1.14

$$\begin{cases} 0 & \text{if } a = 0 \\ \frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(c+d\,x)+\sqrt{a\,\tan(c+d\,x)^2+a}\right)}{d} & \text{if } a \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)^2)^(1/2),x)
```

```
[Out] piecewise(a == 0, 0, a ~= 0, (a^(1/2)*log(a^(1/2)*tan(c + d*x) + (a + a*tan
(c + d*x)^2)^(1/2)))/d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\tan^2(c+dx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*tan(c + d*x)**2 + a), x)
```

3.266 $\int \tan^3(x) \left(a+a \tan^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=32

$$\frac{\left(a \sec^2(x)\right)^{5/2}}{5 a}-\frac{1}{3} \left(a \sec^2(x)\right)^{3/2}$$

```
[Out] -1/3*(a*sec(x)^2)^(3/2)+1/5*(a*sec(x)^2)^(5/2)/a
```

Rubi [A] time = 0.10, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3657, 4124, 43}

$$\frac{\left(a \sec^2(x)\right)^{5/2}}{5 a}-\frac{1}{3} \left(a \sec^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -(a*Sec[x]^2)^(3/2)/3 + (a*Sec[x]^2)^(5/2)/(5*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \tan^3(x) \left(a+a \tan^2(x)\right)^{3/2} \, dx &= \int \left(a \sec^2(x)\right)^{3/2} \tan^3(x) \, dx\\
&= \frac{1}{2} a \operatorname{Subst}\left(\int (-1+x) \sqrt{a x} \, dx,x,\sec^2(x)\right)\\
&= \frac{1}{2} a \operatorname{Subst}\left(\int \left(-\sqrt{a x}+\frac{(a x)^{3/2}}{a}\right) \, dx,x,\sec^2(x)\right)\\
&= -\frac{1}{3} \left(a \sec^2(x)\right)^{3/2}+\frac{\left(a \sec^2(x)\right)^{5/2}}{5 a}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 22, normalized size = 0.69

$$\frac{1}{15} \left(3 \sec^2(x)-5\right) \left(a \sec^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] ((a*Sec[x]^2)^(3/2)*(-5 + 3*Sec[x]^2))/15
```

fricas [A] time = 0.40, size = 29, normalized size = 0.91

$$\frac{1}{15}\left(3\,a\tan(x)^4 + a\tan(x)^2 - 2\,a\right)\sqrt{a\tan(x)^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*tan(x)^4 + a*tan(x)^2 - 2*a)*sqrt(a*tan(x)^2 + a)
```

giac [B] time = 0.30, size = 72, normalized size = 2.25

$$\frac{1}{3}\left(a\tan(x)^2 + a\right)^{\frac{3}{2}} - \sqrt{a\tan(x)^2 + a}\,a + \frac{3\left(a\tan(x)^2 + a\right)^{\frac{5}{2}} - 10\left(a\tan(x)^2 + a\right)^{\frac{3}{2}}a + 15\sqrt{a\tan(x)^2 + a}\,a^2}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(a*tan(x)^2 + a)^(3/2) - sqrt(a*tan(x)^2 + a)*a + 1/15*(3*(a*tan(x)^2 +
 a)^(5/2) - 10*(a*tan(x)^2 + a)^(3/2)*a + 15*sqrt(a*tan(x)^2 + a)*a^2)/a
```

maple [A] time = 0.13, size = 29, normalized size = 0.91

$$\frac{\left(a + a\left(\tan^2(x)\right)\right)^{\frac{5}{2}}}{5a} - \frac{\left(a + a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3*(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] 1/5/a*(a+a*tan(x)^2)^(5/2)-1/3*(a+a*tan(x)^2)^(3/2)
```

maxima [B] time = 1.63, size = 559, normalized size = 17.47

$$-\frac{8\,(50\,a\cos(4\,x)\cos(3\,x) + 50\,a\sin(4\,x)\sin(3\,x) + 25\,a\sin(3\,x)\sin(2\,x) + (5\,a\cos(7\,x) - 2\,a\cos(5\,x) + 5\,a}{15\left(2\,(5\,\cos(8\,x) + 10\,\cos(6\,x) + 10\,\cos(4\,x) + 5\,\cos(2\,x) + 1)\cos(10\,x) + \cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/15*(50*a*cos(4*x)*cos(3*x) + 50*a*sin(4*x)*sin(3*x) + 25*a*sin(3*x)*sin(
2*x) + (5*a*cos(7*x) - 2*a*cos(5*x) + 5*a*cos(3*x))*cos(10*x) + 5*(5*a*cos(
7*x) - 2*a*cos(5*x) + 5*a*cos(3*x))*cos(8*x) + 5*(10*a*cos(6*x) + 10*a*cos(
4*x) + 5*a*cos(2*x) + a)*cos(7*x) - 10*(2*a*cos(5*x) - 5*a*cos(3*x))*cos(6*
x) - 2*(10*a*cos(4*x) + 5*a*cos(2*x) + a)*cos(5*x) + 5*(5*a*cos(2*x) + a)*c
os(3*x) + (5*a*sin(7*x) - 2*a*sin(5*x) + 5*a*sin(3*x))*sin(10*x) + 5*(5*a*s
in(7*x) - 2*a*sin(5*x) + 5*a*sin(3*x))*sin(8*x) + 25*(2*a*sin(6*x) + 2*a*si
n(4*x) + a*sin(2*x))*sin(7*x) - 10*(2*a*sin(5*x) - 5*a*sin(3*x))*sin(6*x) -
 10*(2*a*sin(4*x) + a*sin(2*x))*sin(5*x))*sqrt(a)/(2*(5*cos(8*x) + 10*cos(6
*x) + 10*cos(4*x) + 5*cos(2*x) + 1)*cos(10*x) + cos(10*x)^2 + 10*(10*cos(6*
x) + 10*cos(4*x) + 5*cos(2*x) + 1)*cos(8*x) + 25*cos(8*x)^2 + 20*(10*cos(4*
x) + 5*cos(2*x) + 1)*cos(6*x) + 100*cos(6*x)^2 + 20*(5*cos(2*x) + 1)*cos(4*
x) + 100*cos(4*x)^2 + 25*cos(2*x)^2 + 10*(sin(8*x) + 2*sin(6*x) + 2*sin(4*x
```

```
) + sin(2*x))*sin(10*x) + sin(10*x)^2 + 50*(2*sin(6*x) + 2*sin(4*x) + sin(2
*x))*sin(8*x) + 25*sin(8*x)^2 + 100*(2*sin(4*x) + sin(2*x))*sin(6*x) + 100*
sin(6*x)^2 + 100*sin(4*x)^2 + 100*sin(4*x)*sin(2*x) + 25*sin(2*x)^2 + 10*co
s(2*x) + 1)
```

mupad [B] time = 12.01, size = 19, normalized size = 0.59

$$-\frac{2\sqrt{2}\,a^{3/2}\,\left(10\cos(x)^2-6\right)}{15\left(2\cos(x)^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3*(a + a*tan(x)^2)^(3/2),x)
[Out] -(2*2^(1/2)*a^(3/2)*(10*cos(x)^2 - 6))/(15*(2*cos(x)^2)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\tan^2(x)+1\right)\right)^{\frac{3}{2}}\tan^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3*(a+a*tan(x)**2)**(3/2),x)
[Out] Integral((a*(tan(x)**2 + 1))**(3/2)*tan(x)**3, x)
```

3.267 $\int \tan^2(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{1}{4}a\tan(x)\sec^2(x)\sqrt{a\sec^2(x)}-\frac{1}{8}a\tan(x)\sqrt{a\sec^2(x)}-\frac{1}{8}a\cos(x)\sqrt{a\sec^2(x)}\,\tanh^{-1}(\sin(x))$$

```
[Out] -1/8*a*arctanh(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)-1/8*a*(a*sec(x)^2)^(1/2)*t
an(x)+1/4*a*sec(x)^2*(a*sec(x)^2)^(1/2)*tan(x)
```

Rubi [A] time = 0.12, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3657, 4125, 2611, 3768, 3770}

$$\frac{1}{4}a\tan(x)\sec^2(x)\sqrt{a\sec^2(x)}-\frac{1}{8}a\tan(x)\sqrt{a\sec^2(x)}-\frac{1}{8}a\cos(x)\sqrt{a\sec^2(x)}\,\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -(a*ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2])/8 - (a*Sqrt[a*Sec[x]^2]*Tan[x]
)/8 + (a*Sec[x]^2*Sqrt[a*Sec[x]^2]*Tan[x])/4
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \tan^2(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx = \int \left(a\sec^2(x)\right)^{3/2}\tan^2(x)\,dx$$
$$= \left(a\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec^3(x)\tan^2(x)\,dx$$
$$= \frac{1}{4}a\sec^2(x)\sqrt{a\sec^2(x)}\,\tan(x) - \frac{1}{4}\left(a\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec^3(x)\,dx$$
$$= -\frac{1}{8}a\sqrt{a\sec^2(x)}\,\tan(x) + \frac{1}{4}a\sec^2(x)\sqrt{a\sec^2(x)}\,\tan(x) - \frac{1}{8}\left(a\cos(x)\sqrt{a\sec^2(x)}\right.$$
$$= -\frac{1}{8}a\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} - \frac{1}{8}a\sqrt{a\sec^2(x)}\,\tan(x) + \frac{1}{4}a\sec^2(x)\sqrt{a}$$

Mathematica [A] time = 0.07, size = 34, normalized size = 0.58

$$\frac{1}{8}\left(a\sec^2(x)\right)^{3/2}\left(2\tan(x)-\sin(x)\cos(x)+\cos^3(x)\left(-\tanh^{-1}(\sin(x))\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] ((a*Sec[x]^2)^(3/2)*(-(ArcTanh[Sin[x]]*Cos[x]^3) - Cos[x]*Sin[x] + 2*Tan[x]
))/8
```

fricas [A] time = 0.43, size = 57, normalized size = 0.97

$$\frac{1}{16}a^{\frac{3}{2}}\log\left(2\,a\tan(x)^2 - 2\,\sqrt{a\tan(x)^2+a}\,\sqrt{a}\,\tan(x)+a\right)+\frac{1}{8}\left(2\,a\tan(x)^3+a\tan(x)\right)\sqrt{a\tan(x)^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*a^(3/2)*log(2*a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a)
+ 1/8*(2*a*tan(x)^3 + a*tan(x))*sqrt(a*tan(x)^2 + a)
```

giac [A] time = 0.29, size = 49, normalized size = 0.83

$$\frac{1}{8}\left(\sqrt{a\tan(x)^2+a}\left(2\,\tan(x)^2+1\right)\tan(x)+\sqrt{a}\,\log\left(\left|-\sqrt{a}\,\tan(x)+\sqrt{a\tan(x)^2+a}\right|\right)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 + 1)*tan(x) + sqrt(a)*log(abs(-sqrt(a
)*tan(x) + sqrt(a*tan(x)^2 + a))))*a
```

maple [A] time = 0.14, size = 54, normalized size = 0.92

$$\frac{\tan(x)\left(a+a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}{4}-\frac{a\tan(x)\sqrt{a+a\left(\tan^2(x)\right)}}{8}-\frac{a^{\frac{3}{2}}\ln\left(\sqrt{a}\,\tan(x)+\sqrt{a+a\left(\tan^2(x)\right)}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2*(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] 1/4*tan(x)*(a+a*tan(x)^2)^(3/2)-1/8*a*tan(x)*(a+a*tan(x)^2)^(1/2)-1/8*a^(3/
2)*ln(a^(1/2)*tan(x)+(a+a*tan(x)^2)^(1/2))
```

maxima [B] time = 1.59, size = 934, normalized size = 15.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(112*a*cos(3*x)*sin(2*x) - 16*a*cos(x)*sin(2*x) + 16*a*cos(2*x)*sin(x)
 - 4*(a*sin(7*x) - 7*a*sin(5*x) + 7*a*sin(3*x) - a*sin(x))*cos(8*x) + 8*(2*
a*sin(6*x) + 3*a*sin(4*x) + 2*a*sin(2*x))*cos(7*x) + 16*(7*a*sin(5*x) - 7*a
*sin(3*x) + a*sin(x))*cos(6*x) - 56*(3*a*sin(4*x) + 2*a*sin(2*x))*cos(5*x)
- 24*(7*a*sin(3*x) - a*sin(x))*cos(4*x) - (a*cos(8*x)^2 + 16*a*cos(6*x)^2 +
 36*a*cos(4*x)^2 + 16*a*cos(2*x)^2 + a*sin(8*x)^2 + 16*a*sin(6*x)^2 + 36*a*
sin(4*x)^2 + 48*a*sin(4*x)*sin(2*x) + 16*a*sin(2*x)^2 + 2*(4*a*cos(6*x) + 6
*a*cos(4*x) + 4*a*cos(2*x) + a)*cos(8*x) + 8*(6*a*cos(4*x) + 4*a*cos(2*x) +
 a)*cos(6*x) + 12*(4*a*cos(2*x) + a)*cos(4*x) + 8*a*cos(2*x) + 4*(2*a*sin(6
*x) + 3*a*sin(4*x) + 2*a*sin(2*x))*sin(8*x) + 16*(3*a*sin(4*x) + 2*a*sin(2*
x))*sin(6*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (a*cos(8*x)^2 +
 16*a*cos(6*x)^2 + 36*a*cos(4*x)^2 + 16*a*cos(2*x)^2 + a*sin(8*x)^2 + 16*a*
sin(6*x)^2 + 36*a*sin(4*x)^2 + 48*a*sin(4*x)*sin(2*x) + 16*a*sin(2*x)^2 + 2
*(4*a*cos(6*x) + 6*a*cos(4*x) + 4*a*cos(2*x) + a)*cos(8*x) + 8*(6*a*cos(4*x
) + 4*a*cos(2*x) + a)*cos(6*x) + 12*(4*a*cos(2*x) + a)*cos(4*x) + 8*a*cos(2
*x) + 4*(2*a*sin(6*x) + 3*a*sin(4*x) + 2*a*sin(2*x))*sin(8*x) + 16*(3*a*sin
(4*x) + 2*a*sin(2*x))*sin(6*x) + a)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
 + 4*(a*cos(7*x) - 7*a*cos(5*x) + 7*a*cos(3*x) - a*cos(x))*sin(8*x) - 4*(4*
a*cos(6*x) + 6*a*cos(4*x) + 4*a*cos(2*x) + a)*sin(7*x) - 16*(7*a*cos(5*x) -
 7*a*cos(3*x) + a*cos(x))*sin(6*x) + 28*(6*a*cos(4*x) + 4*a*cos(2*x) + a)*s
in(5*x) + 24*(7*a*cos(3*x) - a*cos(x))*sin(4*x) - 28*(4*a*cos(2*x) + a)*sin
(3*x) + 4*a*sin(x))*sqrt(a)/(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*c
os(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*
x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*
sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x)
+ 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*
x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(x)^2 \left(a \tan(x)^2 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2*(a + a*tan(x)^2)^(3/2),x)
```

```
[Out] int(tan(x)^2*(a + a*tan(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\tan^2(x) + 1\right)\right)^{\frac{3}{2}} \tan^2(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2*(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral((a*(tan(x)**2 + 1))**(3/2)*tan(x)**2, x)
```

3.268 $\int \tan(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}$$

```
[Out] 1/3*(a*sec(x)^2)^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3657, 4124, 32}

$$\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] (a*Sec[x]^2)^(3/2)/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \tan(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx &= \int \left(a\sec^2(x)\right)^{3/2}\tan(x)\,dx\\ &= \frac{1}{2}a\,\text{Subst}\left(\int \sqrt{ax}\,dx, x, \sec^2(x)\right)\\ &= \frac{1}{3}\left(a\sec^2(x)\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] (a*Sec[x]^2)^(3/2)/3
```

fricas [A] time = 0.45, size = 12, normalized size = 0.86

$$\frac{1}{3}\left(a\tan(x)^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/3*(a*tan(x)^2 + a)^(3/2)
```

giac [A] time = 0.34, size = 12, normalized size = 0.86

$$\frac{1}{3}\left(a\tan(x)^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
[Out] 1/3*(a*tan(x)^2 + a)^(3/2)
```

maple [A] time = 0.08, size = 13, normalized size = 0.93

$$\frac{\left(a+a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a+a*tan(x)^2)^(3/2),x)
[Out] 1/3*(a+a*tan(x)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\tan(x)^2+a\right)^{\frac{3}{2}}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((a*tan(x)^2 + a)^(3/2)*tan(x), x)
```

mupad [B] time = 0.18, size = 11, normalized size = 0.79

$$\frac{a^{3/2}}{3\left(\cos(x)^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + a*tan(x)^2)^(3/2),x)
[Out] a^(3/2)/(3*(cos(x)^2)^(3/2))
```

sympy [A] time = 1.94, size = 12, normalized size = 0.86

$$\frac{\left(a\tan^2{(x)}+a\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+a*tan(x)**2)**(3/2),x)
[Out] (a*tan(x)**2 + a)**(3/2)/3
```

3.269 $\int \cot(x) \left(a + a \tan^2(x)\right)^{3/2} dx$

Optimal. Leaf size=37

$$a\sqrt{a\sec^2(x)} - a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)$$

```
[Out] -a^(3/2)*arctanh((a*sec(x)^2)^(1/2)/a^(1/2))+a*(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3657, 4124, 50, 63, 207}

$$a\sqrt{a\sec^2(x)} - a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -(a^(3/2)*ArcTanh[Sqrt[a*Sec[x]^2/Sqrt[a]]) + a*Sqrt[a*Sec[x]^2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(x)\left(a+a\tan^2(x)\right)^{3/2}dx &= \int \cot(x)\left(a\sec^2(x)\right)^{3/2}dx\\ &= \frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt{ax}}{-1+x}dx, x, \sec^2(x)\right)\\ &= a\sqrt{a\sec^2(x)}+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}dx, x, \sec^2(x)\right)\\ &= a\sqrt{a\sec^2(x)}+a\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}dx, x, \sqrt{a\sec^2(x)}\right)\\ &= -a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)+a\sqrt{a\sec^2(x)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 0.92

$$a\sqrt{a\sec^2(x)}\left(\cos(x)\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*(a + a*Tan[x]^2)^(3/2),x]
[Out] a*(1 + Cos[x]*(-Log[Cos[x/2]] + Log[Sin[x/2]]))*Sqrt[a*Sec[x]^2]
```

fricas [A] time = 0.44, size = 49, normalized size = 1.32

$$\frac{1}{2}a^{\frac{3}{2}}\log\left(\frac{a\tan(x)^2-2\sqrt{a\tan(x)^2+a}\sqrt{a}+2a}{\tan(x)^2}\right)+\sqrt{a\tan(x)^2+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/2*a^(3/2)*log((a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a) + 2*a)/tan(x)^
2) + sqrt(a*tan(x)^2 + a)*a
```

giac [A] time = 0.42, size = 42, normalized size = 1.14

$$a^2\left(\frac{\arctan\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\sqrt{a\tan(x)^2+a}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
[Out] a^2*(arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(a*tan(x)^2 + a)/
a)
```

maple [A] time = 0.42, size = 32, normalized size = 0.86

$$\left(\cos(x)\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)+\cos(x)+1\right)\left(\cos^2(x)\right)\left(\frac{a}{\cos(x)^2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] (cos(x)*ln(-(-1+cos(x))/sin(x))+cos(x)+1)*cos(x)^2*(a/cos(x)^2)^(3/2)
```

maxima [B] time = 0.96, size = 134, normalized size = 3.62

$$\frac{\left(4\,a\cos\left(2\,x\right)\cos(x) + 4\,a\sin\left(2\,x\right)\sin(x) + 4\,a\cos(x) - \left(a\cos\left(2\,x\right)^2 + a\sin\left(2\,x\right)^2 + 2\,a\cos\left(2\,x\right) + a\right)\log\left(\cos(x)\right.\right.}{2\left(\cos\left(2\,x\right)^2 + \sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*cos(2*x)*cos(x) + 4*a*sin(2*x)*sin(x) + 4*a*cos(x) - (a*cos(2*x)^2
 + a*sin(2*x)^2 + 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
 + (a*cos(2*x)^2 + a*sin(2*x)^2 + 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2
 - 2*cos(x) + 1))*sqrt(a)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

mupad [B] time = 11.67, size = 33, normalized size = 0.89

$$a\,\sqrt{a\,\tan(x)^2 + a} - a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a\,\tan(x)^2 + a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + a*tan(x)^2)^(3/2),x)
```

```
[Out] a*(a + a*tan(x)^2)^(1/2) - a^(3/2)*atanh((a + a*tan(x)^2)^(1/2)/a^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\tan^2\left(x\right) + 1\right)\right)^{\frac{3}{2}}\cot\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral((a*(tan(x)**2 + 1))**(3/2)*cot(x), x)
```

3.270 $\int \cot^2(x) \left(a + a\tan^2(x)\right)^{3/2} dx$

Optimal. Leaf size=33

$$a\cos(x)\sqrt{a\sec^2(x)}\,\tanh^{-1}(\sin(x)) - a\cot(x)\sqrt{a\sec^2(x)}$$

```
[Out] a*arctanh(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)-a*cot(x)*(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = $\{3657, 4125, 2621, 321, 207\}$

$$a\cos(x)\sqrt{a\sec^2(x)}\,\tanh^{-1}(\sin(x)) - a\cot(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] a*ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2] - a*Cot[x]*Sqrt[a*Sec[x]^2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \cot^2(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx &= \int \cot^2(x)\left(a\sec^2(x)\right)^{3/2}\,dx \\ &= \left(a\cos(x)\sqrt{a\sec^2(x)}\right)\int \csc^2(x)\sec(x)\,dx \\ &= -\left(\left(a\cos(x)\sqrt{a\sec^2(x)}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(x)\right)\right) \\ &= -a\cot(x)\sqrt{a\sec^2(x)} - \left(a\cos(x)\sqrt{a\sec^2(x)}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(x)\right) \\ &= a\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} - a\cot(x)\sqrt{a\sec^2(x)}\end{aligned}$$

Mathematica [C] time = 0.03, size = 27, normalized size = 0.82

$$-a\cot(x)\sqrt{a\sec^2(x)}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -(a*Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[x]^2]*Sqrt[a*Sec[x]^2])
```

fricas [A] time = 0.41, size = 53, normalized size = 1.61

$$\frac{a^{\frac{3}{2}}\log\left(2\,a\tan(x)^2+2\,\sqrt{a\tan(x)^2+a}\,\sqrt{a}\,\tan(x)+a\right)\tan(x)-2\,\sqrt{a\tan(x)^2+a}\,a}{2\,\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a^(3/2)*log(2*a*tan(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a)*
tan(x) - 2*sqrt(a*tan(x)^2 + a)*a)/tan(x)
```

giac [B] time = 0.75, size = 62, normalized size = 1.88

$$-\frac{1}{2}\left(\sqrt{a}\,\log\left(\left(\sqrt{a}\,\tan(x)-\sqrt{a\tan(x)^2+a}\right)^2\right)-\frac{4\,a^{\frac{3}{2}}}{\left(\sqrt{a}\,\tan(x)-\sqrt{a\tan(x)^2+a}\right)^2-a}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(a)*log((sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2) - 4*a^(3/2)/((
sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2 - a))*a
```

maple [A] time = 0.55, size = 55, normalized size = 1.67

$$-\frac{\left(\ln\left(-\frac{-1+\cos(x)+\sin(x)}{\sin(x)}\right)\sin(x)-\ln\left(\frac{1-\cos(x)+\sin(x)}{\sin(x)}\right)\sin(x)+1\right)\left(\cos^3(x)\right)\left(\frac{a}{\cos(x)^2}\right)^{\frac{3}{2}}}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] -(ln(-(-1+cos(x)+sin(x))/sin(x))*sin(x)-ln((1-cos(x)+sin(x))/sin(x))*sin(x)
+1)*cos(x)^3*(a/cos(x)^2)^(3/2)/sin(x)
```

maxima [B] time = 1.01, size = 134, normalized size = 4.06

$$-\frac{\left(4\,a\cos(x)\sin\left(2\,x\right)-4\,a\cos\left(2\,x\right)\sin(x)-\left(a\cos\left(2\,x\right)^{2}+a\sin\left(2\,x\right)^{2}-2\,a\cos\left(2\,x\right)+a\right)\log\left(\cos(x)^{2}+\sin(x\right.}{2\left(\cos\left(2\,x\right)^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(4*a*cos(x)*sin(2*x) - 4*a*cos(2*x)*sin(x) - (a*cos(2*x)^2 + a*sin(2*x
)^2 - 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (a*cos(2*
x)^2 + a*sin(2*x)^2 - 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 - 2*sin(x)
+ 1) + 4*a*sin(x))*sqrt(a)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \cot(x)^2\,\left(a\tan(x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a + a*tan(x)^2)^(3/2),x)
```

```
[Out] int(cot(x)^2*(a + a*tan(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\tan^{2}(x)+1\right)\right)^{\frac{3}{2}}\cot^{2}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2*(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral((a*(tan(x)**2 + 1))**(3/2)*cot(x)**2, x)
```

3.271 $\int \left(a+a\tan^2(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=68

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{2d}+\frac{a\tan(c+dx)\sqrt{a\sec^2(c+dx)}}{2d}$$

```
[Out] 1/2*a^(3/2)*arctanh(a^(1/2)*tan(d*x+c)/(a*sec(d*x+c)^2)^(1/2))/d+1/2*a*(a*s
ec(d*x+c)^2)^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3657, 4122, 195, 217, 206}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{2d}+\frac{a\tan(c+dx)\sqrt{a\sec^2(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a*Sec[c + d*x]^2]])/(2*d) + (a
*Sqrt[a*Sec[c + d*x]^2]*Tan[c + d*x])/(2*d)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+a\tan^2(c+dx)\right)^{3/2} dx &= \int \left(a\sec^2(c+dx)\right)^{3/2} dx \\
&= \frac{a\,\text{Subst}\left(\int \sqrt{a+ax^2}\,dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{a\sqrt{a\sec^2(c+dx)}\,\tan(c+dx)}{2d} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+ax^2}}\,dx, x, \tan(c+dx)\right)}{2d} \\
&= \frac{a\sqrt{a\sec^2(c+dx)}\,\tan(c+dx)}{2d} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{2d} \\
&= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{2d} + \frac{a\sqrt{a\sec^2(c+dx)}\,\tan(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 43, normalized size = 0.63

$$
\frac{a\sqrt{a\sec^2(c+dx)}\left(\tan(c+dx)+\cos(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(3/2),x]
```

```
[Out] (a*Sqrt[a*Sec[c + d*x]^2]*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + Tan[c + d*x
]))/(2*d)
```

fricas [A] time = 0.41, size = 72, normalized size = 1.06

$$
\frac{a^{\frac{3}{2}}\log\left(2\,a\tan\left(dx+c\right)^2+2\sqrt{a\tan\left(dx+c\right)^2+a}\sqrt{a}\,\tan\left(dx+c\right)+a\right)+2\sqrt{a\tan\left(dx+c\right)^2+a}\,a\tan\left(dx+c\right)}{4\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(a^(3/2)*log(2*a*tan(d*x + c)^2 + 2*sqrt(a*tan(d*x + c)^2 + a)*sqrt(a)*
tan(d*x + c) + a) + 2*sqrt(a*tan(d*x + c)^2 + a)*a*tan(d*x + c))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
```

to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)2/d*((-sqrt(a)*a*tan(d*x/2)*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*t
an(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^
3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)-sqrt(a)*a*tan(d*x/2)^3*sign(-4*
tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)
*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c
)^3+1)+2*sqrt(a)*a*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^
4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/
2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^3+2*sqrt(a)*a*sign(-4*tan(d*x/
2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2
*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*t
an(1/2*c)^5-2*sqrt(a)*a*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/
2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan
(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^7-2*sqrt(a)*a*sign(-4*tan
(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*ta
n(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3
+1)*tan(1/2*c)+10*sqrt(a)*a*tan(d*x/2)*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(
d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*t
an(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^2-10*sqrt(
a)*a*tan(d*x/2)*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-t
an(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^
4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^6+sqrt(a)*a*tan(d*x/2)*sign(-4*ta
n(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*t
an(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^
3+1)*tan(1/2*c)^8-18*sqrt(a)*a*tan(d*x/2)^2*sign(-4*tan(d*x/2)^3*tan(1/2*c)
+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2
)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^3-18*
sqrt(a)*a*tan(d*x/2)^2*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2
*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(
d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^5+2*sqrt(a)*a*tan(d*x/2)^2
*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*t
an(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*
tan(1/2*c)^3+1)*tan(1/2*c)^7+2*sqrt(a)*a*tan(d*x/2)^2*sign(-4*tan(d*x/2)^3*
tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4
*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/
2*c)-6*sqrt(a)*a*tan(d*x/2)^3*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*
tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)
^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^2+6*sqrt(a)*a*tan(d
*x/2)^3*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c
)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(
d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^6+sqrt(a)*a*tan(d*x/2)^3*sign(-4*tan(d*x/
2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2
*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*t
an(1/2*c)^8)/(-tan(1/2*c)^2-4*tan(d*x/2)*tan(1/2*c)+tan(d*x/2)^2*tan(1/2*c)
^2-tan(d*x/2)^2+1)^2/(-2*tan(1/2*c)^4+4*tan(1/2*c)^2-2)+(sqrt(a)*a*sign(-4*
tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)
*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c
)^3+1)-sqrt(a)*a*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-
tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)
^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c))*ln(abs(-tan(d*x/2)*tan(1/2*c)+t
an(1/2*c)+tan(d*x/2)+1))/(-4*tan(1/2*c)+4)+(-sqrt(a)*a*sign(-4*tan(d*x/2)^3
*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-
4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)-sqrt(
a)*a*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4
-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x
/2)*tan(1/2*c)^3+1)*tan(1/2*c))*ln(abs(-tan(d*x/2)*tan(1/2*c)-tan(1/2*c)-ta

n(d*x/2)+1))/(4*tan(1/2*c)+4))

maple [A] time = 0.28, size = 62, normalized size = 0.91

$$\frac{a\tan\left(dx+c\right)\sqrt{a+a\left(\tan^{2}\left(dx+c\right)\right)}}{2d}+\frac{a^{\frac{3}{2}}\ln\left(\sqrt{a}\ \tan\left(dx+c\right)+\sqrt{a+a\left(\tan^{2}\left(dx+c\right)\right)}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^(3/2),x)
```

```
[Out] 1/2/d*a*tan(d*x+c)*(a+a*tan(d*x+c)^2)^(1/2)+1/2/d*a^(3/2)*ln(a^(1/2)*tan(d*
x+c)+(a+a*tan(d*x+c)^2)^(1/2))
```

maxima [B] time = 1.01, size = 556, normalized size = 8.18

$$-\frac{\left(8\,a\cos\left(3\,dx+3\,c\right)\sin\left(2\,dx+2\,c\right)-8\,a\cos\left(dx+c\right)\sin\left(2\,dx+2\,c\right)+8\,a\cos\left(2\,dx+2\,c\right)\sin\left(dx+c\right)-4\,(a\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(8*a*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 8*a*cos(d*x + c)*sin(2*d*x +
2*c) + 8*a*cos(2*d*x + 2*c)*sin(d*x + c) - 4*(a*sin(3*d*x + 3*c) - a*sin(d*
x + c))*cos(4*d*x + 4*c) - (a*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 +
 a*sin(4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d
*x + 2*c)^2 + 2*(2*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x
 + 2*c) + a)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + (a
*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin(4*d*x + 4*c)^2 + 4*a*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2*c)^2 + 2*(2*a*cos(2*d*
x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*log(cos(d*x + c)
^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 4*(a*cos(3*d*x + 3*c) - a*cos(d
*x + c))*sin(4*d*x + 4*c) - 4*(2*a*cos(2*d*x + 2*c) + a)*sin(3*d*x + 3*c) +
 4*a*sin(d*x + c))*sqrt(a)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*
d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,{\mathrm{tan}\left(c+d\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)^2)^(3/2),x)
```

```
[Out] int((a + a*tan(c + d*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \tan^{2}{\left(c + dx \right)} + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((a*tan(c + d*x)**2 + a)**(3/2), x)
```

3.272 $\int \left(a+a\tan^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{8d}+\frac{3a^2\tan(c+dx)\sqrt{a\sec^2(c+dx)}}{8d}+\frac{a\tan(c+dx)\left(a\sec^2(c+dx)\right)^{3/2}}{4d}$$

```
[Out] 3/8*a^(5/2)*arctanh(a^(1/2)*tan(d*x+c)/(a*sec(d*x+c)^2)^(1/2))/d+1/4*a*(a*s
ec(d*x+c)^2)^(3/2)*tan(d*x+c)/d+3/8*a^2*(a*sec(d*x+c)^2)^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3657, 4122, 195, 217, 206}

$$\frac{3a^2\tan(c+dx)\sqrt{a\sec^2(c+dx)}}{8d}+\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{8d}+\frac{a\tan(c+dx)\left(a\sec^2(c+dx)\right)^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] (3*a^(5/2)*ArcTanh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a*Sec[c + d*x]^2]])/(8*d) +
(3*a^2*Sqrt[a*Sec[c + d*x]^2]*Tan[c + d*x])/(8*d) + (a*(a*Sec[c + d*x]^2)^(
3/2)*Tan[c + d*x])/(4*d)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+a\tan^2(c+dx)\right)^{5/2}\,dx = \int \left(a\sec^2(c+dx)\right)^{5/2}\,dx$$
$$= \frac{a\,\mathrm{Subst}\left(\int \left(a+ax^2\right)^{3/2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{a\left(a\sec^2(c+dx)\right)^{3/2}\tan(c+dx)}{4d} + \frac{(3a^2)\,\mathrm{Subst}\left(\int \sqrt{a+ax^2}\,dx, x, \tan(c+dx\right.}{4d}$$
$$= \frac{3a^2\sqrt{a\sec^2(c+dx)}\,\tan(c+dx)}{8d} + \frac{a\left(a\sec^2(c+dx)\right)^{3/2}\tan(c+dx)}{4d} + \frac{(3a^3)\,\mathrm{Su}}{}$$
$$= \frac{3a^2\sqrt{a\sec^2(c+dx)}\,\tan(c+dx)}{8d} + \frac{a\left(a\sec^2(c+dx)\right)^{3/2}\tan(c+dx)}{4d} + \frac{(3a^3)\,\mathrm{Su}}{}$$
$$= \frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{8d} + \frac{3a^2\sqrt{a\sec^2(c+dx)}\,\tan(c+dx)}{8d} + \frac{a\left(a\sec^2(c+\right.}{}$$

Mathematica [A] time = 0.22, size = 65, normalized size = 0.66

$$\frac{a^2\cos(c+dx)\sqrt{a\sec^2(c+dx)}\left(3\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\left(2\sec^2(c+dx)+3\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] (a^2*Cos[c + d*x]*Sqrt[a*Sec[c + d*x]^2]*(3*ArcTanh[Sin[c + d*x]] + Sec[c +
 d*x]*(3 + 2*Sec[c + d*x]^2)*Tan[c + d*x]))/(8*d)
```

fricas [A] time = 0.40, size = 91, normalized size = 0.93

$$\frac{3\,a^{\frac{5}{2}}\log\left(2\,a\tan\left(dx+c\right)^2+2\,\sqrt{a\tan\left(dx+c\right)^2+a}\,\sqrt{a}\,\tan\left(dx+c\right)+a\right)+2\left(2\,a^2\tan\left(dx+c\right)^3+5\,a^2\tan\left(dx\right.\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*a^(5/2)*log(2*a*tan(d*x + c)^2 + 2*sqrt(a*tan(d*x + c)^2 + a)*sqrt(
a)*tan(d*x + c) + a) + 2*(2*a^2*tan(d*x + c)^3 + 5*a^2*tan(d*x + c))*sqrt(a
*tan(d*x + c)^2 + a))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
```

```
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)2/d*((5*sqrt(a)*a^2*tan(d*x/2)*sign(-4*tan(d*x/2)^3*tan(1/2*c)
+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2
)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)+3*sqrt(a)*a^2*ta
n(d*x/2)^3*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/
2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*t
an(d*x/2)*tan(1/2*c)^3+1)+3*sqrt(a)*a^2*tan(d*x/2)^5*sign(-4*tan(d*x/2)^3*t
an(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*
tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)+5*sqrt(
a)*a^2*tan(d*x/2)^7*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)
^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x
/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)-34*sqrt(a)*a^2*sign(-4*tan(d*x/2)^3*tan(
1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan
(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)
^3+42*sqrt(a)*a^2*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4
-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2
)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^5-18*sqrt(a)*a^2*sign(-4*tan(d*
x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1
/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)
*tan(1/2*c)^7-18*sqrt(a)*a^2*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*t
an(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^
3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^9+42*sqrt(a)*a^2*sig
n(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d
*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(
```

1/2*c)^3+1)*tan(1/2*c)^11-34*sqrt(a)*a^2*sign(-4*tan(d*x/2)^3*tan(1/2*c)+ta
n(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3
*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^13+10*sq
rt(a)*a^2*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2
*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*ta
n(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^15+10*sqrt(a)*a^2*sign(-4*tan(d*x/2)^3*
tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4
*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/
2*c)-126*sqrt(a)*a^2*tan(d*x/2)*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^
4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*
c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^2+326*sqrt(a)*a^2
*tan(d*x/2)*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1
/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*
tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^4-294*sqrt(a)*a^2*tan(d*x/2)*sign(-4*
tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)
*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c
)^3+1)*tan(1/2*c)^6+294*sqrt(a)*a^2*tan(d*x/2)*sign(-4*tan(d*x/2)^3*tan(1/2
*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*
x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^10
-326*sqrt(a)*a^2*tan(d*x/2)*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*ta
n(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3
-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^12+126*sqrt(a)*a^2*ta
n(d*x/2)*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*
c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan
(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^14-5*sqrt(a)*a^2*tan(d*x/2)*sign(-4*tan(
d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan
(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+
1)*tan(1/2*c)^16+742*sqrt(a)*a^2*tan(d*x/2)^2*sign(-4*tan(d*x/2)^3*tan(1/2*
c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x
/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^3-1
182*sqrt(a)*a^2*tan(d*x/2)^2*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*t
an(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^
3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^5+486*sqrt(a)*a^2*ta
n(d*x/2)^2*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/
2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*t
an(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^7+486*sqrt(a)*a^2*tan(d*x/2)^2*sign(-4
*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2
)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*
c)^3+1)*tan(1/2*c)^9-1182*sqrt(a)*a^2*tan(d*x/2)^2*sign(-4*tan(d*x/2)^3*tan
(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*ta
n(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c
)^11+742*sqrt(a)*a^2*tan(d*x/2)^2*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2
)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/
2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^13-46*sqrt(a)*a
^2*tan(d*x/2)^2*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-t
an(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^
4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^15-46*sqrt(a)*a^2*tan(d*x/2)^2*si
gn(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(
d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan
(1/2*c)^3+1)*tan(1/2*c)+318*sqrt(a)*a^2*tan(d*x/2)^3*sign(-4*tan(d*x/2)^3*t
an(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*
tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2
*c)^2-2198*sqrt(a)*a^2*tan(d*x/2)^3*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x
/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(
1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^4+1382*sqrt(a
)*a^2*tan(d*x/2)^3*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^
4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/
2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^6-1382*sqrt(a)*a^2*tan(d*x/2)^
3*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*

tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)
*tan(1/2*c)^3+1)*tan(1/2*c)^10+2198*sqrt(a)*a^2*tan(d*x/2)^3*sign(-4*tan(d*
x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1
/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)
*tan(1/2*c)^12-318*sqrt(a)*a^2*tan(d*x/2)^3*sign(-4*tan(d*x/2)^3*tan(1/2*c)
+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2
)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^14-3*
sqrt(a)*a^2*tan(d*x/2)^3*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1
/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-ta
n(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^16-1062*sqrt(a)*a^2*tan(
d*x/2)^4*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*
c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan
(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^3+2750*sqrt(a)*a^2*tan(d*x/2)^4*sign(-4*
tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)
*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c
)^3+1)*tan(1/2*c)^5+330*sqrt(a)*a^2*tan(d*x/2)^4*sign(-4*tan(d*x/2)^3*tan(1
/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(
d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^
7+330*sqrt(a)*a^2*tan(d*x/2)^4*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4
*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c
)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^9+2750*sqrt(a)*a^2
*tan(d*x/2)^4*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan
(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-
4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^11-1062*sqrt(a)*a^2*tan(d*x/2)^4*si
gn(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(
d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan
(1/2*c)^3+1)*tan(1/2*c)^13+30*sqrt(a)*a^2*tan(d*x/2)^4*sign(-4*tan(d*x/2)^3
*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-
4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1
/2*c)^15+30*sqrt(a)*a^2*tan(d*x/2)^4*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*
x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan
(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)-162*sqrt(a)*
a^2*tan(d*x/2)^5*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-
tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)
^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^2+1514*sqrt(a)*a^2*tan(d*x/2)^5*
sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*ta
n(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*t
an(1/2*c)^3+1)*tan(1/2*c)^4-506*sqrt(a)*a^2*tan(d*x/2)^5*sign(-4*tan(d*x/2)
^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c
)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan
(1/2*c)^6+506*sqrt(a)*a^2*tan(d*x/2)^5*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(
d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*t
an(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^10-1514*sq
rt(a)*a^2*tan(d*x/2)^5*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2
*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(
d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^12+162*sqrt(a)*a^2*tan(d*x
/2)^5*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^
4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*
x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^14-3*sqrt(a)*a^2*tan(d*x/2)^5*sign(-4*tan(d
*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(
1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1
)*tan(1/2*c)^16+354*sqrt(a)*a^2*tan(d*x/2)^6*sign(-4*tan(d*x/2)^3*tan(1/2*c
)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/
2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^3-33
0*sqrt(a)*a^2*tan(d*x/2)^6*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan
(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-
tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^5-30*sqrt(a)*a^2*tan(d
*x/2)^6*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c
)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(

d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^7-30*sqrt(a)*a^2*tan(d*x/2)^6*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^9-330*sqrt(a)*a^2*tan(d*x/2)^6*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^11+354*sqrt(a)*a^2*tan(d*x/2)^6*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^13+6*sqrt(a)*a^2*tan(d*x/2)^6*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^15+6*sqrt(a)*a^2*tan(d*x/2)^6*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)+34*sqrt(a)*a^2*tan(d*x/2)^7*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^2-58*sqrt(a)*a^2*tan(d*x/2)^7*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^4-6*sqrt(a)*a^2*tan(d*x/2)^7*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^6+6*sqrt(a)*a^2*tan(d*x/2)^7*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^10+58*sqrt(a)*a^2*tan(d*x/2)^7*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^12-34*sqrt(a)*a^2*tan(d*x/2)^7*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^14-5*sqrt(a)*a^2*tan(d*x/2)^7*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c)^16)/(-tan(1/2*c)^2-4*tan(d*x/2)*tan(1/2*c)+tan(d*x/2)^2*tan(1/2*c)^2-tan(d*x/2)^2+1)^4/(8*tan(1/2*c)^8-32*tan(1/2*c)^6+48*tan(1/2*c)^4-32*tan(1/2*c)^2+8)+(3*sqrt(a)*a^2*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)-3*sqrt(a)*a^2*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c))*ln(abs(-tan(d*x/2)*tan(1/2*c)+tan(1/2*c)+tan(d*x/2)+1))/(-16*tan(1/2*c)+16)+(-3*sqrt(a)*a^2*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)-3*sqrt(a)*a^2*sign(-4*tan(d*x/2)^3*tan(1/2*c)+tan(d*x/2)^4*tan(1/2*c)^4-tan(1/2*c)^4-4*tan(d*x/2)*tan(1/2*c)-4*tan(d*x/2)^3*tan(1/2*c)^3-tan(d*x/2)^4-4*tan(d*x/2)*tan(1/2*c)^3+1)*tan(1/2*c))*ln(abs(-tan(d*x/2)*tan(1/2*c)-tan(1/2*c)-tan(d*x/2)+1))/(16*tan(1/2*c)+16))

maple [A] time = 0.26, size = 90, normalized size = 0.92

$$\frac{a\tan(dx+c)\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}{4d}+\frac{3a^2\tan(dx+c)\sqrt{a+a\left(\tan^2(dx+c)\right)}}{8d}+\frac{3a^{\frac{5}{2}}\ln\left(\sqrt{a}\ \tan(dx+c)+\sqrt{}\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^(5/2),x)

[Out] 1/4/d*a*tan(d*x+c)*(a+a*tan(d*x+c)^2)^(3/2)+3/8/d*a^2*tan(d*x+c)*(a+a*tan(d
*x+c)^2)^(1/2)+3/8/d*a^(5/2)*ln(a^(1/2)*tan(d*x+c)+(a+a*tan(d*x+c)^2)^(1/2)
```

)

maxima [B] time = 1.64, size = 1769, normalized size = 18.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*(176*a^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 48*a^2*cos(d*x + c)*sin(2
*d*x + 2*c) - 48*a^2*cos(2*d*x + 2*c)*sin(d*x + c) - 12*a^2*sin(d*x + c) +
4*(3*a^2*sin(7*d*x + 7*c) + 11*a^2*sin(5*d*x + 5*c) - 11*a^2*sin(3*d*x + 3*
c) - 3*a^2*sin(d*x + c))*cos(8*d*x + 8*c) - 24*(2*a^2*sin(6*d*x + 6*c) + 3*
a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*cos(7*d*x + 7*c) + 16*(11*a^
2*sin(5*d*x + 5*c) - 11*a^2*sin(3*d*x + 3*c) - 3*a^2*sin(d*x + c))*cos(6*d*
x + 6*c) - 88*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*cos(5*d*x +
 5*c) - 24*(11*a^2*sin(3*d*x + 3*c) + 3*a^2*sin(d*x + c))*cos(4*d*x + 4*c)
+ 3*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x
+ 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(
6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(
4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) +
a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c)
+ a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c
) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x +
2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) +
 1) - 3*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*
d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*
sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 +
 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c
) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2
*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x +
 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*
x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x +
 2*c))*sin(6*d*x + 6*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x +
c) + 1) - 4*(3*a^2*cos(7*d*x + 7*c) + 11*a^2*cos(5*d*x + 5*c) - 11*a^2*cos(
3*d*x + 3*c) - 3*a^2*cos(d*x + c))*sin(8*d*x + 8*c) + 12*(4*a^2*cos(6*d*x +
 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*sin(7*d*x +
7*c) - 16*(11*a^2*cos(5*d*x + 5*c) - 11*a^2*cos(3*d*x + 3*c) - 3*a^2*cos(d*
x + c))*sin(6*d*x + 6*c) + 44*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2
*c) + a^2)*sin(5*d*x + 5*c) + 24*(11*a^2*cos(3*d*x + 3*c) + 3*a^2*cos(d*x +
 c))*sin(4*d*x + 4*c) - 44*(4*a^2*cos(2*d*x + 2*c) + a^2)*sin(3*d*x + 3*c))
*sqrt(a)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c)
+ 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(
2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c
)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(
8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x +
2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x +
2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\tan\left(c+d\,x\right)^2+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)^2)^(5/2),x)
```

```
[Out] int((a + a*tan(c + d*x)^2)^(5/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\tan^2\left(c+dx\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a*tan(c + d*x)**2 + a)**(5/2), x)
```

3.273 $\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=25

$$\frac{\sqrt{a\sec^2(x)}}{a}+\frac{1}{\sqrt{a\sec^2(x)}}$$

```
[Out] 1/(a*sec(x)^2)^(1/2)+(a*sec(x)^2)^(1/2)/a
```

Rubi [A] time = 0.09, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3657, 4124, 43}

$$\frac{\sqrt{a\sec^2(x)}}{a}+\frac{1}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] 1/Sqrt[a*Sec[x]^2] + Sqrt[a*Sec[x]^2]/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx &= \int \frac{\tan^3(x)}{\sqrt{a\sec^2(x)}}\,dx\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{-1+x}{(ax)^{3/2}}\,dx,x,\sec^2(x)\right)\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \left(-\frac{1}{(ax)^{3/2}}+\frac{1}{a\sqrt{ax}}\right)dx,x,\sec^2(x)\right)\\
&= \frac{1}{\sqrt{a\sec^2(x)}}+\frac{\sqrt{a\sec^2(x)}}{a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 0.68

$$\frac{\sec^2(x)+1}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/Sqrt[a + a*Tan[x]^2],x]
[Out] (1 + Sec[x]^2)/Sqrt[a*Sec[x]^2]
```

fricas [A] time = 0.39, size = 17, normalized size = 0.68

$$\frac{\tan(x)^2+2}{\sqrt{a\tan(x)^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] (tan(x)^2 + 2)/sqrt(a*tan(x)^2 + a)
```

giac [A] time = 0.39, size = 27, normalized size = 1.08

$$\frac{\sqrt{a\tan(x)^2+a}+\frac{a}{\sqrt{a\tan(x)^2+a}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
[Out] (sqrt(a*tan(x)^2 + a) + a/sqrt(a*tan(x)^2 + a))/a
```

maple [A] time = 0.22, size = 26, normalized size = 1.04

$$\frac{\sqrt{a+a\left(\tan^2(x)\right)}}{a}+\frac{1}{\sqrt{a+a\left(\tan^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+a*tan(x)^2)^(1/2),x)
[Out] 1/a*(a+a*tan(x)^2)^(1/2)+1/(a+a*tan(x)^2)^(1/2)
```

maxima [A] time = 0.69, size = 37, normalized size = 1.48

$$\frac{\left(\sin(x)^2-2\right)\sqrt{\sin(x)+1}\,\sqrt{-\sin(x)+1}}{\sqrt{a}\,\sin(x)^2-\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
[Out] (sin(x)^2 - 2)*sqrt(sin(x) + 1)*sqrt(-sin(x) + 1)/(sqrt(a)*sin(x)^2 - sqrt(
a))
```

mupad [B] time = 0.29, size = 22, normalized size = 0.88

$$\frac{\sqrt{2}\,\left(\cos\left(2\,x\right)+3\right)}{2\,\sqrt{a}\,\sqrt{\cos\left(2\,x\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a + a*tan(x)^2)^(1/2),x)
[Out] (2^(1/2)*(cos(2*x) + 3))/(2*a^(1/2)*(cos(2*x) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(x)}{\sqrt{a\left(\tan^2(x)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+a*tan(x)**2)**(1/2),x)
[Out] Integral(tan(x)**3/sqrt(a*(tan(x)**2 + 1)), x)
```

3.274 $\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=31

$$\frac{\sec(x)\tanh^{-1}(\sin(x))}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

```
[Out] arctanh(sin(x))*sec(x)/(a*sec(x)^2)^(1/2)-tan(x)/(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3657, 4125, 2592, 321, 206}

$$\frac{\sec(x)\tanh^{-1}(\sin(x))}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (ArcTanh[Sin[x]]*Sec[x])/Sqrt[a*Sec[x]^2] - Tan[x]/Sqrt[a*Sec[x]^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx &= \int \frac{\tan^2(x)}{\sqrt{a\sec^2(x)}}\,dx\\
&= \frac{\sec(x)\int \sin(x)\tan(x)\,dx}{\sqrt{a\sec^2(x)}}\\
&= \frac{\sec(x)\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(x)\right)}{\sqrt{a\sec^2(x)}}\\
&= -\frac{\tan(x)}{\sqrt{a\sec^2(x)}} + \frac{\sec(x)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)}{\sqrt{a\sec^2(x)}}\\
&= \frac{\tanh^{-1}(\sin(x))\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{\tan(x)}{\sqrt{a\sec^2(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 49, normalized size = 1.58

$$
-\frac{\sec(x)\left(\sin(x)+\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\sec^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2/Sqrt[a + a*Tan[x]^2],x]
[Out] -((Sec[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + Sin[x]))/S
qrt[a*Sec[x]^2])
```

fricas [B] time = 0.42, size = 64, normalized size = 2.06

$$
\frac{\left(\tan(x)^2+1\right)\sqrt{a}\log\left(2\,a\tan(x)^2+2\sqrt{a\tan(x)^2+a}\sqrt{a}\tan(x)+a\right)-2\sqrt{a\tan(x)^2+a}\tan(x)}{2\left(a\tan(x)^2+a\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*((tan(x)^2 + 1)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(
a)*tan(x) + a) - 2*sqrt(a*tan(x)^2 + a)*tan(x))/(a*tan(x)^2 + a)
```

giac [A] time = 0.39, size = 40, normalized size = 1.29

$$
-\frac{\log\left(\left|-\sqrt{a}\tan(x)+\sqrt{a\tan(x)^2+a}\right|\right)}{\sqrt{a}} - \frac{\tan(x)}{\sqrt{a\tan(x)^2+a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
[Out] -log(abs(-sqrt(a)*tan(x) + sqrt(a*tan(x)^2 + a)))/sqrt(a) - tan(x)/sqrt(a*t
an(x)^2 + a)
```

maple [A] time = 0.23, size = 38, normalized size = 1.23

$$
\frac{\ln\left(\sqrt{a}\tan(x)+\sqrt{a+a\left(\tan^2(x)\right)}\right)}{\sqrt{a}} - \frac{\tan(x)}{\sqrt{a+a\left(\tan^2(x)\right)}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+a*tan(x)^2)^(1/2),x)
```

```
[Out] ln(a^(1/2)*tan(x)+(a+a*tan(x)^2)^(1/2))/a^(1/2)-tan(x)/(a+a*tan(x)^2)^(1/2)
```

maxima [A] time = 1.88, size = 42, normalized size = 1.35

$$\frac{\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)-\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)-2\sin(x)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*
sin(x) + 1) - 2*sin(x))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(x)^2}{\sqrt{a\tan(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)^2/(a + a*tan(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(x)}{\sqrt{a\left(\tan^2(x)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)**2/sqrt(a*(tan(x)**2 + 1)), x)
```

3.275 $\int \frac{\tan(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{\sqrt{a\sec^2(x)}}$$

```
[Out] -1/(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3657, 4124, 32}

$$-\frac{1}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(1/Sqrt[a*Sec[x]^2])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\sqrt{a+a\tan^2(x)}}\,dx &= \int \frac{\tan(x)}{\sqrt{a\sec^2(x)}}\,dx\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{(ax)^{3/2}}\,dx, x, \sec^2(x)\right)\\
&= -\frac{1}{\sqrt{a\sec^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\frac{1}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + a*Tan[x]^2],x]
[Out] -(1/Sqrt[a*Sec[x]^2])
```

fricas [A] time = 0.41, size = 12, normalized size = 1.00

$$-\frac{1}{\sqrt{a\tan(x)^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/sqrt(a*tan(x)^2 + a)
```

giac [A] time = 0.29, size = 12, normalized size = 1.00

$$-\frac{1}{\sqrt{a\tan(x)^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
[Out] -1/sqrt(a*tan(x)^2 + a)
```

maple [A] time = 0.15, size = 13, normalized size = 1.08

$$-\frac{1}{\sqrt{a + a\left(\tan^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+a*tan(x)^2)^(1/2),x)
[Out] -1/(a+a*tan(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{a\tan(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(x)/sqrt(a*tan(x)^2 + a), x)
```

mupad [B] time = 11.80, size = 11, normalized size = 0.92

$$-\frac{\sqrt{\cos(x)^2}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + a*tan(x)^2)^(1/2),x)
[Out] -(cos(x)^2)^(1/2)/a^(1/2)
```

sympy [A] time = 0.57, size = 14, normalized size = 1.17

$$-\frac{1}{\sqrt{a\tan^2{(x)} + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] -1/sqrt(a*tan(x)**2 + a)
```

3.276 $\int \frac{\cot(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=35

$$\frac{1}{\sqrt{a\sec^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh((a*sec(x)^2)^(1/2)/a^(1/2))/a^(1/2)+1/(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3657, 4124, 51, 63, 207}

$$\frac{1}{\sqrt{a\sec^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]]/Sqrt[a]) + 1/Sqrt[a*Sec[x]^2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\sqrt{a+a\tan^2(x)}}\,dx &= \int \frac{\cot(x)}{\sqrt{a\sec^2(x)}}\,dx\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{(-1+x)(ax)^{3/2}}\,dx, x, \sec^2(x)\right)\\
&= \frac{1}{\sqrt{a\sec^2(x)}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx, x, \sec^2(x)\right)\\
&= \frac{1}{\sqrt{a\sec^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a\sec^2(x)}\right)}{a}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{1}{\sqrt{a\sec^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.91

$$\frac{\sec(x)\left(\cos(x)+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + a*Tan[x]^2],x]
[Out] ((Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]])*Sec[x])/Sqrt[a*Sec[x]^2]
```

fricas [B] time = 0.44, size = 66, normalized size = 1.89

$$\frac{\left(\tan(x)^2+1\right)\sqrt{a}\,\log\left(\frac{a\tan(x)^2-2\,\sqrt{a\tan(x)^2+a}\,\sqrt{a}+2\,a}{\tan(x)^2}\right)+2\,\sqrt{a\tan(x)^2+a}}{2\left(a\tan(x)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*((tan(x)^2 + 1)*sqrt(a)*log((a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a
) + 2*a)/tan(x)^2) + 2*sqrt(a*tan(x)^2 + a))/(a*tan(x)^2 + a)
```

giac [A] time = 0.35, size = 34, normalized size = 0.97

$$\frac{\arctan\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{\sqrt{a\tan(x)^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/sqrt(-a) + 1/sqrt(a*tan(x)^2 + a)
```

maple [A] time = 0.62, size = 29, normalized size = 0.83

$$\frac{\cos(x)+\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)+1}{\sqrt{\frac{a}{\cos(x)^2}}\,\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+a*tan(x)^2)^(1/2),x)
```

```
[Out] (cos(x)+ln(-(-1+cos(x))/sin(x))+1)/(a/cos(x)^2)^(1/2)/cos(x)
```

maxima [A] time = 0.90, size = 42, normalized size = 1.20

$$\frac{2\cos(x) - \log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*cos(x) - log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1))/sqrt(a)
```

mupad [B] time = 0.14, size = 31, normalized size = 0.89

$$\frac{1}{\sqrt{a\tan(x)^2 + a}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] 1/(a + a*tan(x)^2)^(1/2) - atanh((a + a*tan(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{a\left(\tan^2(x) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a*(tan(x)**2 + 1)), x)
```

3.277 $\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=31

$$-\frac{\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

```
[Out] -csc(x)*sec(x)/(a*sec(x)^2)^(1/2)-tan(x)/(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3657, 4125, 2590, 14}

$$-\frac{\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -((Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2]) - Tan[x]/Sqrt[a*Sec[x]^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \int \frac{\cot^2(x)}{\sqrt{a\sec^2(x)}}\,dx$$
$$= \frac{\sec(x)\int \cos(x)\cot^2(x)\,dx}{\sqrt{a\sec^2(x)}}$$
$$= -\frac{\sec(x)\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, -\sin(x)\right)}{\sqrt{a\sec^2(x)}}$$
$$= -\frac{\sec(x)\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, -\sin(x)\right)}{\sqrt{a\sec^2(x)}}$$
$$= -\frac{\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.71

$$\frac{-\tan(x)-\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/Sqrt[a + a*Tan[x]^2],x]
[Out] (-(Csc[x]*Sec[x]) - Tan[x])/Sqrt[a*Sec[x]^2]
```

fricas [A] time = 0.39, size = 33, normalized size = 1.06

$$-\frac{\sqrt{a\tan(x)^2+a}\left(2\tan(x)^2+1\right)}{a\tan(x)^3+a\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 + 1)/(a*tan(x)^3 + a*tan(x))
```

giac [A] time = 0.32, size = 47, normalized size = 1.52

$$-\frac{\tan(x)}{\sqrt{a\tan(x)^2+a}} + \frac{2\sqrt{a}}{\left(\sqrt{a}\tan(x)-\sqrt{a\tan(x)^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
[Out] -tan(x)/sqrt(a*tan(x)^2 + a) + 2*sqrt(a)/((sqrt(a)*tan(x) - sqrt(a*tan(x)^2
 + a))^2 - a)
```

maple [A] time = 0.58, size = 24, normalized size = 0.77

$$\frac{\cos^2(x)-2}{\sin(x)\cos(x)\sqrt{\frac{a}{\cos(x)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+a*tan(x)^2)^(1/2),x)
```

```
[Out] (cos(x)^2-2)/sin(x)/cos(x)/(a/cos(x)^2)^(1/2)
```

maxima [B] time = 1.48, size = 128, normalized size = 4.13

$$\frac{((\sin(3x) - \sin(x))\cos(4x) - (\cos(3x) - \cos(x))\sin(4x) - (6\cos(2x) - 1)\sin(3x) + 6\cos(3x)\sin(2x) - 6}{2\left(a\cos(3x)^2 - 2a\cos(3x)\cos(x) + a\cos(x)^2 + a\sin(3x)^2 - 2a\sin(3x)\sin(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*sin(4*x) - (6*cos(2
*x) - 1)*sin(3*x) + 6*cos(3*x)*sin(2*x) - 6*cos(x)*sin(2*x) + 6*cos(2*x)*si
n(x) - sin(x))*sqrt(a)/(a*cos(3*x)^2 - 2*a*cos(3*x)*cos(x) + a*cos(x)^2 + a
*sin(3*x)^2 - 2*a*sin(3*x)*sin(x) + a*sin(x)^2)
```

mupad [B] time = 11.82, size = 40, normalized size = 1.29

$$\frac{\sqrt{2}\,\left(6\sin(2x) - 2\sin(2x)\left(2\cos(x)^2 - 1\right)\right)}{8\sqrt{a}\,\sqrt{2\cos(x)^2}\,\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] (2^(1/2)*(6*sin(2*x) - 2*sin(2*x)*(2*cos(x)^2 - 1)))/(8*a^(1/2)*(2*cos(x)^2
)^(1/2)*(cos(x)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{\sqrt{a\left(\tan^2(x) + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)**2/sqrt(a*(tan(x)**2 + 1)), x)
```

3.278 $\int \frac{\tan^3(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}-\frac{1}{a\sqrt{a\sec^2(x)}}$$

```
[Out] 1/3/(a*sec(x)^2)^(3/2)-1/a/(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3657, 4124, 43}

$$\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}-\frac{1}{a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] 1/(3*(a*Sec[x]^2)^(3/2)) - 1/(a*Sqrt[a*Sec[x]^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx &= \int \frac{\tan^3(x)}{\left(a\sec^2(x)\right)^{3/2}}\,dx\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{-1+x}{(ax)^{5/2}}\,dx, x, \sec^2(x)\right)\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \left(-\frac{1}{(ax)^{5/2}}+\frac{1}{a(ax)^{3/2}}\right)dx, x, \sec^2(x)\right)\\
&= \frac{1}{3\left(a\sec^2(x)\right)^{3/2}}-\frac{1}{a\sqrt{a\sec^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.77

$$\frac{\cos(2x) - 5}{6a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] (-5 + Cos[2*x])/(6*a*Sqrt[a*Sec[x]^2])
```

fricas [A] time = 0.41, size = 43, normalized size = 1.43

$$-\frac{\sqrt{a\tan(x)^2 + a}\left(3\tan(x)^2 + 2\right)}{3\left(a^2\tan(x)^4 + 2a^2\tan(x)^2 + a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(a*tan(x)^2 + a)*(3*tan(x)^2 + 2)/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 +
 a^2)
```

giac [A] time = 0.33, size = 26, normalized size = 0.87

$$-\frac{3a\tan(x)^2 + 2a}{3\left(a\tan(x)^2 + a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*tan(x)^2 + 2*a)/((a*tan(x)^2 + a)^(3/2)*a)
```

maple [A] time = 0.17, size = 29, normalized size = 0.97

$$-\frac{1}{a\sqrt{a + a\left(\tan^2(x)\right)}} + \frac{1}{3\left(a + a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] -1/a/(a+a*tan(x)^2)^(1/2)+1/3/(a+a*tan(x)^2)^(3/2)
```

maxima [A] time = 0.54, size = 38, normalized size = 1.27

$$\frac{\left(\sin(x)^2 + 2\right)(\sin(x) + 1)^{\frac{3}{2}}(-\sin(x) + 1)^{\frac{3}{2}}}{3\left(a^{\frac{3}{2}}\sin(x)^2 - a^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sin(x)^2 + 2)*(sin(x) + 1)^(3/2)*(-sin(x) + 1)^(3/2)/(a^(3/2)*sin(x)^2
 - a^(3/2))
```

mupad [B] time = 11.75, size = 29, normalized size = 0.97

$$-\frac{\left(\tan(x)^2 + \frac{2}{3}\right)\sqrt{a\tan(x)^2 + a}}{a^2\left(\tan(x)^2 + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a + a*tan(x)^2)^(3/2),x)
```

```
[Out] -((tan(x)^2 + 2/3)*(a + a*tan(x)^2)^(1/2))/(a^2*(tan(x)^2 + 1)^2)
```

sympy [A] time = 3.57, size = 36, normalized size = 1.20

$$\begin{cases} \frac{\frac{a}{3(a\tan^2(x)+a)^{\frac{3}{2}}} - \frac{1}{\sqrt{a\tan^2(x)+a}}}{a} & \text{for } a \neq 0 \\ \tilde{\infty}\tan^4(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Piecewise(((a/(3*(a*tan(x)**2 + a)**(3/2)) - 1/sqrt(a*tan(x)**2 + a))/a, Ne
(a, 0)), (zoo*tan(x)**4, True))
```

3.279 $\int \frac{\tan^2(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=23

$$\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

[Out] 1/3*sin(x)^2*tan(x)/a/(a*sec(x)^2)^(1/2)

Rubi [A] time = 0.10, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3657, 4125, 2564, 30}

$$\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[Tan[x]^2/(a + a*Tan[x]^2)^(3/2),x]

[Out] (Sin[x]^2*Tan[x])/(3*a*Sqrt[a*Sec[x]^2])

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\tan^2(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx = \int \frac{\tan^2(x)}{\left(a\sec^2(x)\right)^{3/2}}\,dx$$
$$= \frac{\sec(x)\int \cos(x)\sin^2(x)\,dx}{a\sqrt{a\sec^2(x)}}$$
$$= \frac{\sec(x)\,\text{Subst}\left(\int x^2\,dx, x, \sin(x)\right)}{a\sqrt{a\sec^2(x)}}$$
$$= \frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 0.78

$$\frac{\tan^3(x)}{3\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2/(a + a*Tan[x]^2)^(3/2),x]
[Out] Tan[x]^3/(3*(a*Sec[x]^2)^(3/2))
```

fricas [B] time = 0.39, size = 39, normalized size = 1.70

$$\frac{\sqrt{a\tan(x)^2+a}\,\tan(x)^3}{3\left(a^2\tan(x)^4+2\,a^2\tan(x)^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/3*sqrt(a*tan(x)^2 + a)*tan(x)^3/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2)
```

giac [A] time = 0.60, size = 16, normalized size = 0.70

$$\frac{\tan(x)^3}{3\left(a\tan(x)^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
[Out] 1/3*tan(x)^3/(a*tan(x)^2 + a)^(3/2)
```

maple [B] time = 0.18, size = 56, normalized size = 2.43

$$\frac{\tan(x)}{a\sqrt{a+a\left(\tan^2(x)\right)}} - a\left(\frac{\tan(x)}{3a\left(a+a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}} + \frac{2\tan(x)}{3a^2\sqrt{a+a\left(\tan^2(x)\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+a*tan(x)^2)^(3/2),x)
[Out] 1/a*tan(x)/(a+a*tan(x)^2)^(1/2)-a*(1/3/a*tan(x)/(a+a*tan(x)^2)^(3/2)+2/3/a^
2*tan(x)/(a+a*tan(x)^2)^(1/2))
```

maxima [A] time = 0.95, size = 14, normalized size = 0.61

$$-\frac{\sin\left(3\,x\right)-3\,\sin\left(x\right)}{12\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
[Out] -1/12*(sin(3*x) - 3*sin(x))/a^(3/2)
```

mupad [B] time = 11.65, size = 28, normalized size = 1.22

$$\frac{\tan\left(x\right)^3}{\left(3\,a\,\tan\left(x\right)^2+3\,a\right)\,\sqrt{a\,\tan\left(x\right)^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + a*tan(x)^2)^(3/2),x)
[Out] tan(x)^3/((3*a + 3*a*tan(x)^2)*(a + a*tan(x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}\left(x\right)}{\left(a\left(\tan^{2}\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+a*tan(x)**2)**(3/2),x)
[Out] Integral(tan(x)**2/(a*(tan(x)**2 + 1))**(3/2), x)
```

3.280 $\int \frac{\tan(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}$$

```
[Out] -1/3/(a*sec(x)^2)^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3657, 4124, 32}

$$-\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -1/(3*(a*Sec[x]^2)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx &= \int \frac{\tan(x)}{\left(a\sec^2(x)\right)^{3/2}}\,dx \\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{(ax)^{5/2}}\,dx, x, \sec^2(x)\right) \\
&= -\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + a*Tan[x]^2)^(3/2),x]
[Out] -1/3*1/(a*Sec[x]^2)^(3/2)
```

fricas [B] time = 0.40, size = 35, normalized size = 2.50

$$-\frac{\sqrt{a\tan(x)^2+a}}{3\left(a^2\tan(x)^4+2\,a^2\tan(x)^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
[Out] -1/3*sqrt(a*tan(x)^2 + a)/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2)
```

giac [A] time = 0.26, size = 12, normalized size = 0.86

$$-\frac{1}{3\left(a\tan(x)^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/3/(a*tan(x)^2 + a)^(3/2)
```

maple [A] time = 0.12, size = 13, normalized size = 0.93

$$-\frac{1}{3\left(a+a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+a*tan(x)^2)^(3/2),x)
[Out] -1/3/(a+a*tan(x)^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan(x)}{\left(a\tan(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(tan(x)/(a*tan(x)^2 + a)^(3/2), x)
```

mupad [B] time = 0.15, size = 23, normalized size = 1.64

$$-\frac{\sqrt{a\tan(x)^2+a}}{3\,a^2\left(\tan(x)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + a*tan(x)^2)^(3/2),x)
[Out] -(a + a*tan(x)^2)^(1/2)/(3*a^2*(tan(x)^2 + 1)^2)
```

sympy [A] time = 3.32, size = 15, normalized size = 1.07

$$-\frac{1}{3\left(a\tan^{2}\left(x\right)+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] -1/(3*(a*tan(x)**2 + a)**(3/2))
```

3.281 $\int \frac{\cot(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{1}{a\sqrt{a\sec^2(x)}}+\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}$$

```
[Out] -arctanh((a*sec(x)^2)^(1/2)/a^(1/2))/a^(3/2)+1/3/(a*sec(x)^2)^(3/2)+1/a/(a*
sec(x)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3657, 4124, 51, 63, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{1}{a\sqrt{a\sec^2(x)}}+\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]]/a^(3/2)) + 1/(3*(a*Sec[x]^2)^(3/2)) + 1
/(a*Sqrt[a*Sec[x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4124

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
```

```
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx &= \int \frac{\cot(x)}{\left(a\sec^2(x)\right)^{3/2}}\,dx\\ &= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{(-1+x)(ax)^{5/2}}\,dx,x,\sec^2(x)\right)\\ &= \frac{1}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)(ax)^{3/2}}\,dx,x,\sec^2(x)\right)\\ &= \frac{1}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{1}{a\sqrt{a\sec^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx,x,\sec^2(x)\right)}{2a}\\ &= \frac{1}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{1}{a\sqrt{a\sec^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a\sec^2(x)}\right)}{a^2}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{1}{a\sqrt{a\sec^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 47, normalized size = 0.89

$$\frac{\cos(3x)\sec(x)+12\sec(x)\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)+15}{12a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] (15 + Cos[3*x]*Sec[x] + 12*(-Log[Cos[x/2]] + Log[Sin[x/2]])*Sec[x])/(12*a*S
qrt[a*Sec[x]^2])
```

fricas [B] time = 0.42, size = 94, normalized size = 1.77

$$\frac{3\left(\tan(x)^4+2\tan(x)^2+1\right)\sqrt{a}\log\left(\frac{a\tan(x)^2-2\sqrt{a\tan(x)^2+a}\sqrt{a}+2a}{\tan(x)^2}\right)+2\sqrt{a\tan(x)^2+a}\left(3\tan(x)^2+4\right)}{6\left(a^2\tan(x)^4+2a^2\tan(x)^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(tan(x)^4 + 2*tan(x)^2 + 1)*sqrt(a)*log((a*tan(x)^2 - 2*sqrt(a*tan(x
)^2 + a)*sqrt(a) + 2*a)/tan(x)^2) + 2*sqrt(a*tan(x)^2 + a)*(3*tan(x)^2 + 4)
)/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2)
```

giac [A] time = 0.54, size = 53, normalized size = 1.00

$$\frac{\arctan\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{3a\tan(x)^2+4a}{3\left(a\tan(x)^2+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 1/3*(3*a*tan(x)^2 + 4*
a)/((a*tan(x)^2 + a)^(3/2)*a)
```

maple [A] time = 0.46, size = 38, normalized size = 0.72

$$\frac{\cos^3(x)+3\cos(x)+3\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)+4}{3\cos(x)^3\left(\frac{a}{\cos(x)^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] 1/3*(cos(x)^3+3*cos(x)+3*ln(-(-1+cos(x))/sin(x))+4)/cos(x)^3/(a/cos(x)^2)^(
3/2)
```

maxima [A] time = 0.88, size = 48, normalized size = 0.91

$$\frac{\cos(3x)+15\cos(x)-6\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+6\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)}{12a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(cos(3*x) + 15*cos(x) - 6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 6*
log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1))/a^(3/2)
```

mupad [B] time = 11.69, size = 46, normalized size = 0.87

$$\frac{\frac{a\tan(x)^2+a}{a}+\frac{1}{3}}{\left(a\tan(x)^2+a\right)^{3/2}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + a*tan(x)^2)^(3/2),x)
```

```
[Out] ((a + a*tan(x)^2)/a + 1/3)/(a + a*tan(x)^2)^(3/2) - atanh((a + a*tan(x)^2)^
(1/2)/a^(1/2))/a^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot(x)}{\left(a\left(\tan^2(x)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral(cot(x)/(a*(tan(x)**2 + 1))**(3/2), x)
```

3.282 $\int \frac{\cot^2(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{\csc(x)\sec(x)}{a\sqrt{a\sec^2(x)}}-\frac{2\tan(x)}{a\sqrt{a\sec^2(x)}}+\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

```
[Out] -csc(x)*sec(x)/a/(a*sec(x)^2)^(1/2)-2*tan(x)/a/(a*sec(x)^2)^(1/2)+1/3*sin(x
)^2*tan(x)/a/(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3657, 4125, 2590, 270}

$$-\frac{\csc(x)\sec(x)}{a\sqrt{a\sec^2(x)}}-\frac{2\tan(x)}{a\sqrt{a\sec^2(x)}}+\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -((Csc[x]*Sec[x])/(a*Sqrt[a*Sec[x]^2])) - (2*Tan[x])/(a*Sqrt[a*Sec[x]^2]) +
 (Sin[x]^2*Tan[x])/(3*a*Sqrt[a*Sec[x]^2])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4125

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sec[e + f*x]^
n)^FracPart[p])/(Sec[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx &= \int \frac{\cot^2(x)}{\left(a\sec^2(x)\right)^{3/2}}\,dx \\
&= \frac{\sec(x)\int \cos^3(x)\cot^2(x)\,dx}{a\sqrt{a\sec^2(x)}} \\
&= -\frac{\sec(x)\,\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, -\sin(x)\right)}{a\sqrt{a\sec^2(x)}} \\
&= -\frac{\sec(x)\,\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)dx, x, -\sin(x)\right)}{a\sqrt{a\sec^2(x)}} \\
&= -\frac{\csc(x)\sec(x)}{a\sqrt{a\sec^2(x)}} - \frac{2\tan(x)}{a\sqrt{a\sec^2(x)}} + \frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 31, normalized size = 0.52

$$
\frac{\sec^3(x)\left(\sin^3(x)-6\sin(x)-3\csc(x)\right)}{3\left(a\sec^2(x)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/(a + a*Tan[x]^2)^(3/2),x]
[Out] (Sec[x]^3*(-3*Csc[x] - 6*Sin[x] + Sin[x]^3))/(3*(a*Sec[x]^2)^(3/2))
```

fricas [A] time = 0.42, size = 52, normalized size = 0.87

$$
-\frac{\left(8\,\tan(x)^4+12\,\tan(x)^2+3\right)\sqrt{a\tan(x)^2+a}}{3\left(a^2\tan(x)^5+2\,a^2\tan(x)^3+a^2\tan(x)\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
[Out] -1/3*(8*tan(x)^4 + 12*tan(x)^2 + 3)*sqrt(a*tan(x)^2 + a)/(a^2*tan(x)^5 + 2*
a^2*tan(x)^3 + a^2*tan(x))
```

giac [A] time = 0.52, size = 55, normalized size = 0.92

$$
-\frac{\left(5\,\tan(x)^2+6\right)\tan(x)}{3\left(a\tan(x)^2+a\right)^{\frac{3}{2}}} + \frac{2}{\left(\left(\sqrt{a}\,\tan(x)-\sqrt{a\tan(x)^2+a}\right)^2-a\right)\sqrt{a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/3*(5*tan(x)^2 + 6)*tan(x)/(a*tan(x)^2 + a)^(3/2) + 2/(((sqrt(a)*tan(x) -
 sqrt(a*tan(x)^2 + a))^2 - a)*sqrt(a))
```

maple [A] time = 0.53, size = 31, normalized size = 0.52

$$
\frac{\cos^4(x)+4\left(\cos^2(x)\right)-8}{3\sin(x)\cos(x)^3\left(\frac{a}{\cos(x)^2}\right)^{\frac{3}{2}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+a*tan(x)^2)^(3/2),x)
```

```
[Out] 1/3*(cos(x)^4+4*cos(x)^2-8)/sin(x)/cos(x)^3/(a/cos(x)^2)^(3/2)
```

maxima [B] time = 1.34, size = 225, normalized size = 3.75

$$\frac{((\sin(5x) - \sin(3x))\cos(8x) + 20(\sin(5x) - \sin(3x))\cos(6x) + 10(9\sin(4x) - 2\sin(2x))\cos(5x) - (c}{24(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*((sin(5*x) - sin(3*x))*cos(8*x) + 20*(sin(5*x) - sin(3*x))*cos(6*x) +
10*(9*sin(4*x) - 2*sin(2*x))*cos(5*x) - (cos(5*x) - cos(3*x))*sin(8*x) - 20
*(cos(5*x) - cos(3*x))*sin(6*x) - (90*cos(4*x) - 20*cos(2*x) - 1)*sin(5*x)
- 90*cos(3*x)*sin(4*x) - (20*cos(2*x) + 1)*sin(3*x) + 90*cos(4*x)*sin(3*x)
+ 20*cos(3*x)*sin(2*x))*sqrt(a)/(a^2*cos(5*x)^2 - 2*a^2*cos(5*x)*cos(3*x) +
 a^2*cos(3*x)^2 + a^2*sin(5*x)^2 - 2*a^2*sin(5*x)*sin(3*x) + a^2*sin(3*x)^2
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(x)^2}{\left(a\tan(x)^2 + a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a + a*tan(x)^2)^(3/2),x)
```

```
[Out] int(cot(x)^2/(a + a*tan(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{\left(a\left(\tan^2(x) + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral(cot(x)**2/(a*(tan(x)**2 + 1))**(3/2), x)
```

3.283 $\int \frac{1}{\sqrt{a+a\tan^2(c+dx)}}\,dx$

Optimal. Leaf size=24

$$\frac{\tan(c+dx)}{d\sqrt{a\sec^2(c+dx)}}$$

```
[Out] tan(d*x+c)/d/(a*sec(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3657, 4122, 191}

$$\frac{\tan(c+dx)}{d\sqrt{a\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Tan[c + d*x]^2],x]
```

```
[Out] Tan[c + d*x]/(d*Sqrt[a*Sec[c + d*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+a\tan^2(c+dx)}}\,dx &= \int \frac{1}{\sqrt{a\sec^2(c+dx)}}\,dx \\
&= \frac{a\,\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{3/2}}\,dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{\tan(c+dx)}{d\sqrt{a\sec^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 24, normalized size = 1.00

$$\frac{\tan(c+dx)}{d\sqrt{a\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Tan[c + d*x]^2],x]
[Out] Tan[c + d*x]/(d*Sqrt[a*Sec[c + d*x]^2])
```

fricas [A] time = 0.40, size = 38, normalized size = 1.58

$$\frac{\sqrt{a \tan\left(dx+c\right)^{2}+a}\,\tan\left(dx+c\right)}{ad \tan\left(dx+c\right)^{2}+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(a*tan(d*x + c)^2 + a)*tan(d*x + c)/(a*d*tan(d*x + c)^2 + a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \tan\left(dx+c\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*tan(d*x + c)^2 + a), x)
```

maple [A] time = 0.37, size = 25, normalized size = 1.04

$$\frac{\tan\left(dx+c\right)}{d\sqrt{a+a\left(\tan^{2}\left(dx+c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^(1/2),x)
[Out] 1/d*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(1/2)
```

maxima [A] time = 1.44, size = 13, normalized size = 0.54

$$\frac{\sin\left(dx+c\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
[Out] sin(d*x + c)/(sqrt(a)*d)
```

mupad [B] time = 12.07, size = 55, normalized size = 2.29

$$\frac{\sin\left(2\,c+2\,d\,x\right)\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1\right)}{4\,\cos(2\,c+2\,d\,x)+\cos(4\,c+4\,d\,x)+3}}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)^2)^(1/2),x)
[Out] (sin(2*c + 2*d*x)*((a*(cos(2*c + 2*d*x) + 1))/(4*cos(2*c + 2*d*x) + cos(4*c
 + 4*d*x) + 3))^(1/2))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\tan^2(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*tan(c + d*x)**2 + a), x)
```

3.284 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2\tan(c+dx)}{3ad\sqrt{a\sec^2(c+dx)}}+\frac{\tan(c+dx)}{3d\left(a\sec^2(c+dx)\right)^{3/2}}$$

```
[Out] 1/3*tan(d*x+c)/d/(a*sec(d*x+c)^2)^(3/2)+2/3*tan(d*x+c)/a/d/(a*sec(d*x+c)^2)
^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3657, 4122, 192, 191}

$$\frac{2\tan(c+dx)}{3ad\sqrt{a\sec^2(c+dx)}}+\frac{\tan(c+dx)}{3d\left(a\sec^2(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-3/2),x]
```

```
[Out] Tan[c + d*x]/(3*d*(a*Sec[c + d*x]^2)^(3/2)) + (2*Tan[c + d*x])/(3*a*d*Sqrt[
a*Sec[c + d*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx=\int \frac{1}{\left(a\sec^2(c+dx)\right)^{3/2}}\,dx$$
$$=\frac{a\,\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{5/2}}\,dx,x,\tan(c+dx)\right)}{d}$$
$$=\frac{\tan(c+dx)}{3d\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{3/2}}\,dx,x,\tan(c+dx)\right)}{3d}$$
$$=\frac{\tan(c+dx)}{3d\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{2\tan(c+dx)}{3ad\sqrt{a\sec^2(c+dx)}}$$

Mathematica [A] time = 0.06, size = 40, normalized size = 0.69

$$-\frac{\left(\sin^2(c+dx)-3\right)\tan(c+dx)}{3ad\sqrt{a\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-3/2),x]
```

```
[Out] -1/3*((-3 + Sin[c + d*x]^2)*Tan[c + d*x])/(a*d*Sqrt[a*Sec[c + d*x]^2])
```

fricas [A] time = 0.43, size = 70, normalized size = 1.21

$$\frac{\sqrt{a\tan\left(dx+c\right)^2+a}\left(2\,\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)}{3\left(a^2d\tan\left(dx+c\right)^4+2\,a^2d\tan\left(dx+c\right)^2+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*tan(d*x + c)^2 + a)*(2*tan(d*x + c)^3 + 3*tan(d*x + c))/(a^2*d*t
an(d*x + c)^4 + 2*a^2*d*tan(d*x + c)^2 + a^2*d)
```

giac [A] time = 4.02, size = 81, normalized size = 1.40

$$-\frac{2\left(3\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^2-4\sqrt{a}\right)}{3\,a^2d\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*sqrt(a)*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^2 - 4*sqrt(
a))/(a^2*d*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^3*sgn(tan(1/2*d*
x + 1/2*c)^4 - 1))
```

maple [A] time = 0.34, size = 57, normalized size = 0.98

$$\frac{a\left(\frac{\tan(dx+c)}{3a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{2\tan(dx+c)}{3a^2\sqrt{a+a\left(\tan^2(dx+c)\right)}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^(3/2),x)
```

```
[Out] 1/d*a*(1/3/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(3/2)+2/3/a^2*tan(d*x+c)/(a+a*ta
n(d*x+c)^2)^(1/2))
```

maxima [A] time = 0.71, size = 26, normalized size = 0.45

$$\frac{\sin\left(3\,dx+3\,c\right)+9\,\sin\left(dx+c\right)}{12\,a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(d*x + c))/(a^(3/2)*d)
```

mupad [B] time = 11.71, size = 35, normalized size = 0.60

$$\frac{\frac{2\tan(c+d\,x)^3}{3}+\tan\left(c+d\,x\right)}{d\,\left(a\tan\left(c+d\,x\right)^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)^2)^(3/2),x)
```

```
[Out] (tan(c + d*x) + (2*tan(c + d*x)^3)/3)/(d*(a + a*tan(c + d*x)^2)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\tan^2\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((a*tan(c + d*x)**2 + a)**(-3/2), x)
```

3.285 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{8\tan(c+dx)}{15a^2d\sqrt{a\sec^2(c+dx)}}+\frac{4\tan(c+dx)}{15ad\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}}$$

```
[Out] 1/5*tan(d*x+c)/d/(a*sec(d*x+c)^2)^(5/2)+4/15*tan(d*x+c)/a/d/(a*sec(d*x+c)^2
)^(3/2)+8/15*tan(d*x+c)/a^2/d/(a*sec(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3657, 4122, 192, 191}

$$\frac{8\tan(c+dx)}{15a^2d\sqrt{a\sec^2(c+dx)}}+\frac{4\tan(c+dx)}{15ad\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-5/2),x]
```

```
[Out] Tan[c + d*x]/(5*d*(a*Sec[c + d*x]^2)^(5/2)) + (4*Tan[c + d*x])/(15*a*d*(a*S
ec[c + d*x]^2)^(3/2)) + (8*Tan[c + d*x])/(15*a^2*d*Sqrt[a*Sec[c + d*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sec^2(c+dx)\right)^{5/2}}\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{7/2}}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}} + \frac{4\,\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{5/2}}\,dx, x, \tan(c+dx)\right)}{5d}$$
$$= \frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}} + \frac{4\tan(c+dx)}{15ad\left(a\sec^2(c+dx)\right)^{3/2}} + \frac{8\,\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{3/2}}\,dx, x, \right.}{15ad}$$
$$= \frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}} + \frac{4\tan(c+dx)}{15ad\left(a\sec^2(c+dx)\right)^{3/2}} + \frac{8\tan(c+dx)}{15a^2d\sqrt{a\sec^2(c+dx)}}$$

Mathematica [A] time = 0.10, size = 52, normalized size = 0.59

$$\frac{\left(3\sin^4(c+dx)-10\sin^2(c+dx)+15\right)\tan(c+dx)}{15a^2d\sqrt{a\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-5/2),x]
```

```
[Out] ((15 - 10*Sin[c + d*x]^2 + 3*Sin[c + d*x]^4)*Tan[c + d*x])/(15*a^2*d*Sqrt[a
*Sec[c + d*x]^2])
```

fricas [A] time = 0.42, size = 94, normalized size = 1.07

$$\frac{\left(8\,\tan\left(dx+c\right)^5+20\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)\sqrt{a\tan\left(dx+c\right)^2+a}}{15\left(a^3d\tan\left(dx+c\right)^6+3\,a^3d\tan\left(dx+c\right)^4+3\,a^3d\tan\left(dx+c\right)^2+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*tan(d*x + c)^5 + 20*tan(d*x + c)^3 + 15*tan(d*x + c))*sqrt(a*tan(d*
x + c)^2 + a)/(a^3*d*tan(d*x + c)^6 + 3*a^3*d*tan(d*x + c)^4 + 3*a^3*d*tan(
d*x + c)^2 + a^3*d)
```

giac [A] time = 6.25, size = 109, normalized size = 1.24

$$-\frac{2\left(15\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^4-40\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^2+48\sqrt{a}\right)}{15\,a^3d\left(\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^5\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*sqrt(a)*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^4 - 40*sq
rt(a)*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^2 + 48*sqrt(a))/(a^3*
d*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^5*sgn(tan(1/2*d*x + 1/2*c
)^4 - 1))
```

maple [A] time = 0.38, size = 88, normalized size = 1.00

$$\frac{a\left(\frac{\tan(dx+c)}{5a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}}+\frac{\frac{4\tan(dx+c)}{15a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{8\tan(dx+c)}{15a^2\sqrt{a+a\left(\tan^2(dx+c)\right)}}}{a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^(5/2),x)
```

```
[Out] 1/d*a*(1/5/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(5/2)+4/5/a*(1/3/a*tan(d*x+c)/(a
+a*tan(d*x+c)^2)^(3/2)+2/3/a^2*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(1/2)))
```

maxima [A] time = 0.61, size = 39, normalized size = 0.44

$$\frac{3\sin(5\,dx+5\,c)+25\sin(3\,dx+3\,c)+150\sin(dx+c)}{240\,a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/240*(3*sin(5*d*x + 5*c) + 25*sin(3*d*x + 3*c) + 150*sin(d*x + c))/(a^(5/2
)*d)
```

mupad [B] time = 0.20, size = 47, normalized size = 0.53

$$\frac{\tan(c+d\,x)\,\left(8\tan(c+d\,x)^4+20\tan(c+d\,x)^2+15\right)}{15\,d\,\left(a\tan(c+d\,x)^2+a\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)^2)^(5/2),x)
```

```
[Out] (tan(c + d*x)*(20*tan(c + d*x)^2 + 8*tan(c + d*x)^4 + 15))/(15*d*(a + a*tan
(c + d*x)^2)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\tan^2(c+dx)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a*tan(c + d*x)**2 + a)**(-5/2), x)
```

3.286 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{16\tan(c+dx)}{35a^3d\sqrt{a\sec^2(c+dx)}}+\frac{8\tan(c+dx)}{35a^2d\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{6\tan(c+dx)}{35ad\left(a\sec^2(c+dx)\right)^{5/2}}+\frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}}$$

```
[Out] 1/7*tan(d*x+c)/d/(a*sec(d*x+c)^2)^(7/2)+6/35*tan(d*x+c)/a/d/(a*sec(d*x+c)^2
)^(5/2)+8/35*tan(d*x+c)/a^2/d/(a*sec(d*x+c)^2)^(3/2)+16/35*tan(d*x+c)/a^3/d
/(a*sec(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3657, 4122, 192, 191}

$$\frac{16\tan(c+dx)}{35a^3d\sqrt{a\sec^2(c+dx)}}+\frac{8\tan(c+dx)}{35a^2d\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{6\tan(c+dx)}{35ad\left(a\sec^2(c+dx)\right)^{5/2}}+\frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-7/2),x]
```

```
[Out] Tan[c + d*x]/(7*d*(a*Sec[c + d*x]^2)^(7/2)) + (6*Tan[c + d*x])/(35*a*d*(a*S
ec[c + d*x]^2)^(5/2)) + (8*Tan[c + d*x])/(35*a^2*d*(a*Sec[c + d*x]^2)^(3/2)
) + (16*Tan[c + d*x])/(35*a^3*d*Sqrt[a*Sec[c + d*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule steps

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{7/2}}\,dx = \int \frac{1}{\left(a\sec^2(c+dx)\right)^{7/2}}\,dx$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{(a+ax^2)^{9/2}}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}} + \frac{6\,\text{Subst}\left(\int \frac{1}{(a+ax^2)^{7/2}}\,dx, x, \tan(c+dx)\right)}{7d}$$

$$= \frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}} + \frac{6\tan(c+dx)}{35ad\left(a\sec^2(c+dx)\right)^{5/2}} + \frac{24\,\text{Subst}\left(\int \frac{1}{(a+ax^2)^{5/2}}\,dx, x, \text{ta}\right.}{35ad}$$

$$= \frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}} + \frac{6\tan(c+dx)}{35ad\left(a\sec^2(c+dx)\right)^{5/2}} + \frac{8\tan(c+dx)}{35a^2d\left(a\sec^2(c+dx)\right)^{3/2}} + \frac{1}{}$$

$$= \frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}} + \frac{6\tan(c+dx)}{35ad\left(a\sec^2(c+dx)\right)^{5/2}} + \frac{8\tan(c+dx)}{35a^2d\left(a\sec^2(c+dx)\right)^{3/2}} + \frac{}{3}$$

Mathematica [A] time = 0.18, size = 62, normalized size = 0.53

$$\frac{\left(-5\sin^6(c+dx)+21\sin^4(c+dx)-35\sin^2(c+dx)+35\right)\tan(c+dx)}{35a^3d\sqrt{a\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-7/2),x]
```

```
[Out] ((35 - 35*Sin[c + d*x]^2 + 21*Sin[c + d*x]^4 - 5*Sin[c + d*x]^6)*Tan[c + d*
x])/(35*a^3*d*Sqrt[a*Sec[c + d*x]^2])
```

fricas [A] time = 0.39, size = 118, normalized size = 1.00

$$\frac{\left(16\,\tan\left(dx+c\right)^7+56\,\tan\left(dx+c\right)^5+70\,\tan\left(dx+c\right)^3+35\,\tan\left(dx+c\right)\right)\sqrt{a\tan\left(dx+c\right)^2+a}}{35\left(a^4d\tan\left(dx+c\right)^8+4\,a^4d\tan\left(dx+c\right)^6+6\,a^4d\tan\left(dx+c\right)^4+4\,a^4d\tan\left(dx+c\right)^2+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*tan(d*x + c)^7 + 56*tan(d*x + c)^5 + 70*tan(d*x + c)^3 + 35*tan(d*
x + c))*sqrt(a*tan(d*x + c)^2 + a)/(a^4*d*tan(d*x + c)^8 + 4*a^4*d*tan(d*x
+ c)^6 + 6*a^4*d*tan(d*x + c)^4 + 4*a^4*d*tan(d*x + c)^2 + a^4*d)
```

giac [A] time = 8.88, size = 137, normalized size = 1.16

$$-\frac{2\left(35\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^6-140\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^4+336\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\right.\right.}{35\,a^4d\left(\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^7\text{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2/35*(35*sqrt(a)*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^6 - 140*s
qrt(a)*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^4 + 336*sqrt(a)*(1/t
an(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^2 - 320*sqrt(a))/(a^4*d*(1/tan(
1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^7*sgn(tan(1/2*d*x + 1/2*c)^4 - 1))
```

maple [A] time = 0.36, size = 119, normalized size = 1.01

$$\frac{a\left(\frac{\tan(dx+c)}{7a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{7}{2}}}+\frac{\frac{6\tan(dx+c)}{35a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4\tan(dx+c)}{15a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{8\tan(dx+c)}{15a^2\sqrt{a+a\left(\tan^2(dx+c)\right)}}\right)}{7a}}{a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^(7/2),x)
```

```
[Out] 1/d*a*(1/7/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(7/2)+6/7/a*(1/5/a*tan(d*x+c)/(a
+a*tan(d*x+c)^2)^(5/2)+4/5/a*(1/3/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(3/2)+2/3
/a^2*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(1/2))))
```

maxima [A] time = 0.60, size = 50, normalized size = 0.42

$$\frac{5\sin(7dx+7c)+49\sin(5dx+5c)+245\sin(3dx+3c)+1225\sin(dx+c)}{2240a^{\frac{7}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2240*(5*sin(7*d*x + 7*c) + 49*sin(5*d*x + 5*c) + 245*sin(3*d*x + 3*c) + 1
225*sin(d*x + c))/(a^(7/2)*d)
```

mupad [B] time = 11.78, size = 161, normalized size = 1.36

$$\frac{16\tan(c+dx)\sqrt{a\tan(c+dx)^2+a}}{35a^4d\left(\tan(c+dx)^2+1\right)}+\frac{8\tan(c+dx)\sqrt{a\tan(c+dx)^2+a}}{35a^4d\left(\tan(c+dx)^2+1\right)^2}+\frac{6\tan(c+dx)\sqrt{a\tan(c+dx)^2}}{35a^4d\left(\tan(c+dx)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)^2)^(7/2),x)
```

```
[Out] (16*tan(c + d*x)*(a + a*tan(c + d*x)^2)^(1/2))/(35*a^4*d*(tan(c + d*x)^2 +
1)) + (8*tan(c + d*x)*(a + a*tan(c + d*x)^2)^(1/2))/(35*a^4*d*(tan(c + d*x)
^2 + 1)^2) + (6*tan(c + d*x)*(a + a*tan(c + d*x)^2)^(1/2))/(35*a^4*d*(tan(c
 + d*x)^2 + 1)^3) + (tan(c + d*x)*(a + a*tan(c + d*x)^2)^(1/2))/(7*a^4*d*(t
an(c + d*x)^2 + 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\tan^2(c+dx)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**(7/2),x)
```

```
[Out] Integral((a*tan(c + d*x)**2 + a)**(-7/2), x)
```

3.287 $\int \left(1+\tan^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=22

$$\frac{1}{2}\tan(x)\sqrt{\sec^2(x)}+\frac{1}{2}\sinh^{-1}(\tan(x))$$

```
[Out] 1/2*arcsinh(tan(x))+1/2*(sec(x)^2)^(1/2)*tan(x)
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3657, 4122, 195, 215}

$$\frac{1}{2}\tan(x)\sqrt{\sec^2(x)}+\frac{1}{2}\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x]^2)^(3/2),x]
```

```
[Out] ArcSinh[Tan[x]]/2 + (Sqrt[Sec[x]^2]*Tan[x])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(1+\tan^2(x)\right)^{3/2}\,dx &= \int \sec^2(x)^{3/2}\,dx\\ &= \text{Subst}\left(\int \sqrt{1+x^2}\,dx,x,\tan(x)\right)\\ &= \frac{1}{2}\sqrt{\sec^2(x)}\,\tan(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\tan(x)\right)\\ &= \frac{1}{2}\sinh^{-1}(\tan(x))+\frac{1}{2}\sqrt{\sec^2(x)}\,\tan(x)\end{aligned}$$

Mathematica [B] time = 0.07, size = 52, normalized size = 2.36

$$\frac{1}{2}\cos(x)\sqrt{\sec^2(x)}\left(\tan(x)\sec(x)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x]^2)^(3/2),x]
```

```
[Out] (Cos[x]*Sqrt[Sec[x]^2]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]
] + Sec[x]*Tan[x]))/2
```

fricas [B] time = 0.39, size = 72, normalized size = 3.27

$$\frac{1}{2}\sqrt{\tan(x)^2+1}\,\tan(x)+\frac{1}{4}\log\left(\frac{\tan(x)^2+\sqrt{\tan(x)^2+1}\,\tan(x)+1}{\tan(x)^2+1}\right)-\frac{1}{4}\log\left(\frac{\tan(x)^2-\sqrt{\tan(x)^2+1}\,\tan(x)+}{\tan(x)^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(tan(x)^2 + 1)*tan(x) + 1/4*log((tan(x)^2 + sqrt(tan(x)^2 + 1)*tan(
x) + 1)/(tan(x)^2 + 1)) - 1/4*log((tan(x)^2 - sqrt(tan(x)^2 + 1)*tan(x) + 1
)/(tan(x)^2 + 1))
```

giac [A] time = 0.31, size = 29, normalized size = 1.32

$$\frac{1}{2}\sqrt{\tan(x)^2+1}\,\tan(x)-\frac{1}{2}\log\left(\sqrt{\tan(x)^2+1}-\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(tan(x)^2 + 1)*tan(x) - 1/2*log(sqrt(tan(x)^2 + 1) - tan(x))
```

maple [A] time = 0.14, size = 19, normalized size = 0.86

$$\frac{\tan(x)\sqrt{1+\tan^2(x)}}{2}+\frac{\operatorname{arcsinh}(\tan(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(x)^2)^(3/2),x)
```

```
[Out] 1/2*tan(x)*(1+tan(x)^2)^(1/2)+1/2*arcsinh(tan(x))
```

maxima [A] time = 0.55, size = 18, normalized size = 0.82

$$\frac{1}{2}\sqrt{\tan(x)^2+1}\,\tan(x)+\frac{1}{2}\operatorname{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(tan(x)^2 + 1)*tan(x) + 1/2*arcsinh(tan(x))
```

mupad [B] time = 0.11, size = 18, normalized size = 0.82

$$\frac{\operatorname{asinh}(\tan(x))}{2}+\frac{\tan(x)\sqrt{\tan(x)^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)^2 + 1)^(3/2),x)
```

```
[Out] asinh(tan(x))/2 + (tan(x)*(tan(x)^2 + 1)^(1/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\tan^2(x) + 1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)**2)**(3/2),x)
```

```
[Out] Integral((tan(x)**2 + 1)**(3/2), x)
```

3.288 $\int \sqrt{1+\tan^2(x)}\,dx$

Optimal. Leaf size=3

$$\sinh^{-1}(\tan(x))$$

```
[Out] arcsinh(tan(x))
```

Rubi [A] time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3657, 4122, 215}

$$\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Tan[x]^2],x]
```

```
[Out] ArcSinh[Tan[x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+\tan^2(x)}\,dx &= \int \sqrt{\sec^2(x)}\,dx\\
&= \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tan(x)\right)\\
&= \sinh^{-1}(\tan(x))
\end{aligned}$$

Mathematica [B] time = 0.01, size = 44, normalized size = 14.67

$$\cos(x)\sqrt{\sec^2(x)}\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Tan[x]^2],x]
```

```
[Out] Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])*Sqrt[Sec[x]^2
]
```

fricas [B] time = 0.41, size = 60, normalized size = 20.00

$$\frac{1}{2}\log\left(\frac{\tan(x)^2+\sqrt{\tan(x)^2+1}\,\tan(x)+1}{\tan(x)^2+1}\right)-\frac{1}{2}\log\left(\frac{\tan(x)^2-\sqrt{\tan(x)^2+1}\,\tan(x)+1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log((tan(x)^2 + sqrt(tan(x)^2 + 1)*tan(x) + 1)/(tan(x)^2 + 1)) - 1/2*lo
g((tan(x)^2 - sqrt(tan(x)^2 + 1)*tan(x) + 1)/(tan(x)^2 + 1))
```

giac [B] time = 0.39, size = 16, normalized size = 5.33

$$-\log\left(\sqrt{\tan(x)^2+1}-\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(sqrt(tan(x)^2 + 1) - tan(x))
```

maple [A] time = 0.14, size = 4, normalized size = 1.33

$$\operatorname{arcsinh}\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(x)^2)^(1/2),x)
```

```
[Out] arcsinh(tan(x))
```

maxima [A] time = 0.75, size = 3, normalized size = 1.00

$$\operatorname{arsinh}\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(tan(x))
```

mupad [B] time = 0.06, size = 3, normalized size = 1.00

$$\operatorname{asinh}\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)^2 + 1)^(1/2),x)
```

```
[Out] asinh(tan(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\tan^2(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(tan(x)**2 + 1), x)
```

3.289 $\int \frac{1}{\sqrt{1+\tan^2(x)}}\,dx$

Optimal. Leaf size=11

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

```
[Out] tan(x)/(sec(x)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3657, 4122, 191}

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Tan[x]^2],x]
```

```
[Out] Tan[x]/Sqrt[Sec[x]^2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1+\tan^2(x)}}\,dx &= \int \frac{1}{\sqrt{\sec^2(x)}}\,dx\\
&= \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{\sqrt{\sec^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Tan[x]^2],x]
[Out] Tan[x]/Sqrt[Sec[x]^2]
```

fricas [A] time = 0.45, size = 11, normalized size = 1.00

$$\frac{\tan(x)}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] tan(x)/sqrt(tan(x)^2 + 1)
```

giac [A] time = 0.44, size = 11, normalized size = 1.00

$$\frac{\tan(x)}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2)^(1/2),x, algorithm="giac")
[Out] tan(x)/sqrt(tan(x)^2 + 1)
```

maple [A] time = 0.09, size = 12, normalized size = 1.09

$$\frac{\tan(x)}{\sqrt{1+\tan^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tan(x)^2)^(1/2),x)
[Out] 1/(1+tan(x)^2)^(1/2)*tan(x)
```

maxima [A] time = 0.38, size = 11, normalized size = 1.00

$$\frac{\tan(x)}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2)^(1/2),x, algorithm="maxima")
[Out] tan(x)/sqrt(tan(x)^2 + 1)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.82

$$\tan(x)\sqrt{\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x)^2 + 1)^(1/2),x)
[Out] tan(x)*(cos(x)^2)^(1/2)
```

sympy [A] time = 0.36, size = 12, normalized size = 1.09

$$\frac{\tan(x)}{\sqrt{\tan^2(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)**2)**(1/2),x)
[Out] tan(x)/sqrt(tan(x)**2 + 1)
```

3.290 $\int \left(-1-\tan^2(x)\right)^{3/2} dx$

Optimal. Leaf size=35

$$\frac{1}{2}\tan^{-1}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)-\frac{1}{2}\tan(x)\sqrt{-\sec^2(x)}$$

```
[Out] 1/2*arctan(tan(x)/(-sec(x)^2)^(1/2))-1/2*(-sec(x)^2)^(1/2)*tan(x)
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3657, 4122, 195, 217, 203}

$$\frac{1}{2}\tan^{-1}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)-\frac{1}{2}\tan(x)\sqrt{-\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - Tan[x]^2)^(3/2),x]
```

```
[Out] ArcTan[Tan[x]/Sqrt[-Sec[x]^2]]/2 - (Sqrt[-Sec[x]^2]*Tan[x])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(-1-\tan^2(x)\right)^{3/2}\,dx = \int \left(-\sec^2(x)\right)^{3/2}\,dx$$
$$= -\text{Subst}\left(\int \sqrt{-1-x^2}\,dx, x, \tan(x)\right)$$
$$= -\frac{1}{2}\sqrt{-\sec^2(x)}\,\tan(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \tan(x)\right)$$
$$= -\frac{1}{2}\sqrt{-\sec^2(x)}\,\tan(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)$$
$$= \frac{1}{2}\tan^{-1}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right) - \frac{1}{2}\sqrt{-\sec^2(x)}\,\tan(x)$$

Mathematica [B] time = 0.07, size = 72, normalized size = 2.06

$$\frac{1}{4}\cos(x)\sqrt{-\sec^2(x)}\left(\frac{1}{\sin(x)-1} + \frac{1}{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2} + 2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) - 2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Tan[x]^2)^(3/2),x]
```

```
[Out] (Cos[x]*Sqrt[-Sec[x]^2]*(2*Log[Cos[x/2] - Sin[x/2]] - 2*Log[Cos[x/2] + Sin[
x/2]] + (Cos[x/2] + Sin[x/2])^(-2) + (-1 + Sin[x])^(-1)))/4
```

fricas [C] time = 0.45, size = 73, normalized size = 2.09

$$\frac{\left(-i e^{(4ix)} - 2i e^{(2ix)} - i\right)\log\left(e^{(ix)}+i\right) + \left(i e^{(4ix)} + 2i e^{(2ix)} + i\right)\log\left(e^{(ix)}-i\right) - 2e^{(3ix)} + 2e^{(ix)}}{2\left(e^{(4ix)} + 2e^{(2ix)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*e^(4*I*x) - 2*I*e^(2*I*x) - I)*log(e^(I*x) + I) + (I*e^(4*I*x) + 2
*I*e^(2*I*x) + I)*log(e^(I*x) - I) - 2*e^(3*I*x) + 2*e^(I*x))/(e^(4*I*x) +
2*e^(2*I*x) + 1)
```

giac [C] time = 0.30, size = 29, normalized size = 0.83

$$-\frac{1}{2}i\sqrt{\tan(x)^2+1}\,\tan(x) + \frac{1}{2}i\log\left(\sqrt{\tan(x)^2+1} - \tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*I*sqrt(tan(x)^2 + 1)*tan(x) + 1/2*I*log(sqrt(tan(x)^2 + 1) - tan(x))
```

maple [A] time = 0.25, size = 32, normalized size = 0.91

$$-\frac{\tan(x)\sqrt{-1-\left(\tan^2(x)\right)}}{2} + \frac{\arctan\left(\frac{\tan(x)}{\sqrt{-1-\left(\tan^2(x)\right)}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-tan(x)^2)^(3/2),x)
```

```
[Out] -1/2*tan(x)*(-1-tan(x)^2)^(1/2)+1/2*arctan(tan(x)/(-1-tan(x)^2)^(1/2))
```

maxima [C] time = 0.58, size = 20, normalized size = 0.57

$$-\frac{1}{2}\sqrt{-\tan(x)^2-1}\,\tan(x)-\frac{1}{2}i\,\operatorname{arsinh}\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(3/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-tan(x)^2 - 1)*tan(x) - 1/2*I*arcsinh(tan(x))
```

mupad [B] time = 11.59, size = 31, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{\tan(x)}{\sqrt{-\tan(x)^2-1}}\right)}{2}-\frac{\tan(x)\sqrt{-\tan(x)^2-1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- tan(x)^2 - 1)^(3/2),x)
[Out] atan(tan(x)/(- tan(x)^2 - 1)^(1/2))/2 - (tan(x)*(- tan(x)^2 - 1)^(1/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\tan^2(x)-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)**2)**(3/2),x)
[Out] Integral((-tan(x)**2 - 1)**(3/2), x)
```

3.291 $\int \sqrt{-1-\tan^2(x)}\,dx$

Optimal. Leaf size=16

$$-\tan^{-1}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)$$

```
[Out] -arctan(tan(x)/(-sec(x)^2)^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3657, 4122, 217, 203}

$$-\tan^{-1}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Tan[x]^2],x]
```

```
[Out] -ArcTan[Tan[x]/Sqrt[-Sec[x]^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-1-\tan^2(x)}\,dx &= \int \sqrt{-\sec^2(x)}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \tan(x)\right)\\
&= -\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)\\
&= -\tan^{-1}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)
\end{aligned}$$

Mathematica [B] time = 0.01, size = 46, normalized size = 2.88

$$\cos(x)\sqrt{-\sec^2(x)}\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Tan[x]^2],x]
```

```
[Out] Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])*Sqrt[-Sec[x]^
2]
```

fricas [C] time = 0.41, size = 19, normalized size = 1.19

$$i \log\left(e^{(i x)}+i\right)-i \log\left(e^{(i x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] I*log(e^(I*x) + I) - I*log(e^(I*x) - I)
```

giac [C] time = 0.51, size = 16, normalized size = 1.00

$$-i \log\left(\sqrt{\tan(x)^2+1}-\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -I*log(sqrt(tan(x)^2 + 1) - tan(x))
```

maple [A] time = 0.26, size = 17, normalized size = 1.06

$$-\arctan\left(\frac{\tan(x)}{\sqrt{-1-\left(\tan^2(x)\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-tan(x)^2)^(1/2),x)
```

```
[Out] -arctan(tan(x)/(-1-tan(x)^2)^(1/2))
```

maxima [A] time = 0.86, size = 17, normalized size = 1.06

$$\arctan\left(\cos(x), \sin(x)+1\right)+\arctan\left(\cos(x), -\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arctan2(cos(x), sin(x) + 1) + arctan2(cos(x), -sin(x) + 1)
```

mupad [B] time = 0.11, size = 16, normalized size = 1.00

$$-\mathrm{atan}\left(\frac{\tan(x)}{\sqrt{-\tan(x)^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- tan(x)^2 - 1)^(1/2),x)
```

```
[Out] -atan(tan(x)/(- tan(x)^2 - 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\tan^2(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-tan(x)**2 - 1), x)
```

3.292 $\int \frac{1}{\sqrt{-1-\tan^2(x)}}\,dx$

Optimal. Leaf size=13

$$\frac{\tan(x)}{\sqrt{-\sec^2(x)}}$$

[Out] tan(x)/(-sec(x)^2)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3657, 4122, 191}

$$\frac{\tan(x)}{\sqrt{-\sec^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-1 - Tan[x]^2],x]

[Out] Tan[x]/Sqrt[-Sec[x]^2]

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1-\tan^2(x)}}\,dx &= \int \frac{1}{\sqrt{-\sec^2(x)}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{\left(-1-x^2\right)^{3/2}}\,dx, x, \tan(x)\right)\\ &= \frac{\tan(x)}{\sqrt{-\sec^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\tan(x)}{\sqrt{-\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - Tan[x]^2],x]
[Out] Tan[x]/Sqrt[-Sec[x]^2]
```

fricas [C] time = 0.39, size = 12, normalized size = 0.92

$$-\frac{1}{2}\left(e^{(2ix)}-1\right)e^{(-ix)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*(e^(2*I*x) - 1)*e^(-I*x)
```

giac [C] time = 0.35, size = 12, normalized size = 0.92

$$-\frac{i\tan(x)}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tan(x)^2)^(1/2),x, algorithm="giac")
[Out] -I*tan(x)/sqrt(tan(x)^2 + 1)
```

maple [A] time = 0.24, size = 14, normalized size = 1.08

$$\frac{\tan(x)}{\sqrt{-1-\left(\tan^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-tan(x)^2)^(1/2),x)
[Out] tan(x)/(-1-tan(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\tan(x)^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tan(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-tan(x)^2 - 1), x)
```

mupad [B] time = 11.98, size = 13, normalized size = 1.00

$$-\frac{\sqrt{2}\,\sin(2x)\,1\mathrm{i}}{2\sqrt{2\cos(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- tan(x)^2 - 1)^(1/2),x)
[Out] -(2^(1/2)*sin(2*x)*1i)/(2*(2*cos(x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\tan^2(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tan(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-tan(x)**2 - 1), x)
```

3.293 $\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/f+(a+b*tan(f*x+e
)^2)^(1/2)/f-1/3*(a+b)*(a+b*tan(f*x+e)^2)^(3/2)/b^2/f+1/5*(a+b*tan(f*x+e)^2
)^(5/2)/b^2/f
```

Rubi [A] time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3670, 446, 88, 50, 63, 208}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + Sqrt[a
 + b*Tan[e + f*x]^2]/f - ((a + b)*(a + b*Tan[e + f*x]^2)^(3/2))/(3*b^2*f) +
 (a + b*Tan[e + f*x]^2)^(5/2)/(5*b^2*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-b)\sqrt{a+bx}}{b} + \frac{\sqrt{a+bx}}{1+x} + \frac{(a+bx)^{3/2}}{b}\right) dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{\left(a+b\tan^2\right.}{5}$$

$$= \frac{\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{\left(a+b\tan^2\right.}{5}$$

$$= -\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a+b)}{}$$

Mathematica [A] time = 1.38, size = 109, normalized size = 0.93

$$\frac{\frac{\sqrt{a+b\tan^2(e+fx)}\left(-2a^2+b(a-5b)\tan^2(e+fx)-5ab+3b^2\tan^4(e+fx)+15b^2\right)}{b^2}-15\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-15*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + (Sqrt[a
+ b*Tan[e + f*x]^2]*(-2*a^2 - 5*a*b + 15*b^2 + (a - 5*b)*b*Tan[e + f*x]^2 +
 3*b^2*Tan[e + f*x]^4))/b^2)/(15*f)
```

fricas [A] time = 0.59, size = 320, normalized size = 2.74

$$\left[\frac{15\sqrt{a-b}\,b^2\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2-4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)+4\left(3b^2\tan\right.}{60b^2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/60*(15*sqrt(a - b)*b^2*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(
f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4
*(3*b^2*tan(f*x + e)^4 + (a*b - 5*b^2)*tan(f*x + e)^2 - 2*a^2 - 5*a*b + 15*
b^2)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/30*(15*sqrt(-a + b)*b^2*arctan(
2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2
*(3*b^2*tan(f*x + e)^4 + (a*b - 5*b^2)*tan(f*x + e)^2 - 2*a^2 - 5*a*b + 15*
b^2)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f)]
```

giac [A] time = 0.86, size = 143, normalized size = 1.22

$$\frac{(a-b)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}+\frac{3\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}b^8f^4-5\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}ab^8f^4-5\left(b\tan\left(fx+e\right.\right.}{15b^{10}f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] (a - b)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f) +
1/15*(3*(b*tan(f*x + e)^2 + a)^(5/2)*b^8*f^4 - 5*(b*tan(f*x + e)^2 + a)^(3/
2)*a*b^8*f^4 - 5*(b*tan(f*x + e)^2 + a)^(3/2)*b^9*f^4 + 15*sqrt(b*tan(f*x +
 e)^2 + a)*b^10*f^4)/(b^10*f^5)
```

maple [A] time = 0.40, size = 166, normalized size = 1.42

$$\frac{\left(\tan^2\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{5fb}-\frac{2a\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{15f\,b^2}-\frac{\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{3bf}+\frac{\sqrt{a+b\left(\tan^2\left(\right.\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x)
```

```
[Out] 1/5/f*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2)/b-2/15/f*a/b^2*(a+b*tan(f*x+e)^
2)^(3/2)-1/3*(a+b*tan(f*x+e)^2)^(3/2)/b/f+(a+b*tan(f*x+e)^2)^(1/2)/f-1/f*b/
(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+1/f*a/(-a+b)^(1/
2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\tan\left(fx+e\right)^2+a}\,\tan\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^5, x)
```

mupad [B] time = 20.97, size = 157, normalized size = 1.34

$$\frac{\left(b\tan\left(e+f\,x\right)^2+a\right)^{5/2}}{5\,b^2\,f}-\left(\frac{2\,a}{3\,b^2\,f}-\frac{a-b}{3\,b^2\,f}\right)\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}-\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\left(\left(\frac{2\,a}{b^2\,f}-\frac{a-b}{b^2\,f}\right)\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] (atan(((a + b*tan(e + f*x)^2)^(1/2)*1i)/(a - b)^(1/2))*(a - b)^(1/2)*1i)/f
- ((2*a)/(3*b^2*f) - (a - b)/(3*b^2*f))*(a + b*tan(e + f*x)^2)^(3/2) - (a +
 b*tan(e + f*x)^2)^(1/2)*(((2*a)/(b^2*f) - (a - b)/(b^2*f))*(a - b) - a^2/(
b^2*f)) + (a + b*tan(e + f*x)^2)^(5/2)/(5*b^2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\,\tan^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**5, x)
```

3.294 $\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=88

$$\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3bf}-\frac{\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/f-(a+b*tan(f*x+e)
^2)^(1/2)/f+1/3*(a+b*tan(f*x+e)^2)^(3/2)/b/f
```

Rubi [A] time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3670, 446, 80, 50, 63, 208}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3bf}-\frac{\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - Sqrt[a +
b*Tan[e + f*x]^2]/f + (a + b*Tan[e + f*x]^2)^(3/2)/(3*b*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}\\ &= \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3bf} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}\\ &= -\frac{\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3bf} - \frac{(a-b)\,\text{Subst}\left(\int\right.}{}\\ &= -\frac{\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3bf} - \frac{(a-b)\,\text{Subst}\left(\int\right.}{}\\ &= \frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} - \frac{\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{(a+b\,\text{ta}}{}\end{aligned}$$

Mathematica [A] time = 0.35, size = 82, normalized size = 0.93

$$\frac{\sqrt{a+b\tan^2(e+fx)}\left(a+b\tan^2(e+fx)-3b\right)+3b\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{3bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (3*Sqrt[a - b]*b*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a +
 b*Tan[e + f*x]^2]*(a - 3*b + b*Tan[e + f*x]^2))/(3*b*f)
```

fricas [A] time = 0.51, size = 254, normalized size = 2.89

$$\left[\frac{3\sqrt{a-b}\,b\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2+4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+8a^2-8ab+b^2}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)+4\left(b\tan\right.}{12bf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(a - b)*b*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x
 + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
- b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*(b
*tan(f*x + e)^2 + a - 3*b)*sqrt(b*tan(f*x + e)^2 + a))/(b*f), -1/6*(3*sqrt(
-a + b)*b*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^
2 + 2*a - b)) - 2*(b*tan(f*x + e)^2 + a - 3*b)*sqrt(b*tan(f*x + e)^2 + a))/
(b*f)]
```

giac [A] time = 0.63, size = 96, normalized size = 1.09

$$-\frac{(a-b)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}+\frac{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}b^2f^2-3\sqrt{b\tan\left(fx+e\right)^2+a}\,b^3f^2}{3\,b^3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -(a - b)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f) +
 1/3*((b*tan(f*x + e)^2 + a)^(3/2)*b^2*f^2 - 3*sqrt(b*tan(f*x + e)^2 + a)*b
^3*f^2)/(b^3*f^3)
```

maple [A] time = 0.41, size = 114, normalized size = 1.30

$$\frac{\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{3bf}-\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{f}+\frac{b\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}-\frac{a\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x)
```

```
[Out] 1/3*(a+b*tan(f*x+e)^2)^(3/2)/b/f-(a+b*tan(f*x+e)^2)^(1/2)/f+1/f*b/(-a+b)^(1
/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))-1/f*a/(-a+b)^(1/2)*arctan
((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\tan\left(fx+e\right)^2+a}\,\tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^3, x)
```

mupad [B] time = 14.38, size = 76, normalized size = 0.86

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sqrt{a-b}}\right)\sqrt{a-b}}{f}-\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{f}+\frac{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}{3\,b\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] (atanh((a + b*tan(e + f*x)^2)^(1/2)/(a - b)^(1/2))*(a - b)^(1/2))/f - (a +
b*tan(e + f*x)^2)^(1/2)/f + (a + b*tan(e + f*x)^2)^(3/2)/(3*b*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2\left(e + fx\right)}\tan^3\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**3, x)
```

3.295 $\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/f+(a+b*tan(f*x+e
)^2)^(1/2)/f
```

Rubi [A] time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 444, 50, 63, 208}

$$\frac{\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + Sqrt[a
 + b*Tan[e + f*x]^2]/f
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
```

```
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx^2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx,x,\tan^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx,x,\tan^2(e+f
\end{aligned}$$

$$\begin{aligned}
&= \frac{\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\tan^2(e}{bf}\\
&= -\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{\sqrt{a+b\tan^2(e+fx)}}{f}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.95

$$\frac{\sqrt{a+b\tan^2(e+fx)}-\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]) + Sqrt[a +
b*Tan[e + f*x]^2])/f
```

fricas [A] time = 0.51, size = 214, normalized size = 3.45

$$\left[\frac{\sqrt{a-b}\,\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2-4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+8a^2-8ab+b^2}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)+4\sqrt{b\tan\left(\right.}}{4f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)
^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)
+ 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*sqrt(b*
tan(f*x + e)^2 + a))/f, 1/2*(sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 +
```

```
a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2*sqrt(b*tan(f*x + e)^2 + a
))/f]
```

giac [A] time = 0.52, size = 60, normalized size = 0.97

$$\frac{(a-b)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}+\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] (a - b)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f) +
sqrt(b*tan(f*x + e)^2 + a)/f
```

maple [A] time = 0.24, size = 91, normalized size = 1.47

$$\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{f}-\frac{b\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}+\frac{a\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e),x)
```

```
[Out] (a+b*tan(f*x+e)^2)^(1/2)/f-1/f*b/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/
2)/(-a+b)^(1/2))+1/f*a/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^
(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\tan\left(fx+e\right)^2+a}\,\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e), x)
```

mupad [B] time = 12.51, size = 54, normalized size = 0.87

$$\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{f}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sqrt{a-b}}\right)\sqrt{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] (a + b*tan(e + f*x)^2)^(1/2)/f - (atanh((a + b*tan(e + f*x)^2)^(1/2)/(a - b
)^(1/2))*(a - b)^(1/2))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\tan^2\left(e+fx\right)}\,\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x), x)
```

3.296 $\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+arctanh((a+b*tan(f*x+e
)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 446, 83, 63, 208}

$$\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/f) + (Sqrt[a - b]*A
rcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, \ldots\right)}{2f}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}} \ldots\right)}{\ldots}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Mathematica [A] time = 0.06, size = 72, normalized size = 0.97

$$\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right) - \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[a]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]) + Sqrt[a - b]*ArcTa
nh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f
```

fricas [A] time = 0.43, size = 382, normalized size = 5.16

$$\left[\frac{\sqrt{a-b}\log\left(\frac{b\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+2a-b}{\tan\left(fx+e\right)^2+1}\right) + \sqrt{a}\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a}+2a}{\tan\left(fx+e\right)^2}\right)}{2f}, \frac{2\sqrt{-a+b}\,\text{arc}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + sqrt(a)*log((b*tan(f*x + e)^2 -
2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2))/f, 1/2*(2*sqrt
(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) + sqrt(a)
*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*
x + e)^2))/f, 1/2*(2*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)
 + sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
```

```
- b) + 2*a - b)/(tan(f*x + e)^2 + 1)))/f, (sqrt(-a)*arctan(sqrt(b*tan(f*x +
 e)^2 + a)*sqrt(-a)/a) + sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sq
rt(-a + b)/(a - b)))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)2/f*(-1/2*a*atan((-sin(f*x+exp(1))^2*s
qrt(a-b)+sqrt(a*sin(f*x+exp(1))^4-b*sin(f*x+exp(1))^4-2*a*sin(f*x+exp(1))^2
+b*sin(f*x+exp(1))^2+a))/sqrt(-a))/sqrt(-a)+sqrt(a-b)*(-a+b)*ln(abs((2*a-2*
b)*(-sin(f*x+exp(1))^2*sqrt(a-b)+sqrt(a*sin(f*x+exp(1))^4-b*sin(f*x+exp(1))
^4-2*a*sin(f*x+exp(1))^2+b*sin(f*x+exp(1))^2+a))+sqrt(a-b)*(2*a-b)))/(4*a-4
*b))*sign(sin(f*x+exp(1))^2-1)
```

maple [B] time = 1.70, size = 615, normalized size = 8.31

$$-\frac{\sqrt{4}\,\sqrt{\frac{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}{\cos\left(fx+e\right)^2}}\,\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)\left(\ln\left(-\frac{2\left(-1+\cos\left(fx+e\right)\right)\left(\sqrt{a}\,\cos\left(fx+e\right)\sqrt{\frac{a\left(\cos^2\left(fx+\cdots\right.\right.}{\left(1+\cdots\right.}}\cdots\right.}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4/f*4^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f
*x+e)*(-1+cos(f*x+e))*(ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f
*x+e)^2/a^(1/2))*a^(1/2)*(a-b)^(1/2)-a^(1/2)*ln(-4*(((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)
/(-1+cos(f*x+e)))*(a-b)^(1/2)+2*ln(4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a-b)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+4*a*cos(f*x+e)-4*b*cos(f*x+e))*a-
2*ln(4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)*(a-b)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e
))^2)^(1/2)+4*a*cos(f*x+e)-4*b*cos(f*x+e))*b)/sin(f*x+e)^2/((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)/(a-b)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 0.29, size = 83, normalized size = 1.12

$$-\frac{\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sqrt{a}}\right)}{f}-\frac{\operatorname{atanh}\left(\frac{a\,b^3\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\sqrt{a-b}}{a\,b^4-a^2\,b^3}\right)\sqrt{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] - (a^(1/2)*atanh((a + b*tan(e + f*x)^2)^(1/2)/a^(1/2)))/f - (atanh((a*b^3*(
a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(1/2))/(a*b^4 - a^2*b^3))*(a - b)^(1/2)
)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\tan^2\left(e+fx\right)}\,\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x), x)
```

3.297 $\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=115

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

```
[Out] 1/2*(2*a-b)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-arctanh((a+
b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/f-1/2*cot(f*x+e)^2*(a+b*tan(
f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3670, 446, 99, 156, 63, 208}

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*Sqrt[a]*f) - (Sq
rt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - (Cot[e + f*x
]^2*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^3(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-2a+b)-\frac{bx}{2}}{x(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(\right.}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x,\right.}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{\right.}{bf}$$

$$= \frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f} - \frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} - $$

Mathematica [A] time = 0.37, size = 115, normalized size = 1.00

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)-\sqrt{a}\left(2\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)}{2\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] - Sqrt[a]*(2*Sqrt[a
- b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Cot[e + f*x]^2*Sqrt[
a + b*Tan[e + f*x]^2]))/(2*Sqrt[a]*f)
```

fricas [A] time = 0.45, size = 592, normalized size = 5.15

$$\left[\frac{2\sqrt{a-b}\,a\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+2\,a-b}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2-(2\,a-b)\sqrt{a}\,\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+\right.}}{\tan\left(fx+e\right)^2}\right.}{4\,af\tan\left(fx+e\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a - b)*a*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*
sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 - (2*a - b)*sqr
t(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/ta
n(f*x + e)^2)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(a*f*tan(f*x
 + e)^2), -1/4*(4*a*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
 + b)/(a - b))*tan(f*x + e)^2 + (2*a - b)*sqrt(a)*log((b*tan(f*x + e)^2 - 2
*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^2 +
 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(a*f*tan(f*x + e)^2), -1/2*(sqrt(-a)*(2*a
- b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2 - sqrt(a
- b)*a*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2
*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + sqrt(b*tan(f*x + e)^2 + a)*a
)/(a*f*tan(f*x + e)^2), -1/2*(sqrt(-a)*(2*a - b)*arctan(sqrt(b*tan(f*x + e)
^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2 + 2*a*sqrt(-a + b)*arctan(-sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^2 + sqrt(b*tan(f*x + e)^2
+ a)*a)/(a*f*tan(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
```

```
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Warning, integration of abs or sign assumes constant sign by inte
rvals (correct if the argument is real):Check [abs(t_nostep^2-1)]Discontinu
ities at zeroes of t_nostep^2-1 were not checkedUnable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assume
s constant sign by intervals (correct if the argument is real):Check [abs(t
_nostep^2-1)]Evaluation time: 1.54Error: Bad Argument Type
```

maple [B] time = 1.66, size = 2135, normalized size = 18.57

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/8/f*(-1+cos(f*x+e))*(4*cos(f*x+e)^2*a^(5/2)*ln(4*((a*cos(f*x+e)^2-cos(f*
```

```
x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a-b)^(1/2)+4*(a-b)^(1/2)*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+4*a*cos(f*x+e)-4*b
*cos(f*x+e))*4^(1/2)+2*cos(f*x+e)^2*a^(3/2)*(a-b)^(1/2)*((a*cos(f*x+e)^2-co
s(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*4^(1/2)-4*cos(f*x+e)^2*a^(3/2)*ln(4
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a-b
)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^
(1/2)+4*a*cos(f*x+e)-4*b*cos(f*x+e))*4^(1/2)*b-8*cos(f*x+e)^2*a^(1/2)*(a-b)
^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)-4*cos(f*x
+e)^2*a^(1/2)*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))
^2)^(1/2)*4^(1/2)*b+2*cos(f*x+e)^2*(a-b)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+
e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*4^(1/2)*a^2-cos(f*x+e)^2*(a-b)^(1/
2)*ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+
e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*4^(
1/2)*a*b-2*cos(f*x+e)^2*(a-b)^(1/2)*ln(-4*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(
f*x+e)))*4^(1/2)*a^2+cos(f*x+e)^2*(a-b)^(1/2)*ln(-4*(((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-co
s(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b
)/(-1+cos(f*x+e)))*4^(1/2)*a*b-2*cos(f*x+e)*a^(3/2)*(a-b)^(1/2)*((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*4^(1/2)-4*a^(5/2)*ln(4*((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a-b)^(1/
2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)
+4*a*cos(f*x+e)-4*b*cos(f*x+e))*4^(1/2)-16*cos(f*x+e)*a^(1/2)*(a-b)^(1/2)*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)+4*a^(3/2)*ln(4*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a-b)^(
1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/
2)+4*a*cos(f*x+e)-4*b*cos(f*x+e))*4^(1/2)*b-8*a^(1/2)*(a-b)^(1/2)*((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)+4*a^(1/2)*(a-b)^(1/2)*((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*4^(1/2)*b-2*(a-b)^(
1/2)*ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*
x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*4
^(1/2)*a^2+(a-b)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f
*x+e)^2/a^(1/2))*4^(1/2)*a*b+2*(a-b)^(1/2)*ln(-4*(((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(
-1+cos(f*x+e)))*4^(1/2)*a^2-(a-b)^(1/2)*ln(-4*(((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+
cos(f*x+e)))*4^(1/2)*a*b)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos
(f*x+e)^2)^(1/2)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)
/sin(f*x+e)^4/a^(3/2)/(a-b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan\left(fx+e\right)^2 + a}\, \cot\left(fx+e\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^3, x)
```

mupad [B] time = 11.87, size = 238, normalized size = 2.07

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,b^4\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{2\left(\frac{a\,b^4}{2}-\frac{3\,b^5}{4}+\frac{b^6}{4\,a}\right)}-\frac{3\,b^5\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{4\,\sqrt{a}\left(\frac{a\,b^4}{2}-\frac{3\,b^5}{4}+\frac{b^6}{4\,a}\right)}+\frac{b^6\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{4\,a^{3/2}\left(\frac{a\,b^4}{2}-\frac{3\,b^5}{4}+\frac{b^6}{4\,a}\right)}\right)(2\,a-b)}{2\,\sqrt{a}\,f}-\frac{\operatorname{atanh}\left(\frac{b^4\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\sqrt{a-b}}{2\left(\frac{a\,b^4}{2}-\frac{b^5}{2}\right)}\right)\,\sqrt{}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] (atanh((a^(1/2)*b^4*(a + b*tan(e + f*x)^2)^(1/2))/(2*((a*b^4)/2 - (3*b^5)/4
 + b^6/(4*a))) - (3*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*a^(1/2)*((a*b^4)/2
 - (3*b^5)/4 + b^6/(4*a))) + (b^6*(a + b*tan(e + f*x)^2)^(1/2))/(4*a^(3/2)*
((a*b^4)/2 - (3*b^5)/4 + b^6/(4*a))))*(2*a - b))/(2*a^(1/2)*f) - (atanh((b^
4*(a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(1/2))/(2*((a*b^4)/2 - b^5/2)))*(a -
 b)^(1/2))/f - (b*(a + b*tan(e + f*x)^2)^(1/2))/(2*(f*(a + b*tan(e + f*x)^2
) - a*f))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\,\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**3, x)
```

3.298 $\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=163

$$-\frac{\left(8a^2-4ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}+\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx}}{4f}$$

```
[Out] -1/8*(8*a^2-4*a*b-b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+
arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/f+1/8*(4*a-b)*cot
(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)/a/f-1/4*cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^
(1/2)/f
```

Rubi [A] time = 0.21, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3670, 446, 99, 151, 156, 63, 208}

$$-\frac{\left(8a^2-4ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}+\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((8*a^2 - 4*a*b - b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8*a^(
3/2)*f) + (Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f +
 ((4*a - b)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(8*a*f) - (Cot[e + f
*x]^4*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^5(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a+b)-\frac{3bx}{2}}{x^2(1+x)\sqrt{a+bx}}\,dx, x, \tan^2\right.}{4f}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8af} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e}}{4f}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8af} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e}}{4f}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8af} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e}}{4f}$$

$$= -\frac{\left(8a^2-4ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f} + \frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(}}{\sqrt{a-b}}\right.}{f}$$

Mathematica [A] time = 1.32, size = 138, normalized size = 0.85

$$\frac{\left(-8a^2+4ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\left(8a\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)-\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\ldots\right)}{8a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-8*a^2 + 4*a*b + b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] + Sqrt[
a]*(8*a*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] - Cot[e
 + f*x]^2*(-4*a + b + 2*a*Cot[e + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2]))/(8*a
^(3/2)*f)
```

fricas [A] time = 0.45, size = 729, normalized size = 4.47

$$\left[\frac{8\sqrt{a-b}\,a^2\log\left(\frac{b\tan(fx+e)^2+2\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+2a-b}{\tan(fx+e)^2+1}\right)\tan(fx+e)^4-\left(8a^2-4ab-b^2\right)\sqrt{a}\log\left(\frac{b\tan(fx+e)^2+\ldots}{\ldots}\right)\ldots}{16a^2f\tan(fx+\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(a - b)*a^2*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 - (8*a^2 - 4
*a*b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sq
rt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 + 2*((4*a^2 - a*b)*tan(f*x + e)
^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f*tan(f*x + e)^4), 1/16*(16*a^
2*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan
(f*x + e)^4 - (8*a^2 - 4*a*b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(
b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 + 2*((4
*a^2 - a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f*tan(
f*x + e)^4), 1/8*(4*sqrt(a - b)*a^2*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 +
 (8*a^2 - 4*a*b - b^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/
a)*tan(f*x + e)^4 + ((4*a^2 - a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x +
 e)^2 + a))/(a^2*f*tan(f*x + e)^4), 1/8*(8*a^2*sqrt(-a + b)*arctan(-sqrt(b*
tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^4 + (8*a^2 - 4*a*b -
 b^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^4
 + ((4*a^2 - a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*
f*tan(f*x + e)^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
```

e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign

: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^2-1 were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 2.25Error: Bad Argument Type

maple [B] time = 1.28, size = 5676, normalized size = 34.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan\left(fx+e\right)^2 + a}\,\cot\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^5, x)

mupad [B] time = 12.02, size = 542, normalized size = 3.33

$$\frac{\operatorname{atanh}\left(\frac{3\,b^7\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}}{64\,\sqrt{a^3}\,\left(\frac{a\,b^5}{4}-\frac{11\,b^6}{32}+\frac{3\,b^7}{64\,a}+\frac{11\,b^8}{256\,a^2}+\frac{b^9}{256\,a^3}\right)}-\frac{11\,b^6\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}}{32\,\sqrt{a^3}\,\left(\frac{b^5}{4}-\frac{11\,b^6}{32\,a}+\frac{3\,b^7}{64\,a^2}+\frac{11\,b^8}{256\,a^3}+\frac{b^9}{256\,a^4}\right)}+\frac{11\,b^8\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}}{256\,\sqrt{a^3}\,\left(\frac{3\,b^7}{64}-\frac{11\,a\,b^6}{32}+\frac{a^2\,b^5}{4}+\frac{11\,b^8}{256\,a}+\frac{b^9}{256\,a^2}\right)}\right.}{8\,f\,\sqrt{a^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2),x)

[Out] (atanh((3*b^7*(a + b*tan(e + f*x)^2)^(1/2))/(64*(a^3)^(1/2)*((a*b^5)/4 - (11*b^6)/32 + (3*b^7)/(64*a) + (11*b^8)/(256*a^2) + b^9/(256*a^3))) - (11*b^6*(a + b*tan(e + f*x)^2)^(1/2))/(32*(a^3)^(1/2)*(b^5/4 - (11*b^6)/(32*a) + (3*b^7)/(64*a^2) + (11*b^8)/(256*a^3) + b^9/(256*a^4))) + (11*b^8*(a + b*tan(e + f*x)^2)^(1/2))/(256*(a^3)^(1/2)*((3*b^7)/64 - (11*a*b^6)/32 + (a^2*b^5)/4 + (11*b^8)/(256*a) + b^9/(256*a^2))) + (b^9*(a + b*tan(e + f*x)^2)^(1/2))/(256*(a^3)^(1/2)*((3*a*b^7)/64 + (11*b^8)/256 - (11*a^2*b^6)/32 + (a^3*b^5)/4 + b^9/(256*a))) + (a*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*(a^3)^(1/2)*(b^5/4 - (11*b^6)/(32*a) + (3*b^7)/(64*a^2) + (11*b^8)/(256*a^3) + b^9/(256*a^4))))*(4*a*b - 8*a^2 + b^2))/(8*f*(a^3)^(1/2)) - (atanh((b^5*(a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(1/2))/(4*((7*b^6)/32 - (a*b^5)/4 + b^7/(32*a)))

```
 + (b^6*(a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(1/2))/(32*((7*a*b^6)/32 + b^7
/32 - (a^2*b^5)/4)))*(a - b)^(1/2))/f - ((a + b*tan(e + f*x)^2)^(1/2)*((a*b
)/2 + b^2/8) - (b*(a + b*tan(e + f*x)^2)^(3/2)*(4*a - b))/(8*a))/(f*(a + b*
tan(e + f*x)^2)^2 + a^2*f - 2*a*f*(a + b*tan(e + f*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2\left(e + fx\right)}\cot^5\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**5, x)
```

3.299 $\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=222

$$\frac{\left(a^3+2a^2b+8ab^2-16b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16b^{5/2}f}-\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f}-\frac{\sqrt{a-b}}{}$$

```
[Out] 1/16*(a^3+2*a^2*b+8*a*b^2-16*b^3)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e
)^2)^(1/2))/b^(5/2)/f-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2
))*(a-b)^(1/2)/f-1/16*(a-2*b)*(a+4*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b
^2/f+1/24*(a-6*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f+1/6*(a+b*tan(f*
x+e)^2)^(1/2)*tan(f*x+e)^5/f
```

Rubi [A] time = 0.34, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3670, 478, 582, 523, 217, 206, 377, 203}

$$\frac{\left(2a^2b+a^3+8ab^2-16b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16b^{5/2}f}-\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f}+\frac{\tan^5(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) + ((a^3 + 2*a^2*b + 8*a*b^2 - 16*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b*Tan[e + f*x]^2]])/(16*b^(5/2)*f) - ((a - 2*b)*(a + 4*b)*Tan[e +
f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(16*b^2*f) + ((a - 6*b)*Tan[e + f*x]^3*Sqr
t[a + b*Tan[e + f*x]^2])/(24*b*f) + (Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]
^2])/(6*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{6f} - \frac{\text{Subst}\left(\int \frac{x^4\left(5a+(-a+6b)x^2\right)}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \text{t}\right.}{6f}$$

$$= \frac{(a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{24bf} + \frac{\tan^5(e+fx)\sqrt{a+b\,\text{ta}}}{6f}$$

$$= -\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-6b)\tan^3(}{}$$

$$= -\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-6b)\tan^3(}{}$$

$$= -\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-6b)\tan^3(}{}$$

$$= -\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{\left(a^3+2a^2b+8ab^2-16b^3\right)\tanh^{-}}{16b^{5/2}f}$$

Mathematica [C] time = 6.33, size = 823, normalized size = 3.71

$$-\frac{b\left(a^3+2ba^2-8b^3\right)\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{\cos(2(e+fx))+1}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\csc(2(e+fx))F\left(\sin^{-1}\left(\right.\right.}{a(a+b+(a-b)\cos(2(e+fx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-((b*(a^3 + 2*a^2*b - 8*b^3)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 +
Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e +
f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a
- b)*Cos[2*(e + f*x)]))) - (4*b*(-8*a*b^2 + 8*b^3)*Sqrt[1 + Cos[2*(e + f*x)
]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-
((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)
]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*
x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^
2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a +
 b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*
(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e
+ f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSi
n[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*
Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*
```

```
Cos[2*(e + f*x)]]))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/(8*b^2*f) + (S
qrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)]
)]*((Sec[e + f*x]^3*(a*Sin[e + f*x] - 14*b*Sin[e + f*x]))/(24*b) + (Sec[e +
 f*x]*(-3*a^2*Sin[e + f*x] - 8*a*b*Sin[e + f*x] + 44*b^2*Sin[e + f*x]))/(48
*b^2) + (Sec[e + f*x]^4*Tan[e + f*x])/6))/f
```

fricas [A] time = 2.10, size = 826, normalized size = 3.72

$$\left[\frac{48\sqrt{-a+b}\,b^3\log\left(-\frac{(a-2b)\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)-3\left(a^3+2a^2b+8ab^2-16b^3\right)\sqrt{b}\log\left(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] [1/96*(48*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 3*(a^3
 + 2*a^2*b + 8*a*b^2 - 16*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*ta
n(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(8*b^3*tan(f*x + e)^5 + 2*(
a*b^2 - 6*b^3)*tan(f*x + e)^3 - 3*(a^2*b + 2*a*b^2 - 8*b^3)*tan(f*x + e))*s
qrt(b*tan(f*x + e)^2 + a))/(b^3*f), -1/96*(96*sqrt(a - b)*b^3*arctan(-sqrt(
b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + 3*(a^3 + 2*a^2*b + 8*a*
b^2 - 16*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(b)*tan(f*x + e) + a) - 2*(8*b^3*tan(f*x + e)^5 + 2*(a*b^2 - 6*b^3)*ta
n(f*x + e)^3 - 3*(a^2*b + 2*a*b^2 - 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e
)^2 + a))/(b^3*f), 1/48*(24*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)^2
 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e
)^2 + 1)) - 3*(a^3 + 2*a^2*b + 8*a*b^2 - 16*b^3)*sqrt(-b)*arctan(sqrt(b*tan
(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + (8*b^3*tan(f*x + e)^5 + 2*(a*
b^2 - 6*b^3)*tan(f*x + e)^3 - 3*(a^2*b + 2*a*b^2 - 8*b^3)*tan(f*x + e))*sqr
t(b*tan(f*x + e)^2 + a))/(b^3*f), -1/48*(48*sqrt(a - b)*b^3*arctan(-sqrt(b*
tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + 3*(a^3 + 2*a^2*b + 8*a*b^
2 - 16*b^3)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x
+ e))) - (8*b^3*tan(f*x + e)^5 + 2*(a*b^2 - 6*b^3)*tan(f*x + e)^3 - 3*(a^2*
b + 2*a*b^2 - 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^3*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\,\tan\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

maple [B] time = 0.37, size = 451, normalized size = 2.03

$$\frac{\left(\tan^3\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{6fb}-\frac{a\tan\left(fx+e\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{8f\,b^2}+\frac{a^2\tan\left(fx+e\right)\sqrt{a+b\left(\tan^2\left(\right.\right.}}{16f\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x)
```

```
[Out] 1/6/f*tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2)/b-1/8/f/b^2*a*tan(f*x+e)*(a+b*t
an(f*x+e)^2)^(3/2)+1/16/f/b^2*a^2*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)+1/16/
f/b^(5/2)*a^3*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))-1/4/f*tan(f*x
+e)*(a+b*tan(f*x+e)^2)^(3/2)/b+1/8/f/b*a*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2
)+1/8/f/b^(3/2)*a^2*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))+1/2*(a+
b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+1/2/f*a/b^(1/2)*ln(tan(f*x+e)*b^(1/2)+(a
+b*tan(f*x+e)^2)^(1/2))-1/f*b^(1/2)*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2
)^(1/2))+1/f*(b^4*(a-b))^(1/2)/b/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(
a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/f*a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arcta
n((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan\left(fx+e\right)^{2}+a}\, \tan\left(fx+e\right)^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan\left(e+f\,x\right)^{6}\,\sqrt{b\tan\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tan^{2}{\left(e + fx \right)}} \tan^{6}{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**6,x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**6, x)
```

3.300 $\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=169

$$-\frac{\left(a^2+4ab-8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{3/2}f}+\frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8bf}+\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+}{\sqrt{a+b\tan^2(e+}}\right.}{f}$$

```
[Out] -1/8*(a^2+4*a*b-8*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))
/b^(3/2)/f+arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(1
/2)/f+1/8*(a-4*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/4*(a+b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A] time = 0.21, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3670, 478, 582, 523, 217, 206, 377, 203}

$$-\frac{\left(a^2+4ab-8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{3/2}f}+\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}+\frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f - ((a^2 + 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(8*b^(3/2)*f) + ((a - 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x
]^2])/(8*b*f) + (Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
```

```
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a+(-a+4b)x^2\right)}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \tan\ldots\right.}{4f}$$

$$= \frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b\tan\ldots}}{4f}$$

$$= \frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b\tan\ldots}}{4f}$$

$$= \frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b\tan\ldots}}{4f}$$

$$= \frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\left(a^2+4ab-8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e\ldots}{\sqrt{a+b\tan^2\ldots}}\right.}{8b^{3/2}f}$$

Mathematica [C] time = 6.25, size = 767, normalized size = 4.54

$$\frac{\sqrt{\frac{a\cos(2(e+fx))+a-b\cos(2(e+fx))+b}{\cos(2(e+fx))+1}}\left(\frac{\sec(e+fx)(a\sin(e+fx)-6b\sin(e+fx))}{8b}+\frac{1}{4}\tan(e+fx)\sec^2(e+fx)\right)}{f}-\frac{b\left(a^2-4b^2\right)\sin^4(e+fx)\operatorname{cs}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/4*(-((b*(a^2 - 4*b^2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2
*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)]
)*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e +
f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)*
Cos[2*(e + f*x)]))) - (4*b*(-4*a*b + 4*b^2)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt
[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot
[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[
((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*Ell
ipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/S
qrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a
 - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Co
s[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)]
)*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[
((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e +
 f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(
e + f*x)]])))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/(b*f) + (Sqrt[(a + b
+ a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sec[e
+ f*x]*(a*Sin[e + f*x] - 6*b*Sin[e + f*x]))/(8*b) + (Sec[e + f*x]^2*Tan[e +
 f*x])/4))/f
```

fricas [A] time = 1.20, size = 671, normalized size = 3.97

$$\left[\frac{8\sqrt{-a+b}\,b^2\log\left(-\frac{(a-2b)\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\,\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^2+1}\right)-\left(a^2+4ab-8b^2\right)\sqrt{b}\,\log\left(2b\tan\left(fx+e\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(-a + b)*b^2*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (a^2 +
4*a*b - 8*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a
)*sqrt(b)*tan(f*x + e) + a) + 2*(2*b^2*tan(f*x + e)^3 + (a*b - 4*b^2)*tan(f
*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/16*(16*sqrt(a - b)*b^2*arct
an(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (a^2 + 4*a*b -
 8*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
b)*tan(f*x + e) + a) + 2*(2*b^2*tan(f*x + e)^3 + (a*b - 4*b^2)*tan(f*x + e)
)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/8*(4*sqrt(-a + b)*b^2*log(-((a - 2
*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e)
 - a)/(tan(f*x + e)^2 + 1)) + (a^2 + 4*a*b - 8*b^2)*sqrt(-b)*arctan(sqrt(b*
tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + (2*b^2*tan(f*x + e)^3 + (a
```

```
*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/8*(8*sqrt(
a - b)*b^2*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) +
 (a^2 + 4*a*b - 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/
(b*tan(f*x + e))) + (2*b^2*tan(f*x + e)^3 + (a*b - 4*b^2)*tan(f*x + e))*sqr
t(b*tan(f*x + e)^2 + a))/(b^2*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan\left(fx+e\right)^{2}+a}\,\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

maple [B] time = 0.38, size = 323, normalized size = 1.91

$$\frac{\tan\left(fx+e\right)\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{4fb}-\frac{a\tan\left(fx+e\right)\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}{8fb}-\frac{a^{2}\ln\left(\tan\left(fx+e\right)\sqrt{b}+\sqrt{a+}\right.}{8f\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x)
```

```
[Out] 1/4/f*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/b-1/8/f/b*a*tan(f*x+e)*(a+b*tan(f
*x+e)^2)^(1/2)-1/8/f/b^(3/2)*a^2*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(
1/2))-1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-1/2/f*a/b^(1/2)*ln(tan(f*x+
e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))+1/f*b^(1/2)*ln(tan(f*x+e)*b^(1/2)+(a+b
*tan(f*x+e)^2)^(1/2))-1/f*(b^4*(a-b))^(1/2)/b/(a-b)*arctan((a-b)*b^2/(b^4*(
a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/f*a*(b^4*(a-b))^(1/2)/b^
2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x
+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan\left(fx+e\right)^{2}+a}\,\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^{4}\,\sqrt{b\tan\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^{2}\left(e+fx\right)}\,\tan^{4}\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**4, x)
```

3.301 $\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=123

$$\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}-\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{b}\,f}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f+1/2*
(a-2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)+1/2*
(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3670, 478, 523, 217, 206, 377, 203}

$$-\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) + ((a - 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2
]])/(2*Sqrt[b]*f) + (Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
```

```
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{a+(-a+2b)x^2}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+\right.}{2f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan\right.}{2f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan}{\sqrt{a+b}}\right.}{2f}$$

$$= -\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{b}\,f} + $$

Mathematica [C] time = 6.18, size = 708, normalized size = 5.76

$$\frac{b^2\sin^4(e+fx)\csc(2(e+fx))\sqrt{\frac{(a-b)\cos(2(e+fx))+a+b}{\cos(2(e+fx))+1}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2}{b}}}{af((a-b)\cos(2(e+fx))+a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (b^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[-
((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)
]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*
x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^
```

```
2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*f*(a + b + (a - b)*Cos[2*(e + f*x)]))
 + (4*(a - b)*b*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e +
 f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(
1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e +
 f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b +
 (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/
(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (
Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(
e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e +
 f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 +
 Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/(f*Sqrt[a + b
+ (a - b)*Cos[2*(e + f*x)]]) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*
(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Tan[e + f*x])/(2*f)
```

fricas [A] time = 0.72, size = 539, normalized size = 4.38

$$\left[-\frac{(a-2b)\sqrt{b}\log\left(2b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{b}\tan\left(fx+e\right)+a\right)-2\sqrt{-a+b}\,b\log\left(-\frac{(a-2b}{}\right.}{4bf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 +
 a)*sqrt(b)*tan(f*x + e) + a) - 2*sqrt(-a + b)*b*log(-((a - 2*b)*tan(f*x +
e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x
 + e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/(b*f), -1/4*(4
*sqrt(a - b)*b*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e)
)) + (a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a
)*sqrt(b)*tan(f*x + e) + a) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/
(b*f), -1/2*((a - 2*b)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/
(b*tan(f*x + e))) - sqrt(-a + b)*b*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(
b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1))
- sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/(b*f), -1/2*(2*sqrt(a - b)*b*a
rctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (a - 2*b)*s
qrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - sqrt
(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/(b*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\tan\left(fx+e\right)^2+a}\,\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

maple [B] time = 0.30, size = 230, normalized size = 1.87

$$\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\,\tan\left(fx+e\right)}{2f}+\frac{a\ln\left(\tan\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\right)}{2f\sqrt{b}}-\frac{\sqrt{b}\,\ln\left(\tan\left(fx+e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x)
```

```
[Out] 1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+1/2/f*a/b^(1/2)*ln(tan(f*x+e)*b^(
1/2)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*b^(1/2)*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f
*x+e)^2)^(1/2))+1/f*(b^4*(a-b))^(1/2)/b/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^
(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/f*a*(b^4*(a-b))^(1/2)/b^2/(a-b
)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan\left(fx+e\right)^2 + a}\, \tan\left(fx+e\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^2\, \sqrt{b \tan\left(e+f\,x\right)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tan^2\left(e + fx\right)}\, \tan^2\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**2, x)
```

3.302 $\int \sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f+arcta
nh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3661, 402, 217, 206, 377, 203}

$$\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
```

```
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan(e+\right.}{f}$$

$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+f}{\sqrt{a+b\tan^2(}}\right.}{f}$$

$$= \frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

Mathematica [C] time = 0.88, size = 203, normalized size = 2.39

$$\frac{-i\sqrt{a-b}\,\log\left(-\frac{4i\left(\sqrt{a-b}\,\sqrt{a+b\tan^2(e+fx)}+a-ib\tan(e+fx)\right)}{(a-b)^{3/2}(\tan(e+fx)+i)}\right)+i\sqrt{a-b}\,\log\left(\frac{4i\left(\sqrt{a-b}\,\sqrt{a+b\tan^2(e+fx)}+a+ib\tan(e+fx)\right)}{(a-b)^{3/2}(\tan(e+fx)-i)}\right)+2\sqrt{b}\,\text{lo}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[a - b]*Log[((-4*I)*(a - I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a +
b*Tan[e + f*x]^2]))/((a - b)^(3/2)*(I + Tan[e + f*x]))] + I*Sqrt[a - b]*Log
[((4*I)*(a + I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a + b*Tan[e + f*x]^2]))/((
a - b)^(3/2)*(-I + Tan[e + f*x]))] + 2*Sqrt[b]*Log[b*Tan[e + f*x] + Sqrt[b]
*Sqrt[a + b*Tan[e + f*x]^2]])/(2*f)
```

fricas [A] time = 0.51, size = 410, normalized size = 4.82

$$\left[\frac{\sqrt{b}\,\log\left(2\,b\tan\left(fx+e\right)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{b}\,\tan\left(fx+e\right)+a\right)+\sqrt{-a+b}\,\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2+2\,\sqrt{b}}{\tan}\right.}{2\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)
*tan(f*x + e) + a) + sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)))/
f, 1/2*(2*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f
*x + e))) + sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*s
```

```
qrt(b)*tan(f*x + e) + a))/f, -1/2*(2*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2
+ a)*sqrt(-b)/(b*tan(f*x + e))) - sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)
^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
 e)^2 + 1)))/f, (sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b
)*tan(f*x + e))) - sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*t
an(f*x + e))))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan\left(fx+e\right)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [B] time = 0.42, size = 169, normalized size = 1.99

$$\frac{\sqrt{b}\,\ln\left(\tan\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\right)}{f}-\frac{\sqrt{b^4\left(a-b\right)}\,\arctan\left(\frac{(a-b)b^2\tan\left(fx+e\right)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}\right)}{fb\left(a-b\right)}+\frac{a\sqrt{b^4\left(a-\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*b^(1/2)*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*(b^4*(a-b))
^(1/2)/b/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*
tan(f*x+e))+1/f*a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^
(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b \tan\left(e+fx\right)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2\left(e+fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2), x)
```

3.303 $\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f-cot(
f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3670, 475, 12, 377, 203}

$$-\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) - (Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{-a+b}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,d\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x\right.}{f}$$

$$= -\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Mathematica [C] time = 0.28, size = 64, normalized size = 0.85

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{(a-b)\tan^2(e+fx)}{b\tan^2(e+fx)+a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -(((a - b)*Tan[e + f*x]^2)/
(a + b*Tan[e + f*x]^2))]*Sqrt[a + b*Tan[e + f*x]^2])/f)
```

fricas [A] time = 0.53, size = 257, normalized size = 3.43

$$\left[\frac{\sqrt{-a+b}\,\log\left(-\frac{(a^2-8ab+8b^2)\tan(fx+e)^4-2(3a^2-4ab)\tan(fx+e)^2+a^2-4\left((a-2b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan(fx+e)^2+a}\,\sqrt{-a+b}}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)}{4f\tan(fx+e)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 -
4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))
*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^
2 + 1))*tan(f*x + e) - 4*sqrt(b*tan(f*x + e)^2 + a))/(f*tan(f*x + e)), -1/2
*(sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)
/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e) + 2*sqrt(b*tan(f*x + e)^2 + a
))/(f*tan(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan\left(fx+e\right)^{2}+a}\,\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

maple [C] time = 1.62, size = 2233, normalized size = 29.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)*(2^(
1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-
b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/
2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/
2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(
f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)
/a^2)^(1/2))*cos(f*x+e)*sin(f*x+e)*a-2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(
1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(
1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+
e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*
(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*
I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)*sin(f*x+e)*
b-2*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(
f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2
)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))
/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1
/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1
/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)*s
in(f*x+e)*a+2*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1
/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*
(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+c
os(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-
2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^
(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*co
s(f*x+e)*sin(f*x+e)*b+2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1
/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos
(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)
-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(
1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b
^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)*a-2^(1/2)*((I*cos(f*x+e)*(
a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+co
s(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1
/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(
f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1
/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+
e)*b-2*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*c
os(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(
1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+
e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)
^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b
^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e
)*a+2*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*co
```

```
s(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1
/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e
))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^
(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^
(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)
*b+cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a-cos(f*x+e)^2*((
2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b+((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/
a)^(1/2)*b)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/sin(f*x+e)/((2*I*(a-b)^(1/2)*
b^(1/2)+a-2*b)/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan\left(fx+e\right)^2 + a} \cot\left(fx+e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^2\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tan^{2}{\left(e + fx \right)}} \cot^{2}{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**2, x)
```

3.304 $\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f+1/3*(
3*a-b)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a/f-1/3*cot(f*x+e)^3*(a+b*tan(f*
x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3670, 475, 583, 12, 377, 203}

$$\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f + ((3*a - b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a*f) - (Cot[e +
 f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} + \frac{\text{Subst}\left(\int \frac{-3a+b-2bx^2}{x^2(1+x^2)\sqrt{a+bx^2}}\,dx, x, \right.}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2}}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2}}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2}}{3f}$$

$$= \frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e}{3af}$$

Mathematica [C] time = 6.98, size = 241, normalized size = 2.06

$$-\frac{\cos^2(e+fx)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(\frac{b\tan^2(e+fx)}{a}+1\right)\left(\frac{\sec^2(e+fx)\left(a-2b\tan^2(e+fx)\right)\left(\sqrt{\frac{(a-b)\sin^2(e+fx)}{a}}\sin^{-1}\left(\sqrt{\right.\right.}{\left(a+b\tan^2(e+fx)\right)\sqrt{\frac{b\sin^2(e+}{a}}}}{3f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/3*(Cos[e + f*x]^2*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2]*(1 + (b*Tan[
e + f*x]^2)/a)*((Sec[e + f*x]^2*(ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*S
qrt[((a - b)*Sin[e + f*x]^2)/a] + Sqrt[Cos[e + f*x]^2 + (b*Sin[e + f*x]^2)/
a])*(a - 2*b*Tan[e + f*x]^2))/(Sqrt[Cos[e + f*x]^2 + (b*Sin[e + f*x]^2)/a]*
(a + b*Tan[e + f*x]^2)) - (4*(a - b)*Hypergeometric2F1[2, 2, 3/2, ((a - b)*
Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2))/f
```

fricas [A] time = 0.61, size = 311, normalized size = 2.66

$$\left[\frac{3\,a\sqrt{-a+b}\,\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)\tan\left(fx+e\right)^4-2\left(3\,a^2-4\,ab\right)\tan\left(fx+e\right)^2+a^2+4\left(\left(a-2\,b\right)\tan\left(fx+e\right)^3-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2+1}\right)}{12\,af\tan\left(fx+e\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a
^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x
+ e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x
+ e)^2 + 1))*tan(f*x + e)^3 + 4*((3*a - b)*tan(f*x + e)^2 - a)*sqrt(b*tan(f
*x + e)^2 + a))/(a*f*tan(f*x + e)^3), 1/6*(3*sqrt(a - b)*a*arctan(-2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a
))*tan(f*x + e)^3 + 2*((3*a - b)*tan(f*x + e)^2 - a)*sqrt(b*tan(f*x + e)^2
+ a))/(a*f*tan(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\tan\left(fx+e\right)^2+a}\,\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

maple [C] time = 1.42, size = 4518, normalized size = 38.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3/f*(-6*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)
+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-
b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(
f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b
)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/
2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f
*x+e)^3*sin(f*x+e)*a^2+6*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)
^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*
cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x
+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)
*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-
(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a
)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a*b+3*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b
^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)
^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*
x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*
I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-
4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^3*sin(f*x
+e)*a^2-3*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2
)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2
```

```
)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+
b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-
b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(
a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b-6*2^(1/2)*((I*cos(f*x
+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/
(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)
*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-
1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*
(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2
*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a^2+6*2^(1/
2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*
cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)
-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)
*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f
*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b
)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^2*sin(f*x+
e)*a*b+3*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a
*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)
^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*
x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a
)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-
8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a^2-3*sin(f*x+e)*EllipticF
((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I
*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))
*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x
+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b
^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)
^(1/2)*cos(f*x+e)^2*a*b+6*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b
)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I
*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*
x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2
)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(
-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/
a)^(1/2))*cos(f*x+e)*sin(f*x+e)*a^2-6*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^
(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^
(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x
+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*
I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2
)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(
1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)*sin(f*x+e)*a*b-3*2^(1/2)*((I*cos(f*x+e)*(a
-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos
(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/
2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f
*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/
2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e
)*sin(f*x+e)*a^2+3*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)
*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*
x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)
/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2
)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1
/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)*sin(f*x+e)*a*b+4*cos(f*x+e)^4
*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^2-5*cos(f*x+e)^4*((2*I*(a-b)^(
1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b+cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2
*b)/a)^(1/2)*b^2+6*a^2*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(
1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*co
s(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e
)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b
^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2
*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^
```

```
(1/2))*sin(f*x+e)-6*b*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1
/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos
(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)
-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^
(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*
I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(
1/2))*a*sin(f*x+e)-3*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/
2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(
f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-
b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1
/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^
(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)*a^2+3*2^(1/2)*((I*cos(f*x+e
)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1
+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b
^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+c
os(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)
^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*b*a*s
in(f*x+e)-3*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a^2+8*co
s(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b-2*cos(f*x+e)^2*((2
*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2-3*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*
b)/a)^(1/2)*a*b+((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^2)*cos(f*x+e)*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/sin(f*x+e)^3/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\,\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^4\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\,\cot^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**4, x)
```

3.305 $\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=167

$$-\frac{(15a^2-5ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^5(e+fx)\sqrt{a+b\tan}}{5f}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f-1/15
*(15*a^2-5*a*b-2*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f+1/15*(5*a-b
)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a/f-1/5*cot(f*x+e)^5*(a+b*tan(f*x+e
)^2)^(1/2)/f
```

Rubi [A] time = 0.23, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3670, 475, 583, 12, 377, 203}

$$-\frac{(15a^2-5ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^5(e+fx)\sqrt{a+b\tan}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) - ((15*a^2 - 5*a*b - 2*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/
(15*a^2*f) + ((5*a - b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a*f)
 - (Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^6\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5f} + \frac{\text{Subst}\left(\int \frac{-5a+b-4bx^2}{x^4\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \tan\right.}{5f}$$

$$= \frac{(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e}}{5f}$$

$$= -\frac{\left(15a^2-5ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} + \frac{(5a-b)\cot^3(e}{}$$

$$= -\frac{\left(15a^2-5ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} + \frac{(5a-b)\cot^3(e}{}$$

$$= -\frac{\left(15a^2-5ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} + \frac{(5a-b)\cot^3(e}{}$$

$$= -\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\left(15a^2-5ab-2b^2\right)\cot(e+fx)\sqrt{a+}}{15a^2f}$$

Mathematica [C] time = 14.39, size = 339, normalized size = 2.03

$$-\frac{\cos^4(e+fx)\cot^5(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)\left(8(a-b)\tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^3 {}_3F_2\left(2,2,2;1,\frac{3}{2};\frac{(a-b)\sin^2(}{a}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/15*(Cos[e + f*x]^4*Cot[e + f*x]^5*(1 + (b*Tan[e + f*x]^2)/a)*(8*(a - b)*
HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e +
f*x]^2*(a + b*Tan[e + f*x]^2)^3 + 8*Hypergeometric2F1[2, 2, 3/2, ((a - b)*S
in[e + f*x]^2)/a]*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2*(-2*a^2 - 3*b^2*T
an[e + f*x]^2 + a*b*(2 + 3*Tan[e + f*x]^2)) + (a^2*Sec[e + f*x]^4*(3*a^2 -
4*a*b*Tan[e + f*x]^2 + 8*b^2*Tan[e + f*x]^4)*(ArcSin[Sqrt[((a - b)*Sin[e +
f*x]^2)/a]]*Sqrt[((a - b)*Sin[e + f*x]^2)/a] + Sqrt[(Cos[e + f*x]^2*(a + b*
Tan[e + f*x]^2))/a]))/Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]))/(a^
3*f*Sqrt[a + b*Tan[e + f*x]^2])
```

fricas [A] time = 0.54, size = 375, normalized size = 2.25

$$\left[\frac{15\,a^2\sqrt{-a+b}\,\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)\tan\left(fx+e\right)^4-2\left(3\,a^2-4\,ab\right)\tan\left(fx+e\right)^2+a^2-4\left(\left(a-2\,b\right)\tan\left(fx+e\right)^3-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-}}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2+1}\right.}{60\,a^2 f\,\mathrm{t}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*a^2*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(
3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f
*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f
*x + e)^2 + 1))*tan(f*x + e)^5 - 4*((15*a^2 - 5*a*b - 2*b^2)*tan(f*x + e)^4
 - (5*a^2 - a*b)*tan(f*x + e)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f
*tan(f*x + e)^5), -1/30*(15*sqrt(a - b)*a^2*arctan(-2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e)
^5 + 2*((15*a^2 - 5*a*b - 2*b^2)*tan(f*x + e)^4 - (5*a^2 - a*b)*tan(f*x + e
)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f*tan(f*x + e)^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\,\cot\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

maple [C] time = 1.36, size = 6894, normalized size = 41.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan\left(fx+e\right)^2+a}\,\cot\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^6\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\,\cot^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**6, x)
```

3.306 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=145

$$\frac{\left(a+b\tan^2(e+fx)\right)^{7/2}}{7b^2f}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}+\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}-$$

[Out] -(a-b)^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+(a-b)*(a+b*tan(f*x+e)^2)^(1/2)/f+1/3*(a+b*tan(f*x+e)^2)^(3/2)/f-1/5*(a+b)*(a+b*tan(f*x+e)^2)^(5/2)/b^2/f+1/7*(a+b*tan(f*x+e)^2)^(7/2)/b^2/f

Rubi [A] time = 0.17, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3670, 446, 88, 50, 63, 208}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{7/2}}{7b^2f}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}+\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}-$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + ((a
- b)*Sqrt[a + b*Tan[e + f*x]^2])/f + (a + b*Tan[e + f*x]^2)^(3/2)/(3*f) - (
(a + b)*(a + b*Tan[e + f*x]^2)^(5/2))/(5*b^2*f) + (a + b*Tan[e + f*x]^2)^(7
/2)/(7*b^2*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^5\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{3/2}}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-b)(a+bx)^{3/2}}{b}+\frac{(a+bx)^{3/2}}{1+x}+\frac{(a+bx)^{5/2}}{b}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\tan^2(e+fx)\right)^{7/2}}{7b^2f}+\frac{\text{Subst}\left(\right.}{}$$

$$= \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\tan\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+b)\left(a+\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+b)\left(a+\right.}{}$$

$$= -\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+$$

Mathematica [A] time = 1.39, size = 139, normalized size = 0.96

$$\frac{\frac{2\left(a+b\tan^2(e+fx)\right)^{7/2}}{7b^2}-\frac{2(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2}+\frac{2}{3}\left(a+b\tan^2(e+fx)\right)^{3/2}+2(a-b)\left(\sqrt{a+b\tan^2(e+fx)}-\sqrt{a-b}\ \text{ta}\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*(a + b*Tan[e + f*x]^2)^(3/2))/3 - (2*(a + b)*(a + b*Tan[e + f*x]^2)^(5/
2))/(5*b^2) + (2*(a + b*Tan[e + f*x]^2)^(7/2))/(7*b^2) + 2*(a - b)*(-(Sqrt[
a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]) + Sqrt[a + b*Tan[e
+ f*x]^2]))/(2*f)
```

fricas [A] time = 0.55, size = 414, normalized size = 2.86

$$\left[-\frac{105\left(ab^2-b^3\right)\sqrt{a-b}\,\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4\,ab-3\,b^2\right)\tan\left(fx+e\right)^2+4\left(b\tan\left(fx+e\right)^2+2\,a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+8\,a^2-8\,ab+b}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/420*(105*(a*b^2 - b^3)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b
- 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e
)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e
)^2 + 1)) - 4*(15*b^3*tan(f*x + e)^6 + 3*(8*a*b^2 - 7*b^3)*tan(f*x + e)^4 -
 6*a^3 - 21*a^2*b + 140*a*b^2 - 105*b^3 + (3*a^2*b - 42*a*b^2 + 35*b^3)*tan
(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/210*(105*(a*b^2 - b^3)*
sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x +
e)^2 + 2*a - b)) + 2*(15*b^3*tan(f*x + e)^6 + 3*(8*a*b^2 - 7*b^3)*tan(f*x +
 e)^4 - 6*a^3 - 21*a^2*b + 140*a*b^2 - 105*b^3 + (3*a^2*b - 42*a*b^2 + 35*b
^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f)]
```

giac [A] time = 0.89, size = 196, normalized size = 1.35

$$\frac{\left(a^2-2\,ab+b^2\right)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}+\frac{15\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{7}{2}}b^{12}f^6-21\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}ab^{12}f^6-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (a^2 - 2*a*b + b^2)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-
a + b)*f) + 1/105*(15*(b*tan(f*x + e)^2 + a)^(7/2)*b^12*f^6 - 21*(b*tan(f*x
 + e)^2 + a)^(5/2)*a*b^12*f^6 - 21*(b*tan(f*x + e)^2 + a)^(5/2)*b^13*f^6 +
35*(b*tan(f*x + e)^2 + a)^(3/2)*b^14*f^6 + 105*sqrt(b*tan(f*x + e)^2 + a)*a
*b^14*f^6 - 105*sqrt(b*tan(f*x + e)^2 + a)*b^15*f^6)/(b^14*f^7)
```

maple [B] time = 0.34, size = 256, normalized size = 1.77

$$\frac{\left(\tan^2\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{5}{2}}}{7fb}-\frac{2a\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{5}{2}}}{35f\,b^2}-\frac{\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{5}{2}}}{5bf}+\frac{b\left(\tan^2\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/7/f*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(5/2)/b-2/35/f*a/b^2*(a+b*tan(f*x+e)^
2)^(5/2)-1/5*(a+b*tan(f*x+e)^2)^(5/2)/b/f+1/3/f*b*tan(f*x+e)^2*(a+b*tan(f*x
+e)^2)^(1/2)+4/3/f*a*(a+b*tan(f*x+e)^2)^(1/2)-b*(a+b*tan(f*x+e)^2)^(1/2)/f+
1/f*b^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))-2/f*a*b/
```

```
(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+1/f*a^2/(-a+b)^(
1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^5, x)
```

mupad [B] time = 40.84, size = 233, normalized size = 1.61

$$\frac{\left(b\tan\left(e+fx\right)^2+a\right)^{7/2}}{7b^2f}-\left(\frac{2a}{5b^2f}-\frac{a-b}{5b^2f}\right)\left(b\tan\left(e+fx\right)^2+a\right)^{5/2}-\sqrt{b\tan\left(e+fx\right)^2+a}\,(a-b)\left(\left(\frac{2a}{b^2f}-\frac{a-}{b^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (a + b*tan(e + f*x)^2)^(7/2)/(7*b^2*f) - ((2*a)/(5*b^2*f) - (a - b)/(5*b^2*
f))*(a + b*tan(e + f*x)^2)^(5/2) - (a + b*tan(e + f*x)^2)^(1/2)*(a - b)*(((
2*a)/(b^2*f) - (a - b)/(b^2*f))*(a - b) - a^2/(b^2*f)) - (a + b*tan(e + f*x
)^2)^(3/2)*((((2*a)/(b^2*f) - (a - b)/(b^2*f))*(a - b))/3 - a^2/(3*b^2*f))
+ (atan(((a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(3/2)*1i)/(a^2 - 2*a*b + b^2)
)*(a - b)^(3/2)*1i)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**5, x)
```

3.307 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5bf}-\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx}}{\sqrt{a-b}}\right.}{f}$$

```
[Out] (a-b)^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f-(a-b)*(a+b*tan(
f*x+e)^2)^(1/2)/f-1/3*(a+b*tan(f*x+e)^2)^(3/2)/f+1/5*(a+b*tan(f*x+e)^2)^(5/
2)/b/f
```

Rubi [A] time = 0.15, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3670, 446, 80, 50, 63, 208}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5bf}-\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx}}{\sqrt{a-b}}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - ((a - b
)*Sqrt[a + b*Tan[e + f*x]^2])/f - (a + b*Tan[e + f*x]^2)^(3/2)/(3*f) + (a +
 b*Tan[e + f*x]^2)^(5/2)/(5*b*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{x^3\left(a+bx^2\right)^{3/2}}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{1+x}dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5bf} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x}dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5bf} - \frac{(a-b)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(a+b\tan\right.}{5}$$

$$= -\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(a+b\tan\right.}{5}$$

$$= \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} - \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\left(\right.}{}$$

Mathematica [A] time = 0.89, size = 112, normalized size = 0.97

$$\frac{\sqrt{a+b\tan^2(e+fx)}\left(3a^2+b(6a-5b)\tan^2(e+fx)-20ab+3b^2\tan^4(e+fx)+15b^2\right)+15b(a-b)^{3/2}\tanh^{-1}\left(\right.}{15bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (15*(a - b)^(3/2)*b*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Sqrt[
a + b*Tan[e + f*x]^2]*(3*a^2 - 20*a*b + 15*b^2 + (6*a - 5*b)*b*Tan[e + f*x]
^2 + 3*b^2*Tan[e + f*x]^4))/(15*b*f)
```

fricas [A] time = 0.53, size = 334, normalized size = 2.88

$$\left[-\frac{15\left(ab-b^2\right)\sqrt{a-b}\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2-4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)}{60bf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a*b - b^2)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*
b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2
+ a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2
+ 1)) - 4*(3*b^2*tan(f*x + e)^4 + (6*a*b - 5*b^2)*tan(f*x + e)^2 + 3*a^2 -
20*a*b + 15*b^2)*sqrt(b*tan(f*x + e)^2 + a))/(b*f), -1/30*(15*(a*b - b^2)*s
qrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e
)^2 + 2*a - b)) - 2*(3*b^2*tan(f*x + e)^4 + (6*a*b - 5*b^2)*tan(f*x + e)^2
+ 3*a^2 - 20*a*b + 15*b^2)*sqrt(b*tan(f*x + e)^2 + a))/(b*f)]
```

giac [A] time = 0.86, size = 150, normalized size = 1.29

$$-\frac{\left(a^2-2ab+b^2\right)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}f}+\frac{3\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}b^4f^4-5\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}b^5f^4-15}{15b^5f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(a^2 - 2*a*b + b^2)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(
-a + b)*f) + 1/15*(3*(b*tan(f*x + e)^2 + a)^(5/2)*b^4*f^4 - 5*(b*tan(f*x +
e)^2 + a)^(3/2)*b^5*f^4 - 15*sqrt(b*tan(f*x + e)^2 + a)*a*b^5*f^4 + 15*sqrt
(b*tan(f*x + e)^2 + a)*b^6*f^4)/(b^5*f^5)
```

maple [B] time = 0.23, size = 204, normalized size = 1.76

$$\frac{\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{5}{2}}}{5bf}-\frac{b\left(\tan^2\left(fx+e\right)\right)\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{3f}-\frac{4a\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{3f}+\frac{b\sqrt{a+b\left(\tan\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/5*(a+b*tan(f*x+e)^2)^(5/2)/b/f-1/3/f*b*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1
/2)-4/3/f*a*(a+b*tan(f*x+e)^2)^(1/2)+b*(a+b*tan(f*x+e)^2)^(1/2)/f-1/f*b^2/(
-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+2/f*a*b/(-a+b)^(1
/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))-1/f*a^2/(-a+b)^(1/2)*arct
an((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^3, x)
```

mupad [B] time = 22.57, size = 156, normalized size = 1.34

$$\frac{\left(b\tan\left(e+fx\right)^2+a\right)^{5/2}}{5\,b\,f}-\left(\frac{a}{3\,b\,f}-\frac{a-b}{3\,b\,f}\right)\left(b\tan\left(e+fx\right)^2+a\right)^{3/2}-\left(\frac{a}{b\,f}-\frac{a-b}{b\,f}\right)\sqrt{b\tan\left(e+fx\right)^2+a}\,\left(a-b\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (a + b*tan(e + f*x)^2)^(5/2)/(5*b*f) - (a/(3*b*f) - (a - b)/(3*b*f))*(a + b
*tan(e + f*x)^2)^(3/2) - (a/(b*f) - (a - b)/(b*f))*(a + b*tan(e + f*x)^2)^(
1/2)*(a - b) - (atan(((a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(3/2)*1i)/(a^2 -
 2*a*b + b^2))*(a - b)^(3/2)*1i)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**3, x)
```

3.308 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=90

$$\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] -(a-b)^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+(a-b)*(a+b*tan
(f*x+e)^2)^(1/2)/f+1/3*(a+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 444, 50, 63, 208}

$$\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + ((a
- b)*Sqrt[a + b*Tan[e + f*x]^2])/f + (a + b*Tan[e + f*x]^2)^(3/2)/(3*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
```

```
x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{x\left(a+bx^2\right)^{3/2}}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x}dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-b)^2\,\text{Sub}}{}$$

$$= \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-b)^2\,\text{Sub}}{}$$

$$= -\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} +$$

Mathematica [A] time = 0.40, size = 80, normalized size = 0.89

$$\frac{\sqrt{a+b\tan^2(e+fx)}\left(4a+b\tan^2(e+fx)-3b\right)-3(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a
+ b*Tan[e + f*x]^2]*(4*a - 3*b + b*Tan[e + f*x]^2))/(3*f)
```

fricas [A] time = 0.59, size = 255, normalized size = 2.83

$$\left[-\frac{3\,(a-b)^{\frac{3}{2}}\log\left(-\frac{b^2\tan(fx+e)^4+2\left(4\,ab-3\,b^2\right)\tan(fx+e)^2+4\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\,\sqrt{a-b}+8\,a^2-8\,ab+b^2}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)-4\left(b\tan\left(f\right.\right.}{12\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a - b)^(3/2)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*
x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
```

```
- b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(
b*tan(f*x + e)^2 + 4*a - 3*b)*sqrt(b*tan(f*x + e)^2 + a))/f, 1/6*(3*(a - b)
*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x +
 e)^2 + 2*a - b)) + 2*(b*tan(f*x + e)^2 + 4*a - 3*b)*sqrt(b*tan(f*x + e)^2
+ a))/f]
```

giac [A] time = 1.00, size = 114, normalized size = 1.27

$$\frac{\left(a^2-2\,ab+b^2\right)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}+\frac{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}f^2+3\,\sqrt{b\tan\left(fx+e\right)^2+a}\,af^2-3\,\sqrt{b\tan(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (a^2 - 2*a*b + b^2)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-
a + b)*f) + 1/3*((b*tan(f*x + e)^2 + a)^(3/2)*f^2 + 3*sqrt(b*tan(f*x + e)^2
 + a)*a*f^2 - 3*sqrt(b*tan(f*x + e)^2 + a)*b*f^2)/f^3
```

maple [B] time = 0.20, size = 181, normalized size = 2.01

$$\frac{b\left(\tan^2\left(fx+e\right)\right)\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{3f}+\frac{4a\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{3f}-\frac{b\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{f}+\frac{b^2\arctan\Big(}{f\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3/f*b*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)+4/3/f*a*(a+b*tan(f*x+e)^2)^(1
/2)-b*(a+b*tan(f*x+e)^2)^(1/2)/f+1/f*b^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e
)^2)^(1/2)/(-a+b)^(1/2))-2/f*a*b/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/
2)/(-a+b)^(1/2))+1/f*a^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e), x)
```

mupad [B] time = 14.99, size = 91, normalized size = 1.01

$$\frac{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}{3\,f}+\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\left(a-b\right)}{f}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\left(a-b\right)^{3/2}}{a^2-2\,a\,b+b^2}\right)\left(a-b\right)^{3/2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (a + b*tan(e + f*x)^2)^(3/2)/(3*f) + ((a + b*tan(e + f*x)^2)^(1/2)*(a - b))
/f - (atanh(((a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(3/2))/(a^2 - 2*a*b + b^2
))*(a - b)^(3/2))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}} \tan\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x), x)
```

3.309 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=95

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{b\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] -a^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/f+(a-b)^(3/2)*arctanh((a
+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+b*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 446, 84, 156, 63, 208}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{b\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/f) + ((a - b)^(3/2)
*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f + (b*Sqrt[a + b*Tan[e +
 f*x]^2])/f
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x\left(1+x^2\right)}dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x(1+x)}dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{b\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{a^2+(2a-b)bx}{x(1+x)\sqrt{a+bx}}dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{b\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}dx, x, \tan^2(e+fx)\right)}{2f} - \frac{(a$$
$$= \frac{b\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx, x, \sqrt{a+b\tan^2(e+fx)}\right.}{bf}$$
$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} +$$

Mathematica [A] time = 0.25, size = 90, normalized size = 0.95

$$\frac{a^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)\right)+b\sqrt{a+b\tan^2(e+fx)}+(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]

[Out] (-(a^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]) + (a - b)^(3/2)*Arc
Tanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + b*Sqrt[a + b*Tan[e + f*x]^2]
)/f
```

fricas [A] time = 1.22, size = 591, normalized size = 6.22

$$\left[-\frac{(a-b)^{\frac{3}{2}}\log\left(-\frac{b^2\tan(fx+e)^4+2\left(4ab-3b^2\right)\tan(fx+e)^2-4\left(b\tan(fx+e)^2+2a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)-2a^{\frac{3}{2}}\log\left(\frac{b\,t}{}\right.}{4f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((a - b)^(3/2)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x +
 e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a -
b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 2*a^(3
/2)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan
(f*x + e)^2) - 4*sqrt(b*tan(f*x + e)^2 + a)*b)/f, 1/4*(4*sqrt(-a)*a*arctan(
sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) - (a - b)^(3/2)*log(-(b^2*tan(f*x +
e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sq
rt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4
 + 2*tan(f*x + e)^2 + 1)) + 4*sqrt(b*tan(f*x + e)^2 + a)*b)/f, 1/2*((-a + b
)^(3/2)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2
+ 2*a - b)) + a^(3/2)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*
sqrt(a) + 2*a)/tan(f*x + e)^2) + 2*sqrt(b*tan(f*x + e)^2 + a)*b)/f, 1/2*(2*
sqrt(-a)*a*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + (-a + b)^(3/2)*a
rctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b
)) + 2*sqrt(b*tan(f*x + e)^2 + a)*b)/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
```

/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^
2-1 were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_

```
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of ab
s or sign assumes constant sign by intervals (correct if the argument is re
al):Check [abs(t_nostep^2-1)]Evaluation time: 3.86Error: Bad Argument Type
```

maple [B] time = 1.55, size = 1765, normalized size = 18.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/4/f*(-1+cos(f*x+e))^3*(2*ln(4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)*(a-b)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+4*a*cos(f*x+e)-4*b*cos(f*x+e))*cos(f*
x+e)*a^(9/2)-4*ln(4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)*(a-b)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(1+cos(f*x+e))^2)^(1/2)+4*a*cos(f*x+e)-4*b*cos(f*x+e))*cos(f*x+e)*a^(7/2)*
b+2*ln(4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)*(a-b)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x
+e))^2)^(1/2)+4*a*cos(f*x+e)-4*b*cos(f*x+e))*cos(f*x+e)*a^(5/2)*b^2+2*cos(f
*x+e)*a^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*(a
-b)^(1/2)*b+2*b*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*
a^(5/2)*(a-b)^(1/2)+ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+
e)^2/a^(1/2))*cos(f*x+e)*(a-b)^(1/2)*a^4-3*ln(-2*(-1+cos(f*x+e))*(((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b
*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)*(a-b)^(1/2)*a^3*b+6*ln(-2*(
-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/
2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)*(a
-b)^(1/2)*a^2*b^2-3*ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+
e)^2/a^(1/2))*cos(f*x+e)*(a-b)^(1/2)*a*b^3-ln(-4*(((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(
-1+cos(f*x+e)))*cos(f*x+e)*(a-b)^(1/2)*a^4+3*ln(-4*(-1+cos(f*x+e))*(((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)
+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)*(a-b)^(1/2)*a^3*b-6*ln(-4
*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(
1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)*
(a-b)^(1/2)*a^2*b^2+3*ln(-4*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*
x+e)^2/a^(1/2))*cos(f*x+e)*(a-b)^(1/2)*a*b^3)*cos(f*x+e)^2*((a*cos(f*x+e)^2
```

```
-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*4^(1/2)/sin(f*x+e)^6/((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)/a^(5/2)/(a-b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e), x)
```

mupad [B] time = 12.03, size = 546, normalized size = 5.75

$$\frac{b\sqrt{b\tan\left(e+fx\right)^2+a}}{f}+\frac{\mathrm{atanh}\left(\frac{6a^3b^3\sqrt{b\tan\left(e+fx\right)^2+a}\,\sqrt{a^3-3a^2b+3ab^2-b^3}}{6a^5b^3-18a^4b^4+20a^3b^5-10a^2b^6+2ab^7}-\frac{6a^2b^4\sqrt{b\tan\left(e+fx\right)^2+a}\,\sqrt{a^3-3a^2b+3ab^2-b^3}}{6a^5b^3-18a^4b^4+20a^3b^5-10a^2b^6+2ab^7}+\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (b*(a + b*tan(e + f*x)^2)^(1/2))/f + (atanh((6*a^3*b^3*(a + b*tan(e + f*x)^
2)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*
a^3*b^5 - 18*a^4*b^4 + 6*a^5*b^3) - (6*a^2*b^4*(a + b*tan(e + f*x)^2)^(1/2)
*(3*a*b^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*b^5
- 18*a^4*b^4 + 6*a^5*b^3) + (2*a*b^5*(a + b*tan(e + f*x)^2)^(1/2)*(3*a*b^2
- 3*a^2*b + a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*b^5 - 18*a^4*b
^4 + 6*a^5*b^3))*((a - b)^3)^(1/2))/f - (atanh((2*b^6*(a + b*tan(e + f*x)^2
)^(1/2)*(a^3)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12*a^4*b^4 - 6*a^5*b^3) - (8*
a*b^5*(a + b*tan(e + f*x)^2)^(1/2)*(a^3)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12
*a^4*b^4 - 6*a^5*b^3) + (12*a^2*b^4*(a + b*tan(e + f*x)^2)^(1/2)*(a^3)^(1/2
))/(2*a^2*b^6 - 8*a^3*b^5 + 12*a^4*b^4 - 6*a^5*b^3) - (6*a^3*b^3*(a + b*tan
(e + f*x)^2)^(1/2)*(a^3)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12*a^4*b^4 - 6*a^5
*b^3))*(a^3)^(1/2))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x), x)
```

3.310 $\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

```
[Out] -(a-b)^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+1/2*(2*a-3*b)*
arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f-1/2*a*cot(f*x+e)^2*(a+b
*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.17, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3670, 446, 98, 156, 63, 208}

$$\frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*f) - (
(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - (a*Cot[e
 + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^3\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= -\frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(2a-3b)+\frac{1}{2}(a-2b)bx}{x(1+x)\sqrt{a+bx}}\,d\right.}{2f} \\
&= -\frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{(a(2a-3b))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.}{4f} \\
&= -\frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{(a(2a-3b))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{2b} \\
&= \frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2f} - \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e}{\sqrt{a-b}}\right.}{f}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 109, normalized size = 0.94

$$\frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)-2(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)-a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] - 2*(a - b
)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] - a*Cot[e + f*x]^2*
Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

fricas [A] time = 0.48, size = 584, normalized size = 5.03

$$\left[-\frac{2\,(a-b)^{\frac{3}{2}}\log\left(\frac{b\tan\left(fx+e\right)^{2}+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{a-b}+2\,a-b}{\tan\left(fx+e\right)^{2}+1}\right)\tan\left(fx+e\right)^{2}+(2\,a-3\,b)\sqrt{a}\,\log\left(\frac{b\tan\left(fx+e\right)^{2}-2\,\sqrt{b\tan(}{\tan(fx+e}\right.}{4\,f\tan\left(fx+e\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a - b)^(3/2)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + (2*a - 3*b)*
sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)
/tan(f*x + e)^2)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*
x + e)^2), -1/4*(4*(a - b)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*
sqrt(-a + b)/(a - b))*tan(f*x + e)^2 + (2*a - 3*b)*sqrt(a)*log((b*tan(f*x +
 e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x
 + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e)^2), -1/2*(sqrt(-a
)*(2*a - 3*b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2
+ (a - b)^(3/2)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
 - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + sqrt(b*tan(f*x + e)
^2 + a)*a)/(f*tan(f*x + e)^2), -1/2*(sqrt(-a)*(2*a - 3*b)*arctan(sqrt(b*tan
(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2 + 2*(a - b)*sqrt(-a + b)*arctan
(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^2 + sqrt(b*
tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
```

2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign as

```
sumes constant sign by intervals (correct if the argument is real):Check [a
bs(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^2-1 were not checked
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by
intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evalua
tion time: 6.44Error: Bad Argument Type
```

maple [B] time = 1.51, size = 2011, normalized size = 17.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/8/f*4^(1/2)*(-1+cos(f*x+e))^2*(2*(a-b)^(1/2)*a^(3/2)*cos(f*x+e)*ln(-2*(-1
+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*co
s(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)
*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))-2*(a-b)^(1/2)*a
^(3/2)*cos(f*x+e)*ln(-4*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e
))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))-2*a^(3/2)
*ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e)
```

```
)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*(a-b)
^(1/2)+2*a^(3/2)*ln(-4*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e)
)^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*(a-b)^(1/2
)-3*(a-b)^(1/2)*a^(1/2)*cos(f*x+e)*ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x
+e)+b)/sin(f*x+e)^2/a^(1/2))*b+3*(a-b)^(1/2)*a^(1/2)*cos(f*x+e)*ln(-4*(((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x
+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*b+3*a^(1/2)*b*ln(-2*(-1+cos(f*x+e))*((
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2
)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(
f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*(a-b)^(1/2)-3*a^(1/2)*ln(-4*((
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2
)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(
f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*b*(a-b)^(1/2)-2*(a-b)^(1/2)*cos(f*x
+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a+4*cos(f*x+
e)*ln(4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)*(a-b)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+
e))^2)^(1/2)+4*a*cos(f*x+e)-4*b*cos(f*x+e))*a^2-8*cos(f*x+e)*ln(4*((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a-b)^(1/2)+4*
(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+4*a*
cos(f*x+e)-4*b*cos(f*x+e))*a*b+4*cos(f*x+e)*ln(4*((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a-b)^(1/2)+4*(a-b)^(1/2)*((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+4*a*cos(f*x+e)-4*b*co
s(f*x+e))*b^2-4*ln(4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)*(a-b)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/(1+cos(f*x+e))^2)^(1/2)+4*a*cos(f*x+e)-4*b*cos(f*x+e))*a^2+8*ln(4*((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a-b)^(1/2)+
4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+4*
a*cos(f*x+e)-4*b*cos(f*x+e))*a*b-4*ln(4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a-b)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+4*a*cos(f*x+e)-4*b*cos(f*x+e))
*b^2)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(3/2)/sin(f*x+e)^6/(a-b
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^3, x)
```

mupad [B] time = 12.03, size = 447, normalized size = 3.85

$$\frac{\operatorname{atanh}\left(\frac{3\,a^2\,b^4\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{2\left(-\frac{3\,a^4\,b^4}{2}+5\,a^3\,b^5-\frac{11\,a^2\,b^6}{2}+2\,a\,b^7\right)}-\frac{2\,a\,b^5\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{-\frac{3\,a^4\,b^4}{2}+5\,a^3\,b^5-\frac{11\,a^2\,b^6}{2}+2\,a\,b^7}\right)\sqrt{\left(a-b\right)^3}}{f}+\frac{\sqrt{a}\,\operatorname{atanh}\left(\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (atanh((3*a^2*b^4*(a + b*tan(e + f*x)^2)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b
^3)^(1/2))/(2*(2*a*b^7 - (11*a^2*b^6)/2 + 5*a^3*b^5 - (3*a^4*b^4)/2)) - (2*
a*b^5*(a + b*tan(e + f*x)^2)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(
2*a*b^7 - (11*a^2*b^6)/2 + 5*a^3*b^5 - (3*a^4*b^4)/2))*((a - b)^3)^(1/2))/f
 + (a^(1/2)*atanh((3*a^(1/2)*b^7*(a + b*tan(e + f*x)^2)^(1/2))/(3*a*b^7 - (
29*a^2*b^6)/4 + (23*a^3*b^5)/4 - (3*a^4*b^4)/2) - (29*a^(3/2)*b^6*(a + b*ta
n(e + f*x)^2)^(1/2))/(4*(3*a*b^7 - (29*a^2*b^6)/4 + (23*a^3*b^5)/4 - (3*a^4
*b^4)/2)) + (23*a^(5/2)*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*(3*a*b^7 - (29
*a^2*b^6)/4 + (23*a^3*b^5)/4 - (3*a^4*b^4)/2)) - (3*a^(7/2)*b^4*(a + b*tan(
e + f*x)^2)^(1/2))/(2*(3*a*b^7 - (29*a^2*b^6)/4 + (23*a^3*b^5)/4 - (3*a^4*b
^4)/2)))*(2*a - 3*b))/(2*f) - (a*b*(a + b*tan(e + f*x)^2)^(1/2))/(2*(f*(a +
 b*tan(e + f*x)^2) - a*f))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}} \cot^3\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x)**3, x)
```

3.311 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(8a^2-12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{a\cot^4(e+fx)\sqrt{a+b\tan^2(e+}}{4f}$$

```
[Out] (a-b)^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f-1/8*(8*a^2-12*a
*b+3*b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)+1/8*(4*a-5*b)
*cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)/f-1/4*a*cot(f*x+e)^4*(a+b*tan(f*x+e)
^2)^(1/2)/f
```

Rubi [A] time = 0.22, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3670, 446, 98, 151, 156, 63, 208}

$$-\frac{\left(8a^2-12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{a\cot^4(e+fx)\sqrt{a+b\tan^2(e+}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((8*a^2 - 12*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8*
Sqrt[a]*f) + ((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]
)/f + ((4*a - 5*b)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(8*f) - (a*Co
t[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^5\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{a\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(4a-5b)+\frac{1}{2}(3a-4b}{x^2(1+x)\sqrt{a+bx}}\right.}{4f}$$

$$= \frac{(4a-5b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} - \frac{a\cot^4(e+fx)\sqrt{a+}}{4f}$$

$$= \frac{(4a-5b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} - \frac{a\cot^4(e+fx)\sqrt{a+}}{4f}$$

$$= \frac{(4a-5b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} - \frac{a\cot^4(e+fx)\sqrt{a+}}{4f}$$

$$= -\frac{\left(8a^2-12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,f} + \frac{(a-b)^{3/2}\tanh^{-1}}{f}$$

Mathematica [A] time = 1.40, size = 140, normalized size = 0.87

$$\frac{\left(-8a^2+12ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\left(8(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\cot^2(e+fx)\sqrt{a+b\tan}\right.}{8\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-8*a^2 + 12*a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] + Sq
rt[a]*(8*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Co
t[e + f*x]^2*(4*a - 5*b - 2*a*Cot[e + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2]))/
(8*Sqrt[a]*f)
```

fricas [A] time = 0.49, size = 748, normalized size = 4.65

$$\left[-\frac{8\left(a^2-ab\right)\sqrt{a-b}\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+2a-b}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^4-\left(8a^2-12ab+3b^2\right)\sqrt{a}\log\left(\frac{b\tan}{}\right.}{16af\tan\left(f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a^2 - a*b)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x
+ e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 - (
8*a^2 - 12*a*b + 3*b^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x +
e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 - 2*((4*a^2 - 5*a*b
)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x + e)^4),
 1/16*(16*(a^2 - a*b)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b)/(a - b))*tan(f*x + e)^4 + (8*a^2 - 12*a*b + 3*b^2)*sqrt(a)*log((b*t
an(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)
*tan(f*x + e)^4 + 2*((4*a^2 - 5*a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x
 + e)^2 + a))/(a*f*tan(f*x + e)^4), 1/8*((8*a^2 - 12*a*b + 3*b^2)*sqrt(-a)*
arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^4 - 4*(a^2 - a*b
)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a -
 b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + ((4*a^2 - 5*a*b)*tan(
f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x + e)^4), 1/8*(
(8*a^2 - 12*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
)/a)*tan(f*x + e)^4 + 8*(a^2 - a*b)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e
)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^4 + ((4*a^2 - 5*a*b)*tan(f*x +
e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x + e)^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
```

```
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
```

```
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant si
gn by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]
Discontinuities at zeroes of t_nostep^2-1 were not checkedUnable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
```

```
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration
 of abs or sign assumes constant sign by intervals (correct if the argument
 is real):Check [abs(t_nostep^2-1)]Evaluation time: 9.53Error: Bad Argument
 Type
```

maple [B] time = 1.40, size = 5224, normalized size = 32.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^5, x)
```

mupad [B] time = 12.25, size = 578, normalized size = 3.59

$$\frac{\sqrt{b\tan\left(e+fx\right)^2+a}\,\left(\frac{3ab^2}{8}-\frac{a^2b}{2}\right)+\frac{b\left(b\tan\left(e+fx\right)^2+a\right)^{3/2}(4a-5b)}{8}}{f\left(b\tan\left(e+fx\right)^2+a\right)^2+a^2f-2af\left(b\tan\left(e+fx\right)^2+a\right)}-\frac{\mathrm{atanh}\left(\frac{9b^6\sqrt{b\tan\left(e+fx\right)^2+a}\,\sqrt{a^3-3a^2b+3ab^2-b^3}}{32\left(\frac{a^3b^5}{4}-\frac{25a^2b^6}{32}+\frac{13ab^7}{16}-\frac{9b^8}{32}\right)}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] ((a + b*tan(e + f*x)^2)^(1/2)*((3*a*b^2)/8 - (a^2*b)/2) + (b*(a + b*tan(e +
 f*x)^2)^(3/2)*(4*a - 5*b))/8)/(f*(a + b*tan(e + f*x)^2)^2 + a^2*f - 2*a*f*
(a + b*tan(e + f*x)^2)) - (atanh((9*b^6*(a + b*tan(e + f*x)^2)^(1/2)*(3*a*b
^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(32*((13*a*b^7)/16 - (9*b^8)/32 - (25*a^2*
b^6)/32 + (a^3*b^5)/4)) - (a*b^5*(a + b*tan(e + f*x)^2)^(1/2)*(3*a*b^2 - 3*
```

```
a^2*b + a^3 - b^3)^(1/2))/(4*((13*a*b^7)/16 - (9*b^8)/32 - (25*a^2*b^6)/32
+ (a^3*b^5)/4)))*((a - b)^3)^(1/2))/f - (atanh((75*a^(1/2)*b^7*(a + b*tan(e
 + f*x)^2)^(1/2))/(64*((75*a*b^7)/64 - (159*b^8)/256 - (29*a^2*b^6)/32 + (a
^3*b^5)/4 + (27*b^9)/(256*a))) - (159*b^8*(a + b*tan(e + f*x)^2)^(1/2))/(25
6*a^(1/2)*((75*a*b^7)/64 - (159*b^8)/256 - (29*a^2*b^6)/32 + (a^3*b^5)/4 +
(27*b^9)/(256*a))) - (29*a^(3/2)*b^6*(a + b*tan(e + f*x)^2)^(1/2))/(32*((75
*a*b^7)/64 - (159*b^8)/256 - (29*a^2*b^6)/32 + (a^3*b^5)/4 + (27*b^9)/(256*
a))) + (a^(5/2)*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*((75*a*b^7)/64 - (159*
b^8)/256 - (29*a^2*b^6)/32 + (a^3*b^5)/4 + (27*b^9)/(256*a))) + (27*b^9*(a
+ b*tan(e + f*x)^2)^(1/2))/(256*a^(3/2)*((75*a*b^7)/64 - (159*b^8)/256 - (2
9*a^2*b^6)/32 + (a^3*b^5)/4 + (27*b^9)/(256*a))))*(8*a^2 - 12*a*b + 3*b^2))
/(8*a^(1/2)*f)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.312 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=294

$$\frac{\left(3a^2-56ab+48b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{192bf}-\frac{\left(3a^3+8a^2b-80ab^2+64b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(}{128b^2f}$$

```
[Out] -(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/12
8*(3*a^4+8*a^3*b+48*a^2*b^2-192*a*b^3+128*b^4)*arctanh(b^(1/2)*tan(f*x+e)/(
a+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f-1/128*(3*a^3+8*a^2*b-80*a*b^2+64*b^3)*(a
+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/192*(3*a^2-56*a*b+48*b^2)*(a+b*ta
n(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f+1/48*(9*a-8*b)*(a+b*tan(f*x+e)^2)^(1/2)*
tan(f*x+e)^5/f+1/8*b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^7/f
```

Rubi [A] time = 0.45, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3670, 477, 582, 523, 217, 206, 377, 203}

$$\frac{\left(3a^2-56ab+48b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{192bf}-\frac{\left(8a^2b+3a^3-80ab^2+64b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(}{128b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) + ((3*a^4 + 8*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*ArcTanh[(Sq
rt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(128*b^(5/2)*f) - ((3*a^3
+ 8*a^2*b - 80*a*b^2 + 64*b^3)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(12
8*b^2*f) + ((3*a^2 - 56*a*b + 48*b^2)*Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x
]^2])/(192*b*f) + ((9*a - 8*b)*Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(
48*f) + (b*Tan[e + f*x]^7*Sqrt[a + b*Tan[e + f*x]^2])/(8*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^7(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{\text{Subst}\left(\int \frac{x^6\left(a(8a-7b)+(9a-8b)\right.}{\left(1+x^2\right)\sqrt{a+bx^2}}\right.}{8f}$$

$$= \frac{(9a-8b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{48f} + \frac{b\tan^7(e+fx)\sqrt{a}}{8f}$$

$$= \frac{\left(3a^2-56ab+48b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{192bf} + \frac{(9a-8}{}$$

$$= -\frac{\left(3a^3+8a^2b-80ab^2+64b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{128b^2f}$$

$$= -\frac{\left(3a^3+8a^2b-80ab^2+64b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{128b^2f}$$

$$= -\frac{\left(3a^3+8a^2b-80ab^2+64b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{128b^2f}$$

$$= -\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{\left(3a^4+8a^3b+48a^2b^2-192\right.}{}$$

Mathematica [C] time = 6.55, size = 908, normalized size = 3.09

$$-\frac{\frac{b\left(3a^4+8ba^3-16b^2a^2-64b^3a+64b^4\right)\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{\cos(2(e+fx))+1}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\csc(2}{a(a+b+(a-b)\cos(2(e+fx)))}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]

[Out] (-((b*(3*a^4 + 8*a^3*b - 16*a^2*b^2 - 64*a*b^3 + 64*b^4)*Sqrt[(a + b + (a -
 b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]
*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b
)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sq
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[
e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e + f*x)]))) - (4*b*(-64*a^2*b^2 + 1
28*a*b^3 - 64*b^4)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(
e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((
a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(
e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a +
b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^
4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
```

```
- (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f
*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[
2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(
e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[
1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a + b
+ (a - b)*Cos[2*(e + f*x)]])/(64*b^2*f) + (Sqrt[(a + b + a*Cos[2*(e + f*x)]
 - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sec[e + f*x]^5*(9*a*Sin[e
+ f*x] - 26*b*Sin[e + f*x]))/48 + (Sec[e + f*x]^3*(3*a^2*Sin[e + f*x] - 128
*a*b*Sin[e + f*x] + 184*b^2*Sin[e + f*x]))/(192*b) + (Sec[e + f*x]*(-9*a^3*
Sin[e + f*x] - 30*a^2*b*Sin[e + f*x] + 424*a*b^2*Sin[e + f*x] - 400*b^3*Sin
[e + f*x]))/(384*b^2) + (b*Sec[e + f*x]^6*Tan[e + f*x])/8))/f
```

fricas [A] time = 4.94, size = 1059, normalized size = 3.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(3*a^4 + 8*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(b)*log(
2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a)
 - 384*(a*b^3 - b^4)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) +
 2*(48*b^4*tan(f*x + e)^7 + 8*(9*a*b^3 - 8*b^4)*tan(f*x + e)^5 + 2*(3*a^2*b
^2 - 56*a*b^3 + 48*b^4)*tan(f*x + e)^3 - 3*(3*a^3*b + 8*a^2*b^2 - 80*a*b^3
+ 64*b^4)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^3*f), -1/384*(3*(3*a
^4 + 8*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(-b)*arctan(sqrt(b*tan
(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + 192*(a*b^3 - b^4)*sqrt(-a + b
)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b
)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (48*b^4*tan(f*x + e)^7 + 8*(9*a
*b^3 - 8*b^4)*tan(f*x + e)^5 + 2*(3*a^2*b^2 - 56*a*b^3 + 48*b^4)*tan(f*x +
e)^3 - 3*(3*a^3*b + 8*a^2*b^2 - 80*a*b^3 + 64*b^4)*tan(f*x + e))*sqrt(b*tan
(f*x + e)^2 + a))/(b^3*f), -1/768*(768*(a*b^3 - b^4)*sqrt(a - b)*arctan(-sq
rt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - 3*(3*a^4 + 8*a^3*b +
 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(
b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 2*(48*b^4*tan(f*x + e)^7
+ 8*(9*a*b^3 - 8*b^4)*tan(f*x + e)^5 + 2*(3*a^2*b^2 - 56*a*b^3 + 48*b^4)*ta
n(f*x + e)^3 - 3*(3*a^3*b + 8*a^2*b^2 - 80*a*b^3 + 64*b^4)*tan(f*x + e))*sq
rt(b*tan(f*x + e)^2 + a))/(b^3*f), -1/384*(384*(a*b^3 - b^4)*sqrt(a - b)*ar
ctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + 3*(3*a^4 + 8
*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(-b)*arctan(sqrt(b*tan(f*x +
 e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - (48*b^4*tan(f*x + e)^7 + 8*(9*a*b^3
 - 8*b^4)*tan(f*x + e)^5 + 2*(3*a^2*b^2 - 56*a*b^3 + 48*b^4)*tan(f*x + e)^3
 - 3*(3*a^3*b + 8*a^2*b^2 - 80*a*b^3 + 64*b^4)*tan(f*x + e))*sqrt(b*tan(f*x
 + e)^2 + a))/(b^3*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \tan \left(fx+e\right)^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

maple [B] time = 0.28, size = 669, normalized size = 2.28

$$\frac{\left(\tan^{3}\left(fx+e\right)\right)\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{\frac{5}{2}}}{8fb}-\frac{a\tan\left(fx+e\right)\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{\frac{5}{2}}}{16fb^{2}}+\frac{a^{2}\tan\left(fx+e\right)\left(a+b\left(\tan^{2}\left(f\right.\right.\right.}{64fb^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/8/f*tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(5/2)/b-1/16/f/b^2*a*tan(f*x+e)*(a+b*
tan(f*x+e)^2)^(5/2)+1/64/f/b^2*a^2*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+3/12
8/f/b^2*a^3*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)+3/128/f/b^(5/2)*a^4*ln(tan(
f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))-1/6/f*tan(f*x+e)*(a+b*tan(f*x+e)^2
)^(5/2)/b+1/24/f/b*a*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+1/16/f/b*a^2*tan(f
*x+e)*(a+b*tan(f*x+e)^2)^(1/2)+1/16/f/b^(3/2)*a^3*ln(tan(f*x+e)*b^(1/2)+(a+
b*tan(f*x+e)^2)^(1/2))+1/4/f*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+3/8/f*a*ta
n(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)+3/8/f*a^2/b^(1/2)*ln(tan(f*x+e)*b^(1/2)+(
a+b*tan(f*x+e)^2)^(1/2))-1/2*b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-3/2/f*
b^(1/2)*a*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))+1/f*b^(3/2)*ln(ta
n(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*(b^4*(a-b))^(1/2)/(a-b)*arct
an((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+2/f*a/b
*(b^4*(a-b))^(1/2)/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)
^2)^(1/2)*tan(f*x+e))-1/f*a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/
(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan\left(fx+e\right)^{2} + a\right)^{\frac{3}{2}} \tan\left(fx+e\right)^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan\left(e+f\,x\right)^6\,\left(b\,\tan\left(e+f\,x\right)^2+a\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tan^{2}{\left(e + fx \right)}\right)^{\frac{3}{2}} \tan^{6}{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**6, x)
```

3.313 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=224

$$\frac{\left(a^2-10ab+8b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf}-\frac{\left(a^3+6a^2b-24ab^2+16b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16b^{3/2}f}+\frac{(a-b)}{}$$

```
[Out] (a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-1/16*
(a^3+6*a^2*b-24*a*b^2+16*b^3)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)
^(1/2))/b^(3/2)/f+1/16*(a^2-10*a*b+8*b^2)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+
e)/b/f+1/24*(7*a-6*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f+1/6*b*(a+b*ta
n(f*x+e)^2)^(1/2)*tan(f*x+e)^5/f
```

Rubi [A] time = 0.35, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3670, 477, 582, 523, 217, 206, 377, 203}

$$\frac{\left(a^2-10ab+8b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf}-\frac{\left(6a^2b+a^3-24ab^2+16b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16b^{3/2}f}+\frac{b\tan^5}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f - ((a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b*Tan[e + f*x]^2]])/(16*b^(3/2)*f) + ((a^2 - 10*a*b + 8*b^2)*Tan[e
 + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(16*b*f) + ((7*a - 6*b)*Tan[e + f*x]^3*
Sqrt[a + b*Tan[e + f*x]^2])/(24*f) + (b*Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f
*x]^2])/(6*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{6f} + \frac{\text{Subst}\left(\int \frac{x^4\left(a(6a-5b)+(7a-6b)bx^2\right)}{\left(1+x^2\right)\sqrt{a+bx^2}}\right.}{6f}$$

$$= \frac{(7a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{24f} + \frac{b\tan^5(e+fx)\sqrt{a+b}}{6f}$$

$$= \frac{\left(a^2-10ab+8b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf} + \frac{(7a-6b)\tan^3}{}$$

$$= \frac{\left(a^2-10ab+8b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf} + \frac{(7a-6b)\tan^3}{}$$

$$= \frac{\left(a^2-10ab+8b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf} + \frac{(7a-6b)\tan^3}{}$$

$$= \frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\left(a^3+6a^2b-24ab^2+16b^3\right)\tanh}{16b^{3/2}f}$$

Mathematica [C] time = 6.44, size = 833, normalized size = 3.72

$$\frac{\sqrt{\frac{\cos(2(e+fx))a+a+b-b\cos(2(e+fx))}{\cos(2(e+fx))+1}}\left(\frac{1}{6}b\tan(e+fx)\sec^4(e+fx)+\frac{7}{24}(a\sin(e+fx)-2b\sin(e+fx))\sec^3(e+fx)+\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*(-((b*(a^3 - 2*a^2*b - 8*a*b^2 + 8*b^3)*Sqrt[(a + b + (a - b)*Cos[2*(e
 + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(
1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e +
 f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b +
 (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/
(a*(a + b + (a - b)*Cos[2*(e + f*x)]))) - (4*b*(-8*a^2*b + 16*a*b^2 - 8*b^3
)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + C
os[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e +
 f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e
+ f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2
*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 +
Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[
e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[(
(a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*Elli
pticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e
+ f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*
```

```
x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]]))/Sqrt[a + b + (a - b)*Cos[2*(
e + f*x)]]])/(b*f) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])
/(1 + Cos[2*(e + f*x)])]*((7*Sec[e + f*x]^3*(a*Sin[e + f*x] - 2*b*Sin[e + f
*x]))/24 + (Sec[e + f*x]*(3*a^2*Sin[e + f*x] - 44*a*b*Sin[e + f*x] + 44*b^2
*Sin[e + f*x]))/(48*b) + (b*Sec[e + f*x]^4*Tan[e + f*x])/6))/f
```

fricas [A] time = 2.99, size = 861, normalized size = 3.84

$$\left[\frac{3\left(a^3+6a^2b-24ab^2+16b^3\right)\sqrt{b}\log\left(2b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{b}\tan\left(fx+e\right)+a\right)-48}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2
 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 48*(a*b^2 - b^3
)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a
)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*(8*b^3*tan(f*x +
 e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3 + 3*(a^2*b - 10*a*b^2 + 8*b^3)*t
an(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/48*(3*(a^3 + 6*a^2*b -
24*a*b^2 + 16*b^3)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*t
an(f*x + e))) - 24*(a*b^2 - b^3)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^
2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
e)^2 + 1)) + (8*b^3*tan(f*x + e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3 + 3
*(a^2*b - 10*a*b^2 + 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*
f), 1/96*(96*(a*b^2 - b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(
sqrt(a - b)*tan(f*x + e))) + 3*(a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*sqrt(b)*
log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e)
+ a) + 2*(8*b^3*tan(f*x + e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3 + 3*(a^
2*b - 10*a*b^2 + 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f),
1/48*(48*(a*b^2 - b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt
(a - b)*tan(f*x + e))) + 3*(a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*sqrt(-b)*arc
tan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + (8*b^3*tan(f*x
+ e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3 + 3*(a^2*b - 10*a*b^2 + 8*b^3)*
tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

maple [B] time = 0.38, size = 510, normalized size = 2.28

$$\frac{\tan\left(fx+e\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{5}{2}}}{6fb}-\frac{a\tan\left(fx+e\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{24fb}-\frac{a^2\tan\left(fx+e\right)\sqrt{a+b\left(\tan^2\left(\right.\right.}}{16fb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/6/f*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(5/2)/b-1/24/f/b*a*tan(f*x+e)*(a+b*tan(
f*x+e)^2)^(3/2)-1/16/f/b*a^2*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)-1/16/f/b^(
3/2)*a^3*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))-1/4/f*tan(f*x+e)*(
a+b*tan(f*x+e)^2)^(3/2)-3/8/f*a*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)-3/8/f*a
^2/b^(1/2)*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))+1/2*b*(a+b*tan(f
*x+e)^2)^(1/2)*tan(f*x+e)/f+3/2/f*b^(1/2)*a*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(
f*x+e)^2)^(1/2))-1/f*b^(3/2)*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2)
)+1/f*(b^4*(a-b))^(1/2)/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f
*x+e)^2)^(1/2)*tan(f*x+e))-2/f*a/b*(b^4*(a-b))^(1/2)/(a-b)*arctan((a-b)*b^2
/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/f*a^2*(b^4*(a-b))
^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2
)*tan(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \tan\left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan\left(e+f\,x\right)^4\,{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tan^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}} \tan^{4}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**4, x)
```

3.314 $\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{\left(3a^2-12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f}+\frac{(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}-\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-}}{\sqrt{a+}}\right.}{f}$$

```
[Out] -(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/8*
(3*a^2-12*a*b+8*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f
/b^(1/2)+1/8*(5*a-4*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+1/4*b*(a+b*tan
(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A] time = 0.25, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3670, 477, 582, 523, 217, 206, 377, 203}

$$\frac{\left(3a^2-12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f}+\frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}+\frac{(5a-4b)\tan(e+fx)\sqrt{a+}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) + ((3*a^2 - 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b*Tan[e + f*x]^2]])/(8*Sqrt[b]*f) + ((5*a - 4*b)*Tan[e + f*x]*Sqrt[a + b*T
an[e + f*x]^2])/(8*f) + (b*Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
```

```
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2\left(a(4a-3b)+(5a-4b)bx^2\right)}{\left(1+x^2\right)\sqrt{a+bx^2}}\right.}{4f}$$

$$= \frac{(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b\,\mathrm{t}}}{4f}$$

$$= \frac{(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b\,\mathrm{t}}}{4f}$$

$$= \frac{(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b\,\mathrm{t}}}{4f}$$

$$= -\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{\left(3a^2-12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{8\sqrt{b}\,f}$$

Mathematica [C] time = 6.31, size = 771, normalized size = 4.48

$$\frac{b\left(a^2+4ab-4b^2\right)\sin^4(e+fx)\csc(2(e+fx))\sqrt{\frac{(a-b)\cos(2(e+fx))+a+b}{\cos(2(e+fx))+1}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}F\Big(}{a((a-b)\cos(2(e+fx))+a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((b*(a^2 + 4*a*b - 4*b^2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[
2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)
])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x
]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e +
 f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)
*Cos[2*(e + f*x)])) + (4*b*(4*a^2 - 8*a*b + 4*b^2)*Sqrt[1 + Cos[2*(e + f*x)
]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-
((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)
]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*
x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^
2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a +
 b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*
(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e
+ f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSi
n[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*
Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*
Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/(4*f) + (Sqrt[
(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(
(Sec[e + f*x]*(5*a*Sin[e + f*x] - 6*b*Sin[e + f*x]))/8 + (b*Sec[e + f*x]^2*
Tan[e + f*x])/4))/f
```

fricas [A] time = 1.24, size = 708, normalized size = 4.12

$$\left[\frac{\left(3\,a^2-12\,ab+8\,b^2\right)\sqrt{b}\,\log\left(2\,b\tan\left(fx+e\right)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{b}\,\tan\left(fx+e\right)+a\right)-8\left(ab-b^2\right)\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*a^2 - 12*a*b + 8*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*t
an(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 8*(a*b - b^2)*sqrt(-a + b)*l
og(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*t
an(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*(2*b^2*tan(f*x + e)^3 + (5*a*b -
 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b*f), -1/8*((3*a^2 - 12*
a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x
 + e))) + 4*(a*b - b^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqr
t(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)
) - (2*b^2*tan(f*x + e)^3 + (5*a*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x +
e)^2 + a))/(b*f), -1/16*(16*(a*b - b^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x
+ e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a^2 - 12*a*b + 8*b^2)*sqrt(b)*
log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e)
```

```
+ a) - 2*(2*b^2*tan(f*x + e)^3 + (5*a*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f
*x + e)^2 + a))/(b*f), -1/8*(8*(a*b - b^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f
*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (3*a^2 - 12*a*b + 8*b^2)*sqrt(
-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - (2*b^2*t
an(f*x + e)^3 + (5*a*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(
b*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

maple [B] time = 0.31, size = 386, normalized size = 2.24

$$\frac{\tan\left(fx+e\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{4f}+\frac{3a\tan\left(fx+e\right)\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{8f}+\frac{3a^2\ln\left(\tan\left(fx+e\right)\sqrt{b}+\sqrt{a+\right.}{8f\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4/f*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+3/8/f*a*tan(f*x+e)*(a+b*tan(f*x+e
)^2)^(1/2)+3/8/f*a^2/b^(1/2)*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2)
)-1/2*b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-3/2/f*b^(1/2)*a*ln(tan(f*x+e)
*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))+1/f*b^(3/2)*ln(tan(f*x+e)*b^(1/2)+(a+b*t
an(f*x+e)^2)^(1/2))-1/f*(b^4*(a-b))^(1/2)/(a-b)*arctan((a-b)*b^2/(b^4*(a-b)
)^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+2/f*a/b*(b^4*(a-b))^(1/2)/(a-b
)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1
/f*a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*
tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^2\,\left(b\,\tan\left(e+f\,x\right)^2+a\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**2, x)
```

3.315 $\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=125

$$\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}$$

```
[Out] (a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/2*(
3*a-2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2
*b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3661, 416, 523, 217, 206, 377, 203}

$$\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{\sqrt{b}\,(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f + ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a(2a-b)+(3a-2b)bx^2}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\tan(e+$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\tan($$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx,x,\frac{\tan(e}{\sqrt{a+b\tan}}$$

$$= \frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\,\mathrm{t}}{}$$

Mathematica [C] time = 1.43, size = 233, normalized size = 1.86

$$\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)} - i(a-b)^{3/2}\log\left(-\frac{4i\left(\sqrt{a-b}\,\sqrt{a+b\tan^2(e+fx)}+a-ib\tan(e+fx)\right)}{(a-b)^{5/2}(\tan(e+fx)+i)}\right) + i(a-b)^{3/2}\log\left(\frac{4i(}{}\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-I)*(a - b)^(3/2)*Log[((-4*I)*(a - I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a
+ b*Tan[e + f*x]^2]))/((a - b)^(5/2)*(I + Tan[e + f*x]))] + I*(a - b)^(3/2)
*Log[((4*I)*(a + I*b*Tan[e + f*x] + Sqrt[a - b]*Sqrt[a + b*Tan[e + f*x]^2])
)/((a - b)^(5/2)*(-I + Tan[e + f*x]))] + (3*a - 2*b)*Sqrt[b]*Log[b*Tan[e +
f*x] + Sqrt[b]*Sqrt[a + b*Tan[e + f*x]^2]] + b*Tan[e + f*x]*Sqrt[a + b*Tan[
e + f*x]^2])/(2*f)
```

fricas [A] time = 0.93, size = 537, normalized size = 4.30

$$\left[-\frac{(3a-2b)\sqrt{b}\log\left(2b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{b}\tan\left(fx+e\right)+a\right)+2(a-b)\sqrt{-a+b}\log\left(-\cdots\right.}{4f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(a - b)*sqrt(-a + b)*log(-((a - 2*b)*ta
n(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/
(tan(f*x + e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, -1/
2*((3*a - 2*b)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f
*x + e))) - (-a + b)^(3/2)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - sqrt(b
*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/4*(4*(a - b)^(3/2)*arctan(-sqrt(b
*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a - 2*b)*sqrt(b)*log(
2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a)
 + 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/2*(2*(a - b)^(3/2)*arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a - 2*b)*s
qrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + sqrt
(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2), x)
```

maple [B] time = 0.31, size = 297, normalized size = 2.38

$$\frac{b\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\,\tan\left(fx+e\right)}{2f}+\frac{3\sqrt{b}\,a\ln\left(\tan\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\right)}{2f}-\frac{b^{\frac{3}{2}}\ln\left(\tan\left(fx+\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2*b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+3/2/f*b^(1/2)*a*ln(tan(f*x+e)*b
^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*b^(3/2)*ln(tan(f*x+e)*b^(1/2)+(a+b*tan
(f*x+e)^2)^(1/2))+1/f*(b^4*(a-b))^(1/2)/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^
(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-2/f*a/b*(b^4*(a-b))^(1/2)/(a-b)*
arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/f
*a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*ta
n(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int((a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2), x)
```

3.316 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=114

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

```
[Out] -(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+b^(3
/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-a*cot(f*x+e)*(a+
b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.14, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3670, 474, 523, 217, 206, 377, 203}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2
]])/f - (a*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
```

```
1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{-a(a-2b)+b^2x^2}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x\right.}{f}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+}}\right.}{f}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\right.}{f}$$

$$= -\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

Mathematica [C] time = 6.24, size = 724, normalized size = 6.35

$$\frac{b\left(a^2-2ab-b^2\right)\sin^4(e+fx)\csc(2(e+fx))\sqrt{\frac{(a-b)\cos(2(e+fx))+a+b}{\cos(2(e+fx))+1}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{}}{af((a-b)\cos(2(e+fx))+a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e +
 f*x)])]*Cot[e + f*x])/f) + (b*(a^2 - 2*a*b - b^2)*Sqrt[(a + b + (a - b)*Co
s[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[
-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[
```

```
2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a
 + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*
x]^4)/(a*f*(a + b + (a - b)*Cos[2*(e + f*x)])) + (4*b*(a^2 - 2*a*b + b^2)*S
qrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[
2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*
x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f
*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e
 + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos
[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e +
 f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a
+ b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*Ellipti
cPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f
*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]
]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/(f*Sqrt[a + b + (a - b)*Cos[2*(
e + f*x)]])
```

fricas [A] time = 1.28, size = 710, normalized size = 6.23

$$\left[\frac{2\,b^{\frac{3}{2}}\log\left(2\,b\tan\left(fx+e\right)^{2}+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{b}\,\tan\left(fx+e\right)+a\right)\tan\left(fx+e\right)-(a-b)\sqrt{-a+b}\,\log\left(-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b^(3/2)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
b)*tan(f*x + e) + a)*tan(f*x + e) - (a - b)*sqrt(-a + b)*log(-((a^2 - 8*a*b
 + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a -
 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
 b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))*tan(f*x + e) - 4*sqrt(b*tan(f
*x + e)^2 + a)*a)/(f*tan(f*x + e)), -1/4*(4*sqrt(-b)*b*arctan(sqrt(-b)*tan(
f*x + e)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e) + (a - b)*sqrt(-a + b)*lo
g(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2
 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^
2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))*tan(f*x + e)
+ 4*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e)), -1/2*((a - b)^(3/2)*arc
tan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f
*x + e)^2 - a))*tan(f*x + e) - b^(3/2)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*ta
n(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a)*tan(f*x + e) + 2*sqrt(b*tan(f*x
 + e)^2 + a)*a)/(f*tan(f*x + e)), -1/2*((a - b)^(3/2)*arctan(-2*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*ta
n(f*x + e) + 2*sqrt(-b)*b*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^
2 + a))*tan(f*x + e) + 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

maple [C] time = 1.55, size = 3333, normalized size = 29.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f*(2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*co
s(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1
/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e
))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(
1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a
*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)*sin(f*x+e)*a^2-2*2^(1/2)*((I*cos(f*x+e)*(a
-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos
(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/
2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f
*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/
2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e
)*sin(f*x+e)*a*b-2*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)
*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*
x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)
/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/
2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(
a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2
))*cos(f*x+e)*sin(f*x+e)*a^2+4*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I
*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(
-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*c
os(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)
^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b
)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-
2*b)/a)^(1/2))*cos(f*x+e)*sin(f*x+e)*a*b-2*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/
2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e)
)/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*co
s(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))
*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b
^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2
)*b^(1/2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)*cos(f*x+e)+2*2^(1/2)*((I*cos(f*x+
e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(
1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*
b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1
+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*(a
-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I
*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)*cos(f*x+e)+2^(1/2)*((I
*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*
x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-
b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*Ellip
ticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),(
(8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1
/2))*sin(f*x+e)*a^2-2*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1
/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos
(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)
-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(
1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b
^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*b*a*sin(f*x+e)-2*a^2*2^(1/2)*((I*cos(
f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+
b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1
/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi
((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2
*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/
((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)+4*b*2^(1/2)*((I*cos(f
*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b
)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/
2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi(
```

```
(-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*
I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/(
(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*a*sin(f*x+e)-2*2^(1/2)*((I*cos(f*
x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)
/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2
)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((
-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I
*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((
2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)+2*2^(1/2)*((I*cos(f
*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b
)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/
2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi(
(-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I
*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((
2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)+((2*I*(a-b)^(1/2)*b
^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a^2-((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)
^(1/2)*cos(f*x+e)^2*a*b+((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b)*cos(
f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)/
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^2\,\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x)**2, x)
```

3.317 $\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=115

$$\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}+\frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+f}}{3f}$$

```
[Out] (a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/3*(
3*a-4*b)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f-1/3*a*cot(f*x+e)^3*(a+b*tan(
f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3670, 474, 583, 12, 377, 203}

$$\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}+\frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+f}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f + ((3*a - 4*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*f) - (a*Cot
[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^4\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} + \frac{\text{Subst}\left(\int \frac{-a(3a-4b)-(2a-3b)bx^2}{x^2\left(1+x^2\right)\sqrt{a+bx^2}}\right.}{3f}$$

$$= \frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} - \frac{a\cot^3(e+fx)\sqrt{a+b\,\mathrm{ta}}}{3f}$$

$$= \frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} - \frac{a\cot^3(e+fx)\sqrt{a+b\,\mathrm{ta}}}{3f}$$

$$= \frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} - \frac{a\cot^3(e+fx)\sqrt{a+b\,\mathrm{ta}}}{3f}$$

$$= \frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2}}{3f}$$

Mathematica [C] time = 0.34, size = 78, normalized size = 0.68

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(a\cot^2(e+fx)+b\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{(a-b)\tan^2(e+fx)}{b\tan^2(e+fx)+a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*(Cot[e + f*x]*(b + a*Cot[e + f*x]^2)*Hypergeometric2F1[-3/2, 1, -1/2,
-(((a - b)*Tan[e + f*x]^2)/(a + b*Tan[e + f*x]^2))]*Sqrt[a + b*Tan[e + f*x]
^2])/f
```

fricas [A] time = 0.55, size = 308, normalized size = 2.68

$$\left[-\frac{3\,(a-b)\sqrt{-a+b}\,\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)\tan\left(fx+e\right)^4-2\left(3\,a^2-4\,ab\right)\tan\left(fx+e\right)^2+a^2-4\left((a-2\,b)\tan\left(fx+e\right)^3-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^2}}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2+1}\right.}{12\,f\tan\left(fx+e\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a - b)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 -
 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*t
an(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*t
an(f*x + e)^2 + 1))*tan(f*x + e)^3 - 4*((3*a - 4*b)*tan(f*x + e)^2 - a)*sqr
t(b*tan(f*x + e)^2 + a))/(f*tan(f*x + e)^3), 1/6*(3*(a - b)^(3/2)*arctan(-2
*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e
)^2 - a))*tan(f*x + e)^3 + 2*((3*a - 4*b)*tan(f*x + e)^2 - a)*sqrt(b*tan(f*
x + e)^2 + a))/(f*tan(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

maple [C] time = 1.35, size = 6591, normalized size = 57.31

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^4\,\left(b\,\tan\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}}\cot^4\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x)**4, x)
```

3.318 $\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=165

$$-\frac{\left(15a^2-20ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af}-\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot^5(e+fx)\sqrt{a+}}{5f}$$

```
[Out] -(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-1/15
*(15*a^2-20*a*b+3*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a/f+1/15*(5*a-6*
b)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/f-1/5*a*cot(f*x+e)^5*(a+b*tan(f*x+
e)^2)^(1/2)/f
```

Rubi [A] time = 0.24, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3670, 474, 583, 12, 377, 203}

$$-\frac{\left(15a^2-20ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af}-\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot^5(e+fx)\sqrt{a+}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) - ((15*a^2 - 20*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2
])/(15*a*f) + ((5*a - 6*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*f
) - (a*Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^6\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5f} + \frac{\text{Subst}\left(\int \frac{-a(5a-6b)-(4a-5b)bx^2}{x^4\left(1+x^2\right)\sqrt{a+bx^2}}\,d\right.}{5f}$$

$$= \frac{(5a-6b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15f} - \frac{a\cot^5(e+fx)\sqrt{a+b\,\mathrm{t}}}{5f}$$

$$= -\frac{\left(15a^2-20ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af} + \frac{(5a-6b)\,\mathrm{c}}{}$$

$$= -\frac{\left(15a^2-20ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af} + \frac{(5a-6b)\,\mathrm{c}}{}$$

$$= -\frac{\left(15a^2-20ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af} + \frac{(5a-6b)\,\mathrm{c}}{}$$

$$= -\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\left(15a^2-20ab+3b^2\right)\cot(e+fx}{15af}$$

Mathematica [C] time = 9.50, size = 140, normalized size = 0.85

$$-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(a\cot^2(e+fx)+b\right)^2\left(2(a-b)((a-b)\cos(2(e+fx))+a+b)\,{}_2F_1\right(}{15a^3f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*(Cos[e + f*x]*(b + a*Cot[e + f*x]^2)^2*(a*(-2*b + 3*a*Cot[e + f*x]^2)
*Hypergeometric2F1[1, 1, -1/2, ((a - b)*Sin[e + f*x]^2)/a] + 2*(a - b)*(a +
```

```
b + (a - b)*Cos[2*(e + f*x)])*Hypergeometric2F1[2, 2, 1/2, ((a - b)*Sin[e
+ f*x]^2)/a])*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a^3*f)
```

fricas [A] time = 0.57, size = 385, normalized size = 2.33

$$\left[-\frac{15\left(a^{2}-ab\right)\sqrt{-a+b}\,\log\left(-\frac{\left(a^{2}-8\,ab+8\,b^{2}\right)\tan\left(fx+e\right)^{4}-2\left(3\,a^{2}-4\,ab\right)\tan\left(fx+e\right)^{2}+a^{2}+4\left(\left(a-2\,b\right)\tan\left(fx+e\right)^{3}-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(f\right.}}{\tan\left(fx+e\right)^{4}+2\,\tan\left(fx+e\right)^{2}+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a^2 - a*b)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e
)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3
- a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4
+ 2*tan(f*x + e)^2 + 1))*tan(f*x + e)^5 + 4*((15*a^2 - 20*a*b + 3*b^2)*tan(
f*x + e)^4 - (5*a^2 - 6*a*b)*tan(f*x + e)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2
+ a))/(a*f*tan(f*x + e)^5), -1/30*(15*(a^2 - a*b)*sqrt(a - b)*arctan(-2*sqr
t(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2
- a))*tan(f*x + e)^5 + 2*((15*a^2 - 20*a*b + 3*b^2)*tan(f*x + e)^4 - (5*a^2
 - 6*a*b)*tan(f*x + e)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x
+ e)^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

maple [C] time = 1.41, size = 10026, normalized size = 60.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}\left(e+f\,x\right)^6\,{\left(b\,\mathrm{tan}\left(e+f\,x\right)^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.319 $\int \left(a+b\tan^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=170

$$\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{8d}+\frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right)^{3/2}}{4d}+\frac{b(7a-4b)\tan(c+dx}{8}$$

```
[Out] (a-b)^(5/2)*arctan((a-b)^(1/2)*tan(d*x+c)/(a+b*tan(d*x+c)^2)^(1/2))/d+1/8*(
15*a^2-20*a*b+8*b^2)*arctanh(b^(1/2)*tan(d*x+c)/(a+b*tan(d*x+c)^2)^(1/2))*b
^(1/2)/d+1/8*(7*a-4*b)*b*(a+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*b*tan(d*
x+c)*(a+b*tan(d*x+c)^2)^(3/2)/d
```

Rubi [A] time = 0.18, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3661, 416, 528, 523, 217, 206, 377, 203}

$$\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{8d}+\frac{(a-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{d}+\frac{b\tan(c+dx)\left(a+b\tan^2(\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] ((a - b)^(5/2)*ArcTan[(Sqrt[a - b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^2]
])/d + (Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[c + d*x])/Sq
rt[a + b*Tan[c + d*x]^2]])/(8*d) + ((7*a - 4*b)*b*Tan[c + d*x]*Sqrt[a + b*T
an[c + d*x]^2])/(8*d) + (b*Tan[c + d*x]*(a + b*Tan[c + d*x]^2)^(3/2))/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
```

```
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(c+dx)\right)^{5/2}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{5/2}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}$$

$$=\frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right)^{3/2}}{4d}+\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}\left(a(4a-b)+(7a-4b)bx^2\right)}{1+x^2}\,dx,x,\text{ta}\right.}{4d}$$

$$=\frac{(7a-4b)b\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}}{8d}+\frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right)^{3/}}{4d}$$

$$=\frac{(7a-4b)b\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}}{8d}+\frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right)^{3/}}{4d}$$

$$=\frac{(7a-4b)b\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}}{8d}+\frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right)^{3/}}{4d}$$

$$=\frac{(a-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{d}+\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx}}\right.}{8d}$$

Mathematica [C] time = 1.34, size = 259, normalized size = 1.52

$$\sqrt{b}\left(15a^2-20ab+8b^2\right)\log\left(\sqrt{b}\sqrt{a+b\tan^2(c+dx)}+b\tan(c+dx)\right)+b\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}\left(9a+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] ((-4*I)*(a - b)^(5/2)*Log[((-4*I)*(a - I*b*Tan[c + d*x] + Sqrt[a - b]*Sqrt[
a + b*Tan[c + d*x]^2]))/((a - b)^(7/2)*(I + Tan[c + d*x]))] + (4*I)*(a - b)
^(5/2)*Log[((4*I)*(a + I*b*Tan[c + d*x] + Sqrt[a - b]*Sqrt[a + b*Tan[c + d*
x]^2]))/((a - b)^(7/2)*(-I + Tan[c + d*x]))] + Sqrt[b]*(15*a^2 - 20*a*b + 8
*b^2)*Log[b*Tan[c + d*x] + Sqrt[b]*Sqrt[a + b*Tan[c + d*x]^2]] + b*Tan[c +
d*x]*Sqrt[a + b*Tan[c + d*x]^2]*(9*a - 4*b + 2*b*Tan[c + d*x]^2))/(8*d)
```

fricas [A] time = 2.22, size = 703, normalized size = 4.14

$$\left[\frac{\left(15\,a^2 - 20\,ab + 8\,b^2\right)\sqrt{b}\,\log\left(2\,b\tan\left(dx+c\right)^2 + 2\,\sqrt{b\tan\left(dx+c\right)^2 + a}\,\sqrt{b}\,\tan\left(dx+c\right) + a\right) + 8\left(a^2 - 2\,ab +\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*((15*a^2 - 20*a*b + 8*b^2)*sqrt(b)*log(2*b*tan(d*x + c)^2 + 2*sqrt(b*
tan(d*x + c)^2 + a)*sqrt(b)*tan(d*x + c) + a) + 8*(a^2 - 2*a*b + b^2)*sqrt(
-a + b)*log(-((a - 2*b)*tan(d*x + c)^2 + 2*sqrt(b*tan(d*x + c)^2 + a)*sqrt(
-a + b)*tan(d*x + c) - a)/(tan(d*x + c)^2 + 1)) + 2*(2*b^2*tan(d*x + c)^3 +
 (9*a*b - 4*b^2)*tan(d*x + c))*sqrt(b*tan(d*x + c)^2 + a))/d, 1/16*(16*(a^2
 - 2*a*b + b^2)*sqrt(a - b)*arctan(-sqrt(b*tan(d*x + c)^2 + a)/(sqrt(a - b)
*tan(d*x + c))) + (15*a^2 - 20*a*b + 8*b^2)*sqrt(b)*log(2*b*tan(d*x + c)^2
+ 2*sqrt(b*tan(d*x + c)^2 + a)*sqrt(b)*tan(d*x + c) + a) + 2*(2*b^2*tan(d*x
 + c)^3 + (9*a*b - 4*b^2)*tan(d*x + c))*sqrt(b*tan(d*x + c)^2 + a))/d, -1/8
*((15*a^2 - 20*a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(d*x + c)^2 + a)*sqrt
(-b)/(b*tan(d*x + c))) - 4*(a^2 - 2*a*b + b^2)*sqrt(-a + b)*log(-((a - 2*b)
*tan(d*x + c)^2 + 2*sqrt(b*tan(d*x + c)^2 + a)*sqrt(-a + b)*tan(d*x + c) -
a)/(tan(d*x + c)^2 + 1)) - (2*b^2*tan(d*x + c)^3 + (9*a*b - 4*b^2)*tan(d*x
+ c))*sqrt(b*tan(d*x + c)^2 + a))/d, 1/8*(8*(a^2 - 2*a*b + b^2)*sqrt(a - b)
*arctan(-sqrt(b*tan(d*x + c)^2 + a)/(sqrt(a - b)*tan(d*x + c))) - (15*a^2 -
 20*a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(d*x + c)^2 + a)*sqrt(-b)/(b*tan
(d*x + c))) + (2*b^2*tan(d*x + c)^3 + (9*a*b - 4*b^2)*tan(d*x + c))*sqrt(b*
tan(d*x + c)^2 + a))/d]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.57, size = 461, normalized size = 2.71

$$\frac{b^2\left(\tan^3\left(dx+c\right)\right)\sqrt{a+b\left(\tan^2\left(dx+c\right)\right)}}{4d}+\frac{9ba\tan\left(dx+c\right)\sqrt{a+b\left(\tan^2\left(dx+c\right)\right)}}{8d}+\frac{15\sqrt{b}\,a^2\ln\left(\sqrt{b}\,\tan\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^2)^(5/2),x)
```

```
[Out] 1/4/d*b^2*tan(d*x+c)^3*(a+b*tan(d*x+c)^2)^(1/2)+9/8/d*b*a*tan(d*x+c)*(a+b*t
an(d*x+c)^2)^(1/2)+15/8/d*b^(1/2)*a^2*ln(b^(1/2)*tan(d*x+c)+(a+b*tan(d*x+c)
^2)^(1/2))-1/2/d*b^2*tan(d*x+c)*(a+b*tan(d*x+c)^2)^(1/2)-5/2/d*b^(3/2)*a*ln
(b^(1/2)*tan(d*x+c)+(a+b*tan(d*x+c)^2)^(1/2))+1/d*b^(5/2)*ln(b^(1/2)*tan(d*
x+c)+(a+b*tan(d*x+c)^2)^(1/2))-1/d*b*(b^4*(a-b))^(1/2)/(a-b)*arctan((a-b)*b
^2/(b^4*(a-b))^(1/2)/(a+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c))+3/d*a*(b^4*(a-b))
^(1/2)/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(d*x+c)^2)^(1/2)*ta
n(d*x+c))-3/d*a^2/b*(b^4*(a-b))^(1/2)/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1
/2)/(a+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c))+1/d*a^3*(b^4*(a-b))^(1/2)/b^2/(a-b
)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan \left(dx+c\right)^2+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^2+a\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^(5/2),x)
```

```
[Out] int((a + b*tan(c + d*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tan^{2}{\left(c + dx \right)}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**(5/2), x)
```

3.320 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=95

$$\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}-\frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)-(a+b)*(a+b*tan
(f*x+e)^2)^(1/2)/b^2/f+1/3*(a+b*tan(f*x+e)^2)^(3/2)/b^2/f
```

Rubi [A] time = 0.14, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3670, 446, 88, 63, 208}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}-\frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f)) - ((a +
b)*Sqrt[a + b*Tan[e + f*x]^2])/(b^2*f) + (a + b*Tan[e + f*x]^2)^(3/2)/(3*b^
2*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
```

```
x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{-a-b}{b\sqrt{a+bx}} + \frac{1}{(1+x)\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b}\right) dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x\right.}{2f}$$
$$= -\frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \right.}{bf}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f} - \frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}$$

Mathematica [A] time = 2.45, size = 87, normalized size = 0.92

$$-\frac{\frac{2\left(2a-b\tan^2(e+fx)+3b\right)\sqrt{a+b\tan^2(e+fx)}}{3b^2} + \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/2*((2*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a - b] + (2*
(2*a + 3*b - b*Tan[e + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2])/(3*b^2))/f
```

fricas [A] time = 0.55, size = 314, normalized size = 3.31

$$\left[\frac{3\sqrt{a-b}\,b^2\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2-4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+8a^2-8ab+b^2}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)+4\left(\left(ab-b\right.\right.}{12\left(ab^2-b^3\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(a - b)*b^2*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f
*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*
((a*b - b^2)*tan(f*x + e)^2 - 2*a^2 - a*b + 3*b^2)*sqrt(b*tan(f*x + e)^2 +
a))/((a*b^2 - b^3)*f), 1/6*(3*sqrt(-a + b)*b^2*arctan(2*sqrt(b*tan(f*x + e)
^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2*((a*b - b^2)*tan(f*x
 + e)^2 - 2*a^2 - a*b + 3*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a*b^2 - b^3)*f
)]
```

giac [A] time = 2.00, size = 114, normalized size = 1.20

$$\frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}+\frac{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}b^{4}f^{2}-3\sqrt{b\tan\left(fx+e\right)^{2}+a}\,ab^{4}f^{2}-3\sqrt{b\tan\left(fx+e\right)^{2}+a}}{3\,b^{6}f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f) + 1/3*((b*
tan(f*x + e)^2 + a)^(3/2)*b^4*f^2 - 3*sqrt(b*tan(f*x + e)^2 + a)*a*b^4*f^2
- 3*sqrt(b*tan(f*x + e)^2 + a)*b^5*f^2)/(b^6*f^3)
```

maple [A] time = 0.35, size = 111, normalized size = 1.17

$$\frac{\left(\tan^{2}\left(fx+e\right)\right)\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}{3fb}-\frac{2a\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}{3f\,b^{2}}-\frac{\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}{bf}+\frac{\arctan\left(\frac{\sqrt{a+b(\tan}}{\sqrt{}}\right)}{f\sqrt{-a+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f*tan(f*x+e)^2/b*(a+b*tan(f*x+e)^2)^(1/2)-2/3/f*a/b^2*(a+b*tan(f*x+e)^2
)^(1/2)-(a+b*tan(f*x+e)^2)^(1/2)/b/f+1/f/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e
)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 12.88, size = 97, normalized size = 1.02

$$\frac{\left(b\tan\left(e+f\,x\right)^{2}+a\right)^{3/2}}{3\,b^{2}f}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\tan\left(e+f\,x\right)^{2}+a}}{\sqrt{a-b}}\right)}{f\,\sqrt{a-b}}-\left(\frac{2\,a}{b^{2}f}-\frac{a-b}{b^{2}f}\right)\sqrt{b\tan\left(e+f\,x\right)^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] (a + b*tan(e + f*x)^2)^(3/2)/(3*b^2*f) - atanh((a + b*tan(e + f*x)^2)^(1/2)
/(a - b)^(1/2))/(f*(a - b)^(1/2)) - ((2*a)/(b^2*f) - (a - b)/(b^2*f))*(a +
b*tan(e + f*x)^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**5/sqrt(a + b*tan(e + f*x)**2), x)
```

3.321 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{a+b\tan^2(e+fx)}}{bf}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

```
[Out] arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)+(a+b*tan(f*x+e)
^2)^(1/2)/b/f
```

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3670, 446, 80, 63, 208}

$$\frac{\sqrt{a+b\tan^2(e+fx)}}{bf}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f) + Sqrt[a +
b*Tan[e + f*x]^2]/(b*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
```

```
x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\sqrt{a+b\tan^2(e+fx)}}{bf} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\sqrt{a+b\tan^2(e+fx)}}{bf} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f} + \frac{\sqrt{a+b\tan^2(e+fx)}}{bf}$$

Mathematica [A] time = 0.28, size = 62, normalized size = 0.97

$$\frac{\frac{\sqrt{a+b\tan^2(e+fx)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/Sqrt[a - b] + Sqrt[a + b*T
an[e + f*x]^2]/b)/f
```

fricas [A] time = 0.54, size = 248, normalized size = 3.88

$$\left[\frac{\sqrt{a-b}\,b\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2+4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+8a^2-8ab+b^2}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)+4\sqrt{b\tan\left(fx\right.}}{4\left(ab-b^2\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a - b)*b*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x +
e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b
) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*sqrt(
b*tan(f*x + e)^2 + a)*(a - b))/((a*b - b^2)*f), -1/2*(sqrt(-a + b)*b*arctan
```

```
(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) -
2*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a*b - b^2)*f)]
```

giac [A] time = 1.91, size = 62, normalized size = 0.97

$$-\frac{\frac{b\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}-\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{f}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f) - sqrt
(b*tan(f*x + e)^2 + a)/f)/b
```

maple [A] time = 0.30, size = 58, normalized size = 0.91

$$\frac{\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}{bf}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] (a+b*tan(f*x+e)^2)^(1/2)/b/f-1/f/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/
2)/(-a+b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{3}}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)
```

mupad [B] time = 12.33, size = 56, normalized size = 0.88

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{b\tan\left(e+fx\right)^{2}+a}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}+\frac{\sqrt{b\tan\left(e+fx\right)^{2}+a}}{bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] atanh((a + b*tan(e + f*x)^2)^(1/2)/(a - b)^(1/2))/(f*(a - b)^(1/2)) + (a +
b*tan(e + f*x)^2)^(1/2)/(b*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}\left(e+fx\right)}{\sqrt{a+b\tan^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(a + b*tan(e + f*x)**2), x)
```

3.322 $\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3670, 444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

fricas [A] time = 0.57, size = 185, normalized size = 4.51

$$\left[\frac{\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2-4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)}{4\sqrt{a-b}\,f}, \frac{\sqrt{-a+b}\arctan\left(\frac{2\sqrt{b\text{t}}}{b\text{t}}\right.}{2(a-b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan
(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a
*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/(sqrt(a - b)*f), 1/2*sqr
t(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^
2 + 2*a - b))/((a - b)*f)]
```

giac [A] time = 1.97, size = 35, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f)
```

maple [A] time = 0.20, size = 35, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x)
[Out] 1/f/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a), x)
```

mupad [B] time = 12.34, size = 35, normalized size = 0.85

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tan\left(e+f\,x\right)^{2}+a}}{\sqrt{a-b}}\right)}{f\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] -atanh((a + b*tan(e + f*x)^2)^(1/2)/(a - b)^(1/2))/(f*(a - b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(e+fx\right)}{\sqrt{a+b\tan^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

3.323 $\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)+arctanh((a+b*tan(f*x+e
)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 446, 86, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f)) + ArcTanh[Sqrt[a
 + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
```

```
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}$$

Mathematica [A] time = 0.08, size = 72, normalized size = 0.97

$$\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/Sqrt[a]) + ArcTanh[Sqrt[a +
b*Tan[e + f*x]^2]/Sqrt[a - b]]/Sqrt[a - b])/f
```

fricas [A] time = 0.46, size = 446, normalized size = 6.03

$$\left[\frac{\sqrt{a-b}\,a\log\left(\frac{b\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+2a-b}{\tan\left(fx+e\right)^2+1}\right)+(a-b)\sqrt{a}\,\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a}+2a}{\tan\left(fx+e\right)^2}\right)}{2\left(a^2-ab\right)f}, \frac{2a\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*a*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sq
rt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + (a - b)*sqrt(a)*log((b*tan(f*x
 + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2))/((a^
2 - a*b)*f), 1/2*(2*a*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b)/(a - b)) + (a - b)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x
```

```
 + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2))/((a^2 - a*b)*f), 1/2*(2*sqrt(-
a)*(a - b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + sqrt(a - b)*a*lo
g((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(
tan(f*x + e)^2 + 1)))/((a^2 - a*b)*f), (sqrt(-a)*(a - b)*arctan(sqrt(b*tan(
f*x + e)^2 + a)*sqrt(-a)/a) + a*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2
+ a)*sqrt(-a + b)/(a - b)))/((a^2 - a*b)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
```

```
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Warning, integration of abs or sign assumes constant sign by inte
rvals (correct if the argument is real):Check [abs(t_nostep^2-1)]Discontinu
ities at zeroes of t_nostep^2-1 were not checkedWarning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep^2-1)]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b]=[64,89]Warning, need to choose a branch for the root of a polyn
omial with parameters. This might be wrong.The choice was done assuming [a,
b]=[-82,44]Discontinuities at zeroes of t_nostep^2-1 were not checkedWarnin
g, integration of abs or sign assumes constant sign by intervals (correct i
f the argument is real):Check [abs(t_nostep-1)]Evaluation time: 3.3Error: B
ad Argument Type
```

maple [B] time = 1.49, size = 496, normalized size = 6.70

$$-\frac{\sqrt{\frac{a\left(\cos^{2}\left(fx+e\right)\right)-\left(\cos^{2}\left(fx+e\right)\right)b+b}{\left(1+\cos\left(fx+e\right)\right)^{2}}}\left(\ln\left(-\frac{2\left(-1+\cos\left(fx+e\right)\right)\left(\sqrt{a}\,\cos\left(fx+e\right)\sqrt{\frac{a\left(\cos^{2}\left(fx+e\right)\right)-\left(\cos^{2}\left(fx+e\right)\right)b+b}{\left(1+\cos\left(fx+e\right)\right)^{2}}}+\sqrt{\frac{a\left(\cos^{2}\left(fx+e\right)\right)-\left(\cos^{2}\left(fx+e\right)\right)b\cdots}{\left(1+\cos\left(fx+e\right)\right)^{2}}}\cdots\right.}{\sin\left(fx+e\right)^{2}\sqrt{a}}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*(ln(-2*(-
1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2
)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*(a-b)^(1/2)+2*
ln(4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*
(a-b)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))
```

```
^2)^(1/2)+4*a*cos(f*x+e)-4*b*cos(f*x+e))*a^(1/2)-ln(-4*(((a*cos(f*x+e)^2-co
s(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e
)+b)/(-1+cos(f*x+e)))*(a-b)^(1/2))*sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))/a^(1/2)/(a-b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)/sqrt(b*tan(f*x + e)^2 + a), x)
```

mupad [B] time = 12.03, size = 232, normalized size = 3.14

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tan\left(e+fx\right)^{2}+a}}{\sqrt{a}}\right)}{\sqrt{a}\,f}-\frac{\operatorname{atanh}\left(\frac{4\,a\,b^{2}\sqrt{b\tan\left(e+fx\right)^{2}+a}}{\left(\frac{2\,b^{4}\,f^{3}}{a\,f^{3}-b\,f^{3}}-\frac{2\,a\,b^{3}\,f^{3}}{a\,f^{3}-b\,f^{3}}\right)\sqrt{a-b}}-\frac{2\,b^{3}\sqrt{b\tan\left(e+fx\right)^{2}+a}}{\left(\frac{2\,b^{4}\,f^{3}}{a\,f^{3}-b\,f^{3}}-\frac{2\,a\,b^{3}\,f^{3}}{a\,f^{3}-b\,f^{3}}\right)\sqrt{a-b}}+\frac{2\sqrt{b\tan\left(e+fx\right)^{2}+a}\,\left(a\,f^{3}-b\,f^{3}\right)}{b\,f^{3}\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] - atanh((a + b*tan(e + f*x)^2)^(1/2)/a^(1/2))/(a^(1/2)*f) - atanh((4*a*b^2*
(a + b*tan(e + f*x)^2)^(1/2))/(((2*b^4*f^3)/(a*f^3 - b*f^3) - (2*a*b^3*f^3)
/(a*f^3 - b*f^3))*(a - b)^(1/2)) - (2*b^3*(a + b*tan(e + f*x)^2)^(1/2))/(((
2*b^4*f^3)/(a*f^3 - b*f^3) - (2*a*b^3*f^3)/(a*f^3 - b*f^3))*(a - b)^(1/2))
+ (2*(a + b*tan(e + f*x)^2)^(1/2)*(a*f^3 - b*f^3))/(b*f^3*(a - b)^(1/2)))/(
f*(a - b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(e+fx\right)}{\sqrt{a+b\tan^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

3.324 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=116

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}$$

```
[Out] 1/2*(2*a+b)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-arctanh((a+
b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)-1/2*cot(f*x+e)^2*(a+b*tan(
f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3670, 446, 103, 156, 63, 208}

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((2*a + b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*a^(3/2)*f) - Arc
Tanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f) - (Cot[e + f*x
]^2*Sqrt[a + b*Tan[e + f*x]^2])/(2*a*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2a+b)+\frac{bx}{2}}{x(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2af}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} - \cdots$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf} \cdots$$

$$= \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f} - \frac{\cot^2(e+fx)\sqrt{a+b \cdots}}{2af}$$

Mathematica [A] time = 0.76, size = 135, normalized size = 1.16

$$\frac{(2a^2-ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right) + \sqrt{a}\left((b-a)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)} - 2a\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+\cdots}}{\cdots}\right)\right)}{2a^{3/2}f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((2*a^2 - a*b - b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] + Sqrt[a]*
(-2*a*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + (-a + b
)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2]))/(2*a^(3/2)*(a - b)*f)
```

fricas [A] time = 0.46, size = 697, normalized size = 6.01

$$\left[\frac{2\,\sqrt{a-b}\,a^2\log\left(\frac{b\tan\left(fx+e\right)^2-2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+2\,a-b}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2+\left(2\,a^2-ab-b^2\right)\sqrt{a}\,\log\left(\frac{b\tan\left(fx+e\right)^2+2\,\cdots}{\cdots}\right.}{4\left(a^3-a^2b\right)f\tan\left(fx+e\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a - b)*a^2*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a
)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + (2*a^2 - a*
b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*(a^
2 - a*b))/((a^3 - a^2*b)*f*tan(f*x + e)^2), -1/4*(4*a^2*sqrt(-a + b)*arctan
(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^2 - (2*a^2
- a*b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)
*(a^2 - a*b))/((a^3 - a^2*b)*f*tan(f*x + e)^2), 1/2*(sqrt(a - b)*a^2*log((b
*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(
f*x + e)^2 + 1))*tan(f*x + e)^2 - (2*a^2 - a*b - b^2)*sqrt(-a)*arctan(sqrt(
b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2 - sqrt(b*tan(f*x + e)^2 +
a)*(a^2 - a*b))/((a^3 - a^2*b)*f*tan(f*x + e)^2), -1/2*(2*a^2*sqrt(-a + b)*
arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^2 + (
2*a^2 - a*b - b^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*t
an(f*x + e)^2 + sqrt(b*tan(f*x + e)^2 + a)*(a^2 - a*b))/((a^3 - a^2*b)*f*ta
n(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
```

```
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
```

```
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign
assumes constant sign by intervals (correct if the argument is real):Check
[abs(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^2-1 were not check
edWarning, integration of abs or sign assumes constant sign by intervals (c
orrect if the argument is real):Check [abs(t_nostep^2-1)]Warning, need to c
hoose a branch for the root of a polynomial with parameters. This might be
wrong.The choice was done assuming [a,b]=[41,20]Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.The
 choice was done assuming [a,b]=[80,-3]Discontinuities at zeroes of t_noste
p^2-1 were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_nostep^2-1)]Evaluation time: 5.25Error: Bad Argument Type
```

maple [B] time = 1.35, size = 3601, normalized size = 31.04

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4/f*(4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a-b)^(
1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/
2)+4*a*cos(f*x+e)-4*b*cos(f*x+e))*cos(f*x+e)^3*a^(5/2)+4*((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a-b)^(1/2)+4*(a-b)^(1/2)*((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+4*a*cos(f*x+e)-4*b*cos(f
*x+e))*cos(f*x+e)^2*a^(5/2)-2*cos(f*x+e)^3*a^(5/2)*(a-b)^(1/2)-4*((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a-b)^(1/2)+4*(a-b)^(1/2)*
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+4*a*cos(f*x+e)-4
*b*cos(f*x+e))*cos(f*x+e)*a^(5/2)+2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+c
os(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+
e)^2/a^(1/2))*cos(f*x+e)^3*(a-b)^(1/2)*a^2+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(1+cos(f*x+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/s
in(f*x+e)^2/a^(1/2))*cos(f*x+e)^3*(a-b)^(1/2)*a*b-2*((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(
f*x+e)))*cos(f*x+e)^3*(a-b)^(1/2)*a^2-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1
+cos(f*x+e))^2)^(1/2)*ln(-4*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f
*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*cos(f
*x+e)^3*(a-b)^(1/2)*a*b+2*cos(f*x+e)^3*a^(3/2)*(a-b)^(1/2)*b-4*((a*cos(f*x+
e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*(a-b)^(1/2)+4*(a-b)^(1/2)*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)+4*a*cos(f*x+e)-4*b
```

```
*cos(f*x+e))*a^(5/2)+2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)
^(1/2)*ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(
f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))
*cos(f*x+e)^2*(a-b)^(1/2)*a^2+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x
+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a
^(1/2))*cos(f*x+e)^2*(a-b)^(1/2)*a*b-2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(
1+cos(f*x+e))^2)^(1/2)*ln(-4*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(
f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*cos(
f*x+e)^2*(a-b)^(1/2)*a^2-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^
2)^(1/2)*ln(-4*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/
2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*cos(f*x+e)^2*(a-b)
^(1/2)*a*b-2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(
-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)
^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e
)*(a-b)^(1/2)*a^2-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2
)*ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e
))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2))*cos(
f*x+e)*(a-b)^(1/2)*a*b+2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^
2)^(1/2)*ln(-4*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/
2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*cos(f*x+e)*(a-b)^(
1/2)*a^2+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/
2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos
(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*cos(f*x+e)*(a-b)^(1/2)*a*b-2*cos(f
*x+e)*a^(3/2)*(a-b)^(1/2)*b-2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x
+e))^2)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(1+cos(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a
^(1/2))*(a-b)^(1/2)*a^2-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2
)^(1/2)*ln(-2*(-1+cos(f*x+e))*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos
(f*x+e))^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)+b)/sin(f*x+e)^2/a^(1/2)
)*(a-b)^(1/2)*a*b+2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1
/2)*ln(-4*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)*a^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^
(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(-1+cos(f*x+e)))*(a-b)^(1/2)*a^2+((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a^(1/2)+((a*cos(f*x+e
)^2-cos(f*x+e)^2*b+b)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+a*cos(f*x+e)-b*cos(f*
x+e)+b)/(-1+cos(f*x+e)))*(a-b)^(1/2)*a*b)*sin(f*x+e)^2/(-1+cos(f*x+e))^2/co
s(f*x+e)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(1+cos(f*x+
e))^2/a^(5/2)/(a-b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^3}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)
```

mupad [B] time = 0.44, size = 830, normalized size = 7.16

$$\frac{\operatorname{atanh}\left(\frac{b^6\sqrt{b\tan(e+fx)^2+a}}{4\sqrt{a^3}\left(\frac{3ab^4}{2}+\frac{5b^5}{4}+\frac{b^6}{4a}\right)}+\frac{3b^4\sqrt{b\tan(e+fx)^2+a}}{2\sqrt{a^3}\left(\frac{3b^4}{2a}+\frac{5b^5}{4a^2}+\frac{b^6}{4a^3}\right)}+\frac{5b^5\sqrt{b\tan(e+fx)^2+a}}{4\sqrt{a^3}\left(\frac{3b^4}{2}+\frac{5b^5}{4a}+\frac{b^6}{4a^2}\right)}\right)(2a+b)}{2f\sqrt{a^3}}-\frac{b\sqrt{b\tan\left(e+fx\right)^2+a}}{2a\left(f\left(b\tan\left(e+fx\right)^2+a\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] (atan((((((2*a*b^4*f^2 + 2*a^2*b^3*f^2)/(2*a^2*f^3) - ((a + b*tan(e + f*x)^
2)^(1/2)*(16*a^2*b^3*f^2 - 32*a^3*b^2*f^2))/(8*a^2*f^3*(a - b)^(1/2)))/(2*f
*(a - b)^(1/2)) - ((a + b*tan(e + f*x)^2)^(1/2)*(4*a*b^3 + b^4 + 8*a^2*b^2)
)/(4*a^2*f^2))*1i)/(f*(a - b)^(1/2)) - ((((2*a*b^4*f^2 + 2*a^2*b^3*f^2)/(2*
a^2*f^3) + ((a + b*tan(e + f*x)^2)^(1/2)*(16*a^2*b^3*f^2 - 32*a^3*b^2*f^2))
/(8*a^2*f^3*(a - b)^(1/2)))/(2*f*(a - b)^(1/2)) + ((a + b*tan(e + f*x)^2)^(
1/2)*(4*a*b^3 + b^4 + 8*a^2*b^2))/(4*a^2*f^2))*1i)/(f*(a - b)^(1/2)))/((((2
*a*b^4*f^2 + 2*a^2*b^3*f^2)/(2*a^2*f^3) - ((a + b*tan(e + f*x)^2)^(1/2)*(16
*a^2*b^3*f^2 - 32*a^3*b^2*f^2))/(8*a^2*f^3*(a - b)^(1/2)))/(2*f*(a - b)^(1/
2)) - ((a + b*tan(e + f*x)^2)^(1/2)*(4*a*b^3 + b^4 + 8*a^2*b^2))/(4*a^2*f^2
))/(f*(a - b)^(1/2)) + (((2*a*b^4*f^2 + 2*a^2*b^3*f^2)/(2*a^2*f^3) + ((a +
b*tan(e + f*x)^2)^(1/2)*(16*a^2*b^3*f^2 - 32*a^3*b^2*f^2))/(8*a^2*f^3*(a -
b)^(1/2)))/(2*f*(a - b)^(1/2)) + ((a + b*tan(e + f*x)^2)^(1/2)*(4*a*b^3 + b
^4 + 8*a^2*b^2))/(4*a^2*f^2))/(f*(a - b)^(1/2)) - (a*b^3 + b^4/2)/(a^2*f^3)
))*1i)/(f*(a - b)^(1/2)) - (b*(a + b*tan(e + f*x)^2)^(1/2))/(2*a*(f*(a + b*
tan(e + f*x)^2) - a*f)) + (atanh((b^6*(a + b*tan(e + f*x)^2)^(1/2))/(4*(a^3
)^(1/2)*((3*a*b^4)/2 + (5*b^5)/4 + b^6/(4*a))) + (3*b^4*(a + b*tan(e + f*x)
^2)^(1/2))/(2*(a^3)^(1/2)*((3*b^4)/(2*a) + (5*b^5)/(4*a^2) + b^6/(4*a^3)))
+ (5*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*(a^3)^(1/2)*((3*b^4)/2 + (5*b^5)/
(4*a) + b^6/(4*a^2))))*(2*a + b))/(2*f*(a^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/sqrt(a + b*tan(e + f*x)**2), x)
```

3.325 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=166

$$\frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f}-\frac{\left(8a^2+4ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

```
[Out] -1/8*(8*a^2+4*a*b+3*b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/
f+arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)+1/8*(4*a+3*b)
*cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/4*cot(f*x+e)^4*(a+b*tan(f*x+
e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.21, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3670, 446, 103, 151, 156, 63, 208}

$$-\frac{\left(8a^2+4ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}+\frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((8*a^2 + 4*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8*a
^(5/2)*f) + ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f)
 + ((4*a + 3*b)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(8*a^2*f) - (Cot
[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2])/(4*a*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

erQ[m]

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4af} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a+3b)+\frac{3bx}{2}}{x^2(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{4af}$$

$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4af} + \cdots$$

$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4af} - \cdots$$

$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4af} - \cdots$$

$$= -\frac{(8a^2+4ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f} + \frac{(4a+3b\cdots}{\cdots}$$

Mathematica [A] time = 1.96, size = 162, normalized size = 0.98

$$\frac{\sqrt{a}\left(8a^2\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right) + (b-a)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(2a\cot^2(e+fx)-4a-3b\right)\right) \cdots}{8a^{5/2}f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-8*a^3 + 4*a^2*b + a*b^2 + 3*b^3)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt
[a]] + Sqrt[a]*(8*a^2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a
 - b]] + (-a + b)*Cot[e + f*x]^2*(-4*a - 3*b + 2*a*Cot[e + f*x]^2)*Sqrt[a +
 b*Tan[e + f*x]^2]))/(8*a^(5/2)*(a - b)*f)
```

fricas [A] time = 0.49, size = 857, normalized size = 5.16

$$\left[\frac{8\sqrt{a-b}\,a^3\log\left(\frac{b\tan(fx+e)^2+2\sqrt{b\tan(fx+e)^2+a}\,\sqrt{a-b}+2a-b}{\tan(fx+e)^2+1}\right)\tan(fx+e)^4 + \left(8a^3-4a^2b-ab^2-3b^3\right)\sqrt{a}\log\left(\frac{b\tan(f\cdots}{\cdots}\right)\cdots}{16\left(a^4-a^3b\cdots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(a - b)*a^3*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + (8*a^3 - 4
```

```
*a^2*b - a*b^2 - 3*b^3)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x +
e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 - 2*(2*a^3 - 2*a^2*
b - (4*a^3 - a^2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/(
(a^4 - a^3*b)*f*tan(f*x + e)^4), 1/16*(16*a^3*sqrt(-a + b)*arctan(-sqrt(b*t
an(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^4 + (8*a^3 - 4*a^2*b
- a*b^2 - 3*b^3)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 - 2*(2*a^3 - 2*a^2*b - (4*
a^3 - a^2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4 -
a^3*b)*f*tan(f*x + e)^4), 1/8*(4*sqrt(a - b)*a^3*log((b*tan(f*x + e)^2 + 2*
sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan
(f*x + e)^4 + (8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*sqrt(-a)*arctan(sqrt(b*tan(
f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^4 - (2*a^3 - 2*a^2*b - (4*a^3 - a^
2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4 - a^3*b)*f
*tan(f*x + e)^4), 1/8*(8*a^3*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a
)*sqrt(-a + b)/(a - b))*tan(f*x + e)^4 + (8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*
sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^4 - (2*
a^3 - 2*a^2*b - (4*a^3 - a^2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x +
e)^2 + a))/((a^4 - a^3*b)*f*tan(f*x + e)^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
```

```
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning
```

```
, integration of abs or sign assumes constant sign by intervals (correct if
 the argument is real):Check [abs(t_nostep^2-1)]Discontinuities at zeroes o
f t_nostep^2-1 were not checkedWarning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep^2-1)]Warning, need to choose a branch for the root of a polynomial wi
th parameters. This might be wrong.The choice was done assuming [a,b]=[18,-
19]Warning, need to choose a branch for the root of a polynomial with param
eters. This might be wrong.The choice was done assuming [a,b]=[60,-6]Discon
tinuities at zeroes of t_nostep^2-1 were not checkedUnable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of
abs or sign assumes constant sign by intervals (correct if the argument is
real):Check [abs(t_nostep^2-1)]Evaluation time: 7.71Error: Bad Argument Typ
e
```

maple [B] time = 1.96, size = 7641, normalized size = 46.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 12.14, size = 1215, normalized size = 7.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] - (((a + b*tan(e + f*x)^2)^(1/2)*(4*a*b + 5*b^2))/(8*a) - (b*(a + b*tan(e +
 f*x)^2)^(3/2)*(4*a + 3*b))/(8*a^2))/(f*(a + b*tan(e + f*x)^2)^2 + a^2*f -
2*a*f*(a + b*tan(e + f*x)^2)) - (atan((((((3*a^2*b^5*f^2)/2 + (a^3*b^4*f^2
)/2 + 2*a^4*b^3*f^2)/(2*a^4*f^3) - ((a + b*tan(e + f*x)^2)^(1/2)*(256*a^4*b
^3*f^2 - 512*a^5*b^2*f^2))/(128*a^4*f^3*(a - b)^(1/2)))/(2*f*(a - b)^(1/2))
 - ((a + b*tan(e + f*x)^2)^(1/2)*(24*a*b^5 + 9*b^6 + 64*a^2*b^4 + 64*a^3*b^
3 + 128*a^4*b^2))/(64*a^4*f^2))*1i)/(f*(a - b)^(1/2)) - (((((3*a^2*b^5*f^2)
/2 + (a^3*b^4*f^2)/2 + 2*a^4*b^3*f^2)/(2*a^4*f^3) + ((a + b*tan(e + f*x)^2)
^(1/2)*(256*a^4*b^3*f^2 - 512*a^5*b^2*f^2))/(128*a^4*f^3*(a - b)^(1/2)))/(2
*f*(a - b)^(1/2)) + ((a + b*tan(e + f*x)^2)^(1/2)*(24*a*b^5 + 9*b^6 + 64*a^
2*b^4 + 64*a^3*b^3 + 128*a^4*b^2))/(64*a^4*f^2))*1i)/(f*(a - b)^(1/2)))/(((
((3*a^2*b^5*f^2)/2 + (a^3*b^4*f^2)/2 + 2*a^4*b^3*f^2)/(2*a^4*f^3) - ((a + b
*tan(e + f*x)^2)^(1/2)*(256*a^4*b^3*f^2 - 512*a^5*b^2*f^2))/(128*a^4*f^3*(a
 - b)^(1/2)))/(2*f*(a - b)^(1/2)) - ((a + b*tan(e + f*x)^2)^(1/2)*(24*a*b^5
```

```
 + 9*b^6 + 64*a^2*b^4 + 64*a^3*b^3 + 128*a^4*b^2))/(64*a^4*f^2))/(f*(a - b)
^(1/2)) + ((((3*a^2*b^5*f^2)/2 + (a^3*b^4*f^2)/2 + 2*a^4*b^3*f^2)/(2*a^4*f^
3) + ((a + b*tan(e + f*x)^2)^(1/2)*(256*a^4*b^3*f^2 - 512*a^5*b^2*f^2))/(12
8*a^4*f^3*(a - b)^(1/2)))/(2*f*(a - b)^(1/2)) + ((a + b*tan(e + f*x)^2)^(1/
2)*(24*a*b^5 + 9*b^6 + 64*a^2*b^4 + 64*a^3*b^3 + 128*a^4*b^2))/(64*a^4*f^2)
)/(f*(a - b)^(1/2)) - ((3*a*b^5)/4 + (9*b^6)/32 + (5*a^2*b^4)/4 + a^3*b^3)/
(a^4*f^3)))*1i)/(f*(a - b)^(1/2)) - (atanh((35*b^6*(a + b*tan(e + f*x)^2)^(
1/2))/(32*(a^5)^(1/2)*((5*b^5)/(4*a) + (35*b^6)/(32*a^2) + (63*b^7)/(64*a^3
) + (81*b^8)/(256*a^4) + (27*b^9)/(256*a^5))) + (5*b^5*(a + b*tan(e + f*x)^
2)^(1/2))/(4*(a^5)^(1/2)*((5*b^5)/(4*a^2) + (35*b^6)/(32*a^3) + (63*b^7)/(6
4*a^4) + (81*b^8)/(256*a^5) + (27*b^9)/(256*a^6))) + (63*b^7*(a + b*tan(e +
 f*x)^2)^(1/2))/(64*(a^5)^(1/2)*((5*b^5)/4 + (35*b^6)/(32*a) + (63*b^7)/(64
*a^2) + (81*b^8)/(256*a^3) + (27*b^9)/(256*a^4))) + (81*b^8*(a + b*tan(e +
f*x)^2)^(1/2))/(256*(a^5)^(1/2)*((5*a*b^5)/4 + (35*b^6)/32 + (63*b^7)/(64*a
) + (81*b^8)/(256*a^2) + (27*b^9)/(256*a^3))) + (27*b^9*(a + b*tan(e + f*x)
^2)^(1/2))/(256*(a^5)^(1/2)*((35*a*b^6)/32 + (63*b^7)/64 + (5*a^2*b^5)/4 +
(81*b^8)/(256*a) + (27*b^9)/(256*a^2))))*(4*a*b + 8*a^2 + 3*b^2))/(8*f*(a^5
)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/sqrt(a + b*tan(e + f*x)**2), x)
```

3.326 $\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=177

$$\frac{(3a^2+4ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{5/2}f}-\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

```
[Out] 1/8*(3*a^2+4*a*b+8*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)
)/b^(5/2)/f-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)
^(1/2)-1/8*(3*a+4*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/4*(a+b*tan
(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f
```

Rubi [A] time = 0.22, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3670, 479, 582, 523, 217, 206, 377, 203}

$$\frac{(3a^2+4ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{5/2}f}-\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f}+\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) + ((3*a^2 + 4*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*
Tan[e + f*x]^2]])/(8*b^(5/2)*f) - ((3*a + 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[
e + f*x]^2])/(8*b^2*f) + (Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(4*b*f
)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
```

```
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4bf} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a+(3a+4b)x^2\right)}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{4bf}$$

$$= -\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4bf} +$$

$$= -\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4bf} -$$

$$= -\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4bf} -$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f} + \frac{\left(3a^2+4ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{5/2}f} - (3a+4b)$$

Mathematica [C] time = 6.33, size = 768, normalized size = 4.34

$$\frac{16b^3\sqrt{\cos(2(e+fx))+1}\sqrt{\frac{(a-b)\cos(2(e+fx))+a+b}{\cos(2(e+fx))+1}}\left(\frac{\sin^4(e+fx)\csc(2(e+fx))\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}}{4a\sqrt{\cos(2(e+fx))+1}\sqrt{(a-b)\cos(2(e+fx))+a+b}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-((b*(3*a^2 + 4*a*b + 4*b^2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 +
Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e +
f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a
- b)*Cos[2*(e + f*x)]))) + (16*b^3*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b +
 (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]
^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b +
 (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[Ar
cSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]],
1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos
[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e +
f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b +
 (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/
(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]
])))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/(4*b^2*f) + (Sqrt[(a + b + a*C
os[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((-3*Sec[e +
f*x]*(a*Sin[e + f*x] + 2*b*Sin[e + f*x]))/(8*b^2) + (Sec[e + f*x]^2*Tan[e +
 f*x])/(4*b)))/f
```

fricas [A] time = 2.08, size = 817, normalized size = 4.62

$$\left[-\frac{8\sqrt{-a+b}\,b^3\log\left(-\frac{(a-2b)\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\,\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^2+1}\right)-\left(3a^3+a^2b+4ab^2-8b^3\right)\sqrt{b}\log\left(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (3*a^3
 + a^2*b + 4*a*b^2 - 8*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f
*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 2*(2*(a*b^2 - b^3)*tan(f*x + e)^
3 - (3*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a
*b^3 - b^4)*f), -1/8*(4*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)^2 + 2
*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2
+ 1)) + (3*a^3 + a^2*b + 4*a*b^2 - 8*b^3)*sqrt(-b)*arctan(sqrt(b*tan(f*x +
e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - (2*(a*b^2 - b^3)*tan(f*x + e)^3 - (3
*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a*b^3 -
 b^4)*f), -1/16*(16*sqrt(a - b)*b^3*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqr
t(a - b)*tan(f*x + e))) - (3*a^3 + a^2*b + 4*a*b^2 - 8*b^3)*sqrt(b)*log(2*b
```

```
*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) -
2*(2*(a*b^2 - b^3)*tan(f*x + e)^3 - (3*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e))
*sqrt(b*tan(f*x + e)^2 + a))/((a*b^3 - b^4)*f), -1/8*(8*sqrt(a - b)*b^3*arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (3*a^3 + a^2*
b + 4*a*b^2 - 8*b^3)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b
*tan(f*x + e))) - (2*(a*b^2 - b^3)*tan(f*x + e)^3 - (3*a^2*b + a*b^2 - 4*b^
3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a*b^3 - b^4)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^6}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^6/sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [A] time = 0.33, size = 261, normalized size = 1.47

$$\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\,\left(\tan^3\left(fx+e\right)\right)}{4bf}-\frac{3a\tan\left(fx+e\right)\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{8f\,b^2}+\frac{3a^2\ln\left(\tan\left(fx+e\right)\sqrt{b}+\sqrt{a}\right.}{8f\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f-3/8/f/b^2*a*tan(f*x+e)*(a+b*t
an(f*x+e)^2)^(1/2)+3/8/f/b^(5/2)*a^2*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^
2)^(1/2))-1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/2/f*a/b^(3/2)*ln(ta
n(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))+1/f*ln(tan(f*x+e)*b^(1/2)+(a+b*t
an(f*x+e)^2)^(1/2))/b^(1/2)-1/f*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^
2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+fx\right)^6}{\sqrt{b\tan\left(e+fx\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/sqrt(a + b*tan(e + f*x)**2), x)
```

3.327 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=125

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{3/2}f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

```
[Out] -1/2*(a+2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f
+arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)+1/2*
(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f
```

Rubi [A] time = 0.14, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3670, 479, 523, 217, 206, 377, 203}

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{3/2}f}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f) - ((a + 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/(2*b^(3/2)*f) + (Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*b*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
```

```
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf} - \frac{\text{Subst}\left(\int \frac{a+(a+2b)x^2}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{2bf}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} -$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} -$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f} - \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b}}{2bf}$$

Mathematica [C] time = 6.26, size = 713, normalized size = 5.70

$$\frac{\tan(e+fx)\sqrt{\frac{a\cos(2(e+fx))+a-b\cos(2(e+fx))+b}{\cos(2(e+fx))+1}}}{2bf} - \frac{4b^2\sqrt{\cos(2(e+fx))+1}\sqrt{\frac{(a-b)\cos(2(e+fx))+a+b}{\cos(2(e+fx))+1}}\left(\frac{\sin^4(e+fx)\csc(2(e+fx))\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a}{}}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((-((b*(a + b)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x
)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Cs
c[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Cs
c[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e
+ f*x)]))) + (4*b^2*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*
(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-(
(a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a +
 b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]
^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
 - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e +
f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc
[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt
[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a + b
 + (a - b)*Cos[2*(e + f*x)]])/(b*f)) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] -
b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Tan[e + f*x])/(2*b*f)
```

fricas [A] time = 1.19, size = 647, normalized size = 5.18

$$\left[-\frac{2\sqrt{-a+b}\,b^2\log\left(-\frac{(a-2b)\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\,\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^2+1}\right)-\left(a^2+ab-2b^2\right)\sqrt{b}\,\log\left(2b\tan\left(fx+e\right.\right.}{4\left(ab^2-b^3\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-a + b)*b^2*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (a^2 +
a*b - 2*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*
sqrt(b)*tan(f*x + e) + a) - 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*
x + e))/((a*b^2 - b^3)*f), -1/2*(sqrt(-a + b)*b^2*log(-((a - 2*b)*tan(f*x +
 e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*
x + e)^2 + 1)) - (a^2 + a*b - 2*b^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2
+ a)*sqrt(-b)/(b*tan(f*x + e))) - sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*ta
n(f*x + e))/((a*b^2 - b^3)*f), 1/4*(4*sqrt(a - b)*b^2*arctan(-sqrt(b*tan(f*
x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (a^2 + a*b - 2*b^2)*sqrt(b)*log
(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a
) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x + e))/((a*b^2 - b^3)*f
), 1/2*(2*sqrt(a - b)*b^2*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*t
an(f*x + e))) + (a^2 + a*b - 2*b^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 +
 a)*sqrt(-b)/(b*tan(f*x + e))) + sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan
(f*x + e))/((a*b^2 - b^3)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^4}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [A] time = 0.27, size = 165, normalized size = 1.32

$$\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\,\tan\left(fx+e\right)}{2bf}-\frac{a\ln\left(\tan\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\right)}{2f\,b^{\frac{3}{2}}}-\frac{\ln\left(\tan\left(fx+e\right)\sqrt{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x)

[Out] 1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f-1/2/f*a/b^(3/2)*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)+1/f*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^4}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tan(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^4}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2),x)

[Out] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)

[Out] Integral(tan(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)

3.328 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=86

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}\,f}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)+arct
anh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3670, 483, 217, 206, 377, 203}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}\,f}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[b]
*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

Mathematica [C] time = 0.79, size = 149, normalized size = 1.73

$$\frac{a\sin(2(e+fx))\csc^2(e+fx)\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\,\Pi\left(-\frac{b}{a-b};\sin^{-1}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))}{b}}}{\sqrt{2}}\right.\right.}{2bf(a-b)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (a*Csc[e + f*x]^2*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Co
s[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sqrt[(a + b + (a - b)*Cos[2
*(e + f*x)])*Sec[e + f*x]^2]*Sin[2*(e + f*x)])/(2*(a - b)*b*f*Sqrt[((a + b
+ (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b])
```

fricas [A] time = 0.55, size = 479, normalized size = 5.57

$$\left[\frac{(a-b)\sqrt{b}\log\left(2b\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{b}\tan\left(fx+e\right)+a\right)-\sqrt{-a+b}\,b\log\left(-\frac{(a-2b)\tan(f\right.}{}\right.}{2\left(ab-b^2\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a - b)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(b)*tan(f*x + e) + a) - sqrt(-a + b)*b*log(-((a - 2*b)*tan(f*x + e)^2
+ 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)
^2 + 1)))/((a*b - b^2)*f), -1/2*(2*(a - b)*sqrt(-b)*arctan(sqrt(b*tan(f*x +
 e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + sqrt(-a + b)*b*log(-((a - 2*b)*tan(
f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(t
an(f*x + e)^2 + 1)))/((a*b - b^2)*f), -1/2*(2*sqrt(a - b)*b*arctan(-sqrt(b*
tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (a - b)*sqrt(b)*log(2*b*t
an(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a))/((a
*b - b^2)*f), -(sqrt(a - b)*b*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a -
b)*tan(f*x + e))) + (a - b)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt
(-b)/(b*tan(f*x + e))))/((a*b - b^2)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [A] time = 0.33, size = 102, normalized size = 1.19

$$\frac{\ln\left(\tan\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\right)}{f\sqrt{b}}-\frac{\sqrt{b^4\left(a-b\right)}\ \arctan\left(\frac{(a-b)b^2\tan\left(fx+e\right)}{\sqrt{b^4(a-b)}\ \sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}\right)}{f\,b^2\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)-1/f*(b^4*(a-b))
^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2
)*tan(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^2}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

3.329 $\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3661, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f}$$

Mathematica [A] time = 0.07, size = 46, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f)
```

fricas [A] time = 0.44, size = 125, normalized size = 2.72

$$\left[-\frac{\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{-a+b}\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^2+1}\right)}{2(a-b)f}, \frac{\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{a-b}\tan\left(fx+e\right)}\right)}{\sqrt{a-b}\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1))/((a - b)*f), arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e)))/(sqrt(a - b)*f)
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [A] time = 0.36, size = 67, normalized size = 1.46

$$\frac{\sqrt{b^4(a-b)}\ \arctan\left(\frac{(a-b)b^2\tan(fx+e)}{\sqrt{b^4(a-b)}\ \sqrt{a+b\left(\tan^2(fx+e)\right)}}\right)}{f\,b^2(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan
(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 12.69, size = 40, normalized size = 0.87

$$\frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)\,\sqrt{a-b}}{\sqrt{b\,\tan(e+f\,x)^2+a}}\right)}{f\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] atan((tan(e + f*x)*(a - b)^(1/2))/(a + b*tan(e + f*x)^2)^(1/2))/(f*(a - b)^
(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*tan(e + f*x)**2), x)
```

3.330 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)-cot(
f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.12, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3670, 480, 12, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) - (Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
```

```
x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af} - \frac{\text{Subst}\left(\int \frac{a}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af} - \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

Mathematica [C] time = 9.50, size = 212, normalized size = 2.72

$$-\frac{\cos^2(e+fx)\cot(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)\left(\frac{4\sin^2(e+fx)\left(a^2+ab\left(\tan^2(e+fx)-1\right)-b^2\tan^2(e+fx)\right){}_2F_1\left(2,2;\frac{5}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)}{3a^2}+\frac{\text{si}\ldots}{a\sqrt{\frac{\sin^2(\ldots}{\ldots}}}\right.}{f\sqrt{a+b\tan^2(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cos[e + f*x]^2*Cot[e + f*x]*(1 + (b*Tan[e + f*x]^2)/a)*((4*Hypergeometri
c2F1[2, 2, 5/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*(a^2 - b^2*Tan[e
 + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/(3*a^2) + (ArcSin[Sqrt[((a - b)*Sin
[e + f*x]^2)/a]]*(a + 2*b*Tan[e + f*x]^2))/(a*Sqrt[(Cos[e + f*x]^2*Sin[e +
f*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/a^2]))))/(f*S
qrt[a + b*Tan[e + f*x]^2]))
```

fricas [A] time = 0.57, size = 289, normalized size = 3.71

$$\left[-\frac{a\sqrt{-a+b}\,\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)\tan\left(fx+e\right)^4-2\left(3\,a^2-4\,ab\right)\tan\left(fx+e\right)^2+a^2+4\left(\left(a-2\,b\right)\tan\left(fx+e\right)^3-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)\ldots}{4\left(a^2-ab\right)f\tan\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2
 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x +
e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x +
e)^2 + 1))*tan(f*x + e) + 4*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2 - a*b
)*f*tan(f*x + e)), -1/2*(sqrt(a - b)*a*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e) + 2*
sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2 - a*b)*f*tan(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{2}}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [C] time = 1.35, size = 1195, normalized size = 15.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*(2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*co
s(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1
/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e
))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(
1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a
*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)*sin(f*x+e)*a-2*2^(1/2)*((I*cos(f*x+e)*(a-b
)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f
*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)
-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*
x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1
/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)
^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)*sin(f*x+e)*a+2^(1/2)*((I*cos(f*x
+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/
(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)
*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1
+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-
b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin
(f*x+e)*a-2*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2
)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a
-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos
(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*
b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1
/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*sin(
f*x+e)*a+cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a-cos(f*x+e
)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b+((2*I*(a-b)^(1/2)*b^(1/2)+a
-2*b)/a)^(1/2)*b)/cos(f*x+e)/sin(f*x+e)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
cos(f*x+e)^2)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{2}}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^2}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{2}\left(e+fx\right)}{\sqrt{a+b\tan^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

3.331 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=120

$$\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)+1/3*(
3*a+2*b)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/3*cot(f*x+e)^3*(a+b*ta
n(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.16, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3670, 480, 583, 12, 377, 203}

$$\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f) + ((3*a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^2*f) - (Cot
[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} + \frac{\text{Subst}\left(\int \frac{-3a-2b-2bx^2}{x^2(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{3af}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} - \frac{\text{S}}{}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} + \frac{\text{S}}{}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} + \frac{\text{S}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f} + \frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx}{}$$

Mathematica [C] time = 11.07, size = 263, normalized size = 2.19

$$-\frac{\cos^2(e+fx)\cot^3(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)\left(-8(a-b)\sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2 {}_3F_2\left(2,2,2;1,\frac{5}{2};\frac{(a-b)\sin}{a}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/9*(Cos[e + f*x]^2*Cot[e + f*x]^3*(1 + (b*Tan[e + f*x]^2)/a)*(-12*b*(-a +
 b)*(-a - b + (-a + b)*Cos[2*(e + f*x)])*Hypergeometric2F1[2, 2, 5/2, ((a -
```

```
b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^4 - 8*(a - b)*HypergeometricPFQ[{2, 2,
2}, {1, 5/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*(a + b*Tan[e + f*x
]^2)^2 + (6*a*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*(a^2 - 4*a*b*Tan[e +
 f*x]^2 - 8*b^2*Tan[e + f*x]^4))/Sqrt[((a - b)*Sin[2*(e + f*x)]^2*(a + b*Ta
n[e + f*x]^2))/a^2]))/(a^3*f*Sqrt[a + b*Tan[e + f*x]^2])
```

fricas [A] time = 0.56, size = 359, normalized size = 2.99

$$\left[-\frac{3\,a^2\sqrt{-a+b}\,\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)\tan\left(fx+e\right)^4-2\left(3\,a^2-4\,ab\right)\tan\left(fx+e\right)^2+a^2-4\left(\left(a-2\,b\right)\tan\left(fx+e\right)^3-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{\cdots}}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2+1}\right.}{12\left(a^3-a^2b\right)f\tan\left(fx\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*a^2*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(
3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f
*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f
*x + e)^2 + 1))*tan(f*x + e)^3 - 4*((3*a^2 - a*b - 2*b^2)*tan(f*x + e)^2 -
a^2 + a*b)*sqrt(b*tan(f*x + e)^2 + a))/((a^3 - a^2*b)*f*tan(f*x + e)^3), 1/
6*(3*sqrt(a - b)*a^2*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f
*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e)^3 + 2*((3*a^2 - a*b -
2*b^2)*tan(f*x + e)^2 - a^2 + a*b)*sqrt(b*tan(f*x + e)^2 + a))/((a^3 - a^2*
b)*f*tan(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^4}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [C] time = 1.77, size = 2433, normalized size = 20.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3/f*(-6*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)
+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-
b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(
f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b
)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/
2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f
*x+e)^3*sin(f*x+e)*a^2+3*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)
^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*
cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x
+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*
b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2
)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^2-6*2^(1
```

```
/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b
*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2
)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2
)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(
f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*
b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^2*sin(f*x
+e)*a^2+3*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+
a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b
)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f
*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/
a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2
-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a^2+6*2^(1/2)*((I*cos(f*x
+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/
(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)
*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-
1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*
(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2
*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)*sin(f*x+e)*a^2-3*2^(1/2)
*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*co
s(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I
*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*E
llipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+
e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2
)^(1/2))*cos(f*x+e)*sin(f*x+e)*a^2+4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1
/2)*cos(f*x+e)^4*a^2-2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)
^4*a*b-2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*b^2+6*a^2*2
^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e
)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(
1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(
1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/s
in(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a
+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)-3*2^(1
/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b
*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2
)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2
)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f
*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/
a^2)^(1/2))*sin(f*x+e)*a^2-3*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(
f*x+e)^2*a^2+5*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a*b+4
*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b^2-3*((2*I*(a-b)^(
1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b-2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2
)*b^2)/cos(f*x+e)/sin(f*x+e)^3/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e
)^2)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^4}{\sqrt{b\,\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

3.332 $\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=170

$$\frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{\left(15a^2+10ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\ \mathrm{t}}{\sqrt{a+b\,\mathrm{t}}}\right)}{f\sqrt{a-}}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)-1/15
*(15*a^2+10*a*b+8*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^3/f+1/15*(5*a+
4*b)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/5*cot(f*x+e)^5*(a+b*tan(
f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.25, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3670, 480, 583, 12, 377, 203}

$$-\frac{\left(15a^2+10ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f}+\frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{\tan^{-1}\left(\frac{\sqrt{a-}}{\sqrt{a+}}\right)}{f\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) - ((15*a^2 + 10*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])
/(15*a^3*f) + ((5*a + 4*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a
^2*f) - (Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5af} + \frac{\text{Subst}\left(\int \frac{-5a-4b-4bx^2}{x^4(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= \frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5af}$$

$$= -\frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} + \frac{(5a+4b)\cot^3(e+fx)}{15a}$$

$$= -\frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} + \frac{(5a+4b)\cot^3(e+fx)}{15a}$$

$$= -\frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} + \frac{(5a+4b)\cot^3(e+fx)}{15a}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f} - \frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f}$$

Mathematica [C] time = 16.31, size = 794, normalized size = 4.67

$$\frac{\sqrt{\frac{a\cos(2(e+fx))+a-b\cos(2(e+fx))+b}{\cos(2(e+fx))+1}}\left(\frac{\csc^3(e+fx)(11a\cos(e+fx)+4b\cos(e+fx))}{15a^2} + \frac{\csc(e+fx)\left(-23a^2\cos(e+fx)-14ab\cos(e+fx)-8b^2\cos(e+\right.}{15a^3}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x
)])]*(((-23*a^2*Cos[e + f*x] - 14*a*b*Cos[e + f*x] - 8*b^2*Cos[e + f*x])*Cs
c[e + f*x])/(15*a^3) + ((11*a*Cos[e + f*x] + 4*b*Cos[e + f*x])*Csc[e + f*x]
^3)/(15*a^2) - (Cot[e + f*x]*Csc[e + f*x]^4)/(5*a)))/f + (b*Sqrt[(a + b + (
a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/
b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a
- b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin
[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*S
in[e + f*x]^4)/(a*f*(a + b + (a - b)*Cos[2*(e + f*x)])) + (4*b*Sqrt[1 + Cos
[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)
])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e
+ f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*C
sc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*C
sc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x
)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)
]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a -
b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a
- b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/S
qrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a +
b + (a - b)*Cos[2*(e + f*x)]])))/(f*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
```

fricas [A] time = 0.58, size = 437, normalized size = 2.57

$$\left[-\frac{15\,a^{3}\sqrt{-a+b}\,\log\left(-\frac{\left(a^{2}-8\,ab+8\,b^{2}\right)\tan\left(fx+e\right)^{4}-2\left(3\,a^{2}-4\,ab\right)\tan\left(fx+e\right)^{2}+a^{2}+4\left(\left(a-2\,b\right)\tan\left(fx+e\right)^{3}-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-a}}{\tan\left(fx+e\right)^{4}+2\,\tan\left(fx+e\right)^{2}+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*a^3*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*
(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(
f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(
f*x + e)^2 + 1))*tan(f*x + e)^5 + 4*((15*a^3 - 5*a^2*b - 2*a*b^2 - 8*b^3)*t
an(f*x + e)^4 + 3*a^3 - 3*a^2*b - (5*a^3 - a^2*b - 4*a*b^2)*tan(f*x + e)^2)
*sqrt(b*tan(f*x + e)^2 + a))/((a^4 - a^3*b)*f*tan(f*x + e)^5), -1/30*(15*sq
rt(a - b)*a^3*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)
/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e)^5 + 2*((15*a^3 - 5*a^2*b - 2*
a*b^2 - 8*b^3)*tan(f*x + e)^4 + 3*a^3 - 3*a^2*b - (5*a^3 - a^2*b - 4*a*b^2)
*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4 - a^3*b)*f*tan(f*x + e)^
5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{6}}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/sqrt(b*tan(f*x + e)^2 + a), x)
```

maple [C] time = 1.70, size = 3741, normalized size = 22.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15/f*(15*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(
1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*co
s(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e
)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^
(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*
b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^3+34*((2*I*(a-b)^(1/2)*b^(1/2)+a-2
*b)/a)^(1/2)*cos(f*x+e)^4*a^2*b+22*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2
)*cos(f*x+e)^4*a*b^2-9*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)
^6*a^2*b-6*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^6*a*b^2-40*
((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a^2*b-26*((2*I*(a-b)
^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a*b^2+23*((2*I*(a-b)^(1/2)*b^(1
/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^6*a^3-35*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)
^(1/2)*cos(f*x+e)^4*a^3-8*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x
+e)^6*b^3+24*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*b^3-24*
((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b^3+15*((2*I*(a-b)^(
1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a^3-30*sin(f*x+e)*2^(1/2)*((I*cos
(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)
+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(
1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticP
i((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(
2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)
/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*a^3+15*((2*I*(a-b)^(1/2)*b^(1/2
)+a-2*b)/a)^(1/2)*a^2*b+10*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b^2+
8*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^3+15*2^(1/2)*((I*cos(f*x+e)*(
a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+co
s(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1
/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(
f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1
/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+
e)^4*sin(f*x+e)*a^3+60*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(
1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*co
s(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e
)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b
^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2
*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^
(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^3-30*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^
(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^
(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x
+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I
*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4
*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^3*sin(f*x+
e)*a^3-30*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+
a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b
)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f
*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)
/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2
)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f*
x+e)^5*sin(f*x+e)*a^3+60*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)
^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*
cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x
+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)
*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-
```

```
(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a
)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a^3-30*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*
b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a
)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f
*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2
*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)
-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^2*sin(f*
x+e)*a^3-30*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2
)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a
-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos
(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*
b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1
/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(
f*x+e)*sin(f*x+e)*a^3+15*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)
^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*
x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-
a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+
e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*
b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^3+15*cos(f
*x+e)^5*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)
*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*
x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)
/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2
)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1
/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^3-30*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2
)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))
/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos
(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*
((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^
(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)
*b^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^4*sin(f*x+e)*a^3)/cos(f*x+e)/sin(f*x+e
)^5/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/((2*I*(a-b)^(1/2
)*b^(1/2)+a-2*b)/a)^(1/2)/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^6}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**6/sqrt(a + b*tan(e + f*x)**2), x)
```

3.333 $\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{a^2}{b^2f(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{\sqrt{a+b\tan^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+a^2/(a-b)/b^2/
f/(a+b*tan(f*x+e)^2)^(1/2)+(a+b*tan(f*x+e)^2)^(1/2)/b^2/f
```

Rubi [A] time = 0.17, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3670, 446, 87, 63, 208}

$$\frac{a^2}{b^2f(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{\sqrt{a+b\tan^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f)) + a^2/
((a - b)*b^2*f*Sqrt[a + b*Tan[e + f*x]^2]) + Sqrt[a + b*Tan[e + f*x]^2]/(b^
2*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)^{3/2}}\,dx,x,\tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a^2}{(a-b)b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}+\frac{1}{(a-b)(1+x)\sqrt{a+bx}}\right)dx,x,\tan^2(e+fx)\right)}{2f}$$

$$= \frac{a^2}{(a-b)b^2f\sqrt{a+b\tan^2(e+fx)}}+\frac{\sqrt{a+b\tan^2(e+fx)}}{b^2f}+\frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\right.}{2(a}$$

$$= \frac{a^2}{(a-b)b^2f\sqrt{a+b\tan^2(e+fx)}}+\frac{\sqrt{a+b\tan^2(e+fx)}}{b^2f}+\frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx\right.}{(a}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f}+\frac{a^2}{(a-b)b^2f\sqrt{a+b\tan^2(e+fx)}}+\frac{\sqrt{a+b\tan^2(e}}{b^2f}$$

Mathematica [C] time = 0.38, size = 84, normalized size = 0.86

$$\frac{b^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\tan^2(e+fx)+a}{a-b}\right)+(a-b)\left(2a+b\tan^2(e+fx)+b\right)}{b^2f(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (b^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)] + (a -
 b)*(2*a + b + b*Tan[e + f*x]^2))/((a - b)*b^2*f*Sqrt[a + b*Tan[e + f*x]^2]
)
```

fricas [B] time = 0.56, size = 432, normalized size = 4.41

$$\left[-\frac{\left(b^3\tan\left(fx+e\right)^2+ab^2\right)\sqrt{a-b}\,\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2+4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right.}{4\left(\left(a^2b^3-2ab^4+b^5\right)f\tan\left(fx+e\right)^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^3*tan(f*x + e)^2 + a*b^2)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 +
2*(4*a*b - 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*ta
n(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*ta
n(f*x + e)^2 + 1)) - 4*(2*a^3 - 3*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*t
an(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan
(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f), 1/2*((b^3*tan(f*x + e)^2 +
a*b^2)*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan
(f*x + e)^2 + 2*a - b)) + 2*(2*a^3 - 3*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b
^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^2*b^3 - 2*a*b^4 + b^5)*
f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f)]
```

giac [A] time = 3.15, size = 99, normalized size = 1.01

$$\frac{a^2}{\left(ab^2f - b^3f\right)\sqrt{b\tan\left(fx+e\right)^2+a}} + \frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\left(af-bf\right)\sqrt{-a+b}} + \frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] a^2/((a*b^2*f - b^3*f)*sqrt(b*tan(f*x + e)^2 + a)) + arctan(sqrt(b*tan(f*x
+ e)^2 + a)/sqrt(-a + b))/((a*f - b*f)*sqrt(-a + b)) + sqrt(b*tan(f*x + e)^
2 + a)/(b^2*f)
```

maple [A] time = 0.27, size = 141, normalized size = 1.44

$$\frac{\tan^2\left(fx+e\right)}{fb\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}} + \frac{2a}{f\,b^2\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}} + \frac{1}{fb\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}} + \frac{1}{(a-b)\,f\sqrt{a+b\left(\tan^2\left(fx\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/f*tan(f*x+e)^2/b/(a+b*tan(f*x+e)^2)^(1/2)+2/f*a/b^2/(a+b*tan(f*x+e)^2)^(1
/2)+1/f/b/(a+b*tan(f*x+e)^2)^(1/2)+1/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(
a-b)/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 13.73, size = 112, normalized size = 1.14

$$\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{b^2\,f} + \frac{a^2}{b^2\,f\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\left(a-b\right)} + \frac{\mathrm{atan}\left(\frac{a\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,1\mathrm{i}-b\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,1\mathrm{i}}{(a-b)^{3/2}}\right)1\mathrm{i}}{f\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (a + b*tan(e + f*x)^2)^(1/2)/(b^2*f) + (atan((a*(a + b*tan(e + f*x)^2)^(1/2
)*1i - b*(a + b*tan(e + f*x)^2)^(1/2)*1i)/(a - b)^(3/2))*1i)/(f*(a - b)^(3/
2)) + a^2/(b^2*f*(a + b*tan(e + f*x)^2)^(1/2)*(a - b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.334 $\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}-\frac{a}{bf(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f-a/(a-b)/b/f/(a+
b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3670, 446, 78, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}-\frac{a}{bf(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f) - a/((a -
 b)*b*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}\\
&= -\frac{a}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2(a-b)f}\\
&= -\frac{a}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{(a-b)bf}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} - \frac{a}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 75, normalized size = 1.03

$$\frac{\frac{a(b-a)}{b\sqrt{a+b\tan^2(e+fx)}} + \sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + (a*(-a + b))
/(b*Sqrt[a + b*Tan[e + f*x]^2]))/((a - b)^2*f)
```

fricas [B] time = 0.55, size = 358, normalized size = 4.90

$$\left[-\frac{\left(b^2\tan\left(fx+e\right)^2+ab\right)\sqrt{a-b}\,\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2-4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right.}{4\left(\left(a^2b^2-2ab^3+b^4\right)f\tan\left(fx+e\right)^2+\left(a^3b-2a^2b^2+ab^3\right)f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2*tan(f*x + e)^2 + a*b)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*
(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(
f*x + e)^2 + 1)) + 4*sqrt(b*tan(f*x + e)^2 + a)*(a^2 - a*b))/((a^2*b^2 - 2*
a*b^3 + b^4)*f*tan(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f), -1/2*((b^2*
tan(f*x + e)^2 + a*b)*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a^2
- a*b))/((a^2*b^2 - 2*a*b^3 + b^4)*f*tan(f*x + e)^2 + (a^3*b - 2*a^2*b^2 +
a*b^3)*f)]
```

giac [A] time = 2.12, size = 76, normalized size = 1.04

$$-\frac{\frac{b \arctan\left(\frac{\sqrt{b \tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\left(af-bf\right)\sqrt{-a+b}}+\frac{a}{\sqrt{b \tan\left(fx+e\right)^2+a}\left(af-bf\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(b*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a*f - b*f)*sqrt(-a +
b)) + a/(sqrt(b*tan(f*x + e)^2 + a)*(a*f - b*f)))/b
```

maple [A] time = 0.30, size = 92, normalized size = 1.26

$$-\frac{1}{fb\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}-\frac{1}{\left(a-b\right)f\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{f\left(a-b\right)\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/b/(a+b*tan(f*x+e)^2)^(1/2)-1/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-1/f/(a-b
)/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 13.10, size = 90, normalized size = 1.23

$$-\frac{a}{b\,f\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,\left(a-b\right)}-\frac{\mathrm{atan}\left(\frac{a\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,1{}\mathrm{i}-b\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,1{}\mathrm{i}}{{\left(a-b\right)}^{3/2}}\right)\,1{}\mathrm{i}}{f\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] - (atan((a*(a + b*tan(e + f*x)^2)^(1/2)*1i - b*(a + b*tan(e + f*x)^2)^(1/2)
*1i)/(a - b)^(3/2))*1i)/(f*(a - b)^(3/2)) - a/(b*f*(a + b*tan(e + f*x)^2)^(
1/2)*(a - b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.335 $\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{1}{f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+1/(a-b)/f/(a+b
*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 444, 51, 63, 208}

$$\frac{1}{f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f)) + 1/((
a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}\\ &= \frac{1}{(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2(a-b)f}\\ &= \frac{1}{(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{(a-b)bf}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} + \frac{1}{(a-b)f\sqrt{a+b\tan^2(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.07, size = 56, normalized size = 0.81

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\tan^2(e+fx)+a}{a-b}\right)}{f(b-a)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)]/((-a + b)
*f*Sqrt[a + b*Tan[e + f*x]^2]))
```

fricas [B] time = 0.58, size = 332, normalized size = 4.81

$$\left[-\frac{\left(b\tan\left(fx+e\right)^2+a\right)\sqrt{a-b}\,\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2+4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+8a}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right.}{4\left(\left(a^2b-2ab^2+b^3\right)f\tan\left(fx+e\right)^2+\left(a^3-2a^2b+ab^2\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*tan(f*x + e)^2 + a)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a
*b - 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x
+ e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x
+ e)^2 + 1)) - 4*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2*b - 2*a*b^2 + b^
3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f), 1/2*((b*tan(f*x + e)^2 +
a)*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x
 + e)^2 + 2*a - b)) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2*b - 2*a*b
^2 + b^3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f)]
```

giac [A] time = 2.54, size = 69, normalized size = 1.00

$$\frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\left(af-bf\right)\sqrt{-a+b}}+\frac{1}{\sqrt{b\tan\left(fx+e\right)^2+a}\left(af-bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a*f - b*f)*sqrt(-a + b))
+ 1/(sqrt(b*tan(f*x + e)^2 + a)*(a*f - b*f))
```

maple [A] time = 0.15, size = 68, normalized size = 0.99

$$\frac{1}{(a-b)f\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}+\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{f(a-b)\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)/(-a+b)^(1/2)*arctan((a+b*tan(f
*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

mupad [B] time = 13.07, size = 85, normalized size = 1.23

$$\frac{1}{f\sqrt{b\tan\left(e+fx\right)^2+a}\,\left(a-b\right)}+\frac{\mathrm{atan}\left(\frac{a\sqrt{b\tan\left(e+fx\right)^2+a}\,1\mathrm{i}-b\sqrt{b\tan\left(e+fx\right)^2+a}\,1\mathrm{i}}{(a-b)^{3/2}}\right)1\mathrm{i}}{f\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] 1/(f*(a + b*tan(e + f*x)^2)^(1/2)*(a - b)) + (atan((a*(a + b*tan(e + f*x)^2
)^(1/2)*1i - b*(a + b*tan(e + f*x)^2)^(1/2)*1i)/(a - b)^(3/2))*1i)/(f*(a -
b)^(3/2))
```

sympy [A] time = 24.73, size = 56, normalized size = 0.81

$$\frac{1}{f(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] 1/(f*(a - b)*sqrt(a + b*tan(e + f*x)**2)) + atan(sqrt(a + b*tan(e + f*x)**2
)/sqrt(-a + b))/(f*sqrt(-a + b)*(a - b))
```

3.336 $\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+arctanh((a+b*tan(f*x+e
)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f-b/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 446, 85, 156, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + ArcTanh[Sqrt[a
 + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f) - b/(a*(a - b)*f*Sqrt[a
 + b*Tan[e + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{b}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-b-bx}{x(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2a(a-b)f}$$

$$= -\frac{b}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2af} - \text{Sul}$$

$$= -\frac{b}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right.}{abf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} - \frac{b}{a(a-b)f\sqrt{a+b\tan^2(}}$$

Mathematica [C] time = 0.14, size = 91, normalized size = 0.86

$$\frac{(a-b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\tan^2(e+fx)}{a}+1\right) - a\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\tan^2(e+fx)+a}{a-b}\right)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)]) + (a
- b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tan[e + f*x]^2)/a])/(a*(a - b)*
f*Sqrt[a + b*Tan[e + f*x]^2])
```

fricas [B] time = 0.49, size = 920, normalized size = 8.68

$$\left[-\frac{\left(a^2b\tan\left(fx+e\right)^2+a^3\right)\sqrt{a-b}\,\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+2\,a-b}{\tan\left(fx+e\right)^2+1}\right)-\left(a^3-2\,a^2b+ab^2+\left(a^2b-2\,ab^2\cdots\right.\right.}{2\left(\left(a^4b-2\,a^3b^2+a^2b^3\right)f\tan\left(fx+e\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*b*tan(f*x + e)^2 + a^3)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*s
qrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) - (a
^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt(a)*log(
(b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e
)^2) + 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4*b - 2*a^3*b^2 +
a^2*b^3)*f*tan(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f), 1/2*(2*(a^2*b*tan
(f*x + e)^2 + a^3)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
+ b)/(a - b)) + (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x +
e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a)
+ 2*a)/tan(f*x + e)^2) - 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^
4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f),
 1/2*(2*(a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sq
rt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) - (a^2*b*tan(f*x + e)^
2 + a^3)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) - 2*(a^2*b - a*b^2)*sqrt(b*tan(
f*x + e)^2 + a))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a^5 - 2
*a^4*b + a^3*b^2)*f), ((a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan
(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + (a^2*
b*tan(f*x + e)^2 + a^3)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqr
t(-a + b)/(a - b)) - (a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4*b -
2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
```

```
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
```

```
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integratio
n of abs or sign assumes constant sign by intervals (correct if the argumen
t is real):Check [abs(t_nostep^2-1)]Discontinuities at zeroes of t_nostep^2
-1 were not checkedWarning, integration of abs or sign assumes constant sig
n by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]W
arning, need to choose a branch for the root of a polynomial with parameter
s. This might be wrong.The choice was done assuming [a,b]=[-96,-55]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-15,-4]Discontinuities
at zeroes of t_nostep^2-1 were not checkedWarning, integration of abs or si
gn assumes constant sign by intervals (correct if the argument is real):Che
ck [abs(t_nostep-1)]Evaluation time: 7.96Error: Bad Argument Type
```

maple [B] time = 2.19, size = 32888, normalized size = 310.26

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 12.64, size = 1922, normalized size = 18.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] b/(f*(a + b*tan(e + f*x)^2)^(1/2)*(a*b - a^2)) - atanh((2*a^2*b^8*f^2*(a +
b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8*f^2 - 12*a^2*b^7*f^2 + 30*a^
3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5*b^4*f^2 - 6*a^6*b^3*f^2)) - (12*a^3*b^7
*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8*f^2 - 12*a^2*b^7*f
^2 + 30*a^3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5*b^4*f^2 - 6*a^6*b^3*f^2)) + (
30*a^4*b^6*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8*f^2 - 12
*a^2*b^7*f^2 + 30*a^3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5*b^4*f^2 - 6*a^6*b^3
*f^2)) - (38*a^5*b^5*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^
8*f^2 - 12*a^2*b^7*f^2 + 30*a^3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5*b^4*f^2 -
 6*a^6*b^3*f^2)) + (24*a^6*b^4*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/
2)*(2*a*b^8*f^2 - 12*a^2*b^7*f^2 + 30*a^3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5
*b^4*f^2 - 6*a^6*b^3*f^2)) - (6*a^7*b^3*f^2*(a + b*tan(e + f*x)^2)^(1/2))/(
(a^3)^(1/2)*(2*a*b^8*f^2 - 12*a^2*b^7*f^2 + 30*a^3*b^6*f^2 - 38*a^4*b^5*f^2
 + 24*a^5*b^4*f^2 - 6*a^6*b^3*f^2)))/(f*(a^3)^(1/2)) + (atan(((((a + b*tan
(e + f*x)^2)^(1/2)*(2*a^3*b^7*f^3 - 10*a^4*b^6*f^3 + 22*a^5*b^5*f^3 - 26*a^
6*b^4*f^3 + 16*a^7*b^3*f^3 - 4*a^8*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(12*a^5
*b^7*f^4 - 2*a^4*b^8*f^4 - 28*a^6*b^6*f^4 + 32*a^7*b^5*f^4 - 18*a^8*b^4*f^4
 + 4*a^9*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(8*a^5*b
^8*f^5 - 56*a^6*b^7*f^5 + 160*a^7*b^6*f^5 - 240*a^8*b^5*f^5 + 200*a^9*b^4*f
^5 - 88*a^10*b^3*f^5 + 16*a^11*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))
*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3) + ((((a + b*tan(e + f*x)^2)^(1/2)*(2*a
^3*b^7*f^3 - 10*a^4*b^6*f^3 + 22*a^5*b^5*f^3 - 26*a^6*b^4*f^3 + 16*a^7*b^3*
f^3 - 4*a^8*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(2*a^4*b^8*f^4 - 12*a^5*b^7*f^
4 + 28*a^6*b^6*f^4 - 32*a^7*b^5*f^4 + 18*a^8*b^4*f^4 - 4*a^9*b^3*f^4 + ((a
+ b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(8*a^5*b^8*f^5 - 56*a^6*b^7*f^5
 + 160*a^7*b^6*f^5 - 240*a^8*b^5*f^5 + 200*a^9*b^4*f^5 - 88*a^10*b^3*f^5 +
16*a^11*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2)*1i)/
(f*(a - b)^3))/(2*a^3*b^6*f^2 - 6*a^4*b^5*f^2 + 6*a^5*b^4*f^2 - 2*a^6*b^3*f
^2 - ((((a + b*tan(e + f*x)^2)^(1/2)*(2*a^3*b^7*f^3 - 10*a^4*b^6*f^3 + 22*a
^5*b^5*f^3 - 26*a^6*b^4*f^3 + 16*a^7*b^3*f^3 - 4*a^8*b^2*f^3))/2 + (((a - b
)^3)^(1/2)*(12*a^5*b^7*f^4 - 2*a^4*b^8*f^4 - 28*a^6*b^6*f^4 + 32*a^7*b^5*f^
4 - 18*a^8*b^4*f^4 + 4*a^9*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)
^3)^(1/2)*(8*a^5*b^8*f^5 - 56*a^6*b^7*f^5 + 160*a^7*b^6*f^5 - 240*a^8*b^5*f
^5 + 200*a^9*b^4*f^5 - 88*a^10*b^3*f^5 + 16*a^11*b^2*f^5))/(4*f*(a - b)^3))
)/(2*f*(a - b)^3))*((a - b)^3)^(1/2))/(f*(a - b)^3) + ((((a + b*tan(e + f*x
)^2)^(1/2)*(2*a^3*b^7*f^3 - 10*a^4*b^6*f^3 + 22*a^5*b^5*f^3 - 26*a^6*b^4*f^
3 + 16*a^7*b^3*f^3 - 4*a^8*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(2*a^4*b^8*f^4
- 12*a^5*b^7*f^4 + 28*a^6*b^6*f^4 - 32*a^7*b^5*f^4 + 18*a^8*b^4*f^4 - 4*a^9
*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(8*a^5*b^8*f^5 -
 56*a^6*b^7*f^5 + 160*a^7*b^6*f^5 - 240*a^8*b^5*f^5 + 200*a^9*b^4*f^5 - 88*
a^10*b^3*f^5 + 16*a^11*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*((a - b
)^3)^(1/2))/(f*(a - b)^3)))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(e + fx\right)}{\left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.337 $\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=157

$$\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{b(a-3b)}{2a^2f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}-\frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] 1/2*(2*a+3*b)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f-arctanh((
a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f-1/2*(a-3*b)*b/a^2/(a-b)/
f/(a+b*tan(f*x+e)^2)^(1/2)-1/2*cot(f*x+e)^2/a/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3670, 446, 103, 152, 156, 63, 208}

$$-\frac{b(a-3b)}{2a^2f(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}-\frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*a + 3*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*a^(5/2)*f) - A
rcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f) - ((a - 3*
b)*b)/(2*a^2*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) - Cot[e + f*x]^2/(2*a*f*
Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2a+3b)+\frac{3bx}{2}}{x(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2af}$$

$$= -\frac{(a-3b)b}{2a^2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{4}(}{}\right.}{}$$

$$= -\frac{(a-3b)b}{2a^2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{(1+}\right.}{}$$

$$= -\frac{(a-3b)b}{2a^2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{1-\frac{a}{b}}\right.}{}$$

$$= \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} - \frac{(a}{2a^2(a-b)f\sqrt{}}$$

Mathematica [C] time = 0.43, size = 115, normalized size = 0.73

$$\frac{(a-b)\left((2a+3b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\tan^2(e+fx)}{a}+1\right)+a\cot^2(e+fx)\right)-2a^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2a^2f(b-a)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*a^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)] + (
a - b)*(a*Cot[e + f*x]^2 + (2*a + 3*b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 +
(b*Tan[e + f*x]^2)/a]))/(2*a^2*(-a + b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

fricas [B] time = 0.54, size = 1252, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^3*b*tan(f*x + e)^4 + a^4*tan(f*x + e)^2)*sqrt(a - b)*log((b*tan
(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x
+ e)^2 + 1)) - ((2*a^3*b - a^2*b^2 - 4*a*b^3 + 3*b^4)*tan(f*x + e)^4 + (2*a
^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x +
e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) + 2*(a^4
```

```
- 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(b
*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a^
6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)^2), -1/4*(4*(a^3*b*tan(f*x + e)^4 + a
^4*tan(f*x + e)^2)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
+ b)/(a - b)) - ((2*a^3*b - a^2*b^2 - 4*a*b^3 + 3*b^4)*tan(f*x + e)^4 + (2*
a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x +
 e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) + 2*(a^
4 - 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(
b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a
^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)^2), -1/2*(((2*a^3*b - a^2*b^2 - 4*a*
b^3 + 3*b^4)*tan(f*x + e)^4 + (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x
 + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + (a^3*b*ta
n(f*x + e)^4 + a^4*tan(f*x + e)^2)*sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sq
rt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + (a^
4 - 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(
b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a
^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)^2), -1/2*(((2*a^3*b - a^2*b^2 - 4*a*
b^3 + 3*b^4)*tan(f*x + e)^4 + (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x
 + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + 2*(a^3*b*
tan(f*x + e)^4 + a^4*tan(f*x + e)^2)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x +
e)^2 + a)*sqrt(-a + b)/(a - b)) + (a^4 - 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2
*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4
*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)
^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
```

k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*

```
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sig
n by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]D
iscontinuities at zeroes of t_nostep^2-1 were not checkedWarning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(t_nostep^2-1)]Warning, need to choose a branch for t
he root of a polynomial with parameters. This might be wrong.The choice was
 done assuming [a,b]=[24,19]Warning, need to choose a branch for the root o
```

```
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b]=[-23,-97]Discontinuities at zeroes of t_nostep^2-1 were not che
ckedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant si
gn by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]
Evaluation time: 13.83Error: Bad Argument Type
```

maple [**B**] time = 3.12, size = 54353, normalized size = 346.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [**B**] time = 12.54, size = 2483, normalized size = 15.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (b^2/(a*b - a^2) + (b*(a + b*tan(e + f*x)^2)*(a - 3*b))/(2*a*(a*b - a^2)))/
(f*(a + b*tan(e + f*x)^2)^(3/2) - a*f*(a + b*tan(e + f*x)^2)^(1/2)) - (atan
((((((a + b*tan(e + f*x)^2)^(1/2)*(144*a^6*b^9*f^3 - 528*a^7*b^8*f^3 + 544*
a^8*b^7*f^3 + 160*a^9*b^6*f^3 - 496*a^10*b^5*f^3 - 16*a^11*b^4*f^3 + 320*a^
12*b^3*f^3 - 128*a^13*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(512*a^9*b^8*f^4 - 9
6*a^8*b^9*f^4 - 1056*a^10*b^7*f^4 + 1024*a^11*b^6*f^4 - 416*a^12*b^5*f^4 +
32*a^14*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(256*a^10
*b^8*f^5 - 1792*a^11*b^7*f^5 + 5120*a^12*b^6*f^5 - 7680*a^13*b^5*f^5 + 6400
*a^14*b^4*f^5 - 2816*a^15*b^3*f^5 + 512*a^16*b^2*f^5))/(4*f*(a - b)^3)))/(2
*f*(a - b)^3))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3) + ((((a + b*tan(e + f*x)
^2)^(1/2)*(144*a^6*b^9*f^3 - 528*a^7*b^8*f^3 + 544*a^8*b^7*f^3 + 160*a^9*b^
6*f^3 - 496*a^10*b^5*f^3 - 16*a^11*b^4*f^3 + 320*a^12*b^3*f^3 - 128*a^13*b^
2*f^3))/2 + (((a - b)^3)^(1/2)*(96*a^8*b^9*f^4 - 512*a^9*b^8*f^4 + 1056*a^1
0*b^7*f^4 - 1024*a^11*b^6*f^4 + 416*a^12*b^5*f^4 - 32*a^14*b^3*f^4 + ((a +
b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(256*a^10*b^8*f^5 - 1792*a^11*b^7
*f^5 + 5120*a^12*b^6*f^5 - 7680*a^13*b^5*f^5 + 6400*a^14*b^4*f^5 - 2816*a^1
5*b^3*f^5 + 512*a^16*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^
3)^(1/2)*1i)/(f*(a - b)^3))/(144*a^6*b^8*f^2 - 384*a^7*b^7*f^2 + 256*a^8*b^
6*f^2 + 96*a^9*b^5*f^2 - 144*a^10*b^4*f^2 + 32*a^11*b^3*f^2 - ((((a + b*tan
(e + f*x)^2)^(1/2)*(144*a^6*b^9*f^3 - 528*a^7*b^8*f^3 + 544*a^8*b^7*f^3 + 1
60*a^9*b^6*f^3 - 496*a^10*b^5*f^3 - 16*a^11*b^4*f^3 + 320*a^12*b^3*f^3 - 12
8*a^13*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(512*a^9*b^8*f^4 - 96*a^8*b^9*f^4 -
```

```
 1056*a^10*b^7*f^4 + 1024*a^11*b^6*f^4 - 416*a^12*b^5*f^4 + 32*a^14*b^3*f^4
 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(256*a^10*b^8*f^5 - 1792
*a^11*b^7*f^5 + 5120*a^12*b^6*f^5 - 7680*a^13*b^5*f^5 + 6400*a^14*b^4*f^5 -
 2816*a^15*b^3*f^5 + 512*a^16*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*
((a - b)^3)^(1/2))/(f*(a - b)^3) + ((((a + b*tan(e + f*x)^2)^(1/2)*(144*a^6
*b^9*f^3 - 528*a^7*b^8*f^3 + 544*a^8*b^7*f^3 + 160*a^9*b^6*f^3 - 496*a^10*b
^5*f^3 - 16*a^11*b^4*f^3 + 320*a^12*b^3*f^3 - 128*a^13*b^2*f^3))/2 + (((a -
 b)^3)^(1/2)*(96*a^8*b^9*f^4 - 512*a^9*b^8*f^4 + 1056*a^10*b^7*f^4 - 1024*a
^11*b^6*f^4 + 416*a^12*b^5*f^4 - 32*a^14*b^3*f^4 + ((a + b*tan(e + f*x)^2)^
(1/2)*((a - b)^3)^(1/2)*(256*a^10*b^8*f^5 - 1792*a^11*b^7*f^5 + 5120*a^12*b
^6*f^5 - 7680*a^13*b^5*f^5 + 6400*a^14*b^4*f^5 - 2816*a^15*b^3*f^5 + 512*a^
16*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2))/(f*(a -
b)^3)))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3) + (atanh((216*a^5*b^11*f^2*(a +
 b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(216*a^3*b^11*f^2 - 864*a^4*b^10*f^2
 + 936*a^5*b^9*f^2 + 496*a^6*b^8*f^2 - 1464*a^7*b^7*f^2 + 480*a^8*b^6*f^2 +
 440*a^9*b^5*f^2 - 240*a^10*b^4*f^2)) - (864*a^6*b^10*f^2*(a + b*tan(e + f*
x)^2)^(1/2))/((a^5)^(1/2)*(216*a^3*b^11*f^2 - 864*a^4*b^10*f^2 + 936*a^5*b^
9*f^2 + 496*a^6*b^8*f^2 - 1464*a^7*b^7*f^2 + 480*a^8*b^6*f^2 + 440*a^9*b^5*
f^2 - 240*a^10*b^4*f^2)) + (936*a^7*b^9*f^2*(a + b*tan(e + f*x)^2)^(1/2))/(
(a^5)^(1/2)*(216*a^3*b^11*f^2 - 864*a^4*b^10*f^2 + 936*a^5*b^9*f^2 + 496*a^
6*b^8*f^2 - 1464*a^7*b^7*f^2 + 480*a^8*b^6*f^2 + 440*a^9*b^5*f^2 - 240*a^10
*b^4*f^2)) + (496*a^8*b^8*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2
16*a^3*b^11*f^2 - 864*a^4*b^10*f^2 + 936*a^5*b^9*f^2 + 496*a^6*b^8*f^2 - 14
64*a^7*b^7*f^2 + 480*a^8*b^6*f^2 + 440*a^9*b^5*f^2 - 240*a^10*b^4*f^2)) - (
1464*a^9*b^7*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(216*a^3*b^11*f
^2 - 864*a^4*b^10*f^2 + 936*a^5*b^9*f^2 + 496*a^6*b^8*f^2 - 1464*a^7*b^7*f^
2 + 480*a^8*b^6*f^2 + 440*a^9*b^5*f^2 - 240*a^10*b^4*f^2)) + (480*a^10*b^6*
f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(216*a^3*b^11*f^2 - 864*a^4*
b^10*f^2 + 936*a^5*b^9*f^2 + 496*a^6*b^8*f^2 - 1464*a^7*b^7*f^2 + 480*a^8*b
^6*f^2 + 440*a^9*b^5*f^2 - 240*a^10*b^4*f^2)) + (440*a^11*b^5*f^2*(a + b*ta
n(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(216*a^3*b^11*f^2 - 864*a^4*b^10*f^2 + 93
6*a^5*b^9*f^2 + 496*a^6*b^8*f^2 - 1464*a^7*b^7*f^2 + 480*a^8*b^6*f^2 + 440*
a^9*b^5*f^2 - 240*a^10*b^4*f^2)) - (240*a^12*b^4*f^2*(a + b*tan(e + f*x)^2)
^(1/2))/((a^5)^(1/2)*(216*a^3*b^11*f^2 - 864*a^4*b^10*f^2 + 936*a^5*b^9*f^2
 + 496*a^6*b^8*f^2 - 1464*a^7*b^7*f^2 + 480*a^8*b^6*f^2 + 440*a^9*b^5*f^2 -
 240*a^10*b^4*f^2)))*(2*a + 3*b))/(2*f*(a^5)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.338 $\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=215

$$\frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{(8a^2+12ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{b\left(4a^2+3ab-15b^2\right)}{8a^3f(a-b)\sqrt{a+b\tan^2(e+fx)}}+\cdots$$

```
[Out] -1/8*(8*a^2+12*a*b+15*b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/2
)/f+arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+1/8*b*(4*a^
2+3*a*b-15*b^2)/a^3/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/8*(4*a+5*b)*cot(f*x+
e)^2/a^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/4*cot(f*x+e)^4/a/f/(a+b*tan(f*x+e)^2)
^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3670, 446, 103, 151, 152, 156, 63, 208}

$$\frac{b\left(4a^2+3ab-15b^2\right)}{8a^3f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{(8a^2+12ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((8*a^2 + 12*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8
*a^(7/2)*f) + ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2
)*f) + (b*(4*a^2 + 3*a*b - 15*b^2))/(8*a^3*(a - b)*f*Sqrt[a + b*Tan[e + f*x
]^2]) + ((4*a + 5*b)*Cot[e + f*x]^2)/(8*a^2*f*Sqrt[a + b*Tan[e + f*x]^2]) -
 Cot[e + f*x]^4/(4*a*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)}{4af\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a+5b)+\frac{5bx}{2}}{x^2(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{4af}$$

$$= \frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^4(e+fx)}{4af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}\left(8a^2+12ab+1\right.}{x(1+x}\right.}{}$$

$$= \frac{b\left(4a^2+3ab-15b^2\right)}{8a^3(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^4(e-}{4af\sqrt{a+b\,\text{ta}}}$$

$$= \frac{b\left(4a^2+3ab-15b^2\right)}{8a^3(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^4(e-}{4af\sqrt{a+b\,\text{ta}}}$$

$$= \frac{b\left(4a^2+3ab-15b^2\right)}{8a^3(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^4(e-}{4af\sqrt{a+b\,\text{ta}}}$$

$$= -\frac{\left(8a^2+12ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} + \frac{}{8}$$

Mathematica [C] time = 1.23, size = 142, normalized size = 0.66

$$\frac{8a^3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\tan^2(e+fx)+a}{a-b}\right)+(a-b)\left(a\cot^2(e+fx)\left(2a\cot^2(e+fx)-4a-5b\right)-\left(8a^2+12ab+15b^2\right)\,{}_2F\right.}{8a^3f(b-a)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (8*a^3*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)] + (a
 - b)*(a*Cot[e + f*x]^2*(-4*a - 5*b + 2*a*Cot[e + f*x]^2) - (8*a^2 + 12*a*b
 + 15*b^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tan[e + f*x]^2)/a]))/(8*a
^3*(-a + b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

fricas [A] time = 0.56, size = 1522, normalized size = 7.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a^4*b*tan(f*x + e)^6 + a^5*tan(f*x + e)^4)*sqrt(a - b)*log((b*ta
n(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x
 + e)^2 + 1)) - ((8*a^4*b - 4*a^3*b^2 - a^2*b^3 - 18*a*b^4 + 15*b^5)*tan(f*
x + e)^6 + (8*a^5 - 4*a^4*b - a^3*b^2 - 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e)
^4)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) +
2*a)/tan(f*x + e)^2) + 2*(2*a^5 - 4*a^4*b + 2*a^3*b^2 - (4*a^4*b - a^3*b^2
- 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^4 - (4*a^5 - 3*a^4*b - 6*a^3*b^2 + 5*
a^2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 +
a^4*b^3)*f*tan(f*x + e)^6 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^4), 1/
16*(16*(a^4*b*tan(f*x + e)^6 + a^5*tan(f*x + e)^4)*sqrt(-a + b)*arctan(-sqr
t(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) + ((8*a^4*b - 4*a^3*b^2 - a^2
*b^3 - 18*a*b^4 + 15*b^5)*tan(f*x + e)^6 + (8*a^5 - 4*a^4*b - a^3*b^2 - 18*
a^2*b^3 + 15*a*b^4)*tan(f*x + e)^4)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(
b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(2*a^5 - 4*a^4*b +
 2*a^3*b^2 - (4*a^4*b - a^3*b^2 - 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^4 - (
4*a^5 - 3*a^4*b - 6*a^3*b^2 + 5*a^2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e
)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^6 + (a^7 - 2*a^6*b
+ a^5*b^2)*f*tan(f*x + e)^4), 1/8*(((8*a^4*b - 4*a^3*b^2 - a^2*b^3 - 18*a*b
^4 + 15*b^5)*tan(f*x + e)^6 + (8*a^5 - 4*a^4*b - a^3*b^2 - 18*a^2*b^3 + 15*
a*b^4)*tan(f*x + e)^4)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/
a) - 4*(a^4*b*tan(f*x + e)^6 + a^5*tan(f*x + e)^4)*sqrt(a - b)*log((b*tan(f
*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x +
e)^2 + 1)) - (2*a^5 - 4*a^4*b + 2*a^3*b^2 - (4*a^4*b - a^3*b^2 - 18*a^2*b^3
 + 15*a*b^4)*tan(f*x + e)^4 - (4*a^5 - 3*a^4*b - 6*a^3*b^2 + 5*a^2*b^3)*tan
(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*t
an(f*x + e)^6 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^4), 1/8*(((8*a^4*b
 - 4*a^3*b^2 - a^2*b^3 - 18*a*b^4 + 15*b^5)*tan(f*x + e)^6 + (8*a^5 - 4*a^4
*b - a^3*b^2 - 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^4)*sqrt(-a)*arctan(sqrt(
b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + 8*(a^4*b*tan(f*x + e)^6 + a^5*tan(f*x +
 e)^4)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)
) - (2*a^5 - 4*a^4*b + 2*a^3*b^2 - (4*a^4*b - a^3*b^2 - 18*a^2*b^3 + 15*a*b
^4)*tan(f*x + e)^4 - (4*a^5 - 3*a^4*b - 6*a^3*b^2 + 5*a^2*b^3)*tan(f*x + e)
^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x +
e)^6 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
```

```
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
```

```
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
```

```
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep^2-1)]Discontinuities at zeroes of t_nostep^2-1 were not checkedWarnin
g, integration of abs or sign assumes constant sign by intervals (correct i
f the argument is real):Check [abs(t_nostep^2-1)]Warning, need to choose a
branch for the root of a polynomial with parameters. This might be wrong.Th
e choice was done assuming [a,b]=[79,3]Warning, need to choose a branch for
 the root of a polynomial with parameters. This might be wrong.The choice w
as done assuming [a,b]=[30,75]Discontinuities at zeroes of t_nostep^2-1 wer
e not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Warning, integration of abs or sign assumes constant sign by in
tervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluati
on time: 20.84Error: Bad Argument Type
```

maple [B] time = 5.35, size = 79934, normalized size = 371.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 13.13, size = 2118, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (atan((((((a + b*tan(e + f*x)^2)^(1/2)*(230400*a^9*b^11*f^3 - 783360*a^10*b
^10*f^3 + 854016*a^11*b^9*f^3 - 387072*a^12*b^8*f^3 + 480256*a^13*b^7*f^3 -
 680960*a^14*b^6*f^3 + 352256*a^15*b^5*f^3 - 262144*a^16*b^4*f^3 + 327680*a
^17*b^3*f^3 - 131072*a^18*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(638976*a^13*b^9
*f^4 - 122880*a^12*b^10*f^4 - 1318912*a^14*b^8*f^4 + 1376256*a^15*b^7*f^4 -
 794624*a^16*b^6*f^4 + 311296*a^17*b^5*f^4 - 122880*a^18*b^4*f^4 + 32768*a^
19*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(262144*a^15*b
^8*f^5 - 1835008*a^16*b^7*f^5 + 5242880*a^17*b^6*f^5 - 7864320*a^18*b^5*f^5
 + 6553600*a^19*b^4*f^5 - 2883584*a^20*b^3*f^5 + 524288*a^21*b^2*f^5))/(4*f
*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3) + ((((a
+ b*tan(e + f*x)^2)^(1/2)*(230400*a^9*b^11*f^3 - 783360*a^10*b^10*f^3 + 854
016*a^11*b^9*f^3 - 387072*a^12*b^8*f^3 + 480256*a^13*b^7*f^3 - 680960*a^14*
b^6*f^3 + 352256*a^15*b^5*f^3 - 262144*a^16*b^4*f^3 + 327680*a^17*b^3*f^3 -
 131072*a^18*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(122880*a^12*b^10*f^4 - 63897
6*a^13*b^9*f^4 + 1318912*a^14*b^8*f^4 - 1376256*a^15*b^7*f^4 + 794624*a^16*
b^6*f^4 - 311296*a^17*b^5*f^4 + 122880*a^18*b^4*f^4 - 32768*a^19*b^3*f^4 +
((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(262144*a^15*b^8*f^5 - 1835
008*a^16*b^7*f^5 + 5242880*a^17*b^6*f^5 - 7864320*a^18*b^5*f^5 + 6553600*a^
19*b^4*f^5 - 2883584*a^20*b^3*f^5 + 524288*a^21*b^2*f^5))/(4*f*(a - b)^3)))
/(2*f*(a - b)^3))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3))/(230400*a^9*b^10*f^2
 - 552960*a^10*b^9*f^2 + 301056*a^11*b^8*f^2 + 36864*a^12*b^7*f^2 + 123904*
a^13*b^6*f^2 - 147456*a^14*b^5*f^2 - 24576*a^15*b^4*f^2 + 32768*a^16*b^3*f^
2 - ((((a + b*tan(e + f*x)^2)^(1/2)*(230400*a^9*b^11*f^3 - 783360*a^10*b^10
*f^3 + 854016*a^11*b^9*f^3 - 387072*a^12*b^8*f^3 + 480256*a^13*b^7*f^3 - 68
0960*a^14*b^6*f^3 + 352256*a^15*b^5*f^3 - 262144*a^16*b^4*f^3 + 327680*a^17
*b^3*f^3 - 131072*a^18*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(638976*a^13*b^9*f^
4 - 122880*a^12*b^10*f^4 - 1318912*a^14*b^8*f^4 + 1376256*a^15*b^7*f^4 - 79
4624*a^16*b^6*f^4 + 311296*a^17*b^5*f^4 - 122880*a^18*b^4*f^4 + 32768*a^19*
b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(262144*a^15*b^8*
f^5 - 1835008*a^16*b^7*f^5 + 5242880*a^17*b^6*f^5 - 7864320*a^18*b^5*f^5 +
6553600*a^19*b^4*f^5 - 2883584*a^20*b^3*f^5 + 524288*a^21*b^2*f^5))/(4*f*(a
 - b)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2))/(f*(a - b)^3) + ((((a + b*ta
n(e + f*x)^2)^(1/2)*(230400*a^9*b^11*f^3 - 783360*a^10*b^10*f^3 + 854016*a^
11*b^9*f^3 - 387072*a^12*b^8*f^3 + 480256*a^13*b^7*f^3 - 680960*a^14*b^6*f^
3 + 352256*a^15*b^5*f^3 - 262144*a^16*b^4*f^3 + 327680*a^17*b^3*f^3 - 13107
2*a^18*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(122880*a^12*b^10*f^4 - 638976*a^13
*b^9*f^4 + 1318912*a^14*b^8*f^4 - 1376256*a^15*b^7*f^4 + 794624*a^16*b^6*f^
4 - 311296*a^17*b^5*f^4 + 122880*a^18*b^4*f^4 - 32768*a^19*b^3*f^4 + ((a +
b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(262144*a^15*b^8*f^5 - 1835008*a^
16*b^7*f^5 + 5242880*a^17*b^6*f^5 - 7864320*a^18*b^5*f^5 + 6553600*a^19*b^4
*f^5 - 2883584*a^20*b^3*f^5 + 524288*a^21*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*
(a - b)^3))*((a - b)^3)^(1/2))/(f*(a - b)^3)))*((a - b)^3)^(1/2)*1i)/(f*(a
- b)^3) - (b^3/(a*(a - b)) - (b*(a + b*tan(e + f*x)^2)*(5*a*b + 4*a^2 - 25*
```

```
b^2))/(8*(a^2*b - a^3)) + (b*(a + b*tan(e + f*x)^2)^2*(3*a*b + 4*a^2 - 15*b
^2))/(8*(a^3*b - a^4)))/(f*(a + b*tan(e + f*x)^2)^(5/2) + a^2*f*(a + b*tan(
e + f*x)^2)^(1/2) - 2*a*f*(a + b*tan(e + f*x)^2)^(3/2)) - (atan((a^15*b^12*
(a + b*tan(e + f*x)^2)^(1/2)*3375i - a^16*b^11*(a + b*tan(e + f*x)^2)^(1/2)
*12150i + a^17*b^10*(a + b*tan(e + f*x)^2)^(1/2)*13905i - a^18*b^9*(a + b*t
an(e + f*x)^2)^(1/2)*6912i + a^19*b^8*(a + b*tan(e + f*x)^2)^(1/2)*10953i -
 a^20*b^7*(a + b*tan(e + f*x)^2)^(1/2)*16542i + a^21*b^6*(a + b*tan(e + f*x
)^2)^(1/2)*7343i - a^22*b^5*(a + b*tan(e + f*x)^2)^(1/2)*1932i + a^23*b^4*(
a + b*tan(e + f*x)^2)^(1/2)*4200i - a^24*b^3*(a + b*tan(e + f*x)^2)^(1/2)*2
240i)/(a^7*(a^7)^(1/2)*(a^7*(6912*a*b^9 - 13905*b^10 - 10953*a^2*b^8 + 1654
2*a^3*b^7 - 7343*a^4*b^6 + 1932*a^5*b^5 - 4200*a^6*b^4 + 2240*a^7*b^3) - 33
75*a^5*b^12 + 12150*a^6*b^11)))*(12*a*b + 8*a^2 + 15*b^2)*1i)/(8*f*(a^7)^(1
/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.339 $\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{5/2}f}+\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b^2f(a-b)}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{a}{bf(a-b}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-1/2*
(3*a+2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f+1/
2*(3*a-b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/(a-b)/b^2/f-a*tan(f*x+e)^3/(a
-b)/b/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3670, 470, 582, 523, 217, 206, 377, 203}

$$\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b^2f(a-b)}-\frac{(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{5/2}f}-\frac{a\tan^3(e+fx)}{bf(a-b)\sqrt{a+b\tan^2(e+fx)}}-\tan$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) - ((3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*
x]^2]])/(2*b^(5/2)*f) - (a*Tan[e + f*x]^3)/((a - b)*b*f*Sqrt[a + b*Tan[e +
f*x]^2]) + ((3*a - b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*(a - b)*b
^2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{x^2(3a+(3a-b)x^2)}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{(a-b)bf}$$

$$= -\frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)b^2f} - \frac{\text{Su}}{}$$

$$= -\frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)b^2f} - \frac{\text{Su}}{}$$

$$= -\frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)b^2f} - \frac{\text{Su}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{a\tan^3(e}{(a-b)bf\sqrt{a+b}}$$

Mathematica [C] time = 6.44, size = 787, normalized size = 4.32

$$\frac{\sqrt{\frac{a\cos(2(e+fx))+a-b\cos(2(e+fx))+b}{\cos(2(e+fx))+1}}\left(\frac{\tan(e+fx)}{2b^2} - \frac{a^2\sin(2(e+fx))}{b^2(a-b)(a(-\cos(2(e+fx)))-a+b\cos(2(e+fx))-b)}\right)}{f} - \frac{-\frac{b(3a^2-ab-b^2)\sin^4(e+fx)\csc(2(e+fx)}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((-((b*(3*a^2 - a*b - b^2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Co
s[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*
x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f
*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e
 + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a -
b)*Cos[2*(e + f*x)]))) - (4*b^3*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a
 - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)
/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a
 - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSi
n[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*
Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*
(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x
)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*
x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a
 - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*
```

```
(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]))
)/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/((a - b)*b^2*f)) + (Sqrt[(a + b +
 a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(-((a^2*S
in[2*(e + f*x)])/((a - b)*b^2*(-a - b - a*Cos[2*(e + f*x)] + b*Cos[2*(e + f
*x)]))) + Tan[e + f*x]/(2*b^2)))/f
```

fricas [A] time = 2.34, size = 1207, normalized size = 6.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((3*a^4 - 4*a^3*b - a^2*b^2 + 2*a*b^3 + (3*a^3*b - 4*a^2*b^2 - a*b^3 +
 2*b^4)*tan(f*x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x +
 e)^2 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(b^4*tan(f*x + e)^2 + a*b^3)*sqrt(
-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*((a^2*b^2 - 2*a*b^3 + b
^4)*tan(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan
(f*x + e)^2 + a))/((a^2*b^4 - 2*a*b^5 + b^6)*f*tan(f*x + e)^2 + (a^3*b^3 -
2*a^2*b^4 + a*b^5)*f), 1/2*((3*a^4 - 4*a^3*b - a^2*b^2 + 2*a*b^3 + (3*a^3*b
 - 4*a^2*b^2 - a*b^3 + 2*b^4)*tan(f*x + e)^2)*sqrt(-b)*arctan(sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + (b^4*tan(f*x + e)^2 + a*b^3)*sqr
t(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqr
t(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + ((a^2*b^2 - 2*a*b^3 + b
^4)*tan(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan
(f*x + e)^2 + a))/((a^2*b^4 - 2*a*b^5 + b^6)*f*tan(f*x + e)^2 + (a^3*b^3 -
2*a^2*b^4 + a*b^5)*f), -1/4*(4*(b^4*tan(f*x + e)^2 + a*b^3)*sqrt(a - b)*arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a^4 - 4*a^
3*b - a^2*b^2 + 2*a*b^3 + (3*a^3*b - 4*a^2*b^2 - a*b^3 + 2*b^4)*tan(f*x + e
)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*
tan(f*x + e) + a) - 2*((a^2*b^2 - 2*a*b^3 + b^4)*tan(f*x + e)^3 + (3*a^3*b
- 4*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^2*b^4 -
2*a*b^5 + b^6)*f*tan(f*x + e)^2 + (a^3*b^3 - 2*a^2*b^4 + a*b^5)*f), -1/2*(2
*(b^4*tan(f*x + e)^2 + a*b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a
)/(sqrt(a - b)*tan(f*x + e))) - (3*a^4 - 4*a^3*b - a^2*b^2 + 2*a*b^3 + (3*a
^3*b - 4*a^2*b^2 - a*b^3 + 2*b^4)*tan(f*x + e)^2)*sqrt(-b)*arctan(sqrt(b*ta
n(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - ((a^2*b^2 - 2*a*b^3 + b^4)*t
an(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x
+ e)^2 + a))/((a^2*b^4 - 2*a*b^5 + b^6)*f*tan(f*x + e)^2 + (a^3*b^3 - 2*a^2
*b^4 + a*b^5)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{6}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 0.39, size = 286, normalized size = 1.57

$$\frac{\tan^{3}\left(fx+e\right)}{2fb\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}+\frac{3a\tan\left(fx+e\right)}{2f\,b^{2}\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}-\frac{3a\ln\left(\tan\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}\right)}{2f\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/f*tan(f*x+e)^3/b/(a+b*tan(f*x+e)^2)^(1/2)+3/2/f/b^2*a*tan(f*x+e)/(a+b*t
an(f*x+e)^2)^(1/2)-3/2/f/b^(5/2)*a*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)
^(1/2))+1/f*tan(f*x+e)/b/(a+b*tan(f*x+e)^2)^(1/2)-1/f/b^(3/2)*ln(tan(f*x+e)
*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))+1/f*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2
)+b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-1/f/(a-b)^2*(b^4*(a-b))^(
1/2)/b^2*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*
x+e))
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^6}{{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{6}{\left(e + f x \right)}}{\left(a + b \tan^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.340 $\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{3/2}f}-\frac{a\tan(e+fx)}{bf(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f+arcta
nh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f-a*tan(f*x+e)/(a-b
)/b/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3670, 470, 523, 217, 206, 377, 203}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{3/2}f}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{a\tan(e+fx)}{bf(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(b^(3/2)*
f) - (a*Tan[e + f*x])/((a - b)*b*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+(a-b)x^2}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{(a-b)bf}$$

$$= -\frac{a\tan(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{(a-b)f} + \frac{\text{Su}\ldots}{\ldots}$$

$$= -\frac{a\tan(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)f} + \frac{\text{Su}\ldots}{\ldots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a\tan(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+f\ldots}}$$

Mathematica [C] time = 3.20, size = 250, normalized size = 2.03

$$\frac{a\sin(2(e+fx))\sec^2(e+fx)\left(\frac{(a-b)\sqrt{\frac{\csc^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)}{\sqrt{2}} - \frac{b\sqrt{\frac{\csc^2(e+fx)((a-b\ldots}{\ldots}}}{\ldots}\ldots\right.}{\sqrt{2}bf(a-b)^2\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*(-a + b + ((a - b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*
x]^2)/b]/Sqrt[2]], 1])/Sqrt[2] - (b*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)]
)*Csc[e + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)
*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])/Sqrt[2])*Sec[e + f*x]^2
*Sin[2*(e + f*x)])/(Sqrt[2]*(a - b)^2*b*f*Sqrt[(a + b + (a - b)*Cos[2*(e +
f*x)])*Sec[e + f*x]^2])
```

fricas [B] time = 1.36, size = 974, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt
(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x +
 e) + a) + (b^3*tan(f*x + e)^2 + a*b^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*
x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan
(f*x + e)^2 + 1)) - 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a)*tan(f*x +
e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*
b^4)*f), -1/2*(2*(a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x +
 e)^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))
) - (b^3*tan(f*x + e)^2 + a*b^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^
2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
e)^2 + 1)) + 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e))/((a
^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f)
, 1/2*(2*(b^3*tan(f*x + e)^2 + a*b^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x +
e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2
*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan
(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 2*(a^2*b - a*b^2)*sqrt(b*tan(f
*x + e)^2 + a)*tan(f*x + e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 +
(a^3*b^2 - 2*a^2*b^3 + a*b^4)*f), ((b^3*tan(f*x + e)^2 + a*b^2)*sqrt(a - b)
*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (a^3 - 2*
a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt(-b)*arctan(sqr
t(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - (a^2*b - a*b^2)*sqrt(b
*tan(f*x + e)^2 + a)*tan(f*x + e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e
)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{4}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 0.27, size = 193, normalized size = 1.57

$$-\frac{\tan\left(fx+e\right)}{fb\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}+\frac{\ln\left(\tan\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}\right)}{f\,b^{\frac{3}{2}}}-\frac{\tan\left(fx+e\right)}{fa\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}-\frac{}{a(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f*tan(f*x+e)/b/(a+b*tan(f*x+e)^2)^(1/2)+1/f/b^(3/2)*ln(tan(f*x+e)*b^(1/2
)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)-b*tan
(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)^2*(b^4*(a-b))^(1/2)/b^
2*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{4}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^4}{\left(b\,\tan\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.341 $\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=81

$$\frac{\tan(e+fx)}{f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f+tan(
f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3670, 471, 377, 203}

$$\frac{\tan(e+fx)}{f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) + Tan[e + f*x]/((a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{(a-b)f}$$

$$= \frac{\tan(e+fx)}{(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} + \frac{\tan(e+fx)}{(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [A] time = 3.23, size = 154, normalized size = 1.90

$$\frac{\tan(e+fx)\left((a-b)\sqrt{\frac{b\tan^2(e+fx)}{a}+1}+\sqrt{\frac{(b-a)\tan^2(e+fx)}{a}}\left(a\cot^2(e+fx)+b\right)\tanh^{-1}\left(\frac{\sqrt{\frac{(b-a)\tan^2(e+fx)}{a}}}{\sqrt{\frac{b\tan^2(e+fx)}{a}+1}}\right)\right)}{f(a-b)^2\sqrt{a+b\tan^2(e+fx)}\sqrt{\frac{b\tan^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Tan[e + f*x]*(ArcTanh[Sqrt[((-a + b)*Tan[e + f*x]^2)/a]/Sqrt[1 + (b*Tan[e
+ f*x]^2)/a]]*(b + a*Cot[e + f*x]^2)*Sqrt[((-a + b)*Tan[e + f*x]^2)/a] + (a
 - b)*Sqrt[1 + (b*Tan[e + f*x]^2)/a]))/((a - b)^2*f*Sqrt[a + b*Tan[e + f*x]
^2]*Sqrt[1 + (b*Tan[e + f*x]^2)/a])
```

fricas [A] time = 0.47, size = 285, normalized size = 3.52

$$\left[\frac{\left(b\tan\left(fx+e\right)^2+a\right)\sqrt{-a+b}\,\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2-2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\,\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^2+1}\right)+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,(}{2\left(\left(a^2b-2\,ab^2+b^3\right)f\tan\left(fx+e\right)^2+\left(a^3-2\,a^2b+ab^2\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*tan(f*x + e)^2 + a)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 -
2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2
 + 1)) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a - b)*tan(f*x + e))/((a^2*b - 2*a*b
^2 + b^3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f), -((b*tan(f*x + e)^
2 + a)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x
+ e))) - sqrt(b*tan(f*x + e)^2 + a)*(a - b)*tan(f*x + e))/((a^2*b - 2*a*b^2
 + b^3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{2}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 0.24, size = 131, normalized size = 1.62

$$\frac{\tan\left(fx+e\right)}{fa\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}+\frac{b\tan\left(fx+e\right)}{a\left(a-b\right)f\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}-\frac{\sqrt{b^{4}\left(a-b\right)}\ \arctan\left(\frac{(a-b)b^{2}\tan\left(fx+e\right)}{\sqrt{b^{4}(a-b)}\ \sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}\right)}{f\left(a-b\right)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/f*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)+b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f
*x+e)^2)^(1/2)-1/f/(a-b)^2*(b^4*(a-b))^(1/2)/b^2*arctan((a-b)*b^2/(b^4*(a-b
))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^2}{{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}{\left(e+fx\right)}}{\left(a+b\tan^{2}{\left(e+fx\right)}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.342 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}} dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\tan(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*tan
(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3661, 382, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\tan(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) - (b*Tan[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [C] time = 6.28, size = 214, normalized size = 2.52

$$\frac{4\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(\frac{15\left(3a+2b\tan^2(e+fx)\right)\left(a\sqrt{\frac{(a-b)\sin^2(2(e+fx))\left(a+b\tan^2(e+fx)\right)}{a^2}}-2\sin^{-1}\left(\sqrt{\frac{(a-b)\sin^2(e}{a}}\right.\right.}{\left(\frac{(a-b)\sin^2(2(e+fx))\left(a+b\tan^2(e+fx)\right)}{a^2}\right)^{3/2}}\right.}{15a^4f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] (4*Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2]*(a*(a - b)*Hyperg
eometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^2 + (15*(3*
a + 2*b*Tan[e + f*x]^2)*(-2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*(a*Cos
[e + f*x]^2 + b*Sin[e + f*x]^2) + a*Sqrt[((a - b)*Sin[2*(e + f*x)]^2*(a + b
*Tan[e + f*x]^2))/a^2]))/(((a - b)*Sin[2*(e + f*x)]^2*(a + b*Tan[e + f*x]^2
))/a^2)^(3/2)))/(15*a^4*f)
```

fricas [A] time = 0.45, size = 310, normalized size = 3.65

$$\left[\frac{\left(ab\tan\left(fx+e\right)^2+a^2\right)\sqrt{-a+b}\,\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\,\tan\left(fx+e\right)-a}{\tan\left(fx+e\right)^2+1}\right)-2\,\sqrt{b\tan\left(fx+e\right)^2}}{2\left(\left(a^3b-2\,a^2b^2+ab^3\right)f\tan\left(fx+e\right)^2+\left(a^4-2\,a^3b+a^2b^2\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^
2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x + e))/((a^3*b
 - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f), ((a*
b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqr
t(a - b)*tan(f*x + e))) - sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x +
```

```
e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^
2)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(-3/2), x)
```

maple [A] time = 0.44, size = 104, normalized size = 1.22

$$-\frac{b\tan\left(fx+e\right)}{a\left(a-b\right)f\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}+\frac{\sqrt{b^4\left(a-b\right)}\,\arctan\left(\frac{(a-b)b^2\tan\left(fx+e\right)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}\right)}{f\left(a-b\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)^2*(b^4*(a-b))^(1
/2)/b^2*arctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x
+e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(-3/2), x)
```

3.343 $\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a^2f(a-b)}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\cot(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*co
t(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-(a-2*b)*cot(f*x+e)*(a+b*tan(f*x
+e)^2)^(1/2)/a^2/(a-b)/f
```

Rubi [A] time = 0.18, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3670, 472, 583, 12, 377, 203}

$$-\frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a^2f(a-b)}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\cot(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) - (b*Cot[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) - ((a -
 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a^2*(a - b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-2b-2bx^2}{x^2(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{a(a-b)f}$$

$$= -\frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a^2(a-b)f} - \frac{\text{Su}}{}$$

$$= -\frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a^2(a-b)f} - \frac{\text{Su}}{}$$

$$= -\frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a^2(a-b)f} - \frac{\text{Su}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-2b)\cot(e+fx)\sqrt{}}{a^2(a-}$$

Mathematica [C] time = 13.60, size = 882, normalized size = 6.89

$$-\frac{\cos^2(e+fx)\cot(e+fx)\left(\frac{8(a-b)b^2\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)\sin^2(e+fx)\tan^4(e+fx)}{5a^3} + \frac{8(a-b)b^2\,{}_3F_2\left(2,2,2;1,\frac{7}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)\sin^2(e+fx)}{15a^3}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Cos[e + f*x]^2*Cot[e + f*x]*((3*a*Csc[e + f*x]^2)/(a - b) + (12*b*Sec[e
+ f*x]^2)/(a - b) + (16*(a - b)*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Sin[e
 + f*x]^2)/a]*Sin[e + f*x]^2)/(15*a) + (8*(a - b)*HypergeometricPFQ[{2, 2,
2}, {1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/(15*a) + (8*b^2*S
ec[e + f*x]^2*Tan[e + f*x]^2)/(a*(a - b)) + (8*(a - b)*b*Hypergeometric2F1[
2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*a^
2) + (16*(a - b)*b*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Sin[e +
f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(15*a^2) + (8*(a - b)*b^2*Hyperge
ometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*
x]^4)/(5*a^3) + (8*(a - b)*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a -
 b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(15*a^3) - (3*ArcSin[
Sqrt[((a - b)*Sin[e + f*x]^2)/a]])/((((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt
[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) - (12*b*ArcSin[Sqrt[((a - b)*S
in[e + f*x]^2)/a]]*Tan[e + f*x]^2)/(a*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sq
rt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) - (8*b^2*ArcSin[Sqrt[((a - b
)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^4)/(a^2*(((a - b)*Sin[e + f*x]^2)/a)^(3/
2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) + (3*ArcSin[Sqrt[((a -
b)*Sin[e + f*x]^2)/a]])/Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*
Tan[e + f*x]^2))/a^2] + (12*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[
e + f*x]^2)/(a*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f
*x]^2))/a^2]) + (8*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x
]^4)/(a^2*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2
))/a^2])))/(a*f*Sqrt[a + b*Tan[e + f*x]^2]))
```

fricas [A] time = 1.02, size = 471, normalized size = 3.68

$$\left[\frac{\left(a^2 b \tan\left(fx+e\right)^3 + a^3 \tan\left(fx+e\right)\right)\sqrt{-a+b}\,\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)\tan\left(fx+e\right)^4 - 2\left(3\,a^2-4\,ab\right)\tan\left(fx+e\right)^2 + a^2 - 4\left(\left(a-2\,b\right)\tan\left(f\right.\right.}{\tan\left(fx+e\right)^4 + 2\,\tan\left(fx+e\right)^2 +}\right.}{4\left(\left(a^4 b - 2\,a^3 b^2 + a^2 b^3\right)f\tan\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*b*tan(f*x + e)^3 + a^3*tan(f*x + e))*sqrt(-a + b)*log(-((a^2 - 8
*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*(
(a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(a^3 - 2*a^2*b + a*b^
2 + (a^2*b - 3*a*b^2 + 2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/(
(a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*b^2)*
f*tan(f*x + e)), -1/2*((a^2*b*tan(f*x + e)^3 + a^3*tan(f*x + e))*sqrt(a - b
)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*
tan(f*x + e)^2 - a)) + 2*(a^3 - 2*a^2*b + a*b^2 + (a^2*b - 3*a*b^2 + 2*b^3)
*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4*b - 2*a^3*b^2 + a^2*b^3)
*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*b^2)*f*tan(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.43, size = 1305, normalized size = 10.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2
)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))
/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos
(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*(
(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/
2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)*sin(f*
x+e)*a^2-2*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)
+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-
b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(
f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b
)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/
2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f
*x+e)*sin(f*x+e)*a^2+2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/
2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(
f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-
b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1
/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^
(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)*a^2-2*a^2*2^(1/2)*((I*cos(f
*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b
)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/
2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi(
(-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*
I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/(
(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)+((2*I*(a-b)^(1/2)*b^(1
/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a^2-2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^
(1/2)*cos(f*x+e)^2*a*b+2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+
e)^2*b^2+((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b-2*((2*I*(a-b)^(1/2)*
b^(1/2)+a-2*b)/a)^(1/2)*b^2)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)/a^2/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1
/2)/(a-b)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^2}{{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.344 $\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=184

$$\frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f(a-b)}-\frac{(a-4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f(a-b)}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*cot
(f*x+e)^3/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/3*(3*a-4*b)*(a+2*b)*cot(f*x+
e)*(a+b*tan(f*x+e)^2)^(1/2)/a^3/(a-b)/f-1/3*(a-4*b)*cot(f*x+e)^3*(a+b*tan(f
*x+e)^2)^(1/2)/a^2/(a-b)/f
```

Rubi [A] time = 0.26, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3670, 472, 583, 12, 377, 203}

$$-\frac{(a-4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f(a-b)}+\frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f(a-b)}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) - (b*Cot[e + f*x]^3)/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) + ((3*a
- 4*b)*(a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^3*(a - b)*f)
 - ((a - 4*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^2*(a - b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-4b-4bx^2}{x^4\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{a(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2(a-b)f} -$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e}{3a^3(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e}{3a^3(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e}{3a^3(a-b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)(a+2b)\,\mathrm{c}}{}$$

Mathematica [C] time = 16.45, size = 802, normalized size = 4.36

$$-\frac{b\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{\cos(2(e+fx))+1}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\csc(2(e+fx))F\left(\sin^{-1}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+\cdots}{b}}}{\sqrt{2}}\right.\right.}{a(a+b+(a-b)\cos(2(e+fx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-((b*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[
-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b
)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f
*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e + f*x)])))
 - (4*b*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/
(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[
2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*
Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)
*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqr
t[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((
a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*
Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)
]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*
Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(
e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)*C
os[2*(e + f*x)]])/((a - b)*f) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2
*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(((4*a*Cos[e + f*x] + 5*b*Cos[e + f*x]
)*Csc[e + f*x])/(3*a^3) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*a^2) - (b^3*Sin[
2*(e + f*x)])/(a^3*(a - b)*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)]
))))/f
```

fricas [A] time = 0.58, size = 579, normalized size = 3.15

$$\left[\frac{3\left(a^3b\tan\left(fx+e\right)^5+a^4\tan\left(fx+e\right)^3\right)\sqrt{-a+b}\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)\tan\left(fx+e\right)^4-2\left(3\,a^2-4\,ab\right)\tan\left(fx+e\right)^2+a^2+4\left((a-2\,b)\tan\left(f\cdots\right.\right.}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2+\cdots}\right.}{12\left(\left(a^5b\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^3*b*tan(f*x + e)^5 + a^4*tan(f*x + e)^3)*sqrt(-a + b)*log(-((a^
2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2
+ 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*
sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*((3*a^3*b - a^2*
b^2 - 10*a*b^3 + 8*b^4)*tan(f*x + e)^4 - a^4 + 2*a^3*b - a^2*b^2 + (3*a^4 -
 2*a^3*b - 5*a^2*b^2 + 4*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))
/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^4*b^2
)*f*tan(f*x + e)^3), 1/6*(3*(a^3*b*tan(f*x + e)^5 + a^4*tan(f*x + e)^3)*sqr
t(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a
- 2*b)*tan(f*x + e)^2 - a)) + 2*((3*a^3*b - a^2*b^2 - 10*a*b^3 + 8*b^4)*tan
(f*x + e)^4 - a^4 + 2*a^3*b - a^2*b^2 + (3*a^4 - 2*a^3*b - 5*a^2*b^2 + 4*a*
```

```
b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4*b^2 + a^3*
b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.21, size = 2577, normalized size = 14.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(-6*2^(1/2)*((I*cos(f*x+e)*(a-b)
^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*
x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-
a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x
+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/
2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^
(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^3+3*2^(1/2)*((I*co
s(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e
)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^
(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*Elliptic
F((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*
I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2)
)*cos(f*x+e)^3*sin(f*x+e)*a^3-6*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-
I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*
(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*
cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b
)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*
b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a
-2*b)/a)^(1/2))*cos(f*x+e)^2*sin(f*x+e)*a^3+3*2^(1/2)*((I*cos(f*x+e)*(a-b)^
(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x
+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a
*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e
))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b
^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^2*
sin(f*x+e)*a^3+6*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b
^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+
e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(
1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)
+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-
b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))
*cos(f*x+e)*sin(f*x+e)*a^3-3*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(
a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+co
s(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1
/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(
f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1
/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^3+4*((
2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*a^3-3*((2*I*(a-b)^(1/2
```

```
)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*a^2*b-6*((2*I*(a-b)^(1/2)*b^(1/2)+a-
2*b)/a)^(1/2)*cos(f*x+e)^4*a*b^2+8*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2
)*cos(f*x+e)^4*b^3+6*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-
I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*
(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*
cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b
)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*
b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a
-2*b)/a)^(1/2))*a^3-3*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)
-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)
*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b
*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b
)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a
-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^3-3*((2*I*(a-b)^(1/2)*b^
(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a^3+5*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a
)^(1/2)*cos(f*x+e)^2*a^2*b+8*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(
f*x+e)^2*a*b^2-16*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b^
3-3*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^2*b-2*((2*I*(a-b)^(1/2)*b^(
1/2)+a-2*b)/a)^(1/2)*a*b^2+8*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^3)
*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/sin(f*
x+e)^3/a^3/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/(a-b)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^4}{{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{4}{\left(e + f x \right)}}{\left(a + b \tan^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.345 $\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}} dx$

Optimal. Leaf size=252

$$-\frac{(a-6b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a^2f(a-b)}+\frac{\left(5a^2+4ab-24b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f(a-b)}-\frac{\left(15a^3+10\right.}{}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*co
t(f*x+e)^5/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^3+10*a^2*b+8*a*b^2
-48*b^3)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^4/(a-b)/f+1/15*(5*a^2+4*a*b-
24*b^2)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a^3/(a-b)/f-1/5*(a-6*b)*cot(f
*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2)/a^2/(a-b)/f
```

Rubi [A] time = 0.37, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3670, 472, 583, 12, 377, 203}

$$\frac{\left(5a^2+4ab-24b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f(a-b)}-\frac{\left(10a^2b+15a^3+8ab^2-48b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e}}{15a^4f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) - (b*Cot[e + f*x]^5)/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) - ((1
5*a^3 + 10*a^2*b + 8*a*b^2 - 48*b^3)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2
])/(15*a^4*(a - b)*f) + ((5*a^2 + 4*a*b - 24*b^2)*Cot[e + f*x]^3*Sqrt[a + b
*Tan[e + f*x]^2])/(15*a^3*(a - b)*f) - ((a - 6*b)*Cot[e + f*x]^5*Sqrt[a + b
*Tan[e + f*x]^2])/(5*a^2*(a - b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
```

```
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-6b-6bx^2}{x^6(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{a(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-6b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a^2(a-b)f} -$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\left(5a^2+4ab-24b^2\right)\cot^3(e+fx)\sqrt{a+b\tan}}{15a^3(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(15a^3+10a^2b+8ab^2-48b^3\right)\cot(e+fx)\sqrt{}}{15a^4(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(15a^3+10a^2b+8ab^2-48b^3\right)\cot(e+fx)\sqrt{}}{15a^4(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(15a^3+10a^2b+8ab^2-48b^3\right)\cot(e+fx)\sqrt{}}{15a^4(a-b)f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(15a^3+10a^2b+8\right.}{}$$

Mathematica [C] time = 16.56, size = 850, normalized size = 3.37

$$\frac{\sqrt{\frac{\cos(2(e+fx))a+a+b-b\cos(2(e+fx))}{\cos(2(e+fx))+1}}\left(\frac{\sin(2(e+fx))b^4}{a^4(a-b)(\cos(2(e+fx))a+a+b-b\cos(2(e+fx)))} - \frac{\cot(e+fx)\csc^4(e+fx)}{5a^2} + \frac{(11a\cos(e+fx)+9b\cos(e+fx}{15a^3}\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((-((b*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqr
t[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)
/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e +
 f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*
x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e + f*x)])
)) - (4*b*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)]
)/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Co
```

```
s[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)]
)*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a -
b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*S
qrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-
((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)
]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*
x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)]
)*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2
*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)
*Cos[2*(e + f*x)]])/((a - b)*f)) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Co
s[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(((-23*a^2*Cos[e + f*x] - 34*a*b*Co
s[e + f*x] - 33*b^2*Cos[e + f*x])*Csc[e + f*x])/(15*a^4) + ((11*a*Cos[e + f
*x] + 9*b*Cos[e + f*x])*Csc[e + f*x]^3)/(15*a^3) - (Cot[e + f*x]*Csc[e + f*
x]^4)/(5*a^2) + (b^4*Sin[2*(e + f*x)])/(a^4*(a - b)*(a + b + a*Cos[2*(e + f
*x)] - b*Cos[2*(e + f*x)]))))/f
```

fricas [A] time = 0.56, size = 687, normalized size = 2.73

$$\left[\frac{15\left(a^4 b\tan\left(fx+e\right)^7+a^5\tan\left(fx+e\right)^5\right)\sqrt{-a+b}\,\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)\tan\left(fx+e\right)^4-2\left(3\,a^2-4\,ab\right)\tan\left(fx+e\right)^2+a^2-4\left(\left(a-2\,b\right)\tan\left(\right.\right.}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(a^4*b*tan(f*x + e)^7 + a^5*tan(f*x + e)^5)*sqrt(-a + b)*log(-((a
^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2
 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*((15*a^4*b - 5*
a^3*b^2 - 2*a^2*b^3 - 56*a*b^4 + 48*b^5)*tan(f*x + e)^6 + 3*a^5 - 6*a^4*b +
 3*a^3*b^2 + (15*a^5 - 10*a^4*b - a^3*b^2 - 28*a^2*b^3 + 24*a*b^4)*tan(f*x
+ e)^4 - (5*a^5 - 4*a^4*b - 7*a^3*b^2 + 6*a^2*b^3)*tan(f*x + e)^2)*sqrt(b*t
an(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^7 + (a^7
- 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^5), -1/30*(15*(a^4*b*tan(f*x + e)^7 + a
^5*tan(f*x + e)^5)*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
- b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*((15*a^4*b - 5*a^3*b^
2 - 2*a^2*b^3 - 56*a*b^4 + 48*b^5)*tan(f*x + e)^6 + 3*a^5 - 6*a^4*b + 3*a^3
*b^2 + (15*a^5 - 10*a^4*b - a^3*b^2 - 28*a^2*b^3 + 24*a*b^4)*tan(f*x + e)^4
 - (5*a^5 - 4*a^4*b - 7*a^3*b^2 + 6*a^2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x
 + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^7 + (a^7 - 2*a^
6*b + a^5*b^2)*f*tan(f*x + e)^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^6}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

maple [C] time = 1.78, size = 3925, normalized size = 15.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(15*sin(f*x+e)*2^(1/2)*((I*cos(
f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+
b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1
/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF(
(-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*
(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*
a^4+8*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b^3+10*((2*I*(a-b)^(1/2)*
b^(1/2)+a-2*b)/a)^(1/2)*a^2*b^2-30*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^
(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x
+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a
*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+
e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2
)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(
1/2)*b^(1/2)+a-2*b)/a)^(1/2))*a^4+48*cos(f*x+e)^6*((2*I*(a-b)^(1/2)*b^(1/2)
+a-2*b)/a)^(1/2)*b^4+23*cos(f*x+e)^6*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1
/2)*a^4-35*cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^4+15*co
s(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^4-144*cos(f*x+e)^4*(
(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^4+144*cos(f*x+e)^2*((2*I*(a-b)^(
1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^4-12*cos(f*x+e)^6*((2*I*(a-b)^(1/2)*b^(1/2)+
a-2*b)/a)^(1/2)*a^3*b+34*cos(f*x+e)^4*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(
1/2)*a^3*b-40*cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^3*b-
12*cos(f*x+e)^6*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^2*b^2-32*cos(f*
x+e)^6*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b^3+28*cos(f*x+e)^4*((2*
I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^2*b^2+72*cos(f*x+e)^4*((2*I*(a-b)^(
1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a*b^3-26*cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2
)+a-2*b)/a)^(1/2)*a^2*b^2-48*cos(f*x+e)^2*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/
a)^(1/2)*a*b^3+15*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*a^3*b+15*cos(f*
x+e)^5*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*
b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x
+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/
(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)
+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/
2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^4-30*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*
cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x
+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b
)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-
1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1
/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*a^4-30*cos(f*x+e)^4*sin(f*x+
e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f
*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)
*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/
a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/
2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/
2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*a^4-30*cos(f*
x+e)^3*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*
b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x
+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/
(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)
+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/
2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^4+60*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*((
I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f
*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a
-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*Elli
pticPi((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e)
,-1/(2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^
```

```
(1/2)/((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*a^4-30*cos(f*x+e)^2*sin(f*
x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos
(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/
2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e)
)/a)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1
/2)/sin(f*x+e),((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*
b+8*b^2)/a^2)^(1/2))*a^4+60*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*
(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+c
os(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(
1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+co
s(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b
)^(1/2)*b^(1/2)+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(
a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2))*a^4+15*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((
I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f
*x+e)+b)/(1+cos(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a
-b)^(1/2)*b^(1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*Elli
pticF((-1+cos(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),
((8*I*(a-b)^(1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(
1/2))*a^4-30*cos(f*x+e)^5*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*(a-b)^(1/2)*b^(
1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+cos(f*x+e))/a)^(
1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)-a*cos(f*x+
e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I
*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*(a-b)^(1/2)*b^(1/2)
+a-2*b)*a,(-(2*I*(a-b)^(1/2)*b^(1/2)-a+2*b)/a)^(1/2)/((2*I*(a-b)^(1/2)*b^(1
/2)+a-2*b)/a)^(1/2))*a^4+15*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*
(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(1/2)+a*cos(f*x+e)-b*cos(f*x+e)+b)/(1+c
os(f*x+e))/a)^(1/2)*(-2*(I*cos(f*x+e)*(a-b)^(1/2)*b^(1/2)-I*(a-b)^(1/2)*b^(
1/2)-a*cos(f*x+e)+b*cos(f*x+e)-b)/(1+cos(f*x+e))/a)^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*(a-b)^(
1/2)*b^(3/2)-4*I*(a-b)^(1/2)*b^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^4-48*
((2*I*(a-b)^(1/2)*b^(1/2)+a-2*b)/a)^(1/2)*b^4)*cos(f*x+e)^3*((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)^5/a^4/((2*I*(a-b)^(1/2)*
b^(1/2)+a-2*b)/a)^(1/2)/(a-b)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**6/(a + b*tan(e + f*x)**2)**(3/2), x)
```

3.346 $\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=115

$$\frac{a^2}{3b^2f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{a(a-2b)}{b^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f-a*(a-2*b)/(a-b
)^2/b^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1/3*a^2/(a-b)/b^2/f/(a+b*tan(f*x+e)^2)^(
3/2)
```

Rubi [A] time = 0.20, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3670, 446, 87, 63, 208}

$$\frac{a^2}{3b^2f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{a(a-2b)}{b^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f)) + a^2/
(3*(a - b)*b^2*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (a*(a - 2*b))/((a - b)^2*b
^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a^2}{(a-b)b(a+bx)^{5/2}} + \frac{a(a-2b)}{(a-b)^2b(a+bx)^{3/2}} + \frac{1}{(a-b)^2(1+x)\sqrt{a+bx}}\right) dx, x, \tan^2(e+f\text{x}}{2f}$$

$$= \frac{a^2}{3(a-b)b^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2b)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subs}}{}$$

$$= \frac{a^2}{3(a-b)b^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2b)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subs}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \frac{a^2}{3(a-b)b^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{(a-b)^2b^2f}$$

Mathematica [C] time = 0.46, size = 91, normalized size = 0.79

$$\frac{b^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\tan^2(e+fx)+a}{a-b}\right) - (a-b)\left(2a+3b\tan^2(e+fx)-b\right)}{3b^2f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (b^2*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]^2)/(a - b)] - (a
- b)*(2*a - b + 3*b*Tan[e + f*x]^2))/(3*(a - b)*b^2*f*(a + b*Tan[e + f*x]^2
)^(3/2))
```

fricas [B] time = 0.55, size = 608, normalized size = 5.29

$$\left[\frac{3\left(b^4\tan\left(fx+e\right)^4 + 2\,ab^3\tan\left(fx+e\right)^2 + a^2b^2\right)\sqrt{a-b}\,\log\left(-\frac{b^2\tan\left(fx+e\right)^4 + 2\left(4\,ab-3\,b^2\right)\tan\left(fx+e\right)^2 - 4\left(b\tan\left(fx+e\right)^2 + 2\right.}{\tan\left(fx+e\right)^4 + 2\,\tan\left(fx-\right.}\right.}{12\left(\left(a^3b^4 - 3\,a^2b^5 + 3\,ab^6 - b^7\right)f\tan\left(fx+e\right)^4 + 2\left(a^4b^3 - \right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^4*tan(f*x + e)^4 + 2*a*b^3*tan(f*x + e)^2 + a^2*b^2)*sqrt(a - b
)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*
x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b
+ b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(2*a^4 - 7*a^3*b + 5*a^
2*b^2 + 3*(a^3*b - 3*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)
^2 + a))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*f*tan(f*x + e)^4 + 2*(a^4*b
^3 - 3*a^3*b^4 + 3*a^2*b^5 - a*b^6)*f*tan(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3
 + 3*a^3*b^4 - a^2*b^5)*f), 1/6*(3*(b^4*tan(f*x + e)^4 + 2*a*b^3*tan(f*x +
e)^2 + a^2*b^2)*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
b)/(b*tan(f*x + e)^2 + 2*a - b)) - 2*(2*a^4 - 7*a^3*b + 5*a^2*b^2 + 3*(a^3*
b - 3*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^3*
b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*f*tan(f*x + e)^4 + 2*(a^4*b^3 - 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6)*f*tan(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 -
a^2*b^5)*f)]
```

giac [A] time = 4.22, size = 137, normalized size = 1.19

$$\frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\left(a^2f-2\,abf+b^2f\right)\sqrt{-a+b}}-\frac{3\left(b\tan\left(fx+e\right)^2+a\right)a^2-a^3-6\left(b\tan\left(fx+e\right)^2+a\right)ab+a^2b}{3\left(a^2b^2f-2\,ab^3f+b^4f\right)\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a^2*f - 2*a*b*f + b^2*f)*
sqrt(-a + b)) - 1/3*(3*(b*tan(f*x + e)^2 + a)*a^2 - a^3 - 6*(b*tan(f*x + e)
^2 + a)*a*b + a^2*b)/((a^2*b^2*f - 2*a*b^3*f + b^4*f)*(b*tan(f*x + e)^2 + a
)^(3/2))
```

maple [A] time = 0.34, size = 169, normalized size = 1.47

$$-\frac{\tan^2\left(fx+e\right)}{fb\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}-\frac{2a}{3fb^2\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}+\frac{1}{3fb\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}+\frac{1}{3\left(a-b\right)f\left(a+b\left(\tan\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/f*tan(f*x+e)^2/b/(a+b*tan(f*x+e)^2)^(3/2)-2/3/f*a/b^2/(a+b*tan(f*x+e)^2)
^(3/2)+1/3/f/b/(a+b*tan(f*x+e)^2)^(3/2)+1/3/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2
)+1/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)^2/(-a+b)^(1/2)*arctan((a+b
*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 16.03, size = 148, normalized size = 1.29

$$\frac{\frac{a^2}{3\,(a-b)}+\frac{\left(b\tan\left(e+f\,x\right)^2+a\right)\left(2\,a\,b-a^2\right)}{(a-b)^2}}{b^2\,f\,\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}+\frac{\mathrm{atan}\left(\frac{a^2\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,1\mathrm{i}+b^2\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,1\mathrm{i}-a\,b\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,2\mathrm{i}}{(a-b)^{5/2}}\right)1\mathrm{i}}{f\,(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] (atan((a^2*(a + b*tan(e + f*x)^2)^(1/2)*1i + b^2*(a + b*tan(e + f*x)^2)^(1/
2)*1i - a*b*(a + b*tan(e + f*x)^2)^(1/2)*2i)/(a - b)^(5/2))*1i)/(f*(a - b)^
(5/2)) + (a^2/(3*(a - b)) + ((a + b*tan(e + f*x)^2)*(2*a*b - a^2))/(a - b)^
2)/(b^2*f*(a + b*tan(e + f*x)^2)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.347 $\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{a}{3bf(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{1}{f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

```
[Out] arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f-1/(a-b)^2/f/(a+
b*tan(f*x+e)^2)^(1/2)-1/3*a/(a-b)/b/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.16, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3670, 446, 78, 51, 63, 208}

$$-\frac{a}{3bf(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{1}{f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f) - a/(3*(a
 - b)*b*f*(a + b*Tan[e + f*x]^2)^(3/2)) - 1/((a - b)^2*f*Sqrt[a + b*Tan[e +
 f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{a}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx\right.}{2(a-b)f}$$

$$= -\frac{a}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{1}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}}{}$$

$$= -\frac{a}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{1}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} - \frac{a}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{(a-b)^2f\sqrt{a+}}$$

Mathematica [C] time = 0.30, size = 84, normalized size = 0.82

$$\frac{a(b-a)-3b\left(a+b\tan^2(e+fx)\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\tan^2(e+fx)+a}{a-b}\right)}{3bf(a-b)^2\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (a*(-a + b) - 3*b*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a
 - b)]*(a + b*Tan[e + f*x]^2))/(3*(a - b)^2*b*f*(a + b*Tan[e + f*x]^2)^(3/2
))
```

fricas [B] time = 0.58, size = 572, normalized size = 5.55

$$\left[\frac{3\left(b^{3}\tan\left(fx+e\right)^{4}+2\,ab^{2}\tan\left(fx+e\right)^{2}+a^{2}b\right)\sqrt{a-b}\,\log\left(-\frac{b^{2}\tan\left(fx+e\right)^{4}+2\left(4\,ab-3\,b^{2}\right)\tan\left(fx+e\right)^{2}+4\left(b\tan\left(fx+e\right)^{2}+2\,a-b\right)}{\tan\left(fx+e\right)^{4}+2\,\tan\left(fx+e\right)^{2}+}\right.}{12\left(\left(a^{3}b^{3}-3\,a^{2}b^{4}+3\,ab^{5}-b^{6}\right)f\tan\left(fx+e\right)^{4}+2\left(a^{4}b^{2}-3\,a^{3}b^{3}+3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*tan(f*x + e)^4 + 2*a*b^2*tan(f*x + e)^2 + a^2*b)*sqrt(a - b)*
log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x
+ e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b +
b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(a^3 + a^2*b - 2*a*b^2 +
3*(a*b^2 - b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^3 - 3*a
^2*b^4 + 3*a*b^5 - b^6)*f*tan(f*x + e)^4 + 2*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b
^4 - a*b^5)*f*tan(f*x + e)^2 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*f)
, -1/6*(3*(b^3*tan(f*x + e)^4 + 2*a*b^2*tan(f*x + e)^2 + a^2*b)*sqrt(-a + b
)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a
- b)) + 2*(a^3 + a^2*b - 2*a*b^2 + 3*(a*b^2 - b^3)*tan(f*x + e)^2)*sqrt(b*t
an(f*x + e)^2 + a))/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*f*tan(f*x + e)^4
 + 2*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*f*tan(f*x + e)^2 + (a^5*b -
3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*f)]
```

giac [A] time = 4.35, size = 116, normalized size = 1.13

$$-\frac{\frac{3\,b\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{-a+b}}\right)}{\left(a^{2}f-2\,abf+b^{2}f\right)\sqrt{-a+b}}+\frac{a^{2}+3\left(b\tan\left(fx+e\right)^{2}+a\right)b-ab}{\left(a^{2}f-2\,abf+b^{2}f\right)\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a^2*f - 2*a*b*f
 + b^2*f)*sqrt(-a + b)) + (a^2 + 3*(b*tan(f*x + e)^2 + a)*b - a*b)/((a^2*f
- 2*a*b*f + b^2*f)*(b*tan(f*x + e)^2 + a)^(3/2)))/b
```

maple [A] time = 0.28, size = 118, normalized size = 1.15

$$-\frac{1}{3fb\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}}}-\frac{1}{3\left(a-b\right)f\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}}}-\frac{1}{\left(a-b\right)^{2}f\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}-\frac{\arctan\left(\right.}{f\left(a-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f/b/(a+b*tan(f*x+e)^2)^(3/2)-1/3/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-1/(a
-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/f/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*tan(f
*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 15.72, size = 138, normalized size = 1.34

$$-\frac{\frac{a}{3\,(a-b)}+\frac{b\left(b\tan\left(e+f\,x\right)^2+a\right)}{(a-b)^2}}{b\,f\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}-\frac{\mathrm{atan}\left(\frac{a^2\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\;1\mathrm{i}+b^2\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\;1\mathrm{i}-a\,b\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\;2\mathrm{i}}{(a-b)^{5/2}}\right)1\mathrm{i}}{f\,(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] - (atan((a^2*(a + b*tan(e + f*x)^2)^(1/2)*1i + b^2*(a + b*tan(e + f*x)^2)^(
1/2)*1i - a*b*(a + b*tan(e + f*x)^2)^(1/2)*2i)/(a - b)^(5/2))*1i)/(f*(a - b
)^(5/2)) - (a/(3*(a - b)) + (b*(a + b*tan(e + f*x)^2))/(a - b)^2)/(b*f*(a +
 b*tan(e + f*x)^2)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.348 $\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=99

$$\frac{1}{f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{1}{3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f+1/(a-b)^2/f/(a
+b*tan(f*x+e)^2)^(1/2)+1/3/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3670, 444, 51, 63, 208}

$$\frac{1}{f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{1}{3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f)) + 1/(3
*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) + 1/((a - b)^2*f*Sqrt[a + b*Tan[e
+ f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{1}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2(a-b)f}$$

$$= \frac{1}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{1}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \frac{1}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{}{(a-b)^2f\sqrt{a}}$$

Mathematica [C] time = 0.14, size = 58, normalized size = 0.59

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\tan^2(e+fx)+a}{a-b}\right)}{3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]^2)/(a - b)]/(3*(a - b)
*f*(a + b*Tan[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.56, size = 544, normalized size = 5.49

$$\left[\frac{3\left(b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2\right)\sqrt{a-b}\,\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4\,ab-3\,b^2\right)\tan\left(fx+e\right)^2-4\left(b\tan\left(fx+e\right)^2+2\,a-b\right.}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2}\right.}{12\left(\left(a^3b^2-3\,a^2b^3+3\,ab^4-b^5\right)f\tan\left(fx+e\right)^4+2\left(a^4b-3\,a^3b^2+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*log(
-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)
^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)
/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*(3*(a*b - b^2)*tan(f*x + e)^2
 + 4*a^2 - 5*a*b + b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^2 - 3*a^2*b^3 +
 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4
)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f), 1/6*(3*(b^2*
tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*arctan(2*sqrt(b*t
an(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2*(3*(a*b -
 b^2)*tan(f*x + e)^2 + 4*a^2 - 5*a*b + b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a
^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2
 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b
^3)*f)]
```

giac [A] time = 4.16, size = 105, normalized size = 1.06

$$\frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\left(a^2f-2\,abf+b^2f\right)\sqrt{-a+b}}+\frac{3\,b\tan\left(fx+e\right)^2+4\,a-b}{3\left(a^2f-2\,abf+b^2f\right)\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a^2*f - 2*a*b*f + b^2*f)*
sqrt(-a + b)) + 1/3*(3*b*tan(f*x + e)^2 + 4*a - b)/((a^2*f - 2*a*b*f + b^2*
f)*(b*tan(f*x + e)^2 + a)^(3/2))
```

maple [A] time = 0.19, size = 94, normalized size = 0.95

$$\frac{1}{3\left(a-b\right)f\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}+\frac{1}{\left(a-b\right)^2 f\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}+\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{f\left(a-b\right)^2\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)+1/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1
/f/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

mupad [B] time = 15.70, size = 131, normalized size = 1.32

$$\frac{\frac{b\tan\left(e+f\,x\right)^2+a}{(a-b)^2}+\frac{1}{3\,(a-b)}}{f\,\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}+\frac{\mathrm{atan}\left(\frac{a^2\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,1\mathrm{i}+b^2\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,1\mathrm{i}-a\,b\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,2\mathrm{i}}{(a-b)^{5/2}}\right)1\mathrm{i}}{f\,(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] ((a + b*tan(e + f*x)^2)/(a - b)^2 + 1/(3*(a - b)))/(f*(a + b*tan(e + f*x)^2
)^(3/2)) + (atan((a^2*(a + b*tan(e + f*x)^2)^(1/2)*1i + b^2*(a + b*tan(e +
f*x)^2)^(1/2)*1i - a*b*(a + b*tan(e + f*x)^2)^(1/2)*2i)/(a - b)^(5/2))*1i)/
(f*(a - b)^(5/2))
```

sympy [A] time = 31.49, size = 83, normalized size = 0.84

$$\frac{1}{3f\,(a-b)\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}+\frac{1}{f\,(a-b)^2\sqrt{a+b\tan^2\left(e+fx\right)}}+\frac{\mathrm{atan}\left(\frac{\sqrt{a+b\tan^2\left(e+fx\right)}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}\,(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] 1/(3*f*(a - b)*(a + b*tan(e + f*x)**2)**(3/2)) + 1/(f*(a - b)**2*sqrt(a + b
*tan(e + f*x)**2)) + atan(sqrt(a + b*tan(e + f*x)**2)/sqrt(-a + b))/(f*sqrt
(-a + b)*(a - b)**2)
```

3.349 $\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b(2a-b)}{a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{b}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan}}{\sqrt{a}}\right)}{f(a-b)^5}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+arctanh((a+b*tan(f*x+e
)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f-(2*a-b)*b/a^2/(a-b)^2/f/(a+b*tan(f*x+
e)^2)^(1/2)-1/3*b/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3670, 446, 85, 152, 156, 63, 208}

$$-\frac{b(2a-b)}{a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan}}{\sqrt{a}}\right)}{f(a-b)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + ArcTanh[Sqrt[a
 + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f) - b/(3*a*(a - b)*f*(a +
 b*Tan[e + f*x]^2)^(3/2)) - ((2*a - b)*b)/(a^2*(a - b)^2*f*Sqrt[a + b*Tan[e
 + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{b}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a-b-bx}{x(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2a(a-b)f}$$

$$= -\frac{b}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(2a-b)b}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{b}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(2a-b)b}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{b}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(2a-b)b}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} - \frac{b}{3a(a-b)f\left(a+b\tan^2(e+\right.}$$

Mathematica [C] time = 0.37, size = 94, normalized size = 0.64

$$\frac{(a-b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\tan^2(e+fx)}{a}+1\right) - a\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\tan^2(e+fx)+a}{a-b}\right)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(a*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]^2)/(a - b)]) + (a
 - b)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tan[e + f*x]^2)/a])/(3*a*(a -
 b)*f*(a + b*Tan[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.48, size = 1649, normalized size = 11.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^3*b^2*tan(f*x + e)^4 + 2*a^4*b*tan(f*x + e)^2 + a^5)*sqrt(a - b)
*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b
)/(tan(f*x + e)^2 + 1)) + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2
 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(7*a^4*b - 11*a^3*b^2 +
```

```
4*a^2*b^3 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*
x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^
4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^2 + (a^8 - 3
*a^7*b + 3*a^6*b^2 - a^5*b^3)*f), 1/6*(6*(a^3*b^2*tan(f*x + e)^4 + 2*a^4*b*
tan(f*x + e)^2 + a^5)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b)/(a - b)) + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 - 3*a^
2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 -
a*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e
)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(7*a^4*b - 11*a^3*b^2 + 4*a^2*b
^3 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^
2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^4 + 2*(
a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^2 + (a^8 - 3*a^7*b
+ 3*a^6*b^2 - a^5*b^3)*f), 1/6*(6*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a
^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 +
 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 +
 a)*sqrt(-a)/a) + 3*(a^3*b^2*tan(f*x + e)^4 + 2*a^4*b*tan(f*x + e)^2 + a^5)
*sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a -
b) + 2*a - b)/(tan(f*x + e)^2 + 1)) - 2*(7*a^4*b - 11*a^3*b^2 + 4*a^2*b^3 +
 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 +
a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^4 + 2*(a^7*
b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^2 + (a^8 - 3*a^7*b + 3*
a^6*b^2 - a^5*b^3)*f), 1/3*(3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b
^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a
^2*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*
sqrt(-a)/a) + 3*(a^3*b^2*tan(f*x + e)^4 + 2*a^4*b*tan(f*x + e)^2 + a^5)*sqr
t(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) - (7*a^4
*b - 11*a^3*b^2 + 4*a^2*b^3 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e
)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^
5)*f*tan(f*x + e)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x
 + e)^2 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
```

```
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
```

```
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
```

```
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Warning, integration of abs or sign assumes constant sign by interval
s (correct if the argument is real):Check [abs(t_nostep^2-1)]Discontinuitie
s at zeroes of t_nostep^2-1 were not checkedWarning, integration of abs or
sign assumes constant sign by intervals (correct if the argument is real):C
heck [abs(t_nostep^2-1)]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b]=[-29,-86]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [a,b]
=[79,3]Discontinuities at zeroes of t_nostep^2-1 were not checkedUnable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
```

```
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or s
ign assumes constant sign by intervals (correct if the argument is real):Ch
eck [abs(t_nostep-1)]Evaluation time: 14.76Unable to divide, perhaps due to
 rounding error%%%{%%%{2,[4,4]%%%}+%%%{-4,[3,5]%%%}+%%%{2,[2,6]%%%},[4,1]%%
%}+%%%{%%{[%%%{8,[4,4]%%%}+%%%{-8,[3,5]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,
[0,1]%%%}]%%},[3,1]%%%}+%%%{%%%{12,[5,4]%%%}+%%%{-12,[4,5]%%%},[2,1]%%%}+%%
%{%%{[%%%{8,[5,4]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,[0,1]%%%}]%%},[1,1]%%%
}+%%%{%%%{2,[6,4]%%%},[0,1]%%%} / %%%{%%%{1,[4,0]%%%}+%%%{-4,[3,1]%%%}+%%%{
6,[2,2]%%%}+%%%{-4,[1,3]%%%}+%%%{1,[0,4]%%%},[4,0]%%%}+%%%{%%{[%%%{4,[4,0]%
%%}+%%%{-12,[3,1]%%%}+%%%{12,[2,2]%%%}+%%%{-4,[1,3]%%%},0]:[1,0,%%%{-1,[1,0
]%%%}+%%%{1,[0,1]%%%}]%%},[3,0]%%%}+%%%{%%%{6,[5,0]%%%}+%%%{-18,[4,1]%%%}+%
%%{18,[3,2]%%%}+%%%{-6,[2,3]%%%},[2,0]%%%}+%%%{%%{[%%%{4,[5,0]%%%}+%%%{-8,[
4,1]%%%}+%%%{4,[3,2]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{1,[0,1]%%%}]%%},[1,0]
%%%}+%%%{%%%{1,[6,0]%%%}+%%%{-2,[5,1]%%%}+%%%{1,[4,2]%%%},[0,0]%%%} Error:
Bad Argument Value
```

maple [B] time = 21.10, size = 331597, normalized size = 2255.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 12.46, size = 2788, normalized size = 18.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] (b/(3*(a*b - a^2)) - (b*(a + b*tan(e + f*x)^2)*(2*a - b))/(a*b - a^2)^2)/(f
*(a + b*tan(e + f*x)^2)^(3/2)) - atanh((2*a^5*b^13*f^2*(a + b*tan(e + f*x)^
2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*f^2
 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f^2 -
 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^2))
- (22*a^6*b^12*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f
^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^
2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2
 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^2)) + (110*a^7*b^11*f^2*(a + b*tan(e + f
*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11
*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f
^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^
2)) - (330*a^8*b^10*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b
^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b
^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11*b^
5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^2)) + (660*a^9*b^9*f^2*(a + b*tan(e
 + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*
b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b
^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^
3*f^2)) - (922*a^10*b^8*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a
^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a
^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^1
1*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^2)) + (912*a^11*b^7*f^2*(a + b*
tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110
*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*
a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^
13*b^3*f^2)) - (630*a^12*b^6*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)
*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f^2 +
660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 29
0*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^2)) + (290*a^13*b^5*f^2*(a
 + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2
+ 110*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 +
 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 +
10*a^13*b^3*f^2)) - (80*a^14*b^4*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(
1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f^
2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2
+ 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^2)) + (10*a^15*b^3*f^2
*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f
^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^
2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2
 + 10*a^13*b^3*f^2)))/(f*(a^5)^(1/2)) - (atan((a^12*f^3*(a + b*tan(e + f*x)
^2)^(1/2)*2i - a^7*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*
b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*2i + b^7*f*(a + b*
tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a
^2*b^3*f^2 + 10*a^3*b^2*f^2)*1i + a^4*b^8*f^3*(a + b*tan(e + f*x)^2)^(1/2)*
1i - a^5*b^7*f^3*(a + b*tan(e + f*x)^2)^(1/2)*9i + a^6*b^6*f^3*(a + b*tan(e
 + f*x)^2)^(1/2)*35i - a^7*b^5*f^3*(a + b*tan(e + f*x)^2)^(1/2)*77i + a^8*b
^4*f^3*(a + b*tan(e + f*x)^2)^(1/2)*105i - a^9*b^3*f^3*(a + b*tan(e + f*x)^
2)^(1/2)*91i + a^10*b^2*f^3*(a + b*tan(e + f*x)^2)^(1/2)*49i - a^11*b*f^3*(
a + b*tan(e + f*x)^2)^(1/2)*15i + a^2*b^5*f*(a + b*tan(e + f*x)^2)^(1/2)*(a
^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*
f^2)*21i - a^3*b^4*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*
b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*35i + a^4*b^3*f*(a
 + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 -
 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*36i - a^5*b^2*f*(a + b*tan(e + f*x)^2)^(1
/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^
```

```
3*b^2*f^2)*24i - a*b^6*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 +
5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*7i + a^6*b*f*(
a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2
- 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*10i)/((5*a^4*b - 5*a*b^4 + b^5 + 10*a^2*
b^3 - 10*a^3*b^2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b
^3*f^2 + 10*a^3*b^2*f^2)^(3/2)))*1i)/(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a
^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.350 $\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=206

$$\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{b(3a-5b)}{6a^2f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{b\left(a^2-8ab+5b^2\right)}{2a^3f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\text{tanh}$$

```
[Out] 1/2*(2*a+5*b)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/2)/f-arctanh((
a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f-1/2*b*(a^2-8*a*b+5*b^2)/
a^3/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/6*(3*a-5*b)*b/a^2/(a-b)/f/(a+b*tan
(f*x+e)^2)^(3/2)-1/2*cot(f*x+e)^2/a/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.35, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3670, 446, 103, 152, 156, 63, 208}

$$-\frac{b\left(a^2-8ab+5b^2\right)}{2a^3f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{b(3a-5b)}{6a^2f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\text{tan}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*a + 5*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*a^(7/2)*f) - A
rcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f) - ((3*a -
5*b)*b)/(6*a^2*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - Cot[e + f*x]^2/(2*
a*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (b*(a^2 - 8*a*b + 5*b^2))/(2*a^3*(a - b
)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2a+5b)+\frac{5bx}{2}}{x(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2af}$$

$$= -\frac{(3a-5b)b}{6a^2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{(3a-5b)b}{6a^2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b\left(a\right.}{2a^3(a-b)^2}$$

$$= -\frac{(3a-5b)b}{6a^2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b\left(a\right.}{2a^3(a-b)^2}$$

$$= -\frac{(3a-5b)b}{6a^2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{b\left(a\right.}{2a^3(a-b)^2}$$

$$= \frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} - \frac{(3a-}{6a^2(a-b)f\left(a+\right.}$$

Mathematica [C] time = 0.62, size = 138, normalized size = 0.67

$$\frac{\cot^2(e+fx)\left((a-b)\left((2a+5b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\tan^2(e+fx)}{a}+1\right)+3a\cot^2(e+fx)\right)-2a^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\tan^2(e+fx)+}{a-b}\right.\right.}{6a^2f(b-a)\sqrt{a+b\tan^2(e+fx)}\left(a\cot^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cot[e + f*x]^2*(-2*a^2*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x
]^2)/(a - b)] + (a - b)*(3*a*Cot[e + f*x]^2 + (2*a + 5*b)*Hypergeometric2F1
[-3/2, 1, -1/2, 1 + (b*Tan[e + f*x]^2)/a])))/(6*a^2*(-a + b)*f*(b + a*Cot[e
 + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2])
```

fricas [B] time = 0.63, size = 2083, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(6*(a^4*b^2*tan(f*x + e)^6 + 2*a^5*b*tan(f*x + e)^4 + a^6*tan(f*x + e
)^2)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + 3*((2*a^4*b^2 - a^3*b^3 - 9*a^2*b
^4 + 13*a*b^5 - 5*b^6)*tan(f*x + e)^6 + 2*(2*a^5*b - a^4*b^2 - 9*a^3*b^3 +
13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + (2*a^6 - a^5*b - 9*a^4*b^2 + 13*a^3*
b^3 - 5*a^2*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*t
an(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(3*a^6 - 9*a^5*b + 9*
a^4*b^2 - 3*a^3*b^3 + 3*(a^4*b^2 - 9*a^3*b^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*
x + e)^4 + 2*(3*a^5*b - 19*a^4*b^2 + 26*a^3*b^3 - 10*a^2*b^4)*tan(f*x + e)^
2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)
*f*tan(f*x + e)^6 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x +
 e)^4 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^2), -1/12*(12*
(a^4*b^2*tan(f*x + e)^6 + 2*a^5*b*tan(f*x + e)^4 + a^6*tan(f*x + e)^2)*sqrt
(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) - 3*((2*a
^4*b^2 - a^3*b^3 - 9*a^2*b^4 + 13*a*b^5 - 5*b^6)*tan(f*x + e)^6 + 2*(2*a^5*
b - a^4*b^2 - 9*a^3*b^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + (2*a^6 - a
^5*b - 9*a^4*b^2 + 13*a^3*b^3 - 5*a^2*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*t
an(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)
 + 2*(3*a^6 - 9*a^5*b + 9*a^4*b^2 - 3*a^3*b^3 + 3*(a^4*b^2 - 9*a^3*b^3 + 13
*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + 2*(3*a^5*b - 19*a^4*b^2 + 26*a^3*b^3 -
 10*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^2 - 3*a^6*
b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^6 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*
b^3 - a^5*b^4)*f*tan(f*x + e)^4 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*t
an(f*x + e)^2), -1/6*(3*((2*a^4*b^2 - a^3*b^3 - 9*a^2*b^4 + 13*a*b^5 - 5*b^
6)*tan(f*x + e)^6 + 2*(2*a^5*b - a^4*b^2 - 9*a^3*b^3 + 13*a^2*b^4 - 5*a*b^5
)*tan(f*x + e)^4 + (2*a^6 - a^5*b - 9*a^4*b^2 + 13*a^3*b^3 - 5*a^2*b^4)*tan
(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) - 3*(a^
4*b^2*tan(f*x + e)^6 + 2*a^5*b*tan(f*x + e)^4 + a^6*tan(f*x + e)^2)*sqrt(a
- b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a
 - b)/(tan(f*x + e)^2 + 1)) + (3*a^6 - 9*a^5*b + 9*a^4*b^2 - 3*a^3*b^3 + 3*
(a^4*b^2 - 9*a^3*b^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + 2*(3*a^5*b -
19*a^4*b^2 + 26*a^3*b^3 - 10*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2
 + a))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^6 + 2*(a
^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^4 + (a^9 - 3*a^8*b +
 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^2), -1/6*(3*((2*a^4*b^2 - a^3*b^3 - 9*
a^2*b^4 + 13*a*b^5 - 5*b^6)*tan(f*x + e)^6 + 2*(2*a^5*b - a^4*b^2 - 9*a^3*b
^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + (2*a^6 - a^5*b - 9*a^4*b^2 + 13
*a^3*b^3 - 5*a^2*b^4)*tan(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2
 + a)*sqrt(-a)/a) + 6*(a^4*b^2*tan(f*x + e)^6 + 2*a^5*b*tan(f*x + e)^4 + a^
6*tan(f*x + e)^2)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
 b)/(a - b)) + (3*a^6 - 9*a^5*b + 9*a^4*b^2 - 3*a^3*b^3 + 3*(a^4*b^2 - 9*a^
3*b^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + 2*(3*a^5*b - 19*a^4*b^2 + 26
*a^3*b^3 - 10*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^
2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^6 + 2*(a^8*b - 3*a^7*b^
2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^4 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^
6*b^3)*f*tan(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
```

*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s

ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/

2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (

```
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Warning, integration of abs or sign assumes constant sign by intervals
(correct if the argument is real):Check [abs(t_nostep^2-1)]Discontinuities
at zeroes of t_nostep^2-1 were not checkedWarning, integration of abs or si
gn assumes constant sign by intervals (correct if the argument is real):Che
ck [abs(t_nostep^2-1)]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
```

```
[a,b]=[2,-43]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[-3
7,-94]Discontinuities at zeroes of t_nostep^2-1 were not checkedUnable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integrat
ion of abs or sign assumes constant sign by intervals (correct if the argum
ent is real):Check [abs(t_nostep^2-1)]Evaluation time: 27.95Error: Bad Argu
ment Type
```

maple [B] time = 38.08, size = 531560, normalized size = 2580.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

[Out] Timed out

mupad [B] time = 13.10, size = 3429, normalized size = 16.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] (atan((((a + b*tan(e + f*x)^2)^(1/2)*(400*a^9*b^14*f^3 - 3680*a^10*b^13*f^
3 + 14864*a^11*b^12*f^3 - 34240*a^12*b^11*f^3 + 48480*a^13*b^10*f^3 - 41280
*a^14*b^9*f^3 + 16864*a^15*b^8*f^3 + 2688*a^16*b^7*f^3 - 6000*a^17*b^6*f^3
+ 1440*a^18*b^5*f^3 + 1040*a^19*b^4*f^3 - 704*a^20*b^3*f^3 + 128*a^21*b^2*f
^3) - ((2*a + 5*b)*(320*a^12*b^14*f^4 - 3392*a^13*b^13*f^4 + 16192*a^14*b^1
2*f^4 - 45760*a^15*b^11*f^4 + 84608*a^16*b^10*f^4 - 106624*a^17*b^9*f^4 + 9
2288*a^18*b^8*f^4 - 53632*a^19*b^7*f^4 + 19520*a^20*b^6*f^4 - 3648*a^21*b^5
*f^4 + 64*a^22*b^4*f^4 + 64*a^23*b^3*f^4 - ((a + b*tan(e + f*x)^2)^(1/2)*(2
*a + 5*b)*(256*a^15*b^13*f^5 - 3072*a^16*b^12*f^5 + 16640*a^17*b^11*f^5 - 5
3760*a^18*b^10*f^5 + 115200*a^19*b^9*f^5 - 172032*a^20*b^8*f^5 + 182784*a^2
1*b^7*f^5 - 138240*a^22*b^6*f^5 + 72960*a^23*b^5*f^5 - 25600*a^24*b^4*f^5 +
 5376*a^25*b^3*f^5 - 512*a^26*b^2*f^5))/(4*f*(a^7)^(1/2))))/(4*f*(a^7)^(1/2
)))*(2*a + 5*b)*1i)/(4*f*(a^7)^(1/2)) + (((a + b*tan(e + f*x)^2)^(1/2)*(400
*a^9*b^14*f^3 - 3680*a^10*b^13*f^3 + 14864*a^11*b^12*f^3 - 34240*a^12*b^11*
f^3 + 48480*a^13*b^10*f^3 - 41280*a^14*b^9*f^3 + 16864*a^15*b^8*f^3 + 2688*
a^16*b^7*f^3 - 6000*a^17*b^6*f^3 + 1440*a^18*b^5*f^3 + 1040*a^19*b^4*f^3 -
704*a^20*b^3*f^3 + 128*a^21*b^2*f^3) + ((2*a + 5*b)*(320*a^12*b^14*f^4 - 33
92*a^13*b^13*f^4 + 16192*a^14*b^12*f^4 - 45760*a^15*b^11*f^4 + 84608*a^16*b
^10*f^4 - 106624*a^17*b^9*f^4 + 92288*a^18*b^8*f^4 - 53632*a^19*b^7*f^4 + 1
9520*a^20*b^6*f^4 - 3648*a^21*b^5*f^4 + 64*a^22*b^4*f^4 + 64*a^23*b^3*f^4 +
 ((a + b*tan(e + f*x)^2)^(1/2)*(2*a + 5*b)*(256*a^15*b^13*f^5 - 3072*a^16*b
^12*f^5 + 16640*a^17*b^11*f^5 - 53760*a^18*b^10*f^5 + 115200*a^19*b^9*f^5 -
 172032*a^20*b^8*f^5 + 182784*a^21*b^7*f^5 - 138240*a^22*b^6*f^5 + 72960*a^
23*b^5*f^5 - 25600*a^24*b^4*f^5 + 5376*a^25*b^3*f^5 - 512*a^26*b^2*f^5))/(4
*f*(a^7)^(1/2))))/(4*f*(a^7)^(1/2)))*(2*a + 5*b)*1i)/(4*f*(a^7)^(1/2)))/(40
0*a^9*b^12*f^2 - 2880*a^10*b^11*f^2 + 8704*a^11*b^10*f^2 - 14112*a^12*b^9*f
^2 + 12768*a^13*b^8*f^2 - 5600*a^14*b^7*f^2 + 1056*a^16*b^5*f^2 - 368*a^17*
b^4*f^2 + 32*a^18*b^3*f^2 + (((a + b*tan(e + f*x)^2)^(1/2)*(400*a^9*b^14*f^
3 - 3680*a^10*b^13*f^3 + 14864*a^11*b^12*f^3 - 34240*a^12*b^11*f^3 + 48480*
a^13*b^10*f^3 - 41280*a^14*b^9*f^3 + 16864*a^15*b^8*f^3 + 2688*a^16*b^7*f^3
 - 6000*a^17*b^6*f^3 + 1440*a^18*b^5*f^3 + 1040*a^19*b^4*f^3 - 704*a^20*b^3
*f^3 + 128*a^21*b^2*f^3) - ((2*a + 5*b)*(320*a^12*b^14*f^4 - 3392*a^13*b^13
*f^4 + 16192*a^14*b^12*f^4 - 45760*a^15*b^11*f^4 + 84608*a^16*b^10*f^4 - 10
6624*a^17*b^9*f^4 + 92288*a^18*b^8*f^4 - 53632*a^19*b^7*f^4 + 19520*a^20*b^
6*f^4 - 3648*a^21*b^5*f^4 + 64*a^22*b^4*f^4 + 64*a^23*b^3*f^4 - ((a + b*tan
(e + f*x)^2)^(1/2)*(2*a + 5*b)*(256*a^15*b^13*f^5 - 3072*a^16*b^12*f^5 + 16
640*a^17*b^11*f^5 - 53760*a^18*b^10*f^5 + 115200*a^19*b^9*f^5 - 172032*a^20
*b^8*f^5 + 182784*a^21*b^7*f^5 - 138240*a^22*b^6*f^5 + 72960*a^23*b^5*f^5 -
 25600*a^24*b^4*f^5 + 5376*a^25*b^3*f^5 - 512*a^26*b^2*f^5))/(4*f*(a^7)^(1/
2))))/(4*f*(a^7)^(1/2)))*(2*a + 5*b))/(4*f*(a^7)^(1/2)) - (((a + b*tan(e +
f*x)^2)^(1/2)*(400*a^9*b^14*f^3 - 3680*a^10*b^13*f^3 + 14864*a^11*b^12*f^3
- 34240*a^12*b^11*f^3 + 48480*a^13*b^10*f^3 - 41280*a^14*b^9*f^3 + 16864*a^
15*b^8*f^3 + 2688*a^16*b^7*f^3 - 6000*a^17*b^6*f^3 + 1440*a^18*b^5*f^3 + 10
40*a^19*b^4*f^3 - 704*a^20*b^3*f^3 + 128*a^21*b^2*f^3) + ((2*a + 5*b)*(320*
a^12*b^14*f^4 - 3392*a^13*b^13*f^4 + 16192*a^14*b^12*f^4 - 45760*a^15*b^11*
f^4 + 84608*a^16*b^10*f^4 - 106624*a^17*b^9*f^4 + 92288*a^18*b^8*f^4 - 5363
2*a^19*b^7*f^4 + 19520*a^20*b^6*f^4 - 3648*a^21*b^5*f^4 + 64*a^22*b^4*f^4 +
 64*a^23*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*(2*a + 5*b)*(256*a^15*b^13
*f^5 - 3072*a^16*b^12*f^5 + 16640*a^17*b^11*f^5 - 53760*a^18*b^10*f^5 + 115
200*a^19*b^9*f^5 - 172032*a^20*b^8*f^5 + 182784*a^21*b^7*f^5 - 138240*a^22*
b^6*f^5 + 72960*a^23*b^5*f^5 - 25600*a^24*b^4*f^5 + 5376*a^25*b^3*f^5 - 512
```

```
*a^26*b^2*f^5))/(4*f*(a^7)^(1/2))))/(4*f*(a^7)^(1/2)))*(2*a + 5*b))/(4*f*(a
^7)^(1/2))))*(2*a + 5*b)*1i)/(2*f*(a^7)^(1/2)) - (b^2/(3*a*(a - b)) + (b*(a
 + b*tan(e + f*x)^2)*(8*a*b - 5*b^2))/(3*(a^4 - 2*a^3*b + a^2*b^2)) + (b*(a
 + b*tan(e + f*x)^2)^2*(a^2 - 8*a*b + 5*b^2))/(2*(a^5 - 2*a^4*b + a^3*b^2))
)/(f*(a + b*tan(e + f*x)^2)^(5/2) - a*f*(a + b*tan(e + f*x)^2)^(3/2)) - (at
an((a^14*f^3*(a + b*tan(e + f*x)^2)^(1/2)*8i - a^9*f*(a + b*tan(e + f*x)^2)
^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10
*a^3*b^2*f^2)*8i + b^9*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 +
5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*25i + a^6*b^8*
f^3*(a + b*tan(e + f*x)^2)^(1/2)*4i - a^7*b^7*f^3*(a + b*tan(e + f*x)^2)^(1
/2)*36i + a^8*b^6*f^3*(a + b*tan(e + f*x)^2)^(1/2)*140i - a^9*b^5*f^3*(a +
b*tan(e + f*x)^2)^(1/2)*308i + a^10*b^4*f^3*(a + b*tan(e + f*x)^2)^(1/2)*42
0i - a^11*b^3*f^3*(a + b*tan(e + f*x)^2)^(1/2)*364i + a^12*b^2*f^3*(a + b*t
an(e + f*x)^2)^(1/2)*196i - a^13*b*f^3*(a + b*tan(e + f*x)^2)^(1/2)*60i + a
^2*b^7*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*
a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*389i - a^3*b^6*f*(a + b*tan(e
+ f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3
*f^2 + 10*a^3*b^2*f^2)*483i + a^4*b^5*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f
^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)
*259i + a^5*b^4*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4
*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*35i - a^6*b^3*f*(a +
b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10
*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*101i + a^7*b^2*f*(a + b*tan(e + f*x)^2)^(1/2
)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*
b^2*f^2)*19i - a*b^8*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*
a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*155i + a^8*b*f*(
a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2
- 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*20i)/((a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 -
 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)^(3/2)*(105*a*b^6 - 25*b^7 -
 154*a^2*b^5 + 70*a^3*b^4 + 35*a^4*b^3 - 35*a^5*b^2)))*1i)/(a^5*f^2 - b^5*f
^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.351 $\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=272

$$\frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+20ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}+\frac{b\left(12a^2+15ab-35b^2\right)}{24a^3f(a-b)\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] -1/8*(8*a^2+20*a*b+35*b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(9/2
)/f+arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f+1/8*b*(4*a^
3+3*a^2*b-50*a*b^2+35*b^3)/a^4/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1/24*b*(1
2*a^2+15*a*b-35*b^2)/a^3/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)+1/8*(4*a+7*b)*cot
(f*x+e)^2/a^2/f/(a+b*tan(f*x+e)^2)^(3/2)-1/4*cot(f*x+e)^4/a/f/(a+b*tan(f*x+
e)^2)^(3/2)
```

Rubi [A] time = 0.44, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3670, 446, 103, 151, 152, 156, 63, 208}

$$\frac{b\left(3a^2b+4a^3-50ab^2+35b^3\right)}{8a^4f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{b\left(12a^2+15ab-35b^2\right)}{24a^3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+20ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((8*a^2 + 20*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(8
*a^(9/2)*f) + ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2
)*f) + (b*(12*a^2 + 15*a*b - 35*b^2))/(24*a^3*(a - b)*f*(a + b*Tan[e + f*x]
^2)^(3/2)) + ((4*a + 7*b)*Cot[e + f*x]^2)/(8*a^2*f*(a + b*Tan[e + f*x]^2)^(
3/2)) - Cot[e + f*x]^4/(4*a*f*(a + b*Tan[e + f*x]^2)^(3/2)) + (b*(4*a^3 + 3
*a^2*b - 50*a*b^2 + 35*b^3))/(8*a^4*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^5(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(4a+7b)+\frac{7bx}{2}}{x^2(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{4af}$$

$$= \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{4}\left(8a^2+2\ldots\right.\right.}{\ldots}$$

$$= \frac{b\left(12a^2+15ab-35b^2\right)}{24a^3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\ldots}{4af\left(a\ldots\right.}$$

$$= \frac{b\left(12a^2+15ab-35b^2\right)}{24a^3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\ldots}{4af\left(a\ldots\right.}$$

$$= \frac{b\left(12a^2+15ab-35b^2\right)}{24a^3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\ldots}{4af\left(a\ldots\right.}$$

$$= \frac{b\left(12a^2+15ab-35b^2\right)}{24a^3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\ldots}{4af\left(a\ldots\right.}$$

$$= -\frac{\left(8a^2+20ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \ldots$$

Mathematica [C] time = 1.99, size = 165, normalized size = 0.61

$$\frac{\cot^2(e+fx)\left(8a^3\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\tan^2(e+fx)+a}{a-b}\right)+(a-b)\left(3a\cot^2(e+fx)\left(2a\cot^2(e+fx)-4a-7b\right)-\left(8a^2+\ldots\right.\right.\right.}{24a^3f(b-a)\sqrt{a+b\tan^2(e+fx)}\left(a\cot^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cot[e + f*x]^2*(8*a^3*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]
^2)/(a - b)] + (a - b)*(3*a*Cot[e + f*x]^2*(-4*a - 7*b + 2*a*Cot[e + f*x]^2
) - (8*a^2 + 20*a*b + 35*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tan[e
 + f*x]^2)/a])))/(24*a^3*(-a + b)*f*(b + a*Cot[e + f*x]^2)*Sqrt[a + b*Tan[e
 + f*x]^2])
```

fricas [B] time = 0.57, size = 2433, normalized size = 8.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(24*(a^5*b^2*tan(f*x + e)^8 + 2*a^6*b*tan(f*x + e)^6 + a^7*tan(f*x +
e)^4)*sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + 3*((8*a^5*b^2 - 4*a^4*b^3 - a^3*
b^4 - 53*a^2*b^5 + 85*a*b^6 - 35*b^7)*tan(f*x + e)^8 + 2*(8*a^6*b - 4*a^5*b
^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 + (8*a^7
- 4*a^6*b - a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4)
*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a
)/tan(f*x + e)^2) - 2*(6*a^7 - 18*a^6*b + 18*a^5*b^2 - 6*a^4*b^3 - 3*(4*a^5
*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 - 4*(6*
a^6*b - 3*a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4 -
3*(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*tan(f*x + e)^2)*sq
rt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6*b^4 - a^5*b^5)*f*ta
n(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b^4)*f*tan(f*x + e)^6
 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e)^4), 1/48*(48*(a^5*
b^2*tan(f*x + e)^8 + 2*a^6*b*tan(f*x + e)^6 + a^7*tan(f*x + e)^4)*sqrt(-a +
 b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) + 3*((8*a^5*b^
2 - 4*a^4*b^3 - a^3*b^4 - 53*a^2*b^5 + 85*a*b^6 - 35*b^7)*tan(f*x + e)^8 +
2*(8*a^6*b - 4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(
f*x + e)^6 + (8*a^7 - 4*a^6*b - a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*
b^5)*tan(f*x + e)^4)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^
2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(6*a^7 - 18*a^6*b + 18*a^5*b^2 -
6*a^4*b^3 - 3*(4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*ta
n(f*x + e)^6 - 4*(6*a^6*b - 3*a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^
5)*tan(f*x + e)^4 - 3*(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4
)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6
*b^4 - a^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b
^4)*f*tan(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e
)^4), 1/24*(3*((8*a^5*b^2 - 4*a^4*b^3 - a^3*b^4 - 53*a^2*b^5 + 85*a*b^6 - 3
5*b^7)*tan(f*x + e)^8 + 2*(8*a^6*b - 4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*
a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 + (8*a^7 - 4*a^6*b - a^5*b^2 - 53*a^4*b^
3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4)*sqrt(-a)*arctan(sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(-a)/a) + 12*(a^5*b^2*tan(f*x + e)^8 + 2*a^6*b*tan(f*x + e
)^6 + a^7*tan(f*x + e)^4)*sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) - (6*a^7 - 18*
a^6*b + 18*a^5*b^2 - 6*a^4*b^3 - 3*(4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a
^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 - 4*(6*a^6*b - 3*a^5*b^2 - 53*a^4*b^3 + 8
5*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4 - 3*(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 1
7*a^4*b^3 - 7*a^3*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^
2 - 3*a^7*b^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^
2 + 3*a^7*b^3 - a^6*b^4)*f*tan(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a
^7*b^3)*f*tan(f*x + e)^4), 1/24*(3*((8*a^5*b^2 - 4*a^4*b^3 - a^3*b^4 - 53*a
^2*b^5 + 85*a*b^6 - 35*b^7)*tan(f*x + e)^8 + 2*(8*a^6*b - 4*a^5*b^2 - a^4*b
^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 + (8*a^7 - 4*a^6*b
- a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4)*sqrt(-a)*
arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + 24*(a^5*b^2*tan(f*x + e)^8
+ 2*a^6*b*tan(f*x + e)^6 + a^7*tan(f*x + e)^4)*sqrt(-a + b)*arctan(-sqrt(b*
tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) - (6*a^7 - 18*a^6*b + 18*a^5*b^2
- 6*a^4*b^3 - 3*(4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*
tan(f*x + e)^6 - 4*(6*a^6*b - 3*a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*
b^5)*tan(f*x + e)^4 - 3*(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b
^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b^3 + 3*a
^6*b^4 - a^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6
*b^4)*f*tan(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x +
 e)^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
```

check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-

2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check

sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x

```
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
```

```
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Warning, integration of abs or sign assumes constant sign by intervals (c
orrect if the argument is real):Check [abs(t_nostep^2-1)]Discontinuities at
 zeroes of t_nostep^2-1 were not checkedWarning, integration of abs or sign
 assumes constant sign by intervals (correct if the argument is real):Check
 [abs(t_nostep^2-1)]Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming [a
,b]=[-72,77]Warning, need to choose a branch for the root of a polynomial w
ith parameters. This might be wrong.The choice was done assuming [a,b]=[15,
2]Discontinuities at zeroes of t_nostep^2-1 were not checkedUnable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of abs or sign as
sumes constant sign by intervals (correct if the argument is real):Check [a
bs(t_nostep^2-1)]Evaluation time: 42.66Error: Bad Argument Type
```

maple [B] time = 64.74, size = 790286, normalized size = 2905.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [B] time = 14.01, size = 4652, normalized size = 17.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^(5/2),x)
[Out] ((b*(a + b*tan(e + f*x)^2)^2*(15*a^2*b - 250*a*b^2 + 12*a^3 + 175*b^3))/(24
*(a^3*b - a^4)*(a - b)) - b^3/(3*a*(a - b)) + (b*(a + b*tan(e + f*x)^2)^3*(
3*a^2*b - 50*a*b^2 + 4*a^3 + 35*b^3))/(8*(a^3*b - a^4)*(a*b - a^2)) + (b*(1
0*a*b^2 - 7*b^3)*(a + b*tan(e + f*x)^2))/(3*a*(a - b)*(a*b - a^2)))/(f*(a +
 b*tan(e + f*x)^2)^(7/2) + a^2*f*(a + b*tan(e + f*x)^2)^(3/2) - 2*a*f*(a +
b*tan(e + f*x)^2)^(5/2)) - (atan((a^16*f^3*(a + b*tan(e + f*x)^2)^(1/2)*128
i - a^11*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 -
5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*128i + b^11*f*(a + b*tan(e +
 f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*
f^2 + 10*a^3*b^2*f^2)*1225i + a^8*b^8*f^3*(a + b*tan(e + f*x)^2)^(1/2)*64i
- a^9*b^7*f^3*(a + b*tan(e + f*x)^2)^(1/2)*576i + a^10*b^6*f^3*(a + b*tan(e
 + f*x)^2)^(1/2)*2240i - a^11*b^5*f^3*(a + b*tan(e + f*x)^2)^(1/2)*4928i +
a^12*b^4*f^3*(a + b*tan(e + f*x)^2)^(1/2)*6720i - a^13*b^3*f^3*(a + b*tan(e
 + f*x)^2)^(1/2)*5824i + a^14*b^2*f^3*(a + b*tan(e + f*x)^2)^(1/2)*3136i -
a^15*b*f^3*(a + b*tan(e + f*x)^2)^(1/2)*960i + a^2*b^9*f*(a + b*tan(e + f*x
)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2
+ 10*a^3*b^2*f^2)*16885i - a^3*b^8*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2
- b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*19
875i + a^4*b^7*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*
f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*11859i - a^5*b^6*f*(a
+ b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 -
10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*4053i + a^6*b^5*f*(a + b*tan(e + f*x)^2)^(
1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a
^3*b^2*f^2)*2919i - a^7*b^4*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f
^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*2625i + a
^8*b^3*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*
a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*904i - a^9*b^2*f*(a + b*tan(e
+ f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3
*f^2 + 10*a^3*b^2*f^2)*256i - a*b^10*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^
2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*
7175i + a^10*b*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*
f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*320i)/((a^5*f^2 - b^5*
f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)^(3/2)*(1
225*b^9 - 4725*a*b^8 + 6210*a^2*b^7 - 2730*a^3*b^6 + 189*a^4*b^5 - 945*a^5*
b^4 + 840*a^6*b^3)))*1i)/(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 1
0*a^2*b^3*f^2 + 10*a^3*b^2*f^2)^(1/2) - (atanh((156800*a^2*b^17*f^2*(a + b*
tan(e + f*x)^2)^(1/2)*(320*a^12*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3
+ 960*a^11*b^2)^(1/2))/(5488000*a^7*b^19*f^2 - 50960000*a^8*b^18*f^2 + 2074
91200*a^9*b^17*f^2 - 483286400*a^10*b^16*f^2 + 704892160*a^11*b^15*f^2 - 66
8407040*a^12*b^14*f^2 + 435855616*a^13*b^13*f^2 - 248036096*a^14*b^12*f^2 +
 174993280*a^15*b^11*f^2 - 118823040*a^16*b^10*f^2 + 51799680*a^17*b^9*f^2
- 15232896*a^18*b^8*f^2 + 7343616*a^19*b^7*f^2 - 3978240*a^20*b^6*f^2 + 860
```

160*a^21*b^5*f^2) - (1545600*a^3*b^16*f^2*(a + b*tan(e + f*x)^2)^(1/2)*(320
*a^12*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3 + 960*a^11*b^2)^(1/2))/(54
88000*a^7*b^19*f^2 - 50960000*a^8*b^18*f^2 + 207491200*a^9*b^17*f^2 - 48328
6400*a^10*b^16*f^2 + 704892160*a^11*b^15*f^2 - 668407040*a^12*b^14*f^2 + 43
5855616*a^13*b^13*f^2 - 248036096*a^14*b^12*f^2 + 174993280*a^15*b^11*f^2 -
118823040*a^16*b^10*f^2 + 51799680*a^17*b^9*f^2 - 15232896*a^18*b^8*f^2 +
7343616*a^19*b^7*f^2 - 3978240*a^20*b^6*f^2 + 860160*a^21*b^5*f^2) + (67756
80*a^4*b^15*f^2*(a + b*tan(e + f*x)^2)^(1/2)*(320*a^12*b + 64*a^13 + 1225*a
^9*b^4 + 1400*a^10*b^3 + 960*a^11*b^2)^(1/2))/(5488000*a^7*b^19*f^2 - 50960
000*a^8*b^18*f^2 + 207491200*a^9*b^17*f^2 - 483286400*a^10*b^16*f^2 + 70489
2160*a^11*b^15*f^2 - 668407040*a^12*b^14*f^2 + 435855616*a^13*b^13*f^2 - 24
8036096*a^14*b^12*f^2 + 174993280*a^15*b^11*f^2 - 118823040*a^16*b^10*f^2 +
51799680*a^17*b^9*f^2 - 15232896*a^18*b^8*f^2 + 7343616*a^19*b^7*f^2 - 397
8240*a^20*b^6*f^2 + 860160*a^21*b^5*f^2) - (17326720*a^5*b^14*f^2*(a + b*ta
n(e + f*x)^2)^(1/2)*(320*a^12*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3 +
960*a^11*b^2)^(1/2))/(5488000*a^7*b^19*f^2 - 50960000*a^8*b^18*f^2 + 207491
200*a^9*b^17*f^2 - 483286400*a^10*b^16*f^2 + 704892160*a^11*b^15*f^2 - 6684
07040*a^12*b^14*f^2 + 435855616*a^13*b^13*f^2 - 248036096*a^14*b^12*f^2 + 1
74993280*a^15*b^11*f^2 - 118823040*a^16*b^10*f^2 + 51799680*a^17*b^9*f^2 -
15232896*a^18*b^8*f^2 + 7343616*a^19*b^7*f^2 - 3978240*a^20*b^6*f^2 + 86016
0*a^21*b^5*f^2) + (28492032*a^6*b^13*f^2*(a + b*tan(e + f*x)^2)^(1/2)*(320*
a^12*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3 + 960*a^11*b^2)^(1/2))/(548
8000*a^7*b^19*f^2 - 50960000*a^8*b^18*f^2 + 207491200*a^9*b^17*f^2 - 483286
400*a^10*b^16*f^2 + 704892160*a^11*b^15*f^2 - 668407040*a^12*b^14*f^2 + 435
855616*a^13*b^13*f^2 - 248036096*a^14*b^12*f^2 + 174993280*a^15*b^11*f^2 -
118823040*a^16*b^10*f^2 + 51799680*a^17*b^9*f^2 - 15232896*a^18*b^8*f^2 + 7
343616*a^19*b^7*f^2 - 3978240*a^20*b^6*f^2 + 860160*a^21*b^5*f^2) - (314181
12*a^7*b^12*f^2*(a + b*tan(e + f*x)^2)^(1/2)*(320*a^12*b + 64*a^13 + 1225*a
^9*b^4 + 1400*a^10*b^3 + 960*a^11*b^2)^(1/2))/(5488000*a^7*b^19*f^2 - 50960
000*a^8*b^18*f^2 + 207491200*a^9*b^17*f^2 - 483286400*a^10*b^16*f^2 + 70489
2160*a^11*b^15*f^2 - 668407040*a^12*b^14*f^2 + 435855616*a^13*b^13*f^2 - 24
8036096*a^14*b^12*f^2 + 174993280*a^15*b^11*f^2 - 118823040*a^16*b^10*f^2 +
51799680*a^17*b^9*f^2 - 15232896*a^18*b^8*f^2 + 7343616*a^19*b^7*f^2 - 397
8240*a^20*b^6*f^2 + 860160*a^21*b^5*f^2) + (23893760*a^8*b^11*f^2*(a + b*ta
n(e + f*x)^2)^(1/2)*(320*a^12*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3 +
960*a^11*b^2)^(1/2))/(5488000*a^7*b^19*f^2 - 50960000*a^8*b^18*f^2 + 207491
200*a^9*b^17*f^2 - 483286400*a^10*b^16*f^2 + 704892160*a^11*b^15*f^2 - 6684
07040*a^12*b^14*f^2 + 435855616*a^13*b^13*f^2 - 248036096*a^14*b^12*f^2 + 1
74993280*a^15*b^11*f^2 - 118823040*a^16*b^10*f^2 + 51799680*a^17*b^9*f^2 -
15232896*a^18*b^8*f^2 + 7343616*a^19*b^7*f^2 - 3978240*a^20*b^6*f^2 + 86016
0*a^21*b^5*f^2) - (13559040*a^9*b^10*f^2*(a + b*tan(e + f*x)^2)^(1/2)*(320*
a^12*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3 + 960*a^11*b^2)^(1/2))/(548
8000*a^7*b^19*f^2 - 50960000*a^8*b^18*f^2 + 207491200*a^9*b^17*f^2 - 483286
400*a^10*b^16*f^2 + 704892160*a^11*b^15*f^2 - 668407040*a^12*b^14*f^2 + 435
855616*a^13*b^13*f^2 - 248036096*a^14*b^12*f^2 + 174993280*a^15*b^11*f^2 -
118823040*a^16*b^10*f^2 + 51799680*a^17*b^9*f^2 - 15232896*a^18*b^8*f^2 + 7
343616*a^19*b^7*f^2 - 3978240*a^20*b^6*f^2 + 860160*a^21*b^5*f^2) + (728640
0*a^10*b^9*f^2*(a + b*tan(e + f*x)^2)^(1/2)*(320*a^12*b + 64*a^13 + 1225*a^
9*b^4 + 1400*a^10*b^3 + 960*a^11*b^2)^(1/2))/(5488000*a^7*b^19*f^2 - 509600
00*a^8*b^18*f^2 + 207491200*a^9*b^17*f^2 - 483286400*a^10*b^16*f^2 + 704892
160*a^11*b^15*f^2 - 668407040*a^12*b^14*f^2 + 435855616*a^13*b^13*f^2 - 248
036096*a^14*b^12*f^2 + 174993280*a^15*b^11*f^2 - 118823040*a^16*b^10*f^2 +
51799680*a^17*b^9*f^2 - 15232896*a^18*b^8*f^2 + 7343616*a^19*b^7*f^2 - 3978
240*a^20*b^6*f^2 + 860160*a^21*b^5*f^2) - (4459392*a^11*b^8*f^2*(a + b*tan(
e + f*x)^2)^(1/2)*(320*a^12*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3 + 96
0*a^11*b^2)^(1/2))/(5488000*a^7*b^19*f^2 - 50960000*a^8*b^18*f^2 + 20749120
0*a^9*b^17*f^2 - 483286400*a^10*b^16*f^2 + 704892160*a^11*b^15*f^2 - 668407
040*a^12*b^14*f^2 + 435855616*a^13*b^13*f^2 - 248036096*a^14*b^12*f^2 + 174
993280*a^15*b^11*f^2 - 118823040*a^16*b^10*f^2 + 51799680*a^17*b^9*f^2 - 15

```
232896*a^18*b^8*f^2 + 7343616*a^19*b^7*f^2 - 3978240*a^20*b^6*f^2 + 860160*
a^21*b^5*f^2) + (2362752*a^12*b^7*f^2*(a + b*tan(e + f*x)^2)^(1/2)*(320*a^1
2*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3 + 960*a^11*b^2)^(1/2))/(548800
0*a^7*b^19*f^2 - 50960000*a^8*b^18*f^2 + 207491200*a^9*b^17*f^2 - 483286400
*a^10*b^16*f^2 + 704892160*a^11*b^15*f^2 - 668407040*a^12*b^14*f^2 + 435855
616*a^13*b^13*f^2 - 248036096*a^14*b^12*f^2 + 174993280*a^15*b^11*f^2 - 118
823040*a^16*b^10*f^2 + 51799680*a^17*b^9*f^2 - 15232896*a^18*b^8*f^2 + 7343
616*a^19*b^7*f^2 - 3978240*a^20*b^6*f^2 + 860160*a^21*b^5*f^2) - (766080*a^
13*b^6*f^2*(a + b*tan(e + f*x)^2)^(1/2)*(320*a^12*b + 64*a^13 + 1225*a^9*b^
4 + 1400*a^10*b^3 + 960*a^11*b^2)^(1/2))/(5488000*a^7*b^19*f^2 - 50960000*a
^8*b^18*f^2 + 207491200*a^9*b^17*f^2 - 483286400*a^10*b^16*f^2 + 704892160*
a^11*b^15*f^2 - 668407040*a^12*b^14*f^2 + 435855616*a^13*b^13*f^2 - 2480360
96*a^14*b^12*f^2 + 174993280*a^15*b^11*f^2 - 118823040*a^16*b^10*f^2 + 5179
9680*a^17*b^9*f^2 - 15232896*a^18*b^8*f^2 + 7343616*a^19*b^7*f^2 - 3978240*
a^20*b^6*f^2 + 860160*a^21*b^5*f^2) + (107520*a^14*b^5*f^2*(a + b*tan(e + f
*x)^2)^(1/2)*(320*a^12*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3 + 960*a^1
1*b^2)^(1/2))/(5488000*a^7*b^19*f^2 - 50960000*a^8*b^18*f^2 + 207491200*a^9
*b^17*f^2 - 483286400*a^10*b^16*f^2 + 704892160*a^11*b^15*f^2 - 668407040*a
^12*b^14*f^2 + 435855616*a^13*b^13*f^2 - 248036096*a^14*b^12*f^2 + 17499328
0*a^15*b^11*f^2 - 118823040*a^16*b^10*f^2 + 51799680*a^17*b^9*f^2 - 1523289
6*a^18*b^8*f^2 + 7343616*a^19*b^7*f^2 - 3978240*a^20*b^6*f^2 + 860160*a^21*
b^5*f^2))*(320*a^12*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3 + 960*a^11*b
^2)^(1/2))/(8*a^9*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.352 $\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=171

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{5/2}f}-\frac{a(a-2b)\tan(e+fx)}{b^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{a\tan^3(e+fx)}{3bf(a-b)\left(a+b\tan^2(e+f
$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f+arct
anh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f-a*(a-2*b)*tan(f*
x+e)/(a-b)^2/b^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*a*tan(f*x+e)^3/(a-b)/b/f/(a
+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.27, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3670, 470, 578, 523, 217, 206, 377, 203}

$$-\frac{a(a-2b)\tan(e+fx)}{b^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{5/2}f}-\frac{a\tan^3(e+fx)}{3bf(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2}}\right)}{f(a-b)^{5/}}
$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(b^(5/
2)*f) - (a*Tan[e + f*x]^3)/(3*(a - b)*b*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (
a*(a - 2*b)*Tan[e + f*x])/((a - b)^2*b^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
```

```
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{x^2(3a+3(a-b)x^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a-b)bf}$$

$$= -\frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2b)\tan(e+fx)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2b)\tan(e+fx)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2b)\tan(e+fx)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+\right.}$$

Mathematica [C] time = 4.58, size = 295, normalized size = 1.73

$$-\frac{\sqrt{\sec^2(e+fx)((a-b)\cos(2(e+fx))+a+b)}\left(a^2(a-b)\sin(2(e+fx))((3a-7b)((a-b)\cos(2(e+fx))+a+b\right.}{3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(a^2*(a - b)*
(2*a*b + (3*a - 7*b)*(a + b + (a - b)*Cos[2*(e + f*x)]))*Sin[2*(e + f*x)] -
 (3*a^2*b*(((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)^(3/2)*((a
^2 - 3*a*b + 2*b^2)*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)
])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + b^2*EllipticPi[-(b/(a - b)), ArcSin[Sq
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sin
[e + f*x]^2*Sin[2*(e + f*x)])/Sqrt[2]))/(Sqrt[2]*a*(a - b)^3*b^2*f*(a + b +
 (a - b)*Cos[2*(e + f*x)])^2)
```

fricas [B] time = 2.58, size = 1714, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b
^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(f*
x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqr
t(b)*tan(f*x + e) + a) - 3*(b^5*tan(f*x + e)^4 + 2*a*b^4*tan(f*x + e)^2 + a
^2*b^3)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)
^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 2*((4*a^3*b^
2 - 11*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^3 + 3*(a^4*b - 3*a^3*b^2 + 2*a^2*b^3
)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^5 - 3*a^2*b^6 + 3*a*b^7
 - b^8)*f*tan(f*x + e)^4 + 2*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*ta
n(f*x + e)^2 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f), -1/6*(6*(a^5
 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*ta
n(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*sq
rt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + 3*(b^
5*tan(f*x + e)^4 + 2*a*b^4*tan(f*x + e)^2 + a^2*b^3)*sqrt(-a + b)*log(-((a
- 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x +
 e) - a)/(tan(f*x + e)^2 + 1)) + 2*((4*a^3*b^2 - 11*a^2*b^3 + 7*a*b^4)*tan(
f*x + e)^3 + 3*(a^4*b - 3*a^3*b^2 + 2*a^2*b^3)*tan(f*x + e))*sqrt(b*tan(f*x
 + e)^2 + a))/((a^3*b^5 - 3*a^2*b^6 + 3*a*b^7 - b^8)*f*tan(f*x + e)^4 + 2*(
a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^2 + (a^5*b^3 - 3*a^
4*b^4 + 3*a^3*b^5 - a^2*b^6)*f), -1/6*(6*(b^5*tan(f*x + e)^4 + 2*a*b^4*tan(
f*x + e)^2 + a^2*b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(
a - b)*tan(f*x + e))) - 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 -
 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b
^3 - a*b^4)*tan(f*x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f
*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) + 2*((4*a^3*b^2 - 11*a^2*b^3 + 7*a
*b^4)*tan(f*x + e)^3 + 3*(a^4*b - 3*a^3*b^2 + 2*a^2*b^3)*tan(f*x + e))*sqrt
(b*tan(f*x + e)^2 + a))/((a^3*b^5 - 3*a^2*b^6 + 3*a*b^7 - b^8)*f*tan(f*x +
e)^4 + 2*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^2 + (a^5*
b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f), -1/3*(3*(b^5*tan(f*x + e)^4 + 2*
a*b^4*tan(f*x + e)^2 + a^2*b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 +
 a)/(sqrt(a - b)*tan(f*x + e))) + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 +
(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2
 + 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2
 + a)*sqrt(-b)/(b*tan(f*x + e))) + ((4*a^3*b^2 - 11*a^2*b^3 + 7*a*b^4)*tan(
f*x + e)^3 + 3*(a^4*b - 3*a^3*b^2 + 2*a^2*b^3)*tan(f*x + e))*sqrt(b*tan(f*x
 + e)^2 + a))/((a^3*b^5 - 3*a^2*b^6 + 3*a*b^7 - b^8)*f*tan(f*x + e)^4 + 2*(
a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^2 + (a^5*b^3 - 3*a^
4*b^4 + 3*a^3*b^5 - a^2*b^6)*f)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.35, size = 382, normalized size = 2.23

$$-\frac{\tan^{3}\left(fx+e\right)}{3fb\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}}}-\frac{\tan\left(fx+e\right)}{f\,b^{2}\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}}+\frac{\ln\left(\tan\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\tan^{2}\left(fx+e\right)\right)}\right)}{f\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f*tan(f*x+e)^3/b/(a+b*tan(f*x+e)^2)^(3/2)-1/f/b^2*tan(f*x+e)/(a+b*tan(
f*x+e)^2)^(1/2)+1/f/b^(5/2)*ln(tan(f*x+e)*b^(1/2)+(a+b*tan(f*x+e)^2)^(1/2))
```

```
+1/3/f/b*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2)-1/3/f/b/a*tan(f*x+e)/(a+b*tan(
f*x+e)^2)^(1/2)+1/3/f*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/f/a^2*tan(f
*x+e)/(a+b*tan(f*x+e)^2)^(1/2)+1/3*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2
)^(3/2)+2/3/f*b/(a-b)/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)+1/f*b/(a-b)^2
*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)-1/f/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*ar
ctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^6}{\left(b\,\tan\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{6}\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**(5/2),x)
[Out] Integral(tan(e + f*x)**6/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.353 $\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=131

$$\frac{(a-4b)\tan(e+fx)}{3bf(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{a\tan(e+fx)}{3bf(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f+1/3*(
a-4*b)*tan(f*x+e)/(a-b)^2/b/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*a*tan(f*x+e)/(a-
b)/b/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.16, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3670, 470, 527, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}+\frac{(a-4b)\tan(e+fx)}{3bf(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{a\tan(e+fx)}{3bf(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (a*Tan[e + f*x])/(3*(a - b)*b*f*(a + b*Tan[e + f*x]^2)^(3/2)) + ((a
- 4*b)*Tan[e + f*x])/(3*(a - b)^2*b*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+(a-3b)x^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a-b)bf}$$

$$= -\frac{a\tan(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-4b)\tan(e+fx)}{3(a-b)^2bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{a\tan(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-4b)\tan(e+fx)}{3(a-b)^2bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{a\tan(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-4b)\tan(e+fx)}{3(a-b)^2bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{a\tan(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-4b)\tan(}{3(a-b)^2bf\sqrt{a+b}}$$

Mathematica [A] time = 6.05, size = 260, normalized size = 1.98

$$\frac{\tan^5(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)\left(-\frac{\left(\frac{b\tan^2(e+fx)}{a}-\tan^2(e+fx)\right)^2}{3\left(\frac{b\tan^2(e+fx)}{a}+1\right)^2} - \frac{\frac{b\tan^2(e+fx)}{a}-\tan^2(e+fx)}{\frac{b\tan^2(e+fx)}{a}+1} + \frac{\sqrt{\frac{b\tan^2(e+fx)}{a}-\tan^2(e+fx)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{b\tan^2(e+}{a}}}{\sqrt{b\tan}}\right.}{\sqrt{\frac{b\tan^2(e+fx)}{a}+1}}\right.}{a^2f\sqrt{a+b\tan^2(e+fx)}\left(\frac{b\tan^2(e+fx)}{a}-\tan^2(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Tan[e + f*x]^5*(1 + (b*Tan[e + f*x]^2)/a)*((ArcTanh[Sqrt[-Tan[e + f*x]^2 +
 (b*Tan[e + f*x]^2)/a]/Sqrt[1 + (b*Tan[e + f*x]^2)/a]]*Sqrt[-Tan[e + f*x]^2
 + (b*Tan[e + f*x]^2)/a])/Sqrt[1 + (b*Tan[e + f*x]^2)/a] - (-Tan[e + f*x]^2
 + (b*Tan[e + f*x]^2)/a)/(1 + (b*Tan[e + f*x]^2)/a) - (-Tan[e + f*x]^2 + (b
*Tan[e + f*x]^2)/a)^2/(3*(1 + (b*Tan[e + f*x]^2)/a)^2)))/(a^2*f*Sqrt[a + b*
Tan[e + f*x]^2]*(-Tan[e + f*x]^2 + (b*Tan[e + f*x]^2)/a)^3)
```

fricas [A] time = 0.50, size = 498, normalized size = 3.80

$$\left[-\frac{3\left(b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2\right)\sqrt{-a+b}\,\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2-2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\,\tan\left(fx+e\right)-}{\tan\left(fx+e\right)^2+1}\right.}{6\left(\left(a^3b^2-3\,a^2b^3+3\,ab^4-b^5\right)f\tan\left(fx+e\right)^4+2\left(a^4b-3\,a^3b^2+3\,a^2b^3-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*log
(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan
(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 2*((a^2 - 5*a*b + 4*b^2)*tan(f*x + e
)^3 - 3*(a^2 - a*b)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^2 - 3
*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b
^3 - a*b^4)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f), 1/
3*(3*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*arctan(-
sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + ((a^2 - 5*a*b + 4*
b^2)*tan(f*x + e)^3 - 3*(a^2 - a*b)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a
))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 -
 a^2*b^3)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

maple [B] time = 0.32, size = 291, normalized size = 2.22

$$-\frac{\tan\left(fx+e\right)}{3fb\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}+\frac{\tan\left(fx+e\right)}{3fba\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}-\frac{\tan\left(fx+e\right)}{3fa\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}-\frac{2\tan\left(fx\right.}{3f\,a^2\sqrt{a+b\left(\tan\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f/b*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2)+1/3/f/b/a*tan(f*x+e)/(a+b*tan(
f*x+e)^2)^(1/2)-1/3/f*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)-2/3/f/a^2*tan(f
*x+e)/(a+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2
)^(3/2)-2/3/f*b/(a-b)/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)-1/f*b/(a-b)^2
*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*ar
ctan((a-b)*b^2/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^4}{{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}{\left(e + f x \right)}}{\left(a + b \tan^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.354 $\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{(2a+b)\tan(e+fx)}{3af(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan(e+fx)}{3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f+1/3*
(2*a+b)*tan(f*x+e)/a/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1/3*tan(f*x+e)/(a-b
)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.15, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3670, 471, 527, 12, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}+\frac{(2a+b)\tan(e+fx)}{3af(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan(e+fx)}{3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) + Tan[e + f*x]/(3*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) + ((2*a +
 b)*Tan[e + f*x])/(3*a*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1-2x^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a-b)f}$$

$$= \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+b)\tan(e+fx)}{3a(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+b)\tan(e+fx)}{3a(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+b)\tan(e+fx)}{3a(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} + \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+b)\tan}{3a(a-b)^2f\sqrt{a+}}$$

Mathematica [C] time = 7.77, size = 365, normalized size = 2.85

$$\frac{\cos^4(e+fx)\cot(e+fx)\left(12(a-b)^3\tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,{}_2F_1\left(2,2;\frac{9}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)\sqrt{\frac{\sin^2(e+fx)\cos^2(e}{}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cos[e + f*x]^4*Cot[e + f*x]*(12*(a - b)^3*Hypergeometric2F1[2, 2, 9/2, ((a
 - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)*Sqrt[(Cos[e
+ f*x]^2*Sin[e + f*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^
2)))/a^2] + 35*a*Sec[e + f*x]^2*(5*a + 2*b*Tan[e + f*x]^2)*(3*ArcSin[Sqrt[(
(a - b)*Sin[e + f*x]^2)/a]]*(a + b*Tan[e + f*x]^2)^2 + a*Sec[e + f*x]^2*(-4
*b*Tan[e + f*x]^2 + a*(-3 + Tan[e + f*x]^2))*Sqrt[(Cos[e + f*x]^2*Sin[e + f
*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/a^2])))/(315*
a^4*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2]*Sqrt[((a - b)*Cos[e + f*x]^2*Sin
[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]*(1 + (b*Tan[e + f*x]^2)/a))
```

fricas [B] time = 0.55, size = 529, normalized size = 4.13

$$\left[-\frac{3\left(ab^2\tan\left(fx+e\right)^4+2\,a^2b\tan\left(fx+e\right)^2+a^3\right)\sqrt{-a+b}\,\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\,\tan\left(fx+\right.}{\tan\left(fx+e\right)^2+1}\right.}{6\left(\left(a^4b^2-3\,a^3b^3+3\,a^2b^4-ab^5\right)f\tan\left(fx+e\right)^4+2\left(a^5b-3\,a^4b^2+3\,a^3b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a*b^2*tan(f*x + e)^4 + 2*a^2*b*tan(f*x + e)^2 + a^3)*sqrt(-a + b)
*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)
*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 2*((2*a^2*b - a*b^2 - b^3)*tan(f
*x + e)^3 + 3*(a^3 - a^2*b)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^4
*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*f*tan(f*x + e)^4 + 2*(a^5*b - 3*a^4*b
^2 + 3*a^3*b^3 - a^2*b^4)*f*tan(f*x + e)^2 + (a^6 - 3*a^5*b + 3*a^4*b^2 - a
^3*b^3)*f), -1/3*(3*(a*b^2*tan(f*x + e)^4 + 2*a^2*b*tan(f*x + e)^2 + a^3)*s
qrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) -
 ((2*a^2*b - a*b^2 - b^3)*tan(f*x + e)^3 + 3*(a^3 - a^2*b)*tan(f*x + e))*sq
rt(b*tan(f*x + e)^2 + a))/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*f*tan(
f*x + e)^4 + 2*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*f*tan(f*x + e)^2 +
 (a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

maple [B] time = 0.26, size = 232, normalized size = 1.81

$$\frac{\tan\left(fx+e\right)}{3fa\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}+\frac{2\tan\left(fx+e\right)}{3fa^2\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}+\frac{b\tan\left(fx+e\right)}{3a\left(a-b\right)f\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}+\frac{}{3f\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/f/a^2*tan(f*x+e)/(a+b*tan(f
*x+e)^2)^(1/2)+1/3*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)+2/3/f*b/
```

```
(a-b)/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)+1/f*b/(a-b)^2*tan(f*x+e)/a/(a
+b*tan(f*x+e)^2)^(1/2)-1/f/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan((a-b)*b^2/(
b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^2}{{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}{\left(e + f x \right)}}{\left(a + b \tan^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.355 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{b(5a-2b)\tan(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{b\tan(e+fx)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3*(
5*a-2*b)*b*tan(f*x+e)/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+
e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3661, 414, 527, 12, 377, 203}

$$-\frac{b(5a-2b)\tan(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{b\tan(e+fx)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (b*Tan[e + f*x])/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - ((5*
a - 2*b)*b*Tan[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-2b-2bx^2}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}}{}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}}{}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan}{3a^2(a-b)^2f\sqrt{a+}}$$

Mathematica [C] time = 7.60, size = 1331, normalized size = 9.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(1575*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]] -
 (3150*(a - b)*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2)/a +
 (1575*(a - b)^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4)/a
^2 + (2100*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/a - (
4200*(a - b)*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2*Tan[
```

```
e + f*x]^2)/a^2 + (2100*(a - b)^2*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]
]*Sin[e + f*x]^4*Tan[e + f*x]^2)/a^3 + (840*b^2*ArcSin[Sqrt[((a - b)*Sin[e
+ f*x]^2)/a]]*Tan[e + f*x]^4)/a^2 - (1680*(a - b)*b^2*ArcSin[Sqrt[((a - b)*
Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2*Tan[e + f*x]^4)/a^3 + (840*(a - b)^2*b^2
*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4*Tan[e + f*x]^4)/a^
4 + 2100*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan
[e + f*x]^2))/a] + 96*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)
/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e +
f*x]^2))/a] + 24*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*
x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan
[e + f*x]^2))/a] + (2800*b*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Tan[e + f*x]^
2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a + (168*b*Hypergeometri
c2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7
/2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a + (48
*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*(((a
- b)*Sin[e + f*x]^2)/a)^(7/2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Ta
n[e + f*x]^2))/a])/a + (1120*b^2*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Tan[e +
 f*x]^4*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a^2 + (72*b^2*Hype
rgeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]
^2)/a)^(7/2)*Tan[e + f*x]^4*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]
)/a^2 + (24*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x
]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Tan[e + f*x]^4*Sqrt[(Cos[e + f*x
]^2*(a + b*Tan[e + f*x]^2))/a])/a^2 - 1575*Sqrt[((a - b)*Cos[e + f*x]^2*Sin
[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2] - (2100*b*Tan[e + f*x]^2*Sqrt[((a
- b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2])/a - (840*b
^2*Tan[e + f*x]^4*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e
+ f*x]^2))/a^2])/a^2))/(315*a^2*f*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[a
 + b*Tan[e + f*x]^2]*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]*(1 + (
b*Tan[e + f*x]^2)/a))
```

fricas [B] time = 0.64, size = 561, normalized size = 4.19

$$\left[-\frac{3\left(a^2b^2\tan\left(fx+e\right)^4+2\,a^3b\tan\left(fx+e\right)^2+a^4\right)\sqrt{-a+b}\,\log\left(-\frac{(a-2\,b)\tan\left(fx+e\right)^2-2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\,\tan\left(fx\cdots\right.}{\tan\left(fx+e\right)^2+1}\right.}{6\left(\left(a^5b^2-3\,a^4b^3+3\,a^3b^4-a^2b^5\right)f\tan\left(fx+e\right)^4+2\left(a^6b-3\,a^5b\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a^2*b^2*tan(f*x + e)^4 + 2*a^3*b*tan(f*x + e)^2 + a^4)*sqrt(-a +
b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*((5*a^2*b^2 - 7*a*b^3 + 2*b^
4)*tan(f*x + e)^3 + 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*ta
n(f*x + e)^2 + a))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x +
 e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7
 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/3*(3*(a^2*b^2*tan(f*x + e)^4 + 2*a^
3*b*tan(f*x + e)^2 + a^4)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(s
qrt(a - b)*tan(f*x + e))) - ((5*a^2*b^2 - 7*a*b^3 + 2*b^4)*tan(f*x + e)^3 +
 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/
((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b -
3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*
b^2 - a^4*b^3)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(-5/2), x)
```

maple [A] time = 0.43, size = 176, normalized size = 1.31

$$-\frac{b\tan\left(fx+e\right)}{3a\left(a-b\right)f\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}-\frac{2b\tan\left(fx+e\right)}{3f\left(a-b\right)a^2\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}-\frac{b\tan\left(fx+e\right)}{f\left(a-b\right)^2a\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-2/3/f*b/(a-b)/a^2*tan(
f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)-1/f*b/(a-b)^2*tan(f*x+e)/a/(a+b*tan(f*x+e)^
2)^(1/2)+1/f/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan((a-b)*b^2/(b^4*(a-b))^(1/
2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b*tan(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(-5/2), x)
```

3.356 $\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=186

$$-\frac{(a-4b)(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f(a-b)^2}-\frac{b(7a-4b)\cot(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3*
(7*a-4*b)*b*cot(f*x+e)/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*(a-4*b)*(
3*a-2*b)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^3/(a-b)^2/f-1/3*b*cot(f*x+e)
/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.28, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3670, 472, 579, 583, 12, 377, 203}

$$-\frac{(a-4b)(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f(a-b)^2}-\frac{b(7a-4b)\cot(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) - (b*Cot[e + f*x])/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (
(7*a - 4*b)*b*Cot[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
- ((a - 4*b)*(3*a - 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^3*(a
 - b)^2*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
```

```
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-4b-4bx^2}{x^2(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+\right.}{3a(a-b)f}$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(7a-4b)b\cot(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Sul}}{}$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(7a-4b)b\cot(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-}{}$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(7a-4b)b\cot(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-}{}$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(7a-4b)b\cot(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\cot(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(7a-4b}{3a^2(a-b)^2f\sqrt{}}$$

Mathematica [C] time = 15.92, size = 1890, normalized size = 10.16

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((Cos[e + f*x]^2*Cot[e + f*x]*((20*a*Csc[e + f*x]^2)/(3*(a - b)) - (5*a^2*
Csc[e + f*x]^4)/(a - b)^2 + (40*b*Sec[e + f*x]^2)/(a - b) - (30*a*b*Csc[e +
 f*x]^2*Sec[e + f*x]^2)/(a - b)^2 - (40*b^2*Sec[e + f*x]^4)/(a - b)^2 + (92
*(a - b)*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f
*x]^2)/(105*a) + (24*(a - b)*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b
)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/(35*a) + (16*(a - b)*HypergeometricPFQ
[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/(10
5*a) + (160*b^2*Sec[e + f*x]^2*Tan[e + f*x]^2)/(3*a*(a - b)) + (124*(a - b)
*b*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*
Tan[e + f*x]^2)/(35*a^2) + (16*(a - b)*b*HypergeometricPFQ[{2, 2, 2}, {1, 9
/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(7*a^2) + (
16*(a - b)*b*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Sin[e +
f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(35*a^2) + (64*b^3*Sec[e + f*x]^2
*Tan[e + f*x]^4)/(3*a^2*(a - b)) + (152*(a - b)*b^2*Hypergeometric2F1[2, 2,
 9/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(35*a^3) +
 (88*(a - b)*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*
x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(35*a^3) + (16*(a - b)*b^2*Hypergeo
metricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*
```

```
x]^2*Tan[e + f*x]^4)/(35*a^3) + (176*(a - b)*b^3*Hypergeometric2F1[2, 2, 9/
2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^6)/(105*a^4) + (
32*(a - b)*b^3*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x]
^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^6)/(35*a^4) + (16*(a - b)*b^3*Hypergeome
tricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]
^2*Tan[e + f*x]^6)/(105*a^4) + (5*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]])
/((((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*
x]^2))/a]) + (30*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)
/(a*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e +
f*x]^2))/a]) + (40*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x
]^4)/(a^2*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Ta
n[e + f*x]^2))/a]) + (16*b^3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e
 + f*x]^6)/(a^3*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[(Cos[e + f*x]^2*(a
+ b*Tan[e + f*x]^2))/a]) + (5*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]])/Sqr
t[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2] - (10
*a*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Csc[e + f*x]^2)/((a - b)*Sqrt[(
(a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) - (60*b
*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sec[e + f*x]^2)/((a - b)*Sqrt[((a
 - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) + (30*b*A
rcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/(a*Sqrt[((a - b)*Co
s[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) - (80*b^2*ArcSin[
Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sec[e + f*x]^2*Tan[e + f*x]^2)/(a*(a - b)
*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2])
+ (40*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^4)/(a^2*Sqr
t[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) - (3
2*b^3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sec[e + f*x]^2*Tan[e + f*x]^
4)/(a^2*(a - b)*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e +
f*x]^2))/a^2]) + (16*b^3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f
*x]^6)/(a^3*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]
^2))/a^2]) - (16*b^3*(Tan[e + f*x] + Tan[e + f*x]^3)^2)/(a*(a - b)^2)))/(a^
2*f*Sqrt[a + b*Tan[e + f*x]^2]*(1 + (b*Tan[e + f*x]^2)/a)))
```

fricas [B] time = 0.68, size = 753, normalized size = 4.05

$$\left[-\frac{3\left(a^3b^2\tan\left(fx+e\right)^5+2\,a^4b\tan\left(fx+e\right)^3+a^5\tan\left(fx+e\right)\right)\sqrt{-a+b}\,\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)\tan\left(fx+e\right)^4-2\left(3\,a^2-4\,ab\right)\mathrm{ta}\cdots}{\cdots}\right.}{12\left(\left(a^6b^2-3\,a^5b^3\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^3*b^2*tan(f*x + e)^5 + 2*a^4*b*tan(f*x + e)^3 + a^5*tan(f*x +
e))*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*
a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*s
qrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2
+ 1)) + 4*(3*a^5 - 9*a^4*b + 9*a^3*b^2 - 3*a^2*b^3 + (3*a^3*b^2 - 17*a^2*b^
3 + 22*a*b^4 - 8*b^5)*tan(f*x + e)^4 + 3*(2*a^4*b - 9*a^3*b^2 + 11*a^2*b^3
- 4*a*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^
3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^5 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^
3 - a^4*b^4)*f*tan(f*x + e)^3 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f*tan
(f*x + e)), -1/6*(3*(a^3*b^2*tan(f*x + e)^5 + 2*a^4*b*tan(f*x + e)^3 + a^5*
tan(f*x + e))*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*
tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*(3*a^5 - 9*a^4*b + 9*a^3*b
^2 - 3*a^2*b^3 + (3*a^3*b^2 - 17*a^2*b^3 + 22*a*b^4 - 8*b^5)*tan(f*x + e)^4
 + 3*(2*a^4*b - 9*a^3*b^2 + 11*a^2*b^3 - 4*a*b^4)*tan(f*x + e)^2)*sqrt(b*ta
n(f*x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x +
```

```
e)^5 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^3 + (a^8
- 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f*tan(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{2}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

maple [F] time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{\cot^{2}\left(fx+e\right)}{\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
[Out] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{cot}\left(e+f\,x\right)}^2}{{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2),x)
[Out] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{2}{\left(e + f x \right)}}{\left(a + b \tan^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
[Out] Integral(cot(e + f*x)**2/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.357 $\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=249

$$-\frac{b(3a-2b)\cot^3(e+fx)}{a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{(a-2b)\left(3a^2+8ab-8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^4f(a-b)^2}-\frac{\left(a^2-12ab+8b^2\right.}{}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-(3*a-
2*b)*b*cot(f*x+e)^3/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1/3*(a-2*b)*(3*a
^2+8*a*b-8*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^4/(a-b)^2/f-1/3*(a^2-
12*a*b+8*b^2)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a^3/(a-b)^2/f-1/3*b*cot
(f*x+e)^3/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.38, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3670, 472, 579, 583, 12, 377, 203}

$$-\frac{\left(a^2-12ab+8b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f(a-b)^2}+\frac{(a-2b)\left(3a^2+8ab-8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx}}{3a^4f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (b*Cot[e + f*x]^3)/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - ((
3*a - 2*b)*b*Cot[e + f*x]^3)/(a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2]) +
 ((a - 2*b)*(3*a^2 + 8*a*b - 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2]
)/(3*a^4*(a - b)^2*f) - ((a^2 - 12*a*b + 8*b^2)*Cot[e + f*x]^3*Sqrt[a + b*T
an[e + f*x]^2])/(3*a^3*(a - b)^2*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
```

```
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3(a-2b)-6bx^2}{x^4(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx\right.}{3a(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\left(a^2-12\right.}{}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{(a-2b}{}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{(a-2b}{}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{(a-2b}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)b\cot^3}{a^2(a-b)^2f\sqrt{a+b}}$$

Mathematica [C] time = 16.59, size = 871, normalized size = 3.50

$$-\frac{b\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{\cos(2(e+fx))+1}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(\cos(2(e+fx))+1)\csc^2(e+fx)}{b}}\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\csc(2(e+fx))F\left(\sin^{-1}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+\right.}{b}}}{\sqrt{2}}\right.\right.}{a(a+b+(a-b)\cos(2(e+fx)))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (-((b*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[
-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b
)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f
*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e + f*x)])))
 - (4*b*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/
(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[
```

```
2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*
Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)
*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqr
t[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((
a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*
Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)
]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*
Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(
e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]))))/Sqrt[a + b + (a - b)*C
os[2*(e + f*x)]])/((a - b)^2*f) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos
[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((4*(a*Cos[e + f*x] + 2*b*Cos[e + f*
x])*Csc[e + f*x])/(3*a^4) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*a^3) + (2*b^4*
Sin[2*(e + f*x)])/(3*a^3*(a - b)^2*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e
 + f*x)])^2) - (4*(3*a*b^3*Sin[2*(e + f*x)] - 2*b^4*Sin[2*(e + f*x)]))/(3*a
^4*(a - b)^2*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)]))))/f
```

fricas [A] time = 0.73, size = 879, normalized size = 3.53

$$\left[-\frac{3\left(a^4b^2\tan\left(fx+e\right)^7+2\,a^5b\tan\left(fx+e\right)^5+a^6\tan\left(fx+e\right)^3\right)\sqrt{-a+b}\,\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)\tan\left(fx+e\right)^4-2\left(3\,a^2-4\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^4*b^2*tan(f*x + e)^7 + 2*a^5*b*tan(f*x + e)^5 + a^6*tan(f*x +
e)^3)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 -
4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))
*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^
2 + 1)) - 4*((3*a^4*b^2 - a^3*b^3 - 26*a^2*b^4 + 40*a*b^5 - 16*b^6)*tan(f*x
 + e)^6 - a^6 + 3*a^5*b - 3*a^4*b^2 + a^3*b^3 + 3*(2*a^5*b - a^4*b^2 - 13*a
^3*b^3 + 20*a^2*b^4 - 8*a*b^5)*tan(f*x + e)^4 + 3*(a^6 - a^5*b - 3*a^4*b^2
+ 5*a^3*b^3 - 2*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*
b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^7 + 2*(a^8*b - 3*a^7*
b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^5 + (a^9 - 3*a^8*b + 3*a^7*b^2 -
a^6*b^3)*f*tan(f*x + e)^3), 1/6*(3*(a^4*b^2*tan(f*x + e)^7 + 2*a^5*b*tan(f*
x + e)^5 + a^6*tan(f*x + e)^3)*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2
+ a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*((3*a^4*b
^2 - a^3*b^3 - 26*a^2*b^4 + 40*a*b^5 - 16*b^6)*tan(f*x + e)^6 - a^6 + 3*a^5
*b - 3*a^4*b^2 + a^3*b^3 + 3*(2*a^5*b - a^4*b^2 - 13*a^3*b^3 + 20*a^2*b^4 -
 8*a*b^5)*tan(f*x + e)^4 + 3*(a^6 - a^5*b - 3*a^4*b^2 + 5*a^3*b^3 - 2*a^2*b
^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^2 - 3*a^6*b^3 + 3*a
^5*b^4 - a^4*b^5)*f*tan(f*x + e)^7 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5
*b^4)*f*tan(f*x + e)^5 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x +
e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

maple [F] time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{\cot^4\left(fx+e\right)}{\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
[Out] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
[Out] Integral(cot(e + f*x)**4/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.358 $\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=327

$$-\frac{b(11a-8b)\cot^5(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\left(a^2-22ab+16b^2\right)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a^3f(a-b)^2}+\frac{\left(5a^3+4a^2b-88a\right.}{}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3*
(11*a-8*b)*b*cot(f*x+e)^5/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a
^4+10*a^3*b+8*a^2*b^2-176*a*b^3+128*b^4)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2
)/a^5/(a-b)^2/f+1/15*(5*a^3+4*a^2*b-88*a*b^2+64*b^3)*cot(f*x+e)^3*(a+b*tan(
f*x+e)^2)^(1/2)/a^4/(a-b)^2/f-1/5*(a^2-22*a*b+16*b^2)*cot(f*x+e)^5*(a+b*tan
(f*x+e)^2)^(1/2)/a^3/(a-b)^2/f-1/3*b*cot(f*x+e)^5/a/(a-b)/f/(a+b*tan(f*x+e)
^2)^(3/2)
```

Rubi [A] time = 0.50, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3670, 472, 579, 583, 12, 377, 203}

$$-\frac{\left(a^2-22ab+16b^2\right)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a^3f(a-b)^2}+\frac{\left(4a^2b+5a^3-88ab^2+64b^3\right)\cot^3(e+fx)\sqrt{a+b\tan^2}}{15a^4f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) - (b*Cot[e + f*x]^5)/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) -
 ((11*a - 8*b)*b*Cot[e + f*x]^5)/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]
^2]) - ((15*a^4 + 10*a^3*b + 8*a^2*b^2 - 176*a*b^3 + 128*b^4)*Cot[e + f*x]*
Sqrt[a + b*Tan[e + f*x]^2])/(15*a^5*(a - b)^2*f) + ((5*a^3 + 4*a^2*b - 88*a
*b^2 + 64*b^3)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a^4*(a - b)^2
*f) - ((a^2 - 22*a*b + 16*b^2)*Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(
5*a^3*(a - b)^2*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-8b-8bx^2}{x^6(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+\right.}{3a(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \text{Su}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a^2}{}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{(5a}{}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{(15}{}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{(15}{}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{(15}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b}{3a^2(a-b)^2f\sqrt{}}$$

Mathematica [C] time = 16.77, size = 921, normalized size = 2.82

$$\frac{\sqrt{\frac{\cos(2(e+fx))a+a+b-b\cos(2(e+fx))}{\cos(2(e+fx))+1}}\left(-\frac{2\sin(2(e+fx))b^5}{3a^4(a-b)^2(\cos(2(e+fx))a+a+b-b\cos(2(e+fx)))^2} - \frac{\cot(e+fx)\csc^4(e+fx)}{5a^3} + \frac{(11a\cos(e+fx)+14b\cos}{15a}\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((-((b*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqr
t[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)
```

```
/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e +
 f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*
x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e + f*x)])
)) - (4*b*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)]
)/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Co
s[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)]
)*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a -
b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*S
qrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-
((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)
]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*
x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)]
)*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2
*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)
*Cos[2*(e + f*x)]])/((a - b)^2*f)) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*
Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(((-23*a^2*Cos[e + f*x] - 54*a*b*
Cos[e + f*x] - 73*b^2*Cos[e + f*x])*Csc[e + f*x])/(15*a^5) + ((11*a*Cos[e +
 f*x] + 14*b*Cos[e + f*x])*Csc[e + f*x]^3)/(15*a^4) - (Cot[e + f*x]*Csc[e +
 f*x]^4)/(5*a^3) - (2*b^5*Sin[2*(e + f*x)])/(3*a^4*(a - b)^2*(a + b + a*Cos
[2*(e + f*x)] - b*Cos[2*(e + f*x)])^2) + (15*a*b^4*Sin[2*(e + f*x)] - 11*b^
5*Sin[2*(e + f*x)])/(3*a^5*(a - b)^2*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*
(e + f*x)]))))/f
```

fricas [A] time = 0.98, size = 1023, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a^5*b^2*tan(f*x + e)^9 + 2*a^6*b*tan(f*x + e)^7 + a^7*tan(f*x +
 e)^5)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 -
 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e)
)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)
^2 + 1)) + 4*((15*a^5*b^2 - 5*a^4*b^3 - 2*a^3*b^4 - 184*a^2*b^5 + 304*a*b^6
 - 128*b^7)*tan(f*x + e)^8 + 3*a^7 - 9*a^6*b + 9*a^5*b^2 - 3*a^4*b^3 + 3*(1
0*a^6*b - 5*a^5*b^2 - a^4*b^3 - 92*a^3*b^4 + 152*a^2*b^5 - 64*a*b^6)*tan(f*
x + e)^6 + 3*(5*a^7 - 5*a^6*b + a^5*b^2 - 23*a^4*b^3 + 38*a^3*b^4 - 16*a^2*
b^5)*tan(f*x + e)^4 - (5*a^7 - 7*a^6*b - 9*a^5*b^2 + 19*a^4*b^3 - 8*a^3*b^4
)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6
*b^4 - a^5*b^5)*f*tan(f*x + e)^9 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b
^4)*f*tan(f*x + e)^7 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e
)^5), -1/30*(15*(a^5*b^2*tan(f*x + e)^9 + 2*a^6*b*tan(f*x + e)^7 + a^7*tan(
f*x + e)^5)*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*ta
n(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*((15*a^5*b^2 - 5*a^4*b^3 - 2
*a^3*b^4 - 184*a^2*b^5 + 304*a*b^6 - 128*b^7)*tan(f*x + e)^8 + 3*a^7 - 9*a^
6*b + 9*a^5*b^2 - 3*a^4*b^3 + 3*(10*a^6*b - 5*a^5*b^2 - a^4*b^3 - 92*a^3*b^
4 + 152*a^2*b^5 - 64*a*b^6)*tan(f*x + e)^6 + 3*(5*a^7 - 5*a^6*b + a^5*b^2 -
 23*a^4*b^3 + 38*a^3*b^4 - 16*a^2*b^5)*tan(f*x + e)^4 - (5*a^7 - 7*a^6*b -
9*a^5*b^2 + 19*a^4*b^3 - 8*a^3*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 +
 a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^9 + 2*(a^9
*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b^4)*f*tan(f*x + e)^7 + (a^10 - 3*a^9*b +
3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e)^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{6}}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

maple [F] time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{\cot^{6}\left(fx+e\right)}{\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{6}\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**6/(a + b*tan(e + f*x)**2)**(5/2), x)
```

3.359 $\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=72

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(m+2p+1);\frac{1}{2}(m+2p+3);-\tan^2(e+fx)\right)}{f(m+2p+1)}$$

```
[Out] hypergeom([1, 1/2+1/2*m+p],[3/2+1/2*m+p],-tan(f*x+e)^2)*tan(f*x+e)*(d*tan(f
*x+e))^m*(b*tan(f*x+e)^2)^p/f/(1+m+2*p)
```

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3578, 20, 3476, 364}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(m+2p+1);\frac{1}{2}(m+2p+3);-\tan^2(e+fx)\right)}{f(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + 2*p)/2, (3 + m + 2*p)/2, -Tan[e + f*x]^2]*Ta
n[e + f*x]*(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p)/(f*(1 + m + 2*p))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3578

```
Int[((c_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Tan[e + f*x
])^p)^FracPart[n])/(d*Tan[e + f*x])^(p*FracPart[n]), Int[(a + b*Tan[e + f*x
])^m*(d*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int \tan^{2p}(e+fx)(d\tan(e+fx))^m$$
$$= \left(\tan^{-m-2p}(e+fx)(d\tan(e+fx))^m\left(b\tan^2(e+fx)\right)^p\right)\int \tan^{m+2p}(e$$
$$= \frac{\left(\tan^{-m-2p}(e+fx)(d\tan(e+fx))^m\left(b\tan^2(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{m+}}{1+x}\right.}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(1+m+2p);\frac{1}{2}(3+m+2p);-\tan^2(e+fx)\right)\tan(e+fx)(}{f(1+m+2p)}$$

Mathematica [A] time = 0.09, size = 74, normalized size = 1.03

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(m+2p+1);\frac{1}{2}(m+2p+1)+1;-\tan^2(e+fx)\right)}{f(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
[Out] (Hypergeometric2F1[1, (1 + m + 2*p)/2, 1 + (1 + m + 2*p)/2, -Tan[e + f*x]^2
]*Tan[e + f*x]*(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p)/(f*(1 + m + 2*p))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2\right)^p\left(d\tan\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2)^p*(d*tan(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2\right)^p\left(d\tan\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2)^p*(d*tan(f*x + e))^m, x)
```

maple [F] time = 3.06, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(fx+e\right)\right)^m\left(b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
[Out] int((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2\right)^p\left(d\tan\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2)^p*(d*tan(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\tan\left(e+f\,x\right)\right)^m\,\left(b\tan\left(e+f\,x\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
[Out] int((d*tan(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^2\left(e+fx\right)\right)^p\left(d\tan\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
[Out] Integral((b*tan(e + f*x)**2)**p*(d*tan(e + f*x))**m, x)
```

3.360 $\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=100

$$\frac{(d\tan(e+fx))^{m+1}\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};1,-p;\frac{m+3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{df(m+1)}$$

```
[Out] AppellF1(1/2+1/2*m,1,-p,3/2+1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(d*tan(f
*x+e))^(1+m)*(a+b*tan(f*x+e)^2)^p/d/f/(1+m)/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3670, 511, 510}

$$\frac{(d\tan(e+fx))^{m+1}\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};1,-p;\frac{m+3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{df(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 + m)/2, 1, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*(d*Tan[e + f*x])^(1 + m)*(a + b*Tan[e + f*x]^2)^p)/(d*f*(1 + m)*(1 +
(b*Tan[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \frac{\text{Subst}\left(\int \frac{(dx)^m\left(a+bx^2\right)^p}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{(dx)^m\left(1+\frac{bx^2}{a}\right)^p}{1+x^2} d\right.}{f}$$

$$= \frac{F_1\left(\frac{1+m}{2};1,-p;\frac{3+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\tan(e+fx))^{1+}}{df(1+m)}$$

Mathematica [A] time = 0.31, size = 101, normalized size = 1.01

$$\frac{\tan(e+fx)(d\tan(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,1;\frac{m+3}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+\right.}{f(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Tan[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*
x]^2]*Tan[e + f*x]*(d*Tan[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + m)*
(1 + (b*Tan[e + f*x]^2)/a)^p)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\left(d\tan\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\left(d\tan\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

maple [F] time = 2.20, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(fx+e\right)\right)^m\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan\left(fx+e\right)^{2}+a\right)^{p} \left(d \tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d \tan\left(e+f\,x\right)\right)^{m} \left(b \tan\left(e+f\,x\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m*(a + b*tan(e + f*x)^2)^p,x)
[Out] int((d*tan(e + f*x))^m*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

3.361 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=129

$$-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{p+1}}{2b^2f(p+1)}+\frac{\left(a+b\tan^2(e+fx)\right)^{p+2}}{2b^2f(p+2)}-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)}{a-b}\right.}{2f(p+1)(a-b)}$$

```
[Out] -1/2*(a+b)*(a+b*tan(f*x+e)^2)^(1+p)/b^2/f/(1+p)-1/2*hypergeom([1, 1+p],[2+p
],(a+b*tan(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)^(1+p)/(a-b)/f/(1+p)+1/2*(a+b
*tan(f*x+e)^2)^(2+p)/b^2/f/(2+p)
```

Rubi [A] time = 0.17, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3670, 446, 88, 68}

$$-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{p+1}}{2b^2f(p+1)}+\frac{\left(a+b\tan^2(e+fx)\right)^{p+2}}{2b^2f(p+2)}-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)}{a-b}\right.}{2f(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((a + b)*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*b^2*f*(1 + p)) - (Hypergeometr
ic2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^
2)^(1 + p))/(2*(a - b)*f*(1 + p)) + (a + b*Tan[e + f*x]^2)^(2 + p)/(2*b^2*f
*(2 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^5\left(a+bx^2\right)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^p}{1+x}\,dx,x,\tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-b)(a+bx)^p}{b}+\frac{(a+bx)^p}{1+x}+\frac{(a+bx)^{1+p}}{b}\right)dx,x,\tan^2(e+fx)\right)}{2f}$$
$$= -\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2b^2f(1+p)}+\frac{\left(a+b\tan^2(e+fx)\right)^{2+p}}{2b^2f(2+p)}+\frac{\text{Sub}}{}$$
$$= -\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2b^2f(1+p)}-\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx}{a-b}\right.}{2(a-b)f(}$$

Mathematica [A] time = 0.91, size = 106, normalized size = 0.82

$$\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\left(b^2(p+2)\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)+(a-b)\left(a-b(p+1)\tan^2(e+fx)+b(p\right.\right.}{2b^2f(p+1)(p+2)(b-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
[Out] ((a + b*Tan[e + f*x]^2)^(1 + p)*(b^2*(2 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, (a + b*Tan[e + f*x]^2)/(a - b)] + (a - b)*(a + b*(2 + p) - b*(1 + p)*T
an[e + f*x]^2)))/(2*b^2*(-a + b)*f*(1 + p)*(2 + p))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

maple [F] time = 1.16, size = 0, normalized size = 0.00

$$\int \left(\tan^5\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan \left(fx + e\right)^{2} + a\right)^{p} \tan \left(fx + e\right)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e + f\,x\right)^5\,{\left(b\,\tan\left(e + f\,x\right)^2 + a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

3.362 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=95

$$\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}+\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}}{2bf(p+1)}$$

```
[Out] 1/2*(a+b*tan(f*x+e)^2)^(1+p)/b/f/(1+p)+1/2*hypergeom([1, 1+p],[2+p],(a+b*ta
n(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)^(1+p)/(a-b)/f/(1+p)
```

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3670, 446, 80, 68}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}+\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}}{2bf(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (a + b*Tan[e + f*x]^2)^(1 + p)/(2*b*f*(1 + p)) + (Hypergeometric2F1[1, 1 +
p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^2)^(1 + p))/(
2*(a - b)*f*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^3\left(a+bx^2\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x(a+bx)^p}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\left(a+b\tan^2(e+fx)\right)^{1+p}}{2bf(1+p)} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\left(a+b\tan^2(e+fx)\right)^{1+p}}{2bf(1+p)} + \frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan\right.}{2(a-b)f(1+p)}$$

Mathematica [A] time = 0.14, size = 73, normalized size = 0.77

$$-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\left(b\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)+a-b\right)}{2bf(p+1)(b-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
[Out] -1/2*((a - b + b*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/
(a - b)])*(a + b*Tan[e + f*x]^2)^(1 + p))/(b*(-a + b)*f*(1 + p))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int \left(\tan^3\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^{3}\,\left(b\tan\left(e+f\,x\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^{2}\left(e+fx\right)\right)^{p}\tan^{3}\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
[Out] Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x)**3, x)
```

3.363 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=63

$$-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}$$

```
[Out] -1/2*hypergeom([1, 1+p],[2+p],(a+b*tan(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)^
(1+p)/(a-b)/f/(1+p)
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3670, 444, 68}

$$-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b
*Tan[e + f*x]^2)^(1 + p))/(2*(a - b)*f*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{x(a+bx^2)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int\frac{(a+bx)^p}{1+x}\,dx,x,\tan^2(e+fx)\right)}{2f}$$
$$=-\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}$$

Mathematica [A] time = 0.10, size = 63, normalized size = 1.00

$$-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a
 + b*Tan[e + f*x]^2)^(1 + p))/((a - b)*f*(1 + p))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int\tan\left(fx+e\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \tan\left(e+f\,x\right)\,\left(b\tan\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^p, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^p\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x), x)
```

3.364 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=118

$$\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a}\right)}{2af(p+1)}$$

```
[Out] 1/2*hypergeom([1, 1+p],[2+p],(a+b*tan(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)^(
1+p)/(a-b)/f/(1+p)-1/2*hypergeom([1, 1+p],[2+p],1+b*tan(f*x+e)^2/a)*(a+b*ta
n(f*x+e)^2)^(1+p)/a/f/(1+p)
```

Rubi [A] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3670, 446, 86, 65, 68}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a}\right)}{2af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*
Tan[e + f*x]^2)^(1 + p))/(2*(a - b)*f*(1 + p)) - (Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*a*f*
(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
```

```
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \tan^2(e+fx)\right)}{2f} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1+x}\,dx, x, \tan^2(e+\right.}{2f}$$

$$= \frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)} - \frac{{}_2F_1\left(1,1+\right.}{}$$

Mathematica [A] time = 0.17, size = 98, normalized size = 0.83

$$\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\left(a\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)+(b-a)\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)}{a}+1\right)\right)}{2af(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((a*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)] + (-
a + b)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a])*(a + b
*Tan[e + f*x]^2)^(1 + p))/(2*a*(a - b)*f*(1 + p))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

maple [F] time = 1.32, size = 0, normalized size = 0.00

$$\int \cot\left(fx+e\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)\,{\left(b\,\tan\left(e+f\,x\right)^{2}+a\right)}^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^{2}{\left(e+fx\right)}\right)^{p}\cot{\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**p*cot(e + f*x), x)
```

3.365 $\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=158

$$\frac{(a-bp)\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)}{a}+1\right)}{2a^2f(p+1)}-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\right.}{2f(p+1)(a-b)}$$

```
[Out] -1/2*cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1+p)/a/f-1/2*hypergeom([1, 1+p],[2+p]
,(a+b*tan(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)^(1+p)/(a-b)/f/(1+p)+1/2*(-b*p
+a)*hypergeom([1, 1+p],[2+p],1+b*tan(f*x+e)^2/a)*(a+b*tan(f*x+e)^2)^(1+p)/a
^2/f/(1+p)
```

Rubi [A] time = 0.17, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3670, 446, 103, 156, 65, 68}

$$\frac{(a-bp)\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)}{a}+1\right)}{2a^2f(p+1)}-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\right.}{2f(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -(Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*a*f) - (Hypergeometric2
F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^2)^
(1 + p))/(2*(a - b)*f*(1 + p)) + ((a - b*p)*Hypergeometric2F1[1, 1 + p, 2 +
 p, 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*a^2*f*(1 +
 p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x^3\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x^2(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2af} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p(a-bp-bpx)}{x(1+x)}\,\ldots\right.}{2af}$$

$$= -\frac{\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2af} + \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1+x}\,dx, x, \tan\ldots\right.}{2f}$$

$$= -\frac{\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2af} - \frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan\ldots}{a\ldots}\right.}{2(a-\ldots}$$

Mathematica [A] time = 0.74, size = 142, normalized size = 0.90

$$\frac{\tan^2(e+fx)\left(a\cot^2(e+fx)+b\right)\left(a+b\tan^2(e+fx)\right)^p\left(a^2\left(-{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)\right)-(a-b)\left((bp\ldots\right.\right.}{2a^2f(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((b + a*Cot[e + f*x]^2)*(-(a^2*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Ta
n[e + f*x]^2)/(a - b)]) - (a - b)*(a*(1 + p)*Cot[e + f*x]^2 + (-a + b*p)*Hy
pergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]))*Tan[e + f*x]^2
*(a + b*Tan[e + f*x]^2)^p)/(2*a^2*(a - b)*f*(1 + p))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan \left(fx + e\right)^{2} + a\right)^{p} \cot \left(fx + e\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int \left(\cot^{3}\left(fx + e\right)\right) \left(a + b \left(\tan^{2}\left(fx + e\right)\right)\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan \left(fx + e\right)^{2} + a\right)^{p} \cot \left(fx + e\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot \left(e + f\,x\right)^{3}\,\left(b\,\tan \left(e + f\,x\right)^{2} + a\right)^{p} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.366 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=217

$$\frac{(2a-bp+b)\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{p+1}}{4a^2f}-\frac{\left(2a^2-2abp-b^2(1-p)p\right)\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p\right.}{4a^3f(p+1)}$$

```
[Out] 1/4*(-b*p+2*a+b)*cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1+p)/a^2/f-1/4*cot(f*x+e)
^4*(a+b*tan(f*x+e)^2)^(1+p)/a/f+1/2*hypergeom([1, 1+p],[2+p],(a+b*tan(f*x+e
)^2)/(a-b))*(a+b*tan(f*x+e)^2)^(1+p)/(a-b)/f/(1+p)-1/4*(2*a^2-2*a*b*p-b^2*(
1-p)*p)*hypergeom([1, 1+p],[2+p],1+b*tan(f*x+e)^2/a)*(a+b*tan(f*x+e)^2)^(1+
p)/a^3/f/(1+p)
```

Rubi [A] time = 0.25, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3670, 446, 103, 151, 156, 65, 68}

$$-\frac{\left(2a^2-2abp-b^2(1-p)p\right)\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)}{a}+1\right)}{4a^3f(p+1)}+\frac{(2a-bp+b)\cot^2(e+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((2*a + b - b*p)*Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(1 + p))/(4*a^2*f) -
 (Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(1 + p))/(4*a*f) + (Hypergeometric2
F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^2)^
(1 + p))/(2*(a - b)*f*(1 + p)) - ((2*a^2 - 2*a*b*p - b^2*(1 - p)*p)*Hyperge
ometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]^2
)^(1 + p))/(4*a^3*f*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x^5\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x^3(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4af} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p(2a+b-bp+b(1-p}{x^2(1+x)}\right.}{4af}$$

$$= \frac{(2a+b-bp)\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4a^2f} - \frac{\cot^4(e+fx)\left(a+\right.}{4}$$

$$= \frac{(2a+b-bp)\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4a^2f} - \frac{\cot^4(e+fx)\left(a+\right.}{4}$$

$$= \frac{(2a+b-bp)\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4a^2f} - \frac{\cot^4(e+fx)\left(a+\right.}{4}$$

Mathematica [A] time = 2.76, size = 172, normalized size = 0.79

$$-\frac{\tan^2(e+fx)\left(a\cot^2(e+fx)+b\right)\left(a+b\tan^2(e+fx)\right)^p\left((a-b)\left(\left(2a^2-2abp+b^2(p-1)p\right)\,{}_2F_1\left(1,p+1;p+2;\right.\right.\right.}{4a^3f(p+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -1/4*((b + a*Cot[e + f*x]^2)*(-2*a^3*Hypergeometric2F1[1, 1 + p, 2 + p, (a
+ b*Tan[e + f*x]^2)/(a - b)] + (a - b)*(a*(1 + p)*Cot[e + f*x]^2*(-2*a + b*
(-1 + p) + a*Cot[e + f*x]^2) + (2*a^2 - 2*a*b*p + b^2*(-1 + p)*p)*Hypergeom
etric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]))*Tan[e + f*x]^2*(a + b
*Tan[e + f*x]^2)^p)/(a^3*(a - b)*f*(1 + p))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^5, x)
```

maple [F] time = 1.38, size = 0, normalized size = 0.00

$$\int \left(\cot^{5}\left(fx+e\right)\right)\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cot\left(e+f\,x\right)^{5}\,{\left(b\,\tan\left(e+f\,x\right)^{2}+a\right)}^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.367 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan^7(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{7}{2};1,-p;\frac{9}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{7f}$$

```
[Out] 1/7*AppellF1(7/2,1,-p,9/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)^7*(a+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 511, 510}

$$\frac{\tan^7(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{7}{2};1,-p;\frac{9}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[7/2, 1, -p, 9/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^7*(a + b*Tan[e + f*x]^2)^p)/(7*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p dx = \frac{\text{Subst}\left(\int \frac{x^6(a+bx^2)^p}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^6\left(1+\frac{bx^2}{a}\right)^p}{1+x^2}dx, x, \text{ta}\right.}{f}$$

$$= \frac{F_1\left(\frac{7}{2};1,-p;\frac{9}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^7(e+fx)\left(a+b\tan^2(e\right.}{7f}$$

Mathematica [F] time = 3.54, size = 0, normalized size = 0.00

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
[Out] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p, x]
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^6, x)
```

maple [F] time = 1.44, size = 0, normalized size = 0.00

$$\int \left(\tan^6\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^6\,{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.368 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{5f}$$

[Out] 1/5*AppellF1(5/2,1,-p,7/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 511, 510}

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{5f}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]

[Out] (AppellF1[5/2, 1, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/a)^p)

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^4\left(1+\frac{bx^2}{a}\right)^p}{1+x^2}\,dx, x\right.}{f}$$

$$= \frac{F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^5(e+fx)\left(a+b\tan\right.}{5f}$$

Mathematica [B] time = 16.59, size = 1896, normalized size = 22.84

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (-2*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]*(
a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p) + (Tan[e + f*x]*(
a + b*Tan[e + f*x]^2)^p*((-a + b*(3 + 2*p))*Hypergeometric2F1[1/2, -p, 3/2,
 -((b*Tan[e + f*x]^2)/a)] + (a + b*Tan[e + f*x]^2)*(1 + (b*Tan[e + f*x]^2)/
a)^p))/(b*f*(3 + 2*p)*(1 + (b*Tan[e + f*x]^2)/a)^p) + (3*a*AppellF1[1/2, -p
, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*
x]*(a + b*Tan[e + f*x]^2)^(2*p))/(f*(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan
[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((
b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*T
an[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)*((6*a*b*p*AppellF1[1/2
, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Tan[e + f*x]^2*(a +
 b*Tan[e + f*x]^2)^(-1 + p))/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*
x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e
 + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e +
f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (3*a*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2*(a + b*Tan[e +
 f*x]^2)^p)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e
+ f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -T
an[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) - (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p)/(3*a*
AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*
p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] -
a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[
e + f*x]^2) + (3*a*Cos[e + f*x]*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x
])/(3*a) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f
*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3)*(a + b*Tan[e + f*x]^2)^p)/(3*a*Appel
lF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*App
ellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f
*x]^2) - (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f
*x]^2]*Cos[e + f*x]*Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p*(4*(b*p*AppellF1[
3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[
3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Sec[e + f*x]^2*
Tan[e + f*x] + 3*a*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2
)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*AppellF1[3/2
```

```
, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[
e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b
*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b
*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x
]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a)) - a*((6*b*p*AppellF1[5/2, 1 - p, 2
, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x
])/(5*a) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*a*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5
/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2,
 -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

maple [F] time = 1.01, size = 0, normalized size = 0.00

$$\int\left(\tan^{4}\left(fx+e\right)\right)\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\tan\left(e+f\,x\right)^{4}\left(b\tan\left(e+f\,x\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^p\tan^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2)**p,x)
[Out] Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x)**4, x)
```

3.369 $\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

```
[Out] 1/3*AppellF1(3/2,1,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)^3*(a+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 511, 510}

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 1, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{1+x^2}\,dx, x\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^3(e+fx)\left(a+b\tan\right.}{3f}$$

Mathematica [B] time = 15.10, size = 1992, normalized size = 24.00

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(2*p)*(Hypergeometric2F1[1/2, -p, 3/
2, -((b*Tan[e + f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p + (3*a*AppellF1[1/
2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(-
3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2
*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]
^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2
])*Tan[e + f*x]^2)))/(f*(2*b*p*Sec[e + f*x]^2*Tan[e + f*x]^2*(a + b*Tan[e +
 f*x]^2)^(-1 + p)*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)]
/(1 + (b*Tan[e + f*x]^2)/a)^p + (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
 f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(-3*a*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 - p
, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p,
2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + Sec[e
 + f*x]^2*(a + b*Tan[e + f*x]^2)^p*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Ta
n[e + f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p + (3*a*AppellF1[1/2, -p, 1,
3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(-3*a*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*Ap
pellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*A
ppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e +
 f*x]^2)) + Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p*((-2*b*p*Hypergeometric2F
1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (
b*Tan[e + f*x]^2)/a)^(-1 - p))/a - (6*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[
e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*x])/(-3*a*AppellF1[
1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*Appel
lF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*Appe
llF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*
x]^2) + (3*a*Cos[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e
 + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*
x]^2*Tan[e + f*x])/3))/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e +
f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*
x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (Csc[e + f*x]*Sec[e + f*x]*(-
Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)] + (1 + (b*Tan[e +
f*x]^2)/a)^p))/(1 + (b*Tan[e + f*x]^2)/a)^p - (3*a*AppellF1[1/2, -p, 1, 3/2
, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2*(4*(-(b*p*Appell
F1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*Appel
```

```
1F1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Sec[e + f*x
]^2*Tan[e + f*x] - 3*a*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*
x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*AppellF1
[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*
Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(-(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2
, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5
- (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e
 + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a))) + a*((6*b*p*AppellF1[5/2, 1
 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[
e + f*x])/(5*a) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/a), -T
an[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(-3*a*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2,
-p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2)))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

maple [F] time = 0.90, size = 0, normalized size = 0.00

$$\int\left(\tan^2\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\tan\left(e+f\,x\right)^2\,\left(b\,\tan\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^p \tan^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
[Out] Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x)**2, x)
```

3.370 $\int \left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=78

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)*(a+b*tan(
f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(1\right.}{f}$$

Mathematica [B] time = 0.49, size = 192, normalized size = 2.46

$$\frac{3a\sin(2(e+fx))\left(a+b\tan^2(e+fx)\right)^p F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a},-\text{t}\right.}{4f\tan^2(e+fx)\left(bpF_1\left(\frac{3}{2};1-p,1;\frac{5}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)-aF_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Si
n[2*(e + f*x)]*(a + b*Tan[e + f*x]^2)^p)/(6*a*f*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 4*f*(b*p*AppellF1[3/2, 1 - p, 1,
 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*tan(f*x+e)^2)^p,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p,x)
[Out] int((a + b*tan(e + f*x)^2)^p, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**p,x)
[Out] Integral((a + b*tan(e + f*x)**2)**p, x)
```

3.371 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=79

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

```
[Out] -AppellF1(-1/2,1,-p,1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*cot(f*x+e)*(a+b*ta
n(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 511, 510}

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, 1, -p, 1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Cot
[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+bx^2\right)^p}{x^2\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2\left(1+x^2\right)}\,dx,x,\tan\right.}{f}$$

$$=-\frac{F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot(e+fx)\left(a+b\tan^2\right.}{f}$$

Mathematica [B] time = 14.93, size = 1989, normalized size = 25.18

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(2*p)*(-(Hypergeometric2F1[-1/2, -p,
 1/2, -((b*Tan[e + f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p) + (3*a*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2
)/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]
 + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*
x]^2])*Tan[e + f*x]^2)))/(f*(2*b*p*Sec[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(-
1 + p)*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]/(1 + (b
*Tan[e + f*x]^2)/a)^p) + (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b
*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2,
 -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - Csc[e + f*x]
^2*(a + b*Tan[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e
+ f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p) + (3*a*AppellF1[1/2, -p, 1, 3/2
, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(-3*a*AppellF1[
1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*Appel
lF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*Appe
llF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*
x]^2)) + Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^p*((2*b*p*Hypergeometric2F1[-1
/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (b*T
an[e + f*x]^2)/a)^(-1 - p))/a + (6*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
 f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*x])/(-3*a*AppellF1[1/2
, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1
[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF
1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^
2) + (3*a*Sin[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e +
f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*Appell
F1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^
2*Tan[e + f*x])/3))/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a)
, -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x
]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^
2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (Csc[e + f*x]*Sec[e + f*x]*(Hype
rgeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)] - (1 + (b*Tan[e + f*x
]^2)/a)^p))/(1 + (b*Tan[e + f*x]^2)/a)^p - (3*a*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2*(4*(-(b*p*AppellF1[
3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1
```

```
[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Sec[e + f*x]^2
*Tan[e + f*x] - 3*a*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^
2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*AppellF1[3/
2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan
[e + f*x])/3) + 2*Tan[e + f*x]^2*(-(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (
6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a))) + a*((6*b*p*AppellF1[5/2, 1 -
p, 2, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e +
 f*x])/(5*a) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[
e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(-3*a*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 -
p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p,
 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2)))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int\left(\cot^2\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cot\left(e+fx\right)^2\left(b\tan\left(e+fx\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.372 $\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

```
[Out] -1/3*AppellF1(-3/2,1,-p,-1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*cot(f*x+e)^3*
(a+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3670, 511, 510}

$$-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -(AppellF1[-3/2, 1, -p, -1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Cot
[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x^4\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^4\left(1+x^2\right)}\,dx, x, \tan\right.}{f}$$

$$= -\frac{F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot^3(e+fx)\left(a+b\,\text{ta}\right.}{3f}$$

Mathematica [B] time = 6.87, size = 1887, normalized size = 22.73

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (2*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*(
a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p) + (Cot[e + f*x]^3
*(a + b*Tan[e + f*x]^2)^p*(-a - b*Tan[e + f*x]^2 - ((3*a + b*(-1 + 2*p))*Hy
pergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/(1
+ (b*Tan[e + f*x]^2)/a)^p))/(3*a*f) + (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*x]*(a + b*Tan[e
+ f*x]^2)^(2*p))/(f*(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a),
 -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2
)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a
), -Tan[e + f*x]^2])*Tan[e + f*x]^2)*((6*a*b*p*AppellF1[1/2, -p, 1, 3/2, -(
(b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^
2)^(-1 + p))/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e
 + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -
Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan
[e + f*x]^2])*Tan[e + f*x]^2) + (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
 f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p)/(3*a
*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b
*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] -
 a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan
[e + f*x]^2]*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, -p
, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2,
-p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (3
*a*Cos[e + f*x]*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[
e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f
*x]^2*Tan[e + f*x])/3)*(a + b*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, -p, 1,
3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p
, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2
, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*a*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f
*x]*Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p*(4*(b*p*AppellF1[3/2, 1 - p, 1, 5
/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2,
 -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3
*a*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f
*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*AppellF1[3/2, -p, 2, 5/2, -(
```

```
(b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2
*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)
/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF
1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x
]^2*Tan[e + f*x])/(5*a)) - a*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[
e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a) - (12*A
ppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e +
f*x]^2*Tan[e + f*x])/5))))/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]
^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e +
 f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*
x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int\left(\cot^{4}\left(fx+e\right)\right)\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cot\left(e+f\,x\right)^{4}\,\left(b\,\tan\left(e+f\,x\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.373 $\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{5}{2};1,-p;-\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{5f}$$

```
[Out] -1/5*AppellF1(-5/2,1,-p,-3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*cot(f*x+e)^5*
(a+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 511, 510}

$$-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{5}{2};1,-p;-\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -(AppellF1[-5/2, 1, -p, -3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Cot
[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^6(1+x^2)}\,dx, x, \tan\right.}{f}$$

$$= -\frac{F_1\left(-\frac{5}{2};1,-p;-\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot^5(e+fx)\left(a+b\,\text{ta}\right.}{5f}$$

Mathematica [F] time = 4.62, size = 0, normalized size = 0.00

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
[Out] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p, x]
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^6, x)
```

maple [F] time = 1.12, size = 0, normalized size = 0.00

$$\int \left(\cot^6\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] Timed out
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^6\,\left(b\tan\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^p,x)
[Out] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^p, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

3.374 $\int \left(a+b\tan^3(c+dx)\right)^4\,dx$

Optimal. Leaf size=255

$$\frac{b^2\left(6a^2-b^2\right)\tan^5(c+dx)}{5d}-\frac{b^2\left(6a^2-b^2\right)\tan^3(c+dx)}{3d}+\frac{2ab\left(a^2-b^2\right)\tan^2(c+dx)}{d}+\frac{b^2\left(6a^2-b^2\right)\tan(c+dx)}{d}+\frac{4a}{}$$

```
[Out] (a^4-6*a^2*b^2+b^4)*x+4*a*b*(a^2-b^2)*ln(cos(d*x+c))/d+b^2*(6*a^2-b^2)*tan(
d*x+c)/d+2*a*b*(a^2-b^2)*tan(d*x+c)^2/d-1/3*b^2*(6*a^2-b^2)*tan(d*x+c)^3/d+
a*b^3*tan(d*x+c)^4/d+1/5*b^2*(6*a^2-b^2)*tan(d*x+c)^5/d-2/3*a*b^3*tan(d*x+c
)^6/d+1/7*b^4*tan(d*x+c)^7/d+1/2*a*b^3*tan(d*x+c)^8/d-1/9*b^4*tan(d*x+c)^9/
d+1/11*b^4*tan(d*x+c)^11/d
```

Rubi [A] time = 0.15, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3661, 1810, 635, 203, 260}

$$\frac{b^2\left(6a^2-b^2\right)\tan^5(c+dx)}{5d}-\frac{b^2\left(6a^2-b^2\right)\tan^3(c+dx)}{3d}+\frac{2ab\left(a^2-b^2\right)\tan^2(c+dx)}{d}+\frac{b^2\left(6a^2-b^2\right)\tan(c+dx)}{d}+\frac{4a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^4,x]
```

```
[Out] (a^4 - 6*a^2*b^2 + b^4)*x + (4*a*b*(a^2 - b^2)*Log[Cos[c + d*x]])/d + (b^2*
(6*a^2 - b^2)*Tan[c + d*x])/d + (2*a*b*(a^2 - b^2)*Tan[c + d*x]^2)/d - (b^2
*(6*a^2 - b^2)*Tan[c + d*x]^3)/(3*d) + (a*b^3*Tan[c + d*x]^4)/d + (b^2*(6*a
^2 - b^2)*Tan[c + d*x]^5)/(5*d) - (2*a*b^3*Tan[c + d*x]^6)/(3*d) + (b^4*Tan
[c + d*x]^7)/(7*d) + (a*b^3*Tan[c + d*x]^8)/(2*d) - (b^4*Tan[c + d*x]^9)/(9
*d) + (b^4*Tan[c + d*x]^11)/(11*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
```

```
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^3(c+dx)\right)^4\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^3\right)^4}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(6a^2b^2-b^4+4ab\left(a^2-b^2\right)x-b^2\left(6a^2-b^2\right)x^2+4ab^3x^3+b^2\left(6a^2-\right.\right.}{}$$

$$= \frac{b^2\left(6a^2-b^2\right)\tan(c+dx)}{d} + \frac{2ab\left(a^2-b^2\right)\tan^2(c+dx)}{d} - \frac{b^2\left(6a^2-b^2\right)\tan^3(c+}{3d}$$

$$= \frac{b^2\left(6a^2-b^2\right)\tan(c+dx)}{d} + \frac{2ab\left(a^2-b^2\right)\tan^2(c+dx)}{d} - \frac{b^2\left(6a^2-b^2\right)\tan^3(c+}{3d}$$

$$= \left(a^4-6a^2b^2+b^4\right)x + \frac{4ab\left(a^2-b^2\right)\log(\cos(c+dx))}{d} + \frac{b^2\left(6a^2-b^2\right)\tan(c+dx}{d}$$

Mathematica [C] time = 1.04, size = 224, normalized size = 0.88

$$\frac{-1386b^2\left(b^2-6a^2\right)\tan^5(c+dx)+2310b^2\left(b^2-6a^2\right)\tan^3(c+dx)+13860ab\left(a^2-b^2\right)\tan^2(c+dx)-6930b^2\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^4,x]
```

```
[Out] ((-3465*I)*((a - I*b)^4*Log[I - Tan[c + d*x]] - (a + I*b)^4*Log[I + Tan[c +
 d*x]]) - 6930*b^2*(-6*a^2 + b^2)*Tan[c + d*x] + 13860*a*b*(a^2 - b^2)*Tan[
c + d*x]^2 + 2310*b^2*(-6*a^2 + b^2)*Tan[c + d*x]^3 + 6930*a*b^3*Tan[c + d*
x]^4 - 1386*b^2*(-6*a^2 + b^2)*Tan[c + d*x]^5 - 4620*a*b^3*Tan[c + d*x]^6 +
 990*b^4*Tan[c + d*x]^7 + 3465*a*b^3*Tan[c + d*x]^8 - 770*b^4*Tan[c + d*x]^
9 + 630*b^4*Tan[c + d*x]^11)/(6930*d)
```

fricas [A] time = 0.60, size = 225, normalized size = 0.88

$$\frac{630\,b^4\tan\left(dx+c\right)^{11}-770\,b^4\tan\left(dx+c\right)^9+3465\,ab^3\tan\left(dx+c\right)^8+990\,b^4\tan\left(dx+c\right)^7-4620\,ab^3\tan\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^4,x, algorithm="fricas")
```

```
[Out] 1/6930*(630*b^4*tan(d*x + c)^11 - 770*b^4*tan(d*x + c)^9 + 3465*a*b^3*tan(d
*x + c)^8 + 990*b^4*tan(d*x + c)^7 - 4620*a*b^3*tan(d*x + c)^6 + 6930*a*b^3
*tan(d*x + c)^4 + 1386*(6*a^2*b^2 - b^4)*tan(d*x + c)^5 - 2310*(6*a^2*b^2 -
 b^4)*tan(d*x + c)^3 + 6930*(a^4 - 6*a^2*b^2 + b^4)*d*x + 13860*(a^3*b - a*
b^3)*tan(d*x + c)^2 + 13860*(a^3*b - a*b^3)*log(1/(tan(d*x + c)^2 + 1)) + 6
930*(6*a^2*b^2 - b^4)*tan(d*x + c))/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 321, normalized size = 1.26

$$\frac{b^4\left(\tan^{11}(dx+c)\right)}{11d}-\frac{b^4\left(\tan^9(dx+c)\right)}{9d}+\frac{a\,b^3\left(\tan^8(dx+c)\right)}{2d}+\frac{b^4\left(\tan^7(dx+c)\right)}{7d}-\frac{2a\,b^3\left(\tan^6(dx+c)\right)}{3d}+\frac{6\left(\tan^5(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^3)^4,x)
```

```
[Out] 1/11*b^4*tan(d*x+c)^11/d-1/9*b^4*tan(d*x+c)^9/d+1/2*a*b^3*tan(d*x+c)^8/d+1/
7*b^4*tan(d*x+c)^7/d-2/3*a*b^3*tan(d*x+c)^6/d+6/5/d*tan(d*x+c)^5*a^2*b^2-1/
5/d*tan(d*x+c)^5*b^4+a*b^3*tan(d*x+c)^4/d-2/d*tan(d*x+c)^3*a^2*b^2+1/3*b^4*
tan(d*x+c)^3/d+2/d*a^3*b*tan(d*x+c)^2-2*a*b^3*tan(d*x+c)^2/d+6*a^2*b^2*tan(
d*x+c)/d-1/d*b^4*tan(d*x+c)-2/d*ln(1+tan(d*x+c)^2)*a^3*b+2/d*ln(1+tan(d*x+c
)^2)*a*b^3+1/d*arctan(tan(d*x+c))*a^4-6/d*arctan(tan(d*x+c))*a^2*b^2+1/d*ar
ctan(tan(d*x+c))*b^4
```

maxima [A] time = 0.99, size = 260, normalized size = 1.02

$$a^4x+\frac{2\left(3\,\tan(dx+c)^5-5\,\tan(dx+c)^3-15\,dx-15\,c+15\,\tan(dx+c)\right)a^2b^2}{5\,d}+\frac{\left(315\,\tan(dx+c)^{11}-385\,\tan(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 2/5*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d
*x + c))*a^2*b^2/d + 1/3465*(315*tan(d*x + c)^11 - 385*tan(d*x + c)^9 + 495
*tan(d*x + c)^7 - 693*tan(d*x + c)^5 + 1155*tan(d*x + c)^3 + 3465*d*x + 346
5*c - 3465*tan(d*x + c))*b^4/d + 1/6*a*b^3*((48*sin(d*x + c)^6 - 108*sin(d*
x + c)^4 + 88*sin(d*x + c)^2 - 25)/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*s
in(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) - 12*log(sin(d*x + c)^2 - 1))/d - 2*a
^3*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 11.85, size = 310, normalized size = 1.22

$$\frac{\ln\left(\tan(c+d\,x)^2+1\right)\left(2\,a\,b^3-2\,a^3\,b\right)}{d}+\frac{\tan(c+d\,x)^3\left(\frac{b^4}{3}-2\,a^2\,b^2\right)}{d}-\frac{\tan(c+d\,x)^5\left(\frac{b^4}{5}-\frac{6\,a^2\,b^2}{5}\right)}{d}-\frac{\tan(c+d\,x)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^3)^4,x)
```

```
[Out] (log(tan(c + d*x)^2 + 1)*(2*a*b^3 - 2*a^3*b))/d + (tan(c + d*x)^3*(b^4/3 -
2*a^2*b^2))/d - (tan(c + d*x)^5*(b^4/5 - (6*a^2*b^2)/5))/d - (tan(c + d*x)^
2*(2*a*b^3 - 2*a^3*b))/d - (tan(c + d*x)*(b^4 - 6*a^2*b^2))/d + (b^4*tan(c
+ d*x)^7)/(7*d) - (b^4*tan(c + d*x)^9)/(9*d) + (b^4*tan(c + d*x)^11)/(11*d)
 + (atan((tan(c + d*x)*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(a^4 + b^4
- 6*a^2*b^2))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/d + (a*b^3*tan(c + d
*x)^4)/d - (2*a*b^3*tan(c + d*x)^6)/(3*d) + (a*b^3*tan(c + d*x)^8)/(2*d)
```

sympy [A] time = 3.58, size = 301, normalized size = 1.18

$$\begin{cases} a^4x-\frac{2a^3b\log\left(\tan^2(c+dx)+1\right)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}-6a^2b^2x+\frac{6a^2b^2\tan^5(c+dx)}{5d}-\frac{2a^2b^2\tan^3(c+dx)}{d}+\frac{6a^2b^2\tan(c+dx)}{d}+\frac{2ab^3\log\left(\tan\right.}{} \\ x\left(a+b\tan^3(c)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**3)**4,x)
```

```
[Out] Piecewise((a**4*x - 2*a**3*b*log(tan(c + d*x)**2 + 1)/d + 2*a**3*b*tan(c +
d*x)**2/d - 6*a**2*b**2*x + 6*a**2*b**2*tan(c + d*x)**5/(5*d) - 2*a**2*b**2
*tan(c + d*x)**3/d + 6*a**2*b**2*tan(c + d*x)/d + 2*a*b**3*log(tan(c + d*x)
**2 + 1)/d + a*b**3*tan(c + d*x)**8/(2*d) - 2*a*b**3*tan(c + d*x)**6/(3*d)
+ a*b**3*tan(c + d*x)**4/d - 2*a*b**3*tan(c + d*x)**2/d + b**4*x + b**4*tan
(c + d*x)**11/(11*d) - b**4*tan(c + d*x)**9/(9*d) + b**4*tan(c + d*x)**7/(7
*d) - b**4*tan(c + d*x)**5/(5*d) + b**4*tan(c + d*x)**3/(3*d) - b**4*tan(c
+ d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**3)**4, True))
```

3.375 $\int \left(a+b\tan^3(c+dx)\right)^3 \, dx$

Optimal. Leaf size=168

$$\frac{b\left(3a^2-b^2\right)\tan^2(c+dx)}{2d}+\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+ax\left(a^2-3b^2\right)+\frac{3ab^2\tan^5(c+dx)}{5d}-\frac{ab^2\tan^3(c+dx)}{d}+3a$$

```
[Out] a*(a^2-3*b^2)*x+b*(3*a^2-b^2)*ln(cos(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b*(
3*a^2-b^2)*tan(d*x+c)^2/d-a*b^2*tan(d*x+c)^3/d+1/4*b^3*tan(d*x+c)^4/d+3/5*a
*b^2*tan(d*x+c)^5/d-1/6*b^3*tan(d*x+c)^6/d+1/8*b^3*tan(d*x+c)^8/d
```

Rubi [A] time = 0.10, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3661, 1810, 635, 203, 260}

$$\frac{b\left(3a^2-b^2\right)\tan^2(c+dx)}{2d}+\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+ax\left(a^2-3b^2\right)+\frac{3ab^2\tan^5(c+dx)}{5d}-\frac{ab^2\tan^3(c+dx)}{d}+3a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^3,x]
```

```
[Out] a*(a^2 - 3*b^2)*x + (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (3*a*b^2*Tan[c
+ d*x])/d + (b*(3*a^2 - b^2)*Tan[c + d*x]^2)/(2*d) - (a*b^2*Tan[c + d*x]^3)
/d + (b^3*Tan[c + d*x]^4)/(4*d) + (3*a*b^2*Tan[c + d*x]^5)/(5*d) - (b^3*Tan
[c + d*x]^6)/(6*d) + (b^3*Tan[c + d*x]^8)/(8*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^3(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^3\right)^3}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(3ab^2 + b\left(3a^2-b^2\right)x - 3ab^2x^2 + b^3x^3 + 3ab^2x^4 - b^3x^5 + b^3x^7 + \frac{a^3-3a\cdots}{\cdots}\right.\right.}{d}$$

$$= \frac{3ab^2\tan(c+dx)}{d} + \frac{b\left(3a^2-b^2\right)\tan^2(c+dx)}{2d} - \frac{ab^2\tan^3(c+dx)}{d} + \frac{b^3\tan^4(c+\cdots}{4d}$$

$$= \frac{3ab^2\tan(c+dx)}{d} + \frac{b\left(3a^2-b^2\right)\tan^2(c+dx)}{2d} - \frac{ab^2\tan^3(c+dx)}{d} + \frac{b^3\tan^4(c+\cdots}{4d}$$

$$= a\left(a^2-3b^2\right)x + \frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d} + \frac{3ab^2\tan(c+dx)}{d} + \frac{b\left(3a^2-b^2\right)\cdots}{\cdots}$$

Mathematica [C] time = 0.51, size = 160, normalized size = 0.95

$$\frac{-60b\left(b^2-3a^2\right)\tan^2(c+dx) + 72ab^2\tan^5(c+dx) - 120ab^2\tan^3(c+dx) + 360ab^2\tan(c+dx) + 60\left(i(a+ib)^{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^3,x]
```

```
[Out] (60*((-I)*(a - I*b)^3*Log[I - Tan[c + d*x]] + I*(a + I*b)^3*Log[I + Tan[c +
 d*x]]) + 360*a*b^2*Tan[c + d*x] - 60*b*(-3*a^2 + b^2)*Tan[c + d*x]^2 - 120
*a*b^2*Tan[c + d*x]^3 + 30*b^3*Tan[c + d*x]^4 + 72*a*b^2*Tan[c + d*x]^5 - 2
0*b^3*Tan[c + d*x]^6 + 15*b^3*Tan[c + d*x]^8)/(120*d)
```

fricas [A] time = 0.48, size = 148, normalized size = 0.88

$$\frac{15\,b^3\tan\left(dx+c\right)^8 - 20\,b^3\tan\left(dx+c\right)^6 + 72\,ab^2\tan\left(dx+c\right)^5 + 30\,b^3\tan\left(dx+c\right)^4 - 120\,ab^2\tan\left(dx+c\right)^3 + 3\cdots}{120\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(15*b^3*tan(d*x + c)^8 - 20*b^3*tan(d*x + c)^6 + 72*a*b^2*tan(d*x + c
)^5 + 30*b^3*tan(d*x + c)^4 - 120*a*b^2*tan(d*x + c)^3 + 360*a*b^2*tan(d*x
+ c) + 120*(a^3 - 3*a*b^2)*d*x + 60*(3*a^2*b - b^3)*tan(d*x + c)^2 + 60*(3*
a^2*b - b^3)*log(1/(tan(d*x + c)^2 + 1)))/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 201, normalized size = 1.20

$$\frac{b^3\left(\tan^8\left(dx+c\right)\right)}{8d} - \frac{b^3\left(\tan^6\left(dx+c\right)\right)}{6d} + \frac{3a\,b^2\left(\tan^5\left(dx+c\right)\right)}{5d} + \frac{b^3\left(\tan^4\left(dx+c\right)\right)}{4d} - \frac{a\,b^2\left(\tan^3\left(dx+c\right)\right)}{d} + \frac{3a^2b\left(\tan\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^3)^3,x)
```

```
[Out] 1/8*b^3*tan(d*x+c)^8/d-1/6*b^3*tan(d*x+c)^6/d+3/5*a*b^2*tan(d*x+c)^5/d+1/4*
b^3*tan(d*x+c)^4/d-a*b^2*tan(d*x+c)^3/d+3/2/d*a^2*b*tan(d*x+c)^2-1/2*b^3*ta
n(d*x+c)^2/d+3*a*b^2*tan(d*x+c)/d-3/2/d*ln(1+tan(d*x+c)^2)*a^2*b+1/2/d*ln(1
+tan(d*x+c)^2)*b^3+1/d*arctan(tan(d*x+c))*a^3-3/d*arctan(tan(d*x+c))*a*b^2
```

maxima [A] time = 0.66, size = 183, normalized size = 1.09

$$a^3x+\frac{\left(3\,\tan\left(dx+c\right)^5-5\,\tan\left(dx+c\right)^3-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)ab^2}{5\,d}+\frac{b^3\left(\frac{48\,\sin(dx+c)^6-108\,\sin(dx+c)^4+88\,\sin}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/5*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d
*x + c))*a*b^2/d + 1/24*b^3*((48*sin(d*x + c)^6 - 108*sin(d*x + c)^4 + 88*s
in(d*x + c)^2 - 25)/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 -
 4*sin(d*x + c)^2 + 1) - 12*log(sin(d*x + c)^2 - 1))/d - 3/2*a^2*b*(1/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 11.60, size = 174, normalized size = 1.04

$$\frac{\tan\left(c+d\,x\right)^2\,\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right)+\frac{b^3\,\tan(c+d\,x)^4}{4}-\frac{b^3\,\tan(c+d\,x)^6}{6}+\frac{b^3\,\tan(c+d\,x)^8}{8}-\ln\left(\tan\left(c+d\,x\right)^2+1\right)\,\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right)-a\,\mathrm{atan}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^3)^3,x)
```

```
[Out] (tan(c + d*x)^2*((3*a^2*b)/2 - b^3/2) + (b^3*tan(c + d*x)^4)/4 - (b^3*tan(c
 + d*x)^6)/6 + (b^3*tan(c + d*x)^8)/8 - log(tan(c + d*x)^2 + 1)*((3*a^2*b)/
2 - b^3/2) - a*atan((a*tan(c + d*x)*(a^2 - 3*b^2))/(3*a*b^2 - a^3))*(a^2 -
3*b^2) - a*b^2*tan(c + d*x)^3 + (3*a*b^2*tan(c + d*x)^5)/5 + 3*a*b^2*tan(c
+ d*x))/d
```

sympy [A] time = 1.65, size = 194, normalized size = 1.15

$$\begin{cases} a^3x-\frac{3a^2b\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{3a^2b\tan^2(c+dx)}{2d}-3ab^2x+\frac{3ab^2\tan^5(c+dx)}{5d}-\frac{ab^2\tan^3(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\log\left(\tan^2(c+dx\right.}{2d} \\ x\left(a+b\tan^3(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**3)**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*a**2*b*tan(
c + d*x)**2/(2*d) - 3*a*b**2*x + 3*a*b**2*tan(c + d*x)**5/(5*d) - a*b**2*ta
n(c + d*x)**3/d + 3*a*b**2*tan(c + d*x)/d + b**3*log(tan(c + d*x)**2 + 1)/(
2*d) + b**3*tan(c + d*x)**8/(8*d) - b**3*tan(c + d*x)**6/(6*d) + b**3*tan(c
 + d*x)**4/(4*d) - b**3*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c)*
*3)**3, True))
```

3.376 $\int \left(a+b\tan^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=89

$$x\left(a^2-b^2\right)+\frac{ab\tan^2(c+dx)}{d}+\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan^5(c+dx)}{5d}-\frac{b^2\tan^3(c+dx)}{3d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] (a^2-b^2)*x+2*a*b*ln(cos(d*x+c))/d+b^2*tan(d*x+c)/d+a*b*tan(d*x+c)^2/d-1/3*
b^2*tan(d*x+c)^3/d+1/5*b^2*tan(d*x+c)^5/d
```

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3661, 1810, 635, 203, 260}

$$x\left(a^2-b^2\right)+\frac{ab\tan^2(c+dx)}{d}+\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan^5(c+dx)}{5d}-\frac{b^2\tan^3(c+dx)}{3d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^2,x]
```

```
[Out] (a^2 - b^2)*x + (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d + (a*b*T
an[c + d*x]^2)/d - (b^2*Tan[c + d*x]^3)/(3*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^3(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^3)^2}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+2abx-b^2x^2+b^2x^4+\frac{a^2-b^2-2abx}{1+x^2}\right)dx,x,\tan(c+dx)\right)}{d}$$
$$= \frac{b^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}-\frac{b^2\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}+\frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{b^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}-\frac{b^2\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}-\frac{(2ab)\,\text{Subs}}{}$$
$$= \left(a^2-b^2\right)x+\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}-\frac{b^2\tan^3(c}{3d}$$

Mathematica [C] time = 0.52, size = 107, normalized size = 1.20

$$\frac{30ab\tan^2(c+dx)-15i\left((a-ib)^2\log(-\tan(c+dx)+i)-(a+ib)^2\log(\tan(c+dx)+i)\right)+6b^2\tan^5(c+dx)-10b}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^2,x]
```

```
[Out] ((-15*I)*((a - I*b)^2*Log[I - Tan[c + d*x]] - (a + I*b)^2*Log[I + Tan[c + d
*x]]) + 30*b^2*Tan[c + d*x] + 30*a*b*Tan[c + d*x]^2 - 10*b^2*Tan[c + d*x]^3
 + 6*b^2*Tan[c + d*x]^5)/(30*d)
```

fricas [A] time = 0.47, size = 85, normalized size = 0.96

$$\frac{3\,b^2\tan\left(dx+c\right)^5-5\,b^2\tan\left(dx+c\right)^3+15\,ab\tan\left(dx+c\right)^2+15\left(a^2-b^2\right)dx+15\,ab\log\left(\frac{1}{\tan\left(dx+c\right)^2+1}\right)+15\,b^2\tan}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*tan(d*x + c)^5 - 5*b^2*tan(d*x + c)^3 + 15*a*b*tan(d*x + c)^2 +
 15*(a^2 - b^2)*d*x + 15*a*b*log(1/(tan(d*x + c)^2 + 1)) + 15*b^2*tan(d*x +
 c))/d
```

giac [B] time = 14.42, size = 1065, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/15*(15*a^2*d*x*tan(d*x)^5*tan(c)^5 - 15*b^2*d*x*tan(d*x)^5*tan(c)^5 + 15*
a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 -
 75*a^2*d*x*tan(d*x)^4*tan(c)^4 + 75*b^2*d*x*tan(d*x)^4*tan(c)^4 + 15*a*b*t
an(d*x)^5*tan(c)^5 - 75*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c
) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1
))*tan(d*x)^4*tan(c)^4 - 15*b^2*tan(d*x)^5*tan(c)^4 - 15*b^2*tan(d*x)^4*tan
(c)^5 + 150*a^2*d*x*tan(d*x)^3*tan(c)^3 - 150*b^2*d*x*tan(d*x)^3*tan(c)^3 +
 15*a*b*tan(d*x)^5*tan(c)^3 - 45*a*b*tan(d*x)^4*tan(c)^4 + 15*a*b*tan(d*x)^
```

```
3*tan(c)^5 + 5*b^2*tan(d*x)^5*tan(c)^2 + 150*a*b*log(4*(tan(d*x)^4*tan(c)^2
 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(
c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 75*b^2*tan(d*x)^4*tan(c)^3 +
75*b^2*tan(d*x)^3*tan(c)^4 + 5*b^2*tan(d*x)^2*tan(c)^5 - 150*a^2*d*x*tan(d*
x)^2*tan(c)^2 + 150*b^2*d*x*tan(d*x)^2*tan(c)^2 - 45*a*b*tan(d*x)^4*tan(c)^
2 + 60*a*b*tan(d*x)^3*tan(c)^3 - 45*a*b*tan(d*x)^2*tan(c)^4 - 3*b^2*tan(d*x
)^5 - 25*b^2*tan(d*x)^4*tan(c) - 150*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan
(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/
(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 150*b^2*tan(d*x)^3*tan(c)^2 - 150*b^2
*tan(d*x)^2*tan(c)^3 - 25*b^2*tan(d*x)*tan(c)^4 - 3*b^2*tan(c)^5 + 75*a^2*d
*x*tan(d*x)*tan(c) - 75*b^2*d*x*tan(d*x)*tan(c) + 45*a*b*tan(d*x)^3*tan(c)
- 60*a*b*tan(d*x)^2*tan(c)^2 + 45*a*b*tan(d*x)*tan(c)^3 + 5*b^2*tan(d*x)^3
+ 75*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c)
+ 75*b^2*tan(d*x)^2*tan(c) + 75*b^2*tan(d*x)*tan(c)^2 + 5*b^2*tan(c)^3 - 15
*a^2*d*x + 15*b^2*d*x - 15*a*b*tan(d*x)^2 + 45*a*b*tan(d*x)*tan(c) - 15*a*b
*tan(c)^2 - 15*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - 15*b
^2*tan(d*x) - 15*b^2*tan(c) - 15*a*b)/(d*tan(d*x)^5*tan(c)^5 - 5*d*tan(d*x)
^4*tan(c)^4 + 10*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 + 5*d*tan
(d*x)*tan(c) - d)
```

maple [A] time = 0.02, size = 108, normalized size = 1.21

$$\frac{b^2\left(\tan^5(dx+c)\right)}{5d}-\frac{b^2\left(\tan^3(dx+c)\right)}{3d}+\frac{ab\left(\tan^2(dx+c)\right)}{d}+\frac{b^2\tan(dx+c)}{d}-\frac{ab\ln\left(1+\tan^2(dx+c)\right)}{d}+\frac{\arctan(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^3)^2,x)
```

```
[Out] 1/5*b^2*tan(d*x+c)^5/d-1/3*b^2*tan(d*x+c)^3/d+a*b*tan(d*x+c)^2/d+b^2*tan(d*
x+c)/d-1/d*a*b*ln(1+tan(d*x+c)^2)+1/d*arctan(tan(d*x+c))*a^2-1/d*arctan(tan
(d*x+c))*b^2
```

maxima [A] time = 0.92, size = 83, normalized size = 0.93

$$a^2x+\frac{\left(3\tan(dx+c)^5-5\tan(dx+c)^3-15\,dx-15\,c+15\tan(dx+c)\right)b^2}{15\,d}-\frac{ab\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)^2\cdots\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(
d*x + c))*b^2/d - a*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 11.60, size = 117, normalized size = 1.31

$$\frac{b^2\tan(c+d\,x)}{d}-\frac{b^2\tan(c+d\,x)^3}{3\,d}+\frac{b^2\tan(c+d\,x)^5}{5\,d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a+b)\,(a-b)}{a^2-b^2}\right)\,(a+b)\,(a-b)}{d}-\frac{a\,b\,\ln\left(\tan(c+\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^3)^2,x)
```

```
[Out] (b^2*tan(c + d*x))/d - (b^2*tan(c + d*x)^3)/(3*d) + (b^2*tan(c + d*x)^5)/(5
*d) + (atan((tan(c + d*x)*(a + b)*(a - b))/(a^2 - b^2))*(a + b)*(a - b))/d
- (a*b*log(tan(c + d*x)^2 + 1))/d + (a*b*tan(c + d*x)^2)/d
```

sympy [A] time = 0.60, size = 94, normalized size = 1.06

$$\begin{cases} a^2x - \frac{ab\log\left(\tan^2(c+dx)+1\right)}{d} + \frac{ab\tan^2(c+dx)}{d} - b^2x + \frac{b^2\tan^5(c+dx)}{5d} - \frac{b^2\tan^3(c+dx)}{3d} + \frac{b^2\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a+b\tan^3(c)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*x - a*b*log(tan(c + d*x)**2 + 1)/d + a*b*tan(c + d*x)**2/d
- b**2*x + b**2*tan(c + d*x)**5/(5*d) - b**2*tan(c + d*x)**3/(3*d) + b**2*t
an(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**3)**2, True))
```

3.377 $\int \left(a+b\tan^3(c+dx)\right)\,dx$

Optimal. Leaf size=32

$$ax+\frac{b\tan^2(c+dx)}{2d}+\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] a*x+b*ln(cos(d*x+c))/d+1/2*b*tan(d*x+c)^2/d
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3473, 3475}

$$ax+\frac{b\tan^2(c+dx)}{2d}+\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[c + d*x]^3,x]
```

```
[Out] a*x + (b*Log[Cos[c + d*x]])/d + (b*Tan[c + d*x]^2)/(2*d)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^3(c+dx)\right)\,dx &= ax+b\int \tan^3(c+dx)\,dx\\ &= ax+\frac{b\tan^2(c+dx)}{2d}-b\int \tan(c+dx)\,dx\\ &= ax+\frac{b\log(\cos(c+dx))}{d}+\frac{b\tan^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 30, normalized size = 0.94

$$ax+\frac{b\left(\tan^2(c+dx)+2\log(\cos(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[c + d*x]^3,x]
```

```
[Out] a*x + (b*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/(2*d)
```

fricas [A] time = 0.53, size = 36, normalized size = 1.12

$$\frac{2\,adx+b\tan\left(dx+c\right)^2+b\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + b*tan(d*x + c)^2 + b*log(1/(tan(d*x + c)^2 + 1)))/d
```

giac [B] time = 1.62, size = 251, normalized size = 7.84

$$ax+\frac{\left(\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)\tan(dx)^2\tan(c)^2+\tan(dx)^2\tan(c)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] a*x + 1/2*(log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*ta
n(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan
(c)^2 + tan(d*x)^2*tan(c)^2 - 2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*t
an(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
 + 1))*tan(d*x)*tan(c) + tan(d*x)^2 + tan(c)^2 + log(4*(tan(d*x)^4*tan(c)^2
 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(
c) + 1)/(tan(c)^2 + 1)) + 1)*b/(d*tan(d*x)^2*tan(c)^2 - 2*d*tan(d*x)*tan(c)
 + d)
```

maple [A] time = 0.02, size = 36, normalized size = 1.12

$$ax+\frac{b\left(\tan^2(dx+c)\right)}{2d}-\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(d*x+c)^3,x)
```

```
[Out] a*x+1/2*b*tan(d*x+c)^2/d-1/2/d*b*ln(1+tan(d*x+c)^2)
```

maxima [A] time = 0.30, size = 36, normalized size = 1.12

$$ax-\frac{b\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)^2-1\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] a*x - 1/2*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 11.53, size = 34, normalized size = 1.06

$$\frac{\frac{b\tan(c+dx)^2}{2}-\frac{b\ln\left(\tan(c+dx)^2+1\right)}{2}+a\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*tan(c + d*x)^3,x)
```

```
[Out] ((b*tan(c + d*x)^2)/2 - (b*log(tan(c + d*x)^2 + 1))/2 + a*d*x)/d
```

sympy [A] time = 0.18, size = 37, normalized size = 1.16

$$ax+b\left(\begin{cases}-\frac{\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{\tan^2(c+dx)}{2d} & \text{for } d\neq 0\\ x\tan^3(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)**3,x)
```

```
[Out] a*x + b*Piecewise((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**2/(2*d),
 Ne(d, 0)), (x*tan(c)**3, True))
```

3.378 $\int \frac{1}{a+b\tan^3(c+dx)}\,dx$

Optimal. Leaf size=256

$$-\frac{b\log\left(a\cos^3(c+dx)+b\sin^3(c+dx)\right)}{3d\left(a^2+b^2\right)}+\frac{ax}{a^2+b^2}+\frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d\left(a^2+b^2\right)}-\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log$$

```
[Out] a*x/(a^2+b^2)-1/3*b*ln(a*cos(d*x+c)^3+b*sin(d*x+c)^3)/(a^2+b^2)/d+1/3*b^(1/
3)*(a^(4/3)+b^(4/3))*ln(a^(1/3)+b^(1/3)*tan(d*x+c))/a^(2/3)/(a^2+b^2)/d-1/6
*b^(1/3)*(a^(4/3)+b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*tan(d*x+c)+b^(2/3)*ta
n(d*x+c)^2)/a^(2/3)/(a^2+b^2)/d+1/3*b^(1/3)*(a^(4/3)-b^(4/3))*arctan(1/3*(a
^(1/3)-2*b^(1/3)*tan(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(a^2+b^2)/d*3^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3661, 6725, 635, 203, 260, 1871, 1860, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d\left(a^2+b^2\right)}-\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tan(c+dx)+b^{2/3}\tan^2(c+dx)\right)}{6a^{2/3}d\left(a^2+b^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^(-1),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (b^(1/3)*(a^(4/3) - b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3
)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a^2 + b^2)*d) - (b*Lo
g[a*Cos[c + d*x]^3 + b*Sin[c + d*x]^3])/(3*(a^2 + b^2)*d) + (b^(1/3)*(a^(4/
3) + b^(4/3))*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]])/(3*a^(2/3)*(a^2 + b^2)*d
) - (b^(1/3)*(a^(4/3) + b^(4/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x]
 + b^(2/3)*Tan[c + d*x]^2])/(6*a^(2/3)*(a^2 + b^2)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{a+b\tan^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^3)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a+bx}{(a^2+b^2)(1+x^2)} - \frac{b(-b+ax+bx^2)}{(a^2+b^2)(a+bx^3)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a+bx}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)\,d} - \frac{b\,\text{Subst}\left(\int \frac{-b+ax+bx^2}{a+bx^3}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)\,d}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)\,d} + \frac{b\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)\,d} - \frac{b\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{ax}{a^2+b^2} - \frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d} - \frac{b\log\left(a+b\tan^3(c+dx)\right)}{3(a^2+b^2)\,d} - \frac{b^{2/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(a^{4/3}-2b^{4/3}\right.}{a^{2/3}-\sqrt[3]{a}}\right.}{3a^{2/}}$$

$$= \frac{ax}{a^2+b^2} - \frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d} + \frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx)\right)}{3a^{2/3}(a^2+b^2)\,d} - \frac{b\log\left(\right.}{}$$

$$= \frac{ax}{a^2+b^2} - \frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d} + \frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx)\right)}{3a^{2/3}(a^2+b^2)\,d} - \frac{\sqrt[3]{b}\left(a\right.}{}$$

$$= \frac{ax}{a^2+b^2} + \frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tan(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}(a^2+b^2)\,d} - \frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d} + \frac{\sqrt[3]{b}\left(a^{4/3}+b^{4}\right.}{}$$

Mathematica [C] time = 0.66, size = 278, normalized size = 1.09

$$\frac{-b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tan(c+dx)+b^{2/3}\tan^2(c+dx)\right) - 3a^{2/3}b\tan^2(c+dx)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\tan^3(c+dx)}{a}\right) - 2a^{2/3}b\,\text{l}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1
/3))] - (3*I)*a^(5/3)*Log[I - Tan[c + d*x]] + 3*a^(2/3)*b*Log[I - Tan[c + d
*x]] + (3*I)*a^(5/3)*Log[I + Tan[c + d*x]] + 3*a^(2/3)*b*Log[I + Tan[c + d*
x]] + 2*b^(5/3)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]] - b^(5/3)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c + d*x]^2] - 2*a^(2/3)*b*Log[a
 + b*Tan[c + d*x]^3] - 3*a^(2/3)*b*Hypergeometric2F1[2/3, 1, 5/3, -((b*Tan[
c + d*x]^3)/a)]*Tan[c + d*x]^2)/(6*a^(2/3)*(a^2 + b^2)*d)
```

fricas [C] time = 2.10, size = 4817, normalized size = 18.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/24*(2*(a^2 + b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3
) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)
+ 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b
^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^
(1/3)) + 2*b/(a^2*d + b^2*d))*d*log(-1/4*(4*b^2*tan(d*x + c)^2 - ((a^4 + a^
2*b^2)*d^2*tan(d*x + c)^2 - (a^4 + a^2*b^2)*d^2)*((1/2)^(1/3)*(I*sqrt(3) +
1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a
^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d +
b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)
*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))^2 + 2*(a^2*b*d*ta
n(d*x + c)^2 - a^2*b*d + 2*(a^3 - a*b^2)*d*tan(d*x + c))*((1/2)^(1/3)*(I*sq
rt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^
2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((
a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^
2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d)) - 4*a^2)/
(tan(d*x + c)^2 + 1)) - 24*a*d*x - ((a^2 + b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1
)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^
2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b
^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*
b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))*d - 3*sqrt(1/3)*(a
^2 + b^2)*d*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/
(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b
^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)
^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a
^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))^2*d^2 - 4*(a^2*b + b^3)
*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b
^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*
(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2
*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d
+ b^2*d))*d - 12*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 6*b)*log(1/4*(8*a^4
- 16*a^2*b^2 - ((a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*tan(d*x + c)^2 - (a^6 + 2*a
^4*b^2 + a^2*b^4)*d^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d
^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3
) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2
*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3)
)^(1/3)) + 2*b/(a^2*d + b^2*d))^2 + 8*(2*a^2*b^2 - b^4)*tan(d*x + c)^2 + 2*
((a^4*b + a^2*b^3)*d*tan(d*x + c)^2 + 2*(a^5 - a*b^4)*d*tan(d*x + c) - (a^4
*b + a^2*b^3)*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) -
2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*
(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d
^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3
)) + 2*b/(a^2*d + b^2*d)) + 3*sqrt(1/3)*(4*(a^4*b + a^2*b^3)*d*tan(d*x + c)
^2 - 4*(a^5 - a*b^4)*d*tan(d*x + c) - ((a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*tan(
d*x + c)^2 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*
(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2
+ b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2
*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/
((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d)) - 4*(a^4*b + a^2*b^3
)*d)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^
3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a
^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(
a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^
2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))^2*d^2 - 4*(a^2*b + b^3)*((1/2)
^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3
 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqr
t(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^
2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d
))*d - 12*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 24*(a^3*b - a*b^3)*tan(d*x
+ c))/(tan(d*x + c)^2 + 1)) - ((a^2 + b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/
```

```
(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b
^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)
^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a
^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))*d + 3*sqrt(1/3)*(a^2 +
b^2)*d*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*
d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2
*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b
/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 +
b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))^2*d^2 - 4*(a^2*b + b^3)*((1/
2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)
^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*s
qrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d +
b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2
*d))*d - 12*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 6*b)*log(-1/4*(8*a^4 - 16
*a^2*b^2 - ((a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*tan(d*x + c)^2 - (a^6 + 2*a^4*b
^2 + a^2*b^4)*d^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3)
- 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) +
2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2
*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1
/3)) + 2*b/(a^2*d + b^2*d))^2 + 8*(2*a^2*b^2 - b^4)*tan(d*x + c)^2 + 2*((a^
4*b + a^2*b^3)*d*tan(d*x + c)^2 + 2*(a^5 - a*b^4)*d*tan(d*x + c) - (a^4*b +
 a^2*b^3)*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^
3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2
)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3)
- 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) +
 2*b/(a^2*d + b^2*d)) - 3*sqrt(1/3)*(4*(a^4*b + a^2*b^3)*d*tan(d*x + c)^2 -
 4*(a^5 - a*b^4)*d*tan(d*x + c) - ((a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*tan(d*x
+ c)^2 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(
a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^
2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^
2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^
2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d)) - 4*(a^4*b + a^2*b^3)*d)
*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 +
a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d
^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*
d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2
*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))^2*d^2 - 4*(a^2*b + b^3)*((1/2)^(1/
3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (
a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3)
 + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)
^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))*d
 - 12*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 24*(a^3*b - a*b^3)*tan(d*x + c)
)/(tan(d*x + c)^2 + 1)))/((a^2 + b^2)*d)
```

giac [A] time = 1.79, size = 334, normalized size = 1.30

$$\frac{2\left(a^3b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^3-b^5\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\tan(dx+c)\right|\right)}{a^5b+2\,a^3b^3+ab^5}+\frac{6\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\tan(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\right)\left(\left(-ab^2\right)^{\frac{1}{3}}b^2+\right.}{\sqrt{3}\,a^3b+\sqrt{3}\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/6*(2*(a^3*b^2*(-a/b)^(1/3) + a*b^4*(-a/b)^(1/3) - a^2*b^3 - b^5)*(-a/b)^(
1/3)*log(abs(-(-a/b)^(1/3) + tan(d*x + c)))/(a^5*b + 2*a^3*b^3 + a*b^5) + 6
*(pi*floor((d*x + c)/pi + 1/2)*sgn((-a/b)^(1/3)) + arctan(1/3*sqrt(3)*((-a/
b)^(1/3) + 2*tan(d*x + c))/(-a/b)^(1/3)))*((-a*b^2)^(1/3)*b^2 + (-a*b^2)^(2
```

```
/3)*a)/(sqrt(3)*a^3*b + sqrt(3)*a*b^3) + 6*(d*x + c)*a/(a^2 + b^2) + ((-a*b
^2)^(1/3)*b^2 - (-a*b^2)^(2/3)*a)*log(tan(d*x + c)^2 + (-a/b)^(1/3)*tan(d*x
 + c) + (-a/b)^(2/3))/(a^3*b + a*b^3) + 3*b*log(tan(d*x + c)^2 + 1)/(a^2 +
b^2) - 2*b*log(abs(b*tan(d*x + c)^3 + a))/(a^2 + b^2))/d
```

maple [A] time = 0.28, size = 355, normalized size = 1.39

$$\frac{b\ln\left(\tan\left(dx+c\right)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3d\left(a^{2}+b^{2}\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{b\ln\left(\tan^{2}\left(dx+c\right)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\tan\left(dx+c\right)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6d\left(a^{2}+b^{2}\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\tan(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(a^{2}+b^{2}\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^3),x)
```

```
[Out] 1/3/d*b/(a^2+b^2)/(1/b*a)^(2/3)*ln(tan(d*x+c)+(1/b*a)^(1/3))-1/6/d*b/(a^2+b
^2)/(1/b*a)^(2/3)*ln(tan(d*x+c)^2-(1/b*a)^(1/3)*tan(d*x+c)+(1/b*a)^(2/3))+1
/3/d*b/(a^2+b^2)/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*
tan(d*x+c)-1))+1/3/d/(a^2+b^2)*a/(1/b*a)^(1/3)*ln(tan(d*x+c)+(1/b*a)^(1/3))
-1/6/d/(a^2+b^2)*a/(1/b*a)^(1/3)*ln(tan(d*x+c)^2-(1/b*a)^(1/3)*tan(d*x+c)+(
1/b*a)^(2/3))-1/3/d/(a^2+b^2)*a*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2
/(1/b*a)^(1/3)*tan(d*x+c)-1))-1/3/d*b/(a^2+b^2)*ln(a+b*tan(d*x+c)^3)+1/2/d/
(a^2+b^2)*b*ln(1+tan(d*x+c)^2)+1/d/(a^2+b^2)*a*arctan(tan(d*x+c))
```

maxima [A] time = 1.13, size = 291, normalized size = 1.14

$$-\frac{\frac{2\sqrt{3}\left(a\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\right)-b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{2a}{b}\right)\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\tan(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{18\,(dx+c)a}{a^{2}+b^{2}}+\frac{3\left(b\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)+a\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(\tan(dx+c)^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\tan(dx+\cdots\right.}{a^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}\cdots}{18\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/18*(2*sqrt(3)*(a*(3*(a/b)^(2/3) - 2) - b*(3*(a/b)^(1/3) - 2*a/b))*arctan
(-1/3*sqrt(3)*((a/b)^(1/3) - 2*tan(d*x + c))/(a/b)^(1/3))/((a^2*(a/b)^(2/3)
 + b^2*(a/b)^(2/3))*(a/b)^(1/3)) - 18*(d*x + c)*a/(a^2 + b^2) + 3*(b*(2*(a/
b)^(2/3) + 1) + a*(a/b)^(1/3))*log(tan(d*x + c)^2 - (a/b)^(1/3)*tan(d*x + c
) + (a/b)^(2/3))/(a^2*(a/b)^(2/3) + b^2*(a/b)^(2/3)) - 9*b*log(tan(d*x + c)
^2 + 1)/(a^2 + b^2) + 6*(b*((a/b)^(2/3) - 1) - a*(a/b)^(1/3))*log((a/b)^(1/
3) + tan(d*x + c))/(a^2*(a/b)^(2/3) + b^2*(a/b)^(2/3)))/d
```

mupad [B] time = 12.65, size = 342, normalized size = 1.34

$$\frac{\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,a^{2}\,b^{2}\,z^{3}+27\,a^{4}\,z^{3}+27\,a^{2}\,b\,z^{2}-b,z,k\right)\left(\mathrm{root}\left(27\,a^{2}\,b^{2}\,z^{3}+27\,a^{4}\,z^{3}+27\,a^{2}\,b\,z^{2}-b,z,k\right)\left(\mathrm{r}\cdots\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^3),x)
```

```
[Out] symsum(log(root(27*a^2*b^2*z^3 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k)*(root
(27*a^2*b^2*z^3 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k)*(root(27*a^2*b^2*z^3
```

```
 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k)*(tan(c + d*x)*(12*b^6 - 69*a^2*b^4)
 + root(27*a^2*b^2*z^3 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k)*(36*a*b^6 - 1
80*a^3*b^4 + tan(c + d*x)*(162*a^2*b^5 - 54*a^4*b^3)) - 36*a*b^5 + 27*a^3*b
^3) + 13*a*b^4 - 16*b^5*tan(c + d*x)) + 5*b^4*tan(c + d*x)))*root(27*a^2*b^
2*z^3 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k), k, 1, 3)/d + log(tan(c + d*x)
 - 1i)/(2*d*(a*1i + b)) + (log(tan(c + d*x) + 1i)*1i)/(2*d*(a + b*1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\tan^3(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)**3),x)
```

```
[Out] Integral(1/(a + b*tan(c + d*x)**3), x)
```

3.379 $\int \frac{1}{\left(a+b\tan^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=558

$$\frac{b(\tan(c+dx)(b-a\tan(c+dx))+a)}{3ad\left(a^2+b^2\right)\left(a+b\tan^3(c+dx)\right)}-\frac{2ab\log\left(a\cos^3(c+dx)+b\sin^3(c+dx)\right)}{3d\left(a^2+b^2\right)^2}+\frac{x\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}+\frac{\sqrt[3]{b}\left(a^{4/3}-2b^{4/3}\right.}{3\sqrt{3}}$$

```
[Out] (a^2-b^2)*x/(a^2+b^2)^2-2/3*a*b*ln(a*cos(d*x+c)^3+b*sin(d*x+c)^3)/(a^2+b^2)
^2/d+1/3*b^(1/3)*(a^2+2*a^(2/3)*b^(4/3)-b^2)*ln(a^(1/3)+b^(1/3)*tan(d*x+c))
/a^(1/3)/(a^2+b^2)^2/d+1/9*b^(1/3)*(a^(4/3)+2*b^(4/3))*ln(a^(1/3)+b^(1/3)*t
an(d*x+c))/a^(5/3)/(a^2+b^2)/d-1/6*b^(1/3)*(a^2+2*a^(2/3)*b^(4/3)-b^2)*ln(a
^(2/3)-a^(1/3)*b^(1/3)*tan(d*x+c)+b^(2/3)*tan(d*x+c)^2)/a^(1/3)/(a^2+b^2)^2
/d-1/18*b^(1/3)*(a^(4/3)+2*b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*tan(d*x+c)+b
^(2/3)*tan(d*x+c)^2)/a^(5/3)/(a^2+b^2)/d+1/3*b^(1/3)*(a^2-2*a^(2/3)*b^(4/3)
-b^2)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tan(d*x+c))/a^(1/3)*3^(1/2))/a^(1/3)/(a
^2+b^2)^2/d*3^(1/2)+1/9*b^(1/3)*(a^(4/3)-2*b^(4/3))*arctan(1/3*(a^(1/3)-2*b
^(1/3)*tan(d*x+c))/a^(1/3)*3^(1/2))/a^(5/3)/(a^2+b^2)/d*3^(1/2)+1/3*b*(a+ta
n(d*x+c)*(b-a*tan(d*x+c)))/a/(a^2+b^2)/d/(a+b*tan(d*x+c)^3)
```

Rubi [A] time = 0.73, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {3661, 6725, 635, 203, 260, 1854, 1860, 31, 634, 617, 204, 628, 1871}

$$\frac{\sqrt[3]{b}\left(a^{4/3}-2b^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}d\left(a^2+b^2\right)}+\frac{\sqrt[3]{b}\left(-2a^{2/3}b^{4/3}+a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}d\left(a^2+b^2\right)^2}+\frac{b(\tan(c+dx)}{3ad\left(a^2+b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^(-2),x]
```

```
[Out] ((a^2 - b^2)*x)/(a^2 + b^2)^2 + (b^(1/3)*(a^2 - 2*a^(2/3)*b^(4/3) - b^2)*Ar
cTan[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3
)*(a^2 + b^2)^2*d) + (b^(1/3)*(a^(4/3) - 2*b^(4/3))*ArcTan[(a^(1/3) - 2*b^(
1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(a^2 + b^2)*d) -
(2*a*b*Log[a*Cos[c + d*x]^3 + b*Sin[c + d*x]^3])/(3*(a^2 + b^2)^2*d) + (b^(
1/3)*(a^2 + 2*a^(2/3)*b^(4/3) - b^2)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]])/(
3*a^(1/3)*(a^2 + b^2)^2*d) + (b^(1/3)*(a^(4/3) + 2*b^(4/3))*Log[a^(1/3) + b
^(1/3)*Tan[c + d*x]])/(9*a^(5/3)*(a^2 + b^2)*d) - (b^(1/3)*(a^2 + 2*a^(2/3)
*b^(4/3) - b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c
+ d*x]^2])/(6*a^(1/3)*(a^2 + b^2)^2*d) - (b^(1/3)*(a^(4/3) + 2*b^(4/3))*Log
[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c + d*x]^2])/(18*a^(5
/3)*(a^2 + b^2)*d) + (b*(a + Tan[c + d*x]*(b - a*Tan[c + d*x])))/(3*a*(a^2
+ b^2)*d*(a + b*Tan[c + d*x]^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(a+bx^3\right)^2}\,dx,x,\tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-b^2+2abx}{\left(a^2+b^2\right)^2\left(1+x^2\right)}-\frac{b\left(-b+ax+bx^2\right)}{\left(a^2+b^2\right)\left(a+bx^3\right)^2}+\frac{b\left(2ab-\left(a^2-b^2\right)x-2abx^2\right)}{\left(a^2+b^2\right)^2\left(a+bx^3\right)}\right)dx,x,\tan(c+dx\right.}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a^2-b^2+2abx}{1+x^2}\,dx,x,\tan(c+dx)\right)}{\left(a^2+b^2\right)^2 d}+\frac{b\,\text{Subst}\left(\int \frac{2ab-\left(a^2-b^2\right)x-2abx^2}{a+bx^3}\,dx,x,\tan\right.}{\left(a^2+b^2\right)^2 d}$$

$$= \frac{b(a+\tan(c+dx)(b-a\tan(c+dx)))}{3a\left(a^2+b^2\right)d\left(a+b\tan^3(c+dx)\right)}+\frac{b\,\text{Subst}\left(\int \frac{2ab+\left(-a^2+b^2\right)x}{a+bx^3}\,dx,x,\tan(c+d\right.}{\left(a^2+b^2\right)^2 d}$$

$$= \frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2}-\frac{2ab\log(\cos(c+dx))}{\left(a^2+b^2\right)^2 d}-\frac{2ab\log\left(a+b\tan^3(c+dx)\right)}{3\left(a^2+b^2\right)^2 d}+\frac{b(a+\tan(}{3a\left(a^2+\right.}$$

$$= \frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2}-\frac{2ab\log(\cos(c+dx))}{\left(a^2+b^2\right)^2 d}+\frac{\sqrt[3]{b}\left(a^2+2a^{2/3}b^{4/3}-b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\text{ta}\right.}{3\sqrt[3]{a}\left(a^2+b^2\right)^2 d}$$

$$= \frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2}-\frac{2ab\log(\cos(c+dx))}{\left(a^2+b^2\right)^2 d}+\frac{\sqrt[3]{b}\left(a^2+2a^{2/3}b^{4/3}-b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\text{ta}\right.}{3\sqrt[3]{a}\left(a^2+b^2\right)^2 d}$$

$$= \frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2}+\frac{\sqrt[3]{b}\left(a^2-2a^{2/3}b^{4/3}-b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tan(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^2+b^2\right)^2 d}+\frac{\sqrt[3]{b}\left(a^{4/3}-2b^{4/3}\right)}{3\sqrt{3}\,a^{5/}}$$

Mathematica [C] time = 6.31, size = 575, normalized size = 1.03

$$-\frac{b(a-b)(a+b)\tan^2(c+dx)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\tan^3(c+dx)}{a}\right)}{2ad\left(a^2+b^2\right)^2}-\frac{b\tan^2(c+dx)\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};-\frac{b\tan^3(c+dx)}{a}\right)}{2ad\left(a^2+b^2\right)}+\frac{b^2\tan}{3ad\left(a^2+b^2\right)\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^(-2),x]
```

```
[Out] ((-1/2*I)*Log[I - Tan[c + d*x]])/((a - I*b)^2*d) + ((I/2)*Log[I + Tan[c + d
*x]])/((a + I*b)^2*d) + (2*a^(1/3)*b^(5/3)*Log[a^(1/3) + b^(1/3)*Tan[c + d*
x]])/(3*(a^2 + b^2)^2*d) - (a^(1/3)*(2*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*
b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))] + b^(5/3)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c + d*x]^2]))/(3*(a^2 + b^2)^2*d) + ((2*b^
(5/3)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]])/a^(2/3) - (2*Sqrt[3]*b^(5/3)*Arc
Tan[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))] + b^(5/3)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c + d*x]^2])/a^(2/3))/(9*
a*(a^2 + b^2)*d) - (2*a*b*Log[a + b*Tan[c + d*x]^3])/(3*(a^2 + b^2)^2*d) -
((a - b)*b*(a + b)*Hypergeometric2F1[2/3, 1, 5/3, -((b*Tan[c + d*x]^3)/a)]*
Tan[c + d*x]^2)/(2*a*(a^2 + b^2)^2*d) - (b*Hypergeometric2F1[2/3, 2, 5/3, -
((b*Tan[c + d*x]^3)/a)]*Tan[c + d*x]^2)/(2*a*(a^2 + b^2)*d) + b/(3*(a^2 + b
^2)*d*(a + b*Tan[c + d*x]^3)) + (b^2*Tan[c + d*x])/(3*a*(a^2 + b^2)*d*(a +
b*Tan[c + d*x]^3))
```

fricas [C] time = 3.40, size = 11554, normalized size = 20.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/648*(216*a^3*b - 432*a*b^3 + 216*(2*a^2*b^2 - b^4 - 3*(a^3*b - a*b^3)*d*
x)*tan(d*x + c)^3 - 648*(a^4 - a^2*b^2)*d*x + 2*((a^5*b + 2*a^3*b^3 + a*b^5
)*d*tan(d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d)*(4*(9*a^2*b^2/(a^4*d +
2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*s
qrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a
^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/72
9*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 -
28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27
*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2
*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/
(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^
2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^
4*d + 2*a^2*b^2*d + b^4*d))*log(1/324*(10368*a^6 - 12960*a^4*b^2 - 3888*a^2
*b^4 + ((2*a^10 + 5*a^8*b^2 + 4*a^6*b^4 + a^4*b^6)*d^2*tan(d*x + c)^2 - 4*(
a^9*b + 2*a^7*b^3 + a^5*b^5)*d^2*tan(d*x + c) - (2*a^10 + 5*a^8*b^2 + 4*a^6
*b^4 + a^4*b^6)*d^2)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a
^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4
*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^
4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*
a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*
b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d +
 b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*
a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*
b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a
^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))^2 -
 1296*(18*a^4*b^2 + 7*a^2*b^4 + b^6)*tan(d*x + c)^2 - 36*((8*a^7*b - 2*a^5*
b^3 - a^3*b^5)*d*tan(d*x + c)^2 + 2*(4*a^8 - 20*a^6*b^2 - 7*a^4*b^4 - a^2*b
```

```
^6)*d*tan(d*x + c) - (8*a^7*b - 2*a^5*b^3 - a^3*b^5)*d)*(4*(9*a^2*b^2/(a^4*
d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(
-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3
/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) +
4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^
6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-
8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4
*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b
^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 1
0*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b
/(a^4*d + 2*a^2*b^2*d + b^4*d)) + 2592*(4*a^5*b + 2*a^3*b^3 + a*b^5)*tan(d*
x + c))/(tan(d*x + c)^2 + 1)) + 216*(a^3*b + a*b^3)*tan(d*x + c)^2 + (324*a
^2*b^2*tan(d*x + c)^3 + 324*a^3*b - ((a^5*b + 2*a^3*b^3 + a*b^5)*d*tan(d*x
+ c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d
+ b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/
(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a
^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b +
 b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 -
 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^
4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b
^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2
*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)
*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*
b^2*d + b^4*d)) + 3*sqrt(1/3)*((a^5*b + 2*a^3*b^3 + a*b^5)*d*tan(d*x + c)^3
 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d)*sqrt((29808*a^4*b^2 - 10368*a^2*b^4 - 518
4*b^6 - (a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8)*(4*(9*a^2*b^2/
(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^
2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*
a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d
)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*
(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) +
81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 +
2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*
b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^
2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 10
8*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))^2*d^2 + 216*(a^7*b + 2*a^5*b^3 + a^3*b
^5)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^
2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d
+ b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2
*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5
*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*
a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*
a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d
)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*
(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I
*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))*d)/((a^10 + 4*a^8*b^
2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8)*d^2)))*log(1/324*(20736*a^8 - 106272*a
^6*b^2 - 22032*a^4*b^4 - ((2*a^12 + 7*a^10*b^2 + 9*a^8*b^4 + 5*a^6*b^6 + a^
4*b^8)*d^2*tan(d*x + c)^2 - 4*(a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*d^
2*tan(d*x + c) - (2*a^12 + 7*a^10*b^2 + 9*a^8*b^4 + 5*a^6*b^6 + a^4*b^8)*d^
2)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2
*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d +
 b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*
a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*
b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a
^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a
*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)
) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(
8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*
```

sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))^2 + 1296*(42*a^6*b^2 - 59*a^4*b^4 - 22*a^2*b^6 - 2*b^8)*tan(d*x + c)^2 + 36*((8*a^9*b + 6*a^7*b^3 - 3*a^5*b^5 - a^3*b^7)*d*tan(d*x + c)^2 + 2*(4*a^10 - 16*a^8*b^2 - 27*a^6*b^4 - 8*a^4*b^6 - a^2*b^8)*d*tan(d*x + c) - (8*a^9*b + 6*a^7*b^3 - 3*a^5*b^5 - a^3*b^7)*d)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d)) + 3*sqrt(1/3)*(36*(10*a^9*b + 21*a^7*b^3 + 12*a^5*b^5 + a^3*b^7)*d*tan(d*x + c)^2 - 72*(4*a^10 + 2*a^8*b^2 - 9*a^6*b^4 - 8*a^4*b^6 - a^2*b^8)*d*tan(d*x + c) - ((2*a^12 + 7*a^10*b^2 + 9*a^8*b^4 + 5*a^6*b^6 + a^4*b^8)*d^2*tan(d*x + c)^2 - 4*(a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*d^2*tan(d*x + c) - (2*a^12 + 7*a^10*b^2 + 9*a^8*b^4 + 5*a^6*b^6 + a^4*b^8)*d^2)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d)) - 36*(10*a^9*b + 21*a^7*b^3 + 12*a^5*b^5 + a^3*b^7)*d)*sqrt((29808*a^4*b^2 - 10368*a^2*b^4 - 5184*b^6 - (a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))^2*d^2 + 216*(a^7*b + 2*a^5*b^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))*d)/((a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8)*d^2)) - 2592*(28*a^7*b - 78*a^5*b^3 - 27*a^3*b^5 - 2*a*b^7)*tan(d*x + c))/(tan(d*x + c)^2 + 1)) + (324*a^2*b^2*tan(d*x + c)^3 + 324*a^3*b - ((a^5*b + 2*a^3*b^3 + a*b^5)*d*tan(d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2

```
)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4
/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b
^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/
729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3
)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d)) - 3*sqrt(1/3)*((
a^5*b + 2*a^3*b^3 + a*b^5)*d*tan(d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d
)*sqrt((29808*a^4*b^2 - 10368*a^2*b^4 - 5184*b^6 - (a^10 + 4*a^8*b^2 + 6*a^
6*b^4 + 4*a^4*b^6 + a^2*b^8)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2
- b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*
b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2
+ a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*
d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4
 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2
*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^
4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^
3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 +
b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4
*d))^2*d^2 + 216*(a^7*b + 2*a^5*b^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*d + 2*a^2
*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3
) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^
2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*
a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^
4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*
b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2
+ a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*
d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4
 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d +
 2*a^2*b^2*d + b^4*d))*d)/((a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*
b^8)*d^2)))*log(-1/324*(20736*a^8 - 106272*a^6*b^2 - 22032*a^4*b^4 - ((2*a^
12 + 7*a^10*b^2 + 9*a^8*b^4 + 5*a^6*b^6 + a^4*b^8)*d^2*tan(d*x + c)^2 - 4*(
a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*d^2*tan(d*x + c) - (2*a^12 + 7*a^
10*b^2 + 9*a^8*b^4 + 5*a^6*b^6 + a^4*b^8)*d^2)*(4*(9*a^2*b^2/(a^4*d + 2*a^2
*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3
) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^
2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*
a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^
4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*
b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2
+ a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*
d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4
 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d +
 2*a^2*b^2*d + b^4*d))^2 + 1296*(42*a^6*b^2 - 59*a^4*b^4 - 22*a^2*b^6 - 2*b
^8)*tan(d*x + c)^2 + 36*((8*a^9*b + 6*a^7*b^3 - 3*a^5*b^5 - a^3*b^7)*d*tan(
d*x + c)^2 + 2*(4*a^10 - 16*a^8*b^2 - 27*a^6*b^4 - 8*a^4*b^6 - a^2*b^8)*d*t
an(d*x + c) - (8*a^9*b + 6*a^7*b^3 - 3*a^5*b^5 - a^3*b^7)*d)*(4*(9*a^2*b^2/
(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^
2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*
a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d
)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*
(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) +
81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 +
2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*
b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^
2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 10
8*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d)) - 3*sqrt(1/3)*(36*(10*a^9*b + 21*a^7*b
^3 + 12*a^5*b^5 + a^3*b^7)*d*tan(d*x + c)^2 - 72*(4*a^10 + 2*a^8*b^2 - 9*a^
6*b^4 - 8*a^4*b^6 - a^2*b^8)*d*tan(d*x + c) - ((2*a^12 + 7*a^10*b^2 + 9*a^8
*b^4 + 5*a^6*b^6 + a^4*b^8)*d^2*tan(d*x + c)^2 - 4*(a^11*b + 3*a^9*b^3 + 3*
a^7*b^5 + a^5*b^7)*d^2*tan(d*x + c) - (2*a^12 + 7*a^10*b^2 + 9*a^8*b^4 + 5*
```

```
a^6*b^6 + a^4*b^8)*d^2)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2
/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(
a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2
*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 +
 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^
6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*
d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d +
 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a
^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^
4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))
- 36*(10*a^9*b + 21*a^7*b^3 + 12*a^5*b^5 + a^3*b^7)*d)*sqrt((29808*a^4*b^2
- 10368*a^2*b^4 - 5184*b^6 - (a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^
2*b^8)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4
*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2
*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d
+ 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 +
a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)
^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/
81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^
4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/7
29*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)
*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))^2*d^2 + 216*(a^7*
b + 2*a^5*b^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^
2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/
(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^
2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3
+ 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b
^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2
*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d
+ 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 +
a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)
^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))
*d)/((a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8)*d^2)) - 2592*(28*
a^7*b - 78*a^5*b^3 - 27*a^3*b^5 - 2*a*b^7)*tan(d*x + c))/(tan(d*x + c)^2 +
1)) - 216*(a^2*b^2 + b^4)*tan(d*x + c))/((a^5*b + 2*a^3*b^3 + a*b^5)*d*tan(
d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d)
```

giac [A] time = 2.75, size = 605, normalized size = 1.08

$$\frac{9\,ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{6\,ab\log\left(\left|b\tan(dx+c)^3+a\right|\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\left(2\,a^8b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3\,a^6b^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^8\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^7b^3-9\,a^5b^5-6\,a^3b^7-ab^9\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.\right.}{a^{11}b+4\,a^9b^3+6\,a^7b^5+4\,a^5b^7+a^3b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/9*(9*a*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 6*a*b*log(abs(
b*tan(d*x + c)^3 + a))/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*a^8*b^2*(-a/b)^(1/3)
+ 3*a^6*b^4*(-a/b)^(1/3) - a^2*b^8*(-a/b)^(1/3) - 4*a^7*b^3 - 9*a^5*b^5 - 6
*a^3*b^7 - a*b^9)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + tan(d*x + c)))/(a^11
*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*b^7 + a^3*b^9) + 9*(a^2 - b^2)*(d*x + c)
/(a^4 + 2*a^2*b^2 + b^4) + 2*(pi*floor((d*x + c)/pi + 1/2)*sgn((-a/b)^(1/3)
) + arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*tan(d*x + c))/(-a/b)^(1/3)))*((2*s
qrt(3)*a^3 - sqrt(3)*a*b^2)*(-a*b^2)^(2/3) + (4*sqrt(3)*a^2*b^2 + sqrt(3)*b
^4)*(-a*b^2)^(1/3))/(a^6*b + 2*a^4*b^3 + a^2*b^5) - ((2*a^3 - a*b^2)*(-a*b^
2)^(2/3) - (4*a^2*b^2 + b^4)*(-a*b^2)^(1/3))*log(tan(d*x + c)^2 + (-a/b)^(1
```

```
/3)*tan(d*x + c) + (-a/b)^(2/3))/(a^6*b + 2*a^4*b^3 + a^2*b^5) + 3*(2*a^2*b
^2*tan(d*x + c)^3 - a^3*b*tan(d*x + c)^2 - a*b^3*tan(d*x + c)^2 + a^2*b^2*t
an(d*x + c) + b^4*tan(d*x + c) + 3*a^3*b + a*b^3)/((a^5 + 2*a^3*b^2 + a*b^4
)*(b*tan(d*x + c)^3 + a)))/d
```

maple [B] time = 0.35, size = 1086, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^3)^2,x)
```

```
[Out] -1/3/d*b/(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c)^3)*tan(d*x+c)^2*a^2-1/3/d*b^3/
(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c)^3)*tan(d*x+c)^2+1/3/d*b^2/(a^4+2*a^2*b^
2+b^4)/(a+b*tan(d*x+c)^3)*a*tan(d*x+c)+1/3/d*b^4/(a^4+2*a^2*b^2+b^4)/(a+b*t
an(d*x+c)^3)/a*tan(d*x+c)+1/3/d*b/(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c)^3)*a^
2+1/3/d*b^3/(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c)^3)+8/9/d*b/(a^4+2*a^2*b^2+b
^4)*a/(1/b*a)^(2/3)*ln(tan(d*x+c)+(1/b*a)^(1/3))+2/9/d*b^3/(a^4+2*a^2*b^2+b
^4)/a/(1/b*a)^(2/3)*ln(tan(d*x+c)+(1/b*a)^(1/3))-4/9/d*b/(a^4+2*a^2*b^2+b^4
)*a/(1/b*a)^(2/3)*ln(tan(d*x+c)^2-(1/b*a)^(1/3)*tan(d*x+c)+(1/b*a)^(2/3))-1
/9/d*b^3/(a^4+2*a^2*b^2+b^4)/a/(1/b*a)^(2/3)*ln(tan(d*x+c)^2-(1/b*a)^(1/3)*
tan(d*x+c)+(1/b*a)^(2/3))+8/9/d*b/(a^4+2*a^2*b^2+b^4)*a/(1/b*a)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*tan(d*x+c)-1))+2/9/d*b^3/(a^4+2*a^2*
b^2+b^4)/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*tan(d*
x+c)-1))+4/9/d/(a^4+2*a^2*b^2+b^4)*a^2/(1/b*a)^(1/3)*ln(tan(d*x+c)+(1/b*a)^
(1/3))-2/9/d*b^2/(a^4+2*a^2*b^2+b^4)/(1/b*a)^(1/3)*ln(tan(d*x+c)+(1/b*a)^(1
/3))-2/9/d/(a^4+2*a^2*b^2+b^4)*a^2/(1/b*a)^(1/3)*ln(tan(d*x+c)^2-(1/b*a)^(1
/3)*tan(d*x+c)+(1/b*a)^(2/3))+1/9/d*b^2/(a^4+2*a^2*b^2+b^4)/(1/b*a)^(1/3)*l
n(tan(d*x+c)^2-(1/b*a)^(1/3)*tan(d*x+c)+(1/b*a)^(2/3))-4/9/d/(a^4+2*a^2*b^2
+b^4)*a^2*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*tan(d*x
+c)-1))+2/9/d*b^2/(a^4+2*a^2*b^2+b^4)*3^(1/2)/(1/b*a)^(1/3)*arctan(1/3*3^(1
/2)*(2/(1/b*a)^(1/3)*tan(d*x+c)-1))-2/3/d*b/(a^4+2*a^2*b^2+b^4)*a*ln(a+b*ta
n(d*x+c)^3)+1/d/(a^4+2*a^2*b^2+b^4)*a*b*ln(1+tan(d*x+c)^2)+1/d/(a^4+2*a^2*b
^2+b^4)*arctan(tan(d*x+c))*a^2-1/d/(a^4+2*a^2*b^2+b^4)*arctan(tan(d*x+c))*b
^2
```

maxima [A] time = 1.23, size = 502, normalized size = 0.90

$$\frac{\frac{9\,ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} - \frac{2\,\sqrt{3}\left(2\,a^3\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)-2\,a^2b\left(2\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{a}{b}\right)-ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,\tan(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{9\left(a^2-b^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4} - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(9*a*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*sqrt(3)*(2*a
^3*((a/b)^(2/3) - 1) - 2*a^2*b*(2*(a/b)^(1/3) - a/b) - a*b^2*(a/b)^(2/3) -
b^3*(a/b)^(1/3))*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*tan(d*x + c))/(a/b)^(
1/3))/((a^5*(a/b)^(2/3) + 2*a^3*b^2*(a/b)^(2/3) + a*b^4*(a/b)^(2/3))*(a/b)^
(1/3)) + 9*(a^2 - b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (2*a^2*b*(3*(a/b
)^(2/3) + 2) + 2*a^3*(a/b)^(1/3) - a*b^2*(a/b)^(1/3) + b^3)*log(tan(d*x + c
)^2 - (a/b)^(1/3)*tan(d*x + c) + (a/b)^(2/3))/(a^5*(a/b)^(2/3) + 2*a^3*b^2*
(a/b)^(2/3) + a*b^4*(a/b)^(2/3)) - 2*(a^2*b*(3*(a/b)^(2/3) - 4) - 2*a^3*(a/
b)^(1/3) + a*b^2*(a/b)^(1/3) - b^3)*log((a/b)^(1/3) + tan(d*x + c))/(a^5*(a
/b)^(2/3) + 2*a^3*b^2*(a/b)^(2/3) + a*b^4*(a/b)^(2/3)) - 3*(a*b*tan(d*x + c
```

```
)^2 - b^2*tan(d*x + c) - a*b)/(a^4 + a^2*b^2 + (a^3*b + a*b^3)*tan(d*x + c)
^3))/d
```

mupad [B] time = 12.52, size = 988, normalized size = 1.77

$$\sum_{k=1}^{3}\ln\left(-\frac{\frac{16\,a^2\,b^4}{27}+\frac{8\,b^6}{27}}{a^7+2\,a^5\,b^2+a^3\,b^4}+\mathrm{root}\left(1458\,a^7\,b^2\,z^3+729\,a^5\,b^4\,z^3+729\,a^9\,z^3+1458\,a^6\,b\,z^2+108\,a^3\,b^2\,z-64\,a^2\,b-8\,b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^3)^2,x)
```

```
[Out] symsum(log(root(1458*a^7*b^2*z^3 + 729*a^5*b^4*z^3 + 729*a^9*z^3 + 1458*a^6
*b*z^2 + 108*a^3*b^2*z - 64*a^2*b - 8*b^3, z, k)*(((32*a*b^7)/27 - (128*a^3
*b^5)/27)/(a^7 + a^3*b^4 + 2*a^5*b^2) - root(1458*a^7*b^2*z^3 + 729*a^5*b^4
*z^3 + 729*a^9*z^3 + 1458*a^6*b*z^2 + 108*a^3*b^2*z - 64*a^2*b - 8*b^3, z,
k)*(root(1458*a^7*b^2*z^3 + 729*a^5*b^4*z^3 + 729*a^9*z^3 + 1458*a^6*b*z^2
+ 108*a^3*b^2*z - 64*a^2*b - 8*b^3, z, k)*((16*a^3*b^9 + 77*a^5*b^7 + 34*a^
7*b^5 - 27*a^9*b^3)/(a^7 + a^3*b^4 + 2*a^5*b^2) + root(1458*a^7*b^2*z^3 + 7
29*a^5*b^4*z^3 + 729*a^9*z^3 + 1458*a^6*b*z^2 + 108*a^3*b^2*z - 64*a^2*b -
8*b^3, z, k)*((108*a^6*b^8 - 36*a^4*b^10 + 324*a^8*b^6 + 180*a^10*b^4)/(a^7
 + a^3*b^4 + 2*a^5*b^2) - (tan(c + d*x)*(4374*a^5*b^9 + 7290*a^7*b^7 + 1458
*a^9*b^5 - 1458*a^11*b^3))/(27*(a^7 + a^3*b^4 + 2*a^5*b^2))) - (tan(c + d*x
)*(216*a^2*b^10 + 864*a^4*b^8 - 1836*a^6*b^6 - 2484*a^8*b^4))/(27*(a^7 + a^
3*b^4 + 2*a^5*b^2))) - ((64*a^2*b^8)/9 + (353*a^4*b^6)/9 + (388*a^6*b^4)/9)
/(a^7 + a^3*b^4 + 2*a^5*b^2) + (tan(c + d*x)*(96*a*b^9 + 408*a^3*b^7 + 447*
a^5*b^5))/(27*(a^7 + a^3*b^4 + 2*a^5*b^2))) + (tan(c + d*x)*(134*a^2*b^6 -
16*b^8 + 236*a^4*b^4))/(27*(a^7 + a^3*b^4 + 2*a^5*b^2))) - ((8*b^6)/27 + (1
6*a^2*b^4)/27)/(a^7 + a^3*b^4 + 2*a^5*b^2) - (8*a*b^5*tan(c + d*x))/(9*(a^7
 + a^3*b^4 + 2*a^5*b^2)))*root(1458*a^7*b^2*z^3 + 729*a^5*b^4*z^3 + 729*a^9
*z^3 + 1458*a^6*b*z^2 + 108*a^3*b^2*z - 64*a^2*b - 8*b^3, z, k), k, 1, 3)/d
 + (log(tan(c + d*x) - 1i)*1i)/(2*d*(a*b*2i - a^2 + b^2)) + log(tan(c + d*x
) + 1i)/(2*d*(2*a*b - a^2*1i + b^2*1i)) + (b/(3*(a^2 + b^2)) - (b*tan(c + d
*x)^2)/(3*(a^2 + b^2)) + (b^2*tan(c + d*x))/(3*a*(a^2 + b^2)))/(d*(a + b*ta
n(c + d*x)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.380 $\int \frac{1}{1+\tan^3(x)}\,dx$

Optimal. Leaf size=37

$$\frac{x}{2}-\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right)+\frac{1}{6}\log(\tan(x)+1)-\frac{1}{2}\log(\cos(x))$$

```
[Out] 1/2*x-1/2*ln(cos(x))+1/6*ln(1+tan(x))-1/3*ln(1-tan(x)+tan(x)^2)
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3661, 2074, 635, 203, 260, 628}

$$\frac{x}{2}-\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right)+\frac{1}{6}\log(\tan(x)+1)-\frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x]^3)^(-1),x]
```

```
[Out] x/2 - Log[Cos[x]]/2 + Log[1 + Tan[x]]/6 - Log[1 - Tan[x] + Tan[x]^2]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{1+\tan^3(x)}\,dx = \text{Subst}\left(\int \frac{1}{(1+x^2)(1+x^3)}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{6(1+x)} + \frac{1+x}{2(1+x^2)} + \frac{1-2x}{3(1-x+x^2)}\right) dx, x, \tan(x)\right)$$
$$= \frac{1}{6}\log(1+\tan(x)) + \frac{1}{3}\text{Subst}\left(\int \frac{1-2x}{1-x+x^2}\,dx, x, \tan(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{1+x^2}\,dx, x, \tan(x)\right.$$
$$= \frac{1}{6}\log(1+\tan(x)) - \frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right) + \frac{1}{2}\text{Su}$$
$$= \frac{x}{2} - \frac{1}{2}\log(\cos(x)) + \frac{1}{6}\log(1+\tan(x)) - \frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)$$

Mathematica [C] time = 0.03, size = 57, normalized size = 1.54

$$-\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right)+\left(\frac{1}{4}-\frac{i}{4}\right)\log(-\tan(x)+i)+\left(\frac{1}{4}+\frac{i}{4}\right)\log(\tan(x)+i)+\frac{1}{6}\log(\tan(x)+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x]^3)^(-1),x]
[Out] (1/4 - I/4)*Log[I - Tan[x]] + (1/4 + I/4)*Log[I + Tan[x]] + Log[1 + Tan[x]]
/6 - Log[1 - Tan[x] + Tan[x]^2]/3
```

fricas [A] time = 0.54, size = 48, normalized size = 1.30

$$\frac{1}{2}x + \frac{1}{12}\log\left(\frac{\tan(x)^2 + 2\tan(x) + 1}{\tan(x)^2 + 1}\right) - \frac{1}{3}\log\left(\frac{\tan(x)^2 - \tan(x) + 1}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^3),x, algorithm="fricas")
[Out] 1/2*x + 1/12*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) - 1/3*log((tan(x
)^2 - tan(x) + 1)/(tan(x)^2 + 1))
```

giac [A] time = 0.59, size = 34, normalized size = 0.92

$$\frac{1}{2}x - \frac{1}{3}\log\left(\tan(x)^2 - \tan(x) + 1\right) + \frac{1}{4}\log\left(\tan(x)^2 + 1\right) + \frac{1}{6}\log\left(|\tan(x) + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^3),x, algorithm="giac")
[Out] 1/2*x - 1/3*log(tan(x)^2 - tan(x) + 1) + 1/4*log(tan(x)^2 + 1) + 1/6*log(ab
s(tan(x) + 1))
```

maple [A] time = 0.10, size = 34, normalized size = 0.92

$$-\frac{\ln\left(1-\tan(x)+\tan^2(x)\right)}{3} + \frac{\ln(1+\tan(x))}{6} + \frac{\ln\left(1+\tan^2(x)\right)}{4} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tan(x)^3),x)
[Out] -1/3*ln(1-tan(x)+tan(x)^2)+1/6*ln(1+tan(x))+1/4*ln(1+tan(x)^2)+1/2*x
```

maxima [A] time = 0.75, size = 33, normalized size = 0.89

$$\frac{1}{2}x - \frac{1}{3}\log\left(\tan(x)^2 - \tan(x) + 1\right) + \frac{1}{4}\log\left(\tan(x)^2 + 1\right) + \frac{1}{6}\log(\tan(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^3),x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/3*log(tan(x)^2 - tan(x) + 1) + 1/4*log(tan(x)^2 + 1) + 1/6*log(ta
n(x) + 1)
```

mupad [B] time = 11.62, size = 41, normalized size = 1.11

$$\frac{\ln(\tan(x) + 1)}{6} - \frac{\ln\left(\tan(x)^2 - \tan(x) + 1\right)}{3} + \ln(\tan(x) - \mathrm{i})\left(\frac{1}{4} - \frac{1}{4}\mathrm{i}\right) + \ln(\tan(x) + 1\mathrm{i})\left(\frac{1}{4} + \frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x)^3 + 1),x)
```

```
[Out] log(tan(x) + 1)/6 - log(tan(x)^2 - tan(x) + 1)/3 + log(tan(x) - 1i)*(1/4 -
1i/4) + log(tan(x) + 1i)*(1/4 + 1i/4)
```

sympy [A] time = 0.18, size = 34, normalized size = 0.92

$$\frac{x}{2} + \frac{\log(\tan(x) + 1)}{6} + \frac{\log\left(\tan^2(x) + 1\right)}{4} - \frac{\log\left(\tan^2(x) - \tan(x) + 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)**3),x)
```

```
[Out] x/2 + log(tan(x) + 1)/6 + log(tan(x)**2 + 1)/4 - log(tan(x)**2 - tan(x) + 1
)/3
```

3.381 $\int \left(a+b\tan^4(c+dx)\right)^4\,dx$

Optimal. Leaf size=216

$$\frac{b^2\left(6a^2+4ab+b^2\right)\tan^7(c+dx)}{7d}-\frac{b^2\left(6a^2+4ab+b^2\right)\tan^5(c+dx)}{5d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan^3(c+dx)}{3d}-\frac{b(2\cdots}{}$$

```
[Out] (a+b)^4*x-b*(2*a+b)*(2*a^2+2*a*b+b^2)*tan(d*x+c)/d+1/3*b*(2*a+b)*(2*a^2+2*a
*b+b^2)*tan(d*x+c)^3/d-1/5*b^2*(6*a^2+4*a*b+b^2)*tan(d*x+c)^5/d+1/7*b^2*(6*
a^2+4*a*b+b^2)*tan(d*x+c)^7/d-1/9*b^3*(4*a+b)*tan(d*x+c)^9/d+1/11*b^3*(4*a+
b)*tan(d*x+c)^11/d-1/13*b^4*tan(d*x+c)^13/d+1/15*b^4*tan(d*x+c)^15/d
```

Rubi [A] time = 0.13, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3661, 1154, 203}

$$\frac{b^2\left(6a^2+4ab+b^2\right)\tan^7(c+dx)}{7d}-\frac{b^2\left(6a^2+4ab+b^2\right)\tan^5(c+dx)}{5d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan^3(c+dx)}{3d}-\frac{b(2\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^4,x]
```

```
[Out] (a + b)^4*x - (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x])/d + (b*(2*a
+ b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x]^3)/(3*d) - (b^2*(6*a^2 + 4*a*b + b^
2)*Tan[c + d*x]^5)/(5*d) + (b^2*(6*a^2 + 4*a*b + b^2)*Tan[c + d*x]^7)/(7*d)
 - (b^3*(4*a + b)*Tan[c + d*x]^9)/(9*d) + (b^3*(4*a + b)*Tan[c + d*x]^11)/(
11*d) - (b^4*Tan[c + d*x]^13)/(13*d) + (b^4*Tan[c + d*x]^15)/(15*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^4(c+dx)\right)^4\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^4\right)^4}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b(2a+b)\left(2a^2+2ab+b^2\right)+b(2a+b)\left(2a^2+2ab+b^2\right)x^2-b^2\left(6a\right.\right.}{}$$
$$= -\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan^3(c+}{3d}$$
$$= (a+b)^4x-\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)}{3d}$$

Mathematica [A] time = 5.38, size = 196, normalized size = 0.91

$$\frac{b\tan(c+dx)\left(6435b\left(6a^2+4ab+b^2\right)\tan^6(c+dx)-9009b\left(6a^2+4ab+b^2\right)\tan^4(c+dx)+15015\left(4a^3+6a^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^4,x]
```

```
[Out] ((a + b)^4*ArcTan[Tan[c + d*x]])/d + (b*Tan[c + d*x]*(-45045*(4*a^3 + 6*a^2
*b + 4*a*b^2 + b^3) + 15015*(4*a^3 + 6*a^2*b + 4*a*b^2 + b^3)*Tan[c + d*x]^
2 - 9009*b*(6*a^2 + 4*a*b + b^2)*Tan[c + d*x]^4 + 6435*b*(6*a^2 + 4*a*b + b
^2)*Tan[c + d*x]^6 - 5005*b^2*(4*a + b)*Tan[c + d*x]^8 + 4095*b^2*(4*a + b)
*Tan[c + d*x]^10 - 3465*b^3*Tan[c + d*x]^12 + 3003*b^3*Tan[c + d*x]^14))/(4
5045*d)
```

fricas [A] time = 0.68, size = 225, normalized size = 1.04

$$\frac{3003\,b^4\tan\left(dx+c\right)^{15}-3465\,b^4\tan\left(dx+c\right)^{13}+4095\left(4\,ab^3+b^4\right)\tan\left(dx+c\right)^{11}-5005\left(4\,ab^3+b^4\right)\tan\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^4,x, algorithm="fricas")
```

```
[Out] 1/45045*(3003*b^4*tan(d*x + c)^15 - 3465*b^4*tan(d*x + c)^13 + 4095*(4*a*b^
3 + b^4)*tan(d*x + c)^11 - 5005*(4*a*b^3 + b^4)*tan(d*x + c)^9 + 6435*(6*a^
2*b^2 + 4*a*b^3 + b^4)*tan(d*x + c)^7 - 9009*(6*a^2*b^2 + 4*a*b^3 + b^4)*ta
n(d*x + c)^5 + 15015*(4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(d*x + c)^3 +
 45045*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x - 45045*(4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4)*tan(d*x + c))/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 412, normalized size = 1.91

$$-\frac{b^4\left(\tan^9\left(dx+c\right)\right)}{9d}+\frac{4\left(\tan^{11}\left(dx+c\right)\right)a\,b^3}{11d}-\frac{6\left(\tan^5\left(dx+c\right)\right)a^2b^2}{5d}+\frac{2\left(\tan^3\left(dx+c\right)\right)a^2b^2}{d}+\frac{4\left(\tan^7\left(dx+c\right)\right)a}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^4)^4,x)
```

```
[Out] 4/3/d*tan(d*x+c)^3*a*b^3+4/3/d*tan(d*x+c)^3*a^3*b-4/5/d*tan(d*x+c)^5*a*b^3+
4/7/d*tan(d*x+c)^7*a*b^3+6/7/d*tan(d*x+c)^7*a^2*b^2-4/9/d*tan(d*x+c)^9*a*b^
3+4/11/d*tan(d*x+c)^11*a*b^3+4/d*arctan(tan(d*x+c))*a^3*b+4/d*arctan(tan(d*
x+c))*a*b^3-6/5/d*tan(d*x+c)^5*a^2*b^2+2/d*tan(d*x+c)^3*a^2*b^2+6/d*arctan(
tan(d*x+c))*a^2*b^2-4/d*a^3*b*tan(d*x+c)-1/5/d*tan(d*x+c)^5*b^4-1/d*b^4*tan
(d*x+c)+1/d*arctan(tan(d*x+c))*a^4+1/d*arctan(tan(d*x+c))*b^4+1/3*b^4*tan(d
*x+c)^3/d-6*a^2*b^2*tan(d*x+c)/d-4*a*b^3*tan(d*x+c)/d-1/9*b^4*tan(d*x+c)^9/
d+1/11*b^4*tan(d*x+c)^11/d-1/13*b^4*tan(d*x+c)^13/d+1/15*b^4*tan(d*x+c)^15/
d+1/7*b^4*tan(d*x+c)^7/d
```

maxima [A] time = 0.79, size = 265, normalized size = 1.23

$$a^4x+\frac{4\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^3b}{3\,d}+\frac{2\left(15\,\tan(dx+c)^7-21\,\tan(dx+c)^5+35\,\tan(dx+c)^3\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 4/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^3*b/d + 2/35*
(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105*
c - 105*tan(d*x + c))*a^2*b^2/d + 4/3465*(315*tan(d*x + c)^11 - 385*tan(d*x
 + c)^9 + 495*tan(d*x + c)^7 - 693*tan(d*x + c)^5 + 1155*tan(d*x + c)^3 + 3
465*d*x + 3465*c - 3465*tan(d*x + c))*a*b^3/d + 1/45045*(3003*tan(d*x + c)^
15 - 3465*tan(d*x + c)^13 + 4095*tan(d*x + c)^11 - 5005*tan(d*x + c)^9 + 64
35*tan(d*x + c)^7 - 9009*tan(d*x + c)^5 + 15015*tan(d*x + c)^3 + 45045*d*x
+ 45045*c - 45045*tan(d*x + c))*b^4/d
```

mupad [B] time = 11.59, size = 271, normalized size = 1.25

$$\frac{\tan(c+d\,x)^3\,\left(\frac{4\,a^3\,b}{3}+2\,a^2\,b^2+\frac{4\,a\,b^3}{3}+\frac{b^4}{3}\right)}{d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a+b)^4}{a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4}\right)\,(a+b)^4}{d}-\frac{\tan(c+d\,x)\,\left(4\,a^3\,b+6\,a^2\,b^2\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^4)^4,x)
```

```
[Out] (tan(c + d*x)^3*((4*a*b^3)/3 + (4*a^3*b)/3 + b^4/3 + 2*a^2*b^2))/d + (atan(
(tan(c + d*x)*(a + b)^4)/(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))*(a +
b)^4)/d - (tan(c + d*x)*(4*a*b^3 + 4*a^3*b + b^4 + 6*a^2*b^2))/d - (b^4*tan
(c + d*x)^13)/(13*d) + (b^4*tan(c + d*x)^15)/(15*d) - (tan(c + d*x)^5*((4*a
*b^3)/5 + b^4/5 + (6*a^2*b^2)/5))/d + (tan(c + d*x)^7*((4*a*b^3)/7 + b^4/7
+ (6*a^2*b^2)/7))/d - (tan(c + d*x)^9*((4*a*b^3)/9 + b^4/9))/d + (tan(c + d
*x)^11*((4*a*b^3)/11 + b^4/11))/d
```

sympy [A] time = 7.07, size = 386, normalized size = 1.79

$$\begin{cases} a^4x+4a^3bx+\frac{4a^3b\tan^3(c+dx)}{3d}-\frac{4a^3b\tan(c+dx)}{d}+6a^2b^2x+\frac{6a^2b^2\tan^7(c+dx)}{7d}-\frac{6a^2b^2\tan^5(c+dx)}{5d}+\frac{2a^2b^2\tan^3(c+dx)}{d}-\frac{6a^2b^2\tan}{d} \\ x\left(a+b\tan^4(c)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)**4*b)**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*x + 4*a**3*b*tan(c + d*x)**3/(3*d) - 4*a**3*b*
tan(c + d*x)/d + 6*a**2*b**2*x + 6*a**2*b**2*tan(c + d*x)**7/(7*d) - 6*a**2
```

```
*b**2*tan(c + d*x)**5/(5*d) + 2*a**2*b**2*tan(c + d*x)**3/d - 6*a**2*b**2*t
an(c + d*x)/d + 4*a*b**3*x + 4*a*b**3*tan(c + d*x)**11/(11*d) - 4*a*b**3*ta
n(c + d*x)**9/(9*d) + 4*a*b**3*tan(c + d*x)**7/(7*d) - 4*a*b**3*tan(c + d*x
)**5/(5*d) + 4*a*b**3*tan(c + d*x)**3/(3*d) - 4*a*b**3*tan(c + d*x)/d + b**
4*x + b**4*tan(c + d*x)**15/(15*d) - b**4*tan(c + d*x)**13/(13*d) + b**4*ta
n(c + d*x)**11/(11*d) - b**4*tan(c + d*x)**9/(9*d) + b**4*tan(c + d*x)**7/(
7*d) - b**4*tan(c + d*x)**5/(5*d) + b**4*tan(c + d*x)**3/(3*d) - b**4*tan(c
 + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**4)**4, True))
```

3.382 $\int \left(a+b\tan^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=144

$$\frac{b\left(3a^2+3ab+b^2\right)\tan^3(c+dx)}{3d}-\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2(3a+b)\tan^7(c+dx)}{7d}-\frac{b^2(3a+b)\tan^5(c+dx}{5d}$$

```
[Out] (a+b)^3*x-b*(3*a^2+3*a*b+b^2)*tan(d*x+c)/d+1/3*b*(3*a^2+3*a*b+b^2)*tan(d*x+
c)^3/d-1/5*b^2*(3*a+b)*tan(d*x+c)^5/d+1/7*b^2*(3*a+b)*tan(d*x+c)^7/d-1/9*b^
3*tan(d*x+c)^9/d+1/11*b^3*tan(d*x+c)^11/d
```

Rubi [A] time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 1154, 203}

$$\frac{b\left(3a^2+3ab+b^2\right)\tan^3(c+dx)}{3d}-\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2(3a+b)\tan^7(c+dx)}{7d}-\frac{b^2(3a+b)\tan^5(c+dx}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^3,x]
```

```
[Out] (a + b)^3*x - (b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x])/d + (b*(3*a^2 + 3*a*b
+ b^2)*Tan[c + d*x]^3)/(3*d) - (b^2*(3*a + b)*Tan[c + d*x]^5)/(5*d) + (b^2*
(3*a + b)*Tan[c + d*x]^7)/(7*d) - (b^3*Tan[c + d*x]^9)/(9*d) + (b^3*Tan[c +
 d*x]^11)/(11*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^4(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+bx^4\right)^3}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int\left(-b\left(3a^2+3ab+b^2\right)+b\left(3a^2+3ab+b^2\right)x^2-b^2(3a+b)x^4+b^2(3a+\right.\right.}{d}$$
$$=-\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+\frac{b\left(3a^2+3ab+b^2\right)\tan^3(c+dx)}{3d}-\frac{b^2(3a+b)}{}$$
$$=(a+b)^3x-\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+\frac{b\left(3a^2+3ab+b^2\right)\tan^3(c+dx)}{3d}-$$

Mathematica [A] time = 1.07, size = 128, normalized size = 0.89

$$\frac{b\tan(c+dx)\left(1155\left(3a^2+3ab+b^2\right)\tan^2(c+dx)-3465\left(3a^2+3ab+b^2\right)+495b(3a+b)\tan^6(c+dx)-693b\right.}{3465d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^3,x]
```

```
[Out] ((a + b)^3*ArcTan[Tan[c + d*x]])/d + (b*Tan[c + d*x]*(-3465*(3*a^2 + 3*a*b
+ b^2) + 1155*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x]^2 - 693*b*(3*a + b)*Tan[c
+ d*x]^4 + 495*b*(3*a + b)*Tan[c + d*x]^6 - 385*b^2*Tan[c + d*x]^8 + 315*b^
2*Tan[c + d*x]^10))/(3465*d)
```

fricas [A] time = 0.87, size = 145, normalized size = 1.01

$$\frac{315\,b^3\tan(dx+c)^{11}-385\,b^3\tan(dx+c)^9+495\left(3\,ab^2+b^3\right)\tan(dx+c)^7-693\left(3\,ab^2+b^3\right)\tan(dx+c)^5+}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^3*tan(d*x + c)^11 - 385*b^3*tan(d*x + c)^9 + 495*(3*a*b^2 + b
^3)*tan(d*x + c)^7 - 693*(3*a*b^2 + b^3)*tan(d*x + c)^5 + 1155*(3*a^2*b + 3
*a*b^2 + b^3)*tan(d*x + c)^3 + 3465*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 3
465*(3*a^2*b + 3*a*b^2 + b^3)*tan(d*x + c))/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 252, normalized size = 1.75

$$\frac{b^3\left(\tan^{11}(dx+c)\right)}{11d}-\frac{b^3\left(\tan^9(dx+c)\right)}{9d}+\frac{3\left(\tan^7(dx+c)\right)a\,b^2}{7d}+\frac{\left(\tan^7(dx+c)\right)b^3}{7d}-\frac{3a\,b^2\left(\tan^5(dx+c)\right)}{5d}-\frac{b^3\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^4)^3,x)
```

```
[Out] 1/11*b^3*tan(d*x+c)^11/d-1/9*b^3*tan(d*x+c)^9/d+3/7/d*tan(d*x+c)^7*a*b^2+1/
7/d*tan(d*x+c)^7*b^3-3/5*a*b^2*tan(d*x+c)^5/d-1/5*b^3*tan(d*x+c)^5/d+1/d*ta
n(d*x+c)^3*a^2*b+a*b^2*tan(d*x+c)^3/d+1/3/d*b^3*tan(d*x+c)^3-3/d*a^2*b*tan(
d*x+c)-3*a*b^2*tan(d*x+c)/d-1/d*b^3*tan(d*x+c)+1/d*arctan(tan(d*x+c))*a^3+3
/d*arctan(tan(d*x+c))*a^2*b+3/d*arctan(tan(d*x+c))*a*b^2+1/d*arctan(tan(d*x
+c))*b^3
```

maxima [A] time = 0.77, size = 167, normalized size = 1.16

$$a^3x+\frac{\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^2b}{d}+\frac{\left(15\,\tan\left(dx+c\right)^7-21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+1\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2*b/d + 1/35*(15*
tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105*c -
105*tan(d*x + c))*a*b^2/d + 1/3465*(315*tan(d*x + c)^11 - 385*tan(d*x + c)^
9 + 495*tan(d*x + c)^7 - 693*tan(d*x + c)^5 + 1155*tan(d*x + c)^3 + 3465*d*
x + 3465*c - 3465*tan(d*x + c))*b^3/d
```

mupad [B] time = 11.66, size = 180, normalized size = 1.25

$$\frac{\tan\left(c+d\,x\right)^3\,\left(a^2\,b+a\,b^2+\frac{b^3}{3}\right)}{d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a+b)^3}{a^3+3\,a^2\,b+3\,a\,b^2+b^3}\right)\,\left(a+b\right)^3}{d}-\frac{b^3\,\tan\left(c+d\,x\right)^9}{9\,d}+\frac{b^3\,\tan\left(c+d\,x\right)^{11}}{11\,d}-\frac{\tan\left(c+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^4)^3,x)
```

```
[Out] (tan(c + d*x)^3*(a*b^2 + a^2*b + b^3/3))/d + (atan((tan(c + d*x)*(a + b)^3)
/(3*a*b^2 + 3*a^2*b + a^3 + b^3))*(a + b)^3)/d - (b^3*tan(c + d*x)^9)/(9*d)
 + (b^3*tan(c + d*x)^11)/(11*d) - (tan(c + d*x)*(3*a*b^2 + 3*a^2*b + b^3))/
d - (tan(c + d*x)^5*((3*a*b^2)/5 + b^3/5))/d + (tan(c + d*x)^7*((3*a*b^2)/7
 + b^3/7))/d
```

sympy [A] time = 3.18, size = 224, normalized size = 1.56

$$\begin{cases} a^3x+3a^2bx+\frac{a^2b\tan^3(c+dx)}{d}-\frac{3a^2b\tan(c+dx)}{d}+3ab^2x+\frac{3ab^2\tan^7(c+dx)}{7d}-\frac{3ab^2\tan^5(c+dx)}{5d}+\frac{ab^2\tan^3(c+dx)}{d}-\frac{3ab^2\tan(c+dx)}{d} \\ x\left(a+b\tan^4(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)**4*b)**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*x + a**2*b*tan(c + d*x)**3/d - 3*a**2*b*tan(c
+ d*x)/d + 3*a*b**2*x + 3*a*b**2*tan(c + d*x)**7/(7*d) - 3*a*b**2*tan(c + d
*x)**5/(5*d) + a*b**2*tan(c + d*x)**3/d - 3*a*b**2*tan(c + d*x)/d + b**3*x
+ b**3*tan(c + d*x)**11/(11*d) - b**3*tan(c + d*x)**9/(9*d) + b**3*tan(c +
d*x)**7/(7*d) - b**3*tan(c + d*x)**5/(5*d) + b**3*tan(c + d*x)**3/(3*d) - b
**3*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**4)**3, True))
```

3.383 $\int \left(a+b\tan^4(c+dx)\right)^2 dx$

Optimal. Leaf size=82

$$\frac{b(2a+b)\tan^3(c+dx)}{3d}-\frac{b(2a+b)\tan(c+dx)}{d}+x(a+b)^2+\frac{b^2\tan^7(c+dx)}{7d}-\frac{b^2\tan^5(c+dx)}{5d}$$

```
[Out] (a+b)^2*x-b*(2*a+b)*tan(d*x+c)/d+1/3*b*(2*a+b)*tan(d*x+c)^3/d-1/5*b^2*tan(d
*x+c)^5/d+1/7*b^2*tan(d*x+c)^7/d
```

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3661, 1154, 203}

$$\frac{b(2a+b)\tan^3(c+dx)}{3d}-\frac{b(2a+b)\tan(c+dx)}{d}+x(a+b)^2+\frac{b^2\tan^7(c+dx)}{7d}-\frac{b^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^2,x]
```

```
[Out] (a + b)^2*x - (b*(2*a + b)*Tan[c + d*x])/d + (b*(2*a + b)*Tan[c + d*x]^3)/(
3*d) - (b^2*Tan[c + d*x]^5)/(5*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tan^4(c+dx)\right)^2 dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^4\right)^2}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-b(2a+b)+b(2a+b)x^2-b^2x^4+b^2x^6+\frac{(a+b)^2}{1+x^2}\right)dx,x,\tan(c+dx)\right)}{d}\\
&= -\frac{b(2a+b)\tan(c+dx)}{d}+\frac{b(2a+b)\tan^3(c+dx)}{3d}-\frac{b^2\tan^5(c+dx)}{5d}+\frac{b^2\tan^7(c+dx)}{7d}\\
&= (a+b)^2x-\frac{b(2a+b)\tan(c+dx)}{d}+\frac{b(2a+b)\tan^3(c+dx)}{3d}-\frac{b^2\tan^5(c+dx)}{5d}+
\end{aligned}$$

Mathematica [A] time = 0.58, size = 75, normalized size = 0.91

$$\frac{105(a+b)^2\tan^{-1}(\tan(c+dx))+b\tan(c+dx)\left(35(2a+b)\tan^2(c+dx)-105(2a+b)+15b\tan^6(c+dx)-21b\text{ ta}\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^2,x]
```

```
[Out] (105*(a + b)^2*ArcTan[Tan[c + d*x]] + b*Tan[c + d*x]*(-105*(2*a + b) + 35*(
2*a + b)*Tan[c + d*x]^2 - 21*b*Tan[c + d*x]^4 + 15*b*Tan[c + d*x]^6))/(105*
d)
```

fricas [A] time = 0.62, size = 81, normalized size = 0.99

$$\frac{15\,b^2\tan\left(dx+c\right)^7-21\,b^2\tan\left(dx+c\right)^5+35\left(2\,ab+b^2\right)\tan\left(dx+c\right)^3+105\left(a^2+2\,ab+b^2\right)dx-105\left(2\,ab+b^2\right)\text{t}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 - 21*b^2*tan(d*x + c)^5 + 35*(2*a*b + b^2)*tan
(d*x + c)^3 + 105*(a^2 + 2*a*b + b^2)*d*x - 105*(2*a*b + b^2)*tan(d*x + c))
/d
```

giac [B] time = 31.86, size = 1181, normalized size = 14.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^2,x, algorithm="giac")
```

```
[Out] 1/105*(105*a^2*d*x*tan(d*x)^7*tan(c)^7 + 210*a*b*d*x*tan(d*x)^7*tan(c)^7 +
105*b^2*d*x*tan(d*x)^7*tan(c)^7 - 735*a^2*d*x*tan(d*x)^6*tan(c)^6 - 1470*a*
b*d*x*tan(d*x)^6*tan(c)^6 - 735*b^2*d*x*tan(d*x)^6*tan(c)^6 + 210*a*b*tan(d
*x)^7*tan(c)^6 + 105*b^2*tan(d*x)^7*tan(c)^6 + 210*a*b*tan(d*x)^6*tan(c)^7
+ 105*b^2*tan(d*x)^6*tan(c)^7 + 2205*a^2*d*x*tan(d*x)^5*tan(c)^5 + 4410*a*b
*d*x*tan(d*x)^5*tan(c)^5 + 2205*b^2*d*x*tan(d*x)^5*tan(c)^5 - 70*a*b*tan(d*
x)^7*tan(c)^4 - 35*b^2*tan(d*x)^7*tan(c)^4 - 1470*a*b*tan(d*x)^6*tan(c)^5 -
 735*b^2*tan(d*x)^6*tan(c)^5 - 1470*a*b*tan(d*x)^5*tan(c)^6 - 735*b^2*tan(d
*x)^5*tan(c)^6 - 70*a*b*tan(d*x)^4*tan(c)^7 - 35*b^2*tan(d*x)^4*tan(c)^7 -
3675*a^2*d*x*tan(d*x)^4*tan(c)^4 - 7350*a*b*d*x*tan(d*x)^4*tan(c)^4 - 3675*
b^2*d*x*tan(d*x)^4*tan(c)^4 + 21*b^2*tan(d*x)^7*tan(c)^2 + 280*a*b*tan(d*x)
^6*tan(c)^3 + 245*b^2*tan(d*x)^6*tan(c)^3 + 3990*a*b*tan(d*x)^5*tan(c)^4 +
2205*b^2*tan(d*x)^5*tan(c)^4 + 3990*a*b*tan(d*x)^4*tan(c)^5 + 2205*b^2*tan(
d*x)^4*tan(c)^5 + 280*a*b*tan(d*x)^3*tan(c)^6 + 245*b^2*tan(d*x)^3*tan(c)^6
 + 21*b^2*tan(d*x)^2*tan(c)^7 + 3675*a^2*d*x*tan(d*x)^3*tan(c)^3 + 7350*a*b
*d*x*tan(d*x)^3*tan(c)^3 + 3675*b^2*d*x*tan(d*x)^3*tan(c)^3 - 15*b^2*tan(d*
x)^7 - 147*b^2*tan(d*x)^6*tan(c) - 420*a*b*tan(d*x)^5*tan(c)^2 - 735*b^2*ta
n(d*x)^5*tan(c)^2 - 5460*a*b*tan(d*x)^4*tan(c)^3 - 3675*b^2*tan(d*x)^4*tan(
c)^3 - 5460*a*b*tan(d*x)^3*tan(c)^4 - 3675*b^2*tan(d*x)^3*tan(c)^4 - 420*a*
b*tan(d*x)^2*tan(c)^5 - 735*b^2*tan(d*x)^2*tan(c)^5 - 147*b^2*tan(d*x)*tan(
c)^6 - 15*b^2*tan(c)^7 - 2205*a^2*d*x*tan(d*x)^2*tan(c)^2 - 4410*a*b*d*x*ta
n(d*x)^2*tan(c)^2 - 2205*b^2*d*x*tan(d*x)^2*tan(c)^2 + 21*b^2*tan(d*x)^5 +
280*a*b*tan(d*x)^4*tan(c) + 245*b^2*tan(d*x)^4*tan(c) + 3990*a*b*tan(d*x)^3
*tan(c)^2 + 2205*b^2*tan(d*x)^3*tan(c)^2 + 3990*a*b*tan(d*x)^2*tan(c)^3 + 2
205*b^2*tan(d*x)^2*tan(c)^3 + 280*a*b*tan(d*x)*tan(c)^4 + 245*b^2*tan(d*x)*
tan(c)^4 + 21*b^2*tan(c)^5 + 735*a^2*d*x*tan(d*x)*tan(c) + 1470*a*b*d*x*tan
(d*x)*tan(c) + 735*b^2*d*x*tan(d*x)*tan(c) - 70*a*b*tan(d*x)^3 - 35*b^2*tan
(d*x)^3 - 1470*a*b*tan(d*x)^2*tan(c) - 735*b^2*tan(d*x)^2*tan(c) - 1470*a*b
```

```
*tan(d*x)*tan(c)^2 - 735*b^2*tan(d*x)*tan(c)^2 - 70*a*b*tan(c)^3 - 35*b^2*t
an(c)^3 - 105*a^2*d*x - 210*a*b*d*x - 105*b^2*d*x + 210*a*b*tan(d*x) + 105*
b^2*tan(d*x) + 210*a*b*tan(c) + 105*b^2*tan(c))/(d*tan(d*x)^7*tan(c)^7 - 7*
d*tan(d*x)^6*tan(c)^6 + 21*d*tan(d*x)^5*tan(c)^5 - 35*d*tan(d*x)^4*tan(c)^4
 + 35*d*tan(d*x)^3*tan(c)^3 - 21*d*tan(d*x)^2*tan(c)^2 + 7*d*tan(d*x)*tan(c
) - d)
```

maple [A] time = 0.03, size = 134, normalized size = 1.63

$$\frac{b^2\left(\tan^7(dx+c)\right)}{7d}-\frac{b^2\left(\tan^5(dx+c)\right)}{5d}+\frac{2ab\left(\tan^3(dx+c)\right)}{3d}+\frac{b^2\left(\tan^3(dx+c)\right)}{3d}-\frac{2ab\tan(dx+c)}{d}-\frac{b^2\tan(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^4)^2,x)
```

```
[Out] 1/7*b^2*tan(d*x+c)^7/d-1/5*b^2*tan(d*x+c)^5/d+2/3*a*b*tan(d*x+c)^3/d+1/3*b^
2*tan(d*x+c)^3/d-2*a*b*tan(d*x+c)/d-b^2*tan(d*x+c)/d+1/d*arctan(tan(d*x+c))
*a^2+2/d*arctan(tan(d*x+c))*a*b+1/d*arctan(tan(d*x+c))*b^2
```

maxima [A] time = 0.59, size = 91, normalized size = 1.11

$$a^2x+\frac{2\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)ab}{3\,d}+\frac{\left(15\,\tan(dx+c)^7-21\,\tan(dx+c)^5+35\,\tan(dx+c)^3\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a*b/d + 1/105*(
15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105*c
 - 105*tan(d*x + c))*b^2/d
```

mupad [B] time = 11.47, size = 109, normalized size = 1.33

$$\frac{\tan(c+d\,x)^3\,\left(\frac{b^2}{3}+\frac{2\,a\,b}{3}\right)}{d}-\frac{b^2\tan(c+d\,x)^5}{5\,d}+\frac{b^2\tan(c+d\,x)^7}{7\,d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a+b)^2}{a^2+2\,a\,b+b^2}\right)\,(a+b)^2}{d}-\frac{\tan(c+d\,x)\,(b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^4)^2,x)
```

```
[Out] (tan(c + d*x)^3*((2*a*b)/3 + b^2/3))/d - (b^2*tan(c + d*x)^5)/(5*d) + (b^2*
tan(c + d*x)^7)/(7*d) + (atan((tan(c + d*x)*(a + b)^2)/(2*a*b + a^2 + b^2))
*(a + b)^2)/d - (tan(c + d*x)*(2*a*b + b^2))/d
```

sympy [A] time = 1.08, size = 116, normalized size = 1.41

$$\begin{cases} a^2x+2abx+\frac{2ab\tan^3(c+dx)}{3d}-\frac{2ab\tan(c+dx)}{d}+b^2x+\frac{b^2\tan^7(c+dx)}{7d}-\frac{b^2\tan^5(c+dx)}{5d}+\frac{b^2\tan^3(c+dx)}{3d}-\frac{b^2\tan(c+dx)}{d} & \text{for } d \\ x\left(a+b\tan^4(c)\right)^2 & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)**4*b)**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x + 2*a*b*tan(c + d*x)**3/(3*d) - 2*a*b*tan(c + d
*x)/d + b**2*x + b**2*tan(c + d*x)**7/(7*d) - b**2*tan(c + d*x)**5/(5*d) +
b**2*tan(c + d*x)**3/(3*d) - b**2*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(
c)**4)**2, True))
```

3.384 $\int \left(a+b\tan^4(c+dx)\right)\,dx$

Optimal. Leaf size=35

$$ax+\frac{b\tan^3(c+dx)}{3d}-\frac{b\tan(c+dx)}{d}+bx$$

[Out] a*x+b*x-b*tan(d*x+c)/d+1/3*b*tan(d*x+c)^3/d

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3473, 8}

$$ax+\frac{b\tan^3(c+dx)}{3d}-\frac{b\tan(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

[In] Int[a + b*Tan[c + d*x]^4,x]

[Out] a*x + b*x - (b*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tan^4(c+dx)\right)\,dx &= ax+b\int \tan^4(c+dx)\,dx\\
&= ax+\frac{b\tan^3(c+dx)}{3d}-b\int \tan^2(c+dx)\,dx\\
&= ax-\frac{b\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}+b\int 1\,dx\\
&= ax+bx-\frac{b\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 1.26

$$ax+\frac{b\tan^{-1}(\tan(c+dx))}{d}+\frac{b\tan^3(c+dx)}{3d}-\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Tan[c + d*x]^4,x]

[Out] a*x + (b*ArcTan[Tan[c + d*x]])/d - (b*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)

fricas [A] time = 0.64, size = 32, normalized size = 0.91

$$\frac{b\tan\left(dx+c\right)^3+3\,(a+b)dx-3\,b\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+tan(d*x+c)^4*b,x, algorithm="fricas")
```

```
[Out] 1/3*(b*tan(d*x + c)^3 + 3*(a + b)*d*x - 3*b*tan(d*x + c))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+tan(d*x+c)^4*b,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)b*(12*d*x*tan(c)^3*tan(d*x)^
3-36*d*x*tan(c)^2*tan(d*x)^2+36*d*x*tan(c)*tan(d*x)-12*d*x-3*pi*sign(2*tan(
c)^2*tan(d*x)+2*tan(c)*tan(d*x)^2-2*tan(c)-2*tan(d*x))*tan(c)^3*tan(d*x)^3+
9*pi*sign(2*tan(c)^2*tan(d*x)+2*tan(c)*tan(d*x)^2-2*tan(c)-2*tan(d*x))*tan(
c)^2*tan(d*x)^2-9*pi*sign(2*tan(c)^2*tan(d*x)+2*tan(c)*tan(d*x)^2-2*tan(c)-
2*tan(d*x))*tan(c)*tan(d*x)+3*pi*sign(2*tan(c)^2*tan(d*x)+2*tan(c)*tan(d*x)
^2-2*tan(c)-2*tan(d*x))-3*pi*tan(c)^3*tan(d*x)^3+9*pi*tan(c)^2*tan(d*x)^2-9
*pi*tan(c)*tan(d*x)+3*pi+6*atan((tan(c)*tan(d*x)-1)/(tan(c)+tan(d*x)))*tan(
c)^3*tan(d*x)^3-18*atan((tan(c)*tan(d*x)-1)/(tan(c)+tan(d*x)))*tan(c)^2*tan
(d*x)^2+18*atan((tan(c)*tan(d*x)-1)/(tan(c)+tan(d*x)))*tan(c)*tan(d*x)-6*at
an((tan(c)*tan(d*x)-1)/(tan(c)+tan(d*x)))+6*atan((tan(c)+tan(d*x))/(tan(c)*
tan(d*x)-1))*tan(c)^3*tan(d*x)^3-18*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)
-1))*tan(c)^2*tan(d*x)^2+18*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan
(c)*tan(d*x)-6*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))+12*tan(c)^3*tan(
d*x)^2-4*tan(c)^3+12*tan(c)^2*tan(d*x)^3-36*tan(c)^2*tan(d*x)-36*tan(c)*tan
(d*x)^2+12*tan(c)-4*tan(d*x)^3+12*tan(d*x))/(12*d*tan(c)^3*tan(d*x)^3-36*d*
tan(c)^2*tan(d*x)^2+36*d*tan(c)*tan(d*x)-12*d)+a*x
```

maple [A] time = 0.02, size = 43, normalized size = 1.23

$$ax+\frac{b\left(\tan^{3}\left(dx+c\right)\right)}{3d}-\frac{b\tan\left(dx+c\right)}{d}+\frac{b\arctan\left(\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(d*x+c)^4,x)
```

```
[Out] a*x+1/3*b*tan(d*x+c)^3/d-b*tan(d*x+c)/d+1/d*b*arctan(tan(d*x+c))
```

maxima [A] time = 0.46, size = 34, normalized size = 0.97

$$ax+\frac{\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)b}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+tan(d*x+c)^4*b,x, algorithm="maxima")
```

```
[Out] a*x + 1/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*b/d
```

mupad [B] time = 11.59, size = 31, normalized size = 0.89

$$\frac{\frac{b\tan(c+d\,x)^3}{3}-b\tan\left(c+d\,x\right)+d\,x\,\left(a+b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*tan(c + d*x)^4,x)
```

```
[Out] ((b*tan(c + d*x)^3)/3 - b*tan(c + d*x) + d*x*(a + b))/d
```

sympy [A] time = 0.22, size = 32, normalized size = 0.91

$$ax + b\left(\begin{cases} x + \frac{\tan^3(c+dx)}{3d} - \frac{\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\tan^4(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+tan(d*x+c)**4*b,x)
```

```
[Out] a*x + b*Piecewise((x + tan(c + d*x)**3/(3*d) - tan(c + d*x)/d, Ne(d, 0)), (
x*tan(c)**4, True))
```

3.385 $\int \frac{1}{a+b\tan^4(c+dx)}\,dx$

Optimal. Leaf size=302

$$\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}d(a+b)}-\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}d(a+b)}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\right)\log\left(-\right.}{}$$

```
[Out] x/(a+b)+1/4*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)-b
^(1/2))/a^(3/4)/(a+b)/d*2^(1/2)-1/4*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*tan(d*
x+c)/a^(1/4))*(a^(1/2)-b^(1/2))/a^(3/4)/(a+b)/d*2^(1/2)-1/8*b^(1/4)*ln(a^(1
/2)-a^(1/4)*b^(1/4)*2^(1/2)*tan(d*x+c)+b^(1/2)*tan(d*x+c)^2)*(a^(1/2)+b^(1/
2))/a^(3/4)/(a+b)/d*2^(1/2)+1/8*b^(1/4)*ln(a^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*
tan(d*x+c)+b^(1/2)*tan(d*x+c)^2)*(a^(1/2)+b^(1/2))/a^(3/4)/(a+b)/d*2^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3661, 1171, 203, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}d(a+b)}-\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}d(a+b)}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\right)\log\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^(-1),x]
```

```
[Out] x/(a + b) + ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c
+ d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)*d) - ((Sqrt[a] - Sqrt[b])*b^(1
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(a + b)*d) - ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)*d)
+ ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c
+ d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{a+b\tan^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^4)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(1+x^2)} + \frac{b-bx^2}{(a+b)(a+bx^4)}\right)dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a+b)d} + \frac{\text{Subst}\left(\int \frac{b-bx^2}{a+bx^4}\,dx, x, \tan(c+dx)\right)}{(a+b)d}$$
$$= \frac{x}{a+b} - \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx, x, \tan(c+dx)\right)}{2(a+b)d} + \frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{}}{a+}\right.}{2(a}$$
$$= \frac{x}{a+b} - \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \tan(c+dx)\right)}{4(a+b)d} - \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int\right.}{}$$
$$= \frac{x}{a+b} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+dx)\right)}{4\sqrt{2}a^{3/4}(a+b)d} + \left(\right.$$
$$= \frac{x}{a+b} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(a+b)d} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{}}{}\right.}{2\sqrt{2}a^{3/4}(a+b)d}$$

Mathematica [A] time = 0.57, size = 228, normalized size = 0.75

$$\frac{8a^{3/4}\tan^{-1}(\tan(c+dx))+\sqrt{2}\sqrt[4]{b}\left(2\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)-2\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\tan(c+}{\sqrt[4]{a}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^(-1),x]
```

```
[Out] (8*a^(3/4)*ArcTan[Tan[c + d*x]] + Sqrt[2]*b^(1/4)*(2*(Sqrt[a] - Sqrt[b])*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)] - 2*(Sqrt[a] - Sqrt[b])*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)] - (Sqrt[a] + Sqrt[b])*(Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2] -
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^
2])))/(8*a^(3/4)*(a + b)*d)
```

fricas [B] time = 0.74, size = 1541, normalized size = 5.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b),x, algorithm="fricas")
```

```
[Out] 1/8*((a + b)*sqrt(((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3
)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 + 2
*a^2*b + a*b^2)*d^2))*log((2*(a^3 - a*b^2)*d*sqrt(((a^3 + 2*a^2*b + a*b^2)*
d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 +
 a^3*b^4)*d^4)) + 2*b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*tan(d*x + c) + (a*b -
 b^2)*tan(d*x + c)^2 + a^2 - a*b + ((a^4 + 2*a^3*b + a^2*b^2)*d^2*tan(d*x +
 c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d^2)*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7
```

```
+ 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)))/(tan(d*x + c)^2 + 1)) -
 (a + b)*sqrt(((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 + 2*a^2
*b + a*b^2)*d^2))*log(-(2*(a^3 - a*b^2)*d*sqrt(((a^3 + 2*a^2*b + a*b^2)*d^2
*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^
3*b^4)*d^4)) + 2*b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*tan(d*x + c) - (a*b - b^
2)*tan(d*x + c)^2 - a^2 + a*b - ((a^4 + 2*a^3*b + a^2*b^2)*d^2*tan(d*x + c)
^2 - (a^4 + 2*a^3*b + a^2*b^2)*d^2)*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4
*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)))/(tan(d*x + c)^2 + 1)) + (a
 + b)*sqrt(-((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^
7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) - 2*b)/((a^3 + 2*a^2*b
 + a*b^2)*d^2))*log(-(2*(a^3 - a*b^2)*d*sqrt(-((a^3 + 2*a^2*b + a*b^2)*d^2*
sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3
*b^4)*d^4)) - 2*b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*tan(d*x + c) + (a*b - b^2
)*tan(d*x + c)^2 + a^2 - a*b - ((a^4 + 2*a^3*b + a^2*b^2)*d^2*tan(d*x + c)^
2 - (a^4 + 2*a^3*b + a^2*b^2)*d^2)*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*
a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)))/(tan(d*x + c)^2 + 1)) - (a
+ b)*sqrt(-((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7
 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) - 2*b)/((a^3 + 2*a^2*b
+ a*b^2)*d^2))*log((2*(a^3 - a*b^2)*d*sqrt(-((a^3 + 2*a^2*b + a*b^2)*d^2*sq
rt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b
^4)*d^4)) - 2*b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*tan(d*x + c) - (a*b - b^2)*
tan(d*x + c)^2 - a^2 + a*b + ((a^4 + 2*a^3*b + a^2*b^2)*d^2*tan(d*x + c)^2
- (a^4 + 2*a^3*b + a^2*b^2)*d^2)*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^
6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)))/(tan(d*x + c)^2 + 1)) + 8*x)/
(a + b)
```

giac [A] time = 3.19, size = 354, normalized size = 1.17

$$\frac{\frac{2\left(\left(ab^3\right)^{\frac{1}{4}}b^2-\left(ab^3\right)^{\frac{3}{4}}\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\tan\left(dx+c\right)\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{\sqrt{2}a^2b^2+\sqrt{2}ab^3}+\frac{2\left(\left(ab^3\right)^{\frac{1}{4}}b^2-\left(ab^3\right)^{\frac{3}{4}}\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\tan\left(dx+c\right)\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{\sqrt{2}a^2b^2+\sqrt{2}ab^3}+}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b),x, algorithm="giac")
```

```
[Out] 1/4*(2*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*(pi*floor((d*x + c)/pi + 1/2) +
arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*tan(d*x + c))/(a/b)^(1/4)))/(sq
rt(2)*a^2*b^2 + sqrt(2)*a*b^3) + 2*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*(pi*
floor((d*x + c)/pi + 1/2) + arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*ta
n(d*x + c))/(a/b)^(1/4)))/(sqrt(2)*a^2*b^2 + sqrt(2)*a*b^3) + ((a*b^3)^(1/4
)*b^2 + (a*b^3)^(3/4))*log(tan(d*x + c)^2 + sqrt(2)*(a/b)^(1/4)*tan(d*x + c
) + sqrt(a/b))/(sqrt(2)*a^2*b^2 + sqrt(2)*a*b^3) - ((a*b^3)^(1/4)*b^2 + (a*
b^3)^(3/4))*log(tan(d*x + c)^2 - sqrt(2)*(a/b)^(1/4)*tan(d*x + c) + sqrt(a/
b))/(sqrt(2)*a^2*b^2 + sqrt(2)*a*b^3) + 4*(d*x + c)/(a + b))/d
```

maple [A] time = 0.21, size = 374, normalized size = 1.24

$$\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{\tan^2(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)\sqrt{2}+\sqrt{\frac{a}{b}}}{\tan^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{8d\,(a+b)\,a}+\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\tan(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4d\,(a+b)\,a}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\tan(}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{4d\,(a+b)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^4),x)
```

```
[Out] 1/8/d*b/(a+b)*(1/b*a)^(1/4)/a*2^(1/2)*ln((tan(d*x+c)^2+(1/b*a)^(1/4)*tan(d*
x+c)*2^(1/2)+(1/b*a)^(1/2))/(tan(d*x+c)^2-(1/b*a)^(1/4)*tan(d*x+c)*2^(1/2)+
(1/b*a)^(1/2)))+1/4/d*b/(a+b)*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a
)^(1/4)*tan(d*x+c)+1)-1/4/d*b/(a+b)*(1/b*a)^(1/4)/a*2^(1/2)*arctan(-2^(1/2)
/(1/b*a)^(1/4)*tan(d*x+c)+1)-1/8/d/(a+b)/(1/b*a)^(1/4)*2^(1/2)*ln((tan(d*x+
c)^2-(1/b*a)^(1/4)*tan(d*x+c)*2^(1/2)+(1/b*a)^(1/2))/(tan(d*x+c)^2+(1/b*a)^
(1/4)*tan(d*x+c)*2^(1/2)+(1/b*a)^(1/2)))-1/4/d/(a+b)/(1/b*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/b*a)^(1/4)*tan(d*x+c)+1)+1/4/d/(a+b)/(1/b*a)^(1/4)*2^(1/2
)*arctan(-2^(1/2)/(1/b*a)^(1/4)*tan(d*x+c)+1)+1/d/(a+b)*arctan(tan(d*x+c))
```

maxima [A] time = 0.86, size = 261, normalized size = 0.86

$$-\frac{\frac{b\left(\frac{2\sqrt{2}\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\tan(dx+c)+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\tan(dx+c)-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{b}\tan(dx+c)^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right.}{a^{\frac{3}{4}}b^{\frac{3}{4}}}\right.}{a+b}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b),x, algorithm="maxima")
```

```
[Out] -1/8*(b*(2*sqrt(2)*(sqrt(a) - sqrt(b))*arctan(1/2*sqrt(2)*(2*sqrt(b)*tan(d*
x + c) + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*(sqrt(a) - sqrt(b))*arctan(1/2*sqrt(2)*(2
*sqrt(b)*tan(d*x + c) - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sq
rt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*(sqrt(a) + sqrt(b))*log(sqrt
(b)*tan(d*x + c)^2 + sqrt(2)*a^(1/4)*b^(1/4)*tan(d*x + c) + sqrt(a))/(a^(3/
4)*b^(3/4)) + sqrt(2)*(sqrt(a) + sqrt(b))*log(sqrt(b)*tan(d*x + c)^2 - sqrt
(2)*a^(1/4)*b^(1/4)*tan(d*x + c) + sqrt(a))/(a^(3/4)*b^(3/4)))/(a + b) - 8*
(d*x + c)/(a + b))/d
```

mupad [B] time = 15.04, size = 4038, normalized size = 13.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^4),x)
```

```
[Out] (2*atan(((((20*a*b^5 + 4*b^6 - ((((128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 + 2
56*a^4*b^4 - (tan(c + d*x)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 512*a
^5*b^4)*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) + tan(c + d*x)*(32*a*b^6 + 16*b^7
- 240*a^2*b^5))*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) - 6*b^5*tan(c + d*x))/(2*a
 + 2*b) - (((20*a*b^5 + 4*b^6 - ((((128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 +
256*a^4*b^4 + (tan(c + d*x)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 512*
a^5*b^4)*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) - tan(c + d*x)*(32*a*b^6 + 16*b^7
 - 240*a^2*b^5))*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) + 6*b^5*tan(c + d*x))/(2*
a + 2*b))/(((((20*a*b^5 + 4*b^6 - ((((128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5
+ 256*a^4*b^4 - (tan(c + d*x)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 51
2*a^5*b^4)*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) + tan(c + d*x)*(32*a*b^6 + 16*b
^7 - 240*a^2*b^5))*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) - 6*b^5*tan(c + d*x))*1
i)/(2*a + 2*b) + ((((20*a*b^5 + 4*b^6 - ((((128*a^2*b^6 - 64*a*b^7 + 448*a^
3*b^5 + 256*a^4*b^4 + (tan(c + d*x)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^
5 - 512*a^5*b^4)*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) - tan(c + d*x)*(32*a*b^6
+ 16*b^7 - 240*a^2*b^5))*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) + 6*b^5*tan(c + d
*x))*1i)/(2*a + 2*b))))/(d*(2*a + 2*b)) - (atan((((20*a*b^5 - (((2*a^2*b +
a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*
```

```
(128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 + 256*a^4*b^4 + tan(c + d*x)*((2*a^2*
b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1
/2)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 512*a^5*b^4)) - tan(c + d*x)
*(32*a*b^6 + 16*b^7 - 240*a^2*b^5))*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*
b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) + 4*b^6)*((2*a^2*b + a*(-a^
3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) + 6*b^
5*tan(c + d*x))*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4
*b + a^5 + a^3*b^2)))^(1/2)*1i - ((20*a*b^5 - (((2*a^2*b + a*(-a^3*b)^(1/2)
 - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*(128*a^2*b^6 - 6
4*a*b^7 + 448*a^3*b^5 + 256*a^4*b^4 - tan(c + d*x)*((2*a^2*b + a*(-a^3*b)^(
1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*(512*a^2*b^7
 + 512*a^3*b^6 - 512*a^4*b^5 - 512*a^5*b^4)) + tan(c + d*x)*(32*a*b^6 + 16*
b^7 - 240*a^2*b^5))*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2
*a^4*b + a^5 + a^3*b^2)))^(1/2) + 4*b^6)*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(
-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) - 6*b^5*tan(c + d*x))*
((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b
^2)))^(1/2)*1i)/(((20*a*b^5 - (((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1
/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*(128*a^2*b^6 - 64*a*b^7 + 448*a^
3*b^5 + 256*a^4*b^4 + tan(c + d*x)*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b
)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*(512*a^2*b^7 + 512*a^3*b^6 -
 512*a^4*b^5 - 512*a^5*b^4)) - tan(c + d*x)*(32*a*b^6 + 16*b^7 - 240*a^2*b^
5))*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a
^3*b^2)))^(1/2) + 4*b^6)*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(
16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) + 6*b^5*tan(c + d*x))*((2*a^2*b + a*(-
a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) + ((
20*a*b^5 - (((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b +
 a^5 + a^3*b^2)))^(1/2)*(128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 + 256*a^4*b^4
 - tan(c + d*x)*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4
*b + a^5 + a^3*b^2)))^(1/2)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 512*
a^5*b^4)) + tan(c + d*x)*(32*a*b^6 + 16*b^7 - 240*a^2*b^5))*((2*a^2*b + a*(
-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) + 4
*b^6)*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 +
 a^3*b^2)))^(1/2) - 6*b^5*tan(c + d*x))*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-
a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)))*((2*a^2*b + a*(-a^3*b
)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*2i)/d - (
atan((((20*a*b^5 - (((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2
*a^4*b + a^5 + a^3*b^2)))^(1/2)*(128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 + 256
*a^4*b^4 + tan(c + d*x)*((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(1
6*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^
5 - 512*a^5*b^4)) - tan(c + d*x)*(32*a*b^6 + 16*b^7 - 240*a^2*b^5))*((2*a^2
*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(
1/2) + 4*b^6)*((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b
 + a^5 + a^3*b^2)))^(1/2) + 6*b^5*tan(c + d*x))*((2*a^2*b - a*(-a^3*b)^(1/2
) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*1i - ((20*a*b^5
 - (((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a
^3*b^2)))^(1/2)*(128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 + 256*a^4*b^4 - tan(c
 + d*x)*((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5
 + a^3*b^2)))^(1/2)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 512*a^5*b^4)
) + tan(c + d*x)*(32*a*b^6 + 16*b^7 - 240*a^2*b^5))*((2*a^2*b - a*(-a^3*b)^
(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) + 4*b^6)*((
2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2
)))^(1/2) - 6*b^5*tan(c + d*x))*((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(
1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*1i)/(((20*a*b^5 - (((2*a^2*b -
a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*
(128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 + 256*a^4*b^4 + tan(c + d*x)*((2*a^2*
b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1
/2)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 512*a^5*b^4)) - tan(c + d*x)
*(32*a*b^6 + 16*b^7 - 240*a^2*b^5))*((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*
b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) + 4*b^6)*((2*a^2*b - a*(-a^
```

```
3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) + 6*b^
5*tan(c + d*x))*((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4
*b + a^5 + a^3*b^2)))^(1/2) + ((20*a*b^5 - (((2*a^2*b - a*(-a^3*b)^(1/2) +
b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*(128*a^2*b^6 - 64*a
*b^7 + 448*a^3*b^5 + 256*a^4*b^4 - tan(c + d*x)*((2*a^2*b - a*(-a^3*b)^(1/2
) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*(512*a^2*b^7 +
512*a^3*b^6 - 512*a^4*b^5 - 512*a^5*b^4)) + tan(c + d*x)*(32*a*b^6 + 16*b^7
 - 240*a^2*b^5))*((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^
4*b + a^5 + a^3*b^2)))^(1/2) + 4*b^6)*((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^
3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) - 6*b^5*tan(c + d*x))*((2
*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)
))^(1/2)))*((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b +
a^5 + a^3*b^2)))^(1/2)*2i)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\tan^4(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)**4*b),x)
```

```
[Out] Integral(1/(a + b*tan(c + d*x)**4), x)
```

3.386 $\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=648

$$\frac{\sqrt[4]{b}\left(\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}d(a+b)}+\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}d(a+b)^2}-\frac{\sqrt[4]{b}\left(\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\right.}{8\sqrt{2}\,a^{7/4}d(}$$

```
[Out] x/(a+b)^2+1/16*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2
)-3*b^(1/2))/a^(7/4)/(a+b)/d*2^(1/2)-1/16*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*
tan(d*x+c)/a^(1/4))*(a^(1/2)-3*b^(1/2))/a^(7/4)/(a+b)/d*2^(1/2)+1/4*b^(1/4)
*arctan(1-b^(1/4)*2^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/2))/a^(3/4)/(a+
b)^2/d*2^(1/2)-1/4*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*tan(d*x+c)/a^(1/4))*(a^
(1/2)-b^(1/2))/a^(3/4)/(a+b)^2/d*2^(1/2)-1/8*b^(1/4)*ln(a^(1/2)-a^(1/4)*b^(
1/4)*2^(1/2)*tan(d*x+c)+b^(1/2)*tan(d*x+c)^2)*(a^(1/2)+b^(1/2))/a^(3/4)/(a+
b)^2/d*2^(1/2)+1/8*b^(1/4)*ln(a^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*tan(d*x+c)+b^
(1/2)*tan(d*x+c)^2)*(a^(1/2)+b^(1/2))/a^(3/4)/(a+b)^2/d*2^(1/2)-1/32*b^(1/4
)*ln(a^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*tan(d*x+c)+b^(1/2)*tan(d*x+c)^2)*(a^(1
/2)+3*b^(1/2))/a^(7/4)/(a+b)/d*2^(1/2)+1/32*b^(1/4)*ln(a^(1/2)+a^(1/4)*b^(1
/4)*2^(1/2)*tan(d*x+c)+b^(1/2)*tan(d*x+c)^2)*(a^(1/2)+3*b^(1/2))/a^(7/4)/(a
+b)/d*2^(1/2)+1/4*b*tan(d*x+c)*(1-tan(d*x+c)^2)/a/(a+b)/d/(a+b*tan(d*x+c)^4
)
```

Rubi [A] time = 0.66, antiderivative size = 648, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3661, 1239, 203, 1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{b}\left(\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}d(a+b)}+\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}d(a+b)^2}-\frac{\sqrt[4]{b}\left(\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\right.}{8\sqrt{2}\,a^{7/4}d(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^(-2),x]
```

```
[Out] x/(a + b)^2 + ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[
c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)^2*d) + ((Sqrt[a] - 3*Sqrt[b]
)*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*(a + b)*d) - ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)^2*d) - ((Sqrt[a] - 3*Sq
rt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(8*Sqrt[
2]*a^(7/4)*(a + b)*d) - ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*
(a + b)^2*d) - ((Sqrt[a] + 3*Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(16*Sqrt[2]*a^(7/4)*(a + b
)*d) + ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*T
an[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)^2*d) + ((
Sqrt[a] + 3*Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c +
d*x] + Sqrt[b]*Tan[c + d*x]^2])/(16*Sqrt[2]*a^(7/4)*(a + b)*d) + (b*Tan[c +
 d*x]*(1 - Tan[c + d*x]^2))/(4*a*(a + b)*d*(a + b*Tan[c + d*x]^4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1239

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
```

```
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+bx^4\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^2(1+x^2)} + \frac{b-bx^2}{(a+b)\left(a+bx^4\right)^2} + \frac{b-bx^2}{(a+b)^2\left(a+bx^4\right)}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a+b)^2d} + \frac{\text{Subst}\left(\int \frac{b-bx^2}{a+bx^4}\,dx, x, \tan(c+dx)\right)}{(a+b)^2d} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{x}{(a+b)^2} + \frac{b\tan(c+dx)\left(1-\tan^2(c+dx)\right)}{4a(a+b)d\left(a+b\tan^4(c+dx)\right)} - \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx, x, \text{ta}\right.}{2(a+b)^2d}$$

$$= \frac{x}{(a+b)^2} + \frac{b\tan(c+dx)\left(1-\tan^2(c+dx)\right)}{4a(a+b)d\left(a+b\tan^4(c+dx)\right)} - \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,\right.}{4(a+b)^2d}$$

$$= \frac{x}{(a+b)^2} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+dx)\right)}{4\sqrt{2}a^{3/4}(a+b)^2d} -$$

$$= \frac{x}{(a+b)^2} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(a+b)^2d} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1+\right.}{2\sqrt{2}a^{3/4}(a+b}$$

$$= \frac{x}{(a+b)^2} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(a+b)^2d} + \frac{\left(\sqrt{a}-3\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1\right.}{8\sqrt{2}a^{7/4}(a+}$$

Mathematica [C] time = 6.29, size = 598, normalized size = 0.92

$$-\frac{b\tan^3(c+dx)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{b\tan^4(c+dx)}{a}\right)}{3a^2d(a+b)} - \frac{3\left(\frac{2\left(\frac{\sqrt{2}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}} - \frac{\sqrt{2}b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{a}}\right)}{\sqrt{a}} + \frac{\sqrt{2}b^{3/4}\log\left(-\sqrt{2}\right.}{}\right.}{32ad(a+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^(-2),x]
```

```
[Out] ArcTan[Tan[c + d*x]]/((a + b)^2*d) + ((Sqrt[a] - Sqrt[b])*((Sqrt[2]*b^(1/4)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/a^(1/4) - (Sqrt[2]*b^(
1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/a^(1/4)))/(4*Sqrt[
a]*(a + b)^2*d) - ((Sqrt[a] + Sqrt[b])*((Sqrt[2]*b^(1/4)*Log[Sqrt[a] - Sqrt
```

```
[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/a^(1/4) - (Sqrt
[2]*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Ta
n[c + d*x]^2])/a^(1/4)))/(8*Sqrt[a]*(a + b)^2*d) - (3*((2*((Sqrt[2]*b^(3/4)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/a^(1/4) - (Sqrt[2]*b^(
3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/a^(1/4)))/Sqrt[a]
+ ((Sqrt[2]*b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sq
rt[b]*Tan[c + d*x]^2])/a^(1/4) - (Sqrt[2]*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/a^(1/4))/Sqrt[a]))/(32
*a*(a + b)*d) - (b*Hypergeometric2F1[3/4, 2, 7/4, -((b*Tan[c + d*x]^4)/a)]*
Tan[c + d*x]^3)/(3*a^2*(a + b)*d) + (b*Tan[c + d*x])/(4*a*(a + b)*d*(a + b*
Tan[c + d*x]^4))
```

fricas [B] time = 1.15, size = 4291, normalized size = 6.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(32*a*b*d*x*tan(d*x + c)^4 + 32*a^2*d*x - 8*(a*b + b^2)*tan(d*x + c)^3
 + ((a^3*b + 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^4 + (a^4 + 2*a^3*b + a^2*b^2
)*d)*sqrt(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625
*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b
^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 +
56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)) + 70*a^2*b + 44*a*b^2
 + 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2))*log((625
*a^5 - 750*a^4*b - 1376*a^3*b^2 - 594*a^2*b^3 - 81*a*b^4 + (625*a^4*b - 750
*a^3*b^2 - 1376*a^2*b^3 - 594*a*b^4 - 81*b^5)*tan(d*x + c)^2 + 2*(2*(a^11 +
 5*a^10*b + 10*a^9*b^2 + 10*a^8*b^3 + 5*a^7*b^4 + a^6*b^5)*d^3*sqrt(-(625*a
^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6
 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56
*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4))*tan(d*x + c) + (125*a^7
 + 5*a^6*b - 442*a^5*b^2 - 490*a^4*b^3 - 195*a^3*b^4 - 27*a^2*b^5)*d*tan(d*
x + c))*sqrt(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(
625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*
a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4
 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)) + 70*a^2*b + 44*a*
b^2 + 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2)) + ((2
5*a^9 + 109*a^8*b + 186*a^7*b^2 + 154*a^6*b^3 + 61*a^5*b^4 + 9*a^4*b^5)*d^2
*tan(d*x + c)^2 - (25*a^9 + 109*a^8*b + 186*a^7*b^2 + 154*a^6*b^3 + 61*a^5*
b^4 + 9*a^4*b^5)*d^2)*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*
a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^
2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*
b^8)*d^4)))/(tan(d*x + c)^2 + 1)) - ((a^3*b + 2*a^2*b^2 + a*b^3)*d*tan(d*x
+ c)^4 + (a^4 + 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*
a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748
*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b
^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7
*b^8)*d^4)) + 70*a^2*b + 44*a*b^2 + 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*
a^4*b^3 + a^3*b^4)*d^2))*log((625*a^5 - 750*a^4*b - 1376*a^3*b^2 - 594*a^2*
b^3 - 81*a*b^4 + (625*a^4*b - 750*a^3*b^2 - 1376*a^2*b^3 - 594*a*b^4 - 81*b
^5)*tan(d*x + c)^2 - 2*(2*(a^11 + 5*a^10*b + 10*a^9*b^2 + 10*a^8*b^3 + 5*a^
7*b^4 + a^6*b^5)*d^3*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a
^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2
 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b
^8)*d^4))*tan(d*x + c) + (125*a^7 + 5*a^6*b - 442*a^5*b^2 - 490*a^4*b^3 - 1
95*a^3*b^4 - 27*a^2*b^5)*d*tan(d*x + c))*sqrt(((a^7 + 4*a^6*b + 6*a^5*b^2 +
 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2
748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^1
3*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 +
```

```
a^7*b^8)*d^4)) + 70*a^2*b + 44*a*b^2 + 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 +
 4*a^4*b^3 + a^3*b^4)*d^2)) + ((25*a^9 + 109*a^8*b + 186*a^7*b^2 + 154*a^6*
b^3 + 61*a^5*b^4 + 9*a^4*b^5)*d^2*tan(d*x + c)^2 - (25*a^9 + 109*a^8*b + 18
6*a^7*b^2 + 154*a^6*b^3 + 61*a^5*b^4 + 9*a^4*b^5)*d^2)*sqrt(-(625*a^6*b - 1
950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^
7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^
5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)))/(tan(d*x + c)^2 + 1)) - ((a^3*
b + 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^4 + (a^4 + 2*a^3*b + a^2*b^2)*d)*sqrt
(-((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625*a^6*b -
 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*
b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*
b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)) - 70*a^2*b - 44*a*b^2 - 6*b^3
)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2))*log(-(625*a^5 -
750*a^4*b - 1376*a^3*b^2 - 594*a^2*b^3 - 81*a*b^4 + (625*a^4*b - 750*a^3*b^
2 - 1376*a^2*b^3 - 594*a*b^4 - 81*b^5)*tan(d*x + c)^2 + 2*(2*(a^11 + 5*a^10
*b + 10*a^9*b^2 + 10*a^8*b^3 + 5*a^7*b^4 + a^6*b^5)*d^3*sqrt(-(625*a^6*b -
1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b
^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b
^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4))*tan(d*x + c) - (125*a^7 + 5*a^
6*b - 442*a^5*b^2 - 490*a^4*b^3 - 195*a^3*b^4 - 27*a^2*b^5)*d*tan(d*x + c))
*sqrt(-((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625*a^
6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6
+ 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*
a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)) - 70*a^2*b - 44*a*b^2 -
6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2)) - ((25*a^9
+ 109*a^8*b + 186*a^7*b^2 + 154*a^6*b^3 + 61*a^5*b^4 + 9*a^4*b^5)*d^2*tan(d
*x + c)^2 - (25*a^9 + 109*a^8*b + 186*a^7*b^2 + 154*a^6*b^3 + 61*a^5*b^4 +
9*a^4*b^5)*d^2)*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^
4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56
*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d
^4)))/(tan(d*x + c)^2 + 1)) + ((a^3*b + 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^4
 + (a^4 + 2*a^3*b + a^2*b^2)*d)*sqrt(-((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b
^3 + a^3*b^4)*d^2*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*
b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 +
56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)
*d^4)) - 70*a^2*b - 44*a*b^2 - 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b
^3 + a^3*b^4)*d^2))*log(-(625*a^5 - 750*a^4*b - 1376*a^3*b^2 - 594*a^2*b^3
- 81*a*b^4 + (625*a^4*b - 750*a^3*b^2 - 1376*a^2*b^3 - 594*a*b^4 - 81*b^5)*
tan(d*x + c)^2 - 2*(2*(a^11 + 5*a^10*b + 10*a^9*b^2 + 10*a^8*b^3 + 5*a^7*b^
4 + a^6*b^5)*d^3*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b
^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 5
6*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*
d^4))*tan(d*x + c) - (125*a^7 + 5*a^6*b - 442*a^5*b^2 - 490*a^4*b^3 - 195*a
^3*b^4 - 27*a^2*b^5)*d*tan(d*x + c))*sqrt(-((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*
a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748
*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b
^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7
*b^8)*d^4)) - 70*a^2*b - 44*a*b^2 - 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*
a^4*b^3 + a^3*b^4)*d^2)) - ((25*a^9 + 109*a^8*b + 186*a^7*b^2 + 154*a^6*b^3
 + 61*a^5*b^4 + 9*a^4*b^5)*d^2*tan(d*x + c)^2 - (25*a^9 + 109*a^8*b + 186*a
^7*b^2 + 154*a^6*b^3 + 61*a^5*b^4 + 9*a^4*b^5)*d^2)*sqrt(-(625*a^6*b - 1950
*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/
((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 +
 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)))/(tan(d*x + c)^2 + 1)) + 8*(a*b +
b^2)*tan(d*x + c))/((a^3*b + 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^4 + (a^4 + 2
*a^3*b + a^2*b^2)*d)
```

giac [A] time = 3.41, size = 517, normalized size = 0.80

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\tan(dx+c)\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)\left(\left(ab^{3}\right)^{\frac{3}{4}}(5a+b)-\left(ab^{3}\right)^{\frac{1}{4}}\left(7ab^{2}+3b^{3}\right)\right)}{\sqrt{2}a^{4}b^{2}+2\sqrt{2}a^{3}b^{3}+\sqrt{2}a^{2}b^{4}}+\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\tan(dx+c)\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{\sqrt{2}a^{4}b^{2}+2\sqrt{2}a^{3}b^{3}+\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*(pi*floor((d*x + c)/pi + 1/2) + arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^
(1/4) + 2*tan(d*x + c))/(a/b)^(1/4)))*((a*b^3)^(3/4)*(5*a + b) - (a*b^3)^(1
/4)*(7*a*b^2 + 3*b^3))/(sqrt(2)*a^4*b^2 + 2*sqrt(2)*a^3*b^3 + sqrt(2)*a^2*b
^4) + 2*(pi*floor((d*x + c)/pi + 1/2) + arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^
(1/4) - 2*tan(d*x + c))/(a/b)^(1/4)))*((a*b^3)^(3/4)*(5*a + b) - (a*b^3)^(1
/4)*(7*a*b^2 + 3*b^3))/(sqrt(2)*a^4*b^2 + 2*sqrt(2)*a^3*b^3 + sqrt(2)*a^2*b
^4) - ((a*b^3)^(3/4)*(5*a + b) + (a*b^3)^(1/4)*(7*a*b^2 + 3*b^3))*log(tan(d
*x + c)^2 + sqrt(2)*(a/b)^(1/4)*tan(d*x + c) + sqrt(a/b))/(sqrt(2)*a^4*b^2
+ 2*sqrt(2)*a^3*b^3 + sqrt(2)*a^2*b^4) + ((a*b^3)^(3/4)*(5*a + b) + (a*b^3)
^(1/4)*(7*a*b^2 + 3*b^3))*log(tan(d*x + c)^2 - sqrt(2)*(a/b)^(1/4)*tan(d*x
+ c) + sqrt(a/b))/(sqrt(2)*a^4*b^2 + 2*sqrt(2)*a^3*b^3 + sqrt(2)*a^2*b^4) -
 16*(d*x + c)/(a^2 + 2*a*b + b^2) + 4*(b*tan(d*x + c)^3 - b*tan(d*x + c))/(
(b*tan(d*x + c)^4 + a)*(a^2 + a*b)))/d
```

maple [A] time = 0.21, size = 886, normalized size = 1.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^4)^2,x)
```

```
[Out] -1/4/d*b/(a+b)^2/(a+b*tan(d*x+c)^4)*tan(d*x+c)^3-1/4/d*b^2/(a+b)^2/(a+b*tan
(d*x+c)^4)/a*tan(d*x+c)^3+1/4/d*b/(a+b)^2/(a+b*tan(d*x+c)^4)*tan(d*x+c)+1/4
/d*b^2/(a+b)^2/(a+b*tan(d*x+c)^4)/a*tan(d*x+c)+7/32/d*b/(a+b)^2/a*(1/b*a)^(
1/4)*2^(1/2)*ln((tan(d*x+c)^2+(1/b*a)^(1/4)*tan(d*x+c)*2^(1/2)+(1/b*a)^(1/2
))/(tan(d*x+c)^2-(1/b*a)^(1/4)*tan(d*x+c)*2^(1/2)+(1/b*a)^(1/2)))+3/32/d*b^
2/(a+b)^2/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((tan(d*x+c)^2+(1/b*a)^(1/4)*tan(d*x+
c)*2^(1/2)+(1/b*a)^(1/2))/(tan(d*x+c)^2-(1/b*a)^(1/4)*tan(d*x+c)*2^(1/2)+(1
/b*a)^(1/2)))-7/16/d*b/(a+b)^2/a*(1/b*a)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(1/b
*a)^(1/4)*tan(d*x+c)+1)-3/16/d*b^2/(a+b)^2/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan
(-2^(1/2)/(1/b*a)^(1/4)*tan(d*x+c)+1)+7/16/d*b/(a+b)^2/a*(1/b*a)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*tan(d*x+c)+1)+3/16/d*b^2/(a+b)^2/a^2*(1/b*
a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*tan(d*x+c)+1)-5/32/d/(a+b)^2/
(1/b*a)^(1/4)*2^(1/2)*ln((tan(d*x+c)^2-(1/b*a)^(1/4)*tan(d*x+c)*2^(1/2)+(1/
b*a)^(1/2))/(tan(d*x+c)^2+(1/b*a)^(1/4)*tan(d*x+c)*2^(1/2)+(1/b*a)^(1/2)))-
1/32/d*b/(a+b)^2/a/(1/b*a)^(1/4)*2^(1/2)*ln((tan(d*x+c)^2-(1/b*a)^(1/4)*tan
(d*x+c)*2^(1/2)+(1/b*a)^(1/2))/(tan(d*x+c)^2+(1/b*a)^(1/4)*tan(d*x+c)*2^(1/
2)+(1/b*a)^(1/2)))+5/16/d/(a+b)^2/(1/b*a)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(1/
b*a)^(1/4)*tan(d*x+c)+1)+1/16/d*b/(a+b)^2/a/(1/b*a)^(1/4)*2^(1/2)*arctan(-2
^(1/2)/(1/b*a)^(1/4)*tan(d*x+c)+1)-5/16/d/(a+b)^2/(1/b*a)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(1/b*a)^(1/4)*tan(d*x+c)+1)-1/16/d*b/(a+b)^2/a/(1/b*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*tan(d*x+c)+1)+1/d/(a+b)^2*arctan(tan(d*x
+c))
```

maxima [A] time = 0.80, size = 394, normalized size = 0.61

$$-\frac{b\left(\frac{2\sqrt{2}\left(b\left(\sqrt{a}-3\sqrt{b}\right)+5a^{\frac{3}{2}}-7a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\tan(dx+c)+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\left(b\left(\sqrt{a}-3\sqrt{b}\right)+5a^{\frac{3}{2}}-7a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\tan(dx+c)-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\left(b\left(\sqrt{a}+3\right.\right.}{}\right.}{a^3+2a^2b+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(b*(2*sqrt(2)*(b*(sqrt(a) - 3*sqrt(b)) + 5*a^(3/2) - 7*a*sqrt(b))*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*tan(d*x + c) + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*(b*(sqrt
(a) - 3*sqrt(b)) + 5*a^(3/2) - 7*a*sqrt(b))*arctan(1/2*sqrt(2)*(2*sqrt(b)*t
an(d*x + c) - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt
(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*(b*(sqrt(a) + 3*sqrt(b)) + 5*a^(3/2) +
 7*a*sqrt(b))*log(sqrt(b)*tan(d*x + c)^2 + sqrt(2)*a^(1/4)*b^(1/4)*tan(d*x
+ c) + sqrt(a))/(a^(3/4)*b^(3/4)) + sqrt(2)*(b*(sqrt(a) + 3*sqrt(b)) + 5*a^
(3/2) + 7*a*sqrt(b))*log(sqrt(b)*tan(d*x + c)^2 - sqrt(2)*a^(1/4)*b^(1/4)*t
an(d*x + c) + sqrt(a))/(a^(3/4)*b^(3/4)))/(a^3 + 2*a^2*b + a*b^2) + 8*(b*ta
n(d*x + c)^3 - b*tan(d*x + c))/((a^2*b + a*b^2)*tan(d*x + c)^4 + a^3 + a^2*
b) - 32*(d*x + c)/(a^2 + 2*a*b + b^2))/d
```

mupad [B] time = 15.69, size = 11516, normalized size = 17.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^4)^2,x)
```

```
[Out] ((b*tan(c + d*x))/(4*a*(a + b)) - (b*tan(c + d*x)^3)/(4*a*(a + b)))/(d*(a +
 b*tan(c + d*x)^4)) - (2*atan(((((((((((960*a^7*b^8 - 224*a^5*b^10 - 144*a^6
*b^9 - 48*a^4*b^11 + 2480*a^8*b^7 + 2592*a^9*b^6 + 1296*a^10*b^5 + 256*a^11
*b^4)*1i)/(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2) - (tan(c + d*x)
*(65536*a^6*b^11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 - 3276
80*a^10*b^7 - 589824*a^11*b^6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(128*(4*
a*b + 2*a^2 + 2*b^2)*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i
)/(4*a*b + 2*a^2 + 2*b^2) - (tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*b^10 +
20352*a^4*b^9 + 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49024*a^8*b
^5)*1i)/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(4*a*b
 + 2*a^2 + 2*b^2) - (((45*a*b^10)/16 + (305*a^2*b^9)/16 + (385*a^3*b^8)/8 +
 (657*a^4*b^7)/8 + (2081*a^5*b^6)/16 + (1277*a^6*b^5)/16)*1i)/(4*a^7*b + a^
8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2))/(4*a*b + 2*a^2 + 2*b^2) - (tan(c + d*
x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^6 + 1425*a^4*b^5))/(128*
(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/(4*a*b + 2*a^2 + 2*b^2)
 - (((((((960*a^7*b^8 - 224*a^5*b^10 - 144*a^6*b^9 - 48*a^4*b^11 + 2480*a^
8*b^7 + 2592*a^9*b^6 + 1296*a^10*b^5 + 256*a^11*b^4)*1i)/(4*a^7*b + a^8 + a
^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2) + (tan(c + d*x)*(65536*a^6*b^11 + 327680*a^
7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 - 327680*a^10*b^7 - 589824*a^11*b^
6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(128*(4*a*b + 2*a^2 + 2*b^2)*(4*a^7*
b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(4*a*b + 2*a^2 + 2*b^2) +
(tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*b^10 + 20352*a^4*b^9 + 8704*a^5*b^8
 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49024*a^8*b^5)*1i)/(128*(4*a^7*b + a^8
+ a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(4*a*b + 2*a^2 + 2*b^2) - (((45*a*
```

b^10)/16 + (305*a^2*b^9)/16 + (385*a^3*b^8)/8 + (657*a^4*b^7)/8 + (2081*a^5*b^6)/16 + (1277*a^6*b^5)/16)*1i)/(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2))/(4*a*b + 2*a^2 + 2*b^2) + (tan(c + d*x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^6 + 1425*a^4*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/(4*a*b + 2*a^2 + 2*b^2))/((((((((((960*a^7*b^8 - 224*a^5*b^10 - 144*a^6*b^9 - 48*a^4*b^11 + 2480*a^8*b^7 + 2592*a^9*b^6 + 1296*a^10*b^5 + 256*a^11*b^4)*1i)/(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2) - (tan(c + d*x)*(65536*a^6*b^11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 - 327680*a^10*b^7 - 589824*a^11*b^6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(128*(4*a*b + 2*a^2 + 2*b^2)*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(4*a*b + 2*a^2 + 2*b^2) - (tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*b^10 + 20352*a^4*b^9 + 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49024*a^8*b^5)*1i)/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(4*a*b + 2*a^2 + 2*b^2) - (((45*a*b^10)/16 + (305*a^2*b^9)/16 + (385*a^3*b^8)/8 + (657*a^4*b^7)/8 + (2081*a^5*b^6)/16 + (1277*a^6*b^5)/16)*1i)/(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2))*1i)/(4*a*b + 2*a^2 + 2*b^2) - (tan(c + d*x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^6 + 1425*a^4*b^5)*1i)/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/(4*a*b + 2*a^2 + 2*b^2) + ((((((((960*a^7*b^8 - 224*a^5*b^10 - 144*a^6*b^9 - 48*a^4*b^11 + 2480*a^8*b^7 + 2592*a^9*b^6 + 1296*a^10*b^5 + 256*a^11*b^4)*1i)/(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2) + (tan(c + d*x)*(65536*a^6*b^11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 - 327680*a^10*b^7 - 589824*a^11*b^6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(128*(4*a*b + 2*a^2 + 2*b^2)*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(4*a*b + 2*a^2 + 2*b^2) + (tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*b^10 + 20352*a^4*b^9 + 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49024*a^8*b^5)*1i)/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(4*a*b + 2*a^2 + 2*b^2) - (((45*a*b^10)/16 + (305*a^2*b^9)/16 + (385*a^3*b^8)/8 + (657*a^4*b^7)/8 + (2081*a^5*b^6)/16 + (1277*a^6*b^5)/16)*1i)/(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2))*1i)/(4*a*b + 2*a^2 + 2*b^2) + (tan(c + d*x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^6 + 1425*a^4*b^5)*1i)/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/(4*a*b + 2*a^2 + 2*b^2) + ((135*a*b^6)/64 + (81*b^7)/128 + (125*a^2*b^5)/128)/(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2))))/(d*(4*a*b + 2*a^2 + 2*b^2)) + (atan((((((((245760*a^7*b^8 - 57344*a^5*b^10 - 36864*a^6*b^9 - 12288*a^4*b^11 + 634880*a^8*b^7 + 663552*a^9*b^6 + 331776*a^10*b^5 + 65536*a^11*b^4)/(256*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (tan(c + d*x)*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2)*(65536*a^6*b^11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 - 327680*a^10*b^7 - 589824*a^11*b^6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) + (tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*b^10 + 20352*a^4*b^9 + 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49024*a^8*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) - (720*a*b^10 + 4880*a^2*b^9 + 12320*a^3*b^8 + 21024*a^4*b^7 + 33296*a^5*b^6 + 20432*a^6*b^5)/(256*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) + (tan(c + d*x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^6 + 1425*a^4*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 +

```
6*a^9*b^2)))^(1/2)*1i - (((((245760*a^7*b^8 - 57344*a^5*b^10 - 36864*a^6*b^
9 - 12288*a^4*b^11 + 634880*a^8*b^7 + 663552*a^9*b^6 + 331776*a^10*b^5 + 65
536*a^11*b^4)/(256*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) + (ta
n(c + d*x)*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^
4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(25
6*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2)*(65536*a^6*b^
11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 - 327680*a^10*b^7 -
589824*a^11*b^6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(128*(4*a^7*b + a^8 +
a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)
^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a
^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^
2)))^(1/2) - (tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*b^10 + 20352*a^4*b^9 +
 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49024*a^8*b^5))/(128*(4*a^
7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((9*b^3*(-a^7*b)^(1/2) - 25*
a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^
(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^
3 + 6*a^9*b^2)))^(1/2) - (720*a*b^10 + 4880*a^2*b^9 + 12320*a^3*b^8 + 21024
*a^4*b^7 + 33296*a^5*b^6 + 20432*a^6*b^5)/(256*(4*a^7*b + a^8 + a^4*b^4 + 4
*a^5*b^3 + 6*a^6*b^2)))*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70
*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*
b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2)
- (tan(c + d*x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^6 + 1425*a^
4*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((9*b^3*(-
a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 +
41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 +
a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2)*1i)/((((((245760*a^7*b^8 - 57344*a
^5*b^10 - 36864*a^6*b^9 - 12288*a^4*b^11 + 634880*a^8*b^7 + 663552*a^9*b^6
+ 331776*a^10*b^5 + 65536*a^11*b^4)/(256*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b
^3 + 6*a^6*b^2)) - (tan(c + d*x)*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(
1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2
*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)
))^(1/2)*(65536*a^6*b^11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^
8 - 327680*a^10*b^7 - 589824*a^11*b^6 - 327680*a^12*b^5 - 65536*a^13*b^4))/
(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((9*b^3*(-a^7*b)^(
1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2
*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4
+ 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) + (tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*
b^10 + 20352*a^4*b^9 + 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a^7*b^6 - 4902
4*a^8*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((9*b^
3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^
2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^1
1 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) - (720*a*b^10 + 4880*a^2*b^9 +
 12320*a^3*b^8 + 21024*a^4*b^7 + 33296*a^5*b^6 + 20432*a^6*b^5)/(256*(4*a^7
*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((9*b^3*(-a^7*b)^(1/2) - 25*a
^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(
1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3
 + 6*a^9*b^2)))^(1/2) + (tan(c + d*x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 +
2532*a^3*b^6 + 1425*a^4*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6
*a^6*b^2)))*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a
^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(2
56*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) + (((((24576
0*a^7*b^8 - 57344*a^5*b^10 - 36864*a^6*b^9 - 12288*a^4*b^11 + 634880*a^8*b^
7 + 663552*a^9*b^6 + 331776*a^10*b^5 + 65536*a^11*b^4)/(256*(4*a^7*b + a^8
+ a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) + (tan(c + d*x)*((9*b^3*(-a^7*b)^(1/2)
- 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^
7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a
^8*b^3 + 6*a^9*b^2)))^(1/2)*(65536*a^6*b^11 + 327680*a^7*b^10 + 589824*a^8*
b^9 + 327680*a^9*b^8 - 327680*a^10*b^7 - 589824*a^11*b^6 - 327680*a^12*b^5
- 65536*a^13*b^4))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))
```

*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) - (tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*b^10 + 20352*a^4*b^9 + 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49024*a^8*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) - (720*a*b^10 + 4880*a^2*b^9 + 12320*a^3*b^8 + 21024*a^4*b^7 + 33296*a^5*b^6 + 20432*a^6*b^5)/(256*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) - (tan(c + d*x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^6 + 1425*a^4*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) + (270*a*b^6 + 81*b^7 + 125*a^2*b^5)/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2))))*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 41*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2)*2i)/d + (atan(((((((245760*a^7*b^8 - 57344*a^5*b^10 - 36864*a^6*b^9 - 12288*a^4*b^11 + 634880*a^8*b^7 + 663552*a^9*b^6 + 331776*a^10*b^5 + 65536*a^11*b^4)/(256*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (tan(c + d*x)*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2)*(65536*a^6*b^11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 - 327680*a^10*b^7 - 589824*a^11*b^6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) + (tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*b^10 + 20352*a^4*b^9 + 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49024*a^8*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) - (720*a*b^10 + 4880*a^2*b^9 + 12320*a^3*b^8 + 21024*a^4*b^7 + 33296*a^5*b^6 + 20432*a^6*b^5)/(256*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) + (tan(c + d*x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^6 + 1425*a^4*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2)*1i - (((((245760*a^7*b^8 - 57344*a^5*b^10 - 36864*a^6*b^9 - 12288*a^4*b^11 + 634880*a^8*b^7 + 663552*a^9*b^6 + 331776*a^10*b^5 + 65536*a^11*b^4)/(256*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) + (tan(c + d*x)*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2)*(65536*a^6*b^11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 - 327680*a^10*b^7 - 589824*a^11*b^6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) - (tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*b^10 + 20352*a^4*b^9 + 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49024*a^8*

b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) - (720*a*b^10 + 4880*a^2*b^9 + 12320*a^3*b^8 + 21024*a^4*b^7 + 33296*a^5*b^6 + 20432*a^6*b^5)/(256*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) - (tan(c + d*x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^6 + 1425*a^4*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2)*1i)/((((((245760*a^7*b^8 - 57344*a^5*b^10 - 36864*a^6*b^9 - 12288*a^4*b^11 + 634880*a^8*b^7 + 663552*a^9*b^6 + 331776*a^10*b^5 + 65536*a^11*b^4)/(256*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (tan(c + d*x)*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2)*(65536*a^6*b^11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 - 327680*a^10*b^7 - 589824*a^11*b^6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) + (tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*b^10 + 20352*a^4*b^9 + 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49024*a^8*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) - (720*a*b^10 + 4880*a^2*b^9 + 12320*a^3*b^8 + 21024*a^4*b^7 + 33296*a^5*b^6 + 20432*a^6*b^5)/(256*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) + (tan(c + d*x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^6 + 1425*a^4*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) + (((((245760*a^7*b^8 - 57344*a^5*b^10 - 36864*a^6*b^9 - 12288*a^4*b^11 + 634880*a^8*b^7 + 663552*a^9*b^6 + 331776*a^10*b^5 + 65536*a^11*b^4)/(256*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) + (tan(c + d*x)*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2)*(65536*a^6*b^11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 - 327680*a^10*b^7 - 589824*a^11*b^6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) - (tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*b^10 + 20352*a^4*b^9 + 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49024*a^8*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) - (720*a*b^10 + 4880*a^2*b^9 + 12320*a^3*b^8 + 21024*a^4*b^7 + 33296*a^5*b^6 + 20432*a^6*b^5)/(256*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2) - (tan(c + d*x)*(6

```
12*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^6 + 1425*a^4*b^5))/(128*(4*a^
7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*((25*a^3*(-a^7*b)^(1/2) - 9*
b^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^
(1/2) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^
3 + 6*a^9*b^2)))^(1/2) + (270*a*b^6 + 81*b^7 + 125*a^2*b^5)/(128*(4*a^7*b +
 a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2))))*((25*a^3*(-a^7*b)^(1/2) - 9*b^3*
(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 - 41*a*b^2*(-a^7*b)^(1/2
) - 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a^7*b^4 + 4*a^8*b^3 +
6*a^9*b^2)))^(1/2)*2i)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)**4*b)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**4)**(-2), x)
```

3.387 $\int \sqrt{a+b\tan^4(c+dx)}\,dx$

Optimal. Leaf size=650

$$\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2d}+\frac{\sqrt{b}\,\tan(c+dx)\sqrt{a+b\tan^4(c+dx)}}{d\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)}-\frac{\sqrt[4]{b}\,(a+b)\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)\sqrt{\frac{a\cdots}{\left(\sqrt{a}+\cdots\right.}}}{2\sqrt[4]{a}\,d\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\cdots}}$$

```
[Out] 1/2*arctan((a+b)^(1/2)*tan(d*x+c)/(a+b*tan(d*x+c)^4)^(1/2))*(a+b)^(1/2)/d+b
^(1/2)*(a+b*tan(d*x+c)^4)^(1/2)*tan(d*x+c)/d/(a^(1/2)+b^(1/2)*tan(d*x+c)^2)
-a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*tan(d*x+
c)/a^(1/4))),1/2*2^(1/2))*((a+b*tan(d*x+c)^4)/(a^(1/2)+b^(1/2)*tan(d*x+c)^2
)^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(d*x+c)^2)/d/(a+b*tan(d*x+c)^4)^(1/2)-1/2*b^
(1/4)*(a+b)*(cos(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*tan(d*x+c)/a^
(1/4))),1/2*2^(1/2))*((a+b*tan(d*x+c)^4)/(a^(1/2)+b^(1/2)*tan(d*x+c)^2)^2)^
(1/2)*(a^(1/2)+b^(1/2)*tan(d*x+c)^2)/a^(1/4)/d/(a^(1/2)-b^(1/2))/(a+b*tan(d
*x+c)^4)^(1/2)+1/2*b^(1/4)*(cos(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4
)*tan(d*x+c)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)-b^(1/2))*((a+b*tan(d*x+c)^4)/(
a^(1/2)+b^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(d*x+c)^2)/a^(1/
4)/d/(a+b*tan(d*x+c)^4)^(1/2)+1/4*(a+b)*(cos(2*arctan(b^(1/4)*tan(d*x+c)/a^
(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticPi(sin(2
*arctan(b^(1/4)*tan(d*x+c)/a^(1/4))),-1/4*(a^(1/2)-b^(1/2))^2/a^(1/2)/b^(1/
2),1/2*2^(1/2))*(a^(1/2)+b^(1/2))*((a+b*tan(d*x+c)^4)/(a^(1/2)+b^(1/2)*tan(
d*x+c)^2)^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(d*x+c)^2)/a^(1/4)/b^(1/4)/d/(a^(1/2
)-b^(1/2))/(a+b*tan(d*x+c)^4)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3661, 1209, 1198, 220, 1196, 1217, 1707}

$$\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2d}+\frac{\sqrt{b}\,\tan(c+dx)\sqrt{a+b\tan^4(c+dx)}}{d\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)}-\frac{\sqrt[4]{b}\,(a+b)\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)\sqrt{\frac{a\cdots}{\left(\sqrt{a}+\cdots\right.}}}{2\sqrt[4]{a}\,d\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[c + d*x]^4],x]
```

```
[Out] (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^4]])
/(2*d) + (Sqrt[b]*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]^4])/(d*(Sqrt[a] + Sq
rt[b]*Tan[c + d*x]^2)) - (a^(1/4)*b^(1/4)*EllipticE[2*ArcTan[(b^(1/4)*Tan[c
 + d*x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[
c + d*x]^4)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(d*Sqrt[a + b*Tan[c + d*
x]^4]) + ((Sqrt[a] - Sqrt[b])*b^(1/4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[c + d
*x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c +
d*x]^4)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(2*a^(1/4)*d*Sqrt[a + b*Tan[
c + d*x]^4]) - (b^(1/4)*(a + b)*EllipticF[2*ArcTan[(b^(1/4)*Tan[c + d*x])/a
^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4
)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*d*S
qrt[a + b*Tan[c + d*x]^4]) + ((Sqrt[a] + Sqrt[b])*(a + b)*EllipticPi[-(Sqrt
[a] - Sqrt[b])^2/(4*Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(1/
4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/(S
qrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)
*d*Sqrt[a + b*Tan[c + d*x]^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \sqrt{a+b\tan^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^4}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{b-bx^2}{\sqrt{a+bx^4}}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \tan(c+\right.}{d}$$

$$= -\frac{\left(\sqrt{a}\sqrt{b}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \tan(c+dx)\right)}{d} + \frac{\left(\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\right.}{d}$$

$$= \frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2d} + \frac{\sqrt{b}\,\tan(c+dx)\sqrt{a+b\tan^4(c+dx)}}{d\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)} - \frac{\sqrt[4]{a}\,\sqrt[4]{b}\,E\left(2\right.}{}$$

Mathematica [C] time = 0.81, size = 219, normalized size = 0.34

$$\frac{\sqrt{\frac{b\tan^4(c+dx)}{a}+1}\left(\sqrt{a}\sqrt{b}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)\middle|-1\right)+\left(\sqrt{a}-i\sqrt{b}\right)\left(\left(\sqrt{b}-i\sqrt{a}\right)\Pi\left(-\frac{i\sqrt{a}}{\sqrt{b}};i\sinh^{-1}\left(\right.\right.\right.\right.}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\tan^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[c + d*x]^4],x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x
]], -1] + (Sqrt[a] - I*Sqrt[b])*(-(Sqrt[b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt
[b])/Sqrt[a]]*Tan[c + d*x]], -1]) + ((-I)*Sqrt[a] + Sqrt[b])*EllipticPi[((-
I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], -1]
))*Sqrt[1 + (b*Tan[c + d*x]^4)/a])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*d*Sqrt[a + b*
Tan[c + d*x]^4])
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\tan(dx+c)^4+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*tan(d*x + c)^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan(dx+c)^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(d*x + c)^4 + a), x)
```

maple [C] time = 0.57, size = 531, normalized size = 0.82

$$-\frac{b\sqrt{1-\frac{i\sqrt{b}\,(\tan^2(dx+c))}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,(\tan^2(dx+c))}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tan(dx+c)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b(\tan^4(dx+c))}}+\frac{i\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,(\tan^2(dx+c))}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,(\tan^2(dx+c))}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tan\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b(\tan^4(dx+c))}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^4)^(1/2),x)
```

```
[Out] 1/d*(-b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*
(1+I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticF
(tan(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I)+I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan
(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticF(tan(d*x+c)*(I/a^(1/2)*b
^(1/2))^(1/2),I)-I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*ta
n(d*x+c)^4)^(1/2)*EllipticE(tan(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I)+a/(I/a^
(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticPi(tan(d*x+c)*
(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a
^(1/2)*b^(1/2))^(1/2))+b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan
(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4
)^(1/2)*EllipticPi(tan(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(
-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan\left(dx+c\right)^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(d*x + c)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{b\tan\left(c+d\,x\right)^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^4)^(1/2),x)
```

```
[Out] int((a + b*tan(c + d*x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^4\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)**4*b)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(c + d*x)**4), x)
```

3.388 $\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx$

Optimal. Leaf size=348

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2d\sqrt{a+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}d\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(c+dx)}}+\frac{\left(\sqrt{a}+\sqrt{b}\right.}{}$$

```
[Out] 1/2*arctan((a+b)^(1/2)*tan(d*x+c)/(a+b*tan(d*x+c)^4)^(1/2))/d/(a+b)^(1/2)-1
/2*b^(1/4)*(cos(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan
(b^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*tan(d*x+c)/a^(
1/4))),1/2*2^(1/2))*((a+b*tan(d*x+c)^4)/(a^(1/2)+b^(1/2)*tan(d*x+c)^2)^2)^(
1/2)*(a^(1/2)+b^(1/2)*tan(d*x+c)^2)/a^(1/4)/d/(a^(1/2)-b^(1/2))/(a+b*tan(d*
x+c)^4)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*tan(d*
x+c)/a^(1/4))),-1/4*(a^(1/2)-b^(1/2))^2/a^(1/2)/b^(1/2),1/2*2^(1/2))*(a^(1/
2)+b^(1/2))*((a+b*tan(d*x+c)^4)/(a^(1/2)+b^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^
(1/2)+b^(1/2)*tan(d*x+c)^2)/a^(1/4)/b^(1/4)/d/(a^(1/2)-b^(1/2))/(a+b*tan(d*
x+c)^4)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3661, 1217, 220, 1707}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2d\sqrt{a+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}d\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(c+dx)}}+\frac{\left(\sqrt{a}+\sqrt{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Tan[c + d*x]^4],x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^4]]/(2*Sqrt[a + b
]*d) - (b^(1/4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(1/4)], 1/2]*(S
qrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/(Sqrt[a] + Sqr
t[b]*Tan[c + d*x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*d*Sqrt[a + b*Tan[c
+ d*x]^4]) + ((Sqrt[a] + Sqrt[b])*EllipticPi[-(Sqrt[a] - Sqrt[b])^2/(4*Sqrt
[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqr
t[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[b]*Tan[c +
 d*x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)*d*Sqrt[a + b*Tan[c + d*
x]^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \tan(c+dx)\right)}{\left(\sqrt{a}-\sqrt{b}\right)d} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \tan(c\right.}{\left(\sqrt{a}-\sqrt{b}\right)d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2\sqrt{a+b}\,d} - \frac{\sqrt[4]{b}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx\right.}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)d\sqrt{a+b\tan^4(c+}}$$

Mathematica [C] time = 0.41, size = 106, normalized size = 0.30

$$-\frac{i\sqrt{\frac{b\tan^4(c+dx)}{a}+1}\,\Pi\left(-\frac{i\sqrt{a}}{\sqrt{b}};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)\middle|-1\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\tan^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Tan[c + d*x]^4],x]
```

```
[Out] ((-I)*EllipticPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]
]*Tan[c + d*x]], -1]*Sqrt[1 + (b*Tan[c + d*x]^4)/a])/(Sqrt[(I*Sqrt[b])/Sqrt
[a]]*d*Sqrt[a + b*Tan[c + d*x]^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tan\left(dx+c\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tan(d*x + c)^4 + a), x)
```

maple [C] time = 0.31, size = 123, normalized size = 0.35

$$\frac{\sqrt{1-\frac{i\sqrt{b}\,\left(\tan^2(dx+c)\right)}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,\left(\tan^2(dx+c)\right)}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(\tan\left(dx+c\right)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\frac{i\sqrt{a}}{\sqrt{b}},\frac{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\left(\tan^4\left(dx+c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^4)^(1/2),x)
```

```
[Out] 1/d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticPi(ta
n(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(
1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tan\left(dx+c\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*tan(d*x + c)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{b\,{\tan\left(c+d\,x\right)}^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^4)^(1/2),x)
```

```
[Out] int(1/(a + b*tan(c + d*x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan^4\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)**4*b)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*tan(c + d*x)**4), x)
```

3.389 $\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal. Leaf size=103

$$-\frac{1}{4}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{4\sqrt{b}}+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)$$

```
[Out] 1/4*(a+2*b)*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))/b^(1/2)+1/2*arct
anh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))*(a+b)^(1/2)-1/4*(a+b*t
an(x)^4)^(1/2)*(2-tan(x)^2)
```

Rubi [A] time = 0.21, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3670, 1252, 815, 844, 217, 206, 725}

$$-\frac{1}{4}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{4\sqrt{b}}+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/(4*Sqrt[b]) +
(Sqrt[a + b]*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/
2 - ((2 - Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx &= \text{Subst}\left(\int \frac{x^3\sqrt{a+bx^4}}{1+x^2}\,dx,x,\tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{a+bx^2}}{1+x}\,dx,x,\tan^2(x)\right)\\
&= -\frac{1}{4}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{\text{Subst}\left(\int \frac{-ab+b(a+2b)x}{(1+x)\sqrt{a+bx^2}}\,dx,x,\tan^2(x)\right)}{4b}\\
&= -\frac{1}{4}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{2}(-a-b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx,x,\text{ta}\right.\\
&= -\frac{1}{4}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{2}(a+b)\,\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx,x,\frac{a-b\,\text{ta}}{\sqrt{a+b\,\text{ta}}}\right.\\
&= \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{4\sqrt{b}}+\frac{1}{2}\sqrt{a+b}\,\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{4}\left(\right.
\end{aligned}$$

Mathematica [A] time = 4.09, size = 145, normalized size = 1.41

$$\frac{1}{4}\left(\frac{\frac{a^{3/2}\sqrt{\frac{b\tan^4(x)}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a}}\right)}{\sqrt{b}}+\left(\tan^2(x)-2\right)\left(a+b\tan^4(x)\right)}{\sqrt{a+b\tan^4(x)}}+2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+2\sqrt{a+b}\,\text{tanh}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + 2*Sqrt[a + b]
*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] + ((-2 + Tan[
x]^2)*(a + b*Tan[x]^4) + (a^(3/2)*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*Sqrt[
1 + (b*Tan[x]^4)/a])/Sqrt[b])/Sqrt[a + b*Tan[x]^4])/4
```

fricas [A] time = 0.70, size = 555, normalized size = 5.39

$$\left[\frac{(a+2\,b)\sqrt{b}\,\log\left(-2\,b\tan(x)^4-2\,\sqrt{b\tan(x)^4+a}\,\sqrt{b}\,\tan(x)^2-a\right)+2\,\sqrt{a+b}\,b\log\left(\frac{\left(ab+2\,b^2\right)\tan(x)^4-2\,ab\tan(x)^2-2}{\tan(x}\right.}{8\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^3,x, algorithm="fricas")
```

```
[Out] [1/8*((a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*
tan(x)^2 - a) + 2*sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^
2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan
(x)^4 + 2*tan(x)^2 + 1)) + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b))/b, -1
/4*((a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) -
 sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(
x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^
2 + 1)) - sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b))/b, 1/8*(4*sqrt(-a - b)*b
*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan
(x)^4 + a^2 + a*b)) + (a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x)
^4 + a)*sqrt(b)*tan(x)^2 - a) + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b))/
b, 1/4*(2*sqrt(-a - b)*b*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(
-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) - (a + 2*b)*sqrt(-b)*arctan(sqr
t(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + sqrt(b*tan(x)^4 + a)*(b*tan(x)^2
 - 2*b))/b]
```

giac [A] time = 0.46, size = 107, normalized size = 1.04

$$\frac{1}{4}\,\sqrt{b\tan(x)^4+a}\left(\tan(x)^2-2\right)-\frac{(a+b)\arctan\left(-\frac{\sqrt{b}\,\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}-\frac{\left(a\sqrt{b}\,+2\,b^{\frac{3}{2}}\right)\log\left(\left|-\sqrt{b}\,\tan(x)^2\right.\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*tan(x)^4 + a)*(tan(x)^2 - 2) - (a + b)*arctan(-(sqrt(b)*tan(x)^2
 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/sqrt(-a - b) - 1/4*(a*sqrt
(b) + 2*b^(3/2))*log(abs(-sqrt(b)*tan(x)^2 + sqrt(b*tan(x)^4 + a)))/b
```

maple [B] time = 0.31, size = 181, normalized size = 1.76

$$\frac{\sqrt{a+b\left(\tan^4(x)\right)}\,\left(\tan^2(x)\right)}{4}+\frac{a\ln\left(\sqrt{b}\,\left(\tan^2(x)\right)+\sqrt{a+b\left(\tan^4(x)\right)}\right)}{4\sqrt{b}}-\frac{\sqrt{\left(1+\tan^2(x)\right)^2 b-2\left(1+\tan^2(x)\right)b}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(x)^4)^(1/2)*tan(x)^3,x)
```

```
[Out] 1/4*(a+b*tan(x)^4)^(1/2)*tan(x)^2+1/4*a/b^(1/2)*ln(b^(1/2)*tan(x)^2+(a+b*ta
n(x)^4)^(1/2))-1/2*((1+tan(x)^2)^2*b-2*(1+tan(x)^2)*b+a+b)^(1/2)+1/2*b^(1/2
)*ln(((1+tan(x)^2)*b-b)/b^(1/2)+((1+tan(x)^2)^2*b-2*(1+tan(x)^2)*b+a+b)^(1/
2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*(1+tan(x)^2)*b+2*(a+b)^(1/2)*((1+tan(x)^2
)^2*b-2*(1+tan(x)^2)*b+a+b)^(1/2))/(1+tan(x)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\tan(x)^4+a}\,\tan(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*tan(x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(x)^3 \sqrt{b \tan(x)^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3*(a + b*tan(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)^3*(a + b*tan(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tan^4(x)} \tan^3(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)**4)**(1/2)*tan(x)**3,x)
```

```
[Out] Integral(sqrt(a + b*tan(x)**4)*tan(x)**3, x)
```

3.390 $\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal. Leaf size=90

$$\frac{1}{2}\sqrt{a+b\tan^4(x)}-\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)$$

```
[Out] -1/2*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))*b^(1/2)-1/2*arctanh((a-
b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))*(a+b)^(1/2)+1/2*(a+b*tan(x)^4
)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3670, 1248, 735, 844, 217, 206, 725}

$$\frac{1}{2}\sqrt{a+b\tan^4(x)}-\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] -(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/2 - (Sqrt[a + b
]*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/2 + Sqrt[a
+ b*Tan[x]^4]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx &= \text{Subst}\left(\int \frac{x\sqrt{a+bx^4}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x}\,dx, x, \tan^2(x)\right)\\
&= \frac{1}{2}\sqrt{a+b\tan^4(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{a-bx}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\\
&= \frac{1}{2}\sqrt{a+b\tan^4(x)} - \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right) + \frac{1}{2}(a+b)\,\text{Subst}\left(\int \frac{}{(}\right.\\
&= \frac{1}{2}\sqrt{a+b\tan^4(x)} + \frac{1}{2}(-a-b)\,\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}b\,\text{S}\\
&= -\frac{1}{2}\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}\sqrt{a+b}\,\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 86, normalized size = 0.96

$$\frac{1}{2}\left(\sqrt{a+b\tan^4(x)} - \sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) - \sqrt{a+b}\,\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] (-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]) - Sqrt[a + b]*
ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] + Sqrt[a + b*T
an[x]^4])/2
```

fricas [A] time = 0.66, size = 475, normalized size = 5.28

$$\left[\frac{1}{4}\sqrt{b}\log\left(-2b\tan(x)^4 + 2\sqrt{b\tan(x)^4+a}\sqrt{b}\tan(x)^2 - a\right) + \frac{1}{4}\sqrt{a+b}\log\left(\frac{\left(ab+2b^2\right)\tan(x)^4 - 2ab\tan(x)^2 +}{\tan}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 -
a) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 + 2*sqrt(
```

```
b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*t
an(x)^2 + 1)) + 1/2*sqrt(b*tan(x)^4 + a), 1/2*sqrt(-b)*arctan(sqrt(b*tan(x)
^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^
4 - 2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) +
2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/2*sqrt(b*tan(x)^4 + a), -1/2*
sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*
b + b^2)*tan(x)^4 + a^2 + a*b)) + 1/4*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*
tan(x)^4 + a)*sqrt(b)*tan(x)^2 - a) + 1/2*sqrt(b*tan(x)^4 + a), -1/2*sqrt(-
a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^
2)*tan(x)^4 + a^2 + a*b)) + 1/2*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-
b)/(b*tan(x)^2)) + 1/2*sqrt(b*tan(x)^4 + a)]
```

giac [A] time = 0.42, size = 89, normalized size = 0.99

$$\frac{(a+b)\arctan\left(-\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}+\frac{1}{2}\sqrt{b}\log\left(\left|-\sqrt{b}\tan(x)^2+\sqrt{b\tan(x)^4+a}\right|\right)+\frac{1}{2}\sqrt{b\tan(x)^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] (a + b)*arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a
 - b))/sqrt(-a - b) + 1/2*sqrt(b)*log(abs(-sqrt(b)*tan(x)^2 + sqrt(b*tan(x)
^4 + a))) + 1/2*sqrt(b*tan(x)^4 + a)
```

maple [A] time = 0.24, size = 139, normalized size = 1.54

$$\frac{\sqrt{\left(1+\tan^2(x)\right)^2 b-2\left(1+\tan^2(x)\right)b+a+b}}{2}-\frac{\sqrt{b}\ \ln\left(\frac{\left(1+\tan^2(x)\right)b-b}{\sqrt{b}}+\sqrt{\left(1+\tan^2(x)\right)^2 b-2\left(1+\tan^2(x)\right)b+}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(x)^4)^(1/2)*tan(x),x)
```

```
[Out] 1/2*((1+tan(x)^2)^2*b-2*(1+tan(x)^2)*b+a+b)^(1/2)-1/2*b^(1/2)*ln(((1+tan(x)
^2)*b-b)/b^(1/2)+((1+tan(x)^2)^2*b-2*(1+tan(x)^2)*b+a+b)^(1/2))-1/2*(a+b)^(
1/2)*ln((2*a+2*b-2*(1+tan(x)^2)*b+2*(a+b)^(1/2)*((1+tan(x)^2)^2*b-2*(1+tan(
x)^2)*b+a+b)^(1/2))/(1+tan(x)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\tan(x)^4+a}\ \tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*tan(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\tan(x)\sqrt{b\tan(x)^4+a}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*tan(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)*(a + b*tan(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^4(x)}\,\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)**4)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(a + b*tan(x)**4)*tan(x), x)
```

3.391 $\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal. Leaf size=102

$$-\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)$$

```
[Out] -1/2*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))*a^(1/2)+1/2*arctanh(b^(1/2)*tan(
x)^2/(a+b*tan(x)^4)^(1/2))*b^(1/2)+1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(
a+b*tan(x)^4)^(1/2))*(a+b)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3670, 1252, 896, 266, 63, 208, 844, 217, 206, 725}

$$\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/2 + (Sqrt[a + b]
*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/2 - (Sqrt[a]
*ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 896

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[(Simp[c*d*f + a*e*g - c*(e
*f - d*g)*x, x]*(a + c*x^2)^(p - 1))/(f + g*x), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bx^4}}{x\left(1+x^2\right)}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x(1+x)}\,dx, x, \tan^2(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a-bx}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\right) + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x,\right.$$

$$= \frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^4(x)\right) + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right) -$$

$$= -\left(\frac{1}{2}(-a-b)\,\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\right) + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{2b}$$

$$= \frac{1}{2}\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{2}\sqrt{a+b}\,\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}\sqrt{}$$

Mathematica [A] time = 0.09, size = 98, normalized size = 0.96

$$\frac{1}{2}\left(-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right) + \sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) + \sqrt{a+b}\,\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + Sqrt[a + b]*Arc
Tanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] - Sqrt[a]*ArcTanh
[Sqrt[a + b*Tan[x]^4]/Sqrt[a]])/2
```

fricas [A] time = 0.99, size = 1021, normalized size = 10.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a
) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b
*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*ta
n(x)^2 + 1)) + 1/4*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a)
 + 2*a)/tan(x)^4), -1/2*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*ta
n(x)^2)) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2
*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4
 + 2*tan(x)^2 + 1)) + 1/4*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*
sqrt(a) + 2*a)/tan(x)^4), 1/2*sqrt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)
/a) + 1/4*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^
2 + a) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*s
qrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 +
 2*tan(x)^2 + 1)), 1/2*sqrt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) - 1
/2*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*sqrt(a
 + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)
*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)),
1/2*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a))
 + 1/4*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 +
 a) + 1/4*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/t
an(x)^4), 1/2*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(
x)^2 - a)) - 1/2*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)
) + 1/4*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan
(x)^4), 1/2*sqrt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/2*sqrt(-a
- b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a)) + 1/4*sqrt(
b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a), 1/2*sqr
t(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/2*sqrt(-a - b)*arctan(sqr
t(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a)) - 1/2*sqrt(-b)*arctan(sqrt
(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \cot(x)\sqrt{a + b\left(\tan^4(x)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*tan(x)^4)^(1/2),x)
[Out] int(cot(x)*(a+b*tan(x)^4)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan(x)^4 + a}\,\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(x)^4 + a)*cot(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(x)\,\sqrt{b\tan(x)^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*tan(x)^4)^(1/2),x)
[Out] int(cot(x)*(a + b*tan(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^4{(x)}}\,\cot{(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)**4)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(x)**4)*cot(x), x)
```

3.392 $\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal. Leaf size=643

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}+\frac{1}{3}\tan(x)\sqrt{a+b\tan^4(x)}-\frac{1}{2}\sqrt{a+b}\tan^{-1}$$

```
[Out] -1/2*arctan((a+b)^(1/2)*tan(x)/(a+b*tan(x)^4)^(1/2))*(a+b)^(1/2)+1/3*(a+b*t
an(x)^4)^(1/2)*tan(x)-b^(1/2)*(a+b*tan(x)^4)^(1/2)*tan(x)/(a^(1/2)+b^(1/2)*
tan(x)^2)+a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*tan(x)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*tan(x)/
a^(1/4))),1/2*2^(1/2))*((a+b*tan(x)^4)/(a^(1/2)+b^(1/2)*tan(x)^2)^2)^(1/2)*
(a^(1/2)+b^(1/2)*tan(x)^2)/(a+b*tan(x)^4)^(1/2)+1/3*a^(3/4)*(cos(2*arctan(b
^(1/4)*tan(x)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))*Elli
pticF(sin(2*arctan(b^(1/4)*tan(x)/a^(1/4))),1/2*2^(1/2))*((a+b*tan(x)^4)/(a
^(1/2)+b^(1/2)*tan(x)^2)^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(x)^2)/b^(1/4)/(a+b*t
an(x)^4)^(1/2)+1/2*b^(1/4)*(a+b)*(cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*
tan(x)/a^(1/4))),1/2*2^(1/2))*((a+b*tan(x)^4)/(a^(1/2)+b^(1/2)*tan(x)^2)^2)
^(1/2)*(a^(1/2)+b^(1/2)*tan(x)^2)/a^(1/4)/(a^(1/2)-b^(1/2))/(a+b*tan(x)^4)^
(1/2)-1/2*b^(1/4)*(cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*tan(x)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*tan(x)/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)-b^(1/2))*((a+b*tan(x)^4)/(a^(1/2)+b^(1/2)*tan(x)^2)
^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(x)^2)/a^(1/4)/(a+b*tan(x)^4)^(1/2)-1/4*(a+b)
*(cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*tan(x
)/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*tan(x)/a^(1/4))),-1/4*(a^(1/2)-
b^(1/2))^2/a^(1/2)/b^(1/2),1/2*2^(1/2))*(a^(1/2)+b^(1/2))*((a+b*tan(x)^4)/(
a^(1/2)+b^(1/2)*tan(x)^2)^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(x)^2)/a^(1/4)/b^(1/
4)/(a^(1/2)-b^(1/2))/(a+b*tan(x)^4)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3670, 1336, 195, 220, 1209, 1198, 1196, 1217, 1707}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}-\frac{1}{2}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{3}\tan($$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] -(Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a + b*Tan[x]^4]])/2 + (Tan[x
]*Sqrt[a + b*Tan[x]^4])/3 - (Sqrt[b]*Tan[x]*Sqrt[a + b*Tan[x]^4])/(Sqrt[a]
+ Sqrt[b]*Tan[x]^2) + (a^(1/4)*b^(1/4)*EllipticE[2*ArcTan[(b^(1/4)*Tan[x])/
a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a]
+ Sqrt[b]*Tan[x]^2)^2])/Sqrt[a + b*Tan[x]^4] + (a^(3/4)*EllipticF[2*ArcTan[
(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Ta
n[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(3*b^(1/4)*Sqrt[a + b*Tan[x]^4]) -
 ((Sqrt[a] - Sqrt[b])*b^(1/4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)],
 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]
*Tan[x]^2)^2])/(2*a^(1/4)*Sqrt[a + b*Tan[x]^4]) + (b^(1/4)*(a + b)*Elliptic
F[2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqr
t[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - S
qrt[b])*Sqrt[a + b*Tan[x]^4]) - ((Sqrt[a] + Sqrt[b])*(a + b)*EllipticPi[-(S
qrt[a] - Sqrt[b])^2/(4*Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)]
```

```
, 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b
]*Tan[x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)*Sqrt[a + b*Tan[x]^4]
)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
```

```
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx = \text{Subst}\left(\int \frac{x^2\sqrt{a+bx^4}}{1+x^2}\,dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int \left(\sqrt{a+bx^4} - \frac{\sqrt{a+bx^4}}{1+x^2}\right)dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int \sqrt{a+bx^4}\,dx, x, \tan(x)\right) - \text{Subst}\left(\int \frac{\sqrt{a+bx^4}}{1+x^2}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{3}\tan(x)\sqrt{a+b\tan^4(x)} + \frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \tan(x)\right) - (a+b)\,\text{S}$$

$$= \frac{1}{3}\tan(x)\sqrt{a+b\tan^4(x)} + \frac{a^{3/4}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{}}{3\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}$$

$$= -\frac{1}{2}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{3}\tan(x)\sqrt{a+b\tan^4(x)} - \frac{\sqrt{b}\tan(x)\sqrt{a}}{\sqrt{a}+\sqrt{b}}$$

Mathematica [C] time = 17.88, size = 550, normalized size = 0.86

$$\left(\frac{\tan(x)}{3} - \frac{1}{2}\sin(2x)\right)\sqrt{\frac{4a\cos(2x)+a\cos(4x)+3a-4b\cos(2x)+b\cos(4x)+3b}{4\cos(2x)+\cos(4x)+3}} + \frac{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan^5(x)+3a\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] Sqrt[(3*a + 3*b + 4*a*Cos[2*x] - 4*b*Cos[2*x] + a*Cos[4*x] + b*Cos[4*x])/(3
 + 4*Cos[2*x] + Cos[4*x])]*(-1/2*Sin[2*x] + Tan[x]/3) + (3*a*Sqrt[(I*Sqrt[b
])/Sqrt[a]]*Tan[x] + 3*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b*Tan[x]^5 + (3*I)*a*Ellip
ticPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]],
-1]*Sqrt[1 + (b*Tan[x]^4)/a] + (3*I)*b*EllipticPi[((-I)*Sqrt[a])/Sqrt[b], I
*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1]*Sqrt[1 + (b*Tan[x]^4)/a] +
(3*I)*a*EllipticPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[
```

```
a]]*Tan[x]], -1]*Tan[x]^2*Sqrt[1 + (b*Tan[x]^4)/a] + (3*I)*b*EllipticPi[((-
I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1]*Tan[x
]^2*Sqrt[1 + (b*Tan[x]^4)/a] - 3*Sqrt[a]*Sqrt[b]*EllipticE[I*ArcSinh[Sqrt[(
I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1]*(1 + Tan[x]^2)*Sqrt[1 + (b*Tan[x]^4)/a] +
((-2*I)*a + 3*Sqrt[a]*Sqrt[b] - (3*I)*b)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b
])/Sqrt[a]]*Tan[x]], -1]*(1 + Tan[x]^2)*Sqrt[1 + (b*Tan[x]^4)/a])/(3*Sqrt[(
I*Sqrt[b])/Sqrt[a]]*(1 + Tan[x]^2)*Sqrt[a + b*Tan[x]^4])
```

fricas [F] time = 38.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\tan(x)^4+a}\,\tan(x)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*tan(x)^4 + a)*tan(x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\tan(x)^4+a}\,\tan(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*tan(x)^2, x)
```

maple [C] time = 0.23, size = 537, normalized size = 0.84

$$\frac{\sqrt{a+b\left(\tan^4(x)\right)}\,\tan(x)}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\left(\tan^2(x)\right)}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\left(\tan^2(x)\right)}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tan(x)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\left(\tan^4(x)\right)}}+\frac{b\sqrt{1-\frac{i\sqrt{b}\left(\tan^2(x)\right)}{\sqrt{a}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(x)^4)^(1/2)*tan(x)^2,x)
```

```
[Out] 1/3*(a+b*tan(x)^4)^(1/2)*tan(x)+2/3*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2
)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^
4)^(1/2)*EllipticF(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)+b/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)
^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticF(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-I
*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticF(ta
n(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)+I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2
)/(a+b*tan(x)^4)^(1/2)*EllipticE(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-a/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticPi(tan(x)*(I/a^(1/2)*b^(1
/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))
^(1/2))-b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1
+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticPi(tan(x)*(
I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^
(1/2)*b^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\tan(x)^4+a}\,\tan(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*tan(x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(x)^2 \sqrt{b \tan(x)^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2*(a + b*tan(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)^2*(a + b*tan(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tan^4(x)} \tan^2(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)**4)**(1/2)*tan(x)**2,x)
```

```
[Out] Integral(sqrt(a + b*tan(x)**4)*tan(x)**2, x)
```

3.393 $\int \tan^3(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=148

$$\frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{16\sqrt{b}}-\frac{1}{24}\left(4-3\tan^2(x)\right)\left(a+b\tan^4(x)\right)^{3/2}-\frac{1}{16}\left(8(a+b)-(3a+4b)\tan^2(x)\right)$$

```
[Out] 1/2*(a+b)^(3/2)*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))+1/
16*(3*a^2+12*a*b+8*b^2)*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))/b^(1
/2)-1/16*(a+b*tan(x)^4)^(1/2)*(8*a+8*b-(3*a+4*b)*tan(x)^2)-1/24*(4-3*tan(x)
^2)*(a+b*tan(x)^4)^(3/2)
```

Rubi [A] time = 0.31, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3670, 1252, 815, 844, 217, 206, 725}

$$\frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{16\sqrt{b}}-\frac{1}{24}\left(4-3\tan^2(x)\right)\left(a+b\tan^4(x)\right)^{3/2}-\frac{1}{16}\left(8(a+b)-(3a+4b)\tan^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] ((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])
/(16*Sqrt[b]) + ((a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a
 + b*Tan[x]^4])])/2 - ((8*(a + b) - (3*a + 4*b)*Tan[x]^2)*Sqrt[a + b*Tan[x]
^4])/16 - ((4 - 3*Tan[x]^2)*(a + b*Tan[x]^4)^(3/2))/24
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^3(x)\left(a+b\tan^4(x)\right)^{3/2}dx = \text{Subst}\left(\int \frac{x^3\left(a+bx^4\right)^{3/2}}{1+x^2}dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x\left(a+bx^2\right)^{3/2}}{1+x}dx, x, \tan^2(x)\right)$$

$$= -\frac{1}{24}\left(4-3\tan^2(x)\right)\left(a+b\tan^4(x)\right)^{3/2} + \frac{\text{Subst}\left(\int \frac{(-ab+b(3a+4b)x)\sqrt{a+bx^2}}{1+x}dx, x\right.}{8b}$$

$$= -\frac{1}{16}\left(8(a+b)-(3a+4b)\tan^2(x)\right)\sqrt{a+b\tan^4(x)} - \frac{1}{24}\left(4-3\tan^2(x)\right)\left(a\right.$$

$$= -\frac{1}{16}\left(8(a+b)-(3a+4b)\tan^2(x)\right)\sqrt{a+b\tan^4(x)} - \frac{1}{24}\left(4-3\tan^2(x)\right)\left(a\right.$$

$$= -\frac{1}{16}\left(8(a+b)-(3a+4b)\tan^2(x)\right)\sqrt{a+b\tan^4(x)} - \frac{1}{24}\left(4-3\tan^2(x)\right)\left(a\right.$$

$$= \frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{16\sqrt{b}} + \frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\,\text{t}}{\sqrt{a+b}\sqrt{a}}\right.$$

Mathematica [B] time = 6.08, size = 324, normalized size = 2.19

$$\frac{1}{2}a\tan^2(x)\sqrt{a+b\tan^4(x)}\left(\frac{b\tan^4(x)}{a}+1\right)^2\left(\frac{1}{4}\left(\frac{1}{\frac{b\tan^4(x)}{a}+1}+\frac{3}{2\left(\frac{b\tan^4(x)}{a}+1\right)^2}\right)+\frac{3\sqrt{a}\cot^2(x)\sinh^{-1}\left(\frac{\sqrt{b}\tan^2}{\sqrt{a}}\right.}{8\sqrt{b}\left(\frac{b\tan^4(x)}{a}+1\right)^{5/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (-1/3*(a + b*Tan[x]^4)^(3/2) - (a + b)*(-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2
)/Sqrt[a + b*Tan[x]^4]]) - Sqrt[a + b]*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b
]*Sqrt[a + b*Tan[x]^4])] + Sqrt[a + b*Tan[x]^4]) + b*Tan[x]^2*Sqrt[a + b*Ta
n[x]^4]*(1 + (b*Tan[x]^4)/a)*((Sqrt[a]*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*
Cot[x]^2)/(2*Sqrt[b]*(1 + (b*Tan[x]^4)/a)^(3/2)) + 1/(2*(1 + (b*Tan[x]^4)/a
))))/2 + (a*Tan[x]^2*Sqrt[a + b*Tan[x]^4]*(1 + (b*Tan[x]^4)/a)^2*((3*Sqrt[a
]*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*Cot[x]^2)/(8*Sqrt[b]*(1 + (b*Tan[x]^4
)/a)^(5/2)) + (3/(2*(1 + (b*Tan[x]^4)/a)^2) + (1 + (b*Tan[x]^4)/a)^(-1))/4)
)/2
```

fricas [A] time = 0.87, size = 758, normalized size = 5.12

$$\left[\frac{3\left(3\,a^2+12\,ab+8\,b^2\right)\sqrt{b}\,\log\left(-2\,b\tan(x)^4-2\,\sqrt{b\tan(x)^4+a}\,\sqrt{b}\,\tan(x)^2-a\right)+24\left(ab+b^2\right)\sqrt{a+b}\,\log\left(\frac{(ab+2}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(3*a^2 + 12*a*b + 8*b^2)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(
x)^4 + a)*sqrt(b)*tan(x)^2 - a) + 24*(a*b + b^2)*sqrt(a + b)*log(((a*b + 2*
b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sq
rt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*(6*b^2*tan(x)^6 -
 8*b^2*tan(x)^4 + 3*(5*a*b + 4*b^2)*tan(x)^2 - 32*a*b - 24*b^2)*sqrt(b*tan(
x)^4 + a))/b, -1/48*(3*(3*a^2 + 12*a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(
x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) - 12*(a*b + b^2)*sqrt(a + b)*log(((a*b + 2
*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*s
qrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) - (6*b^2*tan(x)^6 -
8*b^2*tan(x)^4 + 3*(5*a*b + 4*b^2)*tan(x)^2 - 32*a*b - 24*b^2)*sqrt(b*tan(x
)^4 + a))/b, 1/96*(48*(a*b + b^2)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*
(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + 3*(3*a^
2 + 12*a*b + 8*b^2)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt
(b)*tan(x)^2 - a) + 2*(6*b^2*tan(x)^6 - 8*b^2*tan(x)^4 + 3*(5*a*b + 4*b^2)*
tan(x)^2 - 32*a*b - 24*b^2)*sqrt(b*tan(x)^4 + a))/b, 1/48*(24*(a*b + b^2)*s
qrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b
 + b^2)*tan(x)^4 + a^2 + a*b)) - 3*(3*a^2 + 12*a*b + 8*b^2)*sqrt(-b)*arctan
(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + (6*b^2*tan(x)^6 - 8*b^2*tan(
x)^4 + 3*(5*a*b + 4*b^2)*tan(x)^2 - 32*a*b - 24*b^2)*sqrt(b*tan(x)^4 + a))/
b]
```

giac [A] time = 0.60, size = 176, normalized size = 1.19

$$\frac{1}{48}\,\sqrt{b\tan(x)^4+a}\left(\left(2\left(3\,b\tan(x)^2-4\,b\right)\tan(x)^2+\frac{3\left(5\,ab^2+4\,b^3\right)}{b^2}\right)\tan(x)^2-\frac{8\left(4\,ab^2+3\,b^3\right)}{b^2}\right)-\frac{\left(a^2+2\,ab+b^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(b*tan(x)^4 + a)*((2*(3*b*tan(x)^2 - 4*b)*tan(x)^2 + 3*(5*a*b^2 +
4*b^3)/b^2)*tan(x)^2 - 8*(4*a*b^2 + 3*b^3)/b^2) - (a^2 + 2*a*b + b^2)*arcta
n(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/sqrt(-
a - b) - 1/16*(3*a^2*sqrt(b) + 12*a*b^(3/2) + 8*b^(5/2))*log(abs(-sqrt(b)*t
an(x)^2 + sqrt(b*tan(x)^4 + a)))/b
```

maple [B] time = 0.21, size = 374, normalized size = 2.53

$$\frac{b\left(\tan^{6}(x)\right)\sqrt{a+b\left(\tan^{4}(x)\right)}}{8}+\frac{5a\left(\tan^{2}(x)\right)\sqrt{a+b\left(\tan^{4}(x)\right)}}{16}+\frac{3a^{2}\ln\left(\sqrt{b}\,\left(\tan^{2}(x)\right)+\sqrt{a+b\left(\tan^{4}(x)\right)}\right)}{16\sqrt{b}}-\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3*(a+b*tan(x)^4)^(3/2),x)
```

```
[Out] 1/8*b*tan(x)^6*(a+b*tan(x)^4)^(1/2)+5/16*a*tan(x)^2*(a+b*tan(x)^4)^(1/2)+3/
16*a^2*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))/b^(1/2)-1/6*b*tan(x)^4*(a+
b*tan(x)^4)^(1/2)-2/3*a*(a+b*tan(x)^4)^(1/2)+1/4*b*tan(x)^2*(a+b*tan(x)^4)^
(1/2)+3/4*a*b^(1/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))-1/2*b*(a+b*ta
n(x)^4)^(1/2)+1/2*b^(3/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))+1/2/(a+
b)^(1/2)*ln((2*a+2*b-2*(1+tan(x)^2)*b+2*(a+b)^(1/2)*((1+tan(x)^2)^2*b-2*(1+
tan(x)^2)*b+a+b)^(1/2))/(1+tan(x)^2))*a^2+1/(a+b)^(1/2)*ln((2*a+2*b-2*(1+ta
n(x)^2)*b+2*(a+b)^(1/2)*((1+tan(x)^2)^2*b-2*(1+tan(x)^2)*b+a+b)^(1/2))/(1+t
an(x)^2))*a*b+1/2/(a+b)^(1/2)*ln((2*a+2*b-2*(1+tan(x)^2)*b+2*(a+b)^(1/2)*((
1+tan(x)^2)^2*b-2*(1+tan(x)^2)*b+a+b)^(1/2))/(1+tan(x)^2))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan(x)^{4}+a\right)^{\frac{3}{2}}\tan(x)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(x)^4 + a)^(3/2)*tan(x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\tan(x)^{3}\left(b\tan(x)^{4}+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3*(a + b*tan(x)^4)^(3/2),x)
```

```
[Out] int(tan(x)^3*(a + b*tan(x)^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{4}(x)\right)^{\frac{3}{2}}\tan^{3}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3*(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral((a + b*tan(x)**4)**(3/2)*tan(x)**3, x)
```

3.394 $\int \tan(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=126

$$\frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2}+\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}-\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{4}\sqrt{b}\,($$

```
[Out] -1/2*(a+b)^(3/2)*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))-1
/4*(3*a+2*b)*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))*b^(1/2)+1/4*(a+
b*tan(x)^4)^(1/2)*(2*a+2*b-b*tan(x)^2)+1/6*(a+b*tan(x)^4)^(3/2)
```

Rubi [A] time = 0.21, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3670, 1248, 735, 815, 844, 217, 206, 725}

$$\frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2}+\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}-\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{4}\sqrt{b}\,($$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] -(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/4 -
 ((a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])
])/2 + ((2*(a + b) - b*Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/4 + (a + b*Tan[x]^4)
^(3/2)/6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan(x)\left(a+b\tan^4(x)\right)^{3/2}dx = \text{Subst}\left(\int \frac{x\left(a+bx^4\right)^{3/2}}{1+x^2}dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1+x}dx, x, \tan^2(x)\right)$$

$$= \frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2} + \frac{1}{2}\text{Subst}\left(\int \frac{(a-bx)\sqrt{a+bx^2}}{1+x}dx, x, \tan^2(x)\right)$$

$$= \frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2} + \frac{\text{Subst}\left(\int \frac{ab}{}\right.}{}$$

$$= \frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2} + \frac{1}{2}(a+b)^2 \text{S}$$

$$= \frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2} - \frac{1}{2}(a+b)^2 \text{S}$$

$$= -\frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\,\text{ta}}{\sqrt{a+b}\,\sqrt{a}\cdots}\right.$$

Mathematica [A] time = 4.84, size = 166, normalized size = 1.32

$$\frac{1}{12}\left(\sqrt{a+b\tan^4(x)}\left(8a+2b\tan^4(x)-3b\tan^2(x)+6b\right)-6(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)-6\sqrt{b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (-6*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] - 6*(a
 + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] +
Sqrt[a + b*Tan[x]^4]*(8*a + 6*b - 3*b*Tan[x]^2 + 2*b*Tan[x]^4) - (3*Sqrt[a]
*Sqrt[b]*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*Sqrt[a + b*Tan[x]^4])/Sqrt[1 +
 (b*Tan[x]^4)/a])/12
```

fricas [A] time = 0.85, size = 593, normalized size = 4.71

$$\left[\frac{1}{8}(3a+2b)\sqrt{b}\log\left(-2b\tan(x)^4+2\sqrt{b\tan(x)^4+a}\sqrt{b}\tan(x)^2-a\right)+\frac{1}{4}(a+b)^{\frac{3}{2}}\log\left(\frac{\left(ab+2b^2\right)\tan(x)^4-2a\cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)
*tan(x)^2 - a) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(
x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(
tan(x)^4 + 2*tan(x)^2 + 1)) + 1/12*(2*b*tan(x)^4 - 3*b*tan(x)^2 + 8*a + 6*b
)*sqrt(b*tan(x)^4 + a), 1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a
)*sqrt(-b)/(b*tan(x)^2)) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 -
2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^
2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/12*(2*b*tan(x)^4 - 3*b*tan(x)^2 +
 8*a + 6*b)*sqrt(b*tan(x)^4 + a), -1/2*(a + b)*sqrt(-a - b)*arctan(sqrt(b*t
an(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*
b)) + 1/8*(3*a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sq
rt(b)*tan(x)^2 - a) + 1/12*(2*b*tan(x)^4 - 3*b*tan(x)^2 + 8*a + 6*b)*sqrt(b
*tan(x)^4 + a), -1/2*(a + b)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*ta
n(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + 1/4*(3*a + 2
*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/12*(2*b
*tan(x)^4 - 3*b*tan(x)^2 + 8*a + 6*b)*sqrt(b*tan(x)^4 + a)]
```

giac [A] time = 0.70, size = 138, normalized size = 1.10

$$\frac{1}{4}\left(3a\sqrt{b}+2b^{\frac{3}{2}}\right)\log\left(\left|-\sqrt{b}\tan(x)^2+\sqrt{b\tan(x)^4+a}\right|\right)+\frac{1}{12}\sqrt{b\tan(x)^4+a}\left(\left(2b\tan(x)^2-3b\right)\tan(x)^2+\frac{2\left(4ab\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*a*sqrt(b) + 2*b^(3/2))*log(abs(-sqrt(b)*tan(x)^2 + sqrt(b*tan(x)^4 +
 a))) + 1/12*sqrt(b*tan(x)^4 + a)*((2*b*tan(x)^2 - 3*b)*tan(x)^2 + 2*(4*a*b
 + 3*b^2)/b) + (a^2 + 2*a*b + b^2)*arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x
)^4 + a) + sqrt(b))/sqrt(-a - b))/sqrt(-a - b)
```

maple [B] time = 0.22, size = 313, normalized size = 2.48

$$\frac{b\left(\tan^4(x)\right)\sqrt{a+b\left(\tan^4(x)\right)}}{6}+\frac{2a\sqrt{a+b\left(\tan^4(x)\right)}}{3}-\frac{b\left(\tan^2(x)\right)\sqrt{a+b\left(\tan^4(x)\right)}}{4}-\frac{3a\sqrt{b}\,\ln\left(\sqrt{b}\,\left(\tan^2(x)\right)+\cdots\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a+b*tan(x)^4)^(3/2),x)
```

```
[Out] 1/6*b*tan(x)^4*(a+b*tan(x)^4)^(1/2)+2/3*a*(a+b*tan(x)^4)^(1/2)-1/4*b*tan(x)
^2*(a+b*tan(x)^4)^(1/2)-3/4*a*b^(1/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1
/2))+1/2*b*(a+b*tan(x)^4)^(1/2)-1/2*b^(3/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)
^4)^(1/2))-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*(1+tan(x)^2)*b+2*(a+b)^(1/2)*((1+t
an(x)^2)^2*b-2*(1+tan(x)^2)*b+a+b)^(1/2))/(1+tan(x)^2))*a^2-1/(a+b)^(1/2)*l
n((2*a+2*b-2*(1+tan(x)^2)*b+2*(a+b)^(1/2)*((1+tan(x)^2)^2*b-2*(1+tan(x)^2)*
b+a+b)^(1/2))/(1+tan(x)^2))*a*b-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*(1+tan(x)^2)*
b+2*(a+b)^(1/2)*((1+tan(x)^2)^2*b-2*(1+tan(x)^2)*b+a+b)^(1/2))/(1+tan(x)^2)
)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan(x)^4 + a\right)^{\frac{3}{2}} \tan(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(x)^4 + a)^(3/2)*tan(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(x) \left(b \tan(x)^4 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*tan(x)^4)^(3/2),x)
```

```
[Out] int(tan(x)*(a + b*tan(x)^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tan^4{(x)}\right)^{\frac{3}{2}} \tan{(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral((a + b*tan(x)**4)**(3/2)*tan(x), x)
```

3.395 $\int \cot(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=155

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)+\frac{1}{2}a\sqrt{a+b\tan^4(x)}-\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{4}\sqrt{b}\,(3a+2b)\tan$$

```
[Out] -1/2*a^(3/2)*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))+1/2*(a+b)^(3/2)*arctanh(
(a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))+1/4*(3*a+2*b)*arctanh(b^(1
/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))*b^(1/2)+1/2*a*(a+b*tan(x)^4)^(1/2)-1/4*(
a+b*tan(x)^4)^(1/2)*(2*a+2*b-b*tan(x)^2)
```

Rubi [A] time = 0.27, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3670, 1252, 896, 266, 50, 63, 208, 815, 844, 217, 206, 725}

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)+\frac{1}{2}a\sqrt{a+b\tan^4(x)}-\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{4}\sqrt{b}\,(3a+2b)\tan$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/4 +
((a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]
)/2 - (a^(3/2)*ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]])/2 + (a*Sqrt[a + b*Tan
[x]^4])/2 - ((2*(a + b) - b*Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/4
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 896

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[(Simp[c*d*f + a*e*g - c*(e
*f - d*g)*x, x]*(a + c*x^2)^(p - 1))/(f + g*x), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot(x)\left(a+b\tan^4(x)\right)^{3/2}dx = \text{Subst}\left(\int \frac{\left(a+bx^4\right)^{3/2}}{x\left(1+x^2\right)}dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x(1+x)}dx, x, \tan^2(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a-bx)\sqrt{a+bx^2}}{1+x}dx, x, \tan^2(x)\right)\right) + \frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x}dx,\right.$$
$$= -\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}dx, x, \tan^4(x\right.$$
$$= \frac{1}{2}a\sqrt{a+b\tan^4(x)} - \frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{1}{4}a^2\,\text{Subst}\left(\int\right.$$
$$= \frac{1}{2}a\sqrt{a+b\tan^4(x)} - \frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+}\right.}{}$$
$$= \frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(}{\sqrt{a+b}\sqrt{a+b\tan}}\right.$$

Mathematica [A] time = 3.23, size = 190, normalized size = 1.23

$$\frac{1}{4}\left(-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right) - 2b\sqrt{a+b\tan^4(x)} + b\tan^2(x)\sqrt{a+b\tan^4(x)} + 2\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (2*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + 2*(a
+ b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] - 2
*a^(3/2)*ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]] - 2*b*Sqrt[a + b*Tan[x]^4] +
 b*Tan[x]^2*Sqrt[a + b*Tan[x]^4] + (Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Tan[x]
^2)/Sqrt[a]]*Sqrt[a + b*Tan[x]^4])/Sqrt[1 + (b*Tan[x]^4)/a])/4
```

fricas [A] time = 37.87, size = 1269, normalized size = 8.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a + 2*b)*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*
tan(x)^2 + a) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x
)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(t
an(x)^4 + 2*tan(x)^2 + 1)) + 1/4*a^(3/2)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^
4 + a)*sqrt(a) + 2*a)/tan(x)^4) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*
b), -1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x
)^2)) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*
sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4
+ 2*tan(x)^2 + 1)) + 1/4*a^(3/2)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*s
```

```
qrt(a) + 2*a)/tan(x)^4) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b), 1/2*
sqrt(-a)*a*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/8*(3*a + 2*b)*sqrt(b
)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a) + 1/4*(a
+ b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4
 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 +
1)) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b), 1/2*sqrt(-a)*a*arctan(sq
rt(b*tan(x)^4 + a)*sqrt(-a)/a) - 1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan
(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*ta
n(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a +
b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/4*sqrt(b*tan(x)^4 + a)*(
b*tan(x)^2 - 2*b), 1/2*(a + b)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sqr
t(-a - b)/(b*tan(x)^2 - a)) + 1/8*(3*a + 2*b)*sqrt(b)*log(2*b*tan(x)^4 + 2*
sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a) + 1/4*a^(3/2)*log((b*tan(x)^4 -
2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) + 1/4*sqrt(b*tan(x)^4 + a)*
(b*tan(x)^2 - 2*b), 1/2*(a + b)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sq
rt(-a - b)/(b*tan(x)^2 - a)) - 1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x
)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*a^(3/2)*log((b*tan(x)^4 - 2*sqrt(b*ta
n(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2
 - 2*b), 1/2*sqrt(-a)*a*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/2*(a +
b)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a))
+ 1/8*(3*a + 2*b)*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)
*tan(x)^2 + a) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b), 1/2*sqrt(-a)*
a*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/2*(a + b)*sqrt(-a - b)*arctan
(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a)) - 1/4*(3*a + 2*b)*sqrt
(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*sqrt(b*tan(x)
^4 + a)*(b*tan(x)^2 - 2*b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \cot(x) \left(a + b \left(\tan^4(x)\right)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*tan(x)^4)^(3/2),x)
```

```
[Out] int(cot(x)*(a+b*tan(x)^4)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan(x)^4 + a\right)^{\frac{3}{2}} \cot(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(x)^4 + a)^(3/2)*cot(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(x) \left(b \tan(x)^4 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*tan(x)^4)^(3/2),x)
```

```
[Out] int(cot(x)*(a + b*tan(x)^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^4(x)\right)^{\frac{3}{2}} \cot(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral((a + b*tan(x)**4)**(3/2)*cot(x), x)
```

3.396 $\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

```
[Out] 1/2*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))/b^(1/2)+1/2*arctanh((a-b
*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 1252, 844, 217, 206, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]/(2*Sqrt[b]) + ArcTanh[(a -
 b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx &= \text{Subst}\left(\int \frac{x^3}{\left(1+x^2\right)\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 74, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]/(2*Sqrt[b]) + ArcTanh[(a -
 b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b])
```

fricas [A] time = 0.71, size = 483, normalized size = 6.53

$$\left[\frac{(a+b)\sqrt{b}\log\left(-2\,b\tan(x)^4-2\sqrt{b\tan(x)^4+a}\sqrt{b}\tan(x)^2-a\right)+\sqrt{a+b}\,b\log\left(\frac{\left(ab+2\,b^2\right)\tan(x)^4-2\,ab\tan(x)^2-2\sqrt{b\tan(}}{\tan(x)^4+2\tan}\right.}{4\left(ab+b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*ta
n(x)^2 - a) + sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 -
2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^
4 + 2*tan(x)^2 + 1)))/(a*b + b^2), -1/4*(2*(a + b)*sqrt(-b)*arctan(sqrt(b*t
an(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) - sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(
x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b)
 + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)))/(a*b + b^2), 1/4*(2*sqrt(-a -
 b)*b*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2
```

```
)*tan(x)^4 + a^2 + a*b)) + (a + b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan
(x)^4 + a)*sqrt(b)*tan(x)^2 - a))/(a*b + b^2), 1/2*(sqrt(-a - b)*b*arctan(s
qrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a
^2 + a*b)) - (a + b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x
)^2)))/(a*b + b^2)]
```

giac [A] time = 0.65, size = 75, normalized size = 1.01

$$-\frac{\arctan\left(-\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}-\frac{\log\left(\left|-\sqrt{b}\tan(x)^2+\sqrt{b\tan(x)^4+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/
sqrt(-a - b) - 1/2*log(abs(-sqrt(b)*tan(x)^2 + sqrt(b*tan(x)^4 + a)))/sqrt(
b)
```

maple [A] time = 0.28, size = 91, normalized size = 1.23

$$\frac{\ln\left(\sqrt{b}\left(\tan^2(x)\right)+\sqrt{a+b\left(\tan^4(x)\right)}\right)}{2\sqrt{b}}+\frac{\ln\left(\frac{2a+2b-2\left(1+\tan^2(x)\right)b+2\sqrt{a+b}\sqrt{\left(1+\tan^2(x)\right)^2b-2\left(1+\tan^2(x)\right)b+a+b}}{1+\tan^2(x)}\right)}{2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+b*tan(x)^4)^(1/2),x)
```

```
[Out] 1/2*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))/b^(1/2)+1/2/(a+b)^(1/2)*ln((2
*a+2*b-2*(1+tan(x)^2)*b+2*(a+b)^(1/2)*((1+tan(x)^2)^2*b-2*(1+tan(x)^2)*b+a+
b)^(1/2))/(1+tan(x)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)^3}{\sqrt{b\tan(x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^3/sqrt(b*tan(x)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tan(x)}^3}{\sqrt{b\,{\tan(x)}^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a + b*tan(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)^3/(a + b*tan(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+b*tan(x)**4)**(1/2),x)
```

```
[Out] Integral(tan(x)**3/sqrt(a + b*tan(x)**4), x)
```

3.397 $\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

[Out] -1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(1/2)

Rubi [A] time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3670, 1248, 725, 206}

$$-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Tan[x]/Sqrt[a + b*Tan[x]^4],x]

[Out] -ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b
])

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx &= \text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] -1/2*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/Sqrt[a +
b]
```

fricas [A] time = 0.64, size = 150, normalized size = 3.66

$$\left[\frac{\log\left(\frac{(ab+2b^2)\tan(x)^4-2ab\tan(x)^2+2\sqrt{b\tan(x)^4+a}\,(b\tan(x)^2-a)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)}{4\sqrt{a+b}}, -\frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\,(b\tan(x)^2-a)\sqrt{-a-b}}{(ab+b^2)\tan(x)^4+a^2+ab}\right)}{2(a+b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*
(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1))/sq
rt(a + b), -1/2*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*s
qrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b))/(a + b)]
```

giac [A] time = 0.46, size = 46, normalized size = 1.12

$$\frac{\arctan\left(-\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/s
qrt(-a - b)
```

maple [A] time = 0.23, size = 65, normalized size = 1.59

$$-\frac{\ln\left(\frac{2a+2b-2(1+\tan^2(x))b+2\sqrt{a+b}\sqrt{(1+\tan^2(x))^2 b-2(1+\tan^2(x))b+a+b}}{1+\tan^2(x)}\right)}{2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*tan(x)^4)^(1/2),x)
```

```
[Out] -1/2/(a+b)^(1/2)*ln((2*a+2*b-2*(1+tan(x)^2)*b+2*(a+b)^(1/2)*((1+tan(x)^2)^2
*b-2*(1+tan(x)^2)*b+a+b)^(1/2))/(1+tan(x)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{b\tan(x)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/sqrt(b*tan(x)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(x)}{\sqrt{b\,{\tan(x)}^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*tan(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)/(a + b*tan(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{a + b\tan^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)**4)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(a + b*tan(x)**4), x)
```

3.398 $\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))/a^(1/2)+1/2*arctanh((a-b*tan(x)^
2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3670, 1252, 961, 725, 206, 266, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b]
) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1-x)\sqrt{a+bx^2}}+\frac{1}{x\sqrt{a+bx^2}}\right)dx, x, \tan^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \tan^2(x\right.\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^4(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a+b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right.\\
&= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^4(x)}\right)}{2b}\\
&= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + b*Tan[x]^4],x]
```

[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b]
) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*Sqrt[a])

fricas **[A]** time = 0.75, size = 475, normalized size = 6.79

$$\left[\frac{\sqrt{a+b}\,a\log\left(\frac{\left(ab+2\,b^2\right)\tan(x)^4-2\,ab\tan(x)^2-2\,\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a+b}+2\,a^2+ab}{\tan(x)^4+2\,\tan(x)^2+1}\right)+(a+b)\sqrt{a}\,\log\left(-\frac{b\tan(x)^4-2\,\sqrt{b\tan(x)^4+}}{\tan(x)^4}\right.}{4\left(a^2+ab\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*a*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b
*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*ta
n(x)^2 + 1)) + (a + b)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sq
rt(a) + 2*a)/tan(x)^4))/(a^2 + a*b), 1/4*(2*sqrt(-a)*(a + b)*arctan(sqrt(b*
tan(x)^4 + a)*sqrt(-a)/a) + sqrt(a + b)*a*log(((a*b + 2*b^2)*tan(x)^4 - 2*a
*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 +
 a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)))/(a^2 + a*b), 1/4*(2*a*sqrt(-a - b)*arct
an(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4
 + a^2 + a*b)) + (a + b)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*
sqrt(a) + 2*a)/tan(x)^4))/(a^2 + a*b), 1/2*(a*sqrt(-a - b)*arctan(sqrt(b*ta
n(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b
)) + sqrt(-a)*(a + b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a))/(a^2 + a*b)]
```

giac **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple **[F]** time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{a+b\left(\tan^4(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*tan(x)^4)^(1/2),x)
```

```
[Out] int(cot(x)/(a+b*tan(x)^4)^(1/2),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{b\tan(x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)/sqrt(b*tan(x)^4 + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(x)}{\sqrt{b\,{\tan(x)}^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*tan(x)^4)^(1/2),x)
```

```
[Out] int(cot(x)/(a + b*tan(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{a + b\tan^4(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)**4)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a + b*tan(x)**4), x)
```

3.399 $\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal. Leaf size=291

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(x)}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{}\right.}{}$$

```
[Out] -1/2*arctan((a+b)^(1/2)*tan(x)/(a+b*tan(x)^4)^(1/2))/(a+b)^(1/2)+1/2*a^(1/4
)*(cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*tan(
x)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*tan(x)/a^(1/4))),1/2*2^(1/2))*(
(a+b*tan(x)^4)/(a^(1/2)+b^(1/2)*tan(x)^2)^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(x)^
2)/b^(1/4)/(a^(1/2)-b^(1/2))/(a+b*tan(x)^4)^(1/2)-1/4*(cos(2*arctan(b^(1/4)
*tan(x)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))*EllipticPi
(sin(2*arctan(b^(1/4)*tan(x)/a^(1/4))),-1/4*(a^(1/2)-b^(1/2))^2/a^(1/2)/b^(
1/2),1/2*2^(1/2))*(a^(1/2)+b^(1/2))*((a+b*tan(x)^4)/(a^(1/2)+b^(1/2)*tan(x)
^2)^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(x)^2)/a^(1/4)/b^(1/4)/(a^(1/2)-b^(1/2))/(
a+b*tan(x)^4)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3670, 1320, 220, 1707}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(x)}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] -ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a + b*Tan[x]^4]]/(2*Sqrt[a + b]) + (a^(1/
4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Ta
n[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(2*(Sqrt[a]
- Sqrt[b])*b^(1/4)*Sqrt[a + b*Tan[x]^4]) - ((Sqrt[a] + Sqrt[b])*EllipticPi[
-(Sqrt[a] - Sqrt[b])^2/(4*Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[x])/a^(1/
4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqr
t[b]*Tan[x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)*Sqrt[a + b*Tan[x]
^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1320

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2]}, -Dist[(a*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a +
c*x^4], x], x] + Dist[(a*d*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)$$
$$= \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)}{\sqrt{a}-\sqrt{b}} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\left(1+x^2\right)\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)}{\sqrt{a}-\sqrt{b}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}} + \frac{\sqrt[4]{a}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\,\tan^2(x)\right)\sqrt{\frac{a+b\tan^4}{\left(\sqrt{a}+\sqrt{b}\,\tan\right.}}}{2\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\,\sqrt{a+b\tan^4(x)}}$$

Mathematica [C] time = 2.36, size = 122, normalized size = 0.42

$$-\frac{i\sqrt{\frac{b\tan^4(x)}{a}+1}\left(F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(x)\right)\middle|-1\right)-\Pi\left(-\frac{i\sqrt{a}}{\sqrt{b}};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(x)\right)\middle|-1\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\tan^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ((-I)*(EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1] - Ellipti
cPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1
])*Sqrt[1 + (b*Tan[x]^4)/a])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*Tan[x]^4
])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tan(x)^2}{\sqrt{b\tan(x)^4+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(tan(x)^2/sqrt(b*tan(x)^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)^2}{\sqrt{b\tan(x)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(x)^2/sqrt(b*tan(x)^4 + a), x)
```

maple [C] time = 0.35, size = 179, normalized size = 0.62

$$\frac{\sqrt{1-\frac{i\sqrt{b}\left(\tan^2(x)\right)}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\left(\tan^2(x)\right)}{\sqrt{a}}}\;\mathrm{EllipticF}\left(\tan(x)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\,i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\left(\tan^4(x)\right)}}-\frac{\sqrt{1-\frac{i\sqrt{b}\left(\tan^2(x)\right)}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\left(\tan^2(x)\right)}{\sqrt{a}}}\;\mathrm{EllipticPi}\left(\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\left(\tan^4(x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+b*tan(x)^4)^(1/2),x)
```

```
[Out] 1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/
2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticF(tan(x)*(I/a^(1/2)
*b^(1/2))^(1/2),I)-1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*Elliptic
Pi(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^
(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)^2}{\sqrt{b\tan(x)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^2/sqrt(b*tan(x)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\tan(x)}^2}{\sqrt{b\,{\tan(x)}^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + b*tan(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)^2/(a + b*tan(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(x)}{\sqrt{a + b\tan^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+b*tan(x)**4)**(1/2),x)
```

```
[Out] Integral(tan(x)**2/sqrt(a + b*tan(x)**4), x)
```

3.400 $\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}}$$

```
[Out] 1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(3/2)+1/
2*(-1+tan(x)^2)/(a+b)/(a+b*tan(x)^4)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 1252, 823, 12, 725, 206}

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(3/
2)) - (1 - Tan[x]^2)/(2*(a + b)*Sqrt[a + b*Tan[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x^3}{\left(1+x^2\right)\left(a+bx^4\right)^{3/2}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan^2(x)\right)\\
&= -\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\text{Subst}\left(\int \frac{ab}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)}{2ab(a+b)}\\
&= -\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)}{2(a+b)}\\
&= -\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}}+\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)}\\
&= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 67, normalized size = 0.94

$$\frac{1}{2}\left(\frac{\tan^2(x)-1}{(a+b)\sqrt{a+b\tan^4(x)}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3/2)
 + (-1 + Tan[x]^2)/((a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

fricas [B] time = 0.78, size = 292, normalized size = 4.11

$$\left[\frac{\left(b\tan(x)^4+a\right)\sqrt{a+b}\,\log\left(\frac{\left(ab+2b^2\right)\tan(x)^4-2ab\tan(x)^2-2\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)+2\sqrt{b\tan(x)^4+a}\left((a+\right.}{4\left(\left(a^2b+2ab^2+b^3\right)\tan(x)^4+a^3+2a^2b+ab^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*tan(x)^4 + a)*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(
x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(
tan(x)^4 + 2*tan(x)^2 + 1)) + 2*sqrt(b*tan(x)^4 + a)*((a + b)*tan(x)^2 - a
- b))/((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*b^2), 1/2*((b*t
an(x)^4 + a)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt
(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + sqrt(b*tan(x)^4 + a)*((a + b
)*tan(x)^2 - a - b))/((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*
b^2)]
```

giac [A] time = 0.55, size = 103, normalized size = 1.45

$$\frac{\frac{(a+b)\tan(x)^2}{a^2+2\,ab+b^2}-\frac{a+b}{a^2+2\,ab+b^2}}{2\,\sqrt{b\tan(x)^4+a}}+\frac{\arctan\left(\frac{\sqrt{b}\,\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{(a+b)\sqrt{-a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((a + b)*tan(x)^2/(a^2 + 2*a*b + b^2) - (a + b)/(a^2 + 2*a*b + b^2))/sq
rt(b*tan(x)^4 + a) + arctan((sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt
(b))/sqrt(-a - b))/((a + b)*sqrt(-a - b))
```

maple [B] time = 0.38, size = 267, normalized size = 3.76

$$\frac{\tan^2(x)}{2a\sqrt{a+b\left(\tan^4(x)\right)}}+\frac{\sqrt{\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)^2 b-2\sqrt{-ab}\,\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)}}{4\left(\sqrt{-ab}\,-b\right)a\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)}-\frac{\sqrt{\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\,\left(\cdots\right.}}{4\left(\sqrt{-ab}\,+b\right)a\left(\tan^2(x)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+b*tan(x)^4)^(3/2),x)
```

```
[Out] 1/2*tan(x)^2/a/(a+b*tan(x)^4)^(1/2)+1/4/((-a*b)^(1/2)-b)/a/(tan(x)^2+(-a*b)
^(1/2)/b)*((tan(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1
/2)/b))^(1/2)-1/4/((-a*b)^(1/2)+b)/a/(tan(x)^2-(-a*b)^(1/2)/b)*((tan(x)^2-(
-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)-1/2*b/((
-a*b)^(1/2)+b)/((-a*b)^(1/2)-b)/(a+b)^(1/2)*ln((2*a+2*b-2*(1+tan(x)^2)*b+2*
(a+b)^(1/2)*((1+tan(x)^2)^2*b-2*(1+tan(x)^2)*b+a+b)^(1/2))/(1+tan(x)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)^3}{\left(b\tan(x)^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^3/(b*tan(x)^4 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)^3}{\left(b\tan(x)^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a + b*tan(x)^4)^(3/2),x)
```

```
[Out] int(tan(x)^3/(a + b*tan(x)^4)^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral(tan(x)**3/(a + b*tan(x)**4)**(3/2), x)
```

3.401 $\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}$$

```
[Out] -1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(3/2)+1
/2*(a+b*tan(x)^2)/a/(a+b)/(a+b*tan(x)^4)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3670, 1248, 741, 12, 725, 206}

$$\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] -ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(3
/2)) + (a + b*Tan[x]^2)/(2*a*(a + b)*Sqrt[a + b*Tan[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x}{\left(1+x^2\right)\left(a+bx^4\right)^{3/2}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan^2(x)\right)\\
&= \frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{\text{Subst}\left(\int \frac{a}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)}{2a(a+b)}\\
&= \frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)}{2(a+b)}\\
&= \frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)}\\
&= -\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}+\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 73, normalized size = 0.99

$$\frac{1}{2}\left(\frac{a+b\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (-(ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3/
2)) + (a + b*Tan[x]^2)/(a*(a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

fricas [B] time = 0.75, size = 319, normalized size = 4.31

$$\left[\frac{\left(ab\tan(x)^4+a^2\right)\sqrt{a+b}\,\log\left(\frac{\left(ab+2\,b^2\right)\tan(x)^4-2\,ab\tan(x)^2+2\,\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a+b}+2\,a^2+ab}{\tan(x)^4+2\,\tan(x)^2+1}\right)+2\,\sqrt{b\tan(x)^4+a}\,\left(\left(\right.\right.}{4\left(\left(a^3b+2\,a^2b^2+ab^3\right)\tan(x)^4+a^4+2\,a^3b+a^2b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*tan(x)^4 + a^2)*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*
tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*
b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*sqrt(b*tan(x)^4 + a)*((a*b + b^2)*tan(x
)^2 + a^2 + a*b))/((a^3*b + 2*a^2*b^2 + a*b^3)*tan(x)^4 + a^4 + 2*a^3*b + a
^2*b^2), -1/2*((a*b*tan(x)^4 + a^2)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a
)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) - sqrt(
b*tan(x)^4 + a)*((a*b + b^2)*tan(x)^2 + a^2 + a*b))/((a^3*b + 2*a^2*b^2 + a
*b^3)*tan(x)^4 + a^4 + 2*a^3*b + a^2*b^2)]
```

giac [A] time = 0.47, size = 119, normalized size = 1.61

$$\frac{\frac{\left(ab+b^2\right)\tan(x)^2}{a^3+2\,a^2b+ab^2}+\frac{a^2+ab}{a^3+2\,a^2b+ab^2}}{2\,\sqrt{b\tan(x)^4+a}}-\frac{\arctan\left(\frac{\sqrt{b}\,\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{(a+b)\sqrt{-a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((a*b + b^2)*tan(x)^2/(a^3 + 2*a^2*b + a*b^2) + (a^2 + a*b)/(a^3 + 2*a^
2*b + a*b^2))/sqrt(b*tan(x)^4 + a) - arctan((sqrt(b)*tan(x)^2 - sqrt(b*tan(
x)^4 + a) + sqrt(b))/sqrt(-a - b))/((a + b)*sqrt(-a - b))
```

maple [B] time = 0.23, size = 248, normalized size = 3.35

$$-\frac{\sqrt{\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)^2 b-2\sqrt{-ab}\,\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)}}{4\left(\sqrt{-ab}\,-b\right)a\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)}+\frac{\sqrt{\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\,\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)}}{4\left(\sqrt{-ab}\,+b\right)a\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)}+-\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*tan(x)^4)^(3/2),x)
```

```
[Out] -1/4/((-a*b)^(1/2)-b)/a/(tan(x)^2+(-a*b)^(1/2)/b)*((tan(x)^2+(-a*b)^(1/2)/b
)^2*b-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)+1/4/((-a*b)^(1/2)+b)/
a/(tan(x)^2-(-a*b)^(1/2)/b)*((tan(x)^2-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(
tan(x)^2-(-a*b)^(1/2)/b))^(1/2)+1/2*b/((-a*b)^(1/2)+b)/((-a*b)^(1/2)-b)/(a+
b)^(1/2)*ln((2*a+2*b-2*(1+tan(x)^2)*b+2*(a+b)^(1/2)*((1+tan(x)^2)^2*b-2*(1+
tan(x)^2)*b+a+b)^(1/2))/(1+tan(x)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(b\tan(x)^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(b*tan(x)^4 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)}{\left(b\,\tan(x)^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*tan(x)^4)^(3/2),x)
```

```
[Out] int(tan(x)/(a + b*tan(x)^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(a + b\tan^4(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral(tan(x)/(a + b*tan(x)**4)**(3/2), x)
```

3.402 $\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{1}{2a\sqrt{a+b\tan^4(x)}}-\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}$$

```
[Out] -1/2*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))/a^(3/2)+1/2*arctanh((a-b*tan(x)^
2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(3/2)+1/2/a/(a+b*tan(x)^4)^(1/2)
+1/2*(-a-b*tan(x)^2)/a/(a+b)/(a+b*tan(x)^4)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {3670, 1252, 961, 741, 12, 725, 206, 266, 51, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{1}{2a\sqrt{a+b\tan^4(x)}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(3/
2)) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*a^(3/2)) + 1/(2*a*Sqrt[a + b
*Tan[x]^4]) - (a + b*Tan[x]^2)/(2*a*(a + b)*Sqrt[a + b*Tan[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\left(a+bx^4\right)^{3/2}}\,dx,x,\tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)\left(a+bx^2\right)^{3/2}}\,dx,x,\tan^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1-x)\left(a+bx^2\right)^{3/2}}+\frac{1}{x\left(a+bx^2\right)^{3/2}}\right)dx,x,\tan^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1-x)\left(a+bx^2\right)^{3/2}}\,dx,x,\tan^2(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx,x,\right.\\
&= -\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,\tan^4(x)\right)+\frac{\text{Subst}\left(\int \frac{}{(-}\right.}{}\\
&= \frac{1}{2a\sqrt{a+b\tan^4(x)}}-\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\tan^4(x)\right)}{4a}+\\
&= \frac{1}{2a\sqrt{a+b\tan^4(x)}}-\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\tan^4}\right.}{2ab}\\
&= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{1}{2a\sqrt{a+b\tan^4(x)}}-\frac{a+}{2a(a+b)}
\end{aligned}$$

Mathematica [C] time = 0.62, size = 108, normalized size = 0.89

$$\frac{1}{2}\left(\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\tan^4(x)}{a}+1\right)}{a\sqrt{a+b\tan^4(x)}}-\frac{a+b\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3/2)
 + Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tan[x]^4)/a]/(a*Sqrt[a + b*Tan[x]
^4]) - (a + b*Tan[x]^2)/(a*(a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

fricas [B] time = 0.99, size = 954, normalized size = 7.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*b*tan(x)^4 + a^3)*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*
b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 +
a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + ((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3
 + 2*a^2*b + a*b^2)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(
a) + 2*a)/tan(x)^4) + 2*sqrt(b*tan(x)^4 + a)*(a^2*b + a*b^2 - (a^2*b + a*b^
```

```
2)*tan(x)^2))/(a^5 + 2*a^4*b + a^3*b^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*tan(
x)^4), 1/4*(2*((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*b^2)*sq
rt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + (a^2*b*tan(x)^4 + a^3)*sqr
t(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 +
 a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)
) + 2*sqrt(b*tan(x)^4 + a)*(a^2*b + a*b^2 - (a^2*b + a*b^2)*tan(x)^2))/(a^5
 + 2*a^4*b + a^3*b^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*tan(x)^4), 1/4*(2*(a^2
*b*tan(x)^4 + a^3)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a
)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + ((a^2*b + 2*a*b^2 + b^
3)*tan(x)^4 + a^3 + 2*a^2*b + a*b^2)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*ta
n(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) + 2*sqrt(b*tan(x)^4 + a)*(a^2*b + a*b^
2 - (a^2*b + a*b^2)*tan(x)^2))/(a^5 + 2*a^4*b + a^3*b^2 + (a^4*b + 2*a^3*b^
2 + a^2*b^3)*tan(x)^4), 1/2*((a^2*b*tan(x)^4 + a^3)*sqrt(-a - b)*arctan(sqr
t(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2
 + a*b)) + ((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*b^2)*sqrt(
-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + sqrt(b*tan(x)^4 + a)*(a^2*b +
 a*b^2 - (a^2*b + a*b^2)*tan(x)^2))/(a^5 + 2*a^4*b + a^3*b^2 + (a^4*b + 2*a
^3*b^2 + a^2*b^3)*tan(x)^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\left(a+b\left(\tan^4(x)\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*tan(x)^4)^(3/2),x)
```

```
[Out] int(cot(x)/(a+b*tan(x)^4)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(x)}{\left(b\tan(x)^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*tan(x)^4)^(3/2),x)
```

```
[Out] int(cot(x)/(a + b*tan(x)^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral(cot(x)/(a + b*tan(x)**4)**(3/2), x)
```

3.403 $\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=109

$$-\frac{(b-2a)\tan^2(x)+3a}{6a(a+b)^2\sqrt{a+b\tan^4(x)}}-\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}$$

```
[Out] 1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(5/2)+1/
6*(-3*a-(-2*a+b)*tan(x)^2)/a/(a+b)^2/(a+b*tan(x)^4)^(1/2)+1/6*(-1+tan(x)^2)
/(a+b)/(a+b*tan(x)^4)^(3/2)
```

Rubi [A] time = 0.23, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3670, 1252, 823, 12, 725, 206}

$$-\frac{(b-2a)\tan^2(x)+3a}{6a(a+b)^2\sqrt{a+b\tan^4(x)}}-\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(5/
2)) - (1 - Tan[x]^2)/(6*(a + b)*(a + b*Tan[x]^4)^(3/2)) - (3*a + (-2*a + b)
*Tan[x]^2)/(6*a*(a + b)^2*Sqrt[a + b*Tan[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^3}{\left(1+x^2\right)\left(a+bx^4\right)^{5/2}}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan^2(x)\right)$$

$$= -\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{ab-2abx}{(1+x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan^2(x)\right)}{6ab(a+b)}$$

$$= -\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{3a-(2a-b)\tan^2(x)}{6a(a+b)^2\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int -\frac{3a^2b^2}{(1+x)\sqrt{a+bx^2}}\,d\right.}{6a^2b^2(a+b}$$

$$= -\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{3a-(2a-b)\tan^2(x)}{6a(a+b)^2\sqrt{a+b\tan^4(x)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx\right.}{2(a+b)^2}$$

$$= -\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{3a-(2a-b)\tan^2(x)}{6a(a+b)^2\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, -\right.}{2(a+b)^2}$$

$$= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{3a-(2a-b)\tan^2(x)}{6a(a+b)^2\sqrt{a+b\tan^4(x)}}$$

Mathematica [A] time = 0.87, size = 104, normalized size = 0.95

$$\frac{1}{6}\left(\frac{3a^2\tan^2(x)+b(2a-b)\tan^6(x)-3ab\tan^4(x)-a(4a+b)}{a(a+b)^2\left(a+b\tan^4(x)\right)^{3/2}} + \frac{3\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ((3*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/(a + b)^(
5/2) + (-(a*(4*a + b)) + 3*a^2*Tan[x]^2 - 3*a*b*Tan[x]^4 + (2*a - b)*b*Tan[
x]^6)/(a*(a + b)^2*(a + b*Tan[x]^4)^(3/2)))/6
```

fricas [B] time = 0.80, size = 556, normalized size = 5.10

$$\left[\frac{3\left(ab^2\tan(x)^8+2a^2b\tan(x)^4+a^3\right)\sqrt{a+b}\log\left(\frac{\left(ab+2b^2\right)\tan(x)^4-2ab\tan(x)^2-2\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)+2\cdots}{12\left(\left(a^4b^2+3a^3b^3+3a^2b^4+ab^5\right)\tan(x)^8+a^6+3a^5b+3\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*b^2*tan(x)^8 + 2*a^2*b*tan(x)^4 + a^3)*sqrt(a + b)*log(((a*b +
2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*
sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*((2*a^2*b + a*b
^2 - b^3)*tan(x)^6 - 3*(a^2*b + a*b^2)*tan(x)^4 - 4*a^3 - 5*a^2*b - a*b^2 +
 3*(a^3 + a^2*b)*tan(x)^2)*sqrt(b*tan(x)^4 + a))/((a^4*b^2 + 3*a^3*b^3 + 3*
a^2*b^4 + a*b^5)*tan(x)^8 + a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + 2*(a^5*b
+ 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(x)^4), 1/6*(3*(a*b^2*tan(x)^8 + 2*a^
2*b*tan(x)^4 + a^3)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 -
a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + ((2*a^2*b + a*b^2 - b
^3)*tan(x)^6 - 3*(a^2*b + a*b^2)*tan(x)^4 - 4*a^3 - 5*a^2*b - a*b^2 + 3*(a^
3 + a^2*b)*tan(x)^2)*sqrt(b*tan(x)^4 + a))/((a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^
4 + a*b^5)*tan(x)^8 + a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + 2*(a^5*b + 3*a^
4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(x)^4)]
```

giac [B] time = 0.69, size = 597, normalized size = 5.48

$$\frac{\left(\left(\left(\frac{\left(2a^7b^2+11a^6b^3+24a^5b^4+25a^4b^5+10a^3b^6-3a^2b^7-4ab^8-b^9\right)\tan(x)^2}{a^9b+8a^8b^2+28a^7b^3+56a^6b^4+70a^5b^5+56a^4b^6+28a^3b^7+8a^2b^8+ab^9}-\frac{3\left(a^7b^2+6a^6b^3+15a^5b^4+20a^4b^5+15a^3b^6+6a^2b^7+ab^8\right)}{a^9b+8a^8b^2+28a^7b^3+56a^6b^4+70a^5b^5+56a^4b^6+28a^3b^7+8a^2b^8+ab^9}\right)\mathrm{ta}\cdots}{6\left(b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((((2*a^7*b^2 + 11*a^6*b^3 + 24*a^5*b^4 + 25*a^4*b^5 + 10*a^3*b^6 - 3*a
^2*b^7 - 4*a*b^8 - b^9)*tan(x)^2/(a^9*b + 8*a^8*b^2 + 28*a^7*b^3 + 56*a^6*b
^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a^3*b^7 + 8*a^2*b^8 + a*b^9) - 3*(a^7*b^2
 + 6*a^6*b^3 + 15*a^5*b^4 + 20*a^4*b^5 + 15*a^3*b^6 + 6*a^2*b^7 + a*b^8)/(a
^9*b + 8*a^8*b^2 + 28*a^7*b^3 + 56*a^6*b^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a
^3*b^7 + 8*a^2*b^8 + a*b^9))*tan(x)^2 + 3*(a^8*b + 6*a^7*b^2 + 15*a^6*b^3 +
 20*a^5*b^4 + 15*a^4*b^5 + 6*a^3*b^6 + a^2*b^7)/(a^9*b + 8*a^8*b^2 + 28*a^7
*b^3 + 56*a^6*b^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a^3*b^7 + 8*a^2*b^8 + a*b^
9))*tan(x)^2 - (4*a^8*b + 25*a^7*b^2 + 66*a^6*b^3 + 95*a^5*b^4 + 80*a^4*b^5
 + 39*a^3*b^6 + 10*a^2*b^7 + a*b^8)/(a^9*b + 8*a^8*b^2 + 28*a^7*b^3 + 56*a^
6*b^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a^3*b^7 + 8*a^2*b^8 + a*b^9))/(b*tan(x
)^4 + a)^(3/2) + arctan((sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))
/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*sqrt(-a - b))
```

maple [B] time = 0.29, size = 654, normalized size = 6.00

$$\frac{\sqrt{a+b\left(\tan^4(x)\right)}\,\left(\tan^2(x)\right)\left(2b\left(\tan^4(x)\right)+3a\right)}{6a^2\left(\left(\tan^8(x)\right)b^2+2\left(\tan^4(x)\right)ab+a^2\right)}-\frac{\left(2\sqrt{-ab}+b\right)\sqrt{\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)^2b+2\sqrt{-ab}\,\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\cdots\right.}}{8\left(\sqrt{-ab}+b\right)^2a^2\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+b*tan(x)^4)^(5/2),x)
```

```
[Out] 1/6*(a+b*tan(x)^4)^(1/2)*tan(x)^2*(2*b*tan(x)^4+3*a)/a^2/(tan(x)^8*b^2+2*ta
n(x)^4*a*b+a^2)-1/8*(2*(-a*b)^(1/2)+b)/((-a*b)^(1/2)+b)^2/a^2/(tan(x)^2-(-a
*b)^(1/2)/b)*((tan(x)^2-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)
^(1/2)/b))^(1/2)+1/8*(2*(-a*b)^(1/2)-b)/((-a*b)^(1/2)-b)^2/a^2/(tan(x)^2+(-
a*b)^(1/2)/b)*((tan(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b
)^(1/2)/b))^(1/2)-1/24/((-a*b)^(1/2)-b)/a/(-a*b)^(1/2)/(tan(x)^2+(-a*b)^(1/
2)/b)^2*((tan(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2
)/b))^(1/2)+1/24/((-a*b)^(1/2)-b)/a^2/(tan(x)^2+(-a*b)^(1/2)/b)*((tan(x)^2+
(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)+1/2*b^2
/((-a*b)^(1/2)+b)^2/((-a*b)^(1/2)-b)^2/(a+b)^(1/2)*ln((2*a+2*b-2*(1+tan(x)^
2)*b+2*(a+b)^(1/2)*((1+tan(x)^2)^2*b-2*(1+tan(x)^2)*b+a+b)^(1/2))/(1+tan(x)
^2))-1/24/((-a*b)^(1/2)+b)/a/(-a*b)^(1/2)/(tan(x)^2-(-a*b)^(1/2)/b)^2*((tan
(x)^2-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)-1
/24/((-a*b)^(1/2)+b)/a^2/(tan(x)^2-(-a*b)^(1/2)/b)*((tan(x)^2-(-a*b)^(1/2)/
b)^2*b+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)^3}{\left(b\tan(x)^4 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^3/(b*tan(x)^4 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)^3}{\left(b\tan(x)^4 + a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a + b*tan(x)^4)^(5/2),x)
```

```
[Out] int(tan(x)^3/(a + b*tan(x)^4)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(x)}{\left(a + b\tan^4(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+b*tan(x)**4)**(5/2),x)
```

```
[Out] Integral(tan(x)**3/(a + b*tan(x)**4)**(5/2), x)
```

3.404 $\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}+\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}$$

```
[Out] -1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(5/2)+1
/6*(3*a^2+b*(5*a+2*b)*tan(x)^2)/a^2/(a+b)^2/(a+b*tan(x)^4)^(1/2)+1/6*(a+b*t
an(x)^2)/a/(a+b)/(a+b*tan(x)^4)^(3/2)
```

Rubi [A] time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3670, 1248, 741, 823, 12, 725, 206}

$$\frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}+\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] -ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(5
/2)) + (a + b*Tan[x]^2)/(6*a*(a + b)*(a + b*Tan[x]^4)^(3/2)) + (3*a^2 + b*(
5*a + 2*b)*Tan[x]^2)/(6*a^2*(a + b)^2*Sqrt[a + b*Tan[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x}{\left(1+x^2\right)\left(a+bx^4\right)^{5/2}}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1+x)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan^2(x)\right)$$

$$= \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a-2b-2bx}{(1+x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan^2(x)\right)}{6a(a+b)}$$

$$= \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{3a^2b}{(1+x)\sqrt{a+bx^2}}\,d\right.}{6a^2b(a+b}$$

$$= \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,d\right.}{2(a+b)^2}$$

$$= \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}} - \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \right.}{2(a+b)^2}$$

$$= -\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}} + \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{3a^2+b(5a+2b)\tan^2}{6a^2(a+b)^2\sqrt{a+b\tan}}$$

Mathematica [A] time = 0.83, size = 113, normalized size = 0.97

$$\frac{1}{6}\left(\frac{3a^2b\tan^4(x)+a^2(4a+b)+b^2(5a+2b)\tan^6(x)+3ab(2a+b)\tan^2(x)}{a^2(a+b)^2\left(a+b\tan^4(x)\right)^{3/2}}-\frac{3\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ((-3*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/(a + b)^
(5/2) + (a^2*(4*a + b) + 3*a*b*(2*a + b)*Tan[x]^2 + 3*a^2*b*Tan[x]^4 + b^2*
(5*a + 2*b)*Tan[x]^6)/(a^2*(a + b)^2*(a + b*Tan[x]^4)^(3/2)))/6
```

fricas [B] time = 0.96, size = 599, normalized size = 5.12

$$\left[\frac{3\left(a^2b^2\tan(x)^8+2a^3b\tan(x)^4+a^4\right)\sqrt{a+b}\,\log\left(\frac{\left(ab+2b^2\right)\tan(x)^4-2ab\tan(x)^2+2\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)+}{12\left(\left(a^5b^2+3a^4b^3+3a^3b^4+a^2b^5\right)\tan(x)^8+a^7\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*tan(x)^8 + 2*a^3*b*tan(x)^4 + a^4)*sqrt(a + b)*log(((a*b
+ 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a
)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*((5*a^2*b^2 +
 7*a*b^3 + 2*b^4)*tan(x)^6 + 3*(a^3*b + a^2*b^2)*tan(x)^4 + 4*a^4 + 5*a^3*b
 + a^2*b^2 + 3*(2*a^3*b + 3*a^2*b^2 + a*b^3)*tan(x)^2)*sqrt(b*tan(x)^4 + a)
)/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*tan(x)^8 + a^7 + 3*a^6*b + 3
*a^5*b^2 + a^4*b^3 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*tan(x)^4),
 -1/6*(3*(a^2*b^2*tan(x)^8 + 2*a^3*b*tan(x)^4 + a^4)*sqrt(-a - b)*arctan(sq
rt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^
2 + a*b)) - ((5*a^2*b^2 + 7*a*b^3 + 2*b^4)*tan(x)^6 + 3*(a^3*b + a^2*b^2)*t
an(x)^4 + 4*a^4 + 5*a^3*b + a^2*b^2 + 3*(2*a^3*b + 3*a^2*b^2 + a*b^3)*tan(x
)^2)*sqrt(b*tan(x)^4 + a))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*tan
(x)^8 + a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*
b^3 + a^3*b^4)*tan(x)^4)]
```

giac [B] time = 0.59, size = 618, normalized size = 5.28

$$\left(\left(\left(\frac{\left(5a^7b^3+32a^6b^4+87a^5b^5+130a^4b^6+115a^3b^7+60a^2b^8+17ab^9+2b^{10}\right)\tan(x)^2}{a^{10}b+8a^9b^2+28a^8b^3+56a^7b^4+70a^6b^5+56a^5b^6+28a^4b^7+8a^3b^8+a^2b^9}+\frac{3\left(a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8\right)}{a^{10}b+8a^9b^2+28a^8b^3+56a^7b^4+70a^6b^5+56a^5b^6+28a^4b^7+8a^3b^8+\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((((5*a^7*b^3 + 32*a^6*b^4 + 87*a^5*b^5 + 130*a^4*b^6 + 115*a^3*b^7 + 6
0*a^2*b^8 + 17*a*b^9 + 2*b^10)*tan(x)^2/(a^10*b + 8*a^9*b^2 + 28*a^8*b^3 +
56*a^7*b^4 + 70*a^6*b^5 + 56*a^5*b^6 + 28*a^4*b^7 + 8*a^3*b^8 + a^2*b^9) +
3*(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 +
 a^2*b^8)/(a^10*b + 8*a^9*b^2 + 28*a^8*b^3 + 56*a^7*b^4 + 70*a^6*b^5 + 56*a
^5*b^6 + 28*a^4*b^7 + 8*a^3*b^8 + a^2*b^9))*tan(x)^2 + 3*(2*a^8*b^2 + 13*a^
7*b^3 + 36*a^6*b^4 + 55*a^5*b^5 + 50*a^4*b^6 + 27*a^3*b^7 + 8*a^2*b^8 + a*b
^9)/(a^10*b + 8*a^9*b^2 + 28*a^8*b^3 + 56*a^7*b^4 + 70*a^6*b^5 + 56*a^5*b^6
```

```
 + 28*a^4*b^7 + 8*a^3*b^8 + a^2*b^9))*tan(x)^2 + (4*a^9*b + 25*a^8*b^2 + 66
*a^7*b^3 + 95*a^6*b^4 + 80*a^5*b^5 + 39*a^4*b^6 + 10*a^3*b^7 + a^2*b^8)/(a^
10*b + 8*a^9*b^2 + 28*a^8*b^3 + 56*a^7*b^4 + 70*a^6*b^5 + 56*a^5*b^6 + 28*a
^4*b^7 + 8*a^3*b^8 + a^2*b^9))/(b*tan(x)^4 + a)^(3/2) - arctan((sqrt(b)*tan
(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*
sqrt(-a - b))
```

maple [B] time = 0.25, size = 602, normalized size = 5.15

$$\frac{\left(2\sqrt{-ab}+b\right)\sqrt{\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)}}{8\left(\sqrt{-ab}+b\right)^2 a^2\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)}-\frac{\left(2\sqrt{-ab}-b\right)\sqrt{\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)^2 b-2\sqrt{-}}}{8\left(\sqrt{-ab}-b\right)^2 a^2\left(\tan^2(x)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*tan(x)^4)^(5/2),x)
```

```
[Out] 1/8*(2*(-a*b)^(1/2)+b)/((-a*b)^(1/2)+b)^2/a^2/(tan(x)^2-(-a*b)^(1/2)/b)*((t
an(x)^2-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)
-1/8*(2*(-a*b)^(1/2)-b)/((-a*b)^(1/2)-b)^2/a^2/(tan(x)^2+(-a*b)^(1/2)/b)*((
tan(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2
)+1/24/((-a*b)^(1/2)-b)/a/(-a*b)^(1/2)/(tan(x)^2+(-a*b)^(1/2)/b)^2*((tan(x)
^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)-1/24
/((-a*b)^(1/2)-b)/a^2/(tan(x)^2+(-a*b)^(1/2)/b)*((tan(x)^2+(-a*b)^(1/2)/b)^
2*b-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)-1/2*b^2/((-a*b)^(1/2)+b
)^2/((-a*b)^(1/2)-b)^2/(a+b)^(1/2)*ln((2*a+2*b-2*(1+tan(x)^2)*b+2*(a+b)^(1/
2)*((1+tan(x)^2)^2*b-2*(1+tan(x)^2)*b+a+b)^(1/2))/(1+tan(x)^2))+1/24/((-a*b
)^(1/2)+b)/a/(-a*b)^(1/2)/(tan(x)^2-(-a*b)^(1/2)/b)^2*((tan(x)^2-(-a*b)^(1/
2)/b)^2*b+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)+1/24/((-a*b)^(1/2
)+b)/a^2/(tan(x)^2-(-a*b)^(1/2)/b)*((tan(x)^2-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^
(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(b\tan(x)^4+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(b*tan(x)^4 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)}{\left(b\tan(x)^4+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*tan(x)^4)^(5/2),x)
```

```
[Out] int(tan(x)/(a + b*tan(x)^4)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)**4)**(5/2),x)
```

```
[Out] Integral(tan(x)/(a + b*tan(x)**4)**(5/2), x)
```

3.405 $\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{1}{2a^2\sqrt{a+b\tan^4(x)}}-\frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}+\frac{1}{6a\left(a+b\tan^4(x)\right)^{3/2}}-\frac{a+b\,\mathrm{t}}{6a(a+b)\left(a+\right.}$$

```
[Out] -1/2*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))/a^(5/2)+1/2*arctanh((a-b*tan(x)^
2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(5/2)+1/2/a^2/(a+b*tan(x)^4)^(1/
2)+1/6*(-3*a^2-b*(5*a+2*b)*tan(x)^2)/a^2/(a+b)^2/(a+b*tan(x)^4)^(1/2)+1/6/a
/(a+b*tan(x)^4)^(3/2)+1/6*(-a-b*tan(x)^2)/a/(a+b)/(a+b*tan(x)^4)^(3/2)
```

Rubi [A] time = 0.30, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3670, 1252, 961, 741, 823, 12, 725, 206, 266, 51, 63, 208}

$$-\frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}+\frac{1}{2a^2\sqrt{a+b\tan^4(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{\quad}{6a\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(5/
2)) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*a^(5/2)) + 1/(6*a*(a + b*Tan
[x]^4)^(3/2)) - (a + b*Tan[x]^2)/(6*a*(a + b)*(a + b*Tan[x]^4)^(3/2)) + 1/(
2*a^2*Sqrt[a + b*Tan[x]^4]) - (3*a^2 + b*(5*a + 2*b)*Tan[x]^2)/(6*a^2*(a +
b)^2*Sqrt[a + b*Tan[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\left(a+bx^4\right)^{5/2}}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1-x)\left(a+bx^2\right)^{5/2}}+\frac{1}{x\left(a+bx^2\right)^{5/2}}\right)dx, x, \tan^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1-x)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan^2(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx, x,$$

$$= -\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \tan^4(x)\right)-\frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{1}{6a\left(a+b\tan^4(x)\right)^{3/2}}-\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}-\frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}+$$

$$= \frac{1}{6a\left(a+b\tan^4(x)\right)^{3/2}}-\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{1}{2a^2\sqrt{a+b\tan^4(x)}}-\frac{3a^2+}{6a^2(a+}$$

$$= \frac{1}{6a\left(a+b\tan^4(x)\right)^{3/2}}-\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{1}{2a^2\sqrt{a+b\tan^4(x)}}-\frac{3a^2+}{6a^2(a+}$$

$$= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{1}{6a\left(a+b\tan^4(x)\right)^{3/2}}-\frac{}{6a(a+}$$

Mathematica [C] time = 1.54, size = 149, normalized size = 0.81

$$\frac{1}{6}\left(-\frac{3a^2b\tan^4(x)+a^2(4a+b)+b^2(5a+2b)\tan^6(x)+3ab(2a+b)\tan^2(x)}{a^2(a+b)^2\left(a+b\tan^4(x)\right)^{3/2}}+\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\tan^4(x)}{a}+1\right)}{a\left(a+b\tan^4(x)\right)^{3/2}}+\frac{3}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ((3*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/(a + b)^(
5/2) + Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tan[x]^4)/a]/(a*(a + b*Tan[x
]^4)^(3/2)) - (a^2*(4*a + b) + 3*a*b*(2*a + b)*Tan[x]^2 + 3*a^2*b*Tan[x]^4
+ b^2*(5*a + 2*b)*Tan[x]^6)/(a^2*(a + b)^2*(a + b*Tan[x]^4)^(3/2)))/6
```

fricas [B] time = 1.19, size = 1749, normalized size = 9.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^3*b^2*tan(x)^8 + 2*a^4*b*tan(x)^4 + a^5)*sqrt(a + b)*log(((a*b
+ 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a
)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 3*((a^3*b^2 + 3
*a^2*b^3 + 3*a*b^4 + b^5)*tan(x)^8 + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 +
2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(x)^4)*sqrt(a)*log(-(b*tan(x)^
4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) - 2*((5*a^3*b^2 + 7*a^2
*b^3 + 2*a*b^4)*tan(x)^6 - 7*a^4*b - 11*a^3*b^2 - 4*a^2*b^3 - 3*(2*a^3*b^2
+ 3*a^2*b^3 + a*b^4)*tan(x)^4 + 3*(2*a^4*b + 3*a^3*b^2 + a^2*b^3)*tan(x)^2)
*sqrt(b*tan(x)^4 + a))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*tan(x)^
8 + a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3
+ a^4*b^4)*tan(x)^4), 1/12*(6*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*tan(x)
^8 + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3
 + a*b^4)*tan(x)^4)*sqrt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 3*(a
^3*b^2*tan(x)^8 + 2*a^4*b*tan(x)^4 + a^5)*sqrt(a + b)*log(((a*b + 2*b^2)*ta
n(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a +
b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) - 2*((5*a^3*b^2 + 7*a^2*b^3
+ 2*a*b^4)*tan(x)^6 - 7*a^4*b - 11*a^3*b^2 - 4*a^2*b^3 - 3*(2*a^3*b^2 + 3*a
^2*b^3 + a*b^4)*tan(x)^4 + 3*(2*a^4*b + 3*a^3*b^2 + a^2*b^3)*tan(x)^2)*sqrt
(b*tan(x)^4 + a))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*tan(x)^8 + a
^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4
*b^4)*tan(x)^4), 1/12*(6*(a^3*b^2*tan(x)^8 + 2*a^4*b*tan(x)^4 + a^5)*sqrt(-
a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^
2)*tan(x)^4 + a^2 + a*b)) + 3*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*tan(x)
^8 + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3
 + a*b^4)*tan(x)^4)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(
a) + 2*a)/tan(x)^4) - 2*((5*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*tan(x)^6 - 7*a^4
*b - 11*a^3*b^2 - 4*a^2*b^3 - 3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(x)^4 +
3*(2*a^4*b + 3*a^3*b^2 + a^2*b^3)*tan(x)^2)*sqrt(b*tan(x)^4 + a))/((a^6*b^2
 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*tan(x)^8 + a^8 + 3*a^7*b + 3*a^6*b^2 +
a^5*b^3 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*tan(x)^4), 1/6*(3*(a^
3*b^2*tan(x)^8 + 2*a^4*b*tan(x)^4 + a^5)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^
4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) +
3*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*tan(x)^8 + a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(x)^4)*sqrt(-a)
*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) - ((5*a^3*b^2 + 7*a^2*b^3 + 2*a*b^
4)*tan(x)^6 - 7*a^4*b - 11*a^3*b^2 - 4*a^2*b^3 - 3*(2*a^3*b^2 + 3*a^2*b^3 +
 a*b^4)*tan(x)^4 + 3*(2*a^4*b + 3*a^3*b^2 + a^2*b^3)*tan(x)^2)*sqrt(b*tan(x
)^4 + a))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*tan(x)^8 + a^8 + 3*a
^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*ta
n(x)^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\left(a+b\left(\tan^4(x)\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*tan(x)^4)^(5/2),x)
[Out] int(cot(x)/(a+b*tan(x)^4)^(5/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*tan(x)^4)^(5/2),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)**4)**(5/2),x)
[Out] Integral(cot(x)/(a + b*tan(x)**4)**(5/2), x)
```

3.406 $\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 \, dx$

Optimal. Leaf size=212

$$\frac{\left(a^2-b^2\sqrt{-c^2}\right)\tan(e+fx)(d\tan(e+fx))^m\,{}_2F_1\left(1,m+1;m+2;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{2f(m+1)}+\frac{\left(a^2+b^2\sqrt{-c^2}\right)\tan(e+fx)(d\tan(}{2}$$

```
[Out] 1/2*hypergeom([1, 1+m],[2+m],-c*tan(f*x+e)/(-c^2)^(1/2))*(a^2-b^2*(-c^2)^(1
/2))*tan(f*x+e)*(d*tan(f*x+e))^m/f/(1+m)+1/2*hypergeom([1, 1+m],[2+m],c*tan
(f*x+e)/(-c^2)^(1/2))*(a^2+b^2*(-c^2)^(1/2))*tan(f*x+e)*(d*tan(f*x+e))^m/f/
(1+m)+4*a*b*hypergeom([1, 3/4+1/2*m],[7/4+1/2*m],-tan(f*x+e)^2)*(c*tan(f*x+
e))^(3/2)*(d*tan(f*x+e))^m/c/f/(3+2*m)
```

Rubi [A] time = 0.71, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3670, 15, 1831, 364, 1286}

$$\frac{\left(a^2-b^2\sqrt{-c^2}\right)\tan(e+fx)(d\tan(e+fx))^m\,{}_2F_1\left(1,m+1;m+2;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{2f(m+1)}+\frac{\left(a^2+b^2\sqrt{-c^2}\right)\tan(e+fx)(d\tan(}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```

```
[Out] ((a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, -((c*Tan[e + f*x
])/Sqrt[-c^2])]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*f*(1 + m)) + ((a^2 + b^
2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, (c*Tan[e + f*x])/Sqrt[-c^2
]]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*f*(1 + m)) + (4*a*b*Hypergeometric2F
1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^(3/2)*(d*T
an[e + f*x])^m)/(c*f*(3 + 2*m))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1286

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (c_.)*(x_)^4), x_Sym
bol] :> With[{q = Rt[-(a*c), 2]}, -Dist[e/2 + (c*d)/(2*q), Int[(f*x)^m/(q -
 c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[(f*x)^m/(q + c*x^2), x], x]]
/; FreeQ[{a, c, d, e, f, m}, x]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int (d\tan(e+fx))^m\left(a+b\sqrt{c\tan(e+fx)}\right)^2\,dx=\frac{c\,\text{Subst}\left(\int\frac{\left(a+b\sqrt{x}\right)^2\left(\frac{dx}{c}\right)^m}{c^2+x^2}\,dx,x,c\tan(e+fx)\right)}{f}$$

$$=\frac{(2c)\,\text{Subst}\left(\int\frac{x\left(\frac{dx^2}{c}\right)^m(a+bx)^2}{c^2+x^4}\,dx,x,\sqrt{c\tan(e+fx)}\right)}{f}$$

$$=\frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int\frac{x^{1+2m}(a+bx)^2}{c^2+x^4}\,d\right.}{f}$$

$$=\frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int\left(\frac{2abx^{2+2m}}{c^2+x^4}+\right.\right.}{f}$$

$$=\frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int\frac{x^{1+2m}\left(a^2+b^2x^2\right.}{c^2+x^4}\right.}{f}$$

$$=\frac{4ab\,{}_2F_1\left(1,\frac{1}{4}(3+2m);\frac{1}{4}(7+2m);-\tan^2(e+fx)\right)(c\tan(e+}{cf(3+2m)}$$

$$=\frac{\left(a^2-b^2\sqrt{-c^2}\right){}_2F_1\left(1,1+m;2+m;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+fx)}{2f(1+m)}$$

Mathematica [A] time = 1.43, size = 151, normalized size = 0.71

$$\frac{\tan(e+fx)(d\tan(e+fx))^m\left(\frac{a^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(e+fx)\right)}{m+1}+b\left(\frac{4a\sqrt{c\tan(e+fx)}\,{}_2F_1\left(1,\frac{1}{4}(2m+3);\frac{1}{4}(2m+7);-\tan^2(e+fx)\right)}{2m+3}+\frac{bc\,t}{}\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```

```
[Out] (Tan[e + f*x]*(d*Tan[e + f*x])^m*((a^2*Hypergeometric2F1[1, (1 + m)/2, (3 +
 m)/2, -Tan[e + f*x]^2])/(1 + m) + b*((b*c*Hypergeometric2F1[1, (2 + m)/2,
(4 + m)/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(2 + m) + (4*a*Hypergeometric2F1[
1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*Sqrt[c*Tan[e + f*x]])/(3 + 2*
m))))/f
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(2\sqrt{c\tan\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^m ab+\left(b^2c\tan\left(fx+e\right)+a^2\right)\left(d\tan\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x, algorithm="frica
s")
```

```
[Out] integral(2*sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*a*b + (b^2*c*tan(f*x + e
) + a^2)*(d*tan(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sqrt{c\tan\left(fx+e\right)}\,b+a\right)^{2}\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x, algorithm="giac"
)
```

```
[Out] integrate((sqrt(c*tan(f*x + e))*b + a)^2*(d*tan(f*x + e))^m, x)
```

maple [F] time = 1.93, size = 0, normalized size = 0.00

$$\int \left(a+b\sqrt{c\tan\left(fx+e\right)}\right)^{2}\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x)
```

```
[Out] int((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sqrt{c\tan\left(fx+e\right)}\,b+a\right)^{2}\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x, algorithm="maxim
a")
```

```
[Out] integrate((sqrt(c*tan(f*x + e))*b + a)^2*(d*tan(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\sqrt{c\tan\left(e+f\,x\right)}\right)^{2}\left(d\tan\left(e+f\,x\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^(1/2))^2*(d*tan(e + f*x))^m,x)
```

```
[Out] int((a + b*(c*tan(e + f*x))^(1/2))^2*(d*tan(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(e+fx\right)\right)^{m}\left(a+b\sqrt{c\tan\left(e+fx\right)}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))**(1/2))**2*(d*tan(f*x+e))**m,x)
```

```
[Out] Integral((d*tan(e + f*x))**m*(a + b*sqrt(c*tan(e + f*x)))**2, x)
```

3.407 $\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right) dx$

Optimal. Leaf size=121

$$\frac{a\tan(e+fx)(d\tan(e+fx))^m\, {}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(e+fx)\right)}{f(m+1)}+\frac{2b(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m\, {}_2F_1\left(1,\frac{1}{4}\right.}{cf(2m+}$$

```
[Out] a*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(f*x+e)^2)*tan(f*x+e)*(d*tan(f*x
+e))^m/f/(1+m)+2*b*hypergeom([1, 3/4+1/2*m],[7/4+1/2*m],-tan(f*x+e)^2)*(c*t
an(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/f/(3+2*m)
```

Rubi [A] time = 0.33, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3670, 15, 1831, 364}

$$\frac{a\tan(e+fx)(d\tan(e+fx))^m\, {}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(e+fx)\right)}{f(m+1)}+\frac{2b(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m\, {}_2F_1\left(1,\frac{1}{4}\right.}{cf(2m+}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[e + f*x]^2]*Tan[e + f*x]
*(d*Tan[e + f*x])^m)/(f*(1 + m)) + (2*b*Hypergeometric2F1[1, (3 + 2*m)/4, (
7 + 2*m)/4, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^(3/2)*(d*Tan[e + f*x])^m)/(c*
f*(3 + 2*m))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right) dx = \frac{c\,\text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)\left(\frac{dx}{c}\right)^m}{c^2+x^2} dx, x, c\tan(e+fx)\right)}{f}$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{x\left(\frac{dx^2}{c}\right)^m (a+bx)}{c^2+x^4} dx, x, \sqrt{c\tan(e+fx)}\right)}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}(a+bx)}{c^2+x^4} dx, x,\right.}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \left(\frac{ax^{1+2m}}{c^2+x^4} + \frac{bx^{2+2m}}{c^2+x^4}\right.\right.}{f}$$

$$= \frac{\left(2ac(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}}{c^2+x^4} dx, x, \sqrt{c\right.}{f}$$

$$= \frac{a\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+fx))^m}{f(1+m)}$$

Mathematica [C] time = 0.65, size = 304, normalized size = 2.51

$$\frac{\tan(e+fx)(d\tan(e+fx))^m\left(\left(a-b\sqrt[4]{-c^2}\right){}_2F_1\left(1,2(m+1);2m+3;-\frac{\sqrt{c\tan(e+fx)}}{\sqrt[4]{-c^2}}\right)+\left(a+ib\sqrt[4]{-c^2}\right){}_2F_1\left(1,2(m+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

```
[Out] (((a - b*(-c^2)^(1/4))*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -(Sqrt[c*Ta
n[e + f*x]]/(-c^2)^(1/4))] + (a + I*b*(-c^2)^(1/4))*Hypergeometric2F1[1, 2*
(1 + m), 3 + 2*m, ((-I)*Sqrt[c*Tan[e + f*x]])/(-c^2)^(1/4)] + a*Hypergeomet
ric2F1[1, 2*(1 + m), 3 + 2*m, (I*Sqrt[c*Tan[e + f*x]])/(-c^2)^(1/4)] - I*b*
(-c^2)^(1/4)*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, (I*Sqrt[c*Tan[e + f*x
]])/(-c^2)^(1/4)] + a*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, Sqrt[c*Tan[e
 + f*x]]/(-c^2)^(1/4)] + b*(-c^2)^(1/4)*Hypergeometric2F1[1, 2*(1 + m), 3 +
 2*m, Sqrt[c*Tan[e + f*x]]/(-c^2)^(1/4)])*Tan[e + f*x]*(d*Tan[e + f*x])^m)/
(4*f*(1 + m))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{c\tan\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^m b + \left(d\tan\left(fx+e\right)\right)^m a, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*b + (d*tan(f*x + e))^m*a,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sqrt{c\tan\left(fx+e\right)}b + a\right)\left(d\tan\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x, algorithm="giac")
[Out] integrate((sqrt(c*tan(f*x + e))*b + a)*(d*tan(f*x + e))^m, x)
```

maple [F] time = 1.33, size = 0, normalized size = 0.00

$$\int \left(a+b\sqrt{c\tan\left(fx+e\right)}\right)\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x)
[Out] int((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sqrt{c\tan\left(fx+e\right)}\,b+a\right)\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x, algorithm="maxima"
)
[Out] integrate((sqrt(c*tan(f*x + e))*b + a)*(d*tan(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\sqrt{c\tan\left(e+f\,x\right)}\right)\,\left(d\tan\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^(1/2))*(d*tan(e + f*x))^m,x)
[Out] int((a + b*(c*tan(e + f*x))^(1/2))*(d*tan(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(e+fx\right)\right)^{m}\left(a+b\sqrt{c\tan\left(e+fx\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))**(1/2))*(d*tan(f*x+e))**m,x)
[Out] Integral((d*tan(e + f*x))**m*(a + b*sqrt(c*tan(e + f*x))), x)
```

3.408 $\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx$

Optimal. Leaf size=460

$$\frac{b^4c^2\tan(e+fx)(d\tan(e+fx))^m\,{}_2F_1\left(1,2(m+1);2m+3;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)}{af(m+1)\left(a^4+b^4c^2\right)}-\frac{b\left(a^2-b^2\sqrt{-c^2}\right)(c\tan(e+fx))^{3/2}(d\tan}{cf(}$$

```
[Out] b^4*c^2*hypergeom([1, 2+2*m],[3+2*m],-b*(c*tan(f*x+e))^(1/2)/a)*tan(f*x+e)*
(d*tan(f*x+e))^m/a/(b^4*c^2+a^4)/f/(1+m)+1/2*a*hypergeom([1, 1+m],[2+m],-c*
tan(f*x+e)/(-c^2)^(1/2))*(a^2-b^2*(-c^2)^(1/2))*tan(f*x+e)*(d*tan(f*x+e))^m
/(b^4*c^2+a^4)/f/(1+m)+1/2*a*hypergeom([1, 1+m],[2+m],c*tan(f*x+e)/(-c^2)^(
1/2))*(a^2+b^2*(-c^2)^(1/2))*tan(f*x+e)*(d*tan(f*x+e))^m/(b^4*c^2+a^4)/f/(1
+m)-b*hypergeom([1, 3/2+m],[5/2+m],-c*tan(f*x+e)/(-c^2)^(1/2))*(a^2-b^2*(-c
^2)^(1/2))*(c*tan(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/(b^4*c^2+a^4)/f/(3+2*m)-
b*hypergeom([1, 3/2+m],[5/2+m],c*tan(f*x+e)/(-c^2)^(1/2))*(a^2+b^2*(-c^2)^(
1/2))*(c*tan(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/(b^4*c^2+a^4)/f/(3+2*m)
```

Rubi [A] time = 1.28, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3670, 15, 6725, 64, 1831, 1286, 364}

$$-\frac{b\left(a^2-b^2\sqrt{-c^2}\right)(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(2m+3);\frac{1}{2}(2m+5);-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{cf(2m+3)\left(a^4+b^4c^2\right)}-\frac{b\left(a^2+b^2\sqrt{-c^2}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

```
[Out] (a*(a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, -((c*Tan[e + f
*x])/Sqrt[-c^2])]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*(a^4 + b^4*c^2)*f*(1
+ m)) + (a*(a^2 + b^2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, (c*Tan
[e + f*x])/Sqrt[-c^2]]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*(a^4 + b^4*c^2)*
f*(1 + m)) + (b^4*c^2*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((b*Sqrt[c*
Tan[e + f*x]])/a)]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(a*(a^4 + b^4*c^2)*f*(1
 + m)) - (b*(a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1[1, (3 + 2*m)/2, (5 + 2
*m)/2, -((c*Tan[e + f*x])/Sqrt[-c^2])]*(c*Tan[e + f*x])^(3/2)*(d*Tan[e + f*
x])^m)/(c*(a^4 + b^4*c^2)*f*(3 + 2*m)) - (b*(a^2 + b^2*Sqrt[-c^2])*Hypergeo
metric2F1[1, (3 + 2*m)/2, (5 + 2*m)/2, (c*Tan[e + f*x])/Sqrt[-c^2]]*(c*Tan[
e + f*x])^(3/2)*(d*Tan[e + f*x])^m)/(c*(a^4 + b^4*c^2)*f*(3 + 2*m))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1286

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (c_.)*(x_)^4), x_Sym
bol] :> With[{q = Rt[-(a*c), 2]}, -Dist[e/2 + (c*d)/(2*q), Int[(f*x)^m/(q -
 c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[(f*x)^m/(q + c*x^2), x], x]]
/; FreeQ[{a, c, d, e, f, m}, x]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx = \frac{c\,\text{Subst}\left(\int \frac{\left(\frac{dx}{c}\right)^m}{\left(a+b\sqrt{x}\right)\left(c^2+x^2\right)}\,dx, x, c\tan(e+fx)\right)}{f}$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{x\left(\frac{dx^2}{c}\right)^m}{(a+bx)\left(c^2+x^4\right)}\,dx, x, \sqrt{c\tan(e+fx)}\right)}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}}{(a+bx)\left(c^2+x^4\right)}\,dx, x, \sqrt{c\tan(e+fx)}\right)}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \left(\frac{b^4x^{1+2m}}{\left(a^4+b^4c^2\right)(a+bx)} + \frac{x^{1+2m}\left(a^3-a^2bx+ab^2x^2\right.}{\left(a^4+b^4c^2\right)\left(c^2+x^4\right.}\right.\right.}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}\left(a^3-a^2bx+ab^2x^2-b^3x^3\right)}{c^2+x^4}\,dx, x, \sqrt{c\,\text{ta}}\right.}{\left(a^4+b^4c^2\right)f}$$

$$= \frac{b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{a\left(a^4+b^4c^2\right)f(1+m)} + \frac{\left(2c(c\,\text{ta}\right.}{}$$

$$= \frac{b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{a\left(a^4+b^4c^2\right)f(1+m)} + \frac{\left(2c(c\,\text{ta}\right.}{}$$

$$= \frac{b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{a\left(a^4+b^4c^2\right)f(1+m)} + \frac{\left(ac\left(b^2\right.\right.}{}$$

$$= \frac{a\left(a^2-b^2\sqrt{-c^2}\right){}_2F_1\left(1,1+m;2+m;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+fx)(d\tan(e+fx))^m}{2\left(a^4+b^4c^2\right)f(1+m)} + \frac{a}{}$$

Mathematica [A] time = 6.31, size = 385, normalized size = 0.84

$$\frac{2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\left(\frac{b^4(c\tan(e+fx))^{m+1}\,{}_2F_1\left(1,2(m+1);2m+3;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)}{2a(m+1)\left(a^4+b^4c^2\right)} - \frac{b^3(c\tan(e+fx))^{\frac{1}{2}(2m+5)}\,{}_2F_1\left(1,\frac{1}{4}(2m+5\right.}{c^2(2m+5)\left(a^4+b^4\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

```
[Out] (2*c*(d*Tan[e + f*x])^m*((a^3*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -T
an[e + f*x]^2]*(c*Tan[e + f*x])^(1 + m))/(2*c^2*(a^4 + b^4*c^2)*(1 + m)) +
(b^4*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)
]*(c*Tan[e + f*x])^(1 + m))/(2*a*(a^4 + b^4*c^2)*(1 + m)) + (a*b^2*Hypergeo
metric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^(2 + m
))/(2*c^2*(a^4 + b^4*c^2)*(2 + m)) - (a^2*b*Hypergeometric2F1[1, (3 + 2*m)/
4, (7 + 2*m)/4, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^((3 + 2*m)/2))/(c^2*(a^4
+ b^4*c^2)*(3 + 2*m)) - (b^3*Hypergeometric2F1[1, (5 + 2*m)/4, (9 + 2*m)/4,
 -Tan[e + f*x]^2]*(c*Tan[e + f*x])^((5 + 2*m)/2))/(c^2*(a^4 + b^4*c^2)*(5 +
 2*m))))/(f*(c*Tan[e + f*x])^m)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c\tan\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{m}b-\left(d\tan\left(fx+e\right)\right)^{m}a}{b^{2}c\tan\left(fx+e\right)-a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="fricas"
)
```

```
[Out] integral((sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*b - (d*tan(f*x + e))^m*a)
/(b^2*c*tan(f*x + e) - a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{m}}{\sqrt{c\tan\left(fx+e\right)}\,b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e))^m/(sqrt(c*tan(f*x + e))*b + a), x)
```

maple [F] time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{m}}{a+b\sqrt{c\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x)
```

```
[Out] int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{m}}{\sqrt{c\tan\left(fx+e\right)}\,b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="maxima"
)
```

```
[Out] integrate((d*tan(f*x + e))^m/(sqrt(c*tan(f*x + e))*b + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\tan\left(e+fx\right)\right)^{m}}{a+b\sqrt{c\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m/(a + b*(c*tan(e + f*x))^(1/2)),x)
```

```
[Out] int((d*tan(e + f*x))^m/(a + b*(c*tan(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\tan\left(e+fx\right)\right)^m}{a+b\sqrt{c\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m/(a+b*(c*tan(f*x+e))**(1/2)),x)
[Out] Integral((d*tan(e + f*x))**m/(a + b*sqrt(c*tan(e + f*x))), x)
```

3.409 $\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx$

Optimal. Leaf size=617

$$\frac{4a^2b^4c^2\tan(e+fx)(d\tan(e+fx))^m\,{}_2F_1\left(1,2(m+1);2m+3;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)}{f(m+1)\left(a^4+b^4c^2\right)^2}+\frac{b^4c^2\tan(e+fx)(d\tan(e+fx))^{}}{a^2f(m}$$

```
[Out] 4*a^2*b^4*c^2*hypergeom([1, 2+2*m],[3+2*m],-b*(c*tan(f*x+e))^(1/2)/a)*tan(f
*x+e)*(d*tan(f*x+e))^m/(b^4*c^2+a^4)^2/f/(1+m)+b^4*c^2*hypergeom([2, 2+2*m]
,[3+2*m],-b*(c*tan(f*x+e))^(1/2)/a)*tan(f*x+e)*(d*tan(f*x+e))^m/a^2/(b^4*c^
2+a^4)/f/(1+m)+1/2*hypergeom([1, 1+m],[2+m],-c*tan(f*x+e)/(-c^2)^(1/2))*(a^
6-3*a^2*b^4*c^2-b^6*(-c^2)^(3/2)-3*a^4*b^2*(-c^2)^(1/2))*tan(f*x+e)*(d*tan(
f*x+e))^m/(b^4*c^2+a^4)^2/f/(1+m)+1/2*hypergeom([1, 1+m],[2+m],c*tan(f*x+e)
/(-c^2)^(1/2))*(a^6-3*a^2*b^4*c^2+b^6*(-c^2)^(3/2)+3*a^4*b^2*(-c^2)^(1/2))*
tan(f*x+e)*(d*tan(f*x+e))^m/(b^4*c^2+a^4)^2/f/(1+m)-2*a*b*hypergeom([1, 3/2
+m],[5/2+m],-c*tan(f*x+e)/(-c^2)^(1/2))*(a^4-b^4*c^2-2*a^2*b^2*(-c^2)^(1/2)
)*(c*tan(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/(b^4*c^2+a^4)^2/f/(3+2*m)-2*a*b*h
ypergeom([1, 3/2+m],[5/2+m],c*tan(f*x+e)/(-c^2)^(1/2))*(a^4-b^4*c^2+2*a^2*b
^2*(-c^2)^(1/2))*(c*tan(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/(b^4*c^2+a^4)^2/f/
(3+2*m)
```

Rubi [A] time = 1.58, antiderivative size = 617, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3670, 15, 6725, 64, 1831, 1286, 364}

$$-\frac{2ab\left(-2a^2b^2\sqrt{-c^2}+a^4-b^4c^2\right)(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(2m+3);\frac{1}{2}(2m+5);-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{cf(2m+3)\left(a^4+b^4c^2\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```

```
[Out] ((a^6 - 3*a^2*b^4*c^2 - 3*a^4*b^2*Sqrt[-c^2] - b^6*(-c^2)^(3/2))*Hypergeome
tric2F1[1, 1 + m, 2 + m, -((c*Tan[e + f*x])/Sqrt[-c^2])]*Tan[e + f*x]*(d*Ta
n[e + f*x])^m)/(2*(a^4 + b^4*c^2)^2*f*(1 + m)) + ((a^6 - 3*a^2*b^4*c^2 + 3*
a^4*b^2*Sqrt[-c^2] + b^6*(-c^2)^(3/2))*Hypergeometric2F1[1, 1 + m, 2 + m, (
c*Tan[e + f*x])/Sqrt[-c^2]]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*(a^4 + b^4*
c^2)^2*f*(1 + m)) + (4*a^2*b^4*c^2*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m,
 -((b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/((a^4 + b^
4*c^2)^2*f*(1 + m)) + (b^4*c^2*Hypergeometric2F1[2, 2*(1 + m), 3 + 2*m, -((
b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(a^2*(a^4 + b^
4*c^2)*f*(1 + m)) - (2*a*b*(a^4 - b^4*c^2 - 2*a^2*b^2*Sqrt[-c^2])*Hypergeom
etric2F1[1, (3 + 2*m)/2, (5 + 2*m)/2, -((c*Tan[e + f*x])/Sqrt[-c^2])]*(c*Ta
n[e + f*x])^(3/2)*(d*Tan[e + f*x])^m)/(c*(a^4 + b^4*c^2)^2*f*(3 + 2*m)) - (
2*a*b*(a^4 - b^4*c^2 + 2*a^2*b^2*Sqrt[-c^2])*Hypergeometric2F1[1, (3 + 2*m)
/2, (5 + 2*m)/2, (c*Tan[e + f*x])/Sqrt[-c^2]]*(c*Tan[e + f*x])^(3/2)*(d*Tan
[e + f*x])^m)/(c*(a^4 + b^4*c^2)^2*f*(3 + 2*m))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1286

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (c_.)*(x_)^4), x_Sym
bol] :> With[{q = Rt[-(a*c), 2]}, -Dist[e/2 + (c*d)/(2*q), Int[(f*x)^m/(q -
 c*x^2), x], x] + Dist[e/2 - (c*d)/(2*q), Int[(f*x)^m/(q + c*x^2), x], x]]
/; FreeQ[{a, c, d, e, f, m}, x]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx = \frac{c\,\text{Subst}\left(\int \frac{\left(\frac{dx}{c}\right)^m}{\left(a+b\sqrt{x}\right)^2(c^2+x^2)}\,dx, x, c\tan(e+fx)\right)}{f}$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{x\left(\frac{dx^2}{c}\right)^m}{(a+bx)^2(c^2+x^4)}\,dx, x, \sqrt{c\tan(e+fx)}\right)}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}}{(a+bx)^2(c^2+x^4)}\,dx, x, \sqrt{c\tan(e-$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int\left(\frac{b^4x^{1+2m}}{(a^4+b^4c^2)(a+bx)^2} + \frac{4a^3b^4x^{1+2m}}{(a^4+b^4c^2)^2(a+}\right.\right.}{}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}\left(a^2(a^4-3b^4c^2)-2ab(a^4-b^4c^2)x+\right.}{c^2+x^4}\right.}{\left(a^4+b^4c^2\right)^2 f}$$

$$= \frac{4a^2b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{\left(a^4+b^4c^2\right)^2 f(1+m)}$$

$$= \frac{4a^2b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{\left(a^4+b^4c^2\right)^2 f(1+m)}$$

$$= \frac{4a^2b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{\left(a^4+b^4c^2\right)^2 f(1+m)}$$

$$= \frac{\left(a^6-3a^2b^4c^2-3a^4b^2\sqrt{-c^2}-b^6\left(-c^2\right)^{3/2}\right){}_2F_1\left(1,1+m;2+m;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\text{t}}{2\left(a^4+b^4c^2\right)^2 f(1+m)}$$

Mathematica [A] time = 6.41, size = 381, normalized size = 0.62

$$\frac{c(d\tan(e+fx))^m\left(\frac{4ab(b^4c^2-a^4)(c\tan(e+fx))^{3/2}\,{}_2F_1\left(1,\frac{1}{4}(2m+3);\frac{1}{4}(2m+7);-\tan^2(e+fx)\right)}{c^2(2m+3)} + \frac{b^2(3a^4-b^4c^2)\tan^2(e+fx)\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\text{ta}\right.}{m+2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```

```
[Out] (c*(d*Tan[e + f*x])^m*((a^2*(a^4 - 3*b^4*c^2)*Hypergeometric2F1[1, (1 + m)/
2, (3 + m)/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(c*(1 + m)) + (4*a^2*b^4*c*Hyp
ergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e
+ f*x])/(1 + m) + (b^4*c*(a^4 + b^4*c^2)*Hypergeometric2F1[2, 2*(1 + m), 3
+ 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e + f*x])/(a^2*(1 + m)) + (b^2*(3
*a^4 - b^4*c^2)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2]
*Tan[e + f*x]^2)/(2 + m) + (4*a*b*(-a^4 + b^4*c^2)*Hypergeometric2F1[1, (3
```

```
+ 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^(3/2))/(c^2*(3 + 2
*m)) - (8*a^3*b^3*Hypergeometric2F1[1, (5 + 2*m)/4, (9 + 2*m)/4, -Tan[e + f
*x]^2]*(c*Tan[e + f*x])^(5/2))/(c^2*(5 + 2*m))))/((a^4 + b^4*c^2)^2*f)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{2\sqrt{c\tan\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{m}ab-\left(b^{2}c\tan\left(fx+e\right)+a^{2}\right)\left(d\tan\left(fx+e\right)\right)^{m}}{b^{4}c^{2}\tan\left(fx+e\right)^{2}-2\,a^{2}b^{2}c\tan\left(fx+e\right)+a^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="frica
s")
```

```
[Out] integral(-(2*sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*a*b - (b^2*c*tan(f*x +
 e) + a^2)*(d*tan(f*x + e))^m)/(b^4*c^2*tan(f*x + e)^2 - 2*a^2*b^2*c*tan(f*
x + e) + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{m}}{\left(\sqrt{c\tan\left(fx+e\right)}\,b+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="giac"
)
```

```
[Out] integrate((d*tan(f*x + e))^m/(sqrt(c*tan(f*x + e))*b + a)^2, x)
```

maple [F] time = 2.12, size = 0, normalized size = 0.00

$$\int \frac{\left(d\tan\left(fx+e\right)\right)^{m}}{\left(a+b\sqrt{c\tan\left(fx+e\right)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x)
```

```
[Out] int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\tan\left(e+f\,x\right)\right)^{m}}{\left(a+b\,\sqrt{c\tan\left(e+f\,x\right)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m/(a + b*(c*tan(e + f*x))^(1/2))^2,x)
```

```
[Out] int((d*tan(e + f*x))^m/(a + b*(c*tan(e + f*x))^(1/2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\tan\left(e+fx\right)\right)^{m}}{\left(a+b\sqrt{c\tan\left(e+fx\right)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m/(a+b*(c*tan(f*x+e))**(1/2))**2,x)
```

```
[Out] Integral((d*tan(e + f*x))**m/(a + b*sqrt(c*tan(e + f*x)))**2, x)
```

3.410 $\int (d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=74

$$\frac{\tan(e+fx)(d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(1,\frac{1}{2}(m+np+1);\frac{1}{2}(m+np+3);-\tan^2(e+fx)\right)}{f(m+np+1)}$$

```
[Out] hypergeom([1, 1/2*n*p+1/2*m+1/2],[1/2*n*p+1/2*m+3/2],-tan(f*x+e)^2)*tan(f*x
+e)*(d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p/f/(n*p+m+1)
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3578, 20, 3476, 364}

$$\frac{\tan(e+fx)(d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(1,\frac{1}{2}(m+np+1);\frac{1}{2}(m+np+3);-\tan^2(e+fx)\right)}{f(m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + n*p)/2, (3 + m + n*p)/2, -Tan[e + f*x]^2]*Ta
n[e + f*x]*(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3578

```
Int[((c_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Tan[e + f*x
])^p)^FracPart[n])/(d*Tan[e + f*x])^(p*FracPart[n]), Int[(a + b*Tan[e + f*x
])^m*(d*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\int (d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{np}(d\ta$$
$$= \left((c\tan(e+fx))^{-m-np}(d\tan(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\right)\int (c$$
$$= \frac{\left(c(c\tan(e+fx))^{-m-np}(d\tan(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\right)\text{Sul}}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(1+m+np);\frac{1}{2}(3+m+np);-\tan^2(e+fx)\right)\tan(e+f}{f(1+m+np)}$$

Mathematica [A] time = 0.08, size = 76, normalized size = 1.03

$$\frac{\tan(e+fx)(d\tan(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(1,\frac{1}{2}(m+np+1);\frac{1}{2}(m+np+1)+1;-\tan^2(e+fx)\right)}{f(m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + n*p)/2, 1 + (1 + m + n*p)/2, -Tan[e + f*x]^2
]*Tan[e + f*x]*(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p
))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \left(d\tan\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*tan(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \left(d\tan\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*tan(f*x + e))^m, x)
```

maple [F] time = 12.61, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(fx+e\right)\right)^m \left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \left(d\tan\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*tan(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\tan\left(e+f\,x\right)\right)^m\,\left(b\,\left(c\tan\left(e+f\,x\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int((d*tan(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p\left(d\tan\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*tan(e + f*x))**m, x)
```

3.411 $\int \tan^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=63

$$\frac{\tan^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+3);\frac{1}{2}(np+5);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}$$

[Out] hypergeom([1, 1/2*n*p+3/2],[1/2*n*p+5/2],-tan(f*x+e)^2)*tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p/f/(n*p+3)

Rubi [A] time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3659, 16, 3476, 364}

$$\frac{\tan^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+3);\frac{1}{2}(np+5);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]

[Out] (Hypergeometric2F1[1, (3 + n*p)/2, (5 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*FracPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \tan^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \tan^2(e+fx)(c\tan(e+fx)$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{2+np}\,dx}{c^2}$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{2+np}}{c^2+x^2}\,dx,x,c\tan(e+\right.}{cf}$$

$$= \frac{{}_2F_1\left(1,\frac{1}{2}(3+np);\frac{1}{2}(5+np);-\tan^2(e+fx)\right)\tan^3(e+fx)\left(b(c\tan(e+\right.}{f(3+np)}$$

Mathematica [A] time = 0.09, size = 65, normalized size = 1.03

$$\frac{\tan^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+3);\frac{1}{2}(np+3)+1;-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (3 + n*p)/2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e
+ f*x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \tan\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^2, x)
```

maple [F] time = 12.82, size = 0, normalized size = 0.00

$$\int \left(\tan^2\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan\left(e+f\,x\right)^2\,{\left(b\,{\left(c\,\tan\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int(tan(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \left(c \tan{\left(e + f x \right)}\right)^{n}\right)^{p} \tan^{2}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*tan(e + f*x)**2, x)
```

3.412 $\int \left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=61

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*(c*ta
n(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{np}\,dx\\ &= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\mathrm{Subst}\left(\int \frac{x^{np}}{c^2+x^2}\,dx,x,c\tan(e+fx)\right)}{f}\\ &= \frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.97

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f + f*n*p)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

maple [F] time = 12.28, size = 0, normalized size = 0.00

$$\int\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p, x)
```

3.413 $\int \cot^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=63

$$-\frac{\cot(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-1);\frac{1}{2}(np+1);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

```
[Out] -cot(f*x+e)*hypergeom([1, 1/2*n*p-1/2],[1/2*n*p+1/2],-tan(f*x+e)^2)*(b*(c*t
an(f*x+e))^n)^p/f/(-n*p+1)
```

Rubi [A] time = 0.11, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3659, 16, 3476, 364}

$$-\frac{\cot(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-1);\frac{1}{2}(np+1);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[1, (-1 + n*p)/2, (1 + n*p)/2, -Tan[e + f*
x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p)))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cot^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cot^2(e+fx)(c\tan(e+fx))$$
$$= \left(c^2(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{-2+np}\,dx$$
$$= \frac{\left(c^3(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{-2+np}}{c^2+x^2}\,dx, x, c\tan(\right.}{f}$$
$$= -\frac{\cot(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-1+np);\frac{1}{2}(1+np);-\tan^2(e+fx)\right)\left(b(c\tan(e+\right.}{f(1-np)}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 0.97

$$\frac{\cot(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-1);\frac{1}{2}(np+1);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]*Hypergeometric2F1[1, (-1 + n*p)/2, (1 + n*p)/2, -Tan[e + f*x]
^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-1 + n*p))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cot\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^2, x)
```

maple [F] time = 12.76, size = 0, normalized size = 0.00

$$\int \left(\cot^2\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cot\left(e+f\,x\right)^2\,{\left(b\,{\left(c\,\tan\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(cot(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \left(c \tan{\left(e + f x \right)}\right)^{n}\right)^{p} \cot^{2}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x)**2, x)
```

3.414 $\int \cot^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=65

$$-\frac{\cot^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-3);\frac{1}{2}(np-1);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}$$

```
[Out] -cot(f*x+e)^3*hypergeom([1, 1/2*n*p-3/2],[1/2*n*p-1/2],-tan(f*x+e)^2)*(b*(c
*tan(f*x+e))^n)^p/f/(-n*p+3)
```

Rubi [A] time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3659, 16, 3476, 364}

$$-\frac{\cot^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-3);\frac{1}{2}(np-1);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]^3*Hypergeometric2F1[1, (-3 + n*p)/2, (-1 + n*p)/2, -Tan[e +
 f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 - n*p)))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cot^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cot^4(e+fx)(c\tan(e+f$$
$$= \left(c^4(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{-4+np}\,dx$$
$$= \frac{\left(c^5(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\operatorname{Subst}\left(\int \frac{x^{-4+np}}{c^2+x^2}\,dx, x, c\,ta\right.}{f}$$
$$= -\frac{\cot^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-3+np);\frac{1}{2}(-1+np);-\tan^2(e+fx)\right)\left(b(c\,ta\right.}{f(3-np)}$$

Mathematica [A] time = 0.08, size = 65, normalized size = 1.00

$$\frac{\cot^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-3);\frac{1}{2}(np-3)+1;-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]^3*Hypergeometric2F1[1, (-3 + n*p)/2, 1 + (-3 + n*p)/2, -Tan[e
 + f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-3 + n*p))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cot\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^4, x)
```

maple [F] time = 12.63, size = 0, normalized size = 0.00

$$\int \left(\cot^4\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cot\left(e+f\,x\right)^4\,\left(b\,\left(c\,\tan\left(e+f\,x\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(cot(e + f*x)^4*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\cot^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x)**4, x)
```

3.415 $\int \cot^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=65

$$-\frac{\cot^5(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-5);\frac{1}{2}(np-3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(5-np)}$$

```
[Out] -cot(f*x+e)^5*hypergeom([1, 1/2*n*p-5/2],[1/2*n*p-3/2],-tan(f*x+e)^2)*(b*(c
*tan(f*x+e))^n)^p/f/(-n*p+5)
```

Rubi [A] time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3659, 16, 3476, 364}

$$-\frac{\cot^5(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-5);\frac{1}{2}(np-3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(5-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]^5*Hypergeometric2F1[1, (-5 + n*p)/2, (-3 + n*p)/2, -Tan[e +
 f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(5 - n*p)))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cot^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cot^6(e+fx)(c\tan(e+fx))$$

$$= \left(c^6(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{-6+np}\,dx$$

$$= \frac{\left(c^7(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{-6+np}}{c^2+x^2}\,dx, x, c\tan(\right.}{f}$$

$$= -\frac{\cot^5(e+fx)\,{}_2F_1\left(1, \frac{1}{2}(-5+np); \frac{1}{2}(-3+np); -\tan^2(e+fx)\right)\left(b(c\tan(\right.}{f(5-np)}$$

Mathematica [A] time = 0.10, size = 65, normalized size = 1.00

$$\frac{\cot^5(e+fx)\,{}_2F_1\left(1, \frac{1}{2}(np-5); \frac{1}{2}(np-5)+1; -\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np-5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]^5*Hypergeometric2F1[1, (-5 + n*p)/2, 1 + (-5 + n*p)/2, -Tan[e
 + f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-5 + n*p))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cot\left(fx+e\right)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cot\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^6, x)
```

maple [F] time = 11.13, size = 0, normalized size = 0.00

$$\int \left(\cot^6\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cot\left(e+f\,x\right)^6\,\left(b\,\left(c\,\tan\left(e+f\,x\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(cot(e + f*x)^6*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

3.416 $\int \tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=63

$$\frac{\tan^4(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+4);\frac{1}{2}(np+6);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+4)}$$

```
[Out] hypergeom([1, 1/2*n*p+2],[1/2*n*p+3],-tan(f*x+e)^2)*tan(f*x+e)^4*(b*(c*tan(
f*x+e))^n)^p/f/(n*p+4)
```

Rubi [A] time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3659, 16, 3476, 364}

$$\frac{\tan^4(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+4);\frac{1}{2}(np+6);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (4 + n*p)/2, (6 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^4*(b*(c*Tan[e + f*x])^n)^p)/(f*(4 + n*p))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \tan^3(e+fx)(c\tan(e+f$$
$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{3+np}\,dx}{c^3}$$
$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{3+np}}{c^2+x^2}\,dx, x, c\tan(\right.}{c^2 f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(4+np);\frac{1}{2}(6+np);-\tan^2(e+fx)\right)\tan^4(e+fx)\left(b(c\tan(\right.}{f(4+np)}$$

Mathematica [A] time = 0.07, size = 61, normalized size = 0.97

$$\frac{\tan^4(e+fx)\,{}_2F_1\left(1,\frac{np}{2}+2;\frac{np}{2}+3;-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
[Out] (Hypergeometric2F1[1, 2 + (n*p)/2, 3 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^4*(b*(c*Tan[e + f*x])^n)^p)/(f*(4 + n*p))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \tan\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^3, x)
```

maple [F] time = 2.01, size = 0, normalized size = 0.00

$$\int \left(\tan^3\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
[Out] int(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan\left(e+f\,x\right)^3\,{\left(b\,{\left(c\,\tan\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(tan(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \left(c \tan{\left(e + f x \right)}\right)^{n}\right)^{p} \tan^{3}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*tan(e + f*x)**3, x)
```

3.417 $\int \tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=63

$$\frac{\tan^2(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+2);\frac{1}{2}(np+4);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+2)}$$

```
[Out] hypergeom([1, 1/2*n*p+1],[1/2*n*p+2],-tan(f*x+e)^2)*tan(f*x+e)^2*(b*(c*tan(
f*x+e))^n)^p/f/(n*p+2)
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3659, 16, 3476, 364}

$$\frac{\tan^2(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+2);\frac{1}{2}(np+4);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (2 + n*p)/2, (4 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^2*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 + n*p))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \tan(e+fx)(c\tan(e+fx))^{np}$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{1+np}\,dx}{c}$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{1+np}}{c^2+x^2}\,dx, x, c\tan(e+$$

$$= \frac{{}_2F_1\left(1,\frac{1}{2}(2+np);\frac{1}{2}(4+np);-\tan^2(e+fx)\right)\tan^2(e+fx)\left(b(c\tan(e+\right.}{f(2+np)}$$

Mathematica [A] time = 0.06, size = 61, normalized size = 0.97

$$\frac{\tan^2(e+fx)\,{}_2F_1\left(1,\frac{np}{2}+1;\frac{np}{2}+2;-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + (n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^2*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 + n*p))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*tan(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e), x)
```

maple [F] time = 12.88, size = 0, normalized size = 0.00

$$\int \tan\left(fx+e\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan\left(e+f\,x\right)\,\left(b\,\left(c\tan\left(e+f\,x\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(tan(e + f*x)*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*tan(e + f*x), x)
```

3.418 $\int \cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=50

$$\frac{{}_2F_1\left(1,\frac{np}{2};\frac{np}{2}+1;-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

[Out] hypergeom([1, 1/2*n*p],[1/2*n*p+1],-tan(f*x+e)^2)*(b*(c*tan(f*x+e))^n)^p/f/n/p

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3659, 16, 3476, 364}

$$\frac{{}_2F_1\left(1,\frac{np}{2};\frac{np}{2}+1;-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]

[Out] (Hypergeometric2F1[1, (n*p)/2, 1 + (n*p)/2, -Tan[e + f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*n*p)

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*FracPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cot(e+fx)(c\tan(e+fx)$$
$$= \left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{-1+np}\,dx$$
$$= \frac{\left(c^2(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{-1+np}}{c^2+x^2}\,dx, x, c\tan\right.}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{np}{2};1+\frac{np}{2};-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.00

$$\frac{{}_2F_1\left(1,\frac{np}{2};\frac{np}{2}+1;-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (n*p)/2, 1 + (n*p)/2, -Tan[e + f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*n*p)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\cot\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e), x)
```

maple [F] time = 13.43, size = 0, normalized size = 0.00

$$\int \cot\left(fx+e\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \cot\left(e+f\,x\right)\,\left(b\,\left(c\tan\left(e+f\,x\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(cot(e + f*x)*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x), x)
```

3.419 $\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=62

$$-\frac{\cot^2(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-2);\frac{np}{2};-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(2-np)}$$

```
[Out] -cot(f*x+e)^2*hypergeom([1, 1/2*n*p-1],[1/2*n*p],-tan(f*x+e)^2)*(b*(c*tan(f
*x+e))^n)^p/f/(-n*p+2)
```

Rubi [A] time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3659, 16, 3476, 364}

$$-\frac{\cot^2(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-2);\frac{np}{2};-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(2-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]^2*Hypergeometric2F1[1, (-2 + n*p)/2, (n*p)/2, -Tan[e + f*x]
^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 - n*p)))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cot^3(e+fx)(c\tan(e+fx))$$
$$= \left(c^3(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{-3+np}\,dx$$
$$= \frac{\left(c^4(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{-3+np}}{c^2+x^2}\,dx,x,c\tan(\right.}{f}$$
$$= -\frac{\cot^2(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-2+np);\frac{np}{2};-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right.}{f(2-np)}$$

Mathematica [A] time = 0.06, size = 59, normalized size = 0.95

$$\frac{\cot^2(e+fx)\,{}_2F_1\left(1,\frac{np}{2}-1;\frac{np}{2};-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]^2*Hypergeometric2F1[1, -1 + (n*p)/2, (n*p)/2, -Tan[e + f*x]^2
]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-2 + n*p))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cot\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cot\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^3, x)
```

maple [F] time = 10.36, size = 0, normalized size = 0.00

$$\int \left(\cot^3\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \cot\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cot\left(e+f\,x\right)^3\,\left(b\,\left(c\tan\left(e+f\,x\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(cot(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x)**3, x)
```

3.420 $\int (d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=30

$$\text{Int}\left((d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p, x\right)$$

[Out] Unintegrable((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p \, dx$$

Verification is Not applicable to the result.

[In] Int[(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]

[Out] Defer[Int][(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]

Rubi steps

$$\int (d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p \, dx = \int (d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p \, dx$$

Mathematica [A] time = 3.76, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p \, dx$$

Verification is Not applicable to the result.

[In] Integrate[(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]

[Out] Integrate[(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b + a\right)^p \left(d\tan\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")

[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b + a\right)^p \left(d\tan\left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")

[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)

maple [A] time = 1.76, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(fx+e\right)\right)^{m}\left(a+b\left(c\tan\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
[Out] int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^m\,{\left(a+b\,{\left(c\,\mathrm{tan}\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p,x)
[Out] int((d*tan(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \tan{\left(e + f x \right)}\right)^{m} \left(a + b \left(c \tan{\left(e + f x \right)}\right)^{n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((d*tan(e + f*x))**m*(a + b*(c*tan(e + f*x))**n)**p, x)
```

3.421 $\int (d\cot(e+fx))^m \left(b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=78

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\cot(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(-m+2p+1);\frac{1}{2}(-m+2p+3);-\tan^2(e+fx)\right)}{f(-m+2p+1)}$$

```
[Out] (d*cot(f*x+e))^m*hypergeom([1, 1/2-1/2*m+p],[3/2-1/2*m+p],-tan(f*x+e)^2)*ta
n(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1-m+2*p)
```

Rubi [A] time = 0.12, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3658, 2604, 3476, 364}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\cot(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(-m+2p+1);\frac{1}{2}(-m+2p+3);-\tan^2(e+fx)\right)}{f(-m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((d*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + 2*p)/2, (3 - m + 2*p)/2,
-Tan[e + f*x]^2]*Tan[e + f*x]*(b*Tan[e + f*x]^2)^p)/(f*(1 - m + 2*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2604

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\cot(e+fx))^m \left(b\tan^2(e+fx)\right)^p dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int (d\cot(e+fx))^m \tan^{2p}(e+fx)$$
$$= \left((d\cot(e+fx))^m \tan^{m-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int \tan^{-m+2p}(e$$
$$= \frac{\left((d\cot(e+fx))^m \tan^{m-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{-m+2}}{1+x^2}\right.}{f}$$
$$= \frac{(d\cot(e+fx))^m\, {}_2F_1\left(1,\frac{1}{2}(1-m+2p);\frac{1}{2}(3-m+2p);-\tan^2(e+fx\right.}{f(1-m+2p)}$$

Mathematica [A] time = 0.16, size = 70, normalized size = 0.90

$$-\frac{d\left(b\tan^2(e+fx)\right)^p (d\cot(e+fx))^{m-1}\, {}_2F_1\left(1,-\frac{m}{2}+p+\frac{1}{2};-\frac{m}{2}+p+\frac{3}{2};-\tan^2(e+fx)\right)}{f(m-2p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
[Out] -((d*(d*Cot[e + f*x])^(-1 + m)*Hypergeometric2F1[1, 1/2 - m/2 + p, 3/2 - m/
2 + p, -Tan[e + f*x]^2]*(b*Tan[e + f*x]^2)^p)/(f*(-1 + m - 2*p)))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2\right)^p \left(d\cot\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2)^p*(d*cot(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\cot\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2)^p*(d*cot(f*x + e))^m, x)
```

maple [F] time = 2.00, size = 0, normalized size = 0.00

$$\int \left(d\cot\left(fx+e\right)\right)^m \left(b\left(\tan^2\left(fx+e\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
[Out] int((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\cot\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*cot(f*x + e))^m, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\cot\left(e+f\,x\right)\right)^m\,\left(b\tan\left(e+f\,x\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
```

```
[Out] int((d*cot(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^2\left(e+fx\right)\right)^p\left(d\cot\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**p*(d*cot(e + f*x))**m, x)
```

3.422 $\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \,dx$

Optimal. Leaf size=107

$$\frac{\tan(e+fx)(d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} F_1\left(\frac{1-m}{2};1,-p;\frac{3-m}{2};-\tan^2(e+fx),-\frac{b\tan^2}{}\right.}{f(1-m)}$$

```
[Out] AppellF1(1/2-1/2*m,1,-p,3/2-1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(d*cot(f
*x+e))^m*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/(1-m)/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.20, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3674, 3670, 511, 510}

$$\frac{\tan(e+fx)(d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} F_1\left(\frac{1-m}{2};1,-p;\frac{3-m}{2};-\tan^2(e+fx),-\frac{b\tan^2}{}\right.}{f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 - m)/2, 1, -p, (3 - m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*(d*Cot[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 - m)*
(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 3674

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(d*Cot[e + f*x])^FracPart[m]*(Tan
[e + f*x]/d)^FracPart[m], Int[(a + b*(c*Tan[e + f*x])^n)^p/(Tan[e + f*x]/d)
^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \left((d\cot(e+fx))^m \left(\frac{\tan(e+fx)}{d}\right)^m\right)\int \left(\frac{\tan(e+fx)}{d}\right)^{-m} \left(a+b\tan^2\right.$$

$$= \frac{\left((d\cot(e+fx))^m \left(\frac{\tan(e+fx)}{d}\right)^m\right)\text{Subst}\left(\int \frac{\left(\frac{x}{d}\right)^{-m}\left(a+bx^2\right)^p}{1+x^2}dx, x, \tan(e+\right.}{f}$$

$$= \frac{\left((d\cot(e+fx))^m \left(\frac{\tan(e+fx)}{d}\right)^m \left(a+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2(e+fx)}{a}\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{1-m}{2};1,-p;\frac{3-m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\cot(e+fx))^m \text{t}}{f(1-m)}$$

Mathematica [B] time = 2.58, size = 265, normalized size = 2.48

$$-\frac{a(m-3)\cos^2(e+fx)\cot(e+fx)(d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p F_1\left(\right.}{f(m-1)\left(-2bpF_1\left(\frac{3-m}{2};1-p,1;\frac{5-m}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)+2aF_1\left(\frac{3-m}{2};-p,2;\frac{5-m}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cot[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((a*(-3 + m)*AppellF1[(1 - m)/2, -p, 1, (3 - m)/2, -((b*Tan[e + f*x]^2)/a)
, -Tan[e + f*x]^2]*Cos[e + f*x]^2*Cot[e + f*x]*(d*Cot[e + f*x])^m*(a + b*Ta
n[e + f*x]^2)^p)/(f*(-1 + m)*(-2*b*p*AppellF1[(3 - m)/2, 1 - p, 1, (5 - m)/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*a*AppellF1[(3 - m)/2, -p,
2, (5 - m)/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + a*(-3 + m)*Appell
F1[(1 - m)/2, -p, 1, (3 - m)/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*C
ot[e + f*x]^2)))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p \left(d\cot\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*cot(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p \left(d\cot\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*cot(f*x + e))^m, x)
```

maple [F] time = 1.92, size = 0, normalized size = 0.00

$$\int \left(d\cot\left(fx+e\right)\right)^m \left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan \left(f x + e\right)^{2} + a\right)^{p} \left(d \cot \left(f x + e\right)\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*cot(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\mathrm{cot}\left(e+f\,x\right)\right)}^m\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^m*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int((d*cot(e + f*x))^m*(a + b*tan(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.423 $\int (d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=80

$$\frac{\tan(e+fx)(d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(1,\frac{1}{2}(-m+np+1);\frac{1}{2}(-m+np+3);-\tan^2(e+fx)\right)}{f(-m+np+1)}$$

```
[Out] (d*cot(f*x+e))^m*hypergeom([1, 1/2*n*p-1/2*m+1/2],[1/2*n*p-1/2*m+3/2],-tan(
f*x+e)^2)*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p-m+1)
```

Rubi [A] time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3659, 2604, 3476, 364}

$$\frac{\tan(e+fx)(d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(1,\frac{1}{2}(-m+np+1);\frac{1}{2}(-m+np+3);-\tan^2(e+fx)\right)}{f(-m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((d*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n*p)/2, (3 - m + n*p)/2,
-Tan[e + f*x]^2]*Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - m + n*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2604

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3659

Rubi steps

$$\int (d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (d\cot(e+fx))^m (c\tan$$

$$= \left((d\cot(e+fx))^m (c\tan(e+fx))^{m-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan$$

$$= \frac{\left(c(d\cot(e+fx))^m (c\tan(e+fx))^{m-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subs}}{f}$$

$$= \frac{(d\cot(e+fx))^m\, {}_2F_1\left(1,\frac{1}{2}(1-m+np);\frac{1}{2}(3-m+np);-\tan^2(e+\right.}{f(1-m+np)}$$

Mathematica [A] time = 0.12, size = 77, normalized size = 0.96

$$\frac{d(d\cot(e+fx))^{m-1}\left(b(c\tan(e+fx))^n\right)^p\, {}_2F_1\left(1,\frac{1}{2}(-m+np+1);\frac{1}{2}(-m+np+3);-\tan^2(e+fx)\right)}{f(-m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (d*(d*Cot[e + f*x])^(-1 + m)*Hypergeometric2F1[1, (1 - m + n*p)/2, (3 - m +
 n*p)/2, -Tan[e + f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - m + n*p))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \left(d\cot\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*cot(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \left(d\cot\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*cot(f*x + e))^m, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(d\cot\left(fx+e\right)\right)^m \left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \left(d\cot\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*cot(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d \cot\left(e+f\,x\right)\right)^m \left(b\left(c \tan\left(e+f\,x\right)\right)^n\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int((d*cot(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c \tan\left(e+fx\right)\right)^n\right)^p \left(d \cot\left(e+fx\right)\right)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*cot(e + f*x))**m, x)
```

3.424 $\int (d\cot(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=57

$$\left(\frac{\tan(e+fx)}{d}\right)^m (d\cot(e+fx))^m \text{Int}\left(\left(\frac{\tan(e+fx)}{d}\right)^{-m} \left(a+b(c\tan(e+fx))^n\right)^p, x\right)$$

```
[Out] (d*cot(f*x+e))^m*(tan(f*x+e)/d)^m*Unintegrable((a+b*(c*tan(f*x+e))^n)^p/((t
an(f*x+e)/d)^m),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d\cot(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Cot[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (d*Cot[e + f*x])^m*(Tan[e + f*x]/d)^m*Defer[Int][(a + b*(c*Tan[e + f*x])^n)
^p/(Tan[e + f*x]/d)^m, x]
```

Rubi steps

$$\int (d\cot(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p \, dx = \left((d\cot(e+fx))^m \left(\frac{\tan(e+fx)}{d}\right)^m\right) \int \left(\frac{\tan(e+fx)}{d}\right)^{-m} (a+$$

Mathematica [A] time = 9.86, size = 0, normalized size = 0.00

$$\int (d\cot(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Cot[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[(d*Cot[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b + a\right)^p \left(d\cot\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*cot(f*x + e))^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b + a\right)^p \left(d\cot\left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cot(f*x + e))^m, x)
```

maple [A] time = 2.00, size = 0, normalized size = 0.00

$$\int \left(d \cot \left(f x+e\right)\right)^{m} \left(a+b \left(c \tan \left(f x+e\right)\right)^{n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
[Out] int((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c \tan \left(f x+e\right)\right)^{n} b+a\right)^{p} \left(d \cot \left(f x+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cot(f*x + e))^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(d\,\mathrm{cot}\left(e+f\,x\right)\right)}^m\,{\left(a+b\,{\left(c\,\mathrm{tan}\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p,x)
[Out] int((d*cot(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
[Out] Timed out
```

3.425 $\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=70

$$\frac{(4a-b)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a-b)\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}$$

```
[Out] 1/8*(4*a-b)*arctanh(sin(d*x+c))/d+1/8*(4*a-b)*sec(d*x+c)*tan(d*x+c)/d+1/4*b
*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3676, 385, 199, 206}

$$\frac{(4a-b)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a-b)\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((4*a - b)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a - b)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{(1-x^2)^3}dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{(4a-b)\,\text{Subst}\left(\int \frac{1}{(1-x^2)^2}dx, x, \sin(c+dx\right.}{4d}$$
$$= \frac{(4a-b)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{(4a-b}{}$$
$$= \frac{(4a-b)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(4a-b)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\,\text{sec}}{}$$

Mathematica [A] time = 0.08, size = 93, normalized size = 1.33

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d} - \frac{b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b\tan(c+dx)\sec^3(c+dx)}{4d} - \frac{b\tan(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) - (b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Sec[
c + d*x]*Tan[c + d*x])/(2*d) - (b*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*Sec
[c + d*x]^3*Tan[c + d*x])/(4*d)
```

fricas [A] time = 0.51, size = 95, normalized size = 1.36

$$\frac{(4\,a-b)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-(4\,a-b)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left((4\,a-b)\cos\left(dx\right.\right.}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/16*((4*a - b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (4*a - b)*cos(d*x +
c)^4*log(-sin(d*x + c) + 1) + 2*((4*a - b)*cos(d*x + c)^2 + 2*b)*sin(d*x +
c))/(d*cos(d*x + c)^4)
```

giac [A] time = 1.59, size = 98, normalized size = 1.40

$$\frac{(4\,a-b)\log\left(|\sin\left(dx+c\right)+1|\right)-(4\,a-b)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\left(4\,a\sin(dx+c)^3-b\sin(dx+c)^3-4\,a\sin(dx+c)-b\sin(dx+c)\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*((4*a - b)*log(abs(sin(d*x + c) + 1)) - (4*a - b)*log(abs(sin(d*x + c)
 - 1)) - 2*(4*a*sin(d*x + c)^3 - b*sin(d*x + c)^3 - 4*a*sin(d*x + c) - b*si
n(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d
```

maple [A] time = 0.57, size = 116, normalized size = 1.66

$$\frac{b\left(\sin^3\left(dx+c\right)\right)}{4d\cos\left(dx+c\right)^4} + \frac{b\left(\sin^3\left(dx+c\right)\right)}{8d\cos\left(dx+c\right)^2} + \frac{b\sin\left(dx+c\right)}{8d} - \frac{b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d} + \frac{a\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/4/d*b*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*b*sin(d*x+c)^3/cos(d*x+c)^2+1/8*b*s
in(d*x+c)/d-1/8/d*b*ln(sec(d*x+c)+tan(d*x+c))+1/2*a*sec(d*x+c)*tan(d*x+c)/d
+1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.52, size = 95, normalized size = 1.36

$$\frac{(4a-b)\log\left(\sin\left(dx+c\right)+1\right)-(4a-b)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left((4a-b)\sin(dx+c)^3-(4a+b)\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/16*((4*a - b)*log(sin(d*x + c) + 1) - (4*a - b)*log(sin(d*x + c) - 1) - 2
*((4*a - b)*sin(d*x + c)^3 - (4*a + b)*sin(d*x + c))/(sin(d*x + c)^4 - 2*si
n(d*x + c)^2 + 1))/d
```

mupad [B] time = 14.41, size = 147, normalized size = 2.10

$$\frac{\left(a+\frac{b}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{7b}{4}-a\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{7b}{4}-a\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(a+\frac{b}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}+\frac{\mathrm{atanh}\left(\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^7*(a + b/4) - tan(c/2 + (d*x)/2)^3*(a - (7*b)/4) - tan(
c/2 + (d*x)/2)^5*(a - (7*b)/4) + tan(c/2 + (d*x)/2)*(a + b/4))/(d*(6*tan(c/
2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2
+ (d*x)/2)^8 + 1)) + (atanh(tan(c/2 + (d*x)/2))*(a - b/4))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(c+dx\right)\right)\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*sec(c + d*x)**3, x)
```

3.426 $\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=42

$$\frac{(2a-b)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 1/2*(2*a-b)*arctanh(sin(d*x+c))/d+1/2*b*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3676, 385, 206}

$$\frac{(2a-b)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((2*a - b)*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]*Tan[c + d*x])/(2*
d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{\left(1-x^2\right)^2}\,dx,x,\sin(c+dx)\right)}{d}\\
&= \frac{b\sec(c+dx)\tan(c+dx)}{2d}+\frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{2d}\\
&= \frac{(2a-b)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 48, normalized size = 1.14

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (b*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c +
d*x]*Tan[c + d*x])/(2*d)
```

fricas [**A**] time = 0.52, size = 76, normalized size = 1.81

$$\frac{(2a-b)\cos(dx+c)^2\log(\sin(dx+c)+1)-(2a-b)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2b\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*a - b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a - b)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) + 2*b*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [**A**] time = 1.39, size = 64, normalized size = 1.52

$$\frac{(2a-b)\log(|\sin(dx+c)+1|)-(2a-b)\log(|\sin(dx+c)-1|)-\frac{2b\sin(dx+c)}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*((2*a - b)*log(abs(sin(d*x + c) + 1)) - (2*a - b)*log(abs(sin(d*x + c)
- 1)) - 2*b*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

maple [**A**] time = 0.29, size = 75, normalized size = 1.79

$$\frac{b\left(\sin^3(dx+c)\right)}{2d\cos(dx+c)^2}+\frac{b\sin(dx+c)}{2d}-\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/2/d*b*sin(d*x+c)^3/cos(d*x+c)^2+1/2*b*sin(d*x+c)/d-1/2/d*b*ln(sec(d*x+c)+
tan(d*x+c))+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [**A**] time = 1.10, size = 62, normalized size = 1.48

$$\frac{(2a-b)\log(\sin(dx+c)+1)-(2a-b)\log(\sin(dx+c)-1)-\frac{2b\sin(dx+c)}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*a - b)*log(sin(d*x + c) + 1) - (2*a - b)*log(sin(d*x + c) - 1) - 2*
b*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 12.29, size = 79, normalized size = 1.88

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(2\,a-b)}{d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)/cos(c + d*x),x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(2*a - b))/d + (b*tan(c/2 + (d*x)/2) + b*tan(c/2
 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*sec(c + d*x), x)
```

3.427 $\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{(a-b)\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] b*arctanh(sin(d*x+c))/d+(a-b)*sin(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3676, 388, 206}

$$\frac{(a-b)\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d + ((a - b)*Sin[c + d*x])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}\\ &= \frac{(a-b)\sin(c+dx)}{d}+\frac{b\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}\\ &= \frac{b\tanh^{-1}(\sin(c+dx))}{d}+\frac{(a-b)\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.68

$$\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
 - (b*Sin[c + d*x])/d
```

fricas [A] time = 0.56, size = 44, normalized size = 1.57

$$\frac{b\log\left(\sin\left(dx+c\right)+1\right)-b\log\left(-\sin\left(dx+c\right)+1\right)+2\left(a-b\right)\sin\left(dx+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(sin(d*x + c) + 1) - b*log(-sin(d*x + c) + 1) + 2*(a - b)*sin(d*x
 + c))/d
```

giac [A] time = 1.49, size = 48, normalized size = 1.71

$$\frac{b\left(\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-2\,\sin\left(dx+c\right)\right)+2\,a\sin\left(dx+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b*(log(abs(sin(d*x + c) + 1)) - log(abs(sin(d*x + c) - 1)) - 2*sin(d*x
 + c)) + 2*a*sin(d*x + c))/d
```

maple [A] time = 0.37, size = 44, normalized size = 1.57

$$\frac{a\sin\left(dx+c\right)}{d}-\frac{b\sin\left(dx+c\right)}{d}+\frac{b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c)^2),x)
```

```
[Out] a*sin(d*x+c)/d-b*sin(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.32, size = 46, normalized size = 1.64

$$\frac{b\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right)\right)+2\,a\sin\left(dx+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) + 2
*a*sin(d*x + c))/d
```

mupad [B] time = 11.87, size = 32, normalized size = 1.14

$$\frac{\sin\left(c+d\,x\right)\,\left(a-b\right)}{d}+\frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*tan(c + d*x)^2),x)
```

```
[Out] (sin(c + d*x)*(a - b))/d + (2*b*atanh(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(c + dx\right)\right)\cos\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x), x)
```

3.428 $\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=32

$$\frac{a\sin(c+dx)}{d}-\frac{(a-b)\sin^3(c+dx)}{3d}$$

```
[Out] a*sin(d*x+c)/d-1/3*(a-b)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3676}

$$\frac{a\sin(c+dx)}{d}-\frac{(a-b)\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Sin[c + d*x])/d - ((a - b)*Sin[c + d*x]^3)/(3*d)
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(a-(a-b)x^2\right)\,dx,x,\sin(c+dx)\right)}{d}\\ &= \frac{a\sin(c+dx)}{d}-\frac{(a-b)\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.38

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.46, size = 30, normalized size = 0.94

$$\frac{\left((a-b)\cos\left(dx+c\right)^2+2\,a+b\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((a - b)*cos(d*x + c)^2 + 2*a + b)*sin(d*x + c)/d
```

giac [A] time = 1.48, size = 36, normalized size = 1.12

$$-\frac{a\sin(dx+c)^3 - b\sin(dx+c)^3 - 3\,a\sin(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="giac")
[Out] -1/3*(a*sin(d*x + c)^3 - b*sin(d*x + c)^3 - 3*a*sin(d*x + c))/d
```

maple [A] time = 0.63, size = 36, normalized size = 1.12

$$\frac{\frac{b\left(\sin^3(dx+c)\right)}{3}+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x)
[Out] 1/d*(1/3*b*sin(d*x+c)^3+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.31, size = 29, normalized size = 0.91

$$-\frac{(a-b)\sin(dx+c)^3 - 3\,a\sin(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
[Out] -1/3*((a - b)*sin(d*x + c)^3 - 3*a*sin(d*x + c))/d
```

mupad [B] time = 12.10, size = 47, normalized size = 1.47

$$\frac{9\,a\,\sin(c+d\,x)+3\,b\,\sin(c+d\,x)+a\,\sin(3\,c+3\,d\,x)-b\,\sin(3\,c+3\,d\,x)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*tan(c + d*x)^2),x)
[Out] (9*a*sin(c + d*x) + 3*b*sin(c + d*x) + a*sin(3*c + 3*d*x) - b*sin(3*c + 3*d
*x))/(12*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2(c+dx)\right)\cos^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c)**2),x)
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**3, x)
```

3.429 $\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)dx$

Optimal. Leaf size=54

$$\frac{(a-b)\sin^5(c+dx)}{5d}-\frac{(2a-b)\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] a*sin(d*x+c)/d-1/3*(2*a-b)*sin(d*x+c)^3/d+1/5*(a-b)*sin(d*x+c)^5/d
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3676, 373}

$$\frac{(a-b)\sin^5(c+dx)}{5d}-\frac{(2a-b)\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Sin[c + d*x])/d - ((2*a - b)*Sin[c + d*x]^3)/(3*d) + ((a - b)*Sin[c + d*
x]^5)/(5*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)dx &= \frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a-(a-b)x^2\right)dx,x,\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int\left(a-(2a-b)x^2+(a-b)x^4\right)dx,x,\sin(c+dx)\right)}{d}\\
&= \frac{a\sin(c+dx)}{d}-\frac{(2a-b)\sin^3(c+dx)}{3d}+\frac{(a-b)\sin^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 52, normalized size = 0.96

$$\frac{\sin(c+dx)(4(7a-2b)\cos(2(c+dx))+3(a-b)\cos(4(c+dx))+89a+11b)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((89*a + 11*b + 4*(7*a - 2*b)*Cos[2*(c + d*x)] + 3*(a - b)*Cos[4*(c + d*x)]
)*Sin[c + d*x])/(120*d)
```

fricas [A] time = 0.55, size = 47, normalized size = 0.87

$$\frac{\left(3\,(a-b)\cos\left(dx+c\right)^{4}+(4\,a+b)\cos\left(dx+c\right)^{2}+8\,a+2\,b\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*(a - b)*cos(d*x + c)^4 + (4*a + b)*cos(d*x + c)^2 + 8*a + 2*b)*sin(
d*x + c)/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.80, size = 72, normalized size = 1.33

$$\frac{\frac{a\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+b\left(-\frac{\sin(dx+c)\left(\cos^{4}(dx+c)\right)}{5}+\frac{\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{15}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+b*(-1/5*sin(d*x+c
)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c)))
```

maxima [A] time = 0.71, size = 47, normalized size = 0.87

$$\frac{3\,(a-b)\sin\left(dx+c\right)^{5}-5\,(2\,a-b)\sin\left(dx+c\right)^{3}+15\,a\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*(a - b)*sin(d*x + c)^5 - 5*(2*a - b)*sin(d*x + c)^3 + 15*a*sin(d*x
+ c))/d
```

mupad [B] time = 12.07, size = 71, normalized size = 1.31

$$\frac{\frac{5\,a\,\sin(c+d\,x)}{8}+\frac{b\,\sin(c+d\,x)}{8}+\frac{5\,a\,\sin(3\,c+3\,d\,x)}{48}+\frac{a\,\sin(5\,c+5\,d\,x)}{80}-\frac{b\,\sin(3\,c+3\,d\,x)}{48}-\frac{b\,\sin(5\,c+5\,d\,x)}{80}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*tan(c + d*x)^2),x)
```

```
[Out] ((5*a*sin(c + d*x))/8 + (b*sin(c + d*x))/8 + (5*a*sin(3*c + 3*d*x))/48 + (a
*sin(5*c + 5*d*x))/80 - (b*sin(3*c + 3*d*x))/48 - (b*sin(5*c + 5*d*x))/80)/
d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{2}\left(c+dx\right)\right)\cos^{5}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**5, x)
```

3.430 $\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)dx$

Optimal. Leaf size=76

$$-\frac{(a-b)\sin^7(c+dx)}{7d}+\frac{(3a-2b)\sin^5(c+dx)}{5d}-\frac{(3a-b)\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] a*sin(d*x+c)/d-1/3*(3*a-b)*sin(d*x+c)^3/d+1/5*(3*a-2*b)*sin(d*x+c)^5/d-1/7*
(a-b)*sin(d*x+c)^7/d
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3676, 373}

$$-\frac{(a-b)\sin^7(c+dx)}{7d}+\frac{(3a-2b)\sin^5(c+dx)}{5d}-\frac{(3a-b)\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Sin[c + d*x])/d - ((3*a - b)*Sin[c + d*x]^3)/(3*d) + ((3*a - 2*b)*Sin[c
+ d*x]^5)/(5*d) - ((a - b)*Sin[c + d*x]^7)/(7*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)dx &= \frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a-(a-b)x^2\right)dx,x,\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a-(3a-b)x^2+(3a-2b)x^4-(a-b)x^6\right)dx,x,\sin(c+dx)\right)}{d}\\ &= \frac{a\sin(c+dx)}{d}-\frac{(3a-b)\sin^3(c+dx)}{3d}+\frac{(3a-2b)\sin^5(c+dx)}{5d}-\frac{(a-b)\sin^7(c+dx)}{7d}\end{aligned}$$

Mathematica [A] time = 0.31, size = 75, normalized size = 0.99

$$\frac{\sin(c+dx)((897a-113b)\cos(2(c+dx))+6(27a-13b)\cos(4(c+dx))+15a\cos(6(c+dx))+2286a-15b\cos\ldots}{3360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((2286*a + 206*b + (897*a - 113*b)*Cos[2*(c + d*x)] + 6*(27*a - 13*b)*Cos[4
*(c + d*x)] + 15*a*Cos[6*(c + d*x)] - 15*b*Cos[6*(c + d*x)])*Sin[c + d*x])/
(3360*d)
```

fricas [A] time = 0.54, size = 63, normalized size = 0.83

$$\frac{\left(15\,(a-b)\cos\left(dx+c\right)^6+3\,(6\,a+b)\cos\left(dx+c\right)^4+4\,(6\,a+b)\cos\left(dx+c\right)^2+48\,a+8\,b\right)\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*(a - b)*cos(d*x + c)^6 + 3*(6*a + b)*cos(d*x + c)^4 + 4*(6*a + b)
*cos(d*x + c)^2 + 48*a + 8*b)*sin(d*x + c)/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.72, size = 92, normalized size = 1.21

$$\frac{\frac{a\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}(dx+c)\right)}{5}+\frac{8\left(\cos^{2}(dx+c)\right)}{5}\right)\sin(dx+c)}{7}+b\left(-\frac{\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
+b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*s
in(d*x+c)))
```

maxima [A] time = 0.44, size = 64, normalized size = 0.84

$$-\frac{15\,(a-b)\sin\left(dx+c\right)^7-21\,(3\,a-2\,b)\sin\left(dx+c\right)^5+35\,(3\,a-b)\sin\left(dx+c\right)^3-105\,a\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/105*(15*(a - b)*sin(d*x + c)^7 - 21*(3*a - 2*b)*sin(d*x + c)^5 + 35*(3*a
 - b)*sin(d*x + c)^3 - 105*a*sin(d*x + c))/d
```

mupad [B] time = 12.00, size = 95, normalized size = 1.25

$$\frac{\frac{35\,a\,\sin(c+d\,x)}{64}+\frac{5\,b\,\sin(c+d\,x)}{64}+\frac{7\,a\,\sin(3\,c+3\,d\,x)}{64}+\frac{7\,a\,\sin(5\,c+5\,d\,x)}{320}+\frac{a\,\sin(7\,c+7\,d\,x)}{448}-\frac{b\,\sin(3\,c+3\,d\,x)}{192}-\frac{3\,b\,\sin(5\,c+5\,d\,x)}{320}-\frac{b\,\sin(7\,c\text{-}}{448}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + b*tan(c + d*x)^2),x)
```

```
[Out] ((35*a*sin(c + d*x))/64 + (5*b*sin(c + d*x))/64 + (7*a*sin(3*c + 3*d*x))/64
 + (7*a*sin(5*c + 5*d*x))/320 + (a*sin(7*c + 7*d*x))/448 - (b*sin(3*c + 3*d
*x))/192 - (3*b*sin(5*c + 5*d*x))/320 - (b*sin(7*c + 7*d*x))/448)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tan^2\left(c + dx\right)\right) \cos^7\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**7, x)
```

3.431 $\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=68

$$\frac{(a+2b)\tan^5(c+dx)}{5d}+\frac{(2a+b)\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\tan^7(c+dx)}{7d}$$

```
[Out] a*tan(d*x+c)/d+1/3*(2*a+b)*tan(d*x+c)^3/d+1/5*(a+2*b)*tan(d*x+c)^5/d+1/7*b*
tan(d*x+c)^7/d
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3675, 373}

$$\frac{(a+2b)\tan^5(c+dx)}{5d}+\frac{(2a+b)\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Tan[c + d*x])/d + ((2*a + b)*Tan[c + d*x]^3)/(3*d) + ((a + 2*b)*Tan[c +
d*x]^5)/(5*d) + (b*Tan[c + d*x]^7)/(7*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)^2\left(a+bx^2\right)dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a+(2a+b)x^2+(a+2b)x^4+bx^6\right)dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{a\tan(c+dx)}{d}+\frac{(2a+b)\tan^3(c+dx)}{3d}+\frac{(a+2b)\tan^5(c+dx)}{5d}+\frac{b\tan^7}{}\end{aligned}$$

Mathematica [A] time = 0.26, size = 75, normalized size = 1.10

$$\frac{\tan(c+dx)\left(21a\tan^4(c+dx)+70a\tan^2(c+dx)+105a+15b\sec^6(c+dx)-3b\sec^4(c+dx)-4b\sec^2(c+dx)-\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (Tan[c + d*x]*(105*a - 8*b - 4*b*Sec[c + d*x]^2 - 3*b*Sec[c + d*x]^4 + 15*b
*Sec[c + d*x]^6 + 70*a*Tan[c + d*x]^2 + 21*a*Tan[c + d*x]^4))/(105*d)
```

fricas [A] time = 0.47, size = 74, normalized size = 1.09

$$\frac{\left(8\,(7\,a-b)\cos\left(dx+c\right)^{6}+4\,(7\,a-b)\cos\left(dx+c\right)^{4}+3\,(7\,a-b)\cos\left(dx+c\right)^{2}+15\,b\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(7*a - b)*cos(d*x + c)^6 + 4*(7*a - b)*cos(d*x + c)^4 + 3*(7*a - b
)*cos(d*x + c)^2 + 15*b)*sin(d*x + c)/(d*cos(d*x + c)^7)
```

giac [A] time = 1.89, size = 70, normalized size = 1.03

$$\frac{15\,b\tan\left(dx+c\right)^{7}+21\,a\tan\left(dx+c\right)^{5}+42\,b\tan\left(dx+c\right)^{5}+70\,a\tan\left(dx+c\right)^{3}+35\,b\tan\left(dx+c\right)^{3}+105\,a\tan}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/105*(15*b*tan(d*x + c)^7 + 21*a*tan(d*x + c)^5 + 42*b*tan(d*x + c)^5 + 70
*a*tan(d*x + c)^3 + 35*b*tan(d*x + c)^3 + 105*a*tan(d*x + c))/d
```

maple [A] time = 0.62, size = 94, normalized size = 1.38

$$\frac{b\left(\frac{\sin^{3}(dx+c)}{7\cos(dx+c)^{7}}+\frac{4\left(\sin^{3}(dx+c)\right)}{35\cos(dx+c)^{5}}+\frac{8\left(\sin^{3}(dx+c)\right)}{105\cos(dx+c)^{3}}\right)-a\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*
sin(d*x+c)^3/cos(d*x+c)^3)-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan
(d*x+c))
```

maxima [A] time = 0.34, size = 56, normalized size = 0.82

$$\frac{15\,b\tan\left(dx+c\right)^{7}+21\,(a+2\,b)\tan\left(dx+c\right)^{5}+35\,(2\,a+b)\tan\left(dx+c\right)^{3}+105\,a\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*b*tan(d*x + c)^7 + 21*(a + 2*b)*tan(d*x + c)^5 + 35*(2*a + b)*tan
(d*x + c)^3 + 105*a*tan(d*x + c))/d
```

mupad [B] time = 12.01, size = 56, normalized size = 0.82

$$\frac{\frac{b\tan(c+d\,x)^{7}}{7}+\left(\frac{a}{5}+\frac{2\,b}{5}\right)\tan\left(c+d\,x\right)^{5}+\left(\frac{2\,a}{3}+\frac{b}{3}\right)\tan\left(c+d\,x\right)^{3}+a\tan\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)/cos(c + d*x)^6,x)
```

```
[Out] (tan(c + d*x)^3*((2*a)/3 + b/3) + tan(c + d*x)^5*(a/5 + (2*b)/5) + a*tan(c
+ d*x) + (b*tan(c + d*x)^7)/7)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(c + dx\right)\right)\sec^6\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*sec(c + d*x)**6, x)
```

3.432 $\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)dx$

Optimal. Leaf size=46

$$\frac{(a+b)\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\tan^5(c+dx)}{5d}$$

```
[Out] a*tan(d*x+c)/d+1/3*(a+b)*tan(d*x+c)^3/d+1/5*b*tan(d*x+c)^5/d
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3675, 373}

$$\frac{(a+b)\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Tan[c + d*x])/d + ((a + b)*Tan[c + d*x]^3)/(3*d) + (b*Tan[c + d*x]^5)/(5
*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+bx^2\right)dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a+(a+b)x^2+bx^4\right)dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{a\tan(c+dx)}{d}+\frac{(a+b)\tan^3(c+dx)}{3d}+\frac{b\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.15, size = 53, normalized size = 1.15

$$\frac{\tan(c+dx)\left(5a\tan^2(c+dx)+15a+3b\sec^4(c+dx)-b\sec^2(c+dx)-2b\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (Tan[c + d*x]*(15*a - 2*b - b*Sec[c + d*x]^2 + 3*b*Sec[c + d*x]^4 + 5*a*Tan
[c + d*x]^2))/(15*d)
```

fricas [A] time = 0.49, size = 56, normalized size = 1.22

$$\frac{\left(2\,(5\,a-b)\cos\left(dx+c\right)^{4}+(5\,a-b)\cos\left(dx+c\right)^{2}+3\,b\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*a - b)*cos(d*x + c)^4 + (5*a - b)*cos(d*x + c)^2 + 3*b)*sin(d*x
+ c)/(d*cos(d*x + c)^5)
```

giac [A] time = 1.52, size = 48, normalized size = 1.04

$$\frac{3\,b\tan\left(dx+c\right)^{5}+5\,a\tan\left(dx+c\right)^{3}+5\,b\tan\left(dx+c\right)^{3}+15\,a\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*b*tan(d*x + c)^5 + 5*a*tan(d*x + c)^3 + 5*b*tan(d*x + c)^3 + 15*a*t
an(d*x + c))/d
```

maple [A] time = 0.59, size = 66, normalized size = 1.43

$$\frac{b\left(\frac{\sin^{3}(dx+c)}{5\cos(dx+c)^{5}}+\frac{2\left(\sin^{3}(dx+c)\right)}{15\cos(dx+c)^{3}}\right)-a\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)-a*(-2
/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

maxima [A] time = 0.31, size = 39, normalized size = 0.85

$$\frac{3\,b\tan\left(dx+c\right)^{5}+5\,(a+b)\tan\left(dx+c\right)^{3}+15\,a\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*tan(d*x + c)^5 + 5*(a + b)*tan(d*x + c)^3 + 15*a*tan(d*x + c))/d
```

mupad [B] time = 11.94, size = 40, normalized size = 0.87

$$\frac{\frac{b\tan(c+d\,x)^{5}}{5}+\left(\frac{a}{3}+\frac{b}{3}\right)\tan\left(c+d\,x\right)^{3}+a\tan\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)/cos(c + d*x)^4,x)
```

```
[Out] (tan(c + d*x)^3*(a/3 + b/3) + a*tan(c + d*x) + (b*tan(c + d*x)^5)/5)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{2}\left(c+dx\right)\right)\sec^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*sec(c + d*x)**4, x)
```

3.433 $\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)dx$

Optimal. Leaf size=28

$$\frac{a\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}$$

```
[Out] a*tan(d*x+c)/d+1/3*b*tan(d*x+c)^3/d
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3675}

$$\frac{a\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)dx &= \frac{\operatorname{Subst}\left(\int \left(a+bx^2\right)dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{a\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{a\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.50, size = 37, normalized size = 1.32

$$\frac{\left((3a-b)\cos(dx+c)^2+b\right)\sin(dx+c)}{3d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*a - b)*cos(d*x + c)^2 + b)*sin(d*x + c)/(d*cos(d*x + c)^3)
```

giac [A] time = 1.74, size = 25, normalized size = 0.89

$$\frac{b\tan\left(dx+c\right)^{3}+3\,a\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="giac")
[Out] 1/3*(b*tan(d*x + c)^3 + 3*a*tan(d*x + c))/d
```

maple [A] time = 0.50, size = 33, normalized size = 1.18

$$\frac{\frac{b\left(\sin^{3}(dx+c)\right)}{3\cos(dx+c)^{3}}+a\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x)
[Out] 1/d*(1/3*b*sin(d*x+c)^3/cos(d*x+c)^3+a*tan(d*x+c))
```

maxima [A] time = 0.50, size = 25, normalized size = 0.89

$$\frac{b\tan\left(dx+c\right)^{3}+3\,a\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
[Out] 1/3*(b*tan(d*x + c)^3 + 3*a*tan(d*x + c))/d
```

mupad [B] time = 11.86, size = 25, normalized size = 0.89

$$\frac{\tan\left(c+d\,x\right)\,\left(b\,{\tan\left(c+d\,x\right)}^2+3\,a\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)/cos(c + d*x)^2,x)
[Out] (tan(c + d*x)*(3*a + b*tan(c + d*x)^2))/(3*d)
```

sympy [A] time = 1.75, size = 36, normalized size = 1.29

$$\begin{cases} \frac{a\tan{\left(c+dx \right)}+\frac{b\tan^{3}{\left(c+dx \right)}}{3}}{d} & \text{for}\: d \neq 0 \\ x\left(a+b\tan^{2}{\left(c \right)}\right)\sec^{2}{\left(c \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c)**2),x)
[Out] Piecewise(((a*tan(c + d*x) + b*tan(c + d*x)**3/3)/d, Ne(d, 0)), (x*(a + b*t
an(c)**2)*sec(c)**2, True))
```

3.434 $\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=33

$$\frac{(a-b)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(a+b)$$

[Out] 1/2*(a+b)*x+1/2*(a-b)*cos(d*x+c)*sin(d*x+c)/d

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3675, 385, 203}

$$\frac{(a-b)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(a+b)$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]

[Out] ((a + b)*x)/2 + ((a - b)*Cos[c + d*x]*Sin[c + d*x])/(2*d)

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a+bx^2}{\left(1+x^2\right)^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{(a-b)\cos(c+dx)\sin(c+dx)}{2d}+\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(c+d\right.}{2d}\\ &= \frac{1}{2}(a+b)x+\frac{(a-b)\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 0.97

$$\frac{2(a+b)(c+dx)+(a-b)\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (2*(a + b)*(c + d*x) + (a - b)*Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.51, size = 30, normalized size = 0.91

$$\frac{(a+b)dx+(a-b)\cos(dx+c)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a + b)*d*x + (a - b)*cos(d*x + c)*sin(d*x + c))/d
```

giac [B] time = 1.50, size = 169, normalized size = 5.12

$$\frac{adx\tan(dx)^2\tan(c)^2+bdx\tan(dx)^2\tan(c)^2+adx\tan(dx)^2+bdx\tan(dx)^2+adx\tan(c)^2+bdx\tan(c)^2-a\,t}{2\left(d\tan(dx)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*d*x*tan(d*x)^2*tan(c)^2 + b*d*x*tan(d*x)^2*tan(c)^2 + a*d*x*tan(d*x)
^2 + b*d*x*tan(d*x)^2 + a*d*x*tan(c)^2 + b*d*x*tan(c)^2 - a*tan(d*x)^2*tan(
c) + b*tan(d*x)^2*tan(c) - a*tan(d*x)*tan(c)^2 + b*tan(d*x)*tan(c)^2 + a*d*
x + b*d*x + a*tan(d*x) - b*tan(d*x) + a*tan(c) - b*tan(c))/(d*tan(d*x)^2*ta
n(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

maple [A] time = 0.34, size = 54, normalized size = 1.64

$$\frac{b\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*(1/2*cos(d*x+c)*sin(d*x
+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.56, size = 39, normalized size = 1.18

$$\frac{(dx+c)(a+b)+\frac{(a-b)\tan(dx+c)}{\tan(dx+c)^2+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x + c)*(a + b) + (a - b)*tan(d*x + c)/(tan(d*x + c)^2 + 1))/d
```

mupad [B] time = 11.94, size = 32, normalized size = 0.97

$$\frac{\sin(2c+2dx)\left(\frac{a}{4}-\frac{b}{4}\right)+dx\left(\frac{a}{2}+\frac{b}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*tan(c + d*x)^2),x)
[Out] (sin(2*c + 2*d*x)*(a/4 - b/4) + d*x*(a/2 + b/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(c + dx\right)\right)\cos^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c)**2),x)
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**2, x)
```

3.435 $\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{(a-b)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{(3a+b)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3a+b)$$

```
[Out] 1/8*(3*a+b)*x+1/8*(3*a+b)*cos(d*x+c)*sin(d*x+c)/d+1/4*(a-b)*cos(d*x+c)^3*si
n(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {3675, 385, 199, 203}

$$\frac{(a-b)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{(3a+b)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3a+b)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((3*a + b)*x)/8 + ((3*a + b)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((a - b)*Co
s[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1+x^2)^3}dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a-b)\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{(3a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)^2}dx, x, \tan(c\right.}{4d}$$
$$= \frac{(3a+b)\cos(c+dx)\sin(c+dx)}{8d} + \frac{(a-b)\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{(3}{}$$
$$= \frac{1}{8}(3a+b)x + \frac{(3a+b)\cos(c+dx)\sin(c+dx)}{8d} + \frac{(a-b)\cos^3(c+dx)\text{si}}{4d}$$

Mathematica [A] time = 0.13, size = 46, normalized size = 0.75

$$\frac{(a-b)\sin(4(c+dx))+12a(c+dx)+8a\sin(2(c+dx))+4bdx}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (4*b*d*x + 12*a*(c + d*x) + 8*a*Sin[2*(c + d*x)] + (a - b)*Sin[4*(c + d*x)]
)/(32*d)
```

fricas [A] time = 0.44, size = 49, normalized size = 0.80

$$\frac{(3\,a+b)dx+\left(2\,(a-b)\cos\left(dx+c\right)^3+(3\,a+b)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((3*a + b)*d*x + (2*(a - b)*cos(d*x + c)^3 + (3*a + b)*cos(d*x + c))*si
n(d*x + c))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
```

eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)(24*a*d*x*tan(c)^4*tan(d*x)^4+48*a*d*x*tan(c)^4*tan(d*x)^2+24*a*d*x*tan
(c)^4+48*a*d*x*tan(c)^2*tan(d*x)^4+96*a*d*x*tan(c)^2*tan(d*x)^2+48*a*d*x*ta
n(c)^2+24*a*d*x*tan(d*x)^4+48*a*d*x*tan(d*x)^2+24*a*d*x-40*a*tan(c)^4*tan(d
*x)^3-24*a*tan(c)^4*tan(d*x)-40*a*tan(c)^3*tan(d*x)^4+48*a*tan(c)^3*tan(d*x
)^2+24*a*tan(c)^3+48*a*tan(c)^2*tan(d*x)^3-48*a*tan(c)^2*tan(d*x)-24*a*tan(
c)*tan(d*x)^4-48*a*tan(c)*tan(d*x)^2+40*a*tan(c)+24*a*tan(d*x)^3+40*a*tan(d
*x)+8*b*d*x*tan(c)^4*tan(d*x)^4+16*b*d*x*tan(c)^4*tan(d*x)^2+8*b*d*x*tan(c)
^4+16*b*d*x*tan(c)^2*tan(d*x)^4+32*b*d*x*tan(c)^2*tan(d*x)^2+16*b*d*x*tan(c
)^2+8*b*d*x*tan(d*x)^4+16*b*d*x*tan(d*x)^2+8*b*d*x+3*b*pi*sign(2*tan(c)^2*t
an(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2
-2)*tan(c)^4*tan(d*x)^4+6*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2
-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^4*tan(d*x)^2+3*b
*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(
2*tan(c)^2*tan(d*x)^2-2)*tan(c)^4+6*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*
tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^2*tan(
d*x)^4+12*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(
d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^2*tan(d*x)^2+6*b*pi*sign(2*tan(c
)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d
*x)^2-2)*tan(c)^2+3*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan
(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(d*x)^4+6*b*pi*sign(2*tan(
c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(
d*x)^2-2)*tan(d*x)^2+3*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*
tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)+3*b*pi*sign(2*tan(c)^2*tan
(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^4*tan(d*x)^4+6*b*pi*s
ign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^4*t
an(d*x)^2+3*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*ta
n(d*x))*tan(c)^4+6*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(
c)+2*tan(d*x))*tan(c)^2*tan(d*x)^4+12*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c
)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^2*tan(d*x)^2+6*b*pi*sign(2*tan(c)^
2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^2+3*b*pi*sign(2*
tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(d*x)^4+6*b*p
i*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(d*x
)^2+3*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x)
)+6*b*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^4*tan(d*x)^4+12*b*
atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^4*tan(d*x)^2+6*b*atan((t
an(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^4+12*b*atan((tan(c)+tan(d*x))/(
tan(c)*tan(d*x)-1))*tan(c)^2*tan(d*x)^4+24*b*atan((tan(c)+tan(d*x))/(tan(c)
*tan(d*x)-1))*tan(c)^2*tan(d*x)^2+12*b*atan((tan(c)+tan(d*x))/(tan(c)*tan(d
*x)-1))*tan(c)^2+6*b*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(d*x)^4
+12*b*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(d*x)^2+6*b*atan((tan(
c)+tan(d*x))/(tan(c)*tan(d*x)-1))-6*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*
x)+1))*tan(c)^4*tan(d*x)^4-12*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))
*tan(c)^4*tan(d*x)^2-6*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)
^4-12*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^2*tan(d*x)^4-24*
b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^2*tan(d*x)^2-12*b*atan
((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^2-6*b*atan((tan(c)-tan(d*x))
/(tan(c)*tan(d*x)+1))*tan(d*x)^4-12*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*
x)+1))*tan(d*x)^2-6*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))+8*b*tan(c
)^4*tan(d*x)^3-8*b*tan(c)^4*tan(d*x)+8*b*tan(c)^3*tan(d*x)^4-48*b*tan(c)^3*
tan(d*x)^2+8*b*tan(c)^3-48*b*tan(c)^2*tan(d*x)^3+48*b*tan(c)^2*tan(d*x)-8*b
*tan(c)*tan(d*x)^4+48*b*tan(c)*tan(d*x)^2-8*b*tan(c)+8*b*tan(d*x)^3-8*b*tan

```
(d*x))/(64*d*tan(c)^4*tan(d*x)^4+128*d*tan(c)^4*tan(d*x)^2+64*d*tan(c)^4+12
8*d*tan(c)^2*tan(d*x)^4+256*d*tan(c)^2*tan(d*x)^2+128*d*tan(c)^2+64*d*tan(d
*x)^4+128*d*tan(d*x)^2+64*d)
```

maple [A] time = 0.59, size = 81, normalized size = 1.33

$$\frac{a\left(\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{\left(\cos^{3}(dx+c)\right)\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+b*(-1/4
*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c))
```

maxima [A] time = 1.69, size = 69, normalized size = 1.13

$$\frac{(dx+c)(3a+b)+\frac{(3a+b)\tan(dx+c)^{3}+(5a-b)\tan(dx+c)}{\tan(dx+c)^{4}+2\tan(dx+c)^{2}+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((d*x + c)*(3*a + b) + ((3*a + b)*tan(d*x + c)^3 + (5*a - b)*tan(d*x +
c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

mupad [B] time = 12.06, size = 67, normalized size = 1.10

$$x\left(\frac{3a}{8}+\frac{b}{8}\right)+\frac{\left(\frac{3a}{8}+\frac{b}{8}\right)\tan(c+dx)^{3}+\left(\frac{5a}{8}-\frac{b}{8}\right)\tan(c+dx)}{d\left(\tan(c+dx)^{4}+2\tan(c+dx)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*tan(c + d*x)^2),x)
```

```
[Out] x*((3*a)/8 + b/8) + (tan(c + d*x)^3*((3*a)/8 + b/8) + tan(c + d*x)*((5*a)/8
 - b/8))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{2}(c+dx)\right)\cos^{4}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**4, x)
```

3.436 $\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=87

$$\frac{(a-b)\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{(5a+b)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(5a+b)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(5a+$$

```
[Out] 1/16*(5*a+b)*x+1/16*(5*a+b)*cos(d*x+c)*sin(d*x+c)/d+1/24*(5*a+b)*cos(d*x+c)
^3*sin(d*x+c)/d+1/6*(a-b)*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3675, 385, 199, 203}

$$\frac{(a-b)\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{(5a+b)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(5a+b)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(5a+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((5*a + b)*x)/16 + ((5*a + b)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((5*a + b
)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + ((a - b)*Cos[c + d*x]^5*Sin[c + d*x
])/(6*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1+x^2)^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{(a-b)\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{(5a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)^3}\,dx, x, \tan(c\right.}{6d}$$

$$= \frac{(5a+b)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{(a-b)\cos^5(c+dx)\sin(c+dx)}{6d} + $$

$$= \frac{(5a+b)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(5a+b)\cos^3(c+dx)\sin(c+dx)}{24d} + $$

$$= \frac{1}{16}(5a+b)x + \frac{(5a+b)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(5a+b)\cos^3(c+dx)}{24d}$$

Mathematica [A] time = 0.21, size = 74, normalized size = 0.85

$$\frac{3(15a+b)\sin(2(c+dx)) + (9a-3b)\sin(4(c+dx)) + a\sin(6(c+dx)) + 60ac + 60adx - b\sin(6(c+dx)) + 12bdx}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (60*a*c + 60*a*d*x + 12*b*d*x + 3*(15*a + b)*Sin[2*(c + d*x)] + (9*a - 3*b)
*Sin[4*(c + d*x)] + a*Sin[6*(c + d*x)] - b*Sin[6*(c + d*x)])/(192*d)
```

fricas [A] time = 0.53, size = 66, normalized size = 0.76

$$\frac{3\,(5\,a+b)dx + \left(8\,(a-b)\cos\left(dx+c\right)^5 + 2\,(5\,a+b)\cos\left(dx+c\right)^3 + 3\,(5\,a+b)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(5*a + b)*d*x + (8*(a - b)*cos(d*x + c)^5 + 2*(5*a + b)*cos(d*x + c
)^3 + 3*(5*a + b)*cos(d*x + c))*sin(d*x + c))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
```

: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)(30*a*d*x*tan(c)^6*tan(d*x)^6+90*a*d*x*
tan(c)^6*tan(d*x)^4+90*a*d*x*tan(c)^6*tan(d*x)^2+30*a*d*x*tan(c)^6+90*a*d*x
*tan(c)^4*tan(d*x)^6+270*a*d*x*tan(c)^4*tan(d*x)^4+270*a*d*x*tan(c)^4*tan(d
*x)^2+90*a*d*x*tan(c)^4+90*a*d*x*tan(c)^2*tan(d*x)^6+270*a*d*x*tan(c)^2*tan
(d*x)^4+270*a*d*x*tan(c)^2*tan(d*x)^2+90*a*d*x*tan(c)^2+30*a*d*x*tan(d*x)^6
+90*a*d*x*tan(d*x)^4+90*a*d*x*tan(d*x)^2+30*a*d*x-66*a*tan(c)^6*tan(d*x)^5-
80*a*tan(c)^6*tan(d*x)^3-30*a*tan(c)^6*tan(d*x)-66*a*tan(c)^5*tan(d*x)^6+90
*a*tan(c)^5*tan(d*x)^4+90*a*tan(c)^5*tan(d*x)^2+30*a*tan(c)^5+90*a*tan(c)^4
*tan(d*x)^5-240*a*tan(c)^4*tan(d*x)^3-90*a*tan(c)^4*tan(d*x)-80*a*tan(c)^3*
tan(d*x)^6-240*a*tan(c)^3*tan(d*x)^4+240*a*tan(c)^3*tan(d*x)^2+80*a*tan(c)^
3+90*a*tan(c)^2*tan(d*x)^5+240*a*tan(c)^2*tan(d*x)^3-90*a*tan(c)^2*tan(d*x)
-30*a*tan(c)*tan(d*x)^6-90*a*tan(c)*tan(d*x)^4-90*a*tan(c)*tan(d*x)^2+66*a*
tan(c)+30*a*tan(d*x)^5+80*a*tan(d*x)^3+66*a*tan(d*x)+6*b*d*x*tan(c)^6*tan(d
*x)^6+18*b*d*x*tan(c)^6*tan(d*x)^4+18*b*d*x*tan(c)^6*tan(d*x)^2+6*b*d*x*tan
(c)^6+18*b*d*x*tan(c)^4*tan(d*x)^6+54*b*d*x*tan(c)^4*tan(d*x)^4+54*b*d*x*ta
n(c)^4*tan(d*x)^2+18*b*d*x*tan(c)^4+18*b*d*x*tan(c)^2*tan(d*x)^6+54*b*d*x*t
an(c)^2*tan(d*x)^4+54*b*d*x*tan(c)^2*tan(d*x)^2+18*b*d*x*tan(c)^2+6*b*d*x*t
an(d*x)^6+18*b*d*x*tan(d*x)^4+18*b*d*x*tan(d*x)^2+6*b*d*x+3*b*pi*sign(2*tan
(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan
(d*x)^2-2)*tan(c)^6*tan(d*x)^6+9*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan
(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^6*tan(d*x
)^4+9*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x)
)*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^6*tan(d*x)^2+3*b*pi*sign(2*tan(c)^2*
tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^
2-2)*tan(c)^6+9*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+
2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^4*tan(d*x)^6+27*b*pi*sign(
2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^
2*tan(d*x)^2-2)*tan(c)^4*tan(d*x)^4+27*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(
c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^4*t
an(d*x)^2+9*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*ta
n(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^4+9*b*pi*sign(2*tan(c)^2*tan(d

*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*
tan(c)^2*tan(d*x)^6+27*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*
tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(c)^2*tan(d*x)^4+27*b*p
i*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*
tan(c)^2*tan(d*x)^2-2)*tan(c)^2*tan(d*x)^2+9*b*pi*sign(2*tan(c)^2*tan(d*x)-
2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(
c)^2+3*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x
))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(d*x)^6+9*b*pi*sign(2*tan(c)^2*tan(d*x)
-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan
(d*x)^4+9*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(
d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)*tan(d*x)^2+3*b*pi*sign(2*tan(c)^2*tan(d
*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*sign(2*tan(c)^2*tan(d*x)^2-2)+
3*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*ta
n(c)^6*tan(d*x)^6+9*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan
(c)+2*tan(d*x))*tan(c)^6*tan(d*x)^4+9*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c
)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^6*tan(d*x)^2+3*b*pi*sign(2*tan(c)^
2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^6+9*b*pi*sign(2*
tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^4*tan(d*x
)^6+27*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x
))*tan(c)^4*tan(d*x)^4+27*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2
-2*tan(c)+2*tan(d*x))*tan(c)^4*tan(d*x)^2+9*b*pi*sign(2*tan(c)^2*tan(d*x)-2
*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^4+9*b*pi*sign(2*tan(c)^2*tan
(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^2*tan(d*x)^6+27*b*pi*
sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^2*
tan(d*x)^4+27*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*
tan(d*x))*tan(c)^2*tan(d*x)^2+9*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(
d*x)^2-2*tan(c)+2*tan(d*x))*tan(c)^2+3*b*pi*sign(2*tan(c)^2*tan(d*x)-2*tan(
c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(d*x)^6+9*b*pi*sign(2*tan(c)^2*tan(d*
x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(d*x)^4+9*b*pi*sign(2*tan(c)
^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))*tan(d*x)^2+3*b*pi*sign
(2*tan(c)^2*tan(d*x)-2*tan(c)*tan(d*x)^2-2*tan(c)+2*tan(d*x))+6*b*atan((tan
(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^6*tan(d*x)^6+18*b*atan((tan(c)+ta
n(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^6*tan(d*x)^4+18*b*atan((tan(c)+tan(d*x)
)/(tan(c)*tan(d*x)-1))*tan(c)^6*tan(d*x)^2+6*b*atan((tan(c)+tan(d*x))/(tan(
c)*tan(d*x)-1))*tan(c)^6+18*b*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*t
an(c)^4*tan(d*x)^6+54*b*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^
4*tan(d*x)^4+54*b*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^4*tan(
d*x)^2+18*b*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^4+18*b*atan(
(tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^2*tan(d*x)^6+54*b*atan((tan(c
)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(c)^2*tan(d*x)^4+54*b*atan((tan(c)+tan(
d*x))/(tan(c)*tan(d*x)-1))*tan(c)^2*tan(d*x)^2+18*b*atan((tan(c)+tan(d*x))/
(tan(c)*tan(d*x)-1))*tan(c)^2+6*b*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1
))*tan(d*x)^6+18*b*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(d*x)^4+1
8*b*atan((tan(c)+tan(d*x))/(tan(c)*tan(d*x)-1))*tan(d*x)^2+6*b*atan((tan(c)
+tan(d*x))/(tan(c)*tan(d*x)-1))-6*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)
+1))*tan(c)^6*tan(d*x)^6-18*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*t
an(c)^6*tan(d*x)^4-18*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^
6*tan(d*x)^2-6*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^6-18*b*
atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^4*tan(d*x)^6-54*b*atan((
tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^4*tan(d*x)^4-54*b*atan((tan(c)
-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^4*tan(d*x)^2-18*b*atan((tan(c)-tan(d
*x))/(tan(c)*tan(d*x)+1))*tan(c)^4-18*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(
d*x)+1))*tan(c)^2*tan(d*x)^6-54*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1
))*tan(c)^2*tan(d*x)^4-54*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan
(c)^2*tan(d*x)^2-18*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(c)^2-
6*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(d*x)^6-18*b*atan((tan(c
)-tan(d*x))/(tan(c)*tan(d*x)+1))*tan(d*x)^4-18*b*atan((tan(c)-tan(d*x))/(ta
n(c)*tan(d*x)+1))*tan(d*x)^2-6*b*atan((tan(c)-tan(d*x))/(tan(c)*tan(d*x)+1)
)+6*b*tan(c)^6*tan(d*x)^5-16*b*tan(c)^6*tan(d*x)^3-6*b*tan(c)^6*tan(d*x)+6*

```
b*tan(c)^5*tan(d*x)^6-78*b*tan(c)^5*tan(d*x)^4+18*b*tan(c)^5*tan(d*x)^2+6*b
*tan(c)^5-78*b*tan(c)^4*tan(d*x)^5+144*b*tan(c)^4*tan(d*x)^3-18*b*tan(c)^4*
tan(d*x)-16*b*tan(c)^3*tan(d*x)^6+144*b*tan(c)^3*tan(d*x)^4-144*b*tan(c)^3*
tan(d*x)^2+16*b*tan(c)^3+18*b*tan(c)^2*tan(d*x)^5-144*b*tan(c)^2*tan(d*x)^3
+78*b*tan(c)^2*tan(d*x)-6*b*tan(c)*tan(d*x)^6-18*b*tan(c)*tan(d*x)^4+78*b*t
an(c)*tan(d*x)^2-6*b*tan(c)+6*b*tan(d*x)^5+16*b*tan(d*x)^3-6*b*tan(d*x))/(9
6*d*tan(c)^6*tan(d*x)^6+288*d*tan(c)^6*tan(d*x)^4+288*d*tan(c)^6*tan(d*x)^2
+96*d*tan(c)^6+288*d*tan(c)^4*tan(d*x)^6+864*d*tan(c)^4*tan(d*x)^4+864*d*ta
n(c)^4*tan(d*x)^2+288*d*tan(c)^4+288*d*tan(c)^2*tan(d*x)^6+864*d*tan(c)^2*t
an(d*x)^4+864*d*tan(c)^2*tan(d*x)^2+288*d*tan(c)^2+96*d*tan(d*x)^6+288*d*ta
n(d*x)^4+288*d*tan(d*x)^2+96*d)
```

maple [A] time = 0.74, size = 102, normalized size = 1.17

$$\frac{a\left(\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+b\left(-\frac{\left(\cos^{5}(dx+c)\right)\sin(dx+c)}{6}+\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)+b*(-1/6*cos(d*x+c)^5*sin(d*x+c)+1/24*(cos(d*x+c)^3+3/2*cos(d*x
+c))*sin(d*x+c)+1/16*d*x+1/16*c))
```

maxima [A] time = 1.04, size = 97, normalized size = 1.11

$$\frac{3\,(dx+c)(5\,a+b)+\frac{3\,(5\,a+b)\tan(dx+c)^{5}+8\,(5\,a+b)\tan(dx+c)^{3}+3\,(11\,a-b)\tan(dx+c)}{\tan(dx+c)^{6}+3\,\tan(dx+c)^{4}+3\,\tan(dx+c)^{2}+1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(d*x + c)*(5*a + b) + (3*(5*a + b)*tan(d*x + c)^5 + 8*(5*a + b)*tan
(d*x + c)^3 + 3*(11*a - b)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4
 + 3*tan(d*x + c)^2 + 1))/d
```

mupad [B] time = 12.54, size = 93, normalized size = 1.07

$$x\left(\frac{5\,a}{16}+\frac{b}{16}\right)+\frac{\left(\frac{5\,a}{16}+\frac{b}{16}\right)\,\tan\left(c+d\,x\right)^5+\left(\frac{5\,a}{6}+\frac{b}{6}\right)\,\tan\left(c+d\,x\right)^3+\left(\frac{11\,a}{16}-\frac{b}{16}\right)\,\tan\left(c+d\,x\right)}{d\,\left({\tan\left(c+d\,x\right)}^6+3\,{\tan\left(c+d\,x\right)}^4+3\,{\tan\left(c+d\,x\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + b*tan(c + d*x)^2),x)
```

```
[Out] x*((5*a)/16 + b/16) + (tan(c + d*x)^3*((5*a)/6 + b/6) + tan(c + d*x)^5*((5*
a)/16 + b/16) + tan(c + d*x)*((11*a)/16 - b/16))/(d*(3*tan(c + d*x)^2 + 3*t
an(c + d*x)^4 + tan(c + d*x)^6 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^{2}\left(c+dx\right)\right)\cos^{6}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**6, x)
```

3.437 $\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=128

$$\frac{(8a^2-4ab+b^2)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(8a^2-4ab+b^2)\tan(c+dx)\sec(c+dx)}{16d}+\frac{b(8a-3b)\tan(c+dx)\sec^3(c+}{24d}$$

```
[Out] 1/16*(8*a^2-4*a*b+b^2)*arctanh(sin(d*x+c))/d+1/16*(8*a^2-4*a*b+b^2)*sec(d*x
+c)*tan(d*x+c)/d+1/24*(8*a-3*b)*b*sec(d*x+c)^3*tan(d*x+c)/d+1/6*b*sec(d*x+c
)^5*(a-(a-b)*sin(d*x+c)^2)*tan(d*x+c)/d
```

Rubi [A] time = 0.16, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3676, 413, 385, 199, 206}

$$\frac{(8a^2-4ab+b^2)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(8a^2-4ab+b^2)\tan(c+dx)\sec(c+dx)}{16d}+\frac{b(8a-3b)\tan(c+dx)\sec^3(c+}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 - 4*a*b + b^2)*ArcTanh[Sin[c + d*x]])/(16*d) + ((8*a^2 - 4*a*b + b^
2)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((8*a - 3*b)*b*Sec[c + d*x]^3*Tan[c
+ d*x])/(24*d) + (b*Sec[c + d*x]^5*(a - (a - b)*Sin[c + d*x]^2)*Tan[c + d*x
])/(6*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^4}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{b\sec^5(c+dx)\left(a-(a-b)\sin^2(c+dx)\right)\tan(c+dx)}{6d} - \frac{\text{Subst}\left(\int \frac{-a(}{}\right.}{}$$

$$= \frac{(8a-3b)b\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{b\sec^5(c+dx)\left(a-(a-b)\sin\right.}{6d}$$

$$= \frac{\left(8a^2-4ab+b^2\right)\sec(c+dx)\tan(c+dx)}{16d} + \frac{(8a-3b)b\sec^3(c+dx)}{24d}$$

$$= \frac{\left(8a^2-4ab+b^2\right)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{\left(8a^2-4ab+b^2\right)\sec(c+d}{16d}$$

Mathematica [C] time = 8.33, size = 875, normalized size = 6.84

$$\frac{\sin(c+dx)\left(380(a-b)^2\,{}_4F_3\left(\frac{3}{2},2,2,2;1,1,\frac{9}{2};\sin^2(c+dx)\right)\sqrt{\sin^2(c+dx)}\,\sin^{10}(c+dx)+128(a-b)^2\,{}_5F_4\left(\frac{3}{2},2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (Sin[c + d*x]*(65625*a^2*ArcTanh[Sqrt[Sin[c + d*x]^2]] - 36855*a^2*ArcTanh[
Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^2 - 91875*a*(a - b)*ArcTanh[Sqrt[Sin[c +
 d*x]^2]]*Sin[c + d*x]^2 + 1680*a^2*ArcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d
*x]^4 + 54180*a*(a - b)*ArcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^4 + 3297
0*(a - b)^2*ArcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^4 - 1365*a*(a - b)*A
rcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^6 - 19845*(a - b)^2*ArcTanh[Sqrt[
Sin[c + d*x]^2]]*Sin[c + d*x]^6 + 525*(a - b)^2*ArcTanh[Sqrt[Sin[c + d*x]^2
]]*Sin[c + d*x]^8 - 65625*a^2*Sqrt[Sin[c + d*x]^2] - 23555*a*(a - b)*Sin[c
+ d*x]^4*Sqrt[Sin[c + d*x]^2] - 32970*(a - b)^2*Sin[c + d*x]^4*Sqrt[Sin[c +
 d*x]^2] + 8855*(a - b)^2*Sin[c + d*x]^6*Sqrt[Sin[c + d*x]^2] + 620*a^2*Hyp
ergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^6*
Sqrt[Sin[c + d*x]^2] + 160*a^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1,
1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^6*Sqrt[Sin[c + d*x]^2] + 16*a^2*Hyper
geometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c
 + d*x]^6*Sqrt[Sin[c + d*x]^2] - 968*a*(a - b)*HypergeometricPFQ[{3/2, 2, 2
, 2}, {1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^8*Sqrt[Sin[c + d*x]^2] - 28
8*a*(a - b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, Sin[c + d*
x]^2]*Sin[c + d*x]^8*Sqrt[Sin[c + d*x]^2] - 32*a*(a - b)*HypergeometricPFQ[
{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^8*Sqr
t[Sin[c + d*x]^2] + 380*(a - b)^2*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1,
9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^10*Sqrt[Sin[c + d*x]^2] + 128*(a - b)^2*
HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c
```

```
+ d*x]^10*Sqrt[Sin[c + d*x]^2] + 16*(a - b)^2*HypergeometricPFQ[{3/2, 2, 2,
 2, 2, 2}, {1, 1, 1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^10*Sqrt[Sin[c +
d*x]^2] + 14980*a^2*(Sin[c + d*x]^2)^(3/2) + 91875*a*(a - b)*(Sin[c + d*x]^
2)^(3/2)))/(2520*d*(Sin[c + d*x]^2)^(5/2))
```

fricas [A] time = 0.53, size = 137, normalized size = 1.07

$$\frac{3\left(8a^2-4ab+b^2\right)\cos(dx+c)^6\log(\sin(dx+c)+1)-3\left(8a^2-4ab+b^2\right)\cos(dx+c)^6\log(-\sin(dx+c)+1)}{96d\cos(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*a^2 - 4*a*b + b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(8*a
^2 - 4*a*b + b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(3*(8*a^2 - 4*a
*b + b^2)*cos(d*x + c)^4 + 2*(12*a*b - 7*b^2)*cos(d*x + c)^2 + 8*b^2)*sin(d
*x + c))/(d*cos(d*x + c)^6)
```

giac [A] time = 2.95, size = 167, normalized size = 1.30

$$\frac{3\left(8a^2-4ab+b^2\right)\log(|\sin(dx+c)+1|)-3\left(8a^2-4ab+b^2\right)\log(|\sin(dx+c)-1|)-\frac{2\left(24a^2\sin(dx+c)^5-12ab\sin(dx\right.}{}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(8*a^2 - 4*a*b + b^2)*log(abs(sin(d*x + c) + 1)) - 3*(8*a^2 - 4*a*b
 + b^2)*log(abs(sin(d*x + c) - 1)) - 2*(24*a^2*sin(d*x + c)^5 - 12*a*b*sin(
d*x + c)^5 + 3*b^2*sin(d*x + c)^5 - 48*a^2*sin(d*x + c)^3 + 8*b^2*sin(d*x +
 c)^3 + 24*a^2*sin(d*x + c) + 12*a*b*sin(d*x + c) - 3*b^2*sin(d*x + c))/(si
n(d*x + c)^2 - 1)^3)/d
```

maple [B] time = 0.61, size = 248, normalized size = 1.94

$$\frac{b^2\left(\sin^5(dx+c)\right)}{6d\cos(dx+c)^6}+\frac{b^2\left(\sin^5(dx+c)\right)}{24d\cos(dx+c)^4}-\frac{b^2\left(\sin^5(dx+c)\right)}{48d\cos(dx+c)^2}-\frac{b^2\left(\sin^3(dx+c)\right)}{48d}-\frac{b^2\sin(dx+c)}{16d}+\frac{b^2\ln(\sec(dx+c)+}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/6/d*b^2*sin(d*x+c)^5/cos(d*x+c)^6+1/24/d*b^2*sin(d*x+c)^5/cos(d*x+c)^4-1/
48/d*b^2*sin(d*x+c)^5/cos(d*x+c)^2-1/48/d*b^2*sin(d*x+c)^3-1/16/d*b^2*sin(d
*x+c)+1/16/d*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*b*sin(d*x+c)^3/cos(d*x+c
)^4+1/4/d*a*b*sin(d*x+c)^3/cos(d*x+c)^2+1/4*a*b*sin(d*x+c)/d-1/4/d*a*b*ln(s
ec(d*x+c)+tan(d*x+c))+1/2*a^2*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^2*ln(sec(d*x+
c)+tan(d*x+c))
```

maxima [A] time = 0.57, size = 156, normalized size = 1.22

$$\frac{3\left(8a^2-4ab+b^2\right)\log(\sin(dx+c)+1)-3\left(8a^2-4ab+b^2\right)\log(\sin(dx+c)-1)-\frac{2\left(3\left(8a^2-4ab+b^2\right)\sin(dx+c)^5-8(\right.}{\sin(dx+c)^6-3}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(8*a^2 - 4*a*b + b^2)*log(sin(d*x + c) + 1) - 3*(8*a^2 - 4*a*b + b^
2)*log(sin(d*x + c) - 1) - 2*(3*(8*a^2 - 4*a*b + b^2)*sin(d*x + c)^5 - 8*(6
```

```
*a^2 - b^2)*sin(d*x + c)^3 + 3*(8*a^2 + 4*a*b - b^2)*sin(d*x + c))/(sin(d*x
 + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1))/d
```

mupad [B] time = 15.64, size = 269, normalized size = 2.10

$$\frac{\left(a^2+\frac{a\,b}{2}-\frac{b^2}{8}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(-3\,a^2+\frac{5\,a\,b}{2}+\frac{17\,b^2}{24}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(2\,a^2-3\,a\,b+\frac{19\,b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-20\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^2/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(2*a^2 - 3*a*b + (19*b^2)/4) + tan(c/2 + (d*x)/2)^7*(
2*a^2 - 3*a*b + (19*b^2)/4) + tan(c/2 + (d*x)/2)^3*((5*a*b)/2 - 3*a^2 + (17
*b^2)/24) + tan(c/2 + (d*x)/2)^9*((5*a*b)/2 - 3*a^2 + (17*b^2)/24) + tan(c/
2 + (d*x)/2)*((a*b)/2 + a^2 - b^2/8) + tan(c/2 + (d*x)/2)^11*((a*b)/2 + a^2
 - b^2/8))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/
2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/
2 + (d*x)/2)^12 + 1)) + (atanh(tan(c/2 + (d*x)/2))*(a^2 - (a*b)/2 + b^2/8))
/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(c+dx\right)\right)^2\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**3, x)
```

3.438 $\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=96

$$\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3b(2a-b)\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)\left(a-(a-b\right.}{4d}$$

```
[Out] 1/8*(8*a^2-8*a*b+3*b^2)*arctanh(sin(d*x+c))/d+3/8*(2*a-b)*b*sec(d*x+c)*tan(
d*x+c)/d+1/4*b*sec(d*x+c)^3*(a-(a-b)*sin(d*x+c)^2)*tan(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3676, 413, 385, 206}

$$\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3b(2a-b)\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)\left(a-(a-b\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (3*(2*a - b)*b*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*(a - (a - b)*Sin[c + d*x]^
2)*Tan[c + d*x])/(4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^3}dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{b\sec^3(c+dx)\left(a-(a-b)\sin^2(c+dx)\right)\tan(c+dx)}{4d} - \frac{\text{Subst}\left(\int \frac{-a(4}{}\right.}{}$$

$$= \frac{3(2a-b)b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\left(a-(a-b)\sin^2\right.}{4d}$$

$$= \frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{3(2a-b)b\sec(c+dx)\tan}{8d}$$

Mathematica [C] time = 7.33, size = 347, normalized size = 3.61

$$\frac{\csc^3(c+dx)\left(128\sin^6(c+dx)\left(\frac{1}{2}a^2(5\cos(2(c+dx))+9)\cos^2(c+dx)+b\sin^2(c+dx)\left(5a\cos(2(c+dx))+7a\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (Csc[c + d*x]^3*(128*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, S
in[c + d*x]^2]*Sin[c + d*x]^6*(a + (-a + b)*Sin[c + d*x]^2)^2 + 128*Hyperge
ometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^6*((a^
2*Cos[c + d*x]^2*(9 + 5*Cos[2*(c + d*x)]))/2 + b*Sin[c + d*x]^2*(7*a + 5*a*
Cos[2*(c + d*x)] + 5*b*Sin[c + d*x]^2)) + 35*(-3375*a^2 + 3*a*(1969*a - 175
0*b)*Sin[c + d*x]^2 + (-3161*a^2 + 5108*a*b - 1947*b^2)*Sin[c + d*x]^4 + 48
5*(a - b)^2*Sin[c + d*x]^6 + (3*ArcTanh[Sqrt[Sin[c + d*x]^2]]*(1125*a^2 - 2
*a*(1172*a - 875*b)*Sin[c + d*x]^2 + (1674*a^2 - 2286*a*b + 649*b^2)*Sin[c
+ d*x]^4 + (-400*a^2 + 778*a*b - 378*b^2)*Sin[c + d*x]^6 + 9*(a - b)^2*Sin[
c + d*x]^8))/Sqrt[Sin[c + d*x]^2])))/(6720*d)
```

fricas [A] time = 0.46, size = 116, normalized size = 1.21

$$\frac{\left(8\,a^2-8\,ab+3\,b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(8\,a^2-8\,ab+3\,b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+\right.}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((8*a^2 - 8*a*b + 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^2
 - 8*a*b + 3*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*((8*a*b - 5*b^2
)*cos(d*x + c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 2.70, size = 120, normalized size = 1.25

$$\frac{\left(8\,a^2-8\,ab+3\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(8\,a^2-8\,ab+3\,b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(8\,ab\sin(dx+c)^3-5\,b^2\sin(\right.}{\left(\sin\right.}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*((8*a^2 - 8*a*b + 3*b^2)*log(abs(sin(d*x + c) + 1)) - (8*a^2 - 8*a*b +
 3*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(8*a*b*sin(d*x + c)^3 - 5*b^2*sin(d*
x + c)^3 - 8*a*b*sin(d*x + c) + 3*b^2*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)
/d
```

maple [A] time = 0.38, size = 178, normalized size = 1.85

$$\frac{b^2\left(\sin^5(dx+c)\right)}{4d\cos(dx+c)^4}-\frac{b^2\left(\sin^5(dx+c)\right)}{8d\cos(dx+c)^2}-\frac{b^2\left(\sin^3(dx+c)\right)}{8d}-\frac{3b^2\sin(dx+c)}{8d}+\frac{3b^2\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/4/d*b^2*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*b^2*sin(d*x+c)^5/cos(d*x+c)^2-1/8
/d*b^2*sin(d*x+c)^3-3/8/d*b^2*sin(d*x+c)+3/8/d*b^2*ln(sec(d*x+c)+tan(d*x+c)
)+1/d*a*b*sin(d*x+c)^3/cos(d*x+c)^2+a*b*sin(d*x+c)/d-1/d*a*b*ln(sec(d*x+c)+
tan(d*x+c))+1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.44, size = 119, normalized size = 1.24

$$\frac{\left(8a^2-8ab+3b^2\right)\log(\sin(dx+c)+1)-\left(8a^2-8ab+3b^2\right)\log(\sin(dx+c)-1)-\frac{2\left(\left(8ab-5b^2\right)\sin(dx+c)^3-\left(8ab-3b^2\right)\cdots\right)}{\sin(dx+c)^4-2\sin(dx+c)^2\cdots}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/16*((8*a^2 - 8*a*b + 3*b^2)*log(sin(d*x + c) + 1) - (8*a^2 - 8*a*b + 3*b^
2)*log(sin(d*x + c) - 1) - 2*((8*a*b - 5*b^2)*sin(d*x + c)^3 - (8*a*b - 3*b
^2)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

mupad [B] time = 14.57, size = 177, normalized size = 1.84

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(2a^2-2ab+\frac{3b^2}{4}\right)}{d}+\frac{\left(2ab-\frac{3b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{11b^2}{4}-2ab\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{11b^2}{4}-\cdots\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\cdots\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^2/cos(c + d*x),x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(2*a^2 - 2*a*b + (3*b^2)/4))/d + (tan(c/2 + (d*x
)/2)^7*(2*a*b - (3*b^2)/4) - tan(c/2 + (d*x)/2)^3*(2*a*b - (11*b^2)/4) - ta
n(c/2 + (d*x)/2)^5*(2*a*b - (11*b^2)/4) + tan(c/2 + (d*x)/2)*(2*a*b - (3*b^
2)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d
*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2(c+dx)\right)^2\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x), x)
```

3.439 $\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=62

$$\frac{(a-b)^2\sin(c+dx)}{d}+\frac{b(4a-3b)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 1/2*(4*a-3*b)*b*arctanh(sin(d*x+c))/d+(a-b)^2*sin(d*x+c)/d+1/2*b^2*sec(d*x+
c)*tan(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3676, 390, 385, 206}

$$\frac{(a-b)^2\sin(c+dx)}{d}+\frac{b(4a-3b)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((4*a - 3*b)*b*ArcTanh[Sin[c + d*x]])/(2*d) + ((a - b)^2*Sin[c + d*x])/d +
(b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^2}dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left((a-b)^2+\frac{(2a-b)b-2(a-b)bx^2}{\left(1-x^2\right)^2}\right)dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{(a-b)^2\sin(c+dx)}{d}+\frac{\text{Subst}\left(\int \frac{(2a-b)b-2(a-b)bx^2}{\left(1-x^2\right)^2}dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{(a-b)^2\sin(c+dx)}{d}+\frac{b^2\sec(c+dx)\tan(c+dx)}{2d}+\frac{((4a-3b)b)\,\text{Subst}}{}$$
$$= \frac{(4a-3b)b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(a-b)^2\sin(c+dx)}{d}+\frac{b^2\sec(c+dx)}{2d}$$

Mathematica [A] time = 0.46, size = 66, normalized size = 1.06

$$\frac{\tan(c+dx)\sec(c+dx)\left(a^2+(a-b)^2\cos(2(c+dx))-2ab+2b^2\right)+b(4a-3b)\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((4*a - 3*b)*b*ArcTanh[Sin[c + d*x]] + (a^2 - 2*a*b + 2*b^2 + (a - b)^2*Cos
[2*(c + d*x)])*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

fricas [A] time = 0.47, size = 106, normalized size = 1.71

$$\frac{\left(4\,ab-3\,b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(4\,ab-3\,b^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\left(a^2-2\right.\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((4*a*b - 3*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (4*a*b - 3*b^2)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*(a^2 - 2*a*b + b^2)*cos(d*x +
 c)^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 2.60, size = 104, normalized size = 1.68

$$\frac{4\,a^2\sin\left(dx+c\right)-8\,ab\sin\left(dx+c\right)+4\,b^2\sin\left(dx+c\right)+\left(4\,ab-3\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(4\,ab-3\,b^2\right)\log\left(\left|s\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(4*a^2*sin(d*x + c) - 8*a*b*sin(d*x + c) + 4*b^2*sin(d*x + c) + (4*a*b
- 3*b^2)*log(abs(sin(d*x + c) + 1)) - (4*a*b - 3*b^2)*log(abs(sin(d*x + c)
- 1)) - 2*b^2*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

maple [B] time = 0.40, size = 125, normalized size = 2.02

$$\frac{b^2\left(\sin^5\left(dx+c\right)\right)}{2d\cos\left(dx+c\right)^2}+\frac{b^2\left(\sin^3\left(dx+c\right)\right)}{2d}+\frac{3b^2\sin\left(dx+c\right)}{2d}-\frac{3b^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}-\frac{2ab\sin\left(dx+c\right)}{d}+\frac{2a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d*b^2*sin(d*x+c)^5/cos(d*x+c)^2+1/2/d*b^2*sin(d*x+c)^3+3/2/d*b^2*sin(d*
x+c)-3/2/d*b^2*ln(sec(d*x+c)+tan(d*x+c))-2*a*b*sin(d*x+c)/d+2/d*a*b*ln(sec(
d*x+c)+tan(d*x+c))+a^2*sin(d*x+c)/d
```

maxima [A] time = 0.47, size = 105, normalized size = 1.69

$$-\frac{b^{2}\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)-4\,\sin\left(dx+c\right)\right)-4\,ab\left(\log\left(\sin\left(dx+c\right)+\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 4*a*b*(log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 4*a^2*sin(d*x + c))/d
```

mupad [B] time = 14.31, size = 148, normalized size = 2.39

$$\frac{b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,a-3\,b\right)}{d}-\frac{\left(2\,a^2-4\,a\,b+3\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-4\,a^2+8\,a\,b-2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] (b*atanh(tan(c/2 + (d*x)/2))*(4*a - 3*b))/d - (tan(c/2 + (d*x)/2)^5*(2*a^2
- 4*a*b + 3*b^2) - tan(c/2 + (d*x)/2)^3*(4*a^2 - 8*a*b + 2*b^2) + tan(c/2 +
 (d*x)/2)*(2*a^2 - 4*a*b + 3*b^2))/(d*(tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*
x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^{2}\left(c+dx\right)\right)^{2}\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x), x)
```

3.440 $\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=56

$$\frac{\left(a^2-b^2\right)\sin(c+dx)}{d}-\frac{(a-b)^2\sin^3(c+dx)}{3d}+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] b^2*arctanh(sin(d*x+c))/d+(a^2-b^2)*sin(d*x+c)/d-1/3*(a-b)^2*sin(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3676, 390, 206}

$$\frac{\left(a^2-b^2\right)\sin(c+dx)}{d}-\frac{(a-b)^2\sin^3(c+dx)}{3d}+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]])/d + ((a^2 - b^2)*Sin[c + d*x])/d - ((a - b)^2*S
in[c + d*x]^3)/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(a^2-b^2-(a-b)^2x^2+\frac{b^2}{1-x^2}\right)dx,x,\sin(c+dx)\right)}{d}\\
&= \frac{\left(a^2-b^2\right)\sin(c+dx)}{d}-\frac{(a-b)^2\sin^3(c+dx)}{3d}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\right.}{d}\\
&= \frac{b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{\left(a^2-b^2\right)\sin(c+dx)}{d}-\frac{(a-b)^2\sin^3(c+dx}{3d}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 71, normalized size = 1.27

$$\frac{\sin(c+dx)\left(\frac{3b^2\tanh^{-1}\left(\sqrt{\sin^2(c+dx)}\right)}{\sqrt{\sin^2(c+dx)}}-(a-b)\left((a-b)\sin^2(c+dx)-3(a+b)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (Sin[c + d*x]*((3*b^2*ArcTanh[Sqrt[Sin[c + d*x]^2]])/Sqrt[Sin[c + d*x]^2] -
 (a - b)*(-3*(a + b) + (a - b)*Sin[c + d*x]^2)))/(3*d)
```

fricas [A] time = 0.51, size = 79, normalized size = 1.41

$$\frac{3\,b^2\log\left(\sin\left(dx+c\right)+1\right)-3\,b^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(\left(a^2-2\,ab+b^2\right)\cos\left(dx+c\right)^2+2\,a^2+2\,ab-4\,b^2\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*log(sin(d*x + c) + 1) - 3*b^2*log(-sin(d*x + c) + 1) + 2*((a^2 -
 2*a*b + b^2)*cos(d*x + c)^2 + 2*a^2 + 2*a*b - 4*b^2)*sin(d*x + c))/d
```

giac [A] time = 3.02, size = 96, normalized size = 1.71

$$-\frac{2\,a^2\sin\left(dx+c\right)^3-4\,ab\sin\left(dx+c\right)^3+2\,b^2\sin\left(dx+c\right)^3-3\,b^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\,b^2\log\left(\left|\sin\left(dx+c\right.\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(2*a^2*sin(d*x + c)^3 - 4*a*b*sin(d*x + c)^3 + 2*b^2*sin(d*x + c)^3 -
3*b^2*log(abs(sin(d*x + c) + 1)) + 3*b^2*log(abs(sin(d*x + c) - 1)) - 6*a^2
*sin(d*x + c) + 6*b^2*sin(d*x + c))/d
```

maple [A] time = 0.62, size = 104, normalized size = 1.86

$$-\frac{b^2\left(\sin^3\left(dx+c\right)\right)}{3d}-\frac{b^2\sin\left(dx+c\right)}{d}+\frac{b^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{2ab\left(\sin^3\left(dx+c\right)\right)}{3d}+\frac{\sin\left(dx+c\right)\left(\cos\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] -1/3/d*b^2*sin(d*x+c)^3-1/d*b^2*sin(d*x+c)+1/d*b^2*ln(sec(d*x+c)+tan(d*x+c)
)+2/3*a*b*sin(d*x+c)^3/d+1/3/d*sin(d*x+c)*cos(d*x+c)^2*a^2+2/3*a^2*sin(d*x+
c)/d
```

maxima [A] time = 0.65, size = 72, normalized size = 1.29

$$-\frac{2\left(a^2-2\,ab+b^2\right)\sin\left(dx+c\right)^3-3\,b^2\log\left(\sin\left(dx+c\right)+1\right)+3\,b^2\log\left(\sin\left(dx+c\right)-1\right)-6\left(a^2-b^2\right)\sin\left(dx\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(a^2 - 2*a*b + b^2)*sin(d*x + c)^3 - 3*b^2*log(sin(d*x + c) + 1) +
3*b^2*log(sin(d*x + c) - 1) - 6*(a^2 - b^2)*sin(d*x + c))/d
```

mupad [B] time = 14.04, size = 136, normalized size = 2.43

$$\frac{2\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{\left(2\,a^2-2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{4\,a^2}{3}+\frac{16\,a\,b}{3}-\frac{20\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a^2-2\,b^2\right)\,\tan\left(\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] (2*b^2*atanh(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)^5*(2*a^2 - 2*b^2)
 + tan(c/2 + (d*x)/2)^3*((16*a*b)/3 + (4*a^2)/3 - (20*b^2)/3) + tan(c/2 + (
d*x)/2)*(2*a^2 - 2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^
4 + tan(c/2 + (d*x)/2)^6 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(c+dx\right)\right)^2\cos^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x)**3, x)
```

3.441 $\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=57

$$\frac{a^2\sin(c+dx)}{d}+\frac{(a-b)^2\sin^5(c+dx)}{5d}-\frac{2a(a-b)\sin^3(c+dx)}{3d}$$

```
[Out] a^2*sin(d*x+c)/d-2/3*a*(a-b)*sin(d*x+c)^3/d+1/5*(a-b)^2*sin(d*x+c)^5/d
```

Rubi [A] time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3676, 194}

$$\frac{a^2\sin(c+dx)}{d}+\frac{(a-b)^2\sin^5(c+dx)}{5d}-\frac{2a(a-b)\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/d - (2*a*(a - b)*Sin[c + d*x]^3)/(3*d) + ((a - b)^2*Sin[
c + d*x]^5)/(5*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx&=\frac{\text{Subst}\left(\int\left(a-(a-b)x^2\right)^2\,dx,x,\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(a^2-2a(a-b)x^2+(a-b)^2x^4\right)dx,x,\sin(c+dx)\right)}{d}\\&=\frac{a^2\sin(c+dx)}{d}-\frac{2a(a-b)\sin^3(c+dx)}{3d}+\frac{(a-b)^2\sin^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.16, size = 52, normalized size = 0.91

$$\frac{15a^2\sin(c+dx)+3(a-b)^2\sin^5(c+dx)-10a(a-b)\sin^3(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (15*a^2*Sin[c + d*x] - 10*a*(a - b)*Sin[c + d*x]^3 + 3*(a - b)^2*Sin[c + d*
x]^5)/(15*d)
```

fricas [A] time = 0.49, size = 71, normalized size = 1.25

$$\frac{\left(3\left(a^2-2\,ab+b^2\right)\cos\left(dx+c\right)^4+2\left(2\,a^2+ab-3\,b^2\right)\cos\left(dx+c\right)^2+8\,a^2+4\,ab+3\,b^2\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*(a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(2*a^2 + a*b - 3*b^2)*cos(d*
x + c)^2 + 8*a^2 + 4*a*b + 3*b^2)*sin(d*x + c)/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.85, size = 89, normalized size = 1.56

$$\frac{\frac{b^2\left(\sin^5(dx+c)\right)}{5}+2ab\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(1/5*b^2*sin(d*x+c)^5+2*a*b*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d
*x+c)^2)*sin(d*x+c))+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
)
```

maxima [A] time = 0.38, size = 56, normalized size = 0.98

$$\frac{3\left(a^2-2\,ab+b^2\right)\sin\left(dx+c\right)^5-10\left(a^2-ab\right)\sin\left(dx+c\right)^3+15\,a^2\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*(a^2 - 2*a*b + b^2)*sin(d*x + c)^5 - 10*(a^2 - a*b)*sin(d*x + c)^3
+ 15*a^2*sin(d*x + c))/d
```

mupad [B] time = 12.20, size = 119, normalized size = 2.09

$$\frac{\frac{5\,a^2\,\sin(c+d\,x)}{8}+\frac{b^2\,\sin(c+d\,x)}{8}+\frac{5\,a^2\,\sin(3\,c+3\,d\,x)}{48}+\frac{a^2\,\sin(5\,c+5\,d\,x)}{80}-\frac{b^2\,\sin(3\,c+3\,d\,x)}{16}+\frac{b^2\,\sin(5\,c+5\,d\,x)}{80}+\frac{a\,b\,\sin(c+d\,x)}{4}-\frac{a\,b\,\sin(3\,c}{24}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] ((5*a^2*sin(c + d*x))/8 + (b^2*sin(c + d*x))/8 + (5*a^2*sin(3*c + 3*d*x))/4
8 + (a^2*sin(5*c + 5*d*x))/80 - (b^2*sin(3*c + 3*d*x))/16 + (b^2*sin(5*c +
5*d*x))/80 + (a*b*sin(c + d*x))/4 - (a*b*sin(3*c + 3*d*x))/24 - (a*b*sin(5*
c + 5*d*x))/40)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2(c + dx)\right)^2 \cos^5(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x)**5, x)
```

3.442 $\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=86

$$\frac{a^2\sin(c+dx)}{d}-\frac{(a-b)^2\sin^7(c+dx)}{7d}+\frac{(a-b)(3a-b)\sin^5(c+dx)}{5d}-\frac{a(3a-2b)\sin^3(c+dx)}{3d}$$

```
[Out] a^2*sin(d*x+c)/d-1/3*a*(3*a-2*b)*sin(d*x+c)^3/d+1/5*(a-b)*(3*a-b)*sin(d*x+c
)^5/d-1/7*(a-b)^2*sin(d*x+c)^7/d
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3676, 373}

$$\frac{a^2\sin(c+dx)}{d}-\frac{(a-b)^2\sin^7(c+dx)}{7d}+\frac{(a-b)(3a-b)\sin^5(c+dx)}{5d}-\frac{a(3a-2b)\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/d - (a*(3*a - 2*b)*Sin[c + d*x]^3)/(3*d) + ((a - b)*(3*a
 - b)*Sin[c + d*x]^5)/(5*d) - ((a - b)^2*Sin[c + d*x]^7)/(7*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx&=\frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a-(a-b)x^2\right)^2\,dx,x,\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(a^2-a(3a-2b)x^2+\left(3a^2-4ab+b^2\right)x^4-(a-b)^2x^6\right)\,dx,x\right.}{d}\\&=\frac{a^2\sin(c+dx)}{d}-\frac{a(3a-2b)\sin^3(c+dx)}{3d}+\frac{(a-b)(3a-b)\sin^5(c+dx}{5d}\end{aligned}$$

Mathematica [A] time = 0.38, size = 77, normalized size = 0.90

$$\frac{21\left(3a^2-4ab+b^2\right)\sin^5(c+dx)+105a^2\sin(c+dx)-15(a-b)^2\sin^7(c+dx)-35a(3a-2b)\sin^3(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (105*a^2*Sin[c + d*x] - 35*a*(3*a - 2*b)*Sin[c + d*x]^3 + 21*(3*a^2 - 4*a*b
 + b^2)*Sin[c + d*x]^5 - 15*(a - b)^2*Sin[c + d*x]^7)/(105*d)
```

fricas [A] time = 0.51, size = 95, normalized size = 1.10

$$\frac{\left(15\left(a^{2}-2\,ab+b^{2}\right)\cos\left(dx+c\right)^{6}+6\left(3\,a^{2}+ab-4\,b^{2}\right)\cos\left(dx+c\right)^{4}+\left(24\,a^{2}+8\,ab+3\,b^{2}\right)\cos\left(dx+c\right)^{2}+48}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*(a^2 - 2*a*b + b^2)*cos(d*x + c)^6 + 6*(3*a^2 + a*b - 4*b^2)*cos(
d*x + c)^4 + (24*a^2 + 8*a*b + 3*b^2)*cos(d*x + c)^2 + 48*a^2 + 16*a*b + 6*
b^2)*sin(d*x + c)/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.98, size = 153, normalized size = 1.78

$$\frac{b^{2}\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{4}(dx+c)\right)}{7}-\frac{3\sin(dx+c)\left(\cos^{4}(dx+c)\right)}{35}+\frac{\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{35}\right)+2ab\left(-\frac{\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^{4}(dx\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*
(2+cos(d*x+c)^2)*sin(d*x+c))+2*a*b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+
cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+1/7*a^2*(16/5+cos(d*x+c)^6+6/5*c
os(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.31, size = 81, normalized size = 0.94

$$-\frac{15\left(a^{2}-2\,ab+b^{2}\right)\sin\left(dx+c\right)^{7}-21\left(3\,a^{2}-4\,ab+b^{2}\right)\sin\left(dx+c\right)^{5}+35\left(3\,a^{2}-2\,ab\right)\sin\left(dx+c\right)^{3}-105\,a^{2}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/105*(15*(a^2 - 2*a*b + b^2)*sin(d*x + c)^7 - 21*(3*a^2 - 4*a*b + b^2)*si
n(d*x + c)^5 + 35*(3*a^2 - 2*a*b)*sin(d*x + c)^3 - 105*a^2*sin(d*x + c))/d
```

mupad [B] time = 12.27, size = 160, normalized size = 1.86

$$\frac{\frac{35\,a^{2}\,\sin(c+d\,x)}{64}+\frac{3\,b^{2}\,\sin(c+d\,x)}{64}+\frac{7\,a^{2}\,\sin(3\,c+3\,d\,x)}{64}+\frac{7\,a^{2}\,\sin(5\,c+5\,d\,x)}{320}+\frac{a^{2}\,\sin(7\,c+7\,d\,x)}{448}-\frac{b^{2}\,\sin(3\,c+3\,d\,x)}{64}-\frac{b^{2}\,\sin(5\,c+5\,d\,x)}{320}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] ((35*a^2*sin(c + d*x))/64 + (3*b^2*sin(c + d*x))/64 + (7*a^2*sin(3*c + 3*d*
x))/64 + (7*a^2*sin(5*c + 5*d*x))/320 + (a^2*sin(7*c + 7*d*x))/448 - (b^2*s
in(3*c + 3*d*x))/64 - (b^2*sin(5*c + 5*d*x))/320 + (b^2*sin(7*c + 7*d*x))/4
48 + (5*a*b*sin(c + d*x))/32 - (a*b*sin(3*c + 3*d*x))/96 - (3*a*b*sin(5*c +
 5*d*x))/160 - (a*b*sin(7*c + 7*d*x))/224)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.443 $\int \cos^9(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=114

$$\frac{\left(6a^2-6ab+b^2\right)\sin^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}+\frac{(a-b)^2\sin^9(c+dx)}{9d}-\frac{2(a-b)(2a-b)\sin^7(c+dx)}{7d}-\frac{2a(2a-b$$

```
[Out] a^2*sin(d*x+c)/d-2/3*a*(2*a-b)*sin(d*x+c)^3/d+1/5*(6*a^2-6*a*b+b^2)*sin(d*x
+c)^5/d-2/7*(a-b)*(2*a-b)*sin(d*x+c)^7/d+1/9*(a-b)^2*sin(d*x+c)^9/d
```

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3676, 373}

$$\frac{\left(6a^2-6ab+b^2\right)\sin^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}+\frac{(a-b)^2\sin^9(c+dx)}{9d}-\frac{2(a-b)(2a-b)\sin^7(c+dx)}{7d}-\frac{2a(2a-b$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/d - (2*a*(2*a - b)*Sin[c + d*x]^3)/(3*d) + ((6*a^2 - 6*a
*b + b^2)*Sin[c + d*x]^5)/(5*d) - (2*(a - b)*(2*a - b)*Sin[c + d*x]^7)/(7*d
) + ((a - b)^2*Sin[c + d*x]^9)/(9*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^9(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\left(1-x^2\right)^2\left(a-(a-b)x^2\right)^2\,dx,x,\sin(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int\left(a^2-2a(2a-b)x^2+\left(6a^2-6ab+b^2\right)x^4-2\left(2a^2-3ab+\right.\right.}{d}$$
$$=\frac{a^2\sin(c+dx)}{d}-\frac{2a(2a-b)\sin^3(c+dx)}{3d}+\frac{\left(6a^2-6ab+b^2\right)\sin^5(c}{5d}$$

Mathematica [A] time = 0.62, size = 116, normalized size = 1.02

$$\frac{630\left(63a^2+14ab+3b^2\right)\sin(c+dx)+420\left(21a^2-b^2\right)\sin(3(c+dx))+252\left(9a^2-4ab-b^2\right)\sin(5(c+dx))+}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (630*(63*a^2 + 14*a*b + 3*b^2)*Sin[c + d*x] + 420*(21*a^2 - b^2)*Sin[3*(c +
 d*x)] + 252*(9*a^2 - 4*a*b - b^2)*Sin[5*(c + d*x)] + 45*(a - b)*(9*a - b)*
Sin[7*(c + d*x)] + 35*(a - b)^2*Sin[9*(c + d*x)])/(80640*d)
```

fricas [A] time = 0.51, size = 117, normalized size = 1.03

$$\frac{\left(35\left(a^2-2\,ab+b^2\right)\cos\left(dx+c\right)^8+10\left(4\,a^2+ab-5\,b^2\right)\cos\left(dx+c\right)^6+3\left(16\,a^2+4\,ab+b^2\right)\cos\left(dx+c\right)^4+4\left(1\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(35*(a^2 - 2*a*b + b^2)*cos(d*x + c)^8 + 10*(4*a^2 + a*b - 5*b^2)*cos
(d*x + c)^6 + 3*(16*a^2 + 4*a*b + b^2)*cos(d*x + c)^4 + 4*(16*a^2 + 4*a*b +
 b^2)*cos(d*x + c)^2 + 128*a^2 + 32*a*b + 8*b^2)*sin(d*x + c)/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.06, size = 183, normalized size = 1.61

$$\frac{b^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{105}\right)+2ab\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{1}{5}\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+2*a*b*(-1/9*sin(d*x+c)*cos
(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*
x+c))+1/9*a^2*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/3
5*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.49, size = 104, normalized size = 0.91

$$\frac{35\left(a^2-2\,ab+b^2\right)\sin\left(dx+c\right)^9-90\left(2\,a^2-3\,ab+b^2\right)\sin\left(dx+c\right)^7+63\left(6\,a^2-6\,ab+b^2\right)\sin\left(dx+c\right)^5-210\left(2\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*(a^2 - 2*a*b + b^2)*sin(d*x + c)^9 - 90*(2*a^2 - 3*a*b + b^2)*sin
(d*x + c)^7 + 63*(6*a^2 - 6*a*b + b^2)*sin(d*x + c)^5 - 210*(2*a^2 - a*b)*s
in(d*x + c)^3 + 315*a^2*sin(d*x + c))/d
```

mupad [B] time = 12.40, size = 188, normalized size = 1.65

$$\frac{\frac{63\,a^2\,\sin(c+d\,x)}{128}+\frac{3\,b^2\,\sin(c+d\,x)}{128}+\frac{7\,a^2\,\sin(3\,c+3\,d\,x)}{64}+\frac{9\,a^2\,\sin(5\,c+5\,d\,x)}{320}+\frac{9\,a^2\,\sin(7\,c+7\,d\,x)}{1792}+\frac{a^2\,\sin(9\,c+9\,d\,x)}{2304}-\frac{b^2\,\sin(3\,c+3\,d\,x)}{192}-\frac{b^{}}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^9*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] ((63*a^2*sin(c + d*x))/128 + (3*b^2*sin(c + d*x))/128 + (7*a^2*sin(3*c + 3*
d*x))/64 + (9*a^2*sin(5*c + 5*d*x))/320 + (9*a^2*sin(7*c + 7*d*x))/1792 + (
a^2*sin(9*c + 9*d*x))/2304 - (b^2*sin(3*c + 3*d*x))/192 - (b^2*sin(5*c + 5*
d*x))/320 + (b^2*sin(7*c + 7*d*x))/1792 + (b^2*sin(9*c + 9*d*x))/2304 + (7*
a*b*sin(c + d*x))/64 - (a*b*sin(5*c + 5*d*x))/80 - (5*a*b*sin(7*c + 7*d*x))
/896 - (a*b*sin(9*c + 9*d*x))/1152)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.444 $\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=96

$$\frac{(a^2+4ab+b^2)\tan^5(c+dx)}{5d}+\frac{a^2\tan(c+dx)}{d}+\frac{2b(a+b)\tan^7(c+dx)}{7d}+\frac{2a(a+b)\tan^3(c+dx)}{3d}+\frac{b^2\tan^9(c+dx)}{9d}$$

```
[Out] a^2*tan(d*x+c)/d+2/3*a*(a+b)*tan(d*x+c)^3/d+1/5*(a^2+4*a*b+b^2)*tan(d*x+c)^
5/d+2/7*b*(a+b)*tan(d*x+c)^7/d+1/9*b^2*tan(d*x+c)^9/d
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3675, 373}

$$\frac{(a^2+4ab+b^2)\tan^5(c+dx)}{5d}+\frac{a^2\tan(c+dx)}{d}+\frac{2b(a+b)\tan^7(c+dx)}{7d}+\frac{2a(a+b)\tan^3(c+dx)}{3d}+\frac{b^2\tan^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (2*a*(a + b)*Tan[c + d*x]^3)/(3*d) + ((a^2 + 4*a*b +
 b^2)*Tan[c + d*x]^5)/(5*d) + (2*b*(a + b)*Tan[c + d*x]^7)/(7*d) + (b^2*Tan
[c + d*x]^9)/(9*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int\left(1+x^2\right)^2\left(a+bx^2\right)^2\,dx,x,\tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int\left(a^2+2a(a+b)x^2+\left(a^2+4ab+b^2\right)x^4+2b(a+b)x^6+b^2x^8\right.\right.}{d}$$
$$= \frac{a^2\tan(c+dx)}{d}+\frac{2a(a+b)\tan^3(c+dx)}{3d}+\frac{\left(a^2+4ab+b^2\right)\tan^5(c+dx)}{5d}$$

Mathematica [A] time = 0.39, size = 106, normalized size = 1.10

$$\frac{\tan(c+dx)\left(3\left(21a^2-6ab+b^2\right)\sec^4(c+dx)+4\left(21a^2-6ab+b^2\right)\sec^2(c+dx)+8\left(21a^2-6ab+b^2\right)+10b(9a\right.}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((8*(21*a^2 - 6*a*b + b^2) + 4*(21*a^2 - 6*a*b + b^2)*Sec[c + d*x]^2 + 3*(2
1*a^2 - 6*a*b + b^2)*Sec[c + d*x]^4 + 10*(9*a - 5*b)*b*Sec[c + d*x]^6 + 35*
b^2*Sec[c + d*x]^8)*Tan[c + d*x])/(315*d)
```

fricas [A] time = 0.51, size = 114, normalized size = 1.19

$$\frac{\left(8\left(21\,a^2-6\,ab+b^2\right)\cos\left(dx+c\right)^8+4\left(21\,a^2-6\,ab+b^2\right)\cos\left(dx+c\right)^6+3\left(21\,a^2-6\,ab+b^2\right)\cos\left(dx+c\right)^4+\right.}{315\,d\cos\left(dx+c\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(21*a^2 - 6*a*b + b^2)*cos(d*x + c)^8 + 4*(21*a^2 - 6*a*b + b^2)*c
os(d*x + c)^6 + 3*(21*a^2 - 6*a*b + b^2)*cos(d*x + c)^4 + 10*(9*a*b - 5*b^2
)*cos(d*x + c)^2 + 35*b^2)*sin(d*x + c)/(d*cos(d*x + c)^9)
```

giac [A] time = 3.14, size = 118, normalized size = 1.23

$$\frac{35\,b^2\tan\left(dx+c\right)^9+90\,ab\tan\left(dx+c\right)^7+90\,b^2\tan\left(dx+c\right)^7+63\,a^2\tan\left(dx+c\right)^5+252\,ab\tan\left(dx+c\right)^5+63}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/315*(35*b^2*tan(d*x + c)^9 + 90*a*b*tan(d*x + c)^7 + 90*b^2*tan(d*x + c)^
7 + 63*a^2*tan(d*x + c)^5 + 252*a*b*tan(d*x + c)^5 + 63*b^2*tan(d*x + c)^5
+ 210*a^2*tan(d*x + c)^3 + 210*a*b*tan(d*x + c)^3 + 315*a^2*tan(d*x + c))/d
```

maple [A] time = 0.76, size = 157, normalized size = 1.64

$$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan\left(dx+c\right)+2ab\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+b^2\left(\frac{\sin^5(dx}{9\cos(dx}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+2*a*b*(1/7*
sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3
/cos(d*x+c)^3)+b^2*(1/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x
+c)^7+8/315*sin(d*x+c)^5/cos(d*x+c)^5))
```

maxima [A] time = 0.49, size = 85, normalized size = 0.89

$$\frac{35\,b^2\tan\left(dx+c\right)^9+90\left(ab+b^2\right)\tan\left(dx+c\right)^7+63\left(a^2+4\,ab+b^2\right)\tan\left(dx+c\right)^5+210\left(a^2+ab\right)\tan\left(dx+c\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^2*tan(d*x + c)^9 + 90*(a*b + b^2)*tan(d*x + c)^7 + 63*(a^2 + 4*
a*b + b^2)*tan(d*x + c)^5 + 210*(a^2 + a*b)*tan(d*x + c)^3 + 315*a^2*tan(d*
x + c))/d
```

mupad [B] time = 12.22, size = 80, normalized size = 0.83

$$\frac{a^2\tan\left(c+d\,x\right)+\frac{b^2\tan(c+d\,x)^9}{9}+\tan\left(c+d\,x\right)^5\left(\frac{a^2}{5}+\frac{4\,a\,b}{5}+\frac{b^2}{5}\right)+\frac{2\,a\tan(c+d\,x)^3\,(a+b)}{3}+\frac{2\,b\tan(c+d\,x)^7\,(a+b)}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^2/cos(c + d*x)^6,x)
```

```
[Out] (a^2*tan(c + d*x) + (b^2*tan(c + d*x)^9)/9 + tan(c + d*x)^5*((4*a*b)/5 + a^
2/5 + b^2/5) + (2*a*tan(c + d*x)^3*(a + b))/3 + (2*b*tan(c + d*x)^7*(a + b)
)/7)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(c + dx\right)\right)^2 \sec^6\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**6, x)
```

3.445 $\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=74

$$\frac{a^2\tan(c+dx)}{d}+\frac{b(2a+b)\tan^5(c+dx)}{5d}+\frac{a(a+2b)\tan^3(c+dx)}{3d}+\frac{b^2\tan^7(c+dx)}{7d}$$

```
[Out] a^2*tan(d*x+c)/d+1/3*a*(a+2*b)*tan(d*x+c)^3/d+1/5*b*(2*a+b)*tan(d*x+c)^5/d+
1/7*b^2*tan(d*x+c)^7/d
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 373}

$$\frac{a^2\tan(c+dx)}{d}+\frac{b(2a+b)\tan^5(c+dx)}{5d}+\frac{a(a+2b)\tan^3(c+dx)}{3d}+\frac{b^2\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (a*(a + 2*b)*Tan[c + d*x]^3)/(3*d) + (b*(2*a + b)*Ta
n[c + d*x]^5)/(5*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int\left(1+x^2\right)\left(a+bx^2\right)^2\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int\left(a^2+a(a+2b)x^2+b(2a+b)x^4+b^2x^6\right)\,dx,x,\tan(c+dx\right.}{d}\\ &= \frac{a^2\tan(c+dx)}{d}+\frac{a(a+2b)\tan^3(c+dx)}{3d}+\frac{b(2a+b)\tan^5(c+dx)}{5d}+\end{aligned}$$

Mathematica [A] time = 0.56, size = 83, normalized size = 1.12

$$\frac{\tan(c+dx)\left(\left(35a^2-14ab+3b^2\right)\sec^2(c+dx)+70a^2+6b(7a-4b)\sec^4(c+dx)-28ab+15b^2\sec^6(c+dx)+\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((70*a^2 - 28*a*b + 6*b^2 + (35*a^2 - 14*a*b + 3*b^2)*Sec[c + d*x]^2 + 6*(7
*a - 4*b)*b*Sec[c + d*x]^4 + 15*b^2*Sec[c + d*x]^6)*Tan[c + d*x])/(105*d)
```

fricas [A] time = 0.56, size = 94, normalized size = 1.27

$$\frac{\left(2\left(35a^2-14ab+3b^2\right)\cos\left(dx+c\right)^6+\left(35a^2-14ab+3b^2\right)\cos\left(dx+c\right)^4+6\left(7ab-4b^2\right)\cos\left(dx+c\right)^2+15b^2}{105d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(35*a^2 - 14*a*b + 3*b^2)*cos(d*x + c)^6 + (35*a^2 - 14*a*b + 3*b^
2)*cos(d*x + c)^4 + 6*(7*a*b - 4*b^2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x + c)
/(d*cos(d*x + c)^7)
```

giac [A] time = 3.99, size = 80, normalized size = 1.08

$$\frac{15b^2\tan\left(dx+c\right)^7+42ab\tan\left(dx+c\right)^5+21b^2\tan\left(dx+c\right)^5+35a^2\tan\left(dx+c\right)^3+70ab\tan\left(dx+c\right)^3+105a^2}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 42*a*b*tan(d*x + c)^5 + 21*b^2*tan(d*x + c)^
5 + 35*a^2*tan(d*x + c)^3 + 70*a*b*tan(d*x + c)^3 + 105*a^2*tan(d*x + c))/d
```

maple [A] time = 0.65, size = 111, normalized size = 1.50

$$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2\left(dx+c\right)\right)}{3}\right)\tan\left(dx+c\right)+2ab\left(\frac{\sin^3\left(dx+c\right)}{5\cos\left(dx+c\right)^5}+\frac{2\left(\sin^3\left(dx+c\right)\right)}{15\cos\left(dx+c\right)^3}\right)+b^2\left(\frac{\sin^5\left(dx+c\right)}{7\cos\left(dx+c\right)^7}+\frac{2\left(\sin^5\left(dx+c\right)\right)}{35\cos\left(dx+c\right)^5}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2*a*b*(1/5*sin(d*x+c)^3/cos(d*
x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+b^2*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2
/35*sin(d*x+c)^5/cos(d*x+c)^5))
```

maxima [A] time = 0.62, size = 66, normalized size = 0.89

$$\frac{15b^2\tan\left(dx+c\right)^7+21\left(2ab+b^2\right)\tan\left(dx+c\right)^5+35\left(a^2+2ab\right)\tan\left(dx+c\right)^3+105a^2\tan\left(dx+c\right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 21*(2*a*b + b^2)*tan(d*x + c)^5 + 35*(a^2 +
2*a*b)*tan(d*x + c)^3 + 105*a^2*tan(d*x + c))/d
```

mupad [B] time = 12.28, size = 60, normalized size = 0.81

$$\frac{a^2\tan\left(c+d\,x\right)+\frac{b^2\tan\left(c+d\,x\right)^7}{7}+\frac{a\tan\left(c+d\,x\right)^3\left(a+2\,b\right)}{3}+\frac{b\tan\left(c+d\,x\right)^5\left(2\,a+b\right)}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^2/cos(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c + d*x) + (b^2*tan(c + d*x)^7)/7 + (a*tan(c + d*x)^3*(a + 2*b))/3
 + (b*tan(c + d*x)^5*(2*a + b))/5)/d
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(c + dx\right)\right)^2 \sec^4\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**4, x)
```

3.446 $\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}$$

```
[Out] a^2*tan(d*x+c)/d+2/3*a*b*tan(d*x+c)^3/d+1/5*b^2*tan(d*x+c)^5/d
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 194}

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (2*a*b*Tan[c + d*x]^3)/(3*d) + (b^2*Tan[c + d*x]^5)/
(5*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx&=\frac{\text{Subst}\left(\int\left(a+bx^2\right)^2\,dx,x,\tan(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(a^2+2abx^2+b^2x^4\right)\,dx,x,\tan(c+dx)\right)}{d}\\&=\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.14, size = 49, normalized size = 1.00

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (2*a*b*Tan[c + d*x]^3)/(3*d) + (b^2*Tan[c + d*x]^5)/
(5*d)
```

fricas [A] time = 0.51, size = 69, normalized size = 1.41

$$\frac{\left(\left(15\,a^{2}-10\,ab+3\,b^{2}\right)\cos\left(dx+c\right)^{4}+2\left(5\,ab-3\,b^{2}\right)\cos\left(dx+c\right)^{2}+3\,b^{2}\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
[Out] 1/15*((15*a^2 - 10*a*b + 3*b^2)*cos(d*x + c)^4 + 2*(5*a*b - 3*b^2)*cos(d*x
+ c)^2 + 3*b^2)*sin(d*x + c)/(d*cos(d*x + c)^5)
```

giac [A] time = 2.75, size = 42, normalized size = 0.86

$$\frac{3\,b^{2}\tan\left(dx+c\right)^{5}+10\,ab\tan\left(dx+c\right)^{3}+15\,a^{2}\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
[Out] 1/15*(3*b^2*tan(d*x + c)^5 + 10*a*b*tan(d*x + c)^3 + 15*a^2*tan(d*x + c))/d
```

maple [A] time = 0.64, size = 57, normalized size = 1.16

$$\frac{a^{2}\tan\left(dx+c\right)+\frac{2ab\left(\sin^{3}\left(dx+c\right)\right)}{3\cos\left(dx+c\right)^{3}}+\frac{b^{2}\left(\sin^{5}\left(dx+c\right)\right)}{5\cos\left(dx+c\right)^{5}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x)
[Out] 1/d*(a^2*tan(d*x+c)+2/3*a*b*sin(d*x+c)^3/cos(d*x+c)^3+1/5*b^2*sin(d*x+c)^5/
cos(d*x+c)^5)
```

maxima [A] time = 0.50, size = 42, normalized size = 0.86

$$\frac{3\,b^{2}\tan\left(dx+c\right)^{5}+10\,ab\tan\left(dx+c\right)^{3}+15\,a^{2}\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
[Out] 1/15*(3*b^2*tan(d*x + c)^5 + 10*a*b*tan(d*x + c)^3 + 15*a^2*tan(d*x + c))/d
```

mupad [B] time = 12.13, size = 40, normalized size = 0.82

$$\frac{a^{2}\tan\left(c+d\,x\right)+\frac{2\,a\,b\tan\left(c+d\,x\right)^{3}}{3}+\frac{b^{2}\tan\left(c+d\,x\right)^{5}}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^2/cos(c + d*x)^2,x)
[Out] (a^2*tan(c + d*x) + (b^2*tan(c + d*x)^5)/5 + (2*a*b*tan(c + d*x)^3)/3)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{2}\left(c+dx\right)\right)^{2}\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c)**2)**2,x)
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**2, x)
```

3.447 $\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2 dx$

Optimal. Leaf size=55

$$\frac{(a-b)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(a+3b)(a-b)+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] 1/2*(a-b)*(a+3*b)*x+1/2*(a-b)^2*cos(d*x+c)*sin(d*x+c)/d+b^2*tan(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3675, 390, 385, 203}

$$\frac{(a-b)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(a+3b)(a-b)+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)*(a + 3*b)*x)/2 + ((a - b)^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^
2*Tan[c + d*x])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{\left(1+x^2\right)^2}dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2-b^2+2(a-b)bx^2}{\left(1+x^2\right)^2}\right)dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{b^2\tan(c+dx)}{d} + \frac{\text{Subst}\left(\int \frac{a^2-b^2+2(a-b)bx^2}{\left(1+x^2\right)^2}dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a-b)^2\cos(c+dx)\sin(c+dx)}{2d} + \frac{b^2\tan(c+dx)}{d} + \frac{((a-b)(a+3b}{}$$
$$= \frac{1}{2}(a-b)(a+3b)x + \frac{(a-b)^2\cos(c+dx)\sin(c+dx)}{2d} + \frac{b^2\tan(c+d}{d}$$

Mathematica [A] time = 0.42, size = 55, normalized size = 1.00

$$\frac{2\left(a^2+2ab-3b^2\right)(c+dx)+(a-b)^2\sin(2(c+dx))+4b^2\tan(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (2*(a^2 + 2*a*b - 3*b^2)*(c + d*x) + (a - b)^2*Sin[2*(c + d*x)] + 4*b^2*Tan
[c + d*x])/(4*d)
```

fricas [A] time = 0.62, size = 69, normalized size = 1.25

$$\frac{\left(a^2+2\,ab-3\,b^2\right)dx\cos\left(dx+c\right)+\left(\left(a^2-2\,ab+b^2\right)\cos\left(dx+c\right)^2+2\,b^2\right)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + 2*a*b - 3*b^2)*d*x*cos(d*x + c) + ((a^2 - 2*a*b + b^2)*cos(d*x
+ c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [B] time = 3.36, size = 594, normalized size = 10.80

$$\frac{a^2dx\tan\left(dx\right)^3\tan(c)^3+2\,abdx\tan\left(dx\right)^3\tan(c)^3-3\,b^2dx\tan\left(dx\right)^3\tan(c)^3+a^2dx\tan\left(dx\right)^3\tan(c)+2\,abdx\,\text{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d*x*tan(d*x)^3*tan(c)^3 + 2*a*b*d*x*tan(d*x)^3*tan(c)^3 - 3*b^2*d*
x*tan(d*x)^3*tan(c)^3 + a^2*d*x*tan(d*x)^3*tan(c) + 2*a*b*d*x*tan(d*x)^3*ta
n(c) - 3*b^2*d*x*tan(d*x)^3*tan(c) - a^2*d*x*tan(d*x)^2*tan(c)^2 - 2*a*b*d*
x*tan(d*x)^2*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(c)^2 + a^2*d*x*tan(d*x)*ta
n(c)^3 + 2*a*b*d*x*tan(d*x)*tan(c)^3 - 3*b^2*d*x*tan(d*x)*tan(c)^3 - a^2*ta
n(d*x)^3*tan(c)^2 + 2*a*b*tan(d*x)^3*tan(c)^2 - 3*b^2*tan(d*x)^3*tan(c)^2 -
 a^2*tan(d*x)^2*tan(c)^3 + 2*a*b*tan(d*x)^2*tan(c)^3 - 3*b^2*tan(d*x)^2*tan
(c)^3 - a^2*d*x*tan(d*x)^2 - 2*a*b*d*x*tan(d*x)^2 + 3*b^2*d*x*tan(d*x)^2 +
```

```
a^2*d*x*tan(d*x)*tan(c) + 2*a*b*d*x*tan(d*x)*tan(c) - 3*b^2*d*x*tan(d*x)*ta
n(c) - a^2*d*x*tan(c)^2 - 2*a*b*d*x*tan(c)^2 + 3*b^2*d*x*tan(c)^2 - 2*b^2*t
an(d*x)^3 + 2*a^2*tan(d*x)^2*tan(c) - 4*a*b*tan(d*x)^2*tan(c) + 2*a^2*tan(d
*x)*tan(c)^2 - 4*a*b*tan(d*x)*tan(c)^2 - 2*b^2*tan(c)^3 - a^2*d*x - 2*a*b*d
*x + 3*b^2*d*x - a^2*tan(d*x) + 2*a*b*tan(d*x) - 3*b^2*tan(d*x) - a^2*tan(c
) + 2*a*b*tan(c) - 3*b^2*tan(c))/(d*tan(d*x)^3*tan(c)^3 + d*tan(d*x)^3*tan(
c) - d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)*tan(c)^3 - d*tan(d*x)^2 + d*tan(d*x
)*tan(c) - d*tan(c)^2 - d)
```

maple [B] time = 0.56, size = 111, normalized size = 2.02

$$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\mathrm{co}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(-1/2*cos(d*x+c)*s
in(d*x+c)+1/2*d*x+1/2*c)+b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin
(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c))
```

maxima [A] time = 0.76, size = 66, normalized size = 1.20

$$\frac{2\,b^2\tan\left(dx+c\right)+\left(a^2+2\,ab-3\,b^2\right)(dx+c)+\frac{\left(a^2-2\,ab+b^2\right)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*tan(d*x + c) + (a^2 + 2*a*b - 3*b^2)*(d*x + c) + (a^2 - 2*a*b +
b^2)*tan(d*x + c)/(tan(d*x + c)^2 + 1))/d
```

mupad [B] time = 12.25, size = 91, normalized size = 1.65

$$\frac{b^2\tan\left(c+d\,x\right)}{d}+\frac{\sin\left(2\,c+2\,d\,x\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{2\,d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a-b)\,(a+3\,b)}{2\left(\frac{a^2}{2}+a\,b-\frac{3\,b^2}{2}\right)}\right)\,\left(a-b\right)\,\left(a+3\,b\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] (b^2*tan(c + d*x))/d + (sin(2*c + 2*d*x)*(a^2/2 - a*b + b^2/2))/(2*d) + (at
an((tan(c + d*x)*(a - b)*(a + 3*b))/(2*(a*b + a^2/2 - (3*b^2)/2)))*(a - b)*
(a + 3*b))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(c+dx\right)\right)^2\cos^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x)**2, x)
```

3.448 $\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=87

$$\frac{3\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(3a^2+2ab+3b^2\right)+\frac{(a-b)\sin(c+dx)\cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)}{4d}$$

```
[Out] 1/8*(3*a^2+2*a*b+3*b^2)*x+3/8*(a^2-b^2)*cos(d*x+c)*sin(d*x+c)/d+1/4*(a-b)*c
os(d*x+c)^3*sin(d*x+c)*(a+b*tan(d*x+c)^2)/d
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3675, 413, 385, 203}

$$\frac{3\left(a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(3a^2+2ab+3b^2\right)+\frac{(a-b)\sin(c+dx)\cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 2*a*b + 3*b^2)*x)/8 + (3*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + ((a - b)*Cos[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x]^2))/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{\left(1+x^2\right)^3} dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a-b)\cos^3(c+dx)\sin(c+dx)\left(a+b\tan^2(c+dx)\right)}{4d} + \frac{\text{Subst}\left(\int \frac{a(3a+}{(}\right.}{}$$
$$= \frac{3\left(a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{(a-b)\cos^3(c+dx)\sin(c+dx)}{4d}$$
$$= \frac{1}{8}\left(3a^2+2ab+3b^2\right)x + \frac{3\left(a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{(a-b)}{}$$

Mathematica [A] time = 0.33, size = 65, normalized size = 0.75

$$\frac{4\left(3a^2+2ab+3b^2\right)(c+dx)+8\left(a^2-b^2\right)\sin(2(c+dx))+(a-b)^2\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 + 2*a*b + 3*b^2)*(c + d*x) + 8*(a^2 - b^2)*Sin[2*(c + d*x)] + (a
- b)^2*Sin[4*(c + d*x)])/(32*d)
```

fricas [A] time = 0.53, size = 75, normalized size = 0.86

$$\frac{\left(3a^2+2ab+3b^2\right)dx+\left(2\left(a^2-2ab+b^2\right)\cos(dx+c)^3+\left(3a^2+2ab-5b^2\right)\cos(dx+c)\right)\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((3*a^2 + 2*a*b + 3*b^2)*d*x + (2*(a^2 - 2*a*b + b^2)*cos(d*x + c)^3 +
(3*a^2 + 2*a*b - 5*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.79, size = 122, normalized size = 1.40

$$\frac{b^2\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+a^2\left(\frac{\left(\cos^3(d\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+2*a*
b*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)+a^
2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.76, size = 97, normalized size = 1.11

$$\frac{\left(3 a^{2}+2 a b+3 b^{2}\right)(d x+c)+\frac{\left(3 a^{2}+2 a b-5 b^{2}\right) \tan (d x+c)^{3}+\left(5 a^{2}-2 a b-3 b^{2}\right) \tan (d x+c)}{\tan (d x+c)^{4}+2 \tan (d x+c)^{2}+1}}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + 2*a*b + 3*b^2)*(d*x + c) + ((3*a^2 + 2*a*b - 5*b^2)*tan(d*x +
 c)^3 + (5*a^2 - 2*a*b - 3*b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x +
 c)^2 + 1))/d
```

mupad [B] time = 12.24, size = 93, normalized size = 1.07

$$x\left(\frac{3 a^{2}}{8}+\frac{a b}{4}+\frac{3 b^{2}}{8}\right)-\frac{\tan (c+d x)\left(-\frac{5 a^{2}}{8}+\frac{a b}{4}+\frac{3 b^{2}}{8}\right)-\tan (c+d x)^{3}\left(\frac{3 a^{2}}{8}+\frac{a b}{4}-\frac{5 b^{2}}{8}\right)}{d\left(\tan (c+d x)^{4}+2 \tan (c+d x)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] x*((a*b)/4 + (3*a^2)/8 + (3*b^2)/8) - (tan(c + d*x)*((a*b)/4 - (5*a^2)/8 +
(3*b^2)/8) - tan(c + d*x)^3*((a*b)/4 + (3*a^2)/8 - (5*b^2)/8))/(d*(2*tan(c
+ d*x)^2 + tan(c + d*x)^4 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b \tan ^{2}(c+d x)\right)^{2} \cos ^{4}(c+d x)\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x)**4, x)
```

3.449 $\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=122

$$\frac{(5a^2+2ab+b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(5a^2+2ab+b^2\right)+\frac{(a-b)(5a+3b)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(a-$$

```
[Out] 1/16*(5*a^2+2*a*b+b^2)*x+1/16*(5*a^2+2*a*b+b^2)*cos(d*x+c)*sin(d*x+c)/d+1/2
4*(a-b)*(5*a+3*b)*cos(d*x+c)^3*sin(d*x+c)/d+1/6*(a-b)*cos(d*x+c)^5*sin(d*x+
c)*(a+b*tan(d*x+c)^2)/d
```

Rubi [A] time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3675, 413, 385, 199, 203}

$$\frac{(5a^2+2ab+b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(5a^2+2ab+b^2\right)+\frac{(a-b)(5a+3b)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(a-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((5*a^2 + 2*a*b + b^2)*x)/16 + ((5*a^2 + 2*a*b + b^2)*Cos[c + d*x]*Sin[c +
d*x])/(16*d) + ((a - b)*(5*a + 3*b)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (
(a - b)*Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Tan[c + d*x]^2))/(6*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{\left(1+x^2\right)^4}\,dx,x,\tan(c+dx)\right)}{d}$$

$$= \frac{(a-b)\cos^5(c+dx)\sin(c+dx)\left(a+b\tan^2(c+dx)\right)}{6d} + \frac{\text{Subst}\left(\int \frac{a(\cdot}{}\right.}{}$$

$$= \frac{(a-b)(5a+3b)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{(a-b)\cos^5(c+dx)\sin}{}$$

$$= \frac{\left(5a^2+2ab+b^2\right)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(a-b)(5a+3b)\cos^3(c}{24d}$$

$$= \frac{1}{16}\left(5a^2+2ab+b^2\right)x + \frac{\left(5a^2+2ab+b^2\right)\cos(c+dx)\sin(c+dx)}{16d} +$$

Mathematica [C] time = 0.37, size = 87, normalized size = 0.71

$$\frac{12(b+(1-2i)a)(b+(1+2i)a)(c+dx)+(a-b)^2\sin(6(c+dx))+3(3a+b)(a-b)\sin(4(c+dx))+3(5a-b)(}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (12*((1 - 2*I)*a + b)*((1 + 2*I)*a + b)*(c + d*x) + 3*(5*a - b)*(3*a + b)*S
in[2*(c + d*x)] + 3*(a - b)*(3*a + b)*Sin[4*(c + d*x)] + (a - b)^2*Sin[6*(c
 + d*x)])/(192*d)
```

fricas [A] time = 0.49, size = 98, normalized size = 0.80

$$\frac{3\left(5a^2+2ab+b^2\right)dx+\left(8\left(a^2-2ab+b^2\right)\cos(dx+c)^5+2\left(5a^2+2ab-7b^2\right)\cos(dx+c)^3+3\left(5a^2+2ab\right.\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(5*a^2 + 2*a*b + b^2)*d*x + (8*(a^2 - 2*a*b + b^2)*cos(d*x + c)^5 +
 2*(5*a^2 + 2*a*b - 7*b^2)*cos(d*x + c)^3 + 3*(5*a^2 + 2*a*b + b^2)*cos(d*x
 + c))*sin(d*x + c))/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 0.83, size = 166, normalized size = 1.36

$$\frac{b^{2}\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{3}(dx+c)\right)}{6}-\frac{\left(\cos^{3}(dx+c)\right)\sin(dx+c)}{8}+\frac{\cos(dx+c)\sin(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+2ab\left(-\frac{\left(\cos^{5}(dx+c)\right)\sin(dx+c)}{6}+\frac{\left(\cos^{3}(dx\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x)

[Out] 1/d*(b^2*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+2*a*b*(-1/6*cos(d*x+c)^5*sin(d*x+c)+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+a^2*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))

maxima [A] time = 0.50, size = 131, normalized size = 1.07

$$\frac{3\left(5a^{2}+2ab+b^{2}\right)(dx+c)+\frac{3\left(5a^{2}+2ab+b^{2}\right)\tan(dx+c)^{5}+8\left(5a^{2}+2ab-b^{2}\right)\tan(dx+c)^{3}+3\left(11a^{2}-2ab-b^{2}\right)\tan(dx+c)}{\tan(dx+c)^{6}+3\tan(dx+c)^{4}+3\tan(dx+c)^{2}+1}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/48*(3*(5*a^2 + 2*a*b + b^2)*(d*x + c) + (3*(5*a^2 + 2*a*b + b^2)*tan(d*x + c)^5 + 8*(5*a^2 + 2*a*b - b^2)*tan(d*x + c)^3 + 3*(11*a^2 - 2*a*b - b^2)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d

mupad [B] time = 13.24, size = 126, normalized size = 1.03

$$x\left(\frac{5a^{2}}{16}+\frac{ab}{8}+\frac{b^{2}}{16}\right)+\frac{\left(\frac{5a^{2}}{16}+\frac{ab}{8}+\frac{b^{2}}{16}\right)\tan(c+dx)^{5}+\left(\frac{5a^{2}}{6}+\frac{ab}{3}-\frac{b^{2}}{6}\right)\tan(c+dx)^{3}+\left(\frac{11a^{2}}{16}-\frac{ab}{8}-\frac{b^{2}}{16}\right)\tan(c+}{d\left(\tan(c+dx)^{6}+3\tan(c+dx)^{4}+3\tan(c+dx)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6*(a + b*tan(c + d*x)^2)^2,x)

[Out] x*((a*b)/8 + (5*a^2)/16 + b^2/16) + (tan(c + d*x)^3*((a*b)/3 + (5*a^2)/6 - b^2/6) - tan(c + d*x)*((a*b)/8 - (11*a^2)/16 + b^2/16) + tan(c + d*x)^5*((a*b)/8 + (5*a^2)/16 + b^2/16))/(d*(3*tan(c + d*x)^2 + 3*tan(c + d*x)^4 + tan(c + d*x)^6 + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*(a+b*tan(d*x+c)**2)**2,x)

[Out] Timed out

3.450 $\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=90

$$\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d}-\frac{(2a-3b)\tanh^{-1}(\sin(c+dx))}{2b^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

```
[Out] -1/2*(2*a-3*b)*arctanh(sin(d*x+c))/b^2/d+(a-b)^(3/2)*arctanh(sin(d*x+c)*(a-
b)^(1/2)/a^(1/2))/b^2/d/a^(1/2)+1/2*sec(d*x+c)*tan(d*x+c)/b/d
```

Rubi [A] time = 0.14, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3676, 414, 522, 206, 208}

$$\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d}-\frac{(2a-3b)\tanh^{-1}(\sin(c+dx))}{2b^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -((2*a - 3*b)*ArcTanh[Sin[c + d*x]])/(2*b^2*d) + ((a - b)^(3/2)*ArcTanh[(Sq
rt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^2*d) + (Sec[c + d*x]*Tan[c + d
*x])/(2*b*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
```

```
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-(a-b)x^2)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\text{Subst}\left(\int \frac{-a+2b+(-a+b)x^2}{(1-x^2)(a+(-a+b)x^2)}\,dx, x, \sin(c+dx)\right)}{2bd}$$

$$= \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{(2a-3b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{2b^2d} + \frac{(a-b)^2\,\text{Subs}}{}$$

$$= -\frac{(2a-3b)\tanh^{-1}(\sin(c+dx))}{2b^2d} + \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d} + \frac{\sec(c+dx)\tan(}{2bd}$$

Mathematica [B] time = 1.38, size = 207, normalized size = 2.30

$$\frac{-\frac{2(a-b)^{3/2}\log\left(\sqrt{a}-\sqrt{a-b}\,\sin(c+dx)\right)}{\sqrt{a}} + \frac{2(a-b)^{3/2}\log\left(\sqrt{a-b}\,\sin(c+dx)+\sqrt{a}\right)}{\sqrt{a}} + 2(2a-3b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (2*(2*a - 3*b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(-2*a + 3*b)*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (2*(a - b)^(3/2)*Log[Sqrt[a] - Sqr
t[a - b]*Sin[c + d*x]])/Sqrt[a] + (2*(a - b)^(3/2)*Log[Sqrt[a] + Sqrt[a - b
]*Sin[c + d*x]])/Sqrt[a] + b/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - b/(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(4*b^2*d)
```

fricas [A] time = 0.50, size = 292, normalized size = 3.24

$$\left[-\frac{2(a-b)\sqrt{\frac{a-b}{a}}\cos(dx+c)^2\log\left(-\frac{(a-b)\cos(dx+c)^2+2a\sqrt{\frac{a-b}{a}}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right) + (2a-3b)\cos(dx+c)^2\log(\sin(dx+}{4b^2d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a - b)*sqrt((a - b)/a)*cos(d*x + c)^2*log(-((a - b)*cos(d*x + c)^
2 + 2*a*sqrt((a - b)/a)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b
)) + (2*a - 3*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a - 3*b)*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) - 2*b*sin(d*x + c))/(b^2*d*cos(d*x + c)^2),
-1/4*(4*(a - b)*sqrt(-(a - b)/a)*arctan(sqrt(-(a - b)/a)*sin(d*x + c))*cos(
d*x + c)^2 + (2*a - 3*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a - 3*b)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*b*sin(d*x + c))/(b^2*d*cos(d*x +
 c)^2)]
```

giac [A] time = 2.96, size = 131, normalized size = 1.46

$$-\frac{\frac{(2a-3b)\log(|\sin(dx+c)+1|)}{b^2}-\frac{(2a-3b)\log(|\sin(dx+c)-1|)}{b^2}-\frac{4\left(a^2-2ab+b^2\right)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b^2}+\frac{2\sin(dx+c)}{\left(\sin(dx+c)^2-1\right)b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/4*((2*a - 3*b)*log(abs(sin(d*x + c) + 1))/b^2 - (2*a - 3*b)*log(abs(sin(
d*x + c) - 1))/b^2 - 4*(a^2 - 2*a*b + b^2)*arctan(-(a*sin(d*x + c) - b*sin(
d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*b^2) + 2*sin(d*x + c)/((sin(d
*x + c)^2 - 1)*b))/d
```

maple [B] time = 0.71, size = 224, normalized size = 2.49

$$\frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)a^2}{d\,b^2\sqrt{a\,(a-b)}}-\frac{2\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)a}{db\sqrt{a\,(a-b)}}+\frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{d\sqrt{a\,(a-b)}}-\frac{1}{4db\,(-1+\sin(dx+c))}-\frac{3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d/b^2/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))*a^2-2/d/b
/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))*a+1/d/(a*(a-b))^
(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))-1/4/d/b/(-1+sin(d*x+c))-3/4
/d/b*ln(-1+sin(d*x+c))+1/2/d/b^2*ln(-1+sin(d*x+c))*a-1/4/d/b/(sin(d*x+c)+1)
+3/4/d/b*ln(sin(d*x+c)+1)-1/2/d/b^2*ln(sin(d*x+c)+1)*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 13.76, size = 268, normalized size = 2.98

$$-\frac{\left(\frac{\operatorname{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)(a-b)^{3/2}\,1\mathrm{i}}{2}-a^{3/2}\,\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-a^{3/2}\,\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(2\,c+2\,d\,x\right)+\frac{\cos(2\,c+2\,d\,x)\,\mathrm{at}}{}\right.}{\sqrt{a}\,b^2\,d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x)^2)),x)
```

```
[Out] - (((atanh((sin(c + d*x)*(a - b)^(1/2))/a^(1/2))*(a - b)^(3/2)*1i)/2 - a^(3
/2)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2)) - a^(3/2)*atan((sin(c/
2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x) + (cos(2*c + 2*d*x)*a
tanh((sin(c + d*x)*(a - b)^(1/2))/a^(1/2))*(a - b)^(3/2)*1i)/2)*1i)/(a^(1/2
```

```
)*b^2*d*(cos(2*c + 2*d*x)/2 + 1/2)) - (((a^(1/2)*sin(c + d*x)*1i)/2 + (3*a^
(1/2)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2)))/2 + (3*a^(1/2)*atan
((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2)*1i)/(a^(1
/2)*b*d*(cos(2*c + 2*d*x)/2 + 1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*tan(c + d*x)**2), x)
```

3.451 $\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{\tanh^{-1}(\sin(c+dx))}{bd}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}$$

```
[Out] arctanh(sin(d*x+c))/b/d-arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))*(a-b)^(1/2)
/b/d/a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3676, 391, 206, 208}

$$\frac{\tanh^{-1}(\sin(c+dx))}{bd}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b*d) - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sin[c + d*x
])/Sqrt[a]])/(Sqrt[a]*b*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-(a-b)x^2)}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{bd} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \sin(c+dx)\right)}{bd}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}$$

Mathematica [A] time = 0.11, size = 53, normalized size = 0.90

$$\frac{\tanh^{-1}(\sin(c+dx)) - \frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]] - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sq
rt[a]])/Sqrt[a])/(b*d)
```

fricas [A] time = 0.49, size = 169, normalized size = 2.86

$$\left[\frac{\sqrt{\frac{a-b}{a}}\log\left(-\frac{(a-b)\cos(dx+c)^2+2a\sqrt{\frac{a-b}{a}}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)+\log(\sin(dx+c)+1)-\log(-\sin(dx+c)+1)}{2bd}, \frac{2\sqrt{-\frac{a-b}{a}}\,\text{arc}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt((a - b)/a)*log(-((a - b)*cos(d*x + c)^2 + 2*a*sqrt((a - b)/a)*si
n(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + log(sin(d*x + c) + 1)
 - log(-sin(d*x + c) + 1))/(b*d), 1/2*(2*sqrt(-(a - b)/a)*arctan(sqrt(-(a -
 b)/a)*sin(d*x + c)) + log(sin(d*x + c) + 1) - log(-sin(d*x + c) + 1))/(b*d
)]
```

giac [A] time = 1.84, size = 88, normalized size = 1.49

$$-\frac{\frac{2(a-b)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b} - \frac{\log(|\sin(dx+c)+1|)}{b} + \frac{\log(|\sin(dx+c)-1|)}{b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a - b)*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))
/(sqrt(-a^2 + a*b)*b) - log(abs(sin(d*x + c) + 1))/b + log(abs(sin(d*x + c)
 - 1))/b)/d
```

maple [B] time = 0.61, size = 111, normalized size = 1.88

$$-\frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)a}{db\sqrt{a(a-b)}}+\frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{d\sqrt{a(a-b)}}-\frac{\ln(-1+\sin(dx+c))}{2db}+\frac{\ln(\sin(dx+c)+1)}{2db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x)
```

```
[Out] -1/d/b/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))*a+1/d/(a*(
a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))-1/2/d/b*ln(-1+sin(d*x
+c))+1/2/d/b*ln(sin(d*x+c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 12.67, size = 67, normalized size = 1.14

$$\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}-\frac{\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,\sqrt{a-b}}{\sqrt{a}\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x)^2)),x)
```

```
[Out] (2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) - (atanh((sin(c + d*
x)*(a - b)^(1/2))/a^(1/2))*(a - b)^(1/2))/(a^(1/2)*b*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}{\left(c + d x \right)}}{a + b \tan^{2}{\left(c + d x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x)**2), x)
```

3.452 $\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d\sqrt{a-b}}$$

```
[Out] arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))/d/a^(1/2)/(a-b)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3676, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx,x,\sin(c+dx)\right)}{d}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a-b}\,d}\end{aligned}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

fricas [**A**] time = 0.44, size = 122, normalized size = 3.05

$$\left[\frac{\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\,\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)}{2\sqrt{a^2-ab}\,d}, -\frac{\sqrt{-a^2+ab}\,\arctan\left(\frac{\sqrt{-a^2+ab}\,\sin(dx+c)}{a}\right)}{(a^2-ab)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a +
b)/((a - b)*cos(d*x + c)^2 + b))/(sqrt(a^2 - a*b)*d), -sqrt(-a^2 + a*b)*arc
tan(sqrt(-a^2 + a*b)*sin(d*x + c)/a)/((a^2 - a*b)*d)]
```

giac [**A**] time = 1.73, size = 47, normalized size = 1.18

$$-\frac{\arctan\left(\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -arctan((a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*
b)*d)
```

maple [**A**] time = 0.55, size = 36, normalized size = 0.90

$$\frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{d\sqrt{a\,(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [**B**] time = 12.57, size = 32, normalized size = 0.80

$$\frac{\operatorname{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)}{\sqrt{a}\,d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*tan(c + d*x)^2)),x)
```

```
[Out] atanh((sin(c + d*x)*(a - b)^(1/2))/a^(1/2))/(a^(1/2)*d*(a - b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x)**2), x)
```

3.453 $\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{\sin(c+dx)}{d(a-b)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{3/2}}$$

```
[Out] sin(d*x+c)/(a-b)/d-b*arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))/(a-b)^(3/2)/d/
a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3676, 388, 208}

$$\frac{\sin(c+dx)}{d(a-b)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d)
) + Sin[c + d*x]/((a - b)*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{a-(a-b)x^2}\,dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{\sin(c+dx)}{(a-b)d}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \sin(c+dx)\right)}{(a-b)d}\\
&= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{3/2}d}+\frac{\sin(c+dx)}{(a-b)d}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 60, normalized size = 1.00

$$\frac{\sin(c+dx)}{d(a-b)} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d)
) + Sin[c + d*x]/((a - b)*d)
```

fricas [A] time = 0.46, size = 182, normalized size = 3.03

$$\left[-\frac{\sqrt{a^2-ab}\,b\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)-2\left(a^2-ab\right)\sin(dx+c)}{2\left(a^3-2a^2b+ab^2\right)d}, \frac{\sqrt{-a^2+ab}\,b\arctan\left(\frac{\sqrt{-a^2+ab}\,s}{a}\right.}{\left(a^3-2a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 - a*b)*b*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*s
in(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) - 2*(a^2 - a*b)*sin(d*
x + c))/((a^3 - 2*a^2*b + a*b^2)*d), (sqrt(-a^2 + a*b)*b*arctan(sqrt(-a^2 +
 a*b)*sin(d*x + c)/a) + (a^2 - a*b)*sin(d*x + c))/((a^3 - 2*a^2*b + a*b^2)*
d)]
```

giac [A] time = 2.12, size = 73, normalized size = 1.22

$$-\frac{\frac{b\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,(a-b)} - \frac{\sin(dx+c)}{a-b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(b*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2
+ a*b)*(a - b)) - sin(d*x + c)/(a - b))/d
```

maple [A] time = 0.55, size = 61, normalized size = 1.02

$$\frac{\frac{\sin(dx+c)}{a-b} - \frac{b\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{(a-b)\sqrt{a(a-b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(1/(a-b)*sin(d*x+c)-b/(a-b)/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a
*(a-b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 12.32, size = 61, normalized size = 1.02

$$\frac{\sin\left(c+d\,x\right)}{d\,\left(a-b\right)}+\frac{b\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,(a-b)^{3/2}}{\sqrt{a}\,b-a^{3/2}}\right)}{\sqrt{a}\,d\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*tan(c + d*x)^2),x)
```

```
[Out] sin(c + d*x)/(d*(a - b)) + (b*atanh((sin(c + d*x)*(a - b)^(3/2))/(a^(1/2)*b
 - a^(3/2))))/(a^(1/2)*d*(a - b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(c+dx\right)}{a+b\tan^{2}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*tan(c + d*x)**2), x)
```

3.454 $\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=88

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{5/2}}-\frac{\sin^3(c+dx)}{3d(a-b)}+\frac{(a-2b)\sin(c+dx)}{d(a-b)^2}$$

```
[Out] (a-2*b)*sin(d*x+c)/(a-b)^2/d-1/3*sin(d*x+c)^3/(a-b)/d+b^2*arctanh(sin(d*x+c
)*(a-b)^(1/2)/a^(1/2))/(a-b)^(5/2)/d/a^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3676, 390, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{5/2}}-\frac{\sin^3(c+dx)}{3d(a-b)}+\frac{(a-2b)\sin(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2)*d)
 + ((a - 2*b)*Sin[c + d*x])/((a - b)^2*d) - Sin[c + d*x]^3/(3*(a - b)*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a-(a-b)x^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a-2b}{(a-b)^2} - \frac{x^2}{a-b} + \frac{b^2}{(a-b)^2(a-(a-b)x^2)}\right) dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{(a-2b)\sin(c+dx)}{(a-b)^2 d} - \frac{\sin^3(c+dx)}{3(a-b)d} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \sin(c+dx)\right)}{(a-b)^2 d}$$
$$= \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{5/2} d} + \frac{(a-2b)\sin(c+dx)}{(a-b)^2 d} - \frac{\sin^3(c+dx)}{3(a-b)d}$$

Mathematica [A] time = 0.51, size = 115, normalized size = 1.31

$$\frac{\frac{6b^2\left(\log\left(\sqrt{a-b}\,\sin(c+dx)+\sqrt{a}\right)-\log\left(\sqrt{a}-\sqrt{a-b}\,\sin(c+dx)\right)\right)}{\sqrt{a}\,(a-b)^{5/2}} + \frac{3(3a-7b)\sin(c+dx)}{(a-b)^2} + \frac{\sin(3(c+dx))}{a-b}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((6*b^2*(-Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d*x]] + Log[Sqrt[a] + Sqrt[a -
b]*Sin[c + d*x]]))/(Sqrt[a]*(a - b)^(5/2)) + (3*(3*a - 7*b)*Sin[c + d*x])/(
a - b)^2 + Sin[3*(c + d*x)]/(a - b))/(12*d)
```

fricas [A] time = 0.50, size = 276, normalized size = 3.14

$$\left[\frac{3\sqrt{a^2-ab}\,b^2\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\,\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)+2\left(2a^3-7a^2b+5ab^2+\left(a^3-2a^2b+ab^2\right)\cos(dx\right.}{6\left(a^4-3a^3b+3a^2b^2-ab^3\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a^2 - a*b)*b^2*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b
)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + 2*(2*a^3 - 7*a^2*
b + 5*a*b^2 + (a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 -
 3*a^3*b + 3*a^2*b^2 - a*b^3)*d), -1/3*(3*sqrt(-a^2 + a*b)*b^2*arctan(sqrt(
-a^2 + a*b)*sin(d*x + c)/a) - (2*a^3 - 7*a^2*b + 5*a*b^2 + (a^3 - 2*a^2*b +
 a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*
d)]
```

giac [B] time = 1.54, size = 161, normalized size = 1.83

$$\frac{\frac{3b^2\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\left(a^2-2ab+b^2\right)\sqrt{-a^2+ab}} - \frac{a^2\sin(dx+c)^3-2ab\sin(dx+c)^3+b^2\sin(dx+c)^3-3a^2\sin(dx+c)+9ab\sin(dx+c)-6b^2\sin(dx+c)}{a^3-3a^2b+3ab^2-b^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/((a^
2 - 2*a*b + b^2)*sqrt(-a^2 + a*b)) - (a^2*sin(d*x + c)^3 - 2*a*b*sin(d*x +
```

```
c)^3 + b^2*sin(d*x + c)^3 - 3*a^2*sin(d*x + c) + 9*a*b*sin(d*x + c) - 6*b^2
*sin(d*x + c))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/d
```

maple [A] time = 0.80, size = 98, normalized size = 1.11

$$\frac{-\frac{\frac{a\left(\sin^3(dx+c)\right)}{3}-\frac{b\left(\sin^3(dx+c)\right)}{3}-a\sin(dx+c)+2b\sin(dx+c)}{(a-b)^2}+\frac{b^2\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{(a-b)^2\sqrt{a(a-b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(-1/(a-b)^2*(1/3*a*sin(d*x+c)^3-1/3*b*sin(d*x+c)^3-a*sin(d*x+c)+2*b*sin
(d*x+c))+b^2/(a-b)^2/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/
2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 15.44, size = 251, normalized size = 2.85

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a-2b)}{a^2-2ab+b^2}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a-4b)}{3\left(a^2-2ab+b^2\right)}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(a-2b)}{a^2-2ab+b^2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{b^2\operatorname{atan}\left(\frac{2\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^3-6\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b+6\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)ab^2-}{\sqrt{a}\,(a-b)^{5/2}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}\right.}{\sqrt{a}\,d\,(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*tan(c + d*x)^2),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)*(a - 2*b))/(a^2 - 2*a*b + b^2) + (4*tan(c/2 + (d*x)/
2)^3*(a - 4*b))/(3*(a^2 - 2*a*b + b^2)) + (2*tan(c/2 + (d*x)/2)^5*(a - 2*b)
)/(a^2 - 2*a*b + b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4
+ tan(c/2 + (d*x)/2)^6 + 1)) - (b^2*atan((a^3*tan(c/2 + (d*x)/2)*2i - b^3*t
an(c/2 + (d*x)/2)*2i + a*b^2*tan(c/2 + (d*x)/2)*6i - a^2*b*tan(c/2 + (d*x)/
2)*6i)/(a^(1/2)*(a - b)^(5/2)*(tan(c/2 + (d*x)/2)^2 + 1)))*1i)/(a^(1/2)*d*(
a - b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.455 $\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{(a^2-3ab+3b^2)\sin(c+dx)}{d(a-b)^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{7/2}}+\frac{\sin^5(c+dx)}{5d(a-b)}-\frac{(2a-3b)\sin^3(c+dx)}{3d(a-b)^2}$$

```
[Out] (a^2-3*a*b+3*b^2)*sin(d*x+c)/(a-b)^3/d-1/3*(2*a-3*b)*sin(d*x+c)^3/(a-b)^2/d
+1/5*sin(d*x+c)^5/(a-b)/d-b^3*arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))/(a-b)
^(7/2)/d/a^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3676, 390, 208}

$$\frac{(a^2-3ab+3b^2)\sin(c+dx)}{d(a-b)^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{7/2}}+\frac{\sin^5(c+dx)}{5d(a-b)}-\frac{(2a-3b)\sin^3(c+dx)}{3d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -((b^3*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(7/2)*
d)) + ((a^2 - 3*a*b + 3*b^2)*Sin[c + d*x])/((a - b)^3*d) - ((2*a - 3*b)*Sin
[c + d*x]^3)/(3*(a - b)^2*d) + Sin[c + d*x]^5/(5*(a - b)*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a-(a-b)x^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+3b^2}{(a-b)^3} - \frac{(2a-3b)x^2}{(a-b)^2} + \frac{x^4}{a-b} - \frac{b^3}{(a-b)^3\left(a-(a-b)x^2\right)}\right) dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\left(a^2-3ab+3b^2\right)\sin(c+dx)}{(a-b)^3 d} - \frac{(2a-3b)\sin^3(c+dx)}{3(a-b)^2 d} + \frac{\sin^5(c+dx)}{5(a-b)d} - \frac{b^3\,\text{Subst}\left(\int \frac{}{a}\right.}{}$$

$$= -\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{7/2}d} + \frac{\left(a^2-3ab+3b^2\right)\sin(c+dx)}{(a-b)^3 d} - \frac{(2a-3b)\sin^3(c+dx)}{3(a-b)^2 d} +$$

Mathematica [A] time = 1.82, size = 148, normalized size = 1.17

$$\frac{\frac{30\left(5a^2-16ab+19b^2\right)\sin(c+dx)}{(a-b)^3} + \frac{120b^3\left(\log\left(\sqrt{a}-\sqrt{a-b}\sin(c+dx)\right)-\log\left(\sqrt{a-b}\sin(c+dx)+\sqrt{a}\right)\right)}{\sqrt{a}\,(a-b)^{7/2}} + \frac{5(5a-9b)\sin(3(c+dx))}{(a-b)^2} + \frac{3\sin(5(c+dx))}{a-b}}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((120*b^3*(Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d*x]] - Log[Sqrt[a] + Sqrt[a -
 b]*Sin[c + d*x]]))/(Sqrt[a]*(a - b)^(7/2)) + (30*(5*a^2 - 16*a*b + 19*b^2)
*Sin[c + d*x])/(a - b)^3 + (5*(5*a - 9*b)*Sin[3*(c + d*x)])/(a - b)^2 + (3*
Sin[5*(c + d*x)])/(a - b))/(240*d)
```

fricas [A] time = 0.55, size = 395, normalized size = 3.13

$$\left[-\frac{15\sqrt{a^2-ab}\,b^3\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right) - 2\left(3\left(a^4-3a^3b+3a^2b^2-ab^3\right)\cos(dx+c)^4 + 8a^4\right.}{30\left(a^5-4a^4b+6a^3b^2-4a^2b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*sqrt(a^2 - a*b)*b^3*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 -
a*b)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) - 2*(3*(a^4 - 3*
a^3*b + 3*a^2*b^2 - a*b^3)*cos(d*x + c)^4 + 8*a^4 - 34*a^3*b + 59*a^2*b^2 -
 33*a*b^3 + (4*a^4 - 17*a^3*b + 22*a^2*b^2 - 9*a*b^3)*cos(d*x + c)^2)*sin(d
*x + c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d), 1/15*(15*sqrt
(-a^2 + a*b)*b^3*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) + (3*(a^4 - 3*a^3*
b + 3*a^2*b^2 - a*b^3)*cos(d*x + c)^4 + 8*a^4 - 34*a^3*b + 59*a^2*b^2 - 33*
a*b^3 + (4*a^4 - 17*a^3*b + 22*a^2*b^2 - 9*a*b^3)*cos(d*x + c)^2)*sin(d*x +
 c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)]
```

giac [B] time = 1.50, size = 319, normalized size = 2.53

$$-\frac{\frac{15b^3\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\left(a^3-3a^2b+3ab^2-b^3\right)\sqrt{-a^2+ab}} - \frac{3a^4\sin(dx+c)^5-12a^3b\sin(dx+c)^5+18a^2b^2\sin(dx+c)^5-12ab^3\sin(dx+c)^5+3b^4\sin(dx+c)^5-10a^4\sin(dx+c)^3}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*b^3*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(-a^2 + a*b)) - (3*a^4*sin(d*x + c)^5 -
 12*a^3*b*sin(d*x + c)^5 + 18*a^2*b^2*sin(d*x + c)^5 - 12*a*b^3*sin(d*x + c
)^5 + 3*b^4*sin(d*x + c)^5 - 10*a^4*sin(d*x + c)^3 + 45*a^3*b*sin(d*x + c)^
3 - 75*a^2*b^2*sin(d*x + c)^3 + 55*a*b^3*sin(d*x + c)^3 - 15*b^4*sin(d*x +
c)^3 + 15*a^4*sin(d*x + c) - 75*a^3*b*sin(d*x + c) + 150*a^2*b^2*sin(d*x +
c) - 135*a*b^3*sin(d*x + c) + 45*b^4*sin(d*x + c))/(a^5 - 5*a^4*b + 10*a^3*
b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5))/d
```

maple [A] time = 0.83, size = 165, normalized size = 1.31

$$\frac{\frac{\frac{\left(\sin^{5}(dx+c)\right)a^{2}}{5}-\frac{2\left(\sin^{5}(dx+c)\right)ab}{5}+\frac{b^{2}\left(\sin^{5}(dx+c)\right)}{5}-\frac{2\left(\sin^{3}(dx+c)\right)a^{2}}{3}+\frac{5\left(\sin^{3}(dx+c)\right)ab}{3}-\left(\sin^{3}(dx+c)\right)b^{2}+a^{2}\sin(dx+c)-3\sin(dx+c)ab+3b^{2}\sin(dx+c)}{(a-b)^{3}}-\frac{b^{3}a}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*(1/(a-b)^3*(1/5*sin(d*x+c)^5*a^2-2/5*sin(d*x+c)^5*a*b+1/5*b^2*sin(d*x+c
)^5-2/3*sin(d*x+c)^3*a^2+5/3*sin(d*x+c)^3*a*b-sin(d*x+c)^3*b^2+a^2*sin(d*x+
c)-3*sin(d*x+c)*a*b+3*b^2*sin(d*x+c))-b^3/(a-b)^3/(a*(a-b))^(1/2)*arctanh((
a-b)*sin(d*x+c)/(a*(a-b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 15.08, size = 1493, normalized size = 11.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*tan(c + d*x)^2),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)*(a^2 - 3*a*b + 3*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^
3) + (tan(c/2 + (d*x)/2)^9*(2*a^2 - 6*a*b + 6*b^2))/(3*a*b^2 - 3*a^2*b + a^
3 - b^3) + (tan(c/2 + (d*x)/2)^3*((8*a^2)/3 - (32*a*b)/3 + 16*b^2))/(3*a*b^
2 - 3*a^2*b + a^3 - b^3) + (tan(c/2 + (d*x)/2)^7*((8*a^2)/3 - (32*a*b)/3 +
16*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (tan(c/2 + (d*x)/2)^5*((116*a^2)
/15 - (332*a*b)/15 + (132*b^2)/5))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(d*(5*t
an(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5
*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) - (b^3*atan(((b^3*((tan
(c/2 + (d*x)/2)*(16*a*b^10 - 96*a^2*b^9 + 240*a^3*b^8 - 320*a^4*b^7 + 240*a
^5*b^6 - 96*a^6*b^5 + 16*a^7*b^4))/2 + (b^3*(tan(c/2 + (d*x)/2)^2*(4*a^12 -
 44*a^11*b + 8*a^2*b^10 - 76*a^3*b^9 + 324*a^4*b^8 - 816*a^5*b^7 + 1344*a^6
*b^6 - 1512*a^7*b^5 + 1176*a^8*b^4 - 624*a^9*b^3 + 216*a^10*b^2) + 36*a^11*
b - 4*a^12 + 4*a^3*b^9 - 36*a^4*b^8 + 144*a^5*b^7 - 336*a^6*b^6 + 504*a^7*b
^5 - 504*a^8*b^4 + 336*a^9*b^3 - 144*a^10*b^2))/(a^(1/2)*(a - b)^(7/2)))*1i
)/(a^(1/2)*(a - b)^(7/2)) + (b^3*((tan(c/2 + (d*x)/2)*(16*a*b^10 - 96*a^2*b
```

```
^9 + 240*a^3*b^8 - 320*a^4*b^7 + 240*a^5*b^6 - 96*a^6*b^5 + 16*a^7*b^4))/2
- (b^3*(tan(c/2 + (d*x)/2)^2*(4*a^12 - 44*a^11*b + 8*a^2*b^10 - 76*a^3*b^9
+ 324*a^4*b^8 - 816*a^5*b^7 + 1344*a^6*b^6 - 1512*a^7*b^5 + 1176*a^8*b^4 -
624*a^9*b^3 + 216*a^10*b^2) + 36*a^11*b - 4*a^12 + 4*a^3*b^9 - 36*a^4*b^8 +
 144*a^5*b^7 - 336*a^6*b^6 + 504*a^7*b^5 - 504*a^8*b^4 + 336*a^9*b^3 - 144*
a^10*b^2))/(a^(1/2)*(a - b)^(7/2)))*1i)/(a^(1/2)*(a - b)^(7/2)))/(tan(c/2 +
 (d*x)/2)^2*(8*a*b^9 - 24*a^2*b^8 + 24*a^3*b^7 - 8*a^4*b^6) - 8*a*b^9 + 24*
a^2*b^8 - 24*a^3*b^7 + 8*a^4*b^6 + (b^3*((tan(c/2 + (d*x)/2)*(16*a*b^10 - 9
6*a^2*b^9 + 240*a^3*b^8 - 320*a^4*b^7 + 240*a^5*b^6 - 96*a^6*b^5 + 16*a^7*b
^4))/2 + (b^3*(tan(c/2 + (d*x)/2)^2*(4*a^12 - 44*a^11*b + 8*a^2*b^10 - 76*a
^3*b^9 + 324*a^4*b^8 - 816*a^5*b^7 + 1344*a^6*b^6 - 1512*a^7*b^5 + 1176*a^8
*b^4 - 624*a^9*b^3 + 216*a^10*b^2) + 36*a^11*b - 4*a^12 + 4*a^3*b^9 - 36*a^
4*b^8 + 144*a^5*b^7 - 336*a^6*b^6 + 504*a^7*b^5 - 504*a^8*b^4 + 336*a^9*b^3
 - 144*a^10*b^2))/(a^(1/2)*(a - b)^(7/2))))/(a^(1/2)*(a - b)^(7/2)) - (b^3*
((tan(c/2 + (d*x)/2)*(16*a*b^10 - 96*a^2*b^9 + 240*a^3*b^8 - 320*a^4*b^7 +
240*a^5*b^6 - 96*a^6*b^5 + 16*a^7*b^4))/2 - (b^3*(tan(c/2 + (d*x)/2)^2*(4*a
^12 - 44*a^11*b + 8*a^2*b^10 - 76*a^3*b^9 + 324*a^4*b^8 - 816*a^5*b^7 + 134
4*a^6*b^6 - 1512*a^7*b^5 + 1176*a^8*b^4 - 624*a^9*b^3 + 216*a^10*b^2) + 36*
a^11*b - 4*a^12 + 4*a^3*b^9 - 36*a^4*b^8 + 144*a^5*b^7 - 336*a^6*b^6 + 504*
a^7*b^5 - 504*a^8*b^4 + 336*a^9*b^3 - 144*a^10*b^2))/(a^(1/2)*(a - b)^(7/2)
)))/(a^(1/2)*(a - b)^(7/2))))*1i)/(a^(1/2)*d*(a - b)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.456 $\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=108

$$\frac{(a^2-3ab+3b^2)\tan(c+dx)}{b^3d}-\frac{(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}d}-\frac{(a-3b)\tan^3(c+dx)}{3b^2d}+\frac{\tan^5(c+dx)}{5bd}$$

```
[Out] -(a-b)^3*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/b^(7/2)/d/a^(1/2)+(a^2-3*a*b+3*
b^2)*tan(d*x+c)/b^3/d-1/3*(a-3*b)*tan(d*x+c)^3/b^2/d+1/5*tan(d*x+c)^5/b/d
```

Rubi [A] time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3675, 390, 205}

$$\frac{(a^2-3ab+3b^2)\tan(c+dx)}{b^3d}-\frac{(a-3b)\tan^3(c+dx)}{3b^2d}-\frac{(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}d}+\frac{\tan^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -(((a - b)^3*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)*d)) +
 ((a^2 - 3*a*b + 3*b^2)*Tan[c + d*x])/(b^3*d) - ((a - 3*b)*Tan[c + d*x]^3)/
(3*b^2*d) + Tan[c + d*x]^5/(5*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+3b^2}{b^3} - \frac{(a-3b)x^2}{b^2} + \frac{x^4}{b} + \frac{-a^3+3a^2b-3ab^2+b^3}{b^3(a+bx^2)}\right) dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a^2-3ab+3b^2)\tan(c+dx)}{b^3d} - \frac{(a-3b)\tan^3(c+dx)}{3b^2d} + \frac{\tan^5(c+dx)}{5bd} - \frac{(a-b)^3\,\text{Subst}}{}$$
$$= -\frac{(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}d} + \frac{(a^2-3ab+3b^2)\tan(c+dx)}{b^3d} - \frac{(a-3b)\tan^3(c+dx)}{3b^2d}$$

Mathematica [A] time = 0.90, size = 103, normalized size = 0.95

$$\frac{\sqrt{b}\tan(c+dx)\left(15a^2-b(5a-9b)\sec^2(c+dx)-40ab+3b^2\sec^4(c+dx)+33b^2\right)-\frac{15(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}}{15b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-15*(a - b)^3*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a] + Sqrt[b]*(
15*a^2 - 40*a*b + 33*b^2 - (5*a - 9*b)*b*Sec[c + d*x]^2 + 3*b^2*Sec[c + d*x
]^4)*Tan[c + d*x])/(15*b^(7/2)*d)
```

fricas [A] time = 0.56, size = 425, normalized size = 3.94

$$\left[\frac{15\left(a^3-3a^2b+3ab^2-b^3\right)\sqrt{-ab}\cos(dx+c)^5\log\left(\frac{\left(a^2+6ab+b^2\right)\cos(dx+c)^4-2\left(3ab+b^2\right)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(d\right.}{\left(a^2-2ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2\right)\cos(dx+c)^2+b^2}\right)}{60\,ab^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(-a*b)*cos(d*x + c)^5*log(((a
^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a +
 b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 -
 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) + 4*((1
5*a^3*b - 40*a^2*b^2 + 33*a*b^3)*cos(d*x + c)^4 + 3*a*b^3 - (5*a^2*b^2 - 9*
a*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)^5), 1/30*(15*(a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 -
 b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c)^5 + 2*((15*a^3*
b - 40*a^2*b^2 + 33*a*b^3)*cos(d*x + c)^4 + 3*a*b^3 - (5*a^2*b^2 - 9*a*b^3)
*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)^5)]
```

giac [A] time = 1.43, size = 151, normalized size = 1.40

$$-\frac{\frac{15\left(a^3-3a^2b+3ab^2-b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}\,b^3} - \frac{3b^4\tan(dx+c)^5-5ab^3\tan(dx+c)^3+15b^4\tan(dx+c)^3+15a^2b^2\tan(dx+c)-45ab^3}{b^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn
(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/(sqrt(a*b)*b^3) - (3*b^4*tan(d*x +
c)^5 - 5*a*b^3*tan(d*x + c)^3 + 15*b^4*tan(d*x + c)^3 + 15*a^2*b^2*tan(d*x
+ c) - 45*a*b^3*tan(d*x + c) + 45*b^4*tan(d*x + c))/b^5)/d
```

maple [B] time = 0.68, size = 206, normalized size = 1.91

$$\frac{\tan^5(dx+c)}{5bd} - \frac{a\left(\tan^3(dx+c)\right)}{3b^2d} + \frac{\tan^3(dx+c)}{bd} + \frac{a^2\tan(dx+c)}{d\,b^3} - \frac{3a\tan(dx+c)}{b^2d} + \frac{3\tan(dx+c)}{bd} - \frac{\arctan\left(\frac{\tan(\cdots}{\sqrt{\cdots}}\right.}{d\,b^3\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/5*tan(d*x+c)^5/b/d-1/3*a*tan(d*x+c)^3/b^2/d+tan(d*x+c)^3/b/d+1/d/b^3*a^2*
tan(d*x+c)-3*a*tan(d*x+c)/b^2/d+3*tan(d*x+c)/b/d-1/d/b^3/(a*b)^(1/2)*arctan
(tan(d*x+c)*b/(a*b)^(1/2))*a^3+3/d/b^2/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b
)^(1/2))*a^2-3/d/b/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))*a+1/d/(a*b)
^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))
```

maxima [A] time = 0.51, size = 110, normalized size = 1.02

$$-\frac{\frac{15\left(a^3-3a^2b+3ab^2-b^3\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} - \frac{3b^2\tan(dx+c)^5-5\left(ab-3b^2\right)\tan(dx+c)^3+15\left(a^2-3ab+3b^2\right)\tan(dx+c)}{b^3}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*arctan(b*tan(d*x + c)/sqrt(a*b))/
(sqrt(a*b)*b^3) - (3*b^2*tan(d*x + c)^5 - 5*(a*b - 3*b^2)*tan(d*x + c)^3 +
15*(a^2 - 3*a*b + 3*b^2)*tan(d*x + c))/b^3)/d
```

mupad [B] time = 12.29, size = 136, normalized size = 1.26

$$\frac{\tan(c+d\,x)\left(\frac{3}{b}+\frac{a\left(\frac{a}{b^2}-\frac{3}{b}\right)}{b}\right)}{d} + \frac{\tan(c+d\,x)^5}{5\,b\,d} - \frac{\tan(c+d\,x)^3\left(\frac{a}{3\,b^2}-\frac{1}{b}\right)}{d} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)\,(a-b)^3}{\sqrt{a}\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}\right)(a-b)^3}{\sqrt{a}\,b^{7/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + b*tan(c + d*x)^2)),x)
```

```
[Out] (tan(c + d*x)*(3/b + (a*(a/b^2 - 3/b))/b))/d + tan(c + d*x)^5/(5*b*d) - (ta
n(c + d*x)^3*(a/(3*b^2) - 1/b))/d - (atan((b^(1/2)*tan(c + d*x)*(a - b)^3)/
(a^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))*(a - b)^3)/(a^(1/2)*b^(7/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**8/(a + b*tan(c + d*x)**2), x)
```

3.457 $\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{(a-2b)\tan(c+dx)}{b^2d}+\frac{\tan^3(c+dx)}{3bd}$$

```
[Out] (a-b)^2*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/b^(5/2)/d/a^(1/2)-(a-2*b)*tan(d*
x+c)/b^2/d+1/3*tan(d*x+c)^3/b/d
```

Rubi [A] time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3675, 390, 205}

$$\frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{(a-2b)\tan(c+dx)}{b^2d}+\frac{\tan^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((a - b)^2*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - ((
a - 2*b)*Tan[c + d*x])/(b^2*d) + Tan[c + d*x]^3/(3*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a-2b}{b^2}+\frac{x^2}{b}+\frac{a^2-2ab+b^2}{b^2(a+bx^2)}\right)dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{(a-2b)\tan(c+dx)}{b^2d}+\frac{\tan^3(c+dx)}{3bd}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{b^2d}$$
$$= \frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{(a-2b)\tan(c+dx)}{b^2d}+\frac{\tan^3(c+dx)}{3bd}$$

Mathematica [A] time = 0.34, size = 74, normalized size = 0.96

$$\frac{\frac{3(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}+\sqrt{b}\tan(c+dx)\left(-3a+b\sec^2(c+dx)+5b\right)}{3b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((3*(a - b)^2*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a] + Sqrt[b]*(-3
*a + 5*b + b*Sec[c + d*x]^2)*Tan[c + d*x])/(3*b^(5/2)*d)
```

fricas [A] time = 0.49, size = 339, normalized size = 4.40

$$\left[-\frac{3\left(a^2-2ab+b^2\right)\sqrt{-ab}\cos(dx+c)^3\log\left(\frac{\left(a^2+6ab+b^2\right)\cos(dx+c)^4-2\left(3ab+b^2\right)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{\cdots}}{\left(a^2-2ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2\right)\cos(dx+c)^2+b^2}\right.\cdots}{12ab^3d\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^2 - 2*a*b + b^2)*sqrt(-a*b)*cos(d*x + c)^3*log(((a^2 + 6*a*b +
 b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x
+ c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2
)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) - 4*(a*b^2 - (3*a^2
*b - 5*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^3*d*cos(d*x + c)^3), -1/6*
(3*(a^2 - 2*a*b + b^2)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sq
rt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c)^3 - 2*(a*b^2 - (3*a^2
*b - 5*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^3*d*cos(d*x + c)^3)]
```

giac [A] time = 1.46, size = 96, normalized size = 1.25

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)\left(a^2-2ab+b^2\right)}{\sqrt{ab}\,b^2}+\frac{b^2\tan(dx+c)^3-3ab\tan(dx+c)+6b^2\tan(dx+c)}{b^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*
b)))*(a^2 - 2*a*b + b^2)/(sqrt(a*b)*b^2) + (b^2*tan(d*x + c)^3 - 3*a*b*tan(
d*x + c) + 6*b^2*tan(d*x + c))/b^3)/d
```

maple [A] time = 0.70, size = 127, normalized size = 1.65

$$\frac{\tan^3(dx+c)}{3bd}-\frac{a\tan(dx+c)}{b^2d}+\frac{2\tan(dx+c)}{bd}+\frac{\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)a^2}{d\,b^2\sqrt{ab}}-\frac{2\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)a}{db\sqrt{ab}}+\frac{\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)}{d\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/3*tan(d*x+c)^3/b/d-a*tan(d*x+c)/b^2/d+2*tan(d*x+c)/b/d+1/d/b^2/(a*b)^(1/2
)*arctan(tan(d*x+c)*b/(a*b)^(1/2))*a^2-2/d/b/(a*b)^(1/2)*arctan(tan(d*x+c)*
b/(a*b)^(1/2))*a+1/d/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))
```

maxima [A] time = 0.66, size = 69, normalized size = 0.90

$$\frac{\frac{3\left(a^2-2\,ab+b^2\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{b\tan(dx+c)^3-3\,(a-2\,b)\tan(dx+c)}{b^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^2 - 2*a*b + b^2)*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*b^2)
 + (b*tan(d*x + c)^3 - 3*(a - 2*b)*tan(d*x + c))/b^2)/d
```

mupad [B] time = 12.23, size = 90, normalized size = 1.17

$$\frac{\tan(c+d\,x)^3}{3\,b\,d}-\frac{\tan(c+d\,x)\,\left(\frac{a}{b^2}-\frac{2}{b}\right)}{d}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)\,(a-b)^2}{\sqrt{a}\,\left(a^2-2\,a\,b+b^2\right)}\right)\,(a-b)^2}{\sqrt{a}\,b^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x)^2)),x)
```

```
[Out] tan(c + d*x)^3/(3*b*d) - (tan(c + d*x)*(a/b^2 - 2/b))/d + (atan((b^(1/2)*ta
n(c + d*x)*(a - b)^2)/(a^(1/2)*(a^2 - 2*a*b + b^2)))*(a - b)^2)/(a^(1/2)*b^
(5/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*tan(c + d*x)**2), x)
```

3.458 $\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{\tan(c+dx)}{bd}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}$$

```
[Out] -(a-b)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/b^(3/2)/d/a^(1/2)+tan(d*x+c)/b/d
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3675, 388, 205}

$$\frac{\tan(c+dx)}{bd}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -(((a - b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d)) + T
an[c + d*x]/(b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{a+bx^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\tan(c+dx)}{bd}-\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\tan(c+dx)\right)}{bd}\\ &= -\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}+\frac{\tan(c+dx)}{bd}\end{aligned}$$

Mathematica [A] time = 0.15, size = 52, normalized size = 1.00

$$\frac{\tan(c+dx)}{bd}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -(((a - b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d)) + T
an[c + d*x]/(b*d)
```

fricas [B] time = 0.56, size = 267, normalized size = 5.13

$$\left[\frac{\sqrt{-ab}\,(a-b)\cos(dx+c)\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(dx+c)^4-2\left(3\,ab+b^2\right)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\,\sin(dx+c)+b^2}{\left(a^2-2\,ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2\right)\cos(dx+c)^2+b^2}\right)}{4\,ab^2d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a*b)*(a - b)*cos(d*x + c)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)
^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x
 + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 +
 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) + 4*a*b*sin(d*x + c))/(a*b^2*d*cos(d*
x + c)), 1/2*(sqrt(a*b)*(a - b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sqr
t(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c) + 2*a*b*sin(d*x + c))/
(a*b^2*d*cos(d*x + c))]
```

giac [A] time = 1.38, size = 62, normalized size = 1.19

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)(a-b)}{\sqrt{ab}\,b}-\frac{\tan(dx+c)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))*
(a - b)/(sqrt(a*b)*b) - tan(d*x + c)/b)/d
```

maple [A] time = 0.59, size = 66, normalized size = 1.27

$$\frac{\tan(dx+c)}{bd}-\frac{\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)a}{db\sqrt{ab}}+\frac{\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)}{d\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x)
```

```
[Out] tan(d*x+c)/b/d-1/d/b/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))*a+1/d/(a*
b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))
```

maxima [A] time = 0.74, size = 45, normalized size = 0.87

$$-\frac{\frac{(a-b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{\tan(dx+c)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -((a - b)*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*b) - tan(d*x + c)/b)/
d
```

mupad [B] time = 12.40, size = 44, normalized size = 0.85

$$\frac{\tan\left(c+d\,x\right)}{b\,d}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan\left(c+d\,x\right)}{\sqrt{a}}\right)\,\left(a-b\right)}{\sqrt{a}\,b^{3/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x)^2)),x)
```

```
[Out] tan(c + d*x)/(b*d) - (atan((b^(1/2)*tan(c + d*x))/a^(1/2))*(a - b))/(a^(1/2
)*b^(3/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(c+dx\right)}{a+b\tan^{2}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x)**2), x)
```

3.459 $\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

```
[Out] arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/d/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

fricas [B] time = 0.54, size = 205, normalized size = 6.41

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(dx+c)^4-2\left(3\,ab+b^2\right)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\,\sin(dx+c)+b^2}{\left(a^2-2\,ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2\right)\cos(dx+c)^2+b^2}\right)}{4\,abd},-\frac{\sqrt{ab}\,\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*
cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin
(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*
x + c)^2 + b^2))/(a*b*d), -1/2*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2
 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))/(a*b*d)]
```

giac [A] time = 1.60, size = 40, normalized size = 1.25

$$\frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/(s
qrt(a*b)*d)
```

maple [A] time = 0.59, size = 24, normalized size = 0.75

$$\frac{\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)}{d\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))
```

maxima [A] time = 0.47, size = 23, normalized size = 0.72

$$\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*d)
```

mupad [B] time = 12.50, size = 24, normalized size = 0.75

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)),x)
```

```
[Out] atan((b^(1/2)*tan(c + d*x))/a^(1/2))/(a^(1/2)*b^(1/2)*d)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

3.460 $\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^2}+\frac{\sin(c+dx)\cos(c+dx)}{2d(a-b)}+\frac{x(a-3b)}{2(a-b)^2}$$

```
[Out] 1/2*(a-3*b)*x/(a-b)^2+1/2*cos(d*x+c)*sin(d*x+c)/(a-b)/d+b^(3/2)*arctan(b^(1
/2)*tan(d*x+c)/a^(1/2))/(a-b)^2/d/a^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3675, 414, 522, 203, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^2}+\frac{\sin(c+dx)\cos(c+dx)}{2d(a-b)}+\frac{x(a-3b)}{2(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((a - 3*b)*x)/(2*(a - b)^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a
]])/(Sqrt[a]*(a - b)^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*(a - b)*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
```

```
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d} - \frac{\text{Subst}\left(\int \frac{-a+2b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(c+dx)\right)}{2(a-b)d}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d} + \frac{(a-3b)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2(a-b)^2d} + \frac{b^2\,\text{Subst}\left(\int \frac{}{a+}\right.}{(}$$

$$= \frac{(a-3b)x}{2(a-b)^2} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^2d} + \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d}$$

Mathematica [A] time = 0.19, size = 78, normalized size = 0.94

$$\frac{4b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right) + \sqrt{a}\,(2(a-3b)(c+dx) + (a-b)\sin(2(c+dx)))}{4\sqrt{a}\,d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (4*b^(3/2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*(2*(a - 3*b)*(c
 + d*x) + (a - b)*Sin[2*(c + d*x)]))/(4*Sqrt[a]*(a - b)^2*d)
```

fricas [A] time = 0.56, size = 290, normalized size = 3.49

$$\left[\frac{2\,(a-3\,b)dx + 2\,(a-b)\cos(dx+c)\sin(dx+c) + b\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(dx+c)^4 - 2\left(3\,ab+b^2\right)\cos(dx+c)^2 - 4\left(\left(a^2+ab\right)\cos\right.}{\left(a^2-2\,ab+b^2\right)\cos(dx+c)^4 + 2\left(ab-b^2\right)}\right.}{4\left(a^2-2\,ab+b^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a - 3*b)*d*x + 2*(a - b)*cos(d*x + c)*sin(d*x + c) + b*sqrt(-b/a)*
log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 -
4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x + c))*sqrt(-b/a)*sin(d*x + c) +
 b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 +
b^2)))/((a^2 - 2*a*b + b^2)*d), 1/2*((a - 3*b)*d*x + (a - b)*cos(d*x + c)*s
in(d*x + c) - b*sqrt(b/a)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sqrt(b/a)
/(b*cos(d*x + c)*sin(d*x + c))))/((a^2 - 2*a*b + b^2)*d)]
```

giac [A] time = 1.52, size = 110, normalized size = 1.33

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)b^2}{\left(a^2-2\,ab+b^2\right)\sqrt{ab}} + \frac{(dx+c)(a-3\,b)}{a^2-2\,ab+b^2} + \frac{\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)(a-b)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*
b)))*b^2/((a^2 - 2*a*b + b^2)*sqrt(a*b)) + (d*x + c)*(a - 3*b)/(a^2 - 2*a*b
 + b^2) + tan(d*x + c)/((tan(d*x + c)^2 + 1)*(a - b)))/d
```

maple [A] time = 0.65, size = 137, normalized size = 1.65

$$\frac{b^2 \arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)}{d(a-b)^2\sqrt{ab}}+\frac{\tan(dx+c)a}{2d(a-b)^2\left(1+\tan^2(dx+c)\right)}-\frac{\tan(dx+c)b}{2d(a-b)^2\left(1+\tan^2(dx+c)\right)}+\frac{\arctan(\tan(dx+c))a}{2d(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x)
```

```
[Out] 1/d*b^2/(a-b)^2/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))+1/2/d/(a-b)^2*
tan(d*x+c)/(1+tan(d*x+c)^2)*a-1/2/d/(a-b)^2*tan(d*x+c)/(1+tan(d*x+c)^2)*b+1
/2/d/(a-b)^2*arctan(tan(d*x+c))*a-3/2/d/(a-b)^2*arctan(tan(d*x+c))*b
```

maxima [A] time = 0.44, size = 95, normalized size = 1.14

$$\frac{\frac{2\,b^2 \arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\left(a^2-2\,ab+b^2\right)\sqrt{ab}}+\frac{(dx+c)(a-3\,b)}{a^2-2\,ab+b^2}+\frac{\tan(dx+c)}{(a-b)\tan(dx+c)^2+a-b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^2 - 2*a*b + b^2)*sqrt(a*b))
 + (d*x + c)*(a - 3*b)/(a^2 - 2*a*b + b^2) + tan(d*x + c)/((a - b)*tan(d*x
+ c)^2 + a - b))/d
```

mupad [B] time = 13.89, size = 254, normalized size = 3.06

$$-\frac{6\,a\,b\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)-a^2\,\sin\left(2\,c+2\,d\,x\right)-2\,a^2\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)+a\,b\,\sin\left(2\,c+2\,d\,x\right)+\mathrm{atan}\left(\frac{a^2\,b^3\,\sin(c+d\,x)}{}\right.}{4\,d\,a^3-8\,d\,a^2\,b+4\,d\,a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*tan(c + d*x)^2),x)
```

```
[Out] -(atan((a^2*b^3*sin(c + d*x)*(-a*b^3)^(1/2)*9i - a^3*b^2*sin(c + d*x)*(-a*b
^3)^(1/2)*6i - a*b^4*sin(c + d*x)*(-a*b^3)^(1/2)*4i + a^4*b*sin(c + d*x)*(-
a*b^3)^(1/2)*1i)/(4*a^2*b^5*cos(c + d*x) - 9*a^3*b^4*cos(c + d*x) + 6*a^4*b
^3*cos(c + d*x) - a^5*b^2*cos(c + d*x)))*(-a*b^3)^(1/2)*4i - 2*a^2*atan(sin
(c + d*x)/cos(c + d*x)) - a^2*sin(2*c + 2*d*x) + 6*a*b*atan(sin(c + d*x)/co
s(c + d*x)) + a*b*sin(2*c + 2*d*x))/(4*a^3*d + 4*a*b^2*d - 8*a^2*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.461 $\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=129

$$\frac{x(3a^2-10ab+15b^2)}{8(a-b)^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^3}+\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a-b)}+\frac{(3a-7b)\sin(c+dx)\cos(c+dx)}{8d(a-b)^2}$$

```
[Out] 1/8*(3*a^2-10*a*b+15*b^2)*x/(a-b)^3+1/8*(3*a-7*b)*cos(d*x+c)*sin(d*x+c)/(a-
b)^2/d+1/4*cos(d*x+c)^3*sin(d*x+c)/(a-b)/d-b^(5/2)*arctan(b^(1/2)*tan(d*x+c
)/a^(1/2))/(a-b)^3/d/a^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3675, 414, 527, 522, 203, 205}

$$\frac{x(3a^2-10ab+15b^2)}{8(a-b)^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^3}+\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a-b)}+\frac{(3a-7b)\sin(c+dx)\cos(c+dx)}{8d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((3*a^2 - 10*a*b + 15*b^2)*x)/(8*(a - b)^3) - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[
c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^3*d) + ((3*a - 7*b)*Cos[c + d*x]*Sin[c
 + d*x])/(8*(a - b)^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*(a - b)*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3\left(a+bx^2\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d} - \frac{\text{Subst}\left(\int \frac{-3a+4b-3bx^2}{\left(1+x^2\right)^2\left(a+bx^2\right)}\,dx, x, \tan(c+dx)\right)}{4(a-b)d}$$

$$= \frac{(3a-7b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d} + \frac{\text{Subst}\left(\int \frac{3a^2-7ab+8b\ldots}{\left(1+x^2\right)\ldots}\right.}{\ldots}$$

$$= \frac{(3a-7b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,d\ldots\right.}{(a-b\ldots}$$

$$= \frac{\left(3a^2-10ab+15b^2\right)x}{8(a-b)^3} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^3d} + \frac{(3a-7b)\cos(c+dx)\sin(c+d\ldots}{8(a-b)^2d}$$

Mathematica [A] time = 0.42, size = 113, normalized size = 0.88

$$\frac{\sqrt{a}\left(4\left(3a^2-10ab+15b^2\right)(c+dx)+8\left(a^2-3ab+2b^2\right)\sin(2(c+dx))+(a-b)^2\sin(4(c+dx))\right)-32b^{5/2}\tan\ldots}{32\sqrt{a}\,d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (-32*b^(5/2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*(4*(3*a^2 - 1
0*a*b + 15*b^2)*(c + d*x) + 8*(a^2 - 3*a*b + 2*b^2)*Sin[2*(c + d*x)] + (a -
 b)^2*Sin[4*(c + d*x)]))/(32*Sqrt[a]*(a - b)^3*d)
```

fricas [A] time = 0.57, size = 401, normalized size = 3.11

$$\left[-\frac{2\,b^2\sqrt{-\frac{b}{a}}\,\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(dx+c)^4-2\left(3\,ab+b^2\right)\cos(dx+c)^2-4\left(\left(a^2+ab\right)\cos(dx+c)^3-ab\cos(dx+c)\right)\sqrt{-\frac{b}{a}}\,\sin(dx+c)+b^2}{\left(a^2-2\,ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2\right)\cos(dx+c)^2+b^2}\right)-\left(3\,a^2-\ldots\right.}{8\left(a^3-3\,a^2b+3\,a\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*b^2*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b
+ b^2)*cos(d*x + c)^2 - 4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x + c))*s
qrt(-b/a)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b
- b^2)*cos(d*x + c)^2 + b^2)) - (3*a^2 - 10*a*b + 15*b^2)*d*x - (2*(a^2 - 2
*a*b + b^2)*cos(d*x + c)^3 + (3*a^2 - 10*a*b + 7*b^2)*cos(d*x + c))*sin(d*x
 + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d), 1/8*(4*b^2*sqrt(b/a)*arctan(1/2
*((a + b)*cos(d*x + c)^2 - b)*sqrt(b/a)/(b*cos(d*x + c)*sin(d*x + c))) + (3
*a^2 - 10*a*b + 15*b^2)*d*x + (2*(a^2 - 2*a*b + b^2)*cos(d*x + c)^3 + (3*a^
2 - 10*a*b + 7*b^2)*cos(d*x + c))*sin(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d)]
```

giac [A] time = 1.46, size = 183, normalized size = 1.42

$$-\frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)b^3}{\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\sqrt{ab}}-\frac{\left(3\,a^2-10\,ab+15\,b^2\right)(dx+c)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{3\,a\tan(dx+c)^3-7\,b\tan(dx+c)^3+5\,a\tan(dx+c)-9\,b\tan(dx+c)}{\left(a^2-2\,ab+b^2\right)\left(\tan(dx+c)^2+1\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*(pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a
*b)))*b^3/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b)) - (3*a^2 - 10*a*b + 1
5*b^2)*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a*tan(d*x + c)^3 - 7*
b*tan(d*x + c)^3 + 5*a*tan(d*x + c) - 9*b*tan(d*x + c))/((a^2 - 2*a*b + b^2
)*(tan(d*x + c)^2 + 1)^2))/d
```

maple [B] time = 0.67, size = 303, normalized size = 2.35

$$-\frac{b^3\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)}{d\,(a-b)^3\sqrt{ab}}+\frac{3\left(\tan^3(dx+c)\right)a^2}{8d\,(a-b)^3\left(1+\tan^2(dx+c)\right)^2}-\frac{5\left(\tan^3(dx+c)\right)ab}{4d\,(a-b)^3\left(1+\tan^2(dx+c)\right)^2}+\frac{7\left(\tan^3(dx+c)\right)}{8d\,(a-b)^3\left(1+\tan^2(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x)
```

```
[Out] -1/d*b^3/(a-b)^3/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))+3/8/d/(a-b)^3
/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^2-5/4/d/(a-b)^3/(1+tan(d*x+c)^2)^2*tan(d
*x+c)^3*a*b+7/8/d/(a-b)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*b^2-7/4/d/(a-b)^3
/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a*b+9/8/d/(a-b)^3/(1+tan(d*x+c)^2)^2*tan(d*x
+c)*b^2+5/8/d/(a-b)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^2+15/8/d/(a-b)^3*arct
an(tan(d*x+c))*b^2+3/8/d/(a-b)^3*arctan(tan(d*x+c))*a^2-5/4/d/(a-b)^3*arcta
n(tan(d*x+c))*a*b
```

maxima [A] time = 0.65, size = 185, normalized size = 1.43

$$-\frac{\frac{8\,b^3\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\sqrt{ab}}-\frac{\left(3\,a^2-10\,ab+15\,b^2\right)(dx+c)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{(3\,a-7\,b)\tan(dx+c)^3+(5\,a-9\,b)\tan(dx+c)}{\left(a^2-2\,ab+b^2\right)\tan(dx+c)^4+2\left(a^2-2\,ab+b^2\right)\tan(dx+c)^2+a^2-2\,ab+b^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*b^3*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*sqrt(a*b)) - (3*a^2 - 10*a*b + 15*b^2)*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^
2 - b^3) - ((3*a - 7*b)*tan(d*x + c)^3 + (5*a - 9*b)*tan(d*x + c))/((a^2 -
2*a*b + b^2)*tan(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*tan(d*x + c)^2 + a^2 -
2*a*b + b^2))/d
```

mupad [B] time = 15.87, size = 3681, normalized size = 28.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*tan(c + d*x)^2),x)
```

```
[Out] ((tan(c + d*x)*(5*a - 9*b))/(8*(a^2 - 2*a*b + b^2)) + (tan(c + d*x)^3*(3*a
- 7*b))/(8*(a^2 - 2*a*b + b^2)))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)
) - (atan(((((tan(c + d*x)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b^4
+ 9*a^4*b^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((256*b^1
0 - 1760*a*b^9 + 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*a^5*b^5
+ 3040*a^6*b^4 - 800*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b
^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) - (tan(c + d*x)*(3*a^2 - 10*a*b
 + 15*b^2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b
^5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i - a^2*b*3i
+ a^3*1i - b^3*1i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(3*a^2 - 1
0*a*b + 15*b^2))/(16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))*(3*a^2 - 10*
a*b + 15*b^2)*1i)/(16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) + (((tan(c +
 d*x)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b^4 + 9*a^4*b^3))/(32*(a^
4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (((256*b^10 - 1760*a*b^9 + 5280
*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*a^5*b^5 + 3040*a^6*b^4 - 800*
a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*
a^3*b^3 + 15*a^4*b^2)) + (tan(c + d*x)*(3*a^2 - 10*a*b + 15*b^2)*(1280*a*b^
8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4 + 1
280*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)*(a
^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(3*a^2 - 10*a*b + 15*b^2))/(16*
(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))*(3*a^2 - 10*a*b + 15*b^2)*1i)/(16
*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))/((115*a*b^7 - 105*b^8 - 51*a^2*b
^6 + 9*a^3*b^5)/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^
3 + 15*a^4*b^2)) - (((tan(c + d*x)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*
a^3*b^4 + 9*a^4*b^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + ((
(256*b^10 - 1760*a*b^9 + 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*
a^5*b^5 + 3040*a^6*b^4 - 800*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a
*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) - (tan(c + d*x)*(3*a^2
- 10*a*b + 15*b^2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 12
80*a^4*b^5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i - a
^2*b*3i + a^3*1i - b^3*1i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(3
*a^2 - 10*a*b + 15*b^2))/(16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))*(3*a
^2 - 10*a*b + 15*b^2))/(16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) + (((ta
n(c + d*x)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b^4 + 9*a^4*b^3))/(3
2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (((256*b^10 - 1760*a*b^9 +
 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*a^5*b^5 + 3040*a^6*b^4 -
 800*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4
- 20*a^3*b^3 + 15*a^4*b^2)) + (tan(c + d*x)*(3*a^2 - 10*a*b + 15*b^2)*(1280
*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^
4 + 1280*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1
i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(3*a^2 - 10*a*b + 15*b^2))
/(16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))*(3*a^2 - 10*a*b + 15*b^2))/(
16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i))))*(3*a^2 - 10*a*b + 15*b^2)*1i)
/(8*d*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) - (atan((((-a*b^5)^(1/2)*((t
an(c + d*x)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b^4 + 9*a^4*b^3))/(
32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + ((-a*b^5)^(1/2)*((256*b^1
0 - 1760*a*b^9 + 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*a^5*b^5
+ 3040*a^6*b^4 - 800*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b
^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) - (tan(c + d*x)*(-a*b^5)^(1/2)*
(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a
^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(64*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^
2)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(2*(a*b^3 + 3*a^3*b - a^4
```

```
 - 3*a^2*b^2)))*1i)/(2*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)) + ((-a*b^5)^(1/
2)*((tan(c + d*x)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b^4 + 9*a^4*b
^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - ((-a*b^5)^(1/2)*((2
56*b^10 - 1760*a*b^9 + 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*a^
5*b^5 + 3040*a^6*b^4 - 800*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b
^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (tan(c + d*x)*(-a*b^5)^
(1/2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 -
2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(64*(a*b^3 + 3*a^3*b - a^4 - 3*
a^2*b^2)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(2*(a*b^3 + 3*a^3*b
 - a^4 - 3*a^2*b^2)))*1i)/(2*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))/((115*a*
b^7 - 105*b^8 - 51*a^2*b^6 + 9*a^3*b^5)/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^6
+ 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) - ((-a*b^5)^(1/2)*((tan(c + d*x)*(
289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b^4 + 9*a^4*b^3))/(32*(a^4 - 4*a
^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + ((-a*b^5)^(1/2)*((256*b^10 - 1760*a*b^
9 + 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*a^5*b^5 + 3040*a^6*b^
4 - 800*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b
^4 - 20*a^3*b^3 + 15*a^4*b^2)) - (tan(c + d*x)*(-a*b^5)^(1/2)*(1280*a*b^8 -
 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4 + 1280
*a^6*b^3 - 256*a^7*b^2))/(64*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)*(a^4 - 4*a
^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(2*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)
)))/(2*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)) + ((-a*b^5)^(1/2)*((tan(c + d*x
)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b^4 + 9*a^4*b^3))/(32*(a^4 -
4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - ((-a*b^5)^(1/2)*((256*b^10 - 1760*a
*b^9 + 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*a^5*b^5 + 3040*a^6
*b^4 - 800*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^
2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (tan(c + d*x)*(-a*b^5)^(1/2)*(1280*a*b^
8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4 + 1
280*a^6*b^3 - 256*a^7*b^2))/(64*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)*(a^4 -
4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(2*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b
^2))))/(2*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2))))*(-a*b^5)^(1/2)*1i)/(d*(a*b
^3 + 3*a^3*b - a^4 - 3*a^2*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.462 $\int \frac{\sec^7(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=167

$$\frac{(4a+b)(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}-\frac{(4a-5b)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{(2a-b)(a-b)\sin(c+dx)}{2ab^2d\left(a-(a-b)\sin^2(c+dx)\right)}+\frac{}{2b}$$

```
[Out] -1/2*(4*a-5*b)*arctanh(sin(d*x+c))/b^3/d+1/2*(a-b)^(3/2)*(4*a+b)*arctanh(si
n(d*x+c)*(a-b)^(1/2)/a^(1/2))/a^(3/2)/b^3/d+1/2*(a-b)*(2*a-b)*sin(d*x+c)/a/
b^2/d/(a-(a-b)*sin(d*x+c)^2)+1/2*sec(d*x+c)*tan(d*x+c)/b/d/(a-(a-b)*sin(d*x
+c)^2)
```

Rubi [A] time = 0.27, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3676, 414, 527, 522, 206, 208}

$$\frac{(4a+b)(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}+\frac{(2a-b)(a-b)\sin(c+dx)}{2ab^2d\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{(4a-5b)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] -((4*a - 5*b)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) + ((a - b)^(3/2)*(4*a + b)*A
rcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + ((a - b)*(2
*a - b)*Sin[c + d*x])/(2*a*b^2*d*(a - (a - b)*Sin[c + d*x]^2)) + (Sec[c + d
*x]*Tan[c + d*x])/(2*b*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a-(a-b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\sec(c+dx)\tan(c+dx)}{2bd\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-a+2b-3(a-b)x^2}{\left(1-x^2\right)\left(a+(-a+b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{2bd}$$

$$= \frac{(a-b)(2a-b)\sin(c+dx)}{2ab^2d\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{\sec(c+dx)\tan(c+dx)}{2bd\left(a-(a-b)\sin^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{2\left(2a^2-\right.}{\left(\right.}\right.}{}$$

$$= \frac{(a-b)(2a-b)\sin(c+dx)}{2ab^2d\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{\sec(c+dx)\tan(c+dx)}{2bd\left(a-(a-b)\sin^2(c+dx)\right)} - \frac{(4a-5b)\,\text{Subst}}{}$$

$$= -\frac{(4a-5b)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(a-b)^{3/2}(4a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{(a}{2ab^2}$$

Mathematica [A] time = 4.08, size = 254, normalized size = 1.52

$$-\frac{(4a+b)(a-b)^{3/2}\log\left(\sqrt{a}-\sqrt{a-b}\sin(c+dx)\right)}{a^{3/2}} + \frac{(4a+b)(a-b)^{3/2}\log\left(\sqrt{a-b}\sin(c+dx)+\sqrt{a}\right)}{a^{3/2}} + \frac{4b(a-b)^2\sin(c+dx)}{a((a-b)\cos(2(c+dx))+a+b)} + 2(4a-5b)\log\left(\text{co}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (2*(4*a - 5*b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(-4*a + 5*b)*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - ((a - b)^(3/2)*(4*a + b)*Log[Sqrt[
a] - Sqrt[a - b]*Sin[c + d*x]])/a^(3/2) + ((a - b)^(3/2)*(4*a + b)*Log[Sqrt
[a] + Sqrt[a - b]*Sin[c + d*x]])/a^(3/2) + b/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^2 - b/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*(a - b)^2*b*Sin[
c + d*x])/(a*(a + b + (a - b)*Cos[2*(c + d*x)])))/(4*b^3*d)
```

fricas [A] time = 0.61, size = 635, normalized size = 3.80

$$\left[-\frac{\left(\left(4\,a^{3}-7\,a^{2}b+2\,ab^{2}+b^{3}\right)\cos\left(dx+c\right)^{4}+\left(4\,a^{2}b-3\,ab^{2}-b^{3}\right)\cos\left(dx+c\right)^{2}\right)\sqrt{\frac{a-b}{a}}\,\log\left(-\frac{(a-b)\cos(dx+c)^{2}+2\,a\sqrt{\cdots}}{(a-b)\cos(\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(((4*a^3 - 7*a^2*b + 2*a*b^2 + b^3)*cos(d*x + c)^4 + (4*a^2*b - 3*a*b
^2 - b^3)*cos(d*x + c)^2)*sqrt((a - b)/a)*log(-((a - b)*cos(d*x + c)^2 + 2*
a*sqrt((a - b)/a)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + (
(4*a^3 - 9*a^2*b + 5*a*b^2)*cos(d*x + c)^4 + (4*a^2*b - 5*a*b^2)*cos(d*x +
c)^2)*log(sin(d*x + c) + 1) - ((4*a^3 - 9*a^2*b + 5*a*b^2)*cos(d*x + c)^4 +
 (4*a^2*b - 5*a*b^2)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a*b^2 + (2
*a^2*b - 3*a*b^2 + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)
^2 + (a^2*b^3 - a*b^4)*d*cos(d*x + c)^4), -1/4*(2*((4*a^3 - 7*a^2*b + 2*a*b
^2 + b^3)*cos(d*x + c)^4 + (4*a^2*b - 3*a*b^2 - b^3)*cos(d*x + c)^2)*sqrt(-
(a - b)/a)*arctan(sqrt(-(a - b)/a)*sin(d*x + c)) + ((4*a^3 - 9*a^2*b + 5*a*
b^2)*cos(d*x + c)^4 + (4*a^2*b - 5*a*b^2)*cos(d*x + c)^2)*log(sin(d*x + c)
+ 1) - ((4*a^3 - 9*a^2*b + 5*a*b^2)*cos(d*x + c)^4 + (4*a^2*b - 5*a*b^2)*co
s(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a*b^2 + (2*a^2*b - 3*a*b^2 + b^3)
*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)^2 + (a^2*b^3 - a*b^4)*
d*cos(d*x + c)^4)]
```

giac [A] time = 2.26, size = 245, normalized size = 1.47

$$-\frac{\frac{(4\,a-5\,b)\log(|\sin(dx+c)+1|)}{b^{3}}-\frac{(4\,a-5\,b)\log(|\sin(dx+c)-1|)}{b^{3}}-\frac{2\left(4\,a^{3}-7\,a^{2}b+2\,ab^{2}+b^{3}\right)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^{2}+ab}}\right)}{\sqrt{-a^{2}+ab}\,ab^{3}}+\frac{2\left(2\,a^{2}\sin(dx+c)^{3}-3\cdots\right.}{\left(a\sin\cdots\right.}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/4*((4*a - 5*b)*log(abs(sin(d*x + c) + 1))/b^3 - (4*a - 5*b)*log(abs(sin(
d*x + c) - 1))/b^3 - 2*(4*a^3 - 7*a^2*b + 2*a*b^2 + b^3)*arctan(-(a*sin(d*x
 + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a*b^3) + 2*(2*a
^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c)^3 + b^2*sin(d*x + c)^3 - 2*a^2*sin(d
*x + c) + 2*a*b*sin(d*x + c) - b^2*sin(d*x + c))/((a*sin(d*x + c)^4 - b*sin
(d*x + c)^4 - 2*a*sin(d*x + c)^2 + b*sin(d*x + c)^2 + a)*a*b^2))/d
```

maple [B] time = 0.74, size = 389, normalized size = 2.33

$$-\frac{a\sin\left(dx+c\right)}{2d\,b^{2}\left(a\left(\sin^{2}\left(dx+c\right)\right)-b\left(\sin^{2}\left(dx+c\right)\right)-a\right)}+\frac{\sin\left(dx+c\right)}{db\left(a\left(\sin^{2}\left(dx+c\right)\right)-b\left(\sin^{2}\left(dx+c\right)\right)-a\right)}-\frac{\cdots}{2da\left(a\left(\sin^{2}\left(\cdots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/b^2*a*sin(d*x+c)/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)+1/d/b*sin(d*x+c)/
(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)-1/2/d/a*sin(d*x+c)/(a*sin(d*x+c)^2-b*sin(
d*x+c)^2-a)+2/d/b^3/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2
))*a^2-7/2/d/b^2/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))*
a+1/d/b/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))+1/2/d/a/(
a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))-1/4/d/b^2/(-1+sin(
```

```
d*x+c))+1/d/b^3*ln(-1+sin(d*x+c))*a-5/4/d/b^2*ln(-1+sin(d*x+c))-1/4/d/b^2/(
sin(d*x+c)+1)-1/d/b^3*ln(sin(d*x+c)+1)*a+5/4/d/b^2*ln(sin(d*x+c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 15.24, size = 4304, normalized size = 25.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + b*tan(c + d*x)^2)^2),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(2*a^2 - 2*a*b + b^2))/(a*b^2) - (tan(c/2 + (d*x)/2)^3
*(2*a^2 - 6*a*b + b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^7*(2*a^2 - 2*a*b + b^
2))/(a*b^2) - (tan(c/2 + (d*x)/2)^5*(2*a^2 - 6*a*b + b^2))/(a*b^2))/(d*(a -
 tan(c/2 + (d*x)/2)^2*(4*a - 4*b) - tan(c/2 + (d*x)/2)^6*(4*a - 4*b) + tan(
c/2 + (d*x)/2)^4*(6*a - 8*b) + a*tan(c/2 + (d*x)/2)^8)) - (atan((((4*a - 5*
b)*((((256*(16*a*b^15 + 92*a^2*b^14 - 8*a^3*b^13 - 2236*a^4*b^12 + 768*a^5*
b^11 + 18228*a^6*b^10 - 41560*a^7*b^9 + 37420*a^8*b^8 - 13552*a^9*b^7 + 64*
a^10*b^6 + 768*a^11*b^5))/(a^3*b^10) + (((((256*(256*a^4*b^16 + 192*a^5*b^1
5 - 1088*a^6*b^14 - 192*a^7*b^13 + 1600*a^8*b^12 - 768*a^9*b^11))/(a^3*b^10
) - (256*tan(c/2 + (d*x)/2)*(4*a - 5*b)*(1024*a^5*b^15 - 2304*a^6*b^14 + 16
64*a^7*b^13 - 384*a^8*b^12))/(a^3*b^11))*(4*a - 5*b))/(2*b^3) - (512*tan(c/
2 + (d*x)/2)*(64*a^2*b^14 + 160*a^3*b^13 - 984*a^4*b^12 - 6560*a^5*b^11 + 2
8720*a^6*b^10 - 42400*a^7*b^9 + 29512*a^8*b^8 - 9664*a^9*b^7 + 1152*a^10*b^
6))/(a^3*b^8))*(4*a - 5*b))/(2*b^3))*(4*a - 5*b))/(2*b^3) + (512*tan(c/2 +
(d*x)/2)*(8*a*b^11 - 8960*a^11*b + 768*a^12 + b^12 + 396*a^2*b^10 + 440*a^3
*b^9 - 7144*a^4*b^8 + 6656*a^5*b^7 + 34712*a^6*b^6 - 106784*a^7*b^5 + 13867
5*a^8*b^4 - 100016*a^9*b^3 + 41248*a^10*b^2))/(a^3*b^8))*1i)/(2*b^3) - ((4*
a - 5*b)*((((256*(16*a*b^15 + 92*a^2*b^14 - 8*a^3*b^13 - 2236*a^4*b^12 + 76
8*a^5*b^11 + 18228*a^6*b^10 - 41560*a^7*b^9 + 37420*a^8*b^8 - 13552*a^9*b^7
 + 64*a^10*b^6 + 768*a^11*b^5))/(a^3*b^10) + (((((256*(256*a^4*b^16 + 192*a
^5*b^15 - 1088*a^6*b^14 - 192*a^7*b^13 + 1600*a^8*b^12 - 768*a^9*b^11))/(a^
3*b^10) + (256*tan(c/2 + (d*x)/2)*(4*a - 5*b)*(1024*a^5*b^15 - 2304*a^6*b^1
4 + 1664*a^7*b^13 - 384*a^8*b^12))/(a^3*b^11))*(4*a - 5*b))/(2*b^3) + (512*
tan(c/2 + (d*x)/2)*(64*a^2*b^14 + 160*a^3*b^13 - 984*a^4*b^12 - 6560*a^5*b^
11 + 28720*a^6*b^10 - 42400*a^7*b^9 + 29512*a^8*b^8 - 9664*a^9*b^7 + 1152*a
^10*b^6))/(a^3*b^8))*(4*a - 5*b))/(2*b^3))*(4*a - 5*b))/(2*b^3) - (512*tan(
c/2 + (d*x)/2)*(8*a*b^11 - 8960*a^11*b + 768*a^12 + b^12 + 396*a^2*b^10 + 4
40*a^3*b^9 - 7144*a^4*b^8 + 6656*a^5*b^7 + 34712*a^6*b^6 - 106784*a^7*b^5 +
 138675*a^8*b^4 - 100016*a^9*b^3 + 41248*a^10*b^2))/(a^3*b^8))*1i)/(2*b^3))
/(((4*a - 5*b)*((((256*(16*a*b^15 + 92*a^2*b^14 - 8*a^3*b^13 - 2236*a^4*b^1
2 + 768*a^5*b^11 + 18228*a^6*b^10 - 41560*a^7*b^9 + 37420*a^8*b^8 - 13552*a
^9*b^7 + 64*a^10*b^6 + 768*a^11*b^5))/(a^3*b^10) + (((((256*(256*a^4*b^16 +
 192*a^5*b^15 - 1088*a^6*b^14 - 192*a^7*b^13 + 1600*a^8*b^12 - 768*a^9*b^11
))/(a^3*b^10) - (256*tan(c/2 + (d*x)/2)*(4*a - 5*b)*(1024*a^5*b^15 - 2304*a
^6*b^14 + 1664*a^7*b^13 - 384*a^8*b^12))/(a^3*b^11))*(4*a - 5*b))/(2*b^3) -
 (512*tan(c/2 + (d*x)/2)*(64*a^2*b^14 + 160*a^3*b^13 - 984*a^4*b^12 - 6560*
a^5*b^11 + 28720*a^6*b^10 - 42400*a^7*b^9 + 29512*a^8*b^8 - 9664*a^9*b^7 +
1152*a^10*b^6))/(a^3*b^8))*(4*a - 5*b))/(2*b^3))*(4*a - 5*b))/(2*b^3) + (51
```

2*tan(c/2 + (d*x)/2)*(8*a*b^11 - 8960*a^11*b + 768*a^12 + b^12 + 396*a^2*b^10 + 440*a^3*b^9 - 7144*a^4*b^8 + 6656*a^5*b^7 + 34712*a^6*b^6 - 106784*a^7*b^5 + 138675*a^8*b^4 - 100016*a^9*b^3 + 41248*a^10*b^2))/(a^3*b^8)))/(2*b^3) - (512*(64*a*b^11 - 27904*a^11*b + 3584*a^12 - 5*b^12 + 467*a^2*b^10 - 1322*a^3*b^9 - 4957*a^4*b^8 + 18148*a^5*b^7 + 165*a^6*b^6 - 81226*a^7*b^5 + 165322*a^8*b^4 - 164368*a^9*b^3 + 92032*a^10*b^2))/(a^3*b^10) + ((4*a - 5*b)*((((256*(16*a*b^15 + 92*a^2*b^14 - 8*a^3*b^13 - 2236*a^4*b^12 + 768*a^5*b^11 + 18228*a^6*b^10 - 41560*a^7*b^9 + 37420*a^8*b^8 - 13552*a^9*b^7 + 64*a^10*b^6 + 768*a^11*b^5))/(a^3*b^10) + (((((256*(256*a^4*b^16 + 192*a^5*b^15 - 1088*a^6*b^14 - 192*a^7*b^13 + 1600*a^8*b^12 - 768*a^9*b^11))/(a^3*b^10) + (256*tan(c/2 + (d*x)/2)*(4*a - 5*b)*(1024*a^5*b^15 - 2304*a^6*b^14 + 1664*a^7*b^13 - 384*a^8*b^12))/(a^3*b^11))*(4*a - 5*b))/(2*b^3) + (512*tan(c/2 + (d*x)/2)*(64*a^2*b^14 + 160*a^3*b^13 - 984*a^4*b^12 - 6560*a^5*b^11 + 28720*a^6*b^10 - 42400*a^7*b^9 + 29512*a^8*b^8 - 9664*a^9*b^7 + 1152*a^10*b^6))/(a^3*b^8))*(4*a - 5*b))/(2*b^3))*(4*a - 5*b))/(2*b^3) - (512*tan(c/2 + (d*x)/2)*(8*a*b^11 - 8960*a^11*b + 768*a^12 + b^12 + 396*a^2*b^10 + 440*a^3*b^9 - 7144*a^4*b^8 + 6656*a^5*b^7 + 34712*a^6*b^6 - 106784*a^7*b^5 + 138675*a^8*b^4 - 100016*a^9*b^3 + 41248*a^10*b^2))/(a^3*b^8)))/(2*b^3)))*(4*a - 5*b)*1i)/(b^3*d) + (atan((((a - b)^(3/2)*(4*a + b)*((128*tan(c/2 + (d*x)/2)*(26*a*b^8 - 1232*a^8*b + 192*a^9 + b^9 + 40*a^2*b^7 - 376*a^3*b^6 + 44*a^4*b^5 + 1964*a^5*b^4 - 3767*a^6*b^3 + 3108*a^7*b^2))/(a^2*b^6) - ((a - b)^(3/2)*(4*a + b)*((16*(80*a^3*b^11 - 52*a^4*b^10 - 912*a^5*b^9 + 488*a^6*b^8 + 3008*a^7*b^7 - 4836*a^8*b^6 + 2784*a^9*b^5 - 576*a^10*b^4))/(a^3*b^8) + (16*tan(c/2 + (d*x)/2)^2*(128*a^2*b^12 + 368*a^3*b^11 - 1164*a^4*b^10 - 7344*a^5*b^9 + 24216*a^6*b^8 - 27808*a^7*b^7 + 15332*a^8*b^6 - 4320*a^9*b^5 + 576*a^10*b^4))/(a^3*b^8) - (((128*tan(c/2 + (d*x)/2)*(16*a^3*b^10 + 340*a^4*b^9 - 952*a^5*b^8 + 836*a^6*b^7 - 240*a^7*b^6))/(a^2*b^6) - (((16*(1024*a^6*b^12 - 1536*a^7*b^11 + 576*a^8*b^10))/(a^3*b^8) + (16*tan(c/2 + (d*x)/2)^2*(2048*a^5*b^13 - 4096*a^6*b^12 + 2688*a^7*b^11 - 576*a^8*b^10))/(a^3*b^8))*(a - b)^(3/2)*(4*a + b))/(4*a^(3/2)*b^3))*(a - b)^(3/2)*(4*a + b))/(4*a^(3/2)*b^3)))/(4*a^(3/2)*b^3))*1i)/(4*a^(3/2)*b^3) + ((a - b)^(3/2)*(4*a + b)*((128*tan(c/2 + (d*x)/2)*(26*a*b^8 - 1232*a^8*b + 192*a^9 + b^9 + 40*a^2*b^7 - 376*a^3*b^6 + 44*a^4*b^5 + 1964*a^5*b^4 - 3767*a^6*b^3 + 3108*a^7*b^2))/(a^2*b^6) + ((a - b)^(3/2)*(4*a + b)*((16*(80*a^3*b^11 - 52*a^4*b^10 - 912*a^5*b^9 + 488*a^6*b^8 + 3008*a^7*b^7 - 4836*a^8*b^6 + 2784*a^9*b^5 - 576*a^10*b^4))/(a^3*b^8) + (16*tan(c/2 + (d*x)/2)^2*(128*a^2*b^12 + 368*a^3*b^11 - 1164*a^4*b^10 - 7344*a^5*b^9 + 24216*a^6*b^8 - 27808*a^7*b^7 + 15332*a^8*b^6 - 4320*a^9*b^5 + 576*a^10*b^4))/(a^3*b^8) + (((128*tan(c/2 + (d*x)/2)*(16*a^3*b^10 + 340*a^4*b^9 - 952*a^5*b^8 + 836*a^6*b^7 - 240*a^7*b^6))/(a^2*b^6) + (((16*(1024*a^6*b^12 - 1536*a^7*b^11 + 576*a^8*b^10))/(a^3*b^8) + (16*tan(c/2 + (d*x)/2)^2*(2048*a^5*b^13 - 4096*a^6*b^12 + 2688*a^7*b^11 - 576*a^8*b^10))/(a^3*b^8))*(a - b)^(3/2)*(4*a + b))/(4*a^(3/2)*b^3))*(a - b)^(3/2)*(4*a + b))/(4*a^(3/2)*b^3)))/(4*a^(3/2)*b^3))*1i)/(4*a^(3/2)*b^3))/((32*(12*a*b^9 - 2592*a^9*b + 448*a^10 - b^10 + 98*a^2*b^8 - 108*a^3*b^7 - 853*a^4*b^6 + 1368*a^5*b^5 + 1648*a^6*b^4 - 5808*a^7*b^3 + 5788*a^8*b^2))/(a^3*b^8) + (32*tan(c/2 + (d*x)/2)^2*(3136*a^10 - 19040*a^9*b - 32*a*b^9 + b^10 + 198*a^2*b^8 + 1244*a^3*b^7 - 4555*a^4*b^6 - 3612*a^5*b^5 + 33032*a^6*b^4 - 56968*a^7*b^3 + 46596*a^8*b^2))/(a^3*b^8) + ((a - b)^(3/2)*(4*a + b)*((128*tan(c/2 + (d*x)/2)*(26*a*b^8 - 1232*a^8*b + 192*a^9 + b^9 + 40*a^2*b^7 - 376*a^3*b^6 + 44*a^4*b^5 + 1964*a^5*b^4 - 3767*a^6*b^3 + 3108*a^7*b^2))/(a^2*b^6) - ((a - b)^(3/2)*(4*a + b)*((16*(80*a^3*b^11 - 52*a^4*b^10 - 912*a^5*b^9 + 488*a^6*b^8 + 3008*a^7*b^7 - 4836*a^8*b^6 + 2784*a^9*b^5 - 576*a^10*b^4))/(a^3*b^8) + (16*tan(c/2 + (d*x)/2)^2*(128*a^2*b^12 + 368*a^3*b^11 - 1164*a^4*b^10 - 7344*a^5*b^9 + 24216*a^6*b^8 - 27808*a^7*b^7 + 15332*a^8*b^6 - 4320*a^9*b^5 + 576*a^10*b^4))/(a^3*b^8) - (((128*tan(c/2 + (d*x)/2)*(16*a^3*b^10 + 340*a^4*b^9 - 952*a^5*b^8 + 836*a^6*b^7 - 240*a^7*b^6))/(a^2*b^6) - (((16*(1024*a^6*b^12 - 1536*a^7*b^11 + 576*a^8*b^10))/(a^3*b^8) + (16*tan(c/2 + (d*x)/2)^2*(2048*a^5*b^13 - 4096*a^6*b^12 + 2688*a^7*b^11 - 576*a^8*b^10))/(a^3*b^8))*(a - b)^(3/2)*(4*a + b))/(4*a^(3/2)*b^3))*(a - b)^(3/2)*

```
(4*a + b))/(4*a^(3/2)*b^3)))/(4*a^(3/2)*b^3)))/(4*a^(3/2)*b^3) - ((a - b)^(
3/2)*(4*a + b)*((128*tan(c/2 + (d*x)/2)*(26*a*b^8 - 1232*a^8*b + 192*a^9 +
b^9 + 40*a^2*b^7 - 376*a^3*b^6 + 44*a^4*b^5 + 1964*a^5*b^4 - 3767*a^6*b^3 +
 3108*a^7*b^2))/(a^2*b^6) + ((a - b)^(3/2)*(4*a + b)*((16*(80*a^3*b^11 - 52
*a^4*b^10 - 912*a^5*b^9 + 488*a^6*b^8 + 3008*a^7*b^7 - 4836*a^8*b^6 + 2784*
a^9*b^5 - 576*a^10*b^4))/(a^3*b^8) + (16*tan(c/2 + (d*x)/2)^2*(128*a^2*b^12
 + 368*a^3*b^11 - 1164*a^4*b^10 - 7344*a^5*b^9 + 24216*a^6*b^8 - 27808*a^7*
b^7 + 15332*a^8*b^6 - 4320*a^9*b^5 + 576*a^10*b^4))/(a^3*b^8) + (((128*tan(
c/2 + (d*x)/2)*(16*a^3*b^10 + 340*a^4*b^9 - 952*a^5*b^8 + 836*a^6*b^7 - 240
*a^7*b^6))/(a^2*b^6) + (((16*(1024*a^6*b^12 - 1536*a^7*b^11 + 576*a^8*b^10)
)/(a^3*b^8) + (16*tan(c/2 + (d*x)/2)^2*(2048*a^5*b^13 - 4096*a^6*b^12 + 268
8*a^7*b^11 - 576*a^8*b^10))/(a^3*b^8))*(a - b)^(3/2)*(4*a + b))/(4*a^(3/2)*
b^3))*(a - b)^(3/2)*(4*a + b))/(4*a^(3/2)*b^3)))/(4*a^(3/2)*b^3)))/(4*a^(3/
2)*b^3)))*(a - b)^(3/2)*(4*a + b)*1i)/(2*a^(3/2)*b^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.463 $\int \frac{\sec^5(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}-\frac{(a-b)\sin(c+dx)}{2abd\left(a-(a-b)\sin^2(c+dx)\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

```
[Out] arctanh(sin(d*x+c))/b^2/d-1/2*(a-b)*sin(d*x+c)/a/b/d/(a-(a-b)*sin(d*x+c)^2)
-1/2*(2*a+b)*arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))*(a-b)^(1/2)/a^(3/2)/b^
2/d
```

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3676, 414, 522, 206, 208}

$$-\frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}-\frac{(a-b)\sin(c+dx)}{2abd\left(a-(a-b)\sin^2(c+dx)\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) - (Sqrt[a - b]*(2*a + b)*ArcTanh[(Sqrt[a - b]
*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^2*d) - ((a - b)*Sin[c + d*x])/(2*a*b*
d*(a - (a - b)*Sin[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a-(a-b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\sin(c+dx)}{2abd\left(a-(a-b)\sin^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-a-b+(-a+b)x^2}{(1-x^2)\left(a+(-a+b)x^2\right)}\,dx, x, \sin(c+dx)\right)}{2abd}$$

$$= -\frac{(a-b)\sin(c+dx)}{2abd\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{b^2d} - \frac{((a-b)(2a\ldots}{}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} - \frac{(a-b)\sin(c\ldots}{2abd\left(a-(a-b)\,\text{s}\ldots\right)}$$

Mathematica [A] time = 0.86, size = 191, normalized size = 1.75

$$\frac{\frac{\sqrt{a-b}\,(2a+b)\log\left(\sqrt{a}-\sqrt{a-b}\,\sin(c+dx)\right)}{a^{3/2}} + \frac{\left(-2a^2+ab+b^2\right)\log\left(\sqrt{a-b}\,\sin(c+dx)+\sqrt{a}\right)}{a^{3/2}\sqrt{a-b}} + \frac{4b(b-a)\sin(c+dx)}{a((a-b)\cos(2(c+dx))+a+b)} - 4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\ldots\right.}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (-4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] + (Sqrt[a - b]*(2*a + b)*Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d
*x]])/a^(3/2) + ((-2*a^2 + a*b + b^2)*Log[Sqrt[a] + Sqrt[a - b]*Sin[c + d*x
]])/(a^(3/2)*Sqrt[a - b]) + (4*b*(-a + b)*Sin[c + d*x])/(a*(a + b + (a - b)
*Cos[2*(c + d*x)])))/(4*b^2*d)
```

fricas [A] time = 0.66, size = 407, normalized size = 3.73

$$\left[\frac{\left(\left(2a^2-ab-b^2\right)\cos(dx+c)^2+2ab+b^2\right)\sqrt{\frac{a-b}{a}}\log\left(-\frac{(a-b)\cos(dx+c)^2+2a\sqrt{\frac{a-b}{a}}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)+2\left(\left(a^2-ab\right)\cos(d\ldots\right.}{4\left(ab^3d+\left(a^2b^2\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a^2 - a*b - b^2)*cos(d*x + c)^2 + 2*a*b + b^2)*sqrt((a - b)/a)*lo
g(-((a - b)*cos(d*x + c)^2 + 2*a*sqrt((a - b)/a)*sin(d*x + c) - 2*a + b)/((
a - b)*cos(d*x + c)^2 + b)) + 2*((a^2 - a*b)*cos(d*x + c)^2 + a*b)*log(sin(
d*x + c) + 1) - 2*((a^2 - a*b)*cos(d*x + c)^2 + a*b)*log(-sin(d*x + c) + 1)
 - 2*(a*b - b^2)*sin(d*x + c))/(a*b^3*d + (a^2*b^2 - a*b^3)*d*cos(d*x + c)^
2), 1/2*(((2*a^2 - a*b - b^2)*cos(d*x + c)^2 + 2*a*b + b^2)*sqrt(-(a - b)/a
```

```
)*arctan(sqrt(-(a - b)/a)*sin(d*x + c)) + ((a^2 - a*b)*cos(d*x + c)^2 + a*b
)*log(sin(d*x + c) + 1) - ((a^2 - a*b)*cos(d*x + c)^2 + a*b)*log(-sin(d*x +
 c) + 1) - (a*b - b^2)*sin(d*x + c))/(a*b^3*d + (a^2*b^2 - a*b^3)*d*cos(d*x
 + c)^2)]
```

giac [A] time = 2.11, size = 153, normalized size = 1.40

$$\frac{\frac{\log(|\sin(dx+c)+1|)}{b^2} - \frac{\log(|\sin(dx+c)-1|)}{b^2} - \frac{\left(2a^2-ab-b^2\right)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,ab^2} + \frac{a\sin(dx+c)-b\sin(dx+c)}{\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)ab}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(sin(d*x + c) + 1))/b^2 - log(abs(sin(d*x + c) - 1))/b^2 - (2*a
^2 - a*b - b^2)*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))
/(sqrt(-a^2 + a*b)*a*b^2) + (a*sin(d*x + c) - b*sin(d*x + c))/((a*sin(d*x +
 c)^2 - b*sin(d*x + c)^2 - a)*a*b))/d
```

maple [B] time = 0.77, size = 236, normalized size = 2.17

$$\frac{\sin(dx+c)}{2db\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)} - \frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)a}{d\,b^2\sqrt{a(a-b)}} + \frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2db\sqrt{a(a-b)}} - \frac{}{2da\left(a\left(\sin\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d/b*sin(d*x+c)/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)-1/d/b^2/(a*(a-b))^(1/2
)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))*a+1/2/d/b/(a*(a-b))^(1/2)*arcta
nh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))-1/2/d/a*sin(d*x+c)/(a*sin(d*x+c)^2-b*s
in(d*x+c)^2-a)+1/2/d/a/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(
1/2))-1/2/d/b^2*ln(-1+sin(d*x+c))+1/2/d/b^2*ln(sin(d*x+c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 14.37, size = 946, normalized size = 8.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x)^2)^2),x)
```

```
[Out] ((a^2*atan((sin(c/2 + (d*x)/2)*(a - 2*b + 2*a*cos(c + d*x) - 2*b*cos(c + d*
x)))/(2*a^(1/2)*cos(c/2 + (d*x)/2)^3*(b - a)^(1/2)))*(b - a)^(1/2)*1i - a^(
5/2)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*2i + (b^2*atan((sin(c/2 +
 (d*x)/2)*(a - 2*b + 2*a*cos(c + d*x) - 2*b*cos(c + d*x)))/(2*a^(1/2)*cos(c
/2 + (d*x)/2)^3*(b - a)^(1/2)))*(b - a)^(1/2)*1i)/2 - a^(3/2)*b*atanh(sin(c
```

```
/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*2i - a^(1/2)*b^2*sin(c + d*x)*1i + a^2*at
an((a^(1/2)*sin(c/2 + (d*x)/2))/(2*cos(c/2 + (d*x)/2)*(b - a)^(1/2)))*(b -
a)^(1/2)*1i + (b^2*atan((a^(1/2)*sin(c/2 + (d*x)/2))/(2*cos(c/2 + (d*x)/2)*
(b - a)^(1/2)))*(b - a)^(1/2)*1i)/2 - a^(5/2)*atanh(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2))*cos(2*c + 2*d*x)*2i + a^(3/2)*b*sin(c + d*x)*1i + a^2*atan(
(a^(1/2)*sin(c/2 + (d*x)/2))/(2*cos(c/2 + (d*x)/2)*(b - a)^(1/2)))*cos(2*c
+ 2*d*x)*(b - a)^(1/2)*1i - (b^2*atan((a^(1/2)*sin(c/2 + (d*x)/2))/(2*cos(c
/2 + (d*x)/2)*(b - a)^(1/2)))*cos(2*c + 2*d*x)*(b - a)^(1/2)*1i)/2 + (a*b*a
tan((sin(c/2 + (d*x)/2)*(a - 2*b + 2*a*cos(c + d*x) - 2*b*cos(c + d*x)))/(2
*a^(1/2)*cos(c/2 + (d*x)/2)^3*(b - a)^(1/2)))*(b - a)^(1/2)*3i)/2 + (a*b*at
an((a^(1/2)*sin(c/2 + (d*x)/2))/(2*cos(c/2 + (d*x)/2)*(b - a)^(1/2)))*(b -
a)^(1/2)*3i)/2 + a^2*atan((sin(c/2 + (d*x)/2)*(a - 2*b + 2*a*cos(c + d*x) -
 2*b*cos(c + d*x)))/(2*a^(1/2)*cos(c/2 + (d*x)/2)^3*(b - a)^(1/2)))*cos(2*c
 + 2*d*x)*(b - a)^(1/2)*1i - (b^2*atan((sin(c/2 + (d*x)/2)*(a - 2*b + 2*a*c
os(c + d*x) - 2*b*cos(c + d*x)))/(2*a^(1/2)*cos(c/2 + (d*x)/2)^3*(b - a)^(1
/2)))*cos(2*c + 2*d*x)*(b - a)^(1/2)*1i)/2 + a^(3/2)*b*atanh(sin(c/2 + (d*x
)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*2i - (a*b*atan((a^(1/2)*sin(c/2 +
 (d*x)/2))/(2*cos(c/2 + (d*x)/2)*(b - a)^(1/2)))*cos(2*c + 2*d*x)*(b - a)^(
1/2)*1i)/2 - (a*b*atan((sin(c/2 + (d*x)/2)*(a - 2*b + 2*a*cos(c + d*x) - 2*
b*cos(c + d*x)))/(2*a^(1/2)*cos(c/2 + (d*x)/2)^3*(b - a)^(1/2)))*cos(2*c +
2*d*x)*(b - a)^(1/2)*1i)/2)*1i)/(2*a^(3/2)*b^2*d*(a/2 + b/2 + (a*cos(2*c +
2*d*x))/2 - (b*cos(2*c + 2*d*x))/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*tan(c + d*x)**2)**2, x)
```

3.464 $\int \frac{\sec^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a-b}}+\frac{\sin(c+dx)}{2ad\left(a-(a-b)\sin^2(c+dx)\right)}$$

```
[Out] 1/2*sin(d*x+c)/a/d/(a-(a-b)*sin(d*x+c)^2)+1/2*arctanh(sin(d*x+c)*(a-b)^(1/2
)/a^(1/2))/a^(3/2)/d/(a-b)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3676, 199, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a-b}}+\frac{\sin(c+dx)}{2ad\left(a-(a-b)\sin^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a - b]*d) + Sin
[c + d*x]/(2*a*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a-(a-b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\sin(c+dx)}{2ad\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \sin(c+dx)\right)}{2ad}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}\,d} + \frac{\sin(c+dx)}{2ad\left(a-(a-b)\sin^2(c+dx)\right)}$$

Mathematica [A] time = 0.25, size = 75, normalized size = 0.95

$$\frac{\frac{\sqrt{a}\sin(c+dx)}{(b-a)\sin^2(c+dx)+a} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/Sqrt[a - b] + (Sqrt[a]*Sin[c +
 d*x])/(a + (-a + b)*Sin[c + d*x]^2))/(2*a^(3/2)*d)
```

fricas [A] time = 0.66, size = 266, normalized size = 3.37

$$\left[\frac{\left((a-b)\cos(dx+c)^2+b\right)\sqrt{a^2-ab}\,\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)+2\left(a^2-ab\right)\sin(dx+c)}{4\left(\left(a^4-2a^3b+a^2b^2\right)d\cos(dx+c)^2+\left(a^3b-a^2b^2\right)d\right)}, -\frac{\left((a-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((a - b)*cos(d*x + c)^2 + b)*sqrt(a^2 - a*b)*log(-((a - b)*cos(d*x +
c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 +
b)) + 2*(a^2 - a*b)*sin(d*x + c))/((a^4 - 2*a^3*b + a^2*b^2)*d*cos(d*x + c)
^2 + (a^3*b - a^2*b^2)*d), -1/2*(((a - b)*cos(d*x + c)^2 + b)*sqrt(-a^2 + a
*b)*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) - (a^2 - a*b)*sin(d*x + c))/((a
^4 - 2*a^3*b + a^2*b^2)*d*cos(d*x + c)^2 + (a^3*b - a^2*b^2)*d)]
```

giac [A] time = 2.33, size = 91, normalized size = 1.15

$$\frac{\frac{\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,a} - \frac{\sin(dx+c)}{\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2
 + a*b)*a) - sin(d*x + c)/((a*sin(d*x + c)^2 - b*sin(d*x + c)^2 - a)*a))/d
```

maple [A] time = 0.68, size = 80, normalized size = 1.01

$$\frac{-\frac{\sin(dx+c)}{2a\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)}+\frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-1/2*sin(d*x+c)/a/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)+1/2/a/(a*(a-b))^(1
/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 12.94, size = 187, normalized size = 2.37

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{a}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+(4b-2a)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}-\frac{\operatorname{atanh}\left(\frac{4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^{3/2}\sqrt{a-b}\left(\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a-b}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a}+\frac{2}{a}-\frac{2}{a-b}+\frac{4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{ab-a^2}\right)}\right)}{2a^{3/2}d\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x)^2)^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3/a + tan(c/2 + (d*x)/2)/a)/(d*(a - tan(c/2 + (d*x)/2)^
2*(2*a - 4*b) + a*tan(c/2 + (d*x)/2)^4)) - atanh((4*b*tan(c/2 + (d*x)/2))/(
a^(3/2)*(a - b)^(1/2)*((2*tan(c/2 + (d*x)/2)^2)/(a - b) - (2*tan(c/2 + (d*x
)/2)^2)/a + 2/a - 2/(a - b) + (4*b*tan(c/2 + (d*x)/2)^2)/(a*b - a^2))))/(2*
a^(3/2)*d*(a - b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x)**2)**2, x)
```

3.465 $\int \frac{\sec(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=94

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}}-\frac{b\sin(c+dx)}{2ad(a-b)\left(a-(a-b)\sin^2(c+dx)\right)}$$

```
[Out] 1/2*(2*a-b)*arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))/a^(3/2)/(a-b)^(3/2)/d-1
/2*b*sin(d*x+c)/a/(a-b)/d/(a-(a-b)*sin(d*x+c)^2)
```

Rubi [A] time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3676, 385, 208}

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}}-\frac{b\sin(c+dx)}{2ad(a-b)\left(a-(a-b)\sin^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((2*a - b)*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^
(3/2)*d) - (b*Sin[c + d*x])/(2*a*(a - b)*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-(a-b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= -\frac{b\sin(c+dx)}{2a(a-b)d\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \sin(c\right.}{2a(a-b)d}$$
$$= \frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{3/2}d} - \frac{b\sin(c+dx)}{2a(a-b)d\left(a-(a-b)\sin^2(c+dx)\right)}$$

Mathematica [A] time = 0.25, size = 92, normalized size = 0.98

$$\frac{\frac{\sqrt{a}\,b\sin(c+dx)}{(a-b)\left((a-b)\sin^2(c+dx)-a\right)} + \frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{(a-b)^{3/2}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (((2*a - b)*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(a - b)^(3/2) + (S
qrt[a]*b*Sin[c + d*x])/((a - b)*(-a + (a - b)*Sin[c + d*x]^2)))/(2*a^(3/2)*
d)
```

fricas [A] time = 0.59, size = 337, normalized size = 3.59

$$\left[\frac{\left(\left(2\,a^2-3\,ab+b^2\right)\cos\left(dx+c\right)^2+2\,ab-b^2\right)\sqrt{a^2-ab}\,\log\left(-\frac{(a-b)\cos(dx+c)^2-2\,\sqrt{a^2-ab}\,\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)-2\left(a^2b-a\right.}{4\left(\left(a^5-3\,a^4b+3\,a^3b^2-a^2b^3\right)d\cos\left(dx+c\right)^2+\left(a^4b-2\,a^3b^2+a^2b^3\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a^2 - 3*a*b + b^2)*cos(d*x + c)^2 + 2*a*b - b^2)*sqrt(a^2 - a*b)*
log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a + b)/((
a - b)*cos(d*x + c)^2 + b)) - 2*(a^2*b - a*b^2)*sin(d*x + c))/((a^5 - 3*a^4
*b + 3*a^3*b^2 - a^2*b^3)*d*cos(d*x + c)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*
d), -1/2*(((2*a^2 - 3*a*b + b^2)*cos(d*x + c)^2 + 2*a*b - b^2)*sqrt(-a^2 +
a*b)*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) + (a^2*b - a*b^2)*sin(d*x + c)
)/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cos(d*x + c)^2 + (a^4*b - 2*a^3*
b^2 + a^2*b^3)*d)]
```

giac [A] time = 1.85, size = 112, normalized size = 1.19

$$-\frac{\frac{(2\,a-b)\arctan\left(\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\left(a^2-ab\right)\sqrt{-a^2+ab}} - \frac{b\sin(dx+c)}{\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)\left(a^2-ab\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((2*a - b)*arctan((a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/
((a^2 - a*b)*sqrt(-a^2 + a*b)) - b*sin(d*x + c)/((a*sin(d*x + c)^2 - b*sin(
d*x + c)^2 - a)*(a^2 - a*b)))/d
```

maple [A] time = 0.59, size = 102, normalized size = 1.09

$$\frac{\frac{b\sin(dx+c)}{2a(a-b)\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)}+\frac{(2a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a(a-b)\sqrt{a(a-b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(1/2*b/a/(a-b)*sin(d*x+c)/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)+1/2*(2*a-b)
/a/(a-b)/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 12.78, size = 239, normalized size = 2.54

$$-\frac{\left(a^2\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,1\mathrm{i}-\frac{b^2\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,1\mathrm{i}}{2}+a^2\,\cos\left(2\,c+2\,d\,x\right)\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,1\mathrm{i}+\frac{b^2\,\cos(2\,c+2}{}\right.}{2\,a^{3/2}\,d\,(a-b)^{3/2}\,\left(\frac{a}{2}+\frac{b}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*tan(c + d*x)^2)^2),x)
```

```
[Out] -((a^2*atanh((sin(c + d*x)*(a - b)^(1/2))/a^(1/2))*1i - (b^2*atanh((sin(c +
 d*x)*(a - b)^(1/2))/a^(1/2))*1i)/2 + a^2*cos(2*c + 2*d*x)*atanh((sin(c + d
*x)*(a - b)^(1/2))/a^(1/2))*1i + (b^2*cos(2*c + 2*d*x)*atanh((sin(c + d*x)*
(a - b)^(1/2))/a^(1/2))*1i)/2 + (a*b*atanh((sin(c + d*x)*(a - b)^(1/2))/a^(
1/2))*1i)/2 - (a*b*cos(2*c + 2*d*x)*atanh((sin(c + d*x)*(a - b)^(1/2))/a^(1
/2))*3i)/2 - a^(1/2)*b*sin(c + d*x)*(a - b)^(1/2)*1i)*1i)/(2*a^(3/2)*d*(a -
 b)^(3/2)*(a/2 + b/2 + (a*cos(2*c + 2*d*x))/2 - (b*cos(2*c + 2*d*x))/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\left(a+b\tan^2\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x)**2)**2, x)
```

3.466 $\int \frac{\cos(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=114

$$-\frac{b(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{5/2}}+\frac{b^2\sin(c+dx)}{2ad(a-b)^2\left(a-(a-b)\sin^2(c+dx)\right)}+\frac{\sin(c+dx)}{d(a-b)^2}$$

```
[Out] -1/2*(4*a-b)*b*arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))/a^(3/2)/(a-b)^(5/2)/
d+sin(d*x+c)/(a-b)^2/d+1/2*b^2*sin(d*x+c)/a/(a-b)^2/d/(a-(a-b)*sin(d*x+c)^2
)
```

Rubi [A] time = 0.18, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3676, 390, 385, 208}

$$-\frac{b(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{5/2}}+\frac{b^2\sin(c+dx)}{2ad(a-b)^2\left(a-(a-b)\sin^2(c+dx)\right)}+\frac{\sin(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] -((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a -
b)^(5/2)*d) + Sin[c + d*x]/((a - b)^2*d) + (b^2*Sin[c + d*x])/(2*a*(a - b)^
2*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-(a-b)x^2\right)^2}dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^2} - \frac{(2a-b)b-2(a-b)bx^2}{(a-b)^2\left(a+(-a+b)x^2\right)^2}\right)dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\sin(c+dx)}{(a-b)^2 d} - \frac{\text{Subst}\left(\int \frac{(2a-b)b-2(a-b)bx^2}{\left(a+(-a+b)x^2\right)^2}dx, x, \sin(c+dx)\right)}{(a-b)^2 d}$$

$$= \frac{\sin(c+dx)}{(a-b)^2 d} + \frac{b^2\sin(c+dx)}{2a(a-b)^2 d\left(a-(a-b)\sin^2(c+dx)\right)} - \frac{((4a-b)b)\,\text{Subst}\left(\int \frac{1}{a+(-a+b}\right.}{2a(a-b)}$$

$$= -\frac{(4a-b)b\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{5/2}d} + \frac{\sin(c+dx)}{(a-b)^2 d} + \frac{b^2\sin(c+dx)}{2a(a-b)^2 d\left(a-(a-b)\sin^2(c+\right.}$$

Mathematica [A] time = 0.56, size = 119, normalized size = 1.04

$$\frac{-\frac{\sqrt{a}\sin(c+dx)\left(a^2+a(a-b)\cos(2(c+dx))+ab+b^2\right)}{(a-b)^2\left((a-b)\sin^2(c+dx)-a\right)} - \frac{b(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{(a-b)^{5/2}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (-(((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(a - b)^(5/2))
 - (Sqrt[a]*(a^2 + a*b + b^2 + a*(a - b)*Cos[2*(c + d*x)])*Sin[c + d*x])/((
a - b)^2*(-a + (a - b)*Sin[c + d*x]^2)))/(2*a^(3/2)*d)
```

fricas [B] time = 0.57, size = 451, normalized size = 3.96

$$\left[-\frac{\left(4\,ab^2-b^3+\left(4\,a^2b-5\,ab^2+b^3\right)\cos\left(dx+c\right)^2\right)\sqrt{a^2-ab}\,\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)-2\left(2\,a^3b\right.}{4\left(\left(a^6-4\,a^5b+6\,a^4b^2-4\,a^3b^3+a^2b^4\right)d\cos\left(dx+c\right)^2+\left(a^5b-3\,a^4b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((4*a*b^2 - b^3 + (4*a^2*b - 5*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(a^2
- a*b)*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a
+ b)/((a - b)*cos(d*x + c)^2 + b)) - 2*(2*a^3*b - a^2*b^2 - a*b^3 + 2*(a^4
- 2*a^3*b + a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 4*a^5*b + 6*a^4*
b^2 - 4*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^2 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^
3 - a^2*b^4)*d), 1/2*((4*a*b^2 - b^3 + (4*a^2*b - 5*a*b^2 + b^3)*cos(d*x +
c)^2)*sqrt(-a^2 + a*b)*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) + (2*a^3*b -
 a^2*b^2 - a*b^3 + 2*(a^4 - 2*a^3*b + a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c)
)/((a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^2 + (a^
5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d)]
```

giac [A] time = 1.89, size = 152, normalized size = 1.33

$$-\frac{\frac{b^2\sin(dx+c)}{\left(a^3-2\,a^2b+ab^2\right)\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)} + \frac{\left(4\,ab-b^2\right)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\left(a^3-2\,a^2b+ab^2\right)\sqrt{-a^2+ab}} - \frac{2\,\sin(dx+c)}{a^2-2\,ab+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*sin(d*x + c)/((a^3 - 2*a^2*b + a*b^2)*(a*sin(d*x + c)^2 - b*sin(d
*x + c)^2 - a)) + (4*a*b - b^2)*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/s
qrt(-a^2 + a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(-a^2 + a*b)) - 2*sin(d*x + c
)/(a^2 - 2*a*b + b^2))/d
```

maple [A] time = 0.62, size = 118, normalized size = 1.04

$$\frac{\frac{\sin(dx+c)}{a^2-2ab+b^2}+\frac{b\left(-\frac{b\sin(dx+c)}{2a\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)}-\frac{(4a-b)\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}\right)}{(a-b)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(1/(a^2-2*a*b+b^2)*sin(d*x+c)+b/(a-b)^2*(-1/2/a*b*sin(d*x+c)/(a*sin(d*x
+c)^2-b*sin(d*x+c)^2-a)-1/2*(4*a-b)/a/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x
+c)/(a*(a-b))^(1/2))))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 15.55, size = 269, normalized size = 2.36

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(2\,a^2+b^2\right)}{a\,(a-b)^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(2\,a^2+b^2\right)}{a\,(a-b)^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(-2\,a^2+4\,a\,b+b^2\right)}{a\,(a-b)^2}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+(4\,b-a)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+(4\,b-a)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}+\frac{b\,\mathrm{atan}\left(\frac{2\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^3-6\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cdots}{\sqrt{a}\,(a-b)^5\cdots}\right)\cdots}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(2*a^2 + b^2))/(a*(a - b)^2) + (tan(c/2 + (d*x)/2)^5*(
2*a^2 + b^2))/(a*(a - b)^2) + (2*tan(c/2 + (d*x)/2)^3*(4*a*b - 2*a^2 + b^2)
)/(a*(a - b)^2))/(d*(a - tan(c/2 + (d*x)/2)^2*(a - 4*b) - tan(c/2 + (d*x)/2
)^4*(a - 4*b) + a*tan(c/2 + (d*x)/2)^6)) + (b*atan((a^3*tan(c/2 + (d*x)/2)*
2i - b^3*tan(c/2 + (d*x)/2)*2i + a*b^2*tan(c/2 + (d*x)/2)*6i - a^2*b*tan(c/
2 + (d*x)/2)*6i)/(a^(1/2)*(a - b)^(5/2)*(tan(c/2 + (d*x)/2)^2 + 1)))*(4*a -
 b)*1i)/(2*a^(3/2)*d*(a - b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(c+dx\right)}{\left(a+b\tan^2\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*tan(c + d*x)**2)**2, x)
```

3.467 $\int \frac{\cos^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=143

$$\frac{b^2(6a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{7/2}}-\frac{b^3\sin(c+dx)}{2ad(a-b)^3\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{\sin^3(c+dx)}{3d(a-b)^2}+\frac{(a-3b)\sin(c+dx)}{d(a-b)^3}$$

```
[Out] 1/2*(6*a-b)*b^2*arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))/a^(3/2)/(a-b)^(7/2)
/d+(a-3*b)*sin(d*x+c)/(a-b)^3/d-1/3*sin(d*x+c)^3/(a-b)^2/d-1/2*b^3*sin(d*x+
c)/a/(a-b)^3/d/(a-(a-b)*sin(d*x+c)^2)
```

Rubi [A] time = 0.21, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3676, 390, 385, 208}

$$\frac{b^2(6a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{7/2}}-\frac{b^3\sin(c+dx)}{2ad(a-b)^3\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{\sin^3(c+dx)}{3d(a-b)^2}+\frac{(a-3b)\sin(c+dx)}{d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a -
 b)^(7/2)*d) + ((a - 3*b)*Sin[c + d*x])/((a - b)^3*d) - Sin[c + d*x]^3/(3*(
a - b)^2*d) - (b^3*Sin[c + d*x])/(2*a*(a - b)^3*d*(a - (a - b)*Sin[c + d*x]
^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-(a-b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a-3b}{(a-b)^3} - \frac{x^2}{(a-b)^2} + \frac{(3a-b)b^2-3(a-b)b^2x^2}{(a-b)^3\left(a+(-a+b)x^2\right)^2}\right) dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{(a-3b)\sin(c+dx)}{(a-b)^3 d} - \frac{\sin^3(c+dx)}{3(a-b)^2 d} + \frac{\text{Subst}\left(\int \frac{(3a-b)b^2-3(a-b)b^2x^2}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \sin(c+dx)\right.}{(a-b)^3 d}$$

$$= \frac{(a-3b)\sin(c+dx)}{(a-b)^3 d} - \frac{\sin^3(c+dx)}{3(a-b)^2 d} - \frac{b^3\sin(c+dx)}{2a(a-b)^3 d\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{((6a}{}$$

$$= \frac{(6a-b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{7/2} d} + \frac{(a-3b)\sin(c+dx)}{(a-b)^3 d} - \frac{\sin^3(c+dx)}{3(a-b)^2 d} - \frac{}{2a(a-b}$$

Mathematica [A] time = 1.62, size = 147, normalized size = 1.03

$$\frac{\frac{3b^2(b-6a)\left(\log\left(\sqrt{a}-\sqrt{a-b}\,\sin(c+dx)\right)-\log\left(\sqrt{a-b}\,\sin(c+dx)+\sqrt{a}\right)\right)}{a^{3/2}(a-b)^{7/2}} + \frac{3\sin(c+dx)\left(-\frac{4b^3}{a((a-b)\cos(2(c+dx))+a+b)}+3a-11b\right)}{(a-b)^3} + \frac{\sin(3(c+dx))}{(a-b)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((3*b^2*(-6*a + b)*(Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d*x]] - Log[Sqrt[a] +
 Sqrt[a - b]*Sin[c + d*x]]))/(a^(3/2)*(a - b)^(7/2)) + (3*(3*a - 11*b - (4*
b^3)/(a*(a + b + (a - b)*Cos[2*(c + d*x)])))*Sin[c + d*x])/(a - b)^3 + Sin[
3*(c + d*x)]/(a - b)^2)/(12*d)
```

fricas [B] time = 0.70, size = 600, normalized size = 4.20

$$\left[\frac{3\left(6\,ab^3-b^4+\left(6\,a^2b^2-7\,ab^3+b^4\right)\cos\left(dx+c\right)^2\right)\sqrt{a^2-ab}\,\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\,\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)+2\left(4\,a^4\right.}{12\left(\left(a^7-5\,a^6b+10\,a^5b^2-10\,a^4b^3+5\,a^3b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(6*a*b^3 - b^4 + (6*a^2*b^2 - 7*a*b^3 + b^4)*cos(d*x + c)^2)*sqrt(
a^2 - a*b)*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) -
2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + 2*(4*a^4*b - 20*a^3*b^2 + 13*a^2*b
^3 + 3*a*b^4 + 2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*cos(d*x + c)^4 + 2*(
2*a^5 - 11*a^4*b + 16*a^3*b^2 - 7*a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((
a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*d*cos(d*x +
c)^2 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d), -1/6*(3*(6
*a*b^3 - b^4 + (6*a^2*b^2 - 7*a*b^3 + b^4)*cos(d*x + c)^2)*sqrt(-a^2 + a*b)
*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) - (4*a^4*b - 20*a^3*b^2 + 13*a^2*b
^3 + 3*a*b^4 + 2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*cos(d*x + c)^4 + 2*(
2*a^5 - 11*a^4*b + 16*a^3*b^2 - 7*a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((
```

```
a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*d*cos(d*x +
c)^2 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d)]
```

giac [B] time = 2.55, size = 329, normalized size = 2.30

$$\frac{\frac{3\,b^3\sin(dx+c)}{\left(a^4-3\,a^3b+3\,a^2b^2-ab^3\right)\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)}+\frac{3\left(6\,ab^2-b^3\right)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\left(a^4-3\,a^3b+3\,a^2b^2-ab^3\right)\sqrt{-a^2+ab}}-\frac{2\left(a^4\sin(dx+c)^3-4\,a^3b\sin(dx+c)^3+6\,a^2\cdots\right.}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*b^3*sin(d*x + c)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*(a*sin(d*x + c
)^2 - b*sin(d*x + c)^2 - a)) + 3*(6*a*b^2 - b^3)*arctan(-(a*sin(d*x + c) -
b*sin(d*x + c))/sqrt(-a^2 + a*b))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt
(-a^2 + a*b)) - 2*(a^4*sin(d*x + c)^3 - 4*a^3*b*sin(d*x + c)^3 + 6*a^2*b^2*
sin(d*x + c)^3 - 4*a*b^3*sin(d*x + c)^3 + b^4*sin(d*x + c)^3 - 3*a^4*sin(d*
x + c) + 18*a^3*b*sin(d*x + c) - 36*a^2*b^2*sin(d*x + c) + 30*a*b^3*sin(d*x
 + c) - 9*b^4*sin(d*x + c))/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a
^2*b^4 - 6*a*b^5 + b^6))/d
```

maple [A] time = 0.81, size = 164, normalized size = 1.15

$$\frac{-\frac{\frac{a\left(\sin^3(dx+c)\right)}{3}-\frac{b\left(\sin^3(dx+c)\right)}{3}-a\sin(dx+c)+3b\sin(dx+c)}{\left(a^2-2ab+b^2\right)(a-b)}-\frac{b^2\left(-\frac{b\sin(dx+c)}{2a\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)}-\frac{(6a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}\right)}{(a-b)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-1/(a^2-2*a*b+b^2)/(a-b)*(1/3*a*sin(d*x+c)^3-1/3*b*sin(d*x+c)^3-a*sin(
d*x+c)+3*b*sin(d*x+c))-b^2/(a-b)^3*(-1/2/a*b*sin(d*x+c)/(a*sin(d*x+c)^2-b*s
in(d*x+c)^2-a)-1/2*(6*a-b)/a/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a
-b))^(1/2))))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 16.01, size = 1690, normalized size = 11.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)*(6*a^2*b - 2*a^3 + b^3))/(a*(3*a*b^2 - 3*a^2*b + a^3
 - b^3)) + (tan(c/2 + (d*x)/2)^9*(6*a^2*b - 2*a^3 + b^3))/(a*(3*a*b^2 - 3*a
^2*b + a^3 - b^3)) + (4*tan(c/2 + (d*x)/2)^3*(18*a*b^2 - 8*a^2*b + 2*a^3 +
```

```
3*b^3))/(3*a*(a - b)*(a^2 - 2*a*b + b^2)) + (4*tan(c/2 + (d*x)/2)^7*(18*a*b
^2 - 8*a^2*b + 2*a^3 + 3*b^3))/(3*a*(a - b)*(a^2 - 2*a*b + b^2)) + (2*tan(c
/2 + (d*x)/2)^5*(56*a*b^2 - 18*a^2*b - 2*a^3 + 9*b^3))/(3*a*(a - b)*(a^2 -
2*a*b + b^2)))/(d*(a + tan(c/2 + (d*x)/2)^2*(a + 4*b) + tan(c/2 + (d*x)/2)^
8*(a + 4*b) - tan(c/2 + (d*x)/2)^4*(2*a - 12*b) - tan(c/2 + (d*x)/2)^6*(2*a
 - 12*b) + a*tan(c/2 + (d*x)/2)^10)) - (b^2*atan(((b^2*(tan(c/2 + (d*x)/2)*
(8*a^3*b^10 - 96*a^4*b^9 + 408*a^5*b^8 - 880*a^6*b^7 + 1080*a^7*b^6 - 768*a
^8*b^5 + 296*a^9*b^4 - 48*a^10*b^3) - (b^2*(6*a - b)*(tan(c/2 + (d*x)/2)^2*
(16*a^15 - 176*a^14*b + 32*a^5*b^10 - 304*a^6*b^9 + 1296*a^7*b^8 - 3264*a^8
*b^7 + 5376*a^9*b^6 - 6048*a^10*b^5 + 4704*a^11*b^4 - 2496*a^12*b^3 + 864*a
^13*b^2) + 144*a^14*b - 16*a^15 + 16*a^6*b^9 - 144*a^7*b^8 + 576*a^8*b^7 -
1344*a^9*b^6 + 2016*a^10*b^5 - 2016*a^11*b^4 + 1344*a^12*b^3 - 576*a^13*b^2
))/(4*a^(3/2)*(a - b)^(7/2)))*(6*a - b)*1i)/(4*a^(3/2)*(a - b)^(7/2)) + (b^
2*(tan(c/2 + (d*x)/2)*(8*a^3*b^10 - 96*a^4*b^9 + 408*a^5*b^8 - 880*a^6*b^7
+ 1080*a^7*b^6 - 768*a^8*b^5 + 296*a^9*b^4 - 48*a^10*b^3) + (b^2*(6*a - b)*
(tan(c/2 + (d*x)/2)^2*(16*a^15 - 176*a^14*b + 32*a^5*b^10 - 304*a^6*b^9 + 1
296*a^7*b^8 - 3264*a^8*b^7 + 5376*a^9*b^6 - 6048*a^10*b^5 + 4704*a^11*b^4 -
 2496*a^12*b^3 + 864*a^13*b^2) + 144*a^14*b - 16*a^15 + 16*a^6*b^9 - 144*a^
7*b^8 + 576*a^8*b^7 - 1344*a^9*b^6 + 2016*a^10*b^5 - 2016*a^11*b^4 + 1344*a
^12*b^3 - 576*a^13*b^2))/(4*a^(3/2)*(a - b)^(7/2)))*(6*a - b)*1i)/(4*a^(3/2
)*(a - b)^(7/2)))/(2*tan(c/2 + (d*x)/2)^2*(a^2*b^9 - 15*a^3*b^8 + 75*a^4*b^
7 - 145*a^5*b^6 + 120*a^6*b^5 - 36*a^7*b^4) - 2*a^2*b^9 + 30*a^3*b^8 - 150*
a^4*b^7 + 290*a^5*b^6 - 240*a^6*b^5 + 72*a^7*b^4 - (b^2*(tan(c/2 + (d*x)/2)
*(8*a^3*b^10 - 96*a^4*b^9 + 408*a^5*b^8 - 880*a^6*b^7 + 1080*a^7*b^6 - 768*
a^8*b^5 + 296*a^9*b^4 - 48*a^10*b^3) - (b^2*(6*a - b)*(tan(c/2 + (d*x)/2)^2
*(16*a^15 - 176*a^14*b + 32*a^5*b^10 - 304*a^6*b^9 + 1296*a^7*b^8 - 3264*a^
8*b^7 + 5376*a^9*b^6 - 6048*a^10*b^5 + 4704*a^11*b^4 - 2496*a^12*b^3 + 864*
a^13*b^2) + 144*a^14*b - 16*a^15 + 16*a^6*b^9 - 144*a^7*b^8 + 576*a^8*b^7 -
 1344*a^9*b^6 + 2016*a^10*b^5 - 2016*a^11*b^4 + 1344*a^12*b^3 - 576*a^13*b^
2))/(4*a^(3/2)*(a - b)^(7/2)))*(6*a - b))/(4*a^(3/2)*(a - b)^(7/2)) + (b^2*
(tan(c/2 + (d*x)/2)*(8*a^3*b^10 - 96*a^4*b^9 + 408*a^5*b^8 - 880*a^6*b^7 +
1080*a^7*b^6 - 768*a^8*b^5 + 296*a^9*b^4 - 48*a^10*b^3) + (b^2*(6*a - b)*(t
an(c/2 + (d*x)/2)^2*(16*a^15 - 176*a^14*b + 32*a^5*b^10 - 304*a^6*b^9 + 129
6*a^7*b^8 - 3264*a^8*b^7 + 5376*a^9*b^6 - 6048*a^10*b^5 + 4704*a^11*b^4 - 2
496*a^12*b^3 + 864*a^13*b^2) + 144*a^14*b - 16*a^15 + 16*a^6*b^9 - 144*a^7*
b^8 + 576*a^8*b^7 - 1344*a^9*b^6 + 2016*a^10*b^5 - 2016*a^11*b^4 + 1344*a^1
2*b^3 - 576*a^13*b^2))/(4*a^(3/2)*(a - b)^(7/2)))*(6*a - b))/(4*a^(3/2)*(a
- b)^(7/2))))*(6*a - b)*1i)/(2*a^(3/2)*d*(a - b)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.468 $\int \frac{\sec^8(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=127

$$\frac{(5a+b)(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}d}-\frac{(a-b)^3\tan(c+dx)}{2ab^3d\left(a+b\tan^2(c+dx)\right)}-\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}$$

```
[Out] 1/2*(a-b)^2*(5*a+b)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/b^(7/2)/d-(2
*a-3*b)*tan(d*x+c)/b^3/d+1/3*tan(d*x+c)^3/b^2/d-1/2*(a-b)^3*tan(d*x+c)/a/b^
3/d/(a+b*tan(d*x+c)^2)
```

Rubi [A] time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3675, 390, 385, 205}

$$\frac{(5a+b)(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}d}-\frac{(a-b)^3\tan(c+dx)}{2ab^3d\left(a+b\tan^2(c+dx)\right)}-\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)^2*(5*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(
7/2)*d) - ((2*a - 3*b)*Tan[c + d*x])/(b^3*d) + Tan[c + d*x]^3/(3*b^2*d) - (
(a - b)^3*Tan[c + d*x])/(2*a*b^3*d*(a + b*Tan[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^8(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{2a-3b}{b^3}+\frac{x^2}{b^2}+\frac{(a-b)^2(2a+b)+3(a-b)^2bx^2}{b^3\left(a+bx^2\right)^2}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}+\frac{\text{Subst}\left(\int \frac{(a-b)^2(2a+b)+3(a-b)^2bx^2}{\left(a+bx^2\right)^2}\,dx, x, \tan(c\right.}{b^3d}$$

$$= -\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}-\frac{(a-b)^3\tan(c+dx)}{2ab^3d\left(a+b\tan^2(c+dx)\right)}+\frac{\left((a-b)^2(5a\right.}{}$$

$$= \frac{(a-b)^2(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}d}-\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}-\frac{(}{2ab^3}$$

Mathematica [A] time = 0.79, size = 135, normalized size = 1.06

$$\frac{\frac{3(a-b)^2(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+4\sqrt{b}\,(4b-3a)\tan(c+dx)+\frac{3\sqrt{b}\,(b-a)^3\sin(2(c+dx))}{a((a-b)\cos(2(c+dx))+a+b)}+2b^{3/2}\tan(c+dx)\sec^2(c+dx)}{6b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((3*(a - b)^2*(5*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/a^(3/2) + (
3*Sqrt[b]*(-a + b)^3*Sin[2*(c + d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)]
)) + 4*Sqrt[b]*(-3*a + 4*b)*Tan[c + d*x] + 2*b^(3/2)*Sec[c + d*x]^2*Tan[c +
 d*x])/(6*b^(7/2)*d)
```

fricas [B] time = 0.62, size = 597, normalized size = 4.70

$$\left[-\frac{3\left(\left(5a^4-14a^3b+12a^2b^2-2ab^3-b^4\right)\cos(dx+c)^5+\left(5a^3b-9a^2b^2+3ab^3+b^4\right)\cos(dx+c)^3\right)\sqrt{-ab}\,\log\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((5*a^4 - 14*a^3*b + 12*a^2*b^2 - 2*a*b^3 - b^4)*cos(d*x + c)^5 +
 (5*a^3*b - 9*a^2*b^2 + 3*a*b^3 + b^4)*cos(d*x + c)^3)*sqrt(-a*b)*log(((a^2
 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b
)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2
*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) - 4*(2*a^
2*b^3 - (15*a^4*b - 37*a^3*b^2 + 25*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^4 - 2*(
5*a^3*b^2 - 7*a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^2*b^5*d*cos(d*x + c
)^3 + (a^3*b^4 - a^2*b^5)*d*cos(d*x + c)^5), -1/12*(3*((5*a^4 - 14*a^3*b +
12*a^2*b^2 - 2*a*b^3 - b^4)*cos(d*x + c)^5 + (5*a^3*b - 9*a^2*b^2 + 3*a*b^3
 + b^4)*cos(d*x + c)^3)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*s
qrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))) - 2*(2*a^2*b^3 - (15*a^4*b - 37*a
^3*b^2 + 25*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^4 - 2*(5*a^3*b^2 - 7*a^2*b^3)*c
```

```
os(d*x + c)^2)*sin(d*x + c))/(a^2*b^5*d*cos(d*x + c)^3 + (a^3*b^4 - a^2*b^5
)*d*cos(d*x + c)^5)]
```

giac [A] time = 2.29, size = 180, normalized size = 1.42

$$\frac{\frac{3\left(5a^3-9a^2b+3ab^2+b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}\,ab^3}-\frac{3\left(a^3\tan(dx+c)-3a^2b\tan(dx+c)+3ab^2\tan(dx+c)-b^3\tan(dx+c)\right)}{\left(b\tan(dx+c)^2+a\right)ab^3}+\frac{2\left(b^4\tan\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(
b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/(sqrt(a*b)*a*b^3) - 3*(a^3*tan(d*x +
 c) - 3*a^2*b*tan(d*x + c) + 3*a*b^2*tan(d*x + c) - b^3*tan(d*x + c))/((b*t
an(d*x + c)^2 + a)*a*b^3) + 2*(b^4*tan(d*x + c)^3 - 6*a*b^3*tan(d*x + c) +
9*b^4*tan(d*x + c))/b^6)/d
```

maple [B] time = 0.92, size = 275, normalized size = 2.17

$$\frac{\tan^3(dx+c)}{3b^2d}-\frac{2a\tan(dx+c)}{d\,b^3}+\frac{3\tan(dx+c)}{b^2d}-\frac{a^2\tan(dx+c)}{2d\,b^3\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{3a\tan(dx+c)}{2d\,b^2\left(a+b\left(\tan^2(dx+c)\right)\right)}-\frac{}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/3*tan(d*x+c)^3/b^2/d-2/d/b^3*a*tan(d*x+c)+3*tan(d*x+c)/b^2/d-1/2/d/b^3*a^
2*tan(d*x+c)/(a+b*tan(d*x+c)^2)+3/2/d/b^2*a*tan(d*x+c)/(a+b*tan(d*x+c)^2)-3
/2/d/b*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2*tan(d*x+c)/a/d/(a+b*tan(d*x+c)^2)+
5/2/d/b^3*a^2/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))-9/2/d/b^2*a/(a*b
)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))+3/2/d/b/(a*b)^(1/2)*arctan(tan(d*x
+c)*b/(a*b)^(1/2))+1/2/d/a/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))
```

maxima [A] time = 0.81, size = 137, normalized size = 1.08

$$-\frac{\frac{3\left(a^3-3a^2b+3ab^2-b^3\right)\tan(dx+c)}{ab^4\tan(dx+c)^2+a^2b^3}-\frac{2\left(b\tan(dx+c)^3-3(2a-3b)\tan(dx+c)\right)}{b^3}-\frac{3\left(5a^3-9a^2b+3ab^2+b^3\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*tan(d*x + c)/(a*b^4*tan(d*x + c)^2
+ a^2*b^3) - 2*(b*tan(d*x + c)^3 - 3*(2*a - 3*b)*tan(d*x + c))/b^3 - 3*(5*a
^3 - 9*a^2*b + 3*a*b^2 + b^3)*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*a
*b^3))/d
```

mupad [B] time = 12.17, size = 167, normalized size = 1.31

$$\frac{\tan(c+d\,x)^3}{3\,b^2\,d}-\frac{\tan(c+d\,x)\,\left(\frac{2\,a}{b^3}-\frac{3}{b^2}\right)}{d}-\frac{\tan(c+d\,x)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,a\,d\,\left(b^4\tan(c+d\,x)^2+a\,b^3\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)\,(a-b)^2\,(5\,a+b)}{\sqrt{a}\,\left(5\,a^3-9\,a^2\,b+3\,a\,b^2+b^3\right)}\right)}{2\,a^{3/2}\,b^{7/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + b*tan(c + d*x)^2)^2),x)
```

```
[Out] tan(c + d*x)^3/(3*b^2*d) - (tan(c + d*x)*((2*a)/b^3 - 3/b^2))/d - (tan(c +
d*x)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*a*d*(a*b^3 + b^4*tan(c + d*x)^2))
+ (atan((b^(1/2)*tan(c + d*x)*(a - b)^2*(5*a + b))/(a^(1/2)*(3*a*b^2 - 9*a^
2*b + 5*a^3 + b^3)))*(a - b)^2*(5*a + b))/(2*a^(3/2)*b^(7/2)*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.469 $\int \frac{\sec^6(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=104

$$-\frac{\left(3a^2-2ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{(a-b)^2\tan(c+dx)}{2ab^2d\left(a+b\tan^2(c+dx)\right)}+\frac{\tan(c+dx)}{b^2d}$$

```
[Out] -1/2*(3*a^2-2*a*b-b^2)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/b^(5/2)/d
+tan(d*x+c)/b^2/d+1/2*(a-b)^2*tan(d*x+c)/a/b^2/d/(a+b*tan(d*x+c)^2)
```

Rubi [A] time = 0.14, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3675, 390, 385, 205}

$$-\frac{\left(3a^2-2ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{(a-b)^2\tan(c+dx)}{2ab^2d\left(a+b\tan^2(c+dx)\right)}+\frac{\tan(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] -((3*a^2 - 2*a*b - b^2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*
b^(5/2)*d) + Tan[c + d*x]/(b^2*d) + ((a - b)^2*Tan[c + d*x])/(2*a*b^2*d*(a
+ b*Tan[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2} - \frac{a^2-b^2+2(a-b)bx^2}{b^2\left(a+bx^2\right)^2}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\tan(c+dx)}{b^2 d} - \frac{\text{Subst}\left(\int \frac{a^2-b^2+2(a-b)bx^2}{\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{b^2 d}$$

$$= \frac{\tan(c+dx)}{b^2 d} + \frac{(a-b)^2\tan(c+dx)}{2ab^2 d\left(a+b\tan^2(c+dx)\right)} - \frac{((a-b)(3a+b))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \right.}{2ab^2 d}$$

$$= -\frac{(a-b)(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{\tan(c+dx)}{b^2 d} + \frac{(a-b)^2\tan(c+dx)}{2ab^2 d\left(a+b\tan^2(c+dx)\right)}$$

Mathematica [A] time = 0.66, size = 104, normalized size = 1.00

$$\frac{-\frac{(3a+b)(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{b}(a-b)^2\sin(2(c+dx))}{a((a-b)\cos(2(c+dx))+a+b)} + 2\sqrt{b}\tan(c+dx)}{2b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (-(((a - b)*(3*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/a^(3/2)) + ((
a - b)^2*Sqrt[b]*Sin[2*(c + d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)])) +
 2*Sqrt[b]*Tan[c + d*x])/(2*b^(5/2)*d)
```

fricas [B] time = 0.54, size = 479, normalized size = 4.61

$$\left[\frac{\left(\left(3a^3 - 5a^2b + ab^2 + b^3\right)\cos(dx+c)^3 + \left(3a^2b - 2ab^2 - b^3\right)\cos(dx+c)\right)\sqrt{-ab}\,\log\left(\frac{\left(a^2+6ab+b^2\right)\cos(dx+c)^4 - 2\left(3ab-\right.}{\left(a^2-2a\right.}\right.}{8\left(a^2b^4d\cos(dx+c) + \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(((3*a^3 - 5*a^2*b + a*b^2 + b^3)*cos(d*x + c)^3 + (3*a^2*b - 2*a*b^2
- b^3)*cos(d*x + c))*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2
*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))
*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*
b - b^2)*cos(d*x + c)^2 + b^2)) + 4*(2*a^2*b^2 + (3*a^3*b - 4*a^2*b^2 + a*b
^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^2*b^4*d*cos(d*x + c) + (a^3*b^3 - a^2*
b^4)*d*cos(d*x + c)^3), 1/4*(((3*a^3 - 5*a^2*b + a*b^2 + b^3)*cos(d*x + c)^
3 + (3*a^2*b - 2*a*b^2 - b^3)*cos(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*c
os(d*x + c)^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))) + 2*(2*a^2*b^
2 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^2*b^4*d*
cos(d*x + c) + (a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^3)]
```

giac [A] time = 2.44, size = 128, normalized size = 1.23

$$\frac{\frac{2\tan(dx+c)}{b^2}-\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)(3a^2-2ab-b^2)}{\sqrt{ab}\,ab^2}+\frac{a^2\tan(dx+c)-2ab\tan(dx+c)+b^2\tan(dx+c)}{\left(b\tan(dx+c)^2+a\right)ab^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*tan(d*x + c)/b^2 - (pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*t
an(d*x + c)/sqrt(a*b)))*(3*a^2 - 2*a*b - b^2)/(sqrt(a*b)*a*b^2) + (a^2*tan(
d*x + c) - 2*a*b*tan(d*x + c) + b^2*tan(d*x + c))/((b*tan(d*x + c)^2 + a)*a
*b^2))/d
```

maple [A] time = 0.65, size = 181, normalized size = 1.74

$$\frac{\tan(dx+c)}{b^2d}+\frac{a\tan(dx+c)}{2d\,b^2\left(a+b\left(\tan^2(dx+c)\right)\right)}-\frac{\tan(dx+c)}{db\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{\tan(dx+c)}{2ad\left(a+b\left(\tan^2(dx+c)\right)\right)}-\frac{3a\arctan}{2d\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] tan(d*x+c)/b^2/d+1/2/d/b^2*a*tan(d*x+c)/(a+b*tan(d*x+c)^2)-1/d/b*tan(d*x+c)
/(a+b*tan(d*x+c)^2)+1/2*tan(d*x+c)/a/d/(a+b*tan(d*x+c)^2)-3/2/d/b^2*a/(a*b)
^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))+1/d/b/(a*b)^(1/2)*arctan(tan(d*x+c)
*b/(a*b)^(1/2))+1/2/d/a/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))
```

maxima [A] time = 0.94, size = 100, normalized size = 0.96

$$\frac{\frac{(a^2-2ab+b^2)\tan(dx+c)}{ab^3\tan(dx+c)^2+a^2b^2}+\frac{2\tan(dx+c)}{b^2}-\frac{(3a^2-2ab-b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 - 2*a*b + b^2)*tan(d*x + c)/(a*b^3*tan(d*x + c)^2 + a^2*b^2) + 2*
tan(d*x + c)/b^2 - (3*a^2 - 2*a*b - b^2)*arctan(b*tan(d*x + c)/sqrt(a*b))/(
sqrt(a*b)*a*b^2))/d
```

mupad [B] time = 12.41, size = 119, normalized size = 1.14

$$\frac{\tan(c+d\,x)}{b^2\,d}+\frac{\tan(c+d\,x)\,\left(a^2-2\,a\,b+b^2\right)}{2\,a\,d\,\left(b^3\tan(c+d\,x)^2+a\,b^2\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)\,(a-b)\,(3\,a+b)}{\sqrt{a}\,\left(-3\,a^2+2\,a\,b+b^2\right)}\right)\,(a-b)\,(3\,a+b)}{2\,a^{3/2}\,b^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x)^2)^2),x)
```

```
[Out] tan(c + d*x)/(b^2*d) + (tan(c + d*x)*(a^2 - 2*a*b + b^2))/(2*a*d*(a*b^2 + b
^3*tan(c + d*x)^2)) + (atan((b^(1/2)*tan(c + d*x)*(a - b)*(3*a + b))/(a^(1/
2)*(2*a*b - 3*a^2 + b^2)))*(a - b)*(3*a + b))/(2*a^(3/2)*b^(5/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^6(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*tan(c + d*x)**2)**2, x)
```

3.470 $\int \frac{\sec^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=77

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\tan(c+dx)}{2abd\left(a+b\tan^2(c+dx)\right)}$$

```
[Out] 1/2*(a+b)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/b^(3/2)/d-1/2*(a-b)*ta
n(d*x+c)/a/b/d/(a+b*tan(d*x+c)^2)
```

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3675, 385, 205}

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\tan(c+dx)}{2abd\left(a+b\tan^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)*d) - ((
a - b)*Tan[c + d*x])/(2*a*b*d*(a + b*Tan[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{(a-b)\tan(c+dx)}{2abd\left(a+b\tan^2(c+dx)\right)} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{2abd}$$
$$= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} - \frac{(a-b)\tan(c+dx)}{2abd\left(a+b\tan^2(c+dx)\right)}$$

Mathematica [A] time = 0.31, size = 83, normalized size = 1.08

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right) + \frac{\sqrt{a}\sqrt{b}(b-a)\sin(2(c+dx))}{(a-b)\cos(2(c+dx))+a+b}}{2a^{3/2}b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] + (Sqrt[a]*Sqrt[b]*(-a + b)
*Sin[2*(c + d*x)])/(a + b + (a - b)*Cos[2*(c + d*x)]))/(2*a^(3/2)*b^(3/2)*d
)
```

fricas [B] time = 0.55, size = 367, normalized size = 4.77

$$\left[-\frac{4\left(a^2b-ab^2\right)\cos(dx+c)\sin(dx+c) + \left(\left(a^2-b^2\right)\cos(dx+c)^2 + ab + b^2\right)\sqrt{-ab}\,\log\left(\frac{\left(a^2+6ab+b^2\right)\cos(dx+c)^4 - 2\left(3a\right.}{\left(a^2-\right.}\right.}{8\left(a^2b^3d + \left(a^3b^2 - a^2b^3\right)d\cos(dx+c)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a^2*b - a*b^2)*cos(d*x + c)*sin(d*x + c) + ((a^2 - b^2)*cos(d*x +
 c)^2 + a*b + b^2)*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(
3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*s
qrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b
- b^2)*cos(d*x + c)^2 + b^2)))/(a^2*b^3*d + (a^3*b^2 - a^2*b^3)*d*cos(d*x +
 c)^2), -1/4*(2*(a^2*b - a*b^2)*cos(d*x + c)*sin(d*x + c) + ((a^2 - b^2)*co
s(d*x + c)^2 + a*b + b^2)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)
*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))))/(a^2*b^3*d + (a^3*b^2 - a^2*b^
3)*d*cos(d*x + c)^2)]
```

giac [A] time = 2.09, size = 92, normalized size = 1.19

$$\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)(a+b)}{\sqrt{ab}\,ab} - \frac{a\tan(dx+c)-b\tan(dx+c)}{\left(b\tan(dx+c)^2+a\right)ab}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)
))*(a + b)/(sqrt(a*b)*a*b) - (a*tan(d*x + c) - b*tan(d*x + c))/((b*tan(d*x
+ c)^2 + a)*a*b))/d
```

maple [A] time = 0.68, size = 112, normalized size = 1.45

$$-\frac{\tan\left(dx+c\right)}{2db\left(a+b\left(\tan^{2}\left(dx+c\right)\right)\right)}+\frac{\tan\left(dx+c\right)}{2ad\left(a+b\left(\tan^{2}\left(dx+c\right)\right)\right)}+\frac{\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)}{2db\sqrt{ab}}+\frac{\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)}{2da\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/b*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2*tan(d*x+c)/a/d/(a+b*tan(d*x+c)^2
)+1/2/d/b/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))+1/2/d/a/(a*b)^(1/2)*
arctan(tan(d*x+c)*b/(a*b)^(1/2))
```

maxima [A] time = 0.57, size = 69, normalized size = 0.90

$$-\frac{\frac{(a-b)\tan(dx+c)}{ab^{2}\tan(dx+c)^{2}+a^{2}b}-\frac{(a+b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((a - b)*tan(d*x + c)/(a*b^2*tan(d*x + c)^2 + a^2*b) - (a + b)*arctan(
b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*a*b))/d
```

mupad [B] time = 12.19, size = 65, normalized size = 0.84

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)}{\sqrt{a}}\right)\,\left(a+b\right)}{2\,a^{3/2}\,b^{3/2}\,d}-\frac{\tan\left(c+d\,x\right)\,\left(a-b\right)}{2\,a\,b\,d\,\left(b\,{\tan\left(c+d\,x\right)}^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x)^2)^2),x)
```

```
[Out] (atan((b^(1/2)*tan(c + d*x))/a^(1/2))*(a + b))/(2*a^(3/2)*b^(3/2)*d) - (tan
(c + d*x)*(a - b))/(2*a*b*d*(a + b*tan(c + d*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{4}\left(c+dx\right)}{\left(a+b\tan^{2}\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x)**2)**2, x)
```

3.471 $\int \frac{\sec^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{\tan(c+dx)}{2ad\left(a+b\tan^2(c+dx)\right)}$$

```
[Out] 1/2*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/d/b^(1/2)+1/2*tan(d*x+c)/a/d
/(a+b*tan(d*x+c)^2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3675, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{\tan(c+dx)}{2ad\left(a+b\tan^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Tan[c + d*x]
/(2*a*d*(a + b*Tan[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\tan(c+dx)}{2ad\left(a+b\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{2ad}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d} + \frac{\tan(c+dx)}{2ad\left(a+b\tan^2(c+dx)\right)}$$

Mathematica [A] time = 0.28, size = 63, normalized size = 0.95

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{\sqrt{a}\tan(c+dx)}{a+b\tan^2(c+dx)}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/Sqrt[b] + (Sqrt[a]*Tan[c + d*x])/(a
 + b*Tan[c + d*x]^2))/(2*a^(3/2)*d)
```

fricas [B] time = 0.53, size = 327, normalized size = 4.95

$$\left[\frac{4\,ab\cos(dx+c)\sin(dx+c) - \left((a-b)\cos(dx+c)^2+b\right)\sqrt{-ab}\,\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(dx+c)^4 - 2\left(3\,ab+b^2\right)\cos(dx+c)^2 + 4\,(}{\left(a^2-2\,ab+b^2\right)\cos(dx+c)^4 +}\right.}{8\left(a^2b^2d + \left(a^3b-a^2b^2\right)d\cos(dx+c)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*a*b*cos(d*x + c)*sin(d*x + c) - ((a - b)*cos(d*x + c)^2 + b)*sqrt(-
a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)
^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) +
b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b
^2)))/(a^2*b^2*d + (a^3*b - a^2*b^2)*d*cos(d*x + c)^2), 1/4*(2*a*b*cos(d*x
+ c)*sin(d*x + c) - ((a - b)*cos(d*x + c)^2 + b)*sqrt(a*b)*arctan(1/2*((a +
 b)*cos(d*x + c)^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))))/(a^2*b^
2*d + (a^3*b - a^2*b^2)*d*cos(d*x + c)^2)]
```

giac [A] time = 2.00, size = 70, normalized size = 1.06

$$\frac{\frac{\pi\left\lfloor \frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor \text{sgn}(b) + \arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} + \frac{\tan(dx+c)}{\left(b\tan(dx+c)^2+a\right)a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)
))/(sqrt(a*b)*a) + tan(d*x + c)/((b*tan(d*x + c)^2 + a)*a))/d
```

maple [A] time = 0.64, size = 57, normalized size = 0.86

$$\frac{\tan(dx+c)}{2ad\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)}{2da\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/2*tan(d*x+c)/a/d/(a+b*tan(d*x+c)^2)+1/2/d/a/(a*b)^(1/2)*arctan(tan(d*x+c)
*b/(a*b)^(1/2))
```

maxima [A] time = 0.48, size = 53, normalized size = 0.80

$$\frac{\frac{\tan(dx+c)}{ab\tan(dx+c)^2+a^2}+\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(tan(d*x + c)/(a*b*tan(d*x + c)^2 + a^2) + arctan(b*tan(d*x + c)/sqrt(a
*b))/(sqrt(a*b)*a))/d
```

mupad [B] time = 12.14, size = 54, normalized size = 0.82

$$\frac{\tan(c+d\,x)}{2\,a\,d\,\left(b\tan(c+d\,x)^2+a\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)^2),x)
```

```
[Out] tan(c + d*x)/(2*a*d*(a + b*tan(c + d*x)^2)) + atan((b^(1/2)*tan(c + d*x))/a
^(1/2))/(2*a^(3/2)*b^(1/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2)**2, x)
```

3.472 $\int \frac{\cos^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=148

$$\frac{b^{3/2}(5a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^3}+\frac{b(a+b)\tan(c+dx)}{2ad(a-b)^2\left(a+b\tan^2(c+dx)\right)}+\frac{\sin(c+dx)\cos(c+dx)}{2d(a-b)\left(a+b\tan^2(c+dx)\right)}+\frac{x(a-5b)}{2(a-b)^3}$$

```
[Out] 1/2*(a-5*b)*x/(a-b)^3+1/2*(5*a-b)*b^(3/2)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2)
)/a^(3/2)/(a-b)^3/d+1/2*cos(d*x+c)*sin(d*x+c)/(a-b)/d/(a+b*tan(d*x+c)^2)+1/
2*b*(a+b)*tan(d*x+c)/a/(a-b)^2/d/(a+b*tan(d*x+c)^2)
```

Rubi [A] time = 0.19, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3675, 414, 527, 522, 203, 205}

$$\frac{b^{3/2}(5a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^3}+\frac{b(a+b)\tan(c+dx)}{2ad(a-b)^2\left(a+b\tan^2(c+dx)\right)}+\frac{\sin(c+dx)\cos(c+dx)}{2d(a-b)\left(a+b\tan^2(c+dx)\right)}+\frac{x(a-5b)}{2(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((a - 5*b)*x)/(2*(a - b)^3) + ((5*a - b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[c + d*
x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^3*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*(a -
 b)*d*(a + b*Tan[c + d*x]^2)) + (b*(a + b)*Tan[c + d*x])/(2*a*(a - b)^2*d*(
a + b*Tan[c + d*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d\left(a+b\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-a+2b-3bx^2}{\left(1+x^2\right)\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{2(a-b)d}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{b(a+b)\tan(c+dx)}{2a(a-b)^2d\left(a+b\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2(a^2}{}\right.}{}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{b(a+b)\tan(c+dx)}{2a(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{(a-5b)\,\text{Subst}}{}$$

$$= \frac{(a-5b)x}{2(a-b)^3} + \frac{(5a-b)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^3d} + \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{}{2a}$$

Mathematica [A] time = 1.25, size = 116, normalized size = 0.78

$$\frac{-\frac{2b^{3/2}(b-5a)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{2b^2(a-b)\sin(2(c+dx))}{a((a-b)\cos(2(c+dx))+a+b)} + 2(a-5b)(c+dx) + (a-b)\sin(2(c+dx))}{4d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (2*(a - 5*b)*(c + d*x) - (2*b^(3/2)*(-5*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x]
)/Sqrt[a]])/a^(3/2) + (a - b)*Sin[2*(c + d*x)] + (2*(a - b)*b^2*Sin[2*(c +
d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)]))))/(4*(a - b)^3*d)
```

fricas [A] time = 0.57, size = 614, normalized size = 4.15

$$\left[\frac{4\left(a^3-6a^2b+5ab^2\right)dx\cos(dx+c)^2 + 4\left(a^2b-5ab^2\right)dx + \left(5ab^2-b^3+\left(5a^2b-6ab^2+b^3\right)\cos(dx+c)^2\right)\sqrt{-\frac{b}{a}}}{8\left(\left(a^5-4a^4b+6\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^3 - 6*a^2*b + 5*a*b^2)*d*x*cos(d*x + c)^2 + 4*(a^2*b - 5*a*b^2)*
d*x + (5*a*b^2 - b^3 + (5*a^2*b - 6*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(-b/a)
*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 -
 4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x + c))*sqrt(-b/a)*sin(d*x + c)
+ b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 +
 b^2)) + 4*((a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^3 + (a^2*b - b^3)*cos(d*x
+ c))*sin(d*x + c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cos(
d*x + c)^2 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d), 1/4*(2*(a^3 - 6*a^
2*b + 5*a*b^2)*d*x*cos(d*x + c)^2 + 2*(a^2*b - 5*a*b^2)*d*x - (5*a*b^2 - b^
3 + (5*a^2*b - 6*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(b/a)*arctan(1/2*((a + b)
*cos(d*x + c)^2 - b)*sqrt(b/a)/(b*cos(d*x + c)*sin(d*x + c))) + 2*((a^3 - 2
*a^2*b + a*b^2)*cos(d*x + c)^3 + (a^2*b - b^3)*cos(d*x + c))*sin(d*x + c))/
((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cos(d*x + c)^2 + (a^4*b
- 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d)]
```

giac [A] time = 2.18, size = 211, normalized size = 1.43

$$\frac{\frac{(dx+c)(a-5b)}{a^3-3a^2b+3ab^2-b^3}+\frac{(5ab^2-b^3)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)}{(a^4-3a^3b+3a^2b^2-ab^3)\sqrt{ab}}+\frac{ab\tan(dx+c)^3+b^2\tan(dx+c)^3+a^2\tan(dx+c)+b^2\tan(dx+c)}{\left(b\tan(dx+c)^4+a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)(a^3-2a^2b+ab^2)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*(a - 5*b)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (5*a*b^2 - b^3)*
(pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/((
a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt(a*b)) + (a*b*tan(d*x + c)^3 + b^2*t
an(d*x + c)^3 + a^2*tan(d*x + c) + b^2*tan(d*x + c))/((b*tan(d*x + c)^4 + a
*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a^3 - 2*a^2*b + a*b^2)))/d
```

maple [A] time = 0.77, size = 248, normalized size = 1.68

$$\frac{b^2\tan(dx+c)}{2d(a-b)^3\left(a+b\left(\tan^2(dx+c)\right)\right)}-\frac{b^3\tan(dx+c)}{2d(a-b)^3a\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{5b^2\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)}{2d(a-b)^3\sqrt{ab}}-\frac{b^3\arctan\left(\frac{\tan}{}\right.}{2d(a-b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d*b^2/(a-b)^3*tan(d*x+c)/(a+b*tan(d*x+c)^2)-1/2/d*b^3/(a-b)^3/a*tan(d*x
+c)/(a+b*tan(d*x+c)^2)+5/2/d*b^2/(a-b)^3/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a
*b)^(1/2))-1/2/d*b^3/(a-b)^3/a/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))
+1/2/d/(a-b)^3*tan(d*x+c)/(1+tan(d*x+c)^2)*a-1/2/d/(a-b)^3*tan(d*x+c)/(1+ta
n(d*x+c)^2)*b+1/2/d/(a-b)^3*arctan(tan(d*x+c))*a-5/2/d/(a-b)^3*arctan(tan(d
*x+c))*b
```

maxima [A] time = 1.05, size = 209, normalized size = 1.41

$$\frac{\frac{(dx+c)(a-5b)}{a^3-3a^2b+3ab^2-b^3}+\frac{(5ab^2-b^3)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^4-3a^3b+3a^2b^2-ab^3)\sqrt{ab}}+\frac{(ab+b^2)\tan(dx+c)^3+(a^2+b^2)\tan(dx+c)}{(a^3b-2a^2b^2+ab^3)\tan(dx+c)^4+a^4-2a^3b+a^2b^2+(a^4-a^3b-a^2b^2+ab^3)\tan(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((d*x + c)*(a - 5*b)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (5*a*b^2 - b^3)*
arctan(b*tan(d*x + c)/sqrt(a*b))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt(
a*b)) + ((a*b + b^2)*tan(d*x + c)^3 + (a^2 + b^2)*tan(d*x + c))/((a^3*b - 2
*a^2*b^2 + a*b^3)*tan(d*x + c)^4 + a^4 - 2*a^3*b + a^2*b^2 + (a^4 - a^3*b -
 a^2*b^2 + a*b^3)*tan(d*x + c)^2))/d
```

mupad [B] time = 16.29, size = 3843, normalized size = 25.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] ((tan(c + d*x)*(a^2 + b^2))/(2*a*(a^2 - 2*a*b + b^2)) + (b*tan(c + d*x)^3*(
a + b))/(2*a*(a^2 - 2*a*b + b^2)))/(d*(a + tan(c + d*x)^2*(a + b) + b*tan(c
 + d*x)^4)) - (atan((((((((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b^7
+ 220*a^5*b^6 - 172*a^6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8 - 6
*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) - (tan
(c + d*x)*(a - 5*b)*(16*a^2*b^9 - 80*a^3*b^8 + 144*a^4*b^7 - 80*a^5*b^6 - 8
0*a^6*b^5 + 144*a^7*b^4 - 80*a^8*b^3 + 16*a^9*b^2))/(8*(a*b^2*3i - a^2*b*3i
 + a^3*1i - b^3*1i)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a
- 5*b))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) - (tan(c + d*x)*(b^7 -
10*a*b^6 + 50*a^2*b^5 - 10*a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*b + a^2*b^4
- 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b)*1i)/(4*(a*b^2*3i - a^2*b*3i + a^3*1i -
 b^3*1i)) - (((((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b^7 + 220*a^5
*b^6 - 172*a^6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8 - 6*a^7*b +
a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) + (tan(c + d*x)
*(a - 5*b)*(16*a^2*b^9 - 80*a^3*b^8 + 144*a^4*b^7 - 80*a^5*b^6 - 80*a^6*b^5
 + 144*a^7*b^4 - 80*a^8*b^3 + 16*a^9*b^2))/(8*(a*b^2*3i - a^2*b*3i + a^3*1i
 - b^3*1i)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b))/(
4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) + (tan(c + d*x)*(b^7 - 10*a*b^6
+ 50*a^2*b^5 - 10*a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b
^3 + 6*a^4*b^2)))*(a - 5*b)*1i)/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i))
)/((((((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b^7 + 220*a^5*b^6 - 17
2*a^6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8 - 6*a^7*b + a^2*b^6 -
 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) - (tan(c + d*x)*(a - 5*b
)*(16*a^2*b^9 - 80*a^3*b^8 + 144*a^4*b^7 - 80*a^5*b^6 - 80*a^6*b^5 + 144*a^
7*b^4 - 80*a^8*b^3 + 16*a^9*b^2))/(8*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i
)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b))/(4*(a*b^2*
3i - a^2*b*3i + a^3*1i - b^3*1i)) - (tan(c + d*x)*(b^7 - 10*a*b^6 + 50*a^2*
b^5 - 10*a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^
4*b^2)))*(a - 5*b))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) - ((21*a*b^
6)/4 - (5*b^7)/4 + (21*a^2*b^5)/4 - (5*a^3*b^4)/4)/(a^8 - 6*a^7*b + a^2*b^6
 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) + (((((2*a*b^10 - 20*a
^2*b^9 + 80*a^3*b^8 - 172*a^4*b^7 + 220*a^5*b^6 - 172*a^6*b^5 + 80*a^7*b^4
- 20*a^8*b^3 + 2*a^9*b^2)/(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4
 - 20*a^5*b^3 + 15*a^6*b^2) + (tan(c + d*x)*(a - 5*b)*(16*a^2*b^9 - 80*a^3*
b^8 + 144*a^4*b^7 - 80*a^5*b^6 - 80*a^6*b^5 + 144*a^7*b^4 - 80*a^8*b^3 + 16
*a^9*b^2))/(8*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)*(a^6 - 4*a^5*b + a^2*
b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i
- b^3*1i)) + (tan(c + d*x)*(b^7 - 10*a*b^6 + 50*a^2*b^5 - 10*a^3*b^4 + a^4*
b^3))/(2*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b))/(4*
(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i))))*(a - 5*b)*1i)/(2*d*(a*b^2*3i - a
^2*b*3i + a^3*1i - b^3*1i)) - (atan((((5*a - b)*(-a^3*b^3)^(1/2)*((tan(c +
d*x)*(b^7 - 10*a*b^6 + 50*a^2*b^5 - 10*a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*
b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)) - (((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b
^8 - 172*a^4*b^7 + 220*a^5*b^6 - 172*a^6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*
a^9*b^2)/(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 1
5*a^6*b^2) - (tan(c + d*x)*(5*a - b)*(-a^3*b^3)^(1/2)*(16*a^2*b^9 - 80*a^3*
b^8 + 144*a^4*b^7 - 80*a^5*b^6 - 80*a^6*b^5 + 144*a^7*b^4 - 80*a^8*b^3 + 16
```

```
*a^9*b^2))/(8*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)*(a^6 - 4*a^5*b + a^2*b^
4 - 4*a^3*b^3 + 6*a^4*b^2)))*(5*a - b)*(-a^3*b^3)^(1/2))/(4*(3*a^5*b - a^6
+ a^3*b^3 - 3*a^4*b^2)))*1i)/(4*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) + ((
5*a - b)*(-a^3*b^3)^(1/2)*((tan(c + d*x)*(b^7 - 10*a*b^6 + 50*a^2*b^5 - 10*
a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)) +
 (((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b^7 + 220*a^5*b^6 - 172*a^
6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8 - 6*a^7*b + a^2*b^6 - 6*a
^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) + (tan(c + d*x)*(5*a - b)*(-
a^3*b^3)^(1/2)*(16*a^2*b^9 - 80*a^3*b^8 + 144*a^4*b^7 - 80*a^5*b^6 - 80*a^6
*b^5 + 144*a^7*b^4 - 80*a^8*b^3 + 16*a^9*b^2))/(8*(3*a^5*b - a^6 + a^3*b^3
- 3*a^4*b^2)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(5*a - b)*
(-a^3*b^3)^(1/2))/(4*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*1i)/(4*(3*a^5*
b - a^6 + a^3*b^3 - 3*a^4*b^2)))/(((21*a*b^6)/4 - (5*b^7)/4 + (21*a^2*b^5)/
4 - (5*a^3*b^4)/4)/(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a
^5*b^3 + 15*a^6*b^2) + ((5*a - b)*(-a^3*b^3)^(1/2)*((tan(c + d*x)*(b^7 - 10
*a*b^6 + 50*a^2*b^5 - 10*a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*b + a^2*b^4 -
4*a^3*b^3 + 6*a^4*b^2)) - (((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b
^7 + 220*a^5*b^6 - 172*a^6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8
- 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) - (
tan(c + d*x)*(5*a - b)*(-a^3*b^3)^(1/2)*(16*a^2*b^9 - 80*a^3*b^8 + 144*a^4*
b^7 - 80*a^5*b^6 - 80*a^6*b^5 + 144*a^7*b^4 - 80*a^8*b^3 + 16*a^9*b^2))/(8*
(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3
+ 6*a^4*b^2)))*(5*a - b)*(-a^3*b^3)^(1/2))/(4*(3*a^5*b - a^6 + a^3*b^3 - 3*
a^4*b^2))))/(4*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) - ((5*a - b)*(-a^3*b^
3)^(1/2)*((tan(c + d*x)*(b^7 - 10*a*b^6 + 50*a^2*b^5 - 10*a^3*b^4 + a^4*b^3
))/(2*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)) + (((2*a*b^10 - 20
*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b^7 + 220*a^5*b^6 - 172*a^6*b^5 + 80*a^7*b^
4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b
^4 - 20*a^5*b^3 + 15*a^6*b^2) + (tan(c + d*x)*(5*a - b)*(-a^3*b^3)^(1/2)*(1
6*a^2*b^9 - 80*a^3*b^8 + 144*a^4*b^7 - 80*a^5*b^6 - 80*a^6*b^5 + 144*a^7*b^
4 - 80*a^8*b^3 + 16*a^9*b^2))/(8*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)*(a^6
 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(5*a - b)*(-a^3*b^3)^(1/2))
/(4*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2))))/(4*(3*a^5*b - a^6 + a^3*b^3 -
3*a^4*b^2))))*(5*a - b)*(-a^3*b^3)^(1/2)*1i)/(2*d*(3*a^5*b - a^6 + a^3*b^3
- 3*a^4*b^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.473 $\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=212

$$-\frac{b^{5/2}(7a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^4}+\frac{x\left(3a^2-14ab+35b^2\right)}{8(a-b)^4}+\frac{b(a-4b)(3a+b)\tan(c+dx)}{8ad(a-b)^3\left(a+b\tan^2(c+dx)\right)}+\frac{\sin(c+dx)\cos^3(}{4d(a-b)\left(a+b\tan\right.}$$

```
[Out] 1/8*(3*a^2-14*a*b+35*b^2)*x/(a-b)^4-1/2*(7*a-b)*b^(5/2)*arctan(b^(1/2)*tan(
d*x+c)/a^(1/2))/a^(3/2)/(a-b)^4/d+3/8*(a-3*b)*cos(d*x+c)*sin(d*x+c)/(a-b)^2
/d/(a+b*tan(d*x+c)^2)+1/4*cos(d*x+c)^3*sin(d*x+c)/(a-b)/d/(a+b*tan(d*x+c)^2
)+1/8*(a-4*b)*b*(3*a+b)*tan(d*x+c)/a/(a-b)^3/d/(a+b*tan(d*x+c)^2)
```

Rubi [A] time = 0.30, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3675, 414, 527, 522, 203, 205}

$$-\frac{b^{5/2}(7a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^4}+\frac{x\left(3a^2-14ab+35b^2\right)}{8(a-b)^4}+\frac{b(a-4b)(3a+b)\tan(c+dx)}{8ad(a-b)^3\left(a+b\tan^2(c+dx)\right)}+\frac{\sin(c+dx)\cos^3(}{4d(a-b)\left(a+b\tan\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 - 14*a*b + 35*b^2)*x)/(8*(a - b)^4) - ((7*a - b)*b^(5/2)*ArcTan[(Sq
rt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^4*d) + (3*(a - 3*b)*Cos[c
+ d*x]*Sin[c + d*x])/(8*(a - b)^2*d*(a + b*Tan[c + d*x]^2)) + (Cos[c + d*x]
^3*Sin[c + d*x])/(4*(a - b)*d*(a + b*Tan[c + d*x]^2)) + ((a - 4*b)*b*(3*a +
 b)*Tan[c + d*x])/(8*a*(a - b)^3*d*(a + b*Tan[c + d*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d\left(a+b\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-3a+4b-5bx^2}{\left(1+x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \tan(c+dx)\right)}{4(a-b)d}$$

$$= \frac{3(a-3b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{3(a-3b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{(a-4b)b\ldots}{8a(a-b)^3\ldots}$$

$$= \frac{3(a-3b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{(a-4b)b\ldots}{8a(a-b)^3\ldots}$$

$$= \frac{\left(3a^2-14ab+35b^2\right)x}{8(a-b)^4} - \frac{(7a-b)b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^4d} + \frac{3(a-3b)\cos(c+dx\ldots}{8(a-b)^2d\left(a+b\tan\ldots\right.}$$

Mathematica [A] time = 2.12, size = 148, normalized size = 0.70

$$\frac{\frac{16b^{5/2}(b-7a)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + 4\left(3a^2-14ab+35b^2\right)(c+dx) - \frac{16b^3(a-b)\sin(2(c+dx))}{a((a-b)\cos(2(c+dx))+a+b)} + 8(a-3b)(a-b)\sin(2(c+\ldots}{32d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 - 14*a*b + 35*b^2)*(c + d*x) + (16*b^(5/2)*(-7*a + b)*ArcTan[(Sqr
t[b]*Tan[c + d*x])/Sqrt[a]])/a^(3/2) + 8*(a - 3*b)*(a - b)*Sin[2*(c + d*x)]
 - (16*(a - b)*b^3*Sin[2*(c + d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)]))
 + (a - b)^2*Sin[4*(c + d*x)])/(32*(a - b)^4*d)
```

fricas [A] time = 0.60, size = 801, normalized size = 3.78

$$\left[\frac{\left(3\,a^{4}-17\,a^{3}b+49\,a^{2}b^{2}-35\,ab^{3}\right)dx\cos\left(dx+c\right)^{2}+\left(3\,a^{3}b-14\,a^{2}b^{2}+35\,ab^{3}\right)dx-\left(7\,ab^{3}-b^{4}+\left(7\,a^{2}b^{2}-8\,ab^{3}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((3*a^4 - 17*a^3*b + 49*a^2*b^2 - 35*a*b^3)*d*x*cos(d*x + c)^2 + (3*a^
3*b - 14*a^2*b^2 + 35*a*b^3)*d*x - (7*a*b^3 - b^4 + (7*a^2*b^2 - 8*a*b^3 +
b^4)*cos(d*x + c)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2
*(3*a*b + b^2)*cos(d*x + c)^2 - 4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x
 + c))*sqrt(-b/a)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 +
 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) + (2*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b
^3)*cos(d*x + c)^5 + 3*(a^4 - 5*a^3*b + 7*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^3
 + (3*a^3*b - 14*a^2*b^2 + 7*a*b^3 + 4*b^4)*cos(d*x + c))*sin(d*x + c))/((a
^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d*cos(d*x + c)^
2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d), 1/8*((3*a^4 - 1
7*a^3*b + 49*a^2*b^2 - 35*a*b^3)*d*x*cos(d*x + c)^2 + (3*a^3*b - 14*a^2*b^2
 + 35*a*b^3)*d*x + 2*(7*a*b^3 - b^4 + (7*a^2*b^2 - 8*a*b^3 + b^4)*cos(d*x +
 c)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sqrt(b/a)/(b*cos(d
*x + c)*sin(d*x + c))) + (2*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*cos(d*x + c
)^5 + 3*(a^4 - 5*a^3*b + 7*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^3 + (3*a^3*b - 1
4*a^2*b^2 + 7*a*b^3 + 4*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6 - 5*a^5*b +
10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*
a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)]
```

giac [A] time = 2.41, size = 269, normalized size = 1.27

$$-\frac{\frac{4\,b^{3}\tan(dx+c)}{\left(a^{4}-3\,a^{3}b+3\,a^{2}b^{2}-ab^{3}\right)\left(b\tan(dx+c)^{2}+a\right)}-\frac{\left(3\,a^{2}-14\,ab+35\,b^{2}\right)(dx+c)}{a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4\,ab^{3}+b^{4}}+\frac{4\left(7\,ab^{3}-b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)}{\left(a^{5}-4\,a^{4}b+6\,a^{3}b^{2}-4\,a^{2}b^{3}+ab^{4}\right)\sqrt{ab}}-\frac{3\,a\tan(dx+c)^{3}\cdots}{(a\cdots}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*tan(d*x + c)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*(b*tan(d*x +
c)^2 + a)) - (3*a^2 - 14*a*b + 35*b^2)*(d*x + c)/(a^4 - 4*a^3*b + 6*a^2*b^2
 - 4*a*b^3 + b^4) + 4*(7*a*b^3 - b^4)*(pi*floor((d*x + c)/pi + 1/2)*sgn(b)
+ arctan(b*tan(d*x + c)/sqrt(a*b)))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3
 + a*b^4)*sqrt(a*b)) - (3*a*tan(d*x + c)^3 - 11*b*tan(d*x + c)^3 + 5*a*tan(
d*x + c) - 13*b*tan(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(tan(d*x + c
)^2 + 1)^2))/d
```

maple [B] time = 0.95, size = 413, normalized size = 1.95

$$-\frac{b^{3}\tan\left(dx+c\right)}{2d\left(a-b\right)^{4}\left(a+b\left(\tan^{2}\left(dx+c\right)\right)\right)}+\frac{b^{4}\tan\left(dx+c\right)}{2d\left(a-b\right)^{4}a\left(a+b\left(\tan^{2}\left(dx+c\right)\right)\right)}-\frac{7b^{3}\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)}{2d\left(a-b\right)^{4}\sqrt{ab}}+\frac{b^{4}\arctan\left(\frac{\tan(\cdots}{\cdots}\right)}{2d\left(a-b\right)^{4}a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d*b^3/(a-b)^4*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2/d*b^4/(a-b)^4/a*tan(d*
x+c)/(a+b*tan(d*x+c)^2)-7/2/d*b^3/(a-b)^4/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(
```

```
a*b)^(1/2))+1/2/d*b^4/(a-b)^4/a/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2)
)+3/8/d/(a-b)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^2-7/4/d/(a-b)^4/(1+tan(d*
x+c)^2)^2*tan(d*x+c)^3*a*b+11/8/d/(a-b)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*b
^2-9/4/d/(a-b)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a*b+13/8/d/(a-b)^4/(1+tan(d*
x+c)^2)^2*tan(d*x+c)*b^2+5/8/d/(a-b)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^2+35
/8/d/(a-b)^4*arctan(tan(d*x+c))*b^2+3/8/d/(a-b)^4*arctan(tan(d*x+c))*a^2-7/
4/d/(a-b)^4*arctan(tan(d*x+c))*a*b
```

maxima [A] time = 0.77, size = 355, normalized size = 1.67

$$\frac{\frac{(3a^2-14ab+35b^2)(dx+c)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4} - \frac{4(7ab^3-b^4)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4)\sqrt{ab}} + \frac{(3a^2b-11ab^2-4b^3)\tan(dx+c)^5+(3a^3-6a^2b-13ab\cdots}{(a^4b-3a^3b^2+3a^2b^3-ab^4)\tan(dx+c)^6+a^5-3a^4b+3a^3b^2-a^2b^3+(a^5-a^4b-3a\cdots}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 - 14*a*b + 35*b^2)*(d*x + c)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b
^3 + b^4) - 4*(7*a*b^3 - b^4)*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^5 - 4*a^
4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sqrt(a*b)) + ((3*a^2*b - 11*a*b^2 - 4*
b^3)*tan(d*x + c)^5 + (3*a^3 - 6*a^2*b - 13*a*b^2 - 8*b^3)*tan(d*x + c)^3 +
 (5*a^3 - 13*a^2*b - 4*b^3)*tan(d*x + c))/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 -
 a*b^4)*tan(d*x + c)^6 + a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^5 - a^4*b
 - 3*a^3*b^2 + 5*a^2*b^3 - 2*a*b^4)*tan(d*x + c)^4 + (2*a^5 - 5*a^4*b + 3*a
^3*b^2 + a^2*b^3 - a*b^4)*tan(d*x + c)^2))/d
```

mupad [B] time = 17.28, size = 5272, normalized size = 24.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] - ((tan(c + d*x)^5*(11*a*b^2 - 3*a^2*b + 4*b^3))/(8*a*(3*a*b^2 - 3*a^2*b +
a^3 - b^3)) + (tan(c + d*x)^3*(13*a*b^2 + 6*a^2*b - 3*a^3 + 8*b^3))/(8*a*(a
 - b)*(a^2 - 2*a*b + b^2)) + (tan(c + d*x)*(13*a^2*b - 5*a^3 + 4*b^3))/(8*a
*(a - b)*(a^2 - 2*a*b + b^2)))/(d*(a + b*tan(c + d*x)^6 + tan(c + d*x)^2*(2
*a + b) + tan(c + d*x)^4*(a + 2*b))) - (atan((((((((2*a*b^13 - 28*a^2*b^12 +
 (315*a^3*b^11)/2 - (987*a^4*b^10)/2 + 978*a^5*b^9 - 1302*a^6*b^8 + 1197*a^
7*b^7 - 765*a^8*b^6 + 336*a^9*b^5 - 98*a^10*b^4 + (35*a^11*b^3)/2 - (3*a^12
*b^2)/2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 +
 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) - (tan(c + d*x)*(a^2*
3i - a*b*14i + b^2*35i)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168
*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792
*a^10*b^3 + 256*a^11*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)
*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^
2)))*(a^2*3i - a*b*14i + b^2*35i))/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a
^2*b^2)) - (tan(c + d*x)*(16*b^9 - 224*a*b^8 + 2009*a^2*b^7 - 980*a^3*b^6 +
 406*a^4*b^5 - 84*a^5*b^4 + 9*a^6*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^
3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a^2*3i - a*b*14i + b^2*35i
)*1i)/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (((((2*a*b^13 - 28
*a^2*b^12 + (315*a^3*b^11)/2 - (987*a^4*b^10)/2 + 978*a^5*b^9 - 1302*a^6*b^
8 + 1197*a^7*b^7 - 765*a^8*b^6 + 336*a^9*b^5 - 98*a^10*b^4 + (35*a^11*b^3)/
2 - (3*a^12*b^2)/2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 8
4*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) + (tan(c +
 d*x)*(a^2*3i - a*b*14i + b^2*35i)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4
*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9
*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 +
 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3
```

+ 15*a^6*b^2)))*(a^2*3i - a*b*14i + b^2*35i))/(16*(a^4 - 4*a^3*b - 4*a*b^3
+ b^4 + 6*a^2*b^2)) + (tan(c + d*x)*(16*b^9 - 224*a*b^8 + 2009*a^2*b^7 - 98
0*a^3*b^6 + 406*a^4*b^5 - 84*a^5*b^4 + 9*a^6*b^3))/(32*(a^8 - 6*a^7*b + a^2
*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a^2*3i - a*b*14
i + b^2*35i)*1i)/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))/((((((2*
a*b^13 - 28*a^2*b^12 + (315*a^3*b^11)/2 - (987*a^4*b^10)/2 + 978*a^5*b^9 -
1302*a^6*b^8 + 1197*a^7*b^7 - 765*a^8*b^6 + 336*a^9*b^5 - 98*a^10*b^4 + (35
*a^11*b^3)/2 - (3*a^12*b^2)/2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*
a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2)
 - (tan(c + d*x)*(a^2*3i - a*b*14i + b^2*35i)*(256*a^2*b^11 - 1792*a^3*b^10
 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5
 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(512*(a^4 - 4*a^3*b - 4*a*
b^3 + b^4 + 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 -
20*a^5*b^3 + 15*a^6*b^2)))*(a^2*3i - a*b*14i + b^2*35i))/(16*(a^4 - 4*a^3*b
 - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (tan(c + d*x)*(16*b^9 - 224*a*b^8 + 2009*a
^2*b^7 - 980*a^3*b^6 + 406*a^4*b^5 - 84*a^5*b^4 + 9*a^6*b^3))/(32*(a^8 - 6*
a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a^2*
3i - a*b*14i + b^2*35i))/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) -
 ((651*a*b^9)/64 - (35*b^10)/16 + (1275*a^2*b^8)/32 - (451*a^3*b^7)/16 + (2
67*a^4*b^6)/32 - (63*a^5*b^5)/64)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 +
36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b
^2) + (((((2*a*b^13 - 28*a^2*b^12 + (315*a^3*b^11)/2 - (987*a^4*b^10)/2 + 9
78*a^5*b^9 - 1302*a^6*b^8 + 1197*a^7*b^7 - 765*a^8*b^6 + 336*a^9*b^5 - 98*a
^10*b^4 + (35*a^11*b^3)/2 - (3*a^12*b^2)/2)/(9*a^10*b - a^11 + a^2*b^9 - 9*
a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3
 - 36*a^9*b^2) + (tan(c + d*x)*(a^2*3i - a*b*14i + b^2*35i)*(256*a^2*b^11 -
1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 -
 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(512*(a^4 - 4
*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 +
15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a^2*3i - a*b*14i + b^2*35i))/(16*(
a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(c + d*x)*(16*b^9 - 224*a
*b^8 + 2009*a^2*b^7 - 980*a^3*b^6 + 406*a^4*b^5 - 84*a^5*b^4 + 9*a^6*b^3))/
(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6
b^2)))(a^2*3i - a*b*14i + b^2*35i))/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 +
6*a^2*b^2))))*(a^2*3i - a*b*14i + b^2*35i)*1i)/(8*d*(a^4 - 4*a^3*b - 4*a*b^
3 + b^4 + 6*a^2*b^2)) - (atan(((((tan(c + d*x)*(16*b^9 - 224*a*b^8 + 2009*a
^2*b^7 - 980*a^3*b^6 + 406*a^4*b^5 - 84*a^5*b^4 + 9*a^6*b^3))/(32*(a^8 - 6*
a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) - ((7*
a - b)*(-a^3*b^5)^(1/2)*((2*a*b^13 - 28*a^2*b^12 + (315*a^3*b^11)/2 - (987*
a^4*b^10)/2 + 978*a^5*b^9 - 1302*a^6*b^8 + 1197*a^7*b^7 - 765*a^8*b^6 + 336
*a^9*b^5 - 98*a^10*b^4 + (35*a^11*b^3)/2 - (3*a^12*b^2)/2)/(9*a^10*b - a^11
 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^
4 + 84*a^8*b^3 - 36*a^9*b^2) - (tan(c + d*x)*(7*a - b)*(-a^3*b^5)^(1/2)*(25
6*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3
584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/
(128*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)*(a^8 - 6*a^7*b + a^2
*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2))))/(4*(a^7 - 4*a^6
*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))*(7*a - b)*(-a^3*b^5)^(1/2)*1i)/(4*(
a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) + (((tan(c + d*x)*(16*b^9
 - 224*a*b^8 + 2009*a^2*b^7 - 980*a^3*b^6 + 406*a^4*b^5 - 84*a^5*b^4 + 9*a^
6*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3
+ 15*a^6*b^2)) + ((7*a - b)*(-a^3*b^5)^(1/2)*((2*a*b^13 - 28*a^2*b^12 + (31
5*a^3*b^11)/2 - (987*a^4*b^10)/2 + 978*a^5*b^9 - 1302*a^6*b^8 + 1197*a^7*b^
7 - 765*a^8*b^6 + 336*a^9*b^5 - 98*a^10*b^4 + (35*a^11*b^3)/2 - (3*a^12*b^2
)/2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126
*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) + (tan(c + d*x)*(7*a - b)
*(-a^3*b^5)^(1/2)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b
^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*
b^3 + 256*a^11*b^2))/(128*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)

```
*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^
2))))/(4*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))*(7*a - b)*(-a^
3*b^5)^(1/2)*1i)/(4*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))/(((
651*a*b^9)/64 - (35*b^10)/16 + (1275*a^2*b^8)/32 - (451*a^3*b^7)/16 + (267*
a^4*b^6)/32 - (63*a^5*b^5)/64)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*
a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2)
 + (((tan(c + d*x)*(16*b^9 - 224*a*b^8 + 2009*a^2*b^7 - 980*a^3*b^6 + 406*a
^4*b^5 - 84*a^5*b^4 + 9*a^6*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5
+ 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) - ((7*a - b)*(-a^3*b^5)^(1/2)*((2*
a*b^13 - 28*a^2*b^12 + (315*a^3*b^11)/2 - (987*a^4*b^10)/2 + 978*a^5*b^9 -
1302*a^6*b^8 + 1197*a^7*b^7 - 765*a^8*b^6 + 336*a^9*b^5 - 98*a^10*b^4 + (35
*a^11*b^3)/2 - (3*a^12*b^2)/2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*
a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2)
 - (tan(c + d*x)*(7*a - b)*(-a^3*b^5)^(1/2)*(256*a^2*b^11 - 1792*a^3*b^10 +
 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 +
 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(128*(a^7 - 4*a^6*b + a^3*b^
4 - 4*a^4*b^3 + 6*a^5*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^
4 - 20*a^5*b^3 + 15*a^6*b^2))))/(4*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6
*a^5*b^2)))*(7*a - b)*(-a^3*b^5)^(1/2))/(4*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4
*b^3 + 6*a^5*b^2)) - (((tan(c + d*x)*(16*b^9 - 224*a*b^8 + 2009*a^2*b^7 - 9
80*a^3*b^6 + 406*a^4*b^5 - 84*a^5*b^4 + 9*a^6*b^3))/(32*(a^8 - 6*a^7*b + a^
2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) + ((7*a - b)*(-a
^3*b^5)^(1/2)*((2*a*b^13 - 28*a^2*b^12 + (315*a^3*b^11)/2 - (987*a^4*b^10)/
2 + 978*a^5*b^9 - 1302*a^6*b^8 + 1197*a^7*b^7 - 765*a^8*b^6 + 336*a^9*b^5 -
 98*a^10*b^4 + (35*a^11*b^3)/2 - (3*a^12*b^2)/2)/(9*a^10*b - a^11 + a^2*b^9
 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8
*b^3 - 36*a^9*b^2) + (tan(c + d*x)*(7*a - b)*(-a^3*b^5)^(1/2)*(256*a^2*b^11
 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^
6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(128*(a^7
- 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a
^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2))))/(4*(a^7 - 4*a^6*b + a^3*b
^4 - 4*a^4*b^3 + 6*a^5*b^2)))*(7*a - b)*(-a^3*b^5)^(1/2))/(4*(a^7 - 4*a^6*b
 + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2))))*(7*a - b)*(-a^3*b^5)^(1/2)*1i)/(2*d*
(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.474 $\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=95

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\sec(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(m+2p+1)}\,{}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(m+2p+1);\frac{1}{2}(2p+3);\sin\right.}{f(2p+1)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+1/2*m+p)*hypergeom([1/2+p, 1/2+1/2*m+p],[3/2+p],sin(f*x
+e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1+2*p)
```

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3658, 2617}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\sec(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(m+2p+1)}\,{}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(m+2p+1);\frac{1}{2}(2p+3);\sin\right.}{f(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + m + 2*p)/2)*Hypergeometric2F1[(1 + 2*p)/2, (1 + m +
 2*p)/2, (3 + 2*p)/2, Sin[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x]*(b*Ta
n[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p\,dx &= \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int (d\sec(e+fx))^m \tan^{2p}(e+fx)\,dx\\ &= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+2p)}\,{}_2F_1\left(\frac{1}{2}(1+2p),\frac{1}{2}(1+m+2p);\frac{1}{2}(3+2p);\sin^2(e\right.}{f(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.19, size = 81, normalized size = 0.85

$$\frac{\cot(e+fx)\left(-\tan^2(e+fx)\right)^{\frac{1}{2}-p}\left(b\tan^2(e+fx)\right)^p (d\sec(e+fx))^m\,{}_2F_1\left(\frac{m}{2},\frac{1}{2}-p;\frac{m+2}{2};\sec^2(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
[Out] (Cot[e + f*x]*Hypergeometric2F1[m/2, 1/2 - p, (2 + m)/2, Sec[e + f*x]^2]*(d
*Sec[e + f*x])^m*(-Tan[e + f*x]^2)^(1/2 - p)*(b*Tan[e + f*x]^2)^p)/(f*m)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2\right)^p\left(d\sec\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2)^p*(d*sec(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2\right)^p\left(d\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2)^p*(d*sec(f*x + e))^m, x)
```

maple [F] time = 2.84, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(fx+e\right)\right)^m\left(b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
[Out] int((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2\right)^p\left(d\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2)^p*(d*sec(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^m\left(b\tan\left(e+f\,x\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
[Out] int((d/cos(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan^2\left(e+fx\right)\right)^p\left(d\sec\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**p*(d*sec(e + f*x))**m, x)
```

3.475 $\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=108

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} F_1\left(\frac{1}{2};1-\frac{m}{2},-p;\frac{3}{2};-\tan\right.}{f}$$

```
[Out] AppellF1(1/2,1-1/2*m,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(d*sec(f*x+e))
^m*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/((sec(f*x+e)^2)^(1/2*m))/((1+b*tan(f*x
+e)^2/a)^p)
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3679, 430, 429}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} F_1\left(\frac{1}{2};1-\frac{m}{2},-p;\frac{3}{2};-\tan\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*
(d*Sec[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(Sec[e + f*x]^
2)^(m/2)*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3679

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(f
f*(d*Sec[e + f*x])^m)/(f*(Sec[e + f*x]^2)^(m/2)), Subst[Int[(1 + ff^2*x^2)^
(m/2 - 1)*(a + b*ff^2*x^2)^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int \left(1+x^2\right)^{-1+\frac{m}{2}} \left(a+bx\right.\right.}{f}$$

$$= \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2} \left(a+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2}{}\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1-\frac{m}{2},-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\sec(e+fx))^m \text{s}}{}$$

Mathematica [B] time = 16.23, size = 2033, normalized size = 18.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
a)]*(d*Sec[e + f*x])^m*(Sec[e + f*x]^2)^(-1 + m/2)*Tan[e + f*x]*(a + b*Tan[
e + f*x]^2)^(2*p))/(f*(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)*((6*a*b*p
*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*
(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(-1 + p))/(3*a
*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]
+ (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (3*a*AppellF1[1/2, 1 - m/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(Sec[e + f*x]^2)^(m/2)*(a + b
*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (6*a*(-1
+ m/2)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*(Sec[e + f*x]^2)^(-1 + m/2)*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p)/
(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (3*a*(Sec[e + f*x]^2)^(-1 + m
/2)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*(1 - m/2
)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]
*Sec[e + f*x]^2*Tan[e + f*x])/3)*(a + b*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/
2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*App
ellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] +
 a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/a)])*Tan[e + f*x]^2) - (3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(Sec[e + f*x]^2)^(-1 + m/2)*Tan[e + f*x]*
(a + b*Tan[e + f*x]^2)^p*(2*(2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Sec[e + f*x]^2*Tan[e + f*
x] + 3*a*((2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*(1 - m/2)*Appell
F1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e +
```

```
f*x]^2*Tan[e + f*x])/3) + Tan[e + f*x]^2*(2*b*p*((-6*b*(1 - p)*AppellF1[5/
2, 1 - m/2, 2 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f
*x]^2*Tan[e + f*x])/(5*a) - (6*(1 - m/2)*AppellF1[5/2, 2 - m/2, 1 - p, 7/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/5)
+ a*(-2 + m)*((6*b*p*AppellF1[5/2, 2 - m/2, 1 - p, 7/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a) - (6*(2 - m/2)*Ap
pellF1[5/2, 3 - m/2, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec
[e + f*x]^2*Tan[e + f*x])/5))))/(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2
 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)
^2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\left(d\sec\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\left(d\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

maple [F] time = 2.87, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(fx+e\right)\right)^m\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2+a\right)^p\left(d\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^p\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p*(d/cos(e + f*x))^m,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^p*(d/cos(e + f*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.476 $\int (d\sec(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=97

$$\frac{\tan(e+fx)(d\sec(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(m+np+1)} \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(\frac{1}{2}(np+1), \frac{1}{2}(m+np+1); \frac{1}{2}(np+\right.}{f(np+1)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+1/2*m+1/2)*hypergeom([1/2*n*p+1/2, 1/2*n*p+1/2*m+1/
2],[1/2*n*p+3/2],sin(f*x+e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)*(b*(c*tan(f*x+e)
)^n)^p/f/(n*p+1)
```

Rubi [A] time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3659, 2617}

$$\frac{\tan(e+fx)(d\sec(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(m+np+1)} \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(\frac{1}{2}(np+1), \frac{1}{2}(m+np+1); \frac{1}{2}(np+\right.}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + m + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (1 + m +
 n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\sec(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx = \left((c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p\right) \int (d\sec(e+fx))^m (c\tan$$

$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+np)} {}_2F_1\left(\frac{1}{2}(1+np), \frac{1}{2}(1+m+np); \frac{1}{2}(3+np);\right.}{f(1+}$$

Mathematica [A] time = 0.18, size = 89, normalized size = 0.92

$$\frac{\cot(e+fx)(d\sec(e+fx))^m \left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)} \left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(\frac{m}{2}, \frac{1}{2}(1-np); \frac{m+2}{2}; \sec^2(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]*Hypergeometric2F1[m/2, (1 - n*p)/2, (2 + m)/2, Sec[e + f*x]^2
]*(d*Sec[e + f*x])^m*(-Tan[e + f*x]^2)^((1 - n*p)/2)*(b*(c*Tan[e + f*x])^n)
^p)/(f*m)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\left(d\sec\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*sec(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\left(d\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*sec(f*x + e))^m, x)
```

maple [F] time = 1.25, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(fx+e\right)\right)^{m}\left(b\left(c\tan\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\left(d\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*sec(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^m\,{\left(b\,{\left(c\,\tan\left(e+f\,x\right)\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int((d/cos(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\left(d\sec\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*sec(e + f*x))**m, x)
```

3.477 $\int \sec^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=99

$$\frac{\tan^5(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+5)}+\frac{2\tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}+\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)+2*tan(f*x+e)^3*(b*(c*tan(f*x+e)
)^n)^p/f/(n*p+3)+tan(f*x+e)^5*(b*(c*tan(f*x+e))^n)^p/f/(n*p+5)
```

Rubi [A] time = 0.13, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3659, 2607, 270}

$$\frac{\tan^5(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+5)}+\frac{2\tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}+\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p)) + (2*Tan[e + f*x]^3*(
b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p)) + (Tan[e + f*x]^5*(b*(c*Tan[e + f*x]
)^n)^p)/(f*(5 + n*p))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sec^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec^6(e+fx)(c\tan(e+f$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int (cx)^{np}\left(1+x^2\right)^2 d\right.}{f}$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int\left((cx)^{np}+\frac{2(cx)^{2+np}}{c^2}\right.\right.}{f}$$

$$= \frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)} + \frac{2\tan^3(e+fx)\left(b(c\tan(e+fx))^n\right.}{f(3+np)}$$

Mathematica [A] time = 2.18, size = 122, normalized size = 1.23

$$\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\left(\tan^2(e+fx)\sec^4(e+fx)\left(2(np+3)\cos(2(e+fx))+\cos(4(e+fx))+n^2p^2\right.\right.}{f(np+1)(np+3)(np+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p*((8 + 6*n*p + n^2*p^2 + 2*(3 + n*p)*
Cos[2*(e + f*x)] + Cos[4*(e + f*x)])*Sec[e + f*x]^4*Tan[e + f*x]^2 + 8*(-Ta
n[e + f*x]^2)^((1 - n*p)/2)))/(f*(1 + n*p)*(3 + n*p)*(5 + n*p))
```

fricas [A] time = 0.45, size = 107, normalized size = 1.08

$$\frac{\left(n^2p^2 + 8\cos\left(fx+e\right)^4 + 4\left(np+1\right)\cos\left(fx+e\right)^2 + 4\,np + 3\right)e^{\left(np\log\left(\frac{c\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)+p\log\left(b\right)\right)}\sin\left(fx+e\right)}{\left(fn^3p^3 + 9\,fn^2p^2 + 23\,fnp + 15\,f\right)\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] (n^2*p^2 + 8*cos(f*x + e)^4 + 4*(n*p + 1)*cos(f*x + e)^2 + 4*n*p + 3)*e^(n*
p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))*sin(f*x + e)/((f*n^3*p^3 + 9
*f*n^2*p^2 + 23*f*n*p + 15*f)*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 6.28Unable to divide, perha
ps due to rounding error%%%{1,[0,1,4,0,0]%%%}+%%%{2,[0,1,2,2,0]%%%}+%%%{1,[
0,1,0,4,0]%%%} / %%%{1,[0,0,5,0,1]%%%} Error: Bad Argument Value
```

maple [F] time = 2.18, size = 0, normalized size = 0.00

$$\int \left(\sec^6\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.74, size = 106, normalized size = 1.07

$$\frac{\frac{b^p c^{np} \left(\tan\left(fx+e\right)^n\right)^p \tan\left(fx+e\right)^5}{np+5} + \frac{2\, b^p c^{np} \left(\tan\left(fx+e\right)^n\right)^p \tan\left(fx+e\right)^3}{np+3} + \frac{b^p c^{np} \left(\tan\left(fx+e\right)^n\right)^p \tan\left(fx+e\right)}{np+1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] (b^p*c^(n*p)*(tan(f*x + e)^n)^p*tan(f*x + e)^5/(n*p + 5) + 2*b^p*c^(n*p)*(t
an(f*x + e)^n)^p*tan(f*x + e)^3/(n*p + 3) + b^p*c^(n*p)*(tan(f*x + e)^n)^p*
tan(f*x + e)/(n*p + 1))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\left(c\tan\left(e+f\,x\right)\right)^n\right)^p}{\cos\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^6,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p \sec^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**6, x)
```

3.478 $\int \sec^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=65

$$\frac{\tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}+\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)+tan(f*x+e)^3*(b*(c*tan(f*x+e))^
n)^p/f/(n*p+3)
```

Rubi [A] time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3659, 2607, 14}

$$\frac{\tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}+\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p)) + (Tan[e + f*x]^3*(b*
(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sec^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec^4(e+fx)(c\tan(e+f\\ &= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int (cx)^{np}\left(1+x^2\right)\,dx\right.}{f}\\ &= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int\left((cx)^{np}+\frac{(cx)^{2+np}}{c^2}\right.\right.}{f}\\ &= \frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}+\frac{\tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)}{f(3+np)}\end{aligned}$$

Mathematica [A] time = 2.13, size = 87, normalized size = 1.34

$$\frac{\cot(e+fx)\left(2\left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)}+\tan^2(e+fx)\left((np+1)\sec^2(e+fx)+2\right)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)(np+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p*((2 + (1 + n*p)*Sec[e + f*x]^2)*Tan[
e + f*x]^2 + 2*(-Tan[e + f*x]^2)^((1 - n*p)/2)))/(f*(1 + n*p)*(3 + n*p))
```

fricas [A] time = 0.51, size = 75, normalized size = 1.15

$$\frac{\left(np+2\,\cos\left(fx+e\right)^2+1\right)e^{\left(np\log\left(\frac{c\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\right)+p\log\left(b\right)\right)}\sin\left(fx+e\right)}{\left(fn^2p^2+4\,fnp+3\,f\right)\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] (n*p + 2*cos(f*x + e)^2 + 1)*e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*lo
g(b))*sin(f*x + e)/((f*n^2*p^2 + 4*f*n*p + 3*f)*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 4.66Unable to divide, perha
ps due to rounding error%%%{1,[0,1,2,0,0]%%%}+%%%{1,[0,1,0,2,0]%%%} / %%%{1
,[0,0,3,0,1]%%%} Error: Bad Argument Value
```

maple [F] time = 2.14, size = 0, normalized size = 0.00

$$\int \left(\sec^4\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.76, size = 71, normalized size = 1.09

$$\frac{\frac{b^pc^{np}\left(\tan\left(fx+e\right)^n\right)^p\tan\left(fx+e\right)^3}{np+3}+\frac{b^pc^{np}\left(\tan\left(fx+e\right)^n\right)^p\tan\left(fx+e\right)}{np+1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] (b^p*c^(n*p)*(tan(f*x + e)^n)^p*tan(f*x + e)^3/(n*p + 3) + b^p*c^(n*p)*(tan
(f*x + e)^n)^p*tan(f*x + e)/(n*p + 1))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p}{\cos\left(e+fx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^4,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p \sec^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**4, x)
```

3.479 $\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=31

$$\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A] time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3659, 2607, 32}

$$\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec^2(e+fx)(c\tan(e+fx))\\ &= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int (cx)^{np}\,dx, x, \tan(e+\right.}{f}\\ &= \frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.00

$$\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

fricas [A] time = 0.55, size = 49, normalized size = 1.58

$$\frac{e^{\left(np\log\left(\frac{c\sin(fx+e)}{\cos(fx+e)}\right)+p\log(b)\right)}\sin\left(fx+e\right)}{\left(fnp+f\right)\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))*sin(f*x + e)/((f*n*p +
f)*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 3.05Unable to divide, perha
ps due to rounding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,1]%%%} Error: Ba
d Argument Value
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(\sec^2\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.89, size = 35, normalized size = 1.13

$$\frac{b^p c^{np}\left(\tan\left(fx+e\right)^n\right)^p\tan\left(fx+e\right)}{\left(np+1\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] b^p*c^(n*p)*(tan(f*x + e)^n)^p*tan(f*x + e)/((n*p + 1)*f)
```

mupad [B] time = 13.29, size = 31, normalized size = 1.00

$$\frac{\tan\left(e+f\,x\right)\,{\left(b\,{\left(c\,\tan\left(e+f\,x\right)\right)}^n\right)}^p}{f\,\left(n\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^2,x)
```

```
[Out] (tan(e + f*x)*(b*(c*tan(e + f*x))^n)^p)/(f*(n*p + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\sec^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**2, x)
```

3.480 $\int \left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=61

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*(c*ta
n(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{np}\,dx\\
&= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{np}}{c^2+x^2}\,dx,x,c\tan(e+fx)\right)}{f}\\
&= \frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.97

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f + f*n*p)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(b\left(c\tan\left(e+fx\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \left(c \tan \left(e + fx\right)\right)^{n}\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p, x)
```

3.481 $\int \cos^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=61

$$\frac{\tan(e+fx)\,{}_2F_1\left(2,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] hypergeom([2, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*(c*ta
n(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3659, 2607, 364}

$$\frac{\tan(e+fx)\,{}_2F_1\left(2,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[2, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cos^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^2(e+fx)(c\tan(e+fx)$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{(cx)^{np}}{\left(1+x^2\right)^2}\,dx,x,\tan(e+\right.}{f}$$

$$= \frac{{}_2F_1\left(2,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\left(b(c\tan(e+\right.}{f(1+np)}$$

Mathematica [C] time = 5.51, size = 1060, normalized size = 17.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (2*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] - 4*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + 4*AppellF1[(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[e + f*x]^2*Tan[(e + f*x)/2]*(b*
(c*Tan[e + f*x])^n)^p)/(f*((AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[(1 + n*p)/2, n*p, 2, (3 +
 n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*AppellF1[(1 + n*p)/2,
 n*p, 3, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*
x)/2]^2 + n*p*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*AppellF1[(1 + n*p)/2, n*p, 3, (3 +
 n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Sec[e
 + f*x] + (2*(1 + n*p)*(-AppellF1[(3 + n*p)/2, n*p, 2, (5 + n*p)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 8*AppellF1[(3 + n*p)/2, n*p, 3, (5 + n*
p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 12*AppellF1[(3 + n*p)/2, n
*p, 4, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*p*AppellF1
[(3 + n*p)/2, 1 + n*p, 1, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] - 4*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + 4*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 3, (5 + n*
p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2]^2)/(3 + n*p) - 2*n*p*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[(1 + n*p)/2, n*p, 2, (
3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*AppellF1[(1 + n*p)
/2, n*p, 3, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[e +
f*x]*Tan[(e + f*x)/2]^2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^2, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int\left(\cos^{2}\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n} b\right)^{p} \cos\left(fx+e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(e+f\,x\right)^{2} \left(b\left(c\tan\left(e+f\,x\right)\right)^{n}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(cos(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p} \cos^{2}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cos(e + f*x)**2, x)
```

3.482 $\int \sec^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=93

$$\frac{\tan(e+fx)\sec^3(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+4)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+4);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+f\right.}{f(np+1)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+2)*hypergeom([1/2*n*p+2, 1/2*n*p+1/2],[1/2*n*p+3/2]
,sin(f*x+e)^2)*sec(f*x+e)^3*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A] time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3659, 2617}

$$\frac{\tan(e+fx)\sec^3(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+4)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+4);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+f\right.}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((4 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (4 + n*p)/2,
 (3 + n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*Tan[e + f*x]*(b*(c*Tan[e + f*x
])^n)^p)/(f*(1 + n*p))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sec^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec^3(e+fx)(c\tan(e+f\\ &= \frac{\cos^2(e+fx)^{\frac{1}{2}(4+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(4+np);\frac{1}{2}(3+np);\sin^2(e+f\right.}{f(1+np)}\end{aligned}$$

Mathematica [A] time = 0.11, size = 81, normalized size = 0.87

$$\frac{\csc(e+fx)\sec^2(e+fx)\left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(1-np);\frac{5}{2};\sec^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Csc[e + f*x]*Hypergeometric2F1[3/2, (1 - n*p)/2, 5/2, Sec[e + f*x]^2]*Sec[
e + f*x]^2*(-Tan[e + f*x]^2)^((1 - n*p)/2)*(b*(c*Tan[e + f*x])^n)^p)/(3*f)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\sec\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*sec(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e)^3, x)
```

maple [F] time = 2.06, size = 0, normalized size = 0.00

$$\int\left(\sec^{3}\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\left(c\tan\left(e+f\,x\right)\right)^{n}\right)^{p}}{\cos\left(e+f\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^3,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\sec^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**3, x)
```

3.483 $\int \sec(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=91

$$\frac{\tan(e+fx)\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+2)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+2);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+1)*hypergeom([1/2*n*p+1, 1/2*n*p+1/2],[1/2*n*p+3/2]
,sin(f*x+e)^2)*sec(f*x+e)*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A] time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3659, 2617}

$$\frac{\tan(e+fx)\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+2)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+2);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((2 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (2 + n*p)/2,
 (3 + n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*Tan[e + f*x]*(b*(c*Tan[e + f*x])
^n)^p)/(f*(1 + n*p))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec(e+fx)(c\tan(e+fx))^{np}\\ &= \frac{\cos^2(e+fx)^{\frac{1}{2}(2+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(2+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\mathrm{s}}{f(1+np)}\end{aligned}$$

Mathematica [A] time = 0.08, size = 70, normalized size = 0.77

$$\frac{\csc(e+fx)\left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{3}{2};\sec^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Csc[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, 3/2, Sec[e + f*x]^2]*(-Ta
n[e + f*x]^2)^((1 - n*p)/2)*(b*(c*Tan[e + f*x])^n)^p)/f
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*sec(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e), x)
```

maple [F] time = 12.00, size = 0, normalized size = 0.00

$$\int\sec\left(fx+e\right)\left(b\left(c\tan\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\left(c\tan\left(e+f\,x\right)\right)^{n}\right)^{p}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x),x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x), x)
```

3.484 $\int \cos(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=79

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p)*hypergeom([1/2*n*p, 1/2*n*p+1/2],[1/2*n*p+3/2],sin
(f*x+e)^2)*sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3659, 2617}

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((n*p)/2)*Hypergeometric2F1[(n*p)/2, (1 + n*p)/2, (3 + n*
p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cos(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos(e+fx)(c\tan(e+fx$$

$$= \frac{\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+f}{f(1+np)}$$

Mathematica [C] time = 3.61, size = 482, normalized size = 6.10

$$\frac{}{2f(np+1)\left((np+3)F_1\left(\frac{1}{2}(np+1);np,1;\frac{1}{2}(np+3);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\left(\tan^2\left(\frac{1}{2}(e+fx\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((3 + n*p)*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] - 2*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[2*(e + f*x)]*(b*(c*Tan[e + f*x])^n)^
p)/(2*f*(1 + n*p)*((3 + n*p)*AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((3 + n*p)*AppellF1[(1 + n*p)/2,
n*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[(
3 + n*p)/2, n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] -
 4*AppellF1[(3 + n*p)/2, n*p, 3, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] - n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 1, (5 + n*p)/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 2, (5
 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e), x)
```

maple [F] time = 15.82, size = 0, normalized size = 0.00

$$\int\cos\left(fx+e\right)\left(b\left(c\tan\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(e+f\,x\right)\,{\left(b\,{\left(c\,\tan\left(e+f\,x\right)\right)}^{n}\right)}^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(cos(e + f*x)*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\cos\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cos(e + f*x), x)
```

3.485 $\int \cos^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=82

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{1}{2}(np-2),\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p)*hypergeom([1/2*n*p-1, 1/2*n*p+1/2],[1/2*n*p+3/2],s
in(f*x+e)^2)*sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3659, 2617}

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{1}{2}(np-2),\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((n*p)/2)*Hypergeometric2F1[(-2 + n*p)/2, (1 + n*p)/2, (3
 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n
*p))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cos^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^3(e+fx)(c\tan(e+fx)$$

$$= \frac{\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{1}{2}(-2+np),\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin}{f(1+np)}$$

Mathematica [C] time = 6.37, size = 1552, normalized size = 18.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((6 + 2*n*p)*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] - 6*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 12*AppellF1[(1 + n*p)/2, n*p, 3, (3 +
 n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 8*AppellF1[(1 + n*p)/2,
 n*p, 4, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*
x)/2]^3*Cos[e + f*x]^3*Sin[(e + f*x)/2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n
*p)*(-AppellF1[(3 + n*p)/2, n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + 12*AppellF1[(3 + n*p)/2, n*p, 3, (5 + n*p)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] - 36*AppellF1[(3 + n*p)/2, n*p, 4, (5 + n*p)/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 32*AppellF1[(3 + n*p)/2, n*p,
5, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*p*AppellF1[(3
+ n*p)/2, 1 + n*p, 1, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 - 6*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + 12*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 3, (5 + n*p)/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 8*n*p*AppellF1[(3 + n*p)/2, 1
 + n*p, 4, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + n*p
)*AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Cos[(e + f*x)/2]^2 - 18*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 - 6*n*p*Appe
llF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Cos[(e + f*x)/2]^2 + 36*AppellF1[(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 12*n*p*AppellF1[
(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
Cos[(e + f*x)/2]^2 - 8*(3 + n*p)*AppellF1[(1 + n*p)/2, n*p, 4, (3 + n*p)/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + AppellF1[(3
+ n*p)/2, n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos
[e + f*x] - 12*AppellF1[(3 + n*p)/2, n*p, 3, (5 + n*p)/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] + 36*AppellF1[(3 + n*p)/2, n*p, 4, (5
+ n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] - 32*Appell
F1[(3 + n*p)/2, n*p, 5, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Cos[e + f*x] - n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 1, (5 + n*p)/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] + 6*n*p*AppellF1[(3 + n*p)/
2, 1 + n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e
+ f*x] - 12*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 3, (5 + n*p)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] + 8*n*p*AppellF1[(3 + n*p)/2, 1 +
 n*p, 4, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]
))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\cos\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b\right)^{p}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^3, x)
```

maple [F] time = 24.82, size = 0, normalized size = 0.00

$$\int \left(\cos^{3}\left(fx+e\right)\right)\left(b\left(c\tan\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
[Out] int(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^{n} b\right)^{p}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^{3}\left(b\left(c\tan\left(e+f\,x\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int(cos(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Timed out
```

3.486 $\int (d\sec(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=30

$$\text{Int}\left((d\sec(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

[Out] Unintegrable((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d\sec(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

[In] Int[(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]

[Out] Defer[Int][(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]

Rubi steps

$$\int (d\sec(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int (d\sec(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 2.84, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]

[Out] Integrate[(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p\left(d\sec\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")

[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*sec(f*x + e))^m, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p\left(d\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")

[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*sec(f*x + e))^m, x)

maple [A] time = 5.48, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(f x+e\right)\right)^{m} \left(a+b \left(c \tan \left(f x+e\right)\right)^{n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c \tan \left(f x+e\right)\right)^{n} b+a\right)^{p} \left(d \sec \left(f x+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*sec(f*x + e))^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(a+b\,{\left(c\,\mathrm{tan}\left(e+f\,x\right)\right)}^n\right)}^p\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p*(d/cos(e + f*x))^m,x)
```

```
[Out] int((a + b*(c*tan(e + f*x))^n)^p*(d/cos(e + f*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

3.487 $\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Defer[Int][Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 6.08, size = 0, normalized size = 0.00

$$\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Integrate[Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p \sec\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p \sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^3, x)
```

maple [**A**] time = 1.73, size = 0, normalized size = 0.00

$$\int \left(\sec^3\left(fx+e\right)\right)\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
[Out] int(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p \sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^3, x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\left(c\tan\left(e+f\,x\right)\right)^n\right)^p}{\cos\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^3,x)
[Out] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*(c*tan(f*x+e))**n)**p,x)
[Out] Timed out
```

3.488 $\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Defer[Int][Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 1.87, size = 0, normalized size = 0.00

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Integrate[Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e), x)
```

maple [A] time = 1.29, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+b\left(c\tan\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
[Out] int(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\left(a+b\left(c\tan\left(e+f\,x\right)\right)^{n}\right)^{p}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x),x)
[Out] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((a + b*(c*tan(e + f*x))**n)**p*sec(e + f*x), x)
```

3.489 $\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Defer[Int][Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 3.48, size = 0, normalized size = 0.00

$$\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Integrate[Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.44, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
[Out] int(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c \tan\left(fx + e\right)\right)^{n} b + a\right)^{p} \cos\left(fx + e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e + f\,x\right)\,\left(a + b\,\left(c\tan\left(e + f\,x\right)\right)^{n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b*(c*tan(e + f*x))^n)^p,x)
[Out] int(cos(e + f*x)*(a + b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))**n)**p,x)
[Out] Timed out
```

3.490 $\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Defer[Int][Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 11.87, size = 0, normalized size = 0.00

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Integrate[Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p\cos\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^3, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 4.51, size = 0, normalized size = 0.00

$$\int \left(\cos^3\left(fx+e\right)\right)\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(c\tan\left(fx+e\right)\right)^{n}b+a\right)^{p}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int\cos\left(e+f\,x\right)^{3}\,\left(a+b\,\left(c\,\tan\left(e+f\,x\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(cos(e + f*x)^3*(a + b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

3.491 $\int \sec^6(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=244

$$\frac{\tan^5(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{n},-p;\frac{n+5}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{5f}+\frac{2\tan^3(e+fx)\left(a+\right.}{}$$

```
[Out] hypergeom([1/n, -p],[1+1/n],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)*(a+b*(c*tan(f
*x+e))^n)^p/f/((1+b*(c*tan(f*x+e))^n/a)^p)+2/3*hypergeom([-p, 3/n],[(3+n)/n
],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p/f/((1+b*(c*t
an(f*x+e))^n/a)^p)+1/5*hypergeom([-p, 5/n],[(5+n)/n],-b*(c*tan(f*x+e))^n/a)
*tan(f*x+e)^5*(a+b*(c*tan(f*x+e))^n)^p/f/((1+b*(c*tan(f*x+e))^n/a)^p)
```

Rubi [A] time = 0.19, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3675, 1893, 246, 245, 365, 364}

$$\frac{\tan^5(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{n},-p;\frac{n+5}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{5f}+\frac{2\tan^3(e+fx)\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
) + (2*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*x])^n)/a)]*T
an[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p)/(3*f*(1 + (b*(c*Tan[e + f*x])^n
)/a)^p) + (Hypergeometric2F1[5/n, -p, (5 + n)/n, -((b*(c*Tan[e + f*x])^n)/a
)]*Tan[e + f*x]^5*(a + b*(c*Tan[e + f*x])^n)^p)/(5*f*(1 + (b*(c*Tan[e + f*x
])^n)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
```

```
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \sec^6(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx = \frac{\text{Subst}\left(\int \left(c^2+x^2\right)^2(a+bx^n)^p\,dx,x,c\tan(e+fx)\right)}{c^5f}$$

$$= \frac{\text{Subst}\left(\int \left(c^4(a+bx^n)^p+2c^2x^2(a+bx^n)^p+x^4(a+bx^n)^p\right)dx,x,c$$

$$= \frac{\text{Subst}\left(\int x^4(a+bx^n)^p\,dx,x,c\tan(e+fx)\right)}{c^5f}+\frac{2\,\text{Subst}\left(\int x^2(a+b$$

$$= \frac{\left(\left(a+b(c\tan(e+fx))^n\right)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}\right)\text{Subst}\left(\int x^4\left(1+\frac{bx}{a}\right.\right.}{c^5f}$$

$$= \frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)\left(a+b(c\tan(e+fx))\right.}{f}$$

Mathematica [A] time = 3.13, size = 165, normalized size = 0.68

$$\frac{\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}\left(3\tan^4(e+fx)\,{}_2F_1\left(\frac{5}{n},-p;\frac{n+5}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)+10\tan\right.}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(15*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e +
 f*x])^n)/a)] + 10*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*
x])^n)/a)]*Tan[e + f*x]^2 + 3*Hypergeometric2F1[5/n, -p, (5 + n)/n, -((b*(c
*Tan[e + f*x])^n)/a)]*Tan[e + f*x]^4)*(a + b*(c*Tan[e + f*x])^n)^p)/(15*f*(
1 + (b*(c*Tan[e + f*x])^n)/a)^p)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p\sec\left(fx+e\right)^6,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^6, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 6.51Unable to divide, perha
ps due to rounding error%%%{1,[0,1,4,0,0]%%%}+%%%{2,[0,1,2,2,0]%%%}+%%%{1,[
0,1,0,4,0]%%%} / %%%{1,[0,0,5,0,1]%%%} Error: Bad Argument Value
```

maple [F] time = 1.80, size = 0, normalized size = 0.00

$$\int \left(\sec^{6}\left(f x +e \right)\right) \left(a +b \left(c \tan \left(f x +e \right)\right)^{n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c \tan \left(f x + e\right)\right)^{n} b + a\right)^{p} \sec \left(f x + e\right)^{6}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,{\left(c\,\mathrm{tan}\left(e+f\,x\right)\right)}^n\right)}^p}{{\cos\left(e+f\,x\right)}^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^6,x)
```

```
[Out] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

3.492 $\int \sec^4(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=160

$$\frac{\tan^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{n},-p;\frac{n+3}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{3f}+\frac{\tan(e+fx)\left(a+b(c\tan\right.}{}$$

```
[Out] hypergeom([1/n, -p],[1+1/n],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)*(a+b*(c*tan(f
*x+e))^n)^p/f/((1+b*(c*tan(f*x+e))^n/a)^p)+1/3*hypergeom([-p, 3/n],[(3+n)/n
],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p/f/((1+b*(c*t
an(f*x+e))^n/a)^p)
```

Rubi [A] time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3675, 1893, 246, 245, 365, 364}

$$\frac{\tan^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{n},-p;\frac{n+3}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{3f}+\frac{\tan(e+fx)\left(a+b(c\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
) + (Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p)/(3*f*(1 + (b*(c*Tan[e + f*x])^n)/
a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx = \frac{\text{Subst}\left(\int \left(c^2+x^2\right)(a+bx^n)^p\,dx,x,c\tan(e+fx)\right)}{c^3f}$$
$$= \frac{\text{Subst}\left(\int \left(c^2(a+bx^n)^p+x^2(a+bx^n)^p\right)dx,x,c\tan(e+fx)\right)}{c^3f}$$
$$= \frac{\text{Subst}\left(\int x^2(a+bx^n)^p\,dx,x,c\tan(e+fx)\right)}{c^3f} + \frac{\text{Subst}\left(\int (a+bx^{\text{[cut off]}}\right.}{}$$
$$= \frac{\left(\left(a+b(c\tan(e+fx))^n\right)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}\right)\text{Subst}\left(\int x^2\left(1+\text{[cut off]}\right.\right.}{c^3f}$$
$$= \frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)\left(a+b(c\tan(e+f\text{[cut off]}\right.}{f}$$

Mathematica [A] time = 4.12, size = 122, normalized size = 0.76

$$\frac{\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}\left(\tan^2(e+fx)\,{}_2F_1\left(\frac{3}{n},-p;\frac{n+3}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)+3\,{}_2F_1\text{[cut off]}\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(3*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e +
f*x])^n)/a)] + Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*x])^
n)/a)]*Tan[e + f*x]^2)*(a + b*(c*Tan[e + f*x])^n)^p)/(3*f*(1 + (b*(c*Tan[e
+ f*x])^n)/a)^p)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p \sec\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 4.9Unable to divide, perhap
s due to rounding error%%%{1,[0,1,2,0,0]%%%}+%%%{1,[0,1,0,2,0]%%%} / %%%{1,
[0,0,3,0,1]%%%} Error: Bad Argument Value
```

maple [F] time = 1.82, size = 0, normalized size = 0.00

$$\int \left(\sec^4\left(fx+e\right)\right)\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p \sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\left(c\tan\left(e+f\,x\right)\right)^n\right)^p}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^4,x)
```

```
[Out] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

3.493 $\int \sec^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=75

$$\frac{\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{f}$$

```
[Out] hypergeom([1/n, -p],[1+1/n],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)*(a+b*(c*tan(f
*x+e))^n)^p/f/((1+b*(c*tan(f*x+e))^n/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3675, 246, 245}

$$\frac{\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx = \frac{\text{Subst}\left(\int (a+bx^n)^p\,dx, x, c\tan(e+fx)\right)}{cf}$$
$$= \frac{\left((a+b(c\tan(e+fx))^n)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^n}{a}\right)^{p}\right.}{cf}$$
$$= \frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)\left(a+b(c\tan(e+fx))^{n}\right.}{f}$$

Mathematica [A] time = 0.13, size = 75, normalized size = 1.00

$$\frac{\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p \sec\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 3.32Unable to divide, perha
ps due to rounding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,1]%%%} Error: Ba
d Argument Value
```

maple [F] time = 1.61, size = 0, normalized size = 0.00

$$\int \left(\sec^2\left(fx+e\right)\right)\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b + a\right)^p \sec\left(fx+e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^2, x)
```

mupad [B] time = 14.05, size = 76, normalized size = 1.01

$$\frac{\tan\left(e+fx\right)\left(a+b\left(c\tan\left(e+fx\right)\right)^n\right)^p {}_2F_1\left(\frac{1}{n},-p;\ \frac{1}{n}+1;\ -\frac{b\left(c\tan\left(e+fx\right)\right)^n}{a}\right)}{f\left(\frac{b\left(c\tan\left(e+fx\right)\right)^n}{a}+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^2,x)
```

```
[Out] (tan(e + f*x)*(a + b*(c*tan(e + f*x))^n)^p*hypergeom([1/n, -p], 1/n + 1, -(
b*(c*tan(e + f*x))^n)/a))/(f*((b*(c*tan(e + f*x))^n)/a + 1)^p)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

3.494 $\int \left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*(c*tan(f*x+e))^n)^p,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Defer[Int][(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 0.81, size = 0, normalized size = 0.00

$$\int \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Integrate[(a + b*(c*Tan[e + f*x])^n)^p, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p, x)
```

maple [A] time = 1.46, size = 0, normalized size = 0.00

$$\int \left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((a+b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c \tan \left(fx+e\right)\right)^{n} b + a\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \left(a+b\left(c \tan \left(e+f\,x\right)\right)^{n}\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int((a + b*(c*tan(e + f*x))^n)^p, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\left(c \tan \left(e+fx\right)\right)^{n}\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((a + b*(c*tan(e + f*x))**n)**p, x)
```

3.495 $\int \cos^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\cos^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \cos^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 8.62, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Integrate[Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p\cos\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^2, x)
```

maple [A] time = 2.77, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p \cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^2\left(a+b\left(c\tan\left(e+f\,x\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(cos(e + f*x)^2*(a + b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

3.496 $\int (d\csc(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=98

$$\frac{\tan(e+fx)\cos^2(e+fx)^{p+\frac{1}{2}}\left(b\tan^2(e+fx)\right)^p(d\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(-m+2p+1);\frac{1}{2}(-m+2p+3);s\right.}{f(-m+2p+1)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+p)*(d*csc(f*x+e))^m*hypergeom([1/2+p, 1/2-1/2*m+p],[3/2
-1/2*m+p],sin(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1-m+2*p)
```

Rubi [A] time = 0.19, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3658, 2618, 2602, 2577}

$$\frac{\tan(e+fx)\cos^2(e+fx)^{p+\frac{1}{2}}\left(b\tan^2(e+fx)\right)^p(d\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(-m+2p+1);\frac{1}{2}(-m+2p+3);s\right.}{f(-m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^(1/2 + p)*(d*Csc[e + f*x])^m*Hypergeometric2F1[(1 + 2*p)/
2, (1 - m + 2*p)/2, (3 - m + 2*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*Tan[e
+ f*x]^2)^p)/(f*(1 - m + 2*p))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2618

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\csc(e+fx))^m\left(b\tan^2(e+fx)\right)^p\,dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int (d\csc(e+fx))^m\tan^{2p}(e+fx)$$

$$= \left((d\csc(e+fx))^m\left(\frac{\sin(e+fx)}{d}\right)^m\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)$$

$$= \frac{\left(\cos^{2p}(e+fx)(d\csc(e+fx))^{1+m}\sin(e+fx)\left(\frac{\sin(e+fx)}{d}\right)^{m-2p}\left(b\tan^2\right.\right.}{d}$$

$$= \frac{\cos^2(e+fx)^{\frac{1}{2}+p}(d\csc(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2}(1+2p),\frac{1}{2}(1-m+2p);\frac{1}{2}\right.}{df(1-}$$

Mathematica [C] time = 2.02, size = 299, normalized size = 3.05

$$-\frac{d(m-2p-3)\left(b\tan\right.}{f(m-2p-1)\left(2\tan^2\left(\frac{1}{2}(e+fx)\right)\left(-\left((m-1)F_1\left(-\frac{m}{2}+p+\frac{3}{2};2p,2-m;-\frac{m}{2}+p+\frac{5}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((d*(-3 + m - 2*p)*AppellF1[1/2 - m/2 + p, 2*p, 1 - m, 3/2 - m/2 + p, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(d*Csc[e + f*x])^(-1 + m)*(b*Tan[e + f
*x]^2)^p)/(f*(-1 + m - 2*p)*((-3 + m - 2*p)*AppellF1[1/2 - m/2 + p, 2*p, 1
- m, 3/2 - m/2 + p, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*(-((-1 + m
)*AppellF1[3/2 - m/2 + p, 2*p, 2 - m, 5/2 - m/2 + p, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]) - 2*p*AppellF1[3/2 - m/2 + p, 1 + 2*p, 1 - m, 5/2 - m/2
 + p, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2\right)^p\left(d\csc\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p*(d*csc(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan\left(fx+e\right)^2\right)^p\left(d\csc\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*csc(f*x + e))^m, x)
```

maple [F] time = 3.36, size = 0, normalized size = 0.00

$$\int\left(d\csc\left(fx+e\right)\right)^m\left(b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\csc\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2)^p*(d*csc(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan\left(e+f\,x\right)^2\right)^p\,\left(\frac{d}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^2)^p*(d/sin(e + f*x))^m,x)
[Out] int((b*tan(e + f*x)^2)^p*(d/sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^2\left(e+fx\right)\right)^p \left(d\csc\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
[Out] Integral((b*tan(e + f*x)**2)**p*(d*csc(e + f*x))**m, x)
```

3.497 $\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=127

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} F_1\left(\frac{1-m}{2};1-\frac{m}{2},-p;\frac{3-m}{2};\right.}{f(1-m)}$$

```
[Out] AppellF1(1/2-1/2*m,1-1/2*m,-p,3/2-1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(d
*csc(f*x+e))^m*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/(1-m)/((sec(f*x+e)^2)^(1/2
*m))/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3681, 3667, 511, 510}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} F_1\left(\frac{1-m}{2};1-\frac{m}{2},-p;\frac{3-m}{2};\right.}{f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 - m)/2, 1 - m/2, -p, (3 - m)/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/a)]*(d*Csc[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1
 - m)*(Sec[e + f*x]^2)^(m/2)*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3667

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(f
f*(d*Sin[e + f*x])^m*(Sec[e + f*x]^2)^(m/2))/(f*Tan[e + f*x]^m), Subst[Int[
((ff*x)^m*(a + b*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]
/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3681

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(d*Csc[e + f*x])^FracPart[m]*(Sin
[e + f*x]/d)^FracPart[m], Int[(a + b*(c*Tan[e + f*x])^n)^p/(Sin[e + f*x]/d)
^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \left((d\csc(e+fx))^m \left(\frac{\sin(e+fx)}{d}\right)^m\right)\int \left(\frac{\sin(e+fx)}{d}\right)^{-m} \left(a+b\tan^2(\right.$$

$$= \frac{\left((d\csc(e+fx))^m \sec^2(e+fx)^{-m/2}\tan^m(e+fx)\right)\text{Subst}\left(\int x^{-m}\left(1\right.\right.}{f}$$

$$= \frac{\left((d\csc(e+fx))^m \sec^2(e+fx)^{-m/2}\tan^m(e+fx)\left(a+b\tan^2(e+fx\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1-m}{2};1-\frac{m}{2},-p;\frac{3-m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\csc(e+fx}{f}$$

Mathematica [B] time = 3.67, size = 292, normalized size = 2.30

$$-\frac{a(m-3)\cos^2(e+fx)\cot(e+fx)(d\csc(e+fx))^m\left(a+b\tan^2(e\right.}{f(m-1)\left(-2bpF_1\left(\frac{3}{2}-\frac{m}{2};1-\frac{m}{2},1-p;\frac{5}{2}-\frac{m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)-a(m-2)F_1\left(\frac{3}{2}-\frac{m}{2};2-\frac{m}{2},-p;\frac{5}{2}-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((a*(-3 + m)*AppellF1[1/2 - m/2, 1 - m/2, -p, 3/2 - m/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/a)]*Cos[e + f*x]^2*Cot[e + f*x]*(d*Csc[e + f*x])^m*(a
+ b*Tan[e + f*x]^2)^p)/(f*(-1 + m)*(-2*b*p*AppellF1[3/2 - m/2, 1 - m/2, 1 -
 p, 5/2 - m/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(-2 + m)*Appel
lF1[3/2 - m/2, 2 - m/2, -p, 5/2 - m/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)] + a*(-3 + m)*AppellF1[1/2 - m/2, 1 - m/2, -p, 3/2 - m/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/a)]*Cot[e + f*x]^2)))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\tan\left(fx+e\right)^2+a\right)^p\left(d\csc\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*csc(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^2+a\right)^p\left(d\csc\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*csc(f*x + e))^m, x)
```

maple [F] time = 2.71, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(fx+e\right)\right)^m\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
[Out] int((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{p}\left(d\csc\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*csc(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan\left(e+f\,x\right)^{2}+a\right)^{p}\left(\frac{d}{\sin\left(e+f\,x\right)}\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p*(d/sin(e + f*x))^m,x)
[Out] int((a + b*tan(e + f*x)^2)^p*(d/sin(e + f*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

3.498 $\int (d\csc(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=104

$$\frac{\tan(e+fx)(d\csc(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(np+1)}\left(b(c\tan(e+fx))^n\right)^p\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(-m+np+1);\frac{1}{2}(-m+np}{f(-m+np+1)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+1/2)*(d*csc(f*x+e))^m*hypergeom([1/2*n*p+1/2, 1/2*n
*p-1/2*m+1/2],[1/2*n*p-1/2*m+3/2],sin(f*x+e)^2)*tan(f*x+e)*(b*(c*tan(f*x+e)
)^n)^p/f/(n*p-m+1)
```

Rubi [A] time = 0.21, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3659, 2618, 2602, 2577}

$$\frac{\tan(e+fx)(d\csc(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(np+1)}\left(b(c\tan(e+fx))^n\right)^p\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(-m+np+1);\frac{1}{2}(-m+np}{f(-m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*(d*Csc[e + f*x])^m*Hypergeometric2F1[(1 + n
*p)/2, (1 - m + n*p)/2, (3 - m + n*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 - m + n*p))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2618

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\csc(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (d\csc(e+fx))^m (c\tan$$

$$= \left((d\csc(e+fx))^m\left(\frac{\sin(e+fx)}{d}\right)^m (c\tan(e+fx))^{-np}\left(b(c\tan(e+\right.\right.$$

$$= \frac{\left(\cos^{np}(e+fx)(d\csc(e+fx))^{1+m}\sin(e+fx)\left(\frac{\sin(e+fx)}{d}\right)^{m-np}\left(b(c\right.\right.}{d}$$

$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}(d\csc(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(1-m+\right.}{d}$$

Mathematica [C] time = 2.19, size = 319, normalized size = 3.07

$$-\frac{d(m-np-3)}{f(m-np-1)\left((m-np-3)F_1\left(\frac{1}{2}(-m+np+1);np,1-m;\frac{1}{2}(-m+np+3);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((d*(-3 + m - n*p)*AppellF1[(1 - m + n*p)/2, n*p, 1 - m, (3 - m + n*p)/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(d*Csc[e + f*x])^(-1 + m)*(b*(c*Ta
n[e + f*x])^n)^p)/(f*(-1 + m - n*p)*((-3 + m - n*p)*AppellF1[(1 - m + n*p)/
2, n*p, 1 - m, (3 - m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] -
2*((-1 + m)*AppellF1[(3 - m + n*p)/2, n*p, 2 - m, (5 - m + n*p)/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*p*AppellF1[(3 - m + n*p)/2, 1 + n*p, 1
 - m, (5 - m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2]^2)))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \left(d\csc\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*csc(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p \left(d\csc\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*csc(f*x + e))^m, x)
```

maple [F] time = 1.37, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(fx+e\right)\right)^m \left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
[Out] int((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c \tan\left(fx+e\right)\right)^{n} b\right)^{p} \left(d \csc\left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*csc(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\sin\left(e+f\,x\right)}\right)^{m} \left(b\left(c \tan\left(e+f\,x\right)\right)^{n}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int((d/sin(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c \tan\left(e+fx\right)\right)^{n}\right)^{p} \left(d \csc\left(e+fx\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*csc(e + f*x))**m, x)
```

3.499 $\int (d\csc(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=57

$$\left(\frac{\sin(e+fx)}{d}\right)^m (d\csc(e+fx))^m \text{Int}\left(\left(\frac{\sin(e+fx)}{d}\right)^{-m}\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] (d*csc(f*x+e))^m*(sin(f*x+e)/d)^m*Unintegrable((a+b*(c*tan(f*x+e))^n)^p/((s
in(f*x+e)/d)^m),x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d\csc(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Csc[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (d*Csc[e + f*x])^m*(Sin[e + f*x]/d)^m*Defer[Int][(a + b*(c*Tan[e + f*x])^n)
^p/(Sin[e + f*x]/d)^m, x]
```

Rubi steps

$$\int (d\csc(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx = \left((d\csc(e+fx))^m \left(\frac{\sin(e+fx)}{d}\right)^m\right)\int \left(\frac{\sin(e+fx)}{d}\right)^{-m} (a+b$$

Mathematica [A] time = 2.91, size = 0, normalized size = 0.00

$$\int (d\csc(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Csc[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[(d*Csc[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p \left(d\csc\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*csc(f*x + e))^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c\tan\left(fx+e\right)\right)^n b+a\right)^p \left(d\csc\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*csc(f*x + e))^m, x)
```

maple [A] time = 5.11, size = 0, normalized size = 0.00

$$\int \left(d \csc \left(fx + e\right)\right)^{m} \left(a + b \left(c \tan \left(fx + e\right)\right)^{n}\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
[Out] int((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(c \tan \left(fx + e\right)\right)^{n} b + a\right)^{p} \left(d \csc \left(fx + e\right)\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*csc(f*x + e))^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a+b\,{\left(c\,\mathrm{tan}\left(e+f\,x\right)\right)}^n\right)}^p\,{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p*(d/sin(e + f*x))^m,x)
[Out] int((a + b*(c*tan(e + f*x))^n)^p*(d/sin(e + f*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
    not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```